

If n is an integer and $\underbrace{n^3+5}_{(p)}$ is odd, then
 n is even.
(by contradiction) $\neg q := n \text{ is odd.}$

Assume that n^3+5 is odd. Assume, for a contradiction, that n is odd,

• $\exists k \in \mathbb{Z}$ s.t. $n^3+5 = 2k+1$ (1)

• $\exists m \in \mathbb{Z}$ s.t. $n = 2m+1$. (2)

(2) \Rightarrow (1) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(2m+1)^3 + 5 = 2k+1$

$(2m)^3 + 3(2m)^2(1) + 3(2m)(1)^2 + 1^3 + 5 = 2k+1$

$8m^3 + 12m^2 + 6m + 6 = 2k+1$

$2(4m^3 + 6m^2 + 3m + 3) = 2k+1$

Bec. \mathbb{Z} closed under $+$ and \cdot ,

$r = 4m^3 + 6m^2 + 3m + 3 \in \mathbb{Z}$

$2r = 2k+1$

Which is absurd, because even=odd.

$\Rightarrow n$ must be even.