**Question 1.** *What is the fundamental difference between groups and rings?*

**Solution.** In Group Theory, we study a set $G$ with a *single* binary operation $* : G \times G \to G$, in which we call the pair $(G, *)$ a group. Here in Ring Theory, we study a set $R$ with *two* binary operations, namely $+$ and $\cdot$, in which we call $R$ a ring.

**Question 2.** *Give two characterizations of an integral domain.*

**Solution.** Two characterizations of an integral domain is as follows:

(1) The Cancellation Law: If $D$ is a commutative ring with identity, then $D$ is an integral domain if and only if for every nonzero $d \in D$ with $da = db$, then $a = b$.
(2) $R$ is an integral domain if for every $x, y \in R$ such that $xy = 0$, then either $x = 0$ or $y = 0$.

**Question 3.** *Provide two examples of fields, one infinite, one finite.*

**Solution.** We have the two examples of fields below:

(1) For the example where a field is finite, take $F$ to be the set of matrices
$$F = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$
This is a (finite) field with entries in $\mathbb{Z}_2$.
(2) The set $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a (infinite) field.