Question 1. *Let* $f_n(x) = \dfrac{\sin nx}{n}$ *for* $x \in [0, 1]$. *Show that* $(f_n)_n$ *converges uniformly to a continuous function on* $[0, 1]$, *and that the limit function is differentiable. Moreover, show that* $(f_n')_n$ *does not converge uniformly on* $[0, 1]$.

Question 2. *Let* $f_n(x) = \dfrac{x^n}{n}$ *for* $x \in [0, 1]$. *Determine whether the sequence* $(f_n)_n$ *converges uniformly on* $[0, 1]$, *and if so, show that the function is differentiable. Moreover, show that* $(f_n')_n$ *converges uniformly on* $[0, 1]$.

Question 3. *Let* $f_n(x) = \dfrac{\cos nx}{n}$ *for* $x \in [0, 1]$. *Show that* $(f_n)_n$ *converges uniformly to a continuous function on* $[0, 1]$ *and that the limit function is differentiable. Moreover, show that* $(f_n')_n$ *converges uniformly on* $[0, 1]$.

Question 4. *Let* $f_n(x) = \dfrac{\ln(1 + nx)}{n}$ *for* $x \in [0, 1]$. *Determine whether the sequence* $(f_n)_n$ *converges uniformly on* $[0, 1]$, *and if so, show that the limit function is differentiable. Moreover, show that* $(f_n')_n$ *converges uniformly on* $[0, 1]$.

Question 5. *Let* $f_n(x) = \dfrac{1}{n}\sin(nx^2)$ *for* $x \in [0, 1]$. *Show that* $(f_n)_n$ *converges uniformly to a continuous function on* $[0, 1]$ *and that the limit function is differentiable. Moreover, show that* $(f_n')_n$ *converges uniformly on* $[0, 1]$.