

MATH 3022 KRITIK ASSIGNMENT 3

Question 1. *Why do the axioms of a vector space appear to only have four conditions, rather than the ten you may have seen the first time you saw an axiomatic definition?*

Vector spaces have a similar structure compared to groups and rings, in which we are able to use the axioms defined for groups and rings and apply it for vector spaces.

Question 2. *The set $V = \mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{11} : a, b \in \mathbb{Q}\}$ is a vector space. Carefully define the operations on this set that will make this possible. Describe the subspace spanned by $S = \{\vec{u}\}$, where $\vec{u} = 3 + \frac{2}{7}\sqrt{11} \in V$.*

Let $\vec{u}, \vec{v} \in V$. Then we have the following operations defined by the mappings: $+: V \times V \rightarrow V$ and $\cdot: V \times V \rightarrow V$ where,

$$\vec{u} + \vec{v} = (a + b\sqrt{11}) + (c + d\sqrt{11}) = (a + c) + (b + d)\sqrt{11}$$

and

$$\vec{u} \cdot \vec{v} = (a + b\sqrt{11})(c + d\sqrt{11}) = (ac + 11bd) + (ad + bc)\sqrt{11}$$

where $a + c, b + d, ac + 11bd$, and $ad + bc \in \mathbb{Q}$.

The subspace spanned by $S = \{3 + \frac{2}{7}\sqrt{11}\}$ is the set of the form

$$W = \left\{ 3a + \frac{2}{7}b\sqrt{11} : a, b \in \mathbb{Q} \right\}$$

for some coefficients $a, b \in \mathbb{Q}$.