Math 1200 Tutorial 7: Midterm Review

Joe Tran

October 23, 2024

What we have covered so far:

- True or false statements
- Definitions: Odd, even, divides
- Proof Methods: Direct, contrapositive, contradiction, cases, counterexamples
- Mathematical induction

Question 1. Let $P(x, y) :=$ "$x + 2y = xy$" where the domain of x and y are the real numbers. Determine the truth value of the following statements:

(i) $\forall x \exists y : P(x, y)$

(ii) $\exists y \exists x : P(x, y)$

(iii) $\exists y \forall x : \neg P(x, y)$

Question 2. Prove that if x is odd and y is even, then $3x + 2y$ is odd.

Question 3. Prove that if $a \mid b$ and $b \mid a$, where $a, b \in \mathbb{Z}$, then either $a = b$ or $a = -b$.

Question 4. Prove by mathematical induction:

$$2 + 2(-7) + 2(-7)^2 + \cdots + 2(-7)^n = \frac{1 - (-7)^{n+1}}{4}$$

Question 5. Consider the sequence of numbers $x_0, x_1, x_2, ...$ given by

$$x_k = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k = 1 \\ \frac{x_{k-2}}{4} & \text{if } k \geq 2 \end{cases}$$

Prove that for all integers $n \in \mathbb{N}$,

$$x_n = \left(\frac{1}{2}\right)^{n+1} - \left(-\frac{1}{2}\right)^{n+1}$$