

Question 1. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges absolutely. Prove that $\sum_{n=1}^{\infty} \Re(z_n)$ and $\sum_{n=1}^{\infty} \Im(z_n)$ converge absolutely.

Question 2. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} |z_n|^2$ converges. Prove that $\lim_{n \rightarrow \infty} z_n = 0$.

Question 3. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges. Prove that $\lim_{n \rightarrow \infty} z_n = 0$.

Question 4. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges. Prove that $\sum_{n=1}^{\infty} |z_n|$ converges.

Question 5. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges absolutely. Prove that $\sum_{n=1}^{\infty} z_n^2$ converges.

Question 6. Let $(z_n)_n$ be a sequence of complex numbers such that $\sum_{n=1}^{\infty} z_n$ converges conditionally. Prove that there exists infinitely many $n \in \mathbb{N}$ such that $\Re(z_n) = 0$.