Question 1. *Determine whether the following sequence of functions converges pointwise on* $\mathbb{R}$ *and if so, fid the limit function* $f_n(x) = \dfrac{x}{n+x}$ *for* $n \in \mathbb{N}$.

Question 2. *Determine whether the series of functions converges uniformly on* $[0,1]$*:* $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$.

Question 3. *Show that the sequence of functions* $f_n(x) = x^n$ *converges pointwise to the function*

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x = 1 \end{cases}$$

on $[0,1]$ *but does not converge uniformly.*

Question 4. *Let* $f_n(x) = \dfrac{x^n}{n^2}$ *for* $x \in [0,1]$ *and* $n \in \mathbb{N}$. *Use the Weierstrass M-Test to show that the series* $\displaystyle\sum_{n=1}^{\infty} f_n(x)$ *converges uniformly on* $[0,1]$.

Question 5. *Show that the sequence of functions* $f_n(x) = \dfrac{nx}{1+n^2x^2}$ *converges uniformly to the function* $f(x) = 0$ *on* $[0,1]$.

Question 6. *Determine whether the following series of functions converges uniformly on* $\mathbb{R}$*:*

$$\sum_{n=1}^{\infty} \frac{1}{n(1+nx^2)}$$

Question 7. *Consider the series of functions* $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$. *Use the Weierstrass M-Test to show that the series converges uniformly on any bounded interval* $[-a,a]$.