MATH 4081 TOPOLOGY I ASSIGNMENT 1

JOE TRAN

Question 1. *Let (X, d) be a discrete metric space, (Y, ρ) be any metric space, and $f : X \to Y$ be any function. Then, always $f : (X, d) \to (Y, \rho)$ is continuous.*

Question 2. *Let (X, d) and (Y, ρ) be metric spaces and let $f : (X, d) \to (Y, \rho)$ be an isometry.*

(a) *Prove that f is one-to-one.*
(b) *If f is onto, prove that $f^{-1} : (Y, \rho) \to (X, d)$ exists and it is an onto isometry.*

Question 3. *Let (X, d) be a metric space.*

(a) *For $x_0 \in X$ and $\varepsilon > 0$ prove that the set $B(x_0, \varepsilon)$ is open.*
(b) *If $U_1, U_2, ..., U_n$ are open subsets of X, prove that $\bigcap_{i=1}^{n} U_i$ is open.*
(c) *If $(U_i)_{i\in I}$ is an arbitrary collection of open subsets of X, prove that $\bigcup_{i\in I} U_i$ is open.*

Question 4. *Let X be an uncountable set equipped with the co-countable topology*

$$\mathcal{T} = \{A \subset X : X \setminus A \text{ is countable}\} \cup \{\emptyset\}$$

Prove that $(X, \mathcal{T})$ is not metrizable.

Question 5. *In $\mathbb{R}$, show that the family*

$$\mathcal{F} = \{(-\infty, a) : a \in \mathbb{R}\} \cup \{(b, \infty) : b \in \mathbb{R}\}$$

is a subbasis for the usual topology of $\mathbb{R}$.

Question 6. *In $\mathbb{N}$ with the cofinite topology show that*

$$\mathcal{B} = \{\{m\} \cup \{n, n+1, ...\} : m, n \in \mathbb{N}\}$$

is a basis for the topology.

Question 7. *Let $(X, \mathcal{T})$ be a topological space and $\mathcal{B}$ be a basis for $\mathcal{T}$. Prove a subset U of X is open if and only if for every $x \in U$, there exists a $V \in \mathcal{B}$ with $x \in V$ and $V \subset U$.*

Date: September 15, 2023, Due: September 29, 2023 @ 11:59 PM.