Day 7

1. Exam on Thurs in SLH B
   open book, open note, open calculator no phone/tablets/laptops
2. Assignment 1-8 due Jan 31 (tomorrow!) Early deadline: today!
3. disease test paradox
4. dependence/independence wheels
5. craps

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Say that there is a disease which only about 5% of the population has and that there is a test for this disease which is roughly 95% accurate (that is someone who has the disease will test positive 95% of the time and negative 5% of the time, while someone who does not have the disease will test negative 95% of the time and positive 5% of the time).

Given that a patient tests positive for the disease, what is the probability that he or she actually has it?

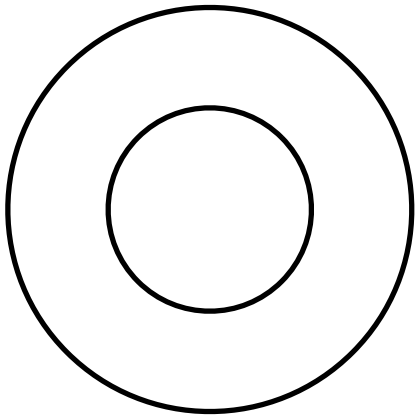
Is the answer?

A) 95%
B) 90%
C) 75%
D) 50%

"X is dependent on Y" and "X is independent of Y" are not opposite statements of each other, rather they are on opposite sides of a spectrum of possibilities.

"X is not dependent on Y" does not mean "X is independent of Y"

X is independent of Y
X is dependent on Y
Y is dependent on X



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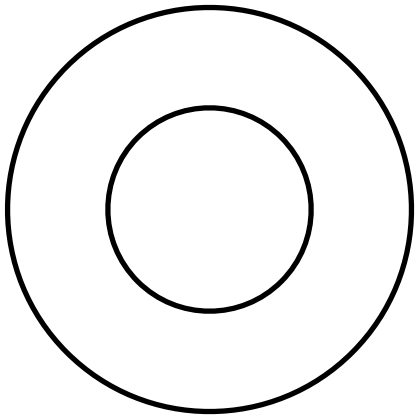

X is independent of Y
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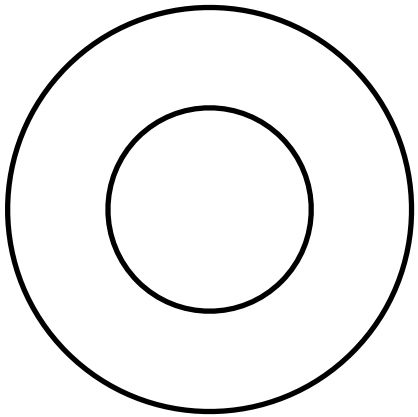

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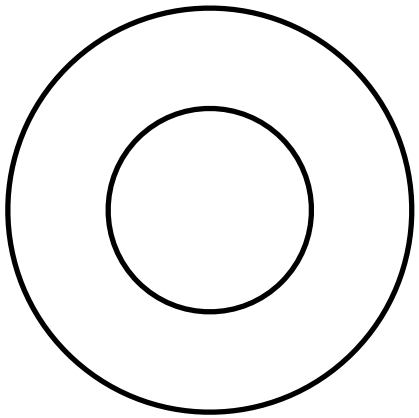

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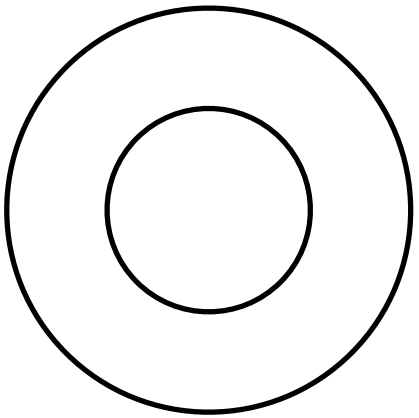

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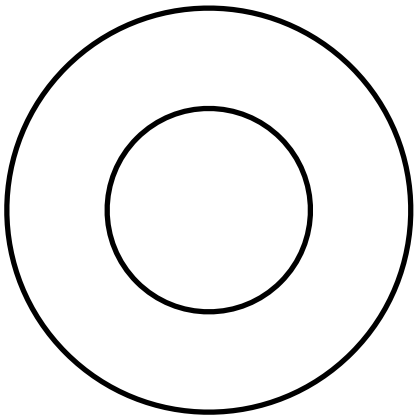
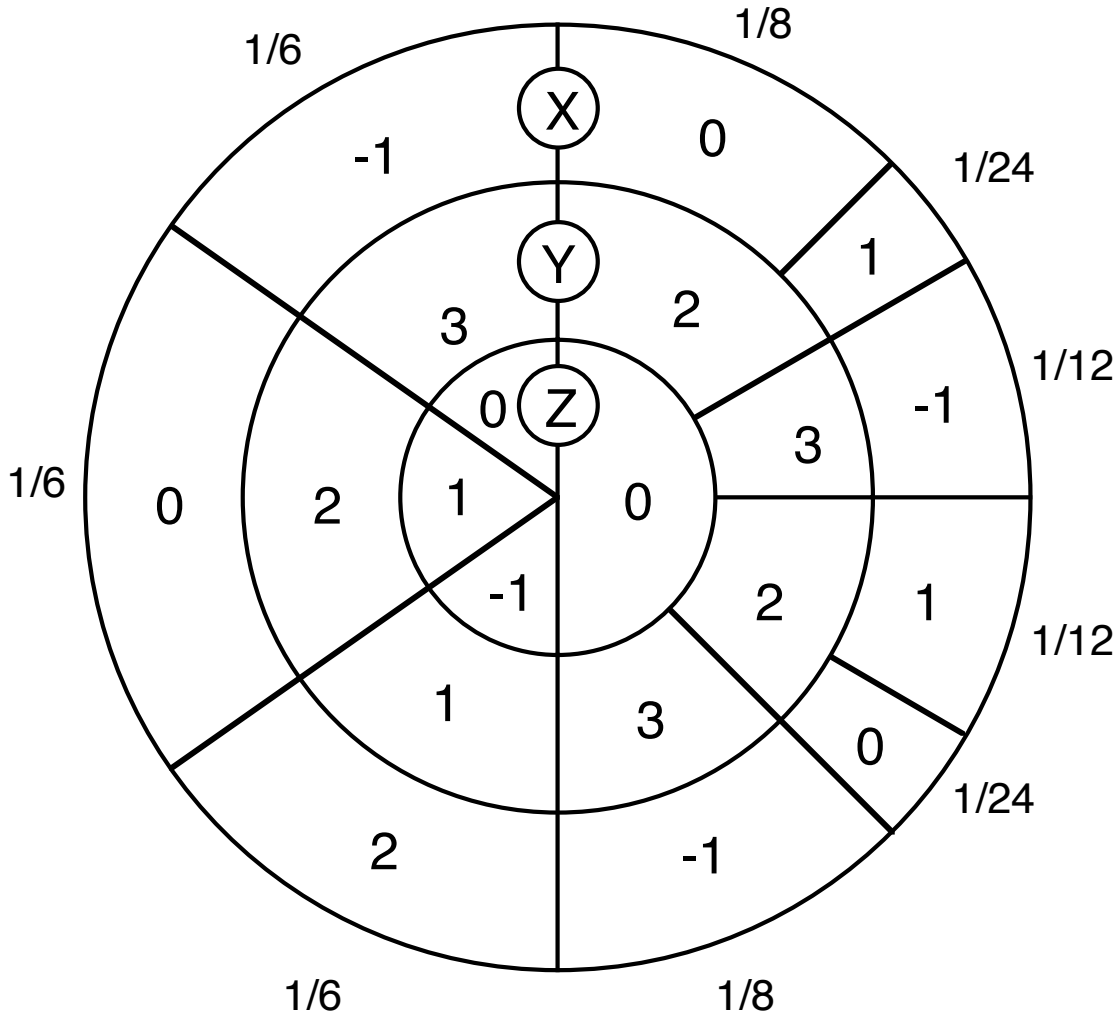


1. The wheel below represents the random variables X, Y and Z.



Calculate:

a) P(X=0)

b) P(X=1)

c) P(Z=-1 or X=0)

d) P(Y=2)

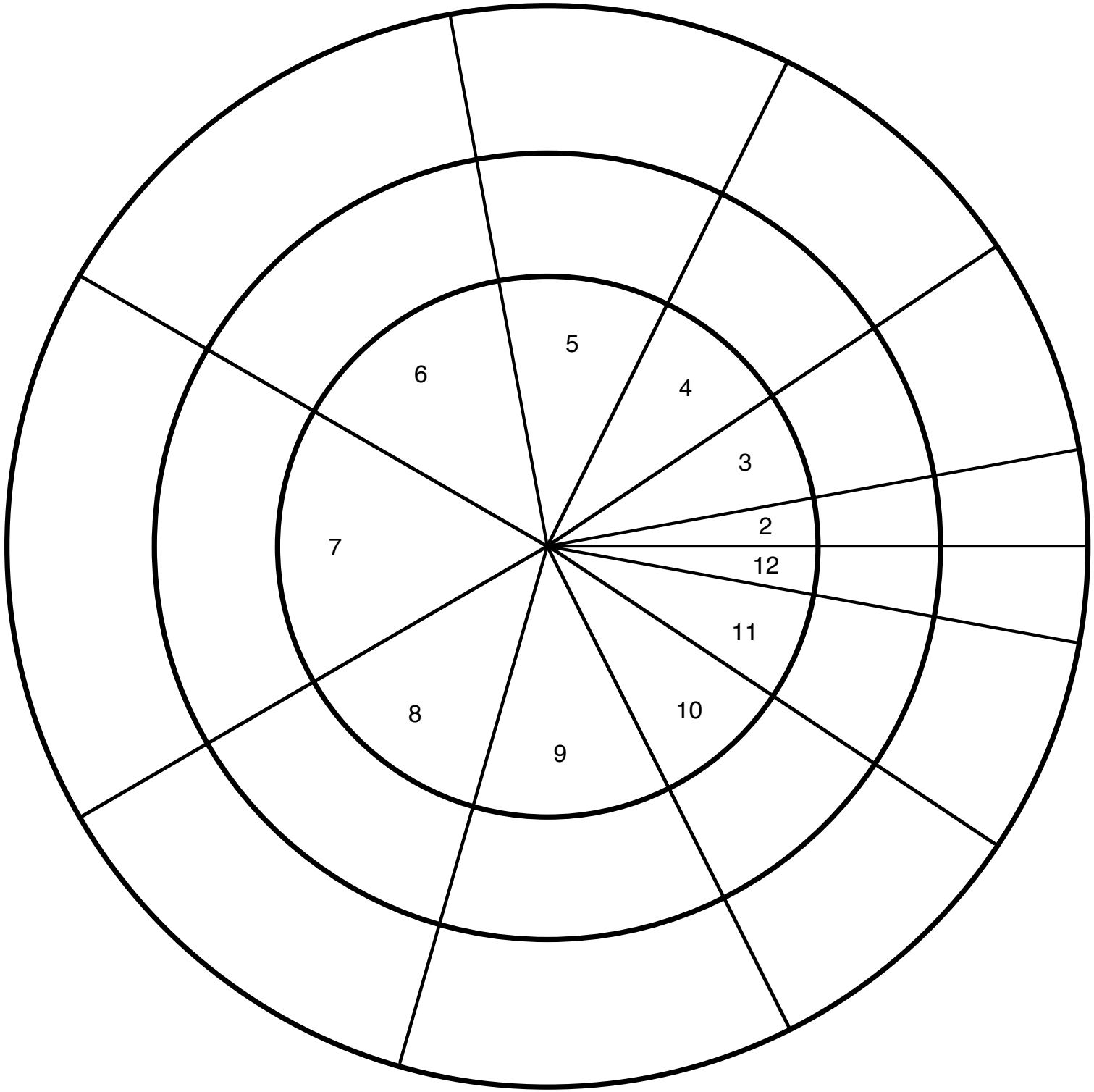
e) P(Y=2 or X=0)

f) P(Y=2 and X=0)

g) P(X=0 | Y=2)

h) P(X=0 | Z=-1)

i) P(X=2 | Z=-1)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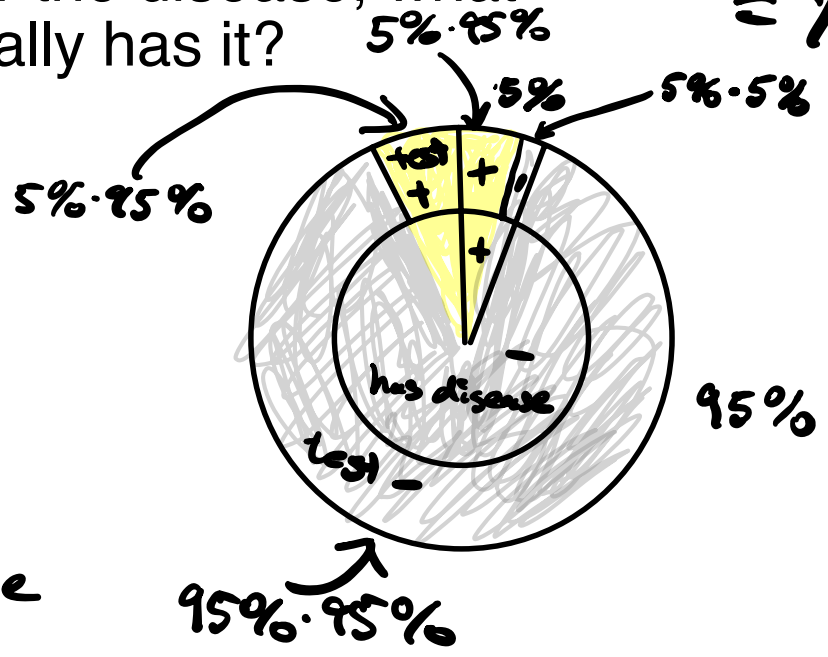
Say that there is a disease which only about 5% of the population has and that there is a test for this disease which is roughly 95% accurate (that is someone who has the disease will test positive 95% of the time and negative 5% of the time, while someone who does not have the disease will test negative 95% of the time and positive 5% of the time).

$$P(\text{have disease} \mid \text{test positive}) = \frac{P(\text{have} \,\&\, \text{test}+)}{P(\text{test}+)} = \frac{.95 \cdot .05}{.95 \cdot .05 + .05 \cdot .95} = 1/2$$

Given that a patient tests positive for the disease, what is the probability that he or she actually has it?

Is the answer?

1 A) 95%
3 B) 90%
4 C) 75%
5 D) 50%
1 E) don't know/don't care



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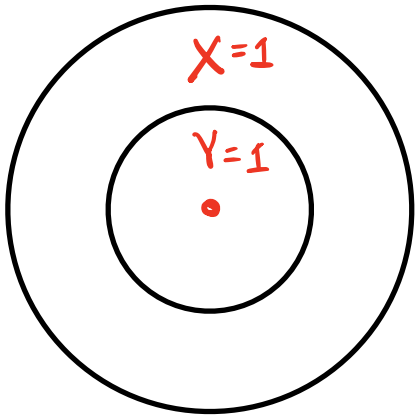

X is independent of Y
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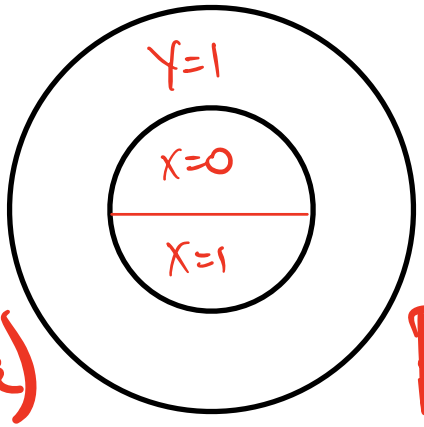

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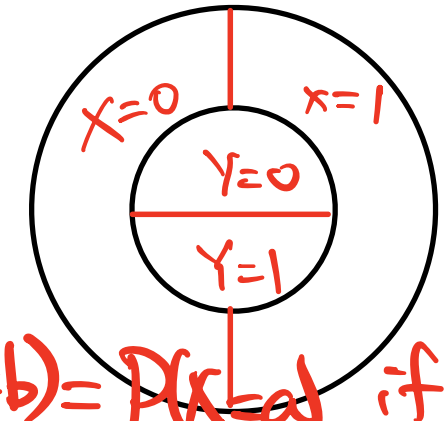
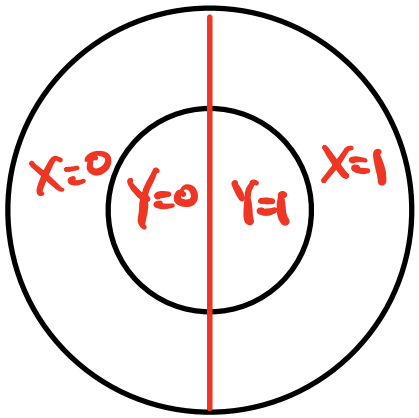


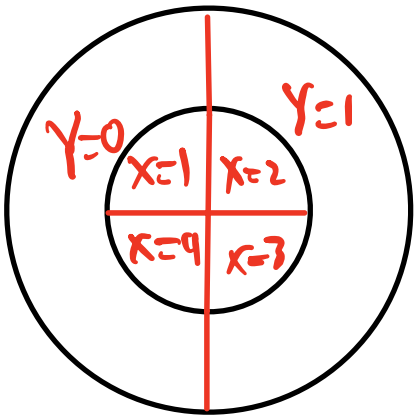
$P(X=a)$

$P(X=a|Y=b)=P(X=a)$ if X & Y are ind.

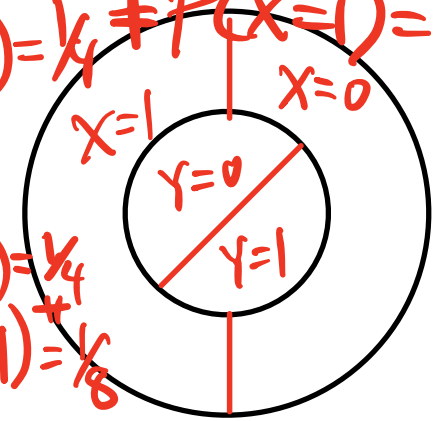
X is not independent of Y
X is dependent on Y
Y is dependent on X



X is not independent of Y
X is not dependent on Y
Y is dependent on X



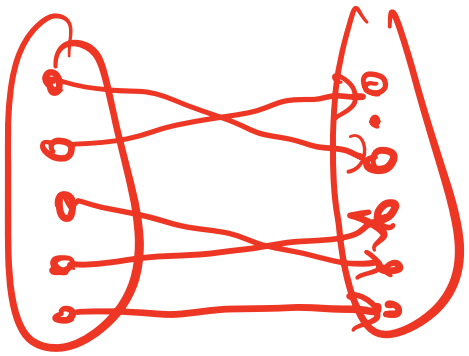
X is not independent of Y
X is not dependent on Y
Y is not dependent on X



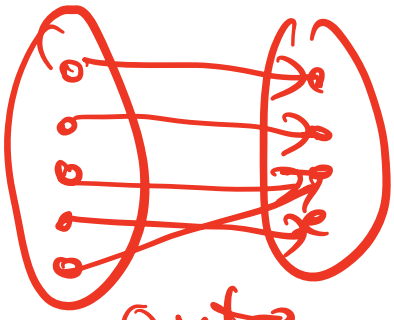
$P(X=1|Y=1)=1/4 \neq P(X=1)=1/2$

$P(X=1)P(Y=1)=1/4$

$P(X=1 \& Y=1)=1/8$



1-1 no arrows point to same point



Onto is every point on the right has an arrow.

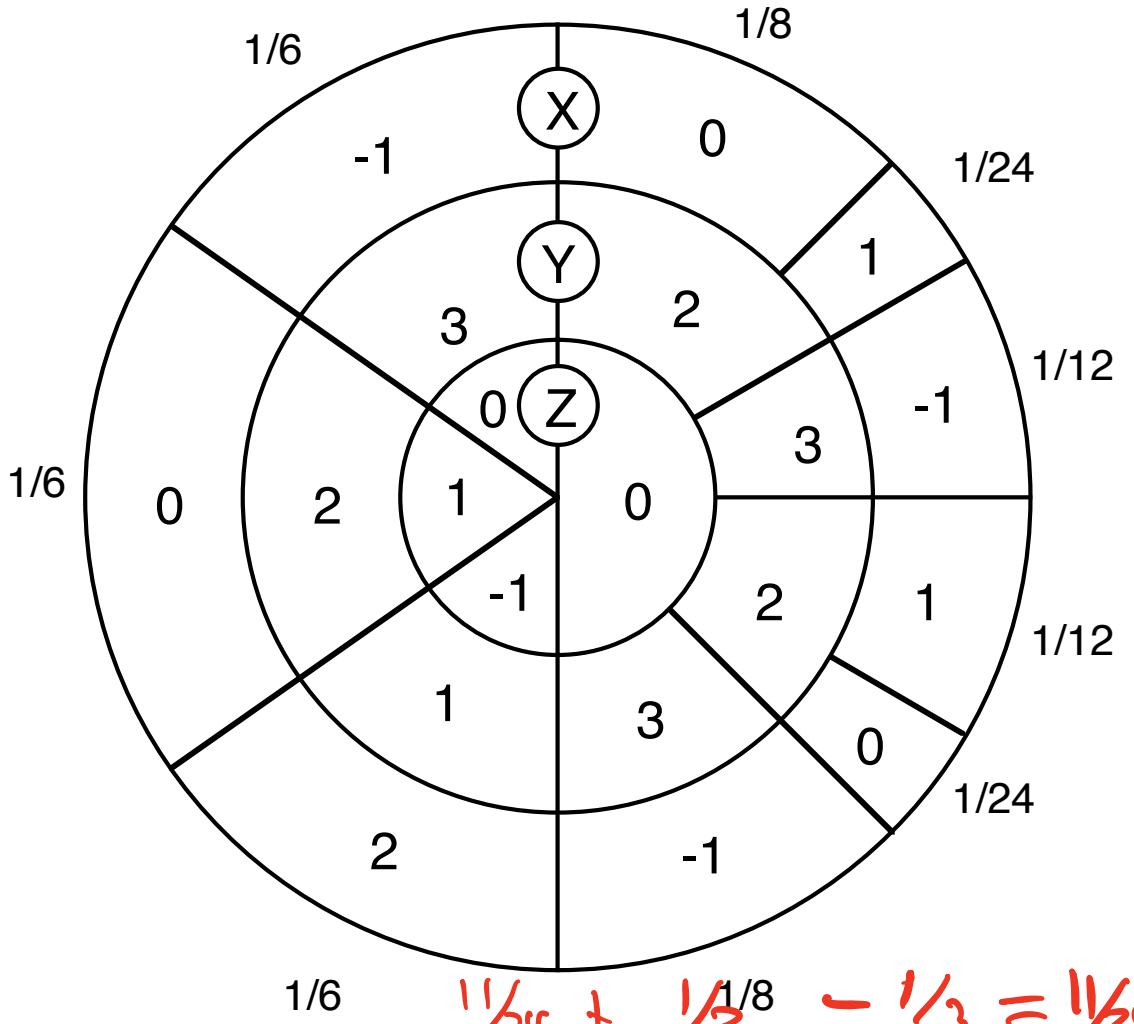
X & Y are independent

$$P(X=a \,\&\, Y=b \mid Y=b) = P(X=a \,\&\, Y=b)$$

$$\| $$

$$\frac{P(X=a \,\&\, Y=b)}{P(Y=b)} = \frac{P(X=a)P(Y=b)}{P(Y=b)}$$

1. The wheel below represents the random variables X, Y and Z.



Calculate:

a) P(X=0) $= 1/8 + 1/6 + 1/24 = 1/3$

b) P(X=1) $= 1/12 + 1/24 = 3/24 = 1/8$

c) P(Z=-1 or X=0) $= 1/3 + 1/6 = 1/2$

d) P(Y=2) $= 1/6 + 1/8 + 1/24 + 1/12 + 1/24 = 11/24$

e) P(Y=2 or X=0) $= 11/24$

f) P(Y=2 and X=0) $= 1/3$

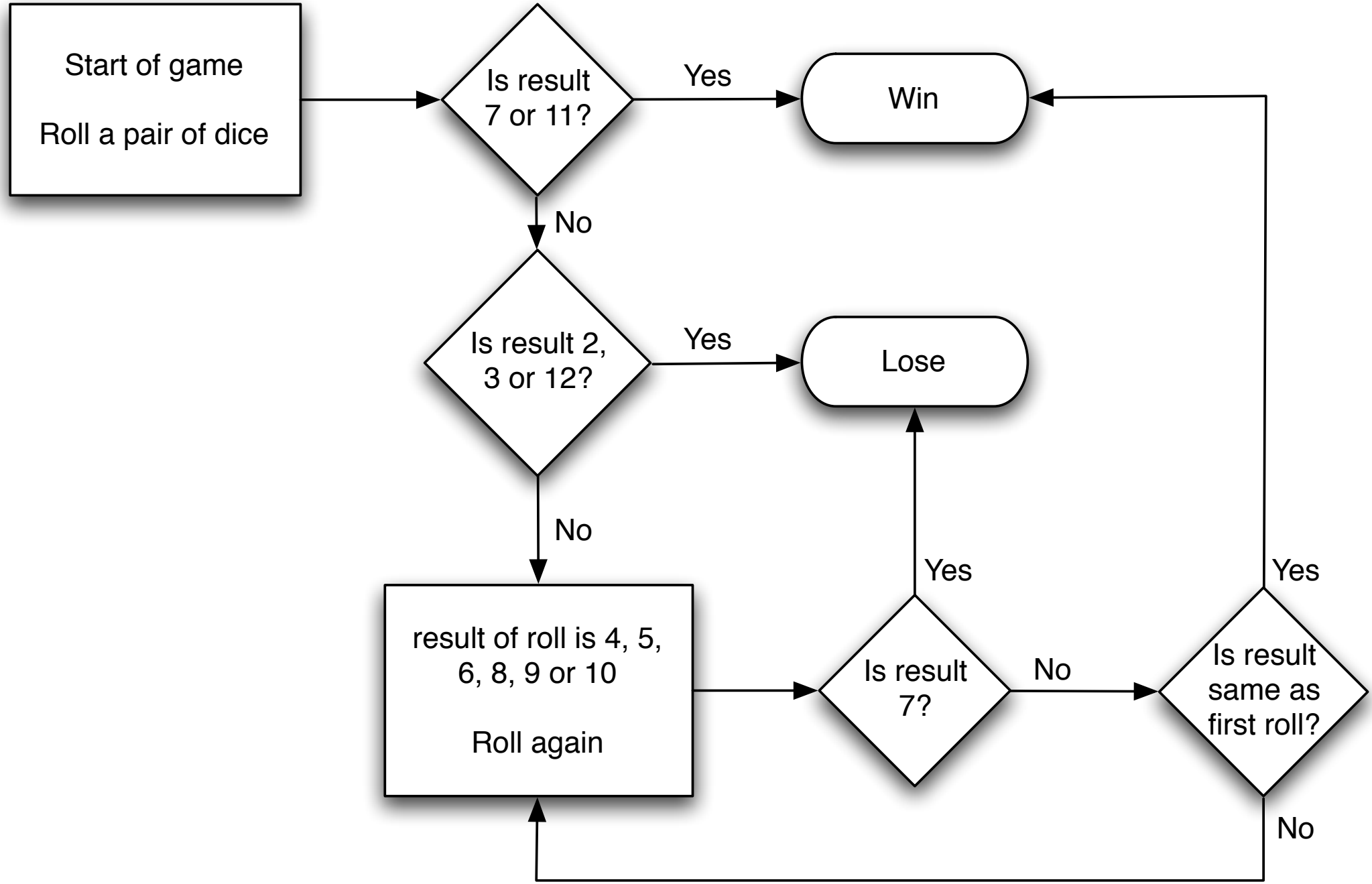
g) P(X=0 | Y=2) $= \frac{P(X=0 \text{ and } Y=2)}{P(Y=2)} = \frac{1/3}{11/24} = 8/11$

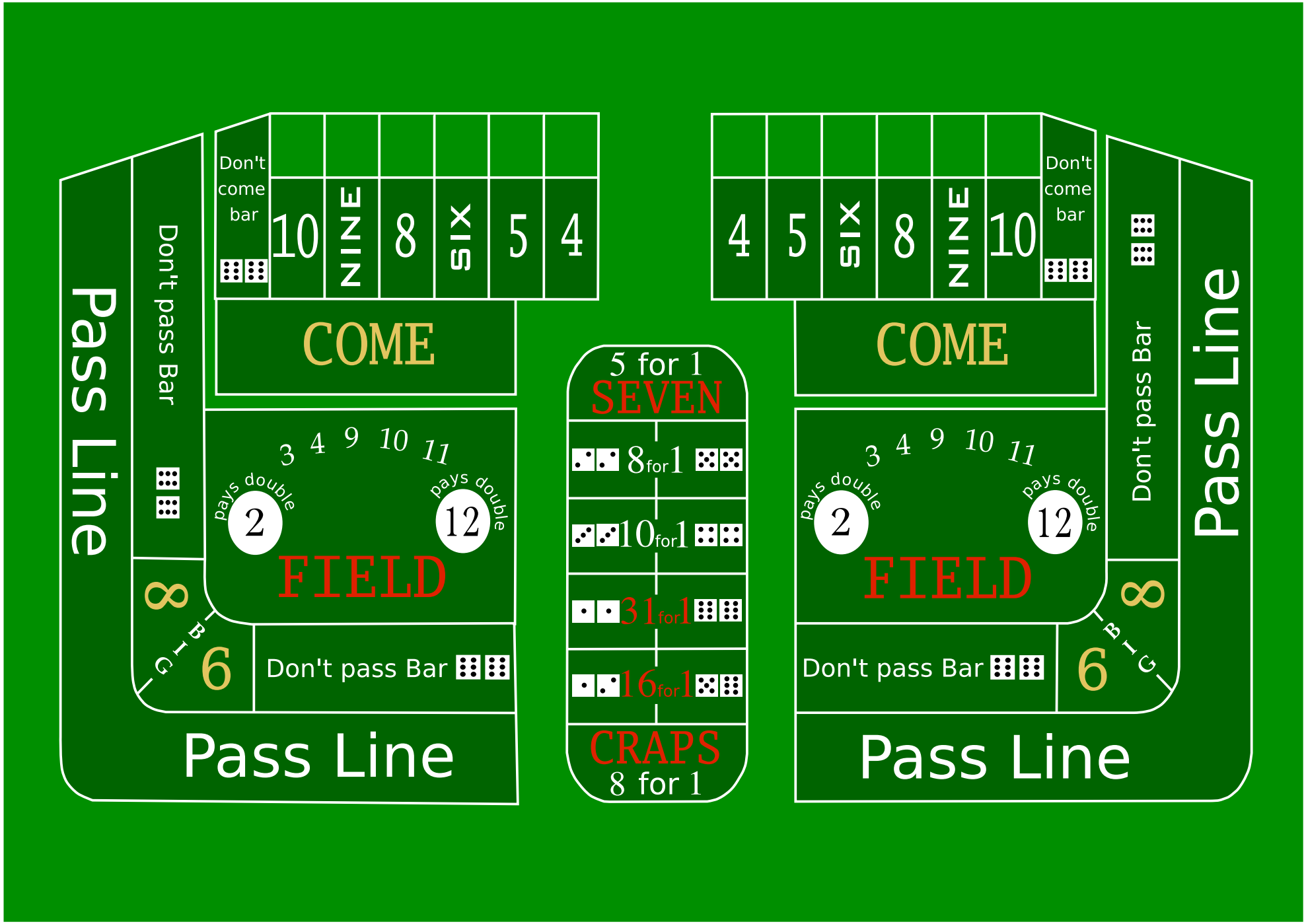
h) P(X=0 | Z=-1) $= 0$

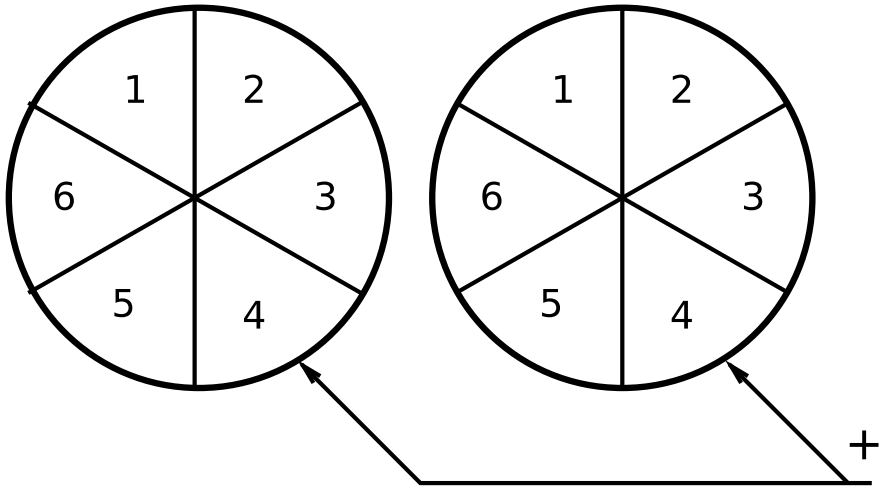
i) P(X=2 | Z=-1) $= 1$

$P(Y=2 \text{ OR } X=0) = P(Y=2) + P(X=0) - P(Y=2 \& X=0)$

$11/24 + 1/3 - 1/3 = 11/24$


Start of game
Roll a pair of dice
Is result 7 or 11?
Yes
Win
No
Is result 2, 3 or 12?
Yes
Lose
No
result of roll is 4, 5, 6, 8, 9 or 10
Roll again
Is result 7?
Yes
No
Is result same as first roll?
Yes
No


Don't come bar
10
NINE
8
SIX
5
4
COME
Pass Line
Don't pass Bar
3 4 9 10 11
pays double
2
12
pays double
FIELD
8
BIG
6
Don't pass Bar
Pass Line
5 for 1
SEVEN
8 for 1
10 for 1
31 for 1
16 for 1
CRAPS
8 for 1
4
5
SIX
8
NINE
10
Don't come bar
COME
Don't pass Bar
Pass Line
3 4 9 10 11
pays double
2
12
pays double
FIELD
8
BIG
6
Don't pass Bar
Pass Line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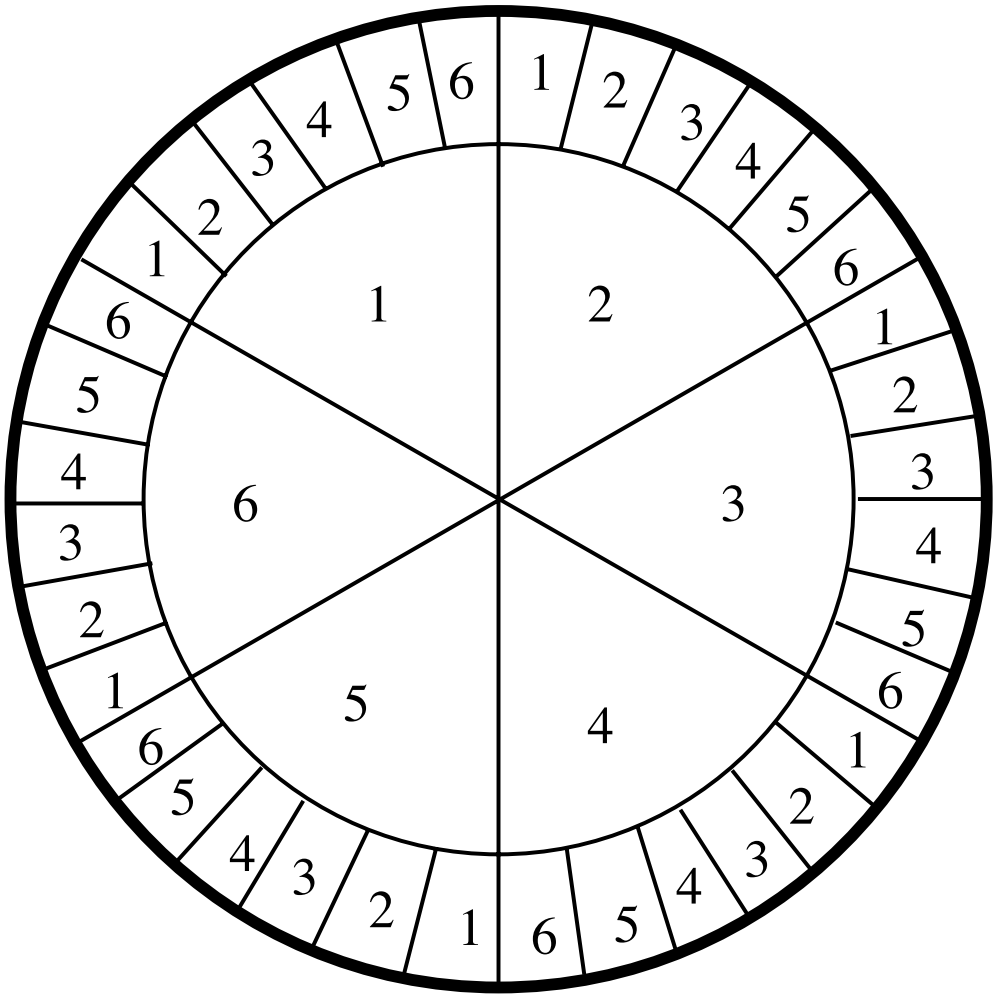
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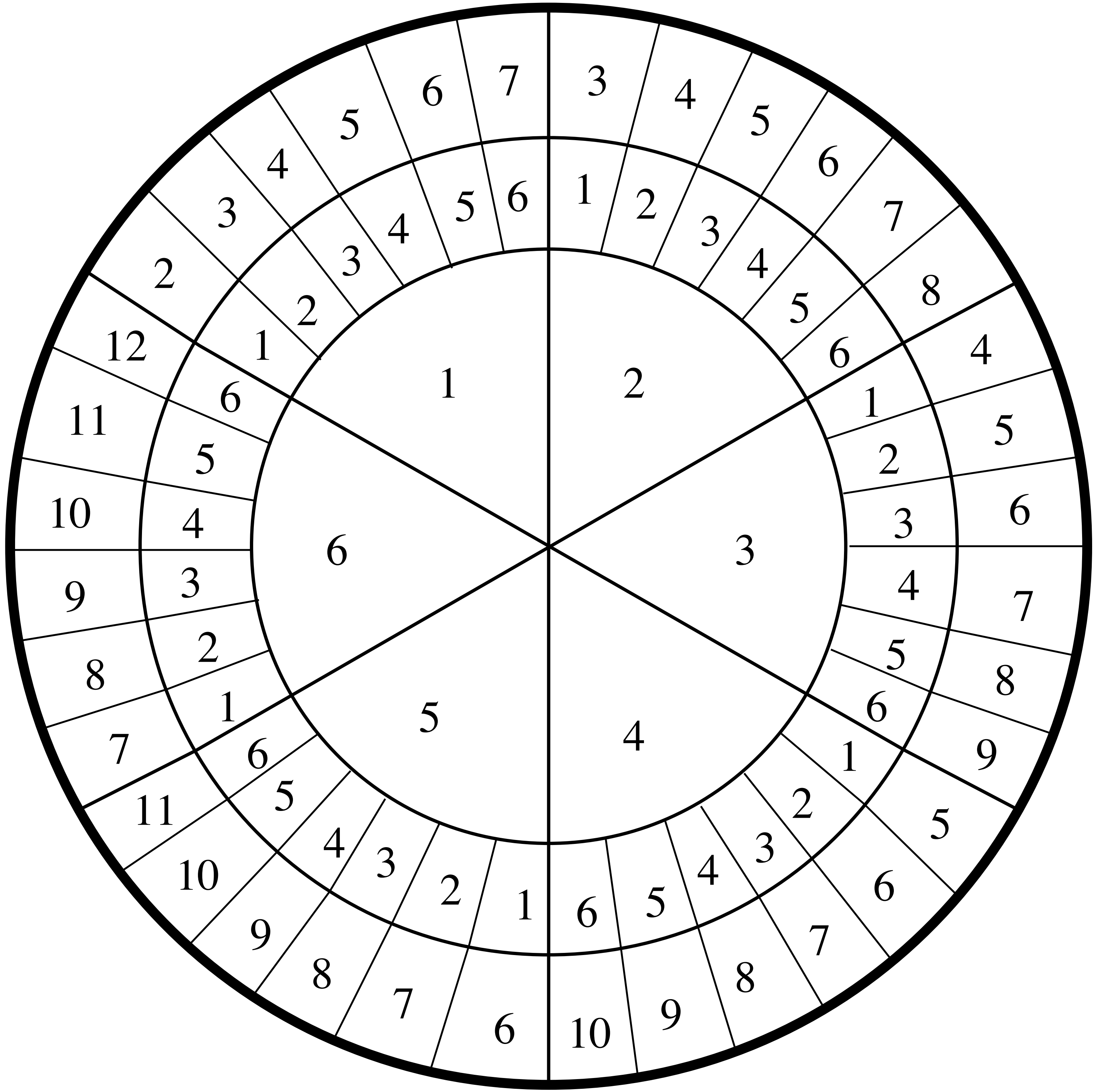

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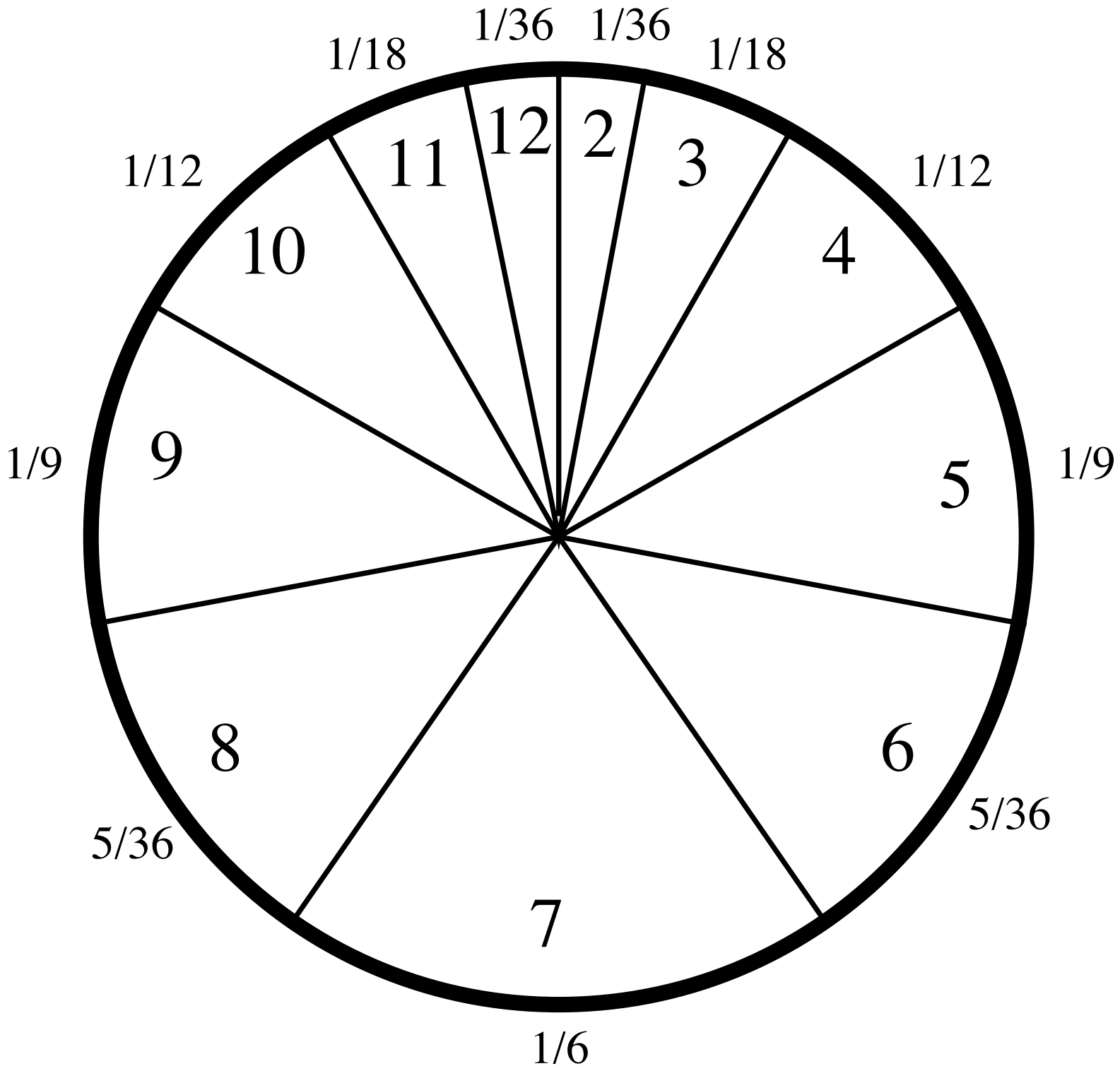
7 8 9 10 11 12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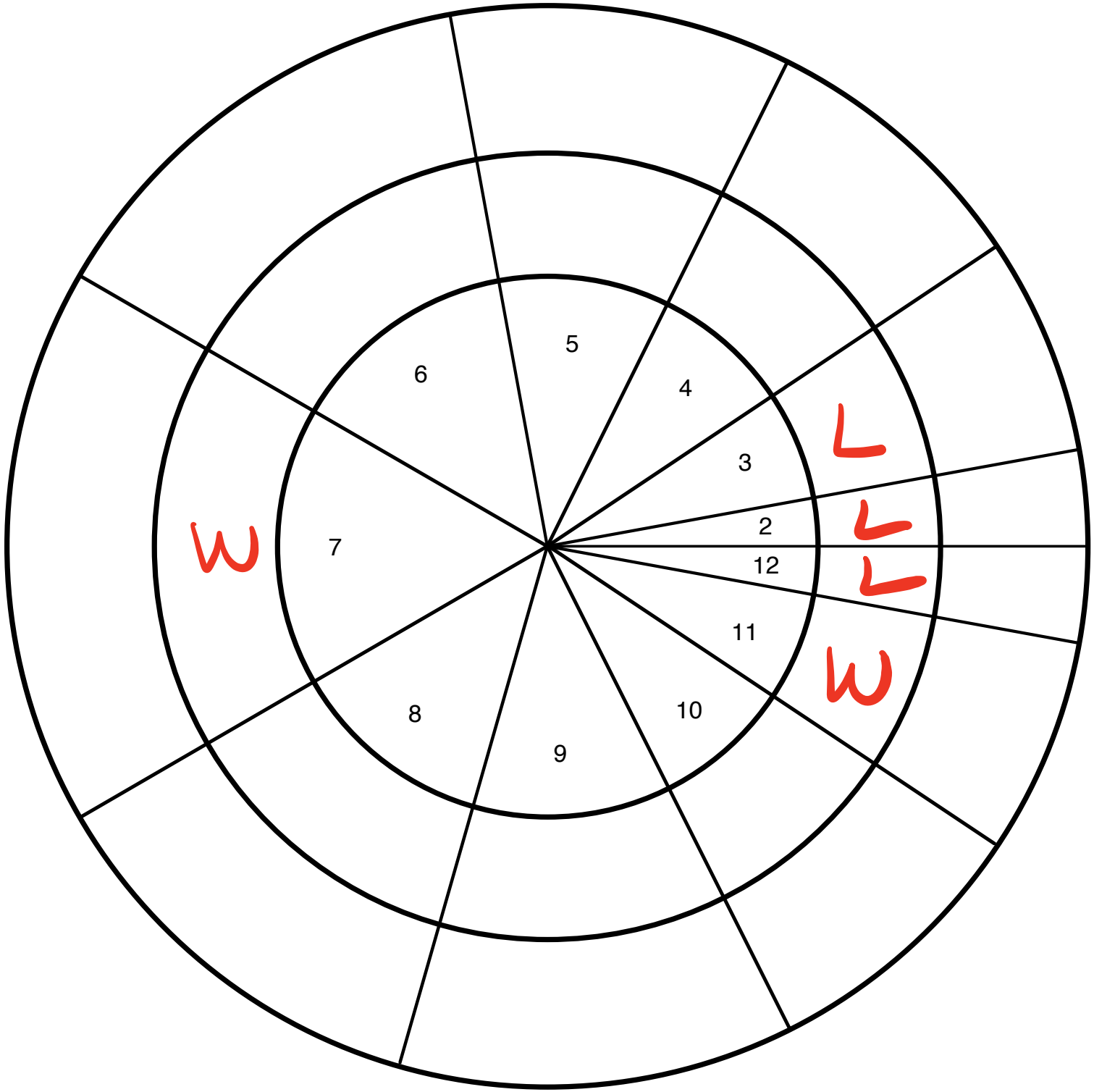
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