

Question 1. Let $(z_n)_n$ be a sequence of complex numbers such that $\lim_{n \rightarrow \infty} |z_n| = 0$. Prove that $\lim_{n \rightarrow \infty} z_n = 0$.

Question 2. Let $(z_n)_n$ be a sequence of complex numbers such that $\lim_{n \rightarrow \infty} z_n = z$. Prove that $\lim_{n \rightarrow \infty} |z_n| = |z|$.

Question 3. Let $(z_n)_n$ be a sequence of complex numbers such that $\lim_{n \rightarrow \infty} z_n = z$ and $\lim_{n \rightarrow \infty} \Re(z_n) = a$. Prove that $\Re(z) = a$.

Question 4. Let $(z_n)_n$ be a sequence of complex numbers such that $\lim_{n \rightarrow \infty} z_n = z$. Prove that $\lim_{n \rightarrow \infty} \Im(z_n) = \Im(z)$.

Question 5. Let $(z_n)_n$ be a sequence of complex numbers such that $|z_n| \leq a_n$ for all $n \in \mathbb{N}$ where $(a_n)_n$ is a convergent sequence with $\lim_{n \rightarrow \infty} a_n = 0$. Prove that $\lim_{n \rightarrow \infty} z_n = 0$.

Question 6. Let $(z_n)_n$ be a sequence of complex numbers such that $\lim_{n \rightarrow \infty} (z_n - \alpha)^2 = 0$, where $\alpha \in \mathbb{C}$. Prove that $\lim_{n \rightarrow \infty} z_n = \alpha$.

Question 7. Let $(z_n)_n$ be a sequence of complex numbers such that $|z_n - z_{n-1}| \leq 2^{-n}$ for all $n \geq 1$. Prove that $(z_n)_n$ is a Cauchy sequence.