Discussion Prompt

Using the same method as above, prove by mathematical induction that $6 \mid 3n^4 + 2n^3 + 7n$ for all $n \in \mathbb{N}$.

Prove that for all $n \in \mathbb{N}$ such that $n \geq 2$, $n! < n^n$.
$P(n)$

Recall: $n! = n(n-1)(n-2)\cdots(2)(1)$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Note: $k! = k(k-1)!$

Proof: by Induction.

- **Base Case:** For $n = 2$,

  $LS = 2! = 2$

  $RS = 2^2 = 4$

  $2! < 2^2$ which is true. ✓

- **Inductive Step:** Assume $\forall k \in \mathbb{N}$ such that $k \geq 2$,

  $k! < k^k$ (Inductive Hypothesis, IH)

  We want to show for $k+1 \in \mathbb{N}$, $(k+1)! < (k+1)^{k+1}$

  $$\begin{aligned} (k+1)! &= (k+1)k! \\ &< (k+1)k^k \quad \text{(IH)} \quad k < k+1 \\ &< (k+1)^1(k+1)^k \\ &= (k+1)^{k+1} \end{aligned}$$

  Thereby completing the inductive step.

Therefore, by mathematical induction

$n! < n^n \quad \forall n \in \mathbb{N},\ n \geq 2.$

Prove that $\forall n \in \mathbb{N}, n > 6.\ 3^n < n!$

Proof: by Induction.

<u>Base Case:</u> For $n = 7$

$$\left.\begin{aligned} 3^7 &= 2187 \\ 7! &= 5040 \end{aligned}\right\} \quad 3^7 < 7! \quad \checkmark$$

<u>Inductive Step:</u> Assume $\forall k \in \mathbb{N}, k \geq 7,$

$$3^k < k! \qquad (IH)$$

Then we show for $k+1 \in \mathbb{N}$, $3^{k+1} < (k+1)!$

$$\begin{aligned} 3^{k+1} &= 3 \cdot 3^k \\ &< 3 \cdot k! \quad (IH) \\ &< (k+1)k! \quad (3 < k+1 \text{ for } k \geq 7) \\ &= (k+1)! \end{aligned}$$

Therefore, the inductive step is complete.

Therefore $\forall n \in \mathbb{N}$ w. $n > 6$, $3^n < n!$. ∎