

Question 1. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges absolutely. Prove that for any bijection $\sigma : \mathbb{N} \rightarrow \mathbb{N}$ the series $\sum_{n=1}^{\infty} z_{\sigma(n)}$ converges absolutely to the same limit.

Question 2. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges conditionally. Prove that for any $\alpha \in \mathbb{C}$, there exists a bijection $\sigma : \mathbb{N} \rightarrow \mathbb{N}$ such that $\sum_{n=1}^{\infty} z_{\sigma(n)} = \alpha$.

Question 3. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} |z_n| = \infty$. Prove that for any bijection $\sigma : \mathbb{N} \rightarrow \mathbb{N}$ there exists a rearrangement $(w_n)_n$ of $(z_n)_n$ such that $\sum_{n=1}^{\infty} w_n = \infty$.