Question 1. *Let* $f_n(x) = \dfrac{1}{1+n^2x^2}$ *for* $x \in [0,1]$. *Show that* $\lim_{n\to\infty} \int_0^1 f_n(x)\ dx = \int_0^1 \lim_{n\to\infty} f_n(x)\ dx$, *but the convergence is not uniform.*

Question 2. *Let* $f_n(x) = nxe^{-nx}$ *for* $x \in [0,1]$. *Determine whether the sequence* $(f_n)_n$ *converges uniformly on* $[0,1]$ *and if so, compute* $\lim_{n\to\infty} \int_0^1 f_n(x)dx$.

Question 3. *Let* $f_n(x) = \dfrac{x}{1+nx^2}$ *for* $x \in [0,1]$. *Determine whether the sequence* $(f_n(x))$ *converges uniformly on* $[0,1]$, *and if so, compute* $\lim_{n\to\infty} \int_0^1 f_n(x)dx$.

Question 4. *Let* $f_n(x) = \dfrac{\sin nx}{\sqrt{n}}$ *for* $x \in [0,\pi]$. *Show that* $\lim_{n\to\infty} \int_0^\pi f_n(x)dx = 0$ *and the convergence is uniform.*

Question 5. *Let* $f_n(x) = \dfrac{x^n}{n}$ *for* $x \in [0,1]$. *Determine whether the sequence* $(f_n)_n$ *converges uniformly on* $[0,1]$, *and if so, compute* $\lim_{n\to\infty} \int_0^1 f_n(x)dx$.