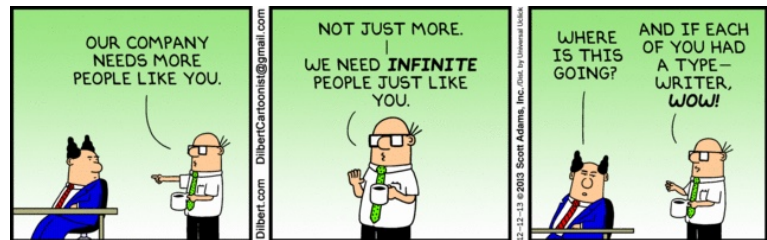


Day 10

1. Extra Verncom assignment due today Feb 8
Extra Hill assignment due next week Feb 15
2. Probability paradox
3. Craps bets
4. Model English w/ probability
5. Break Ceasar/Vigenere

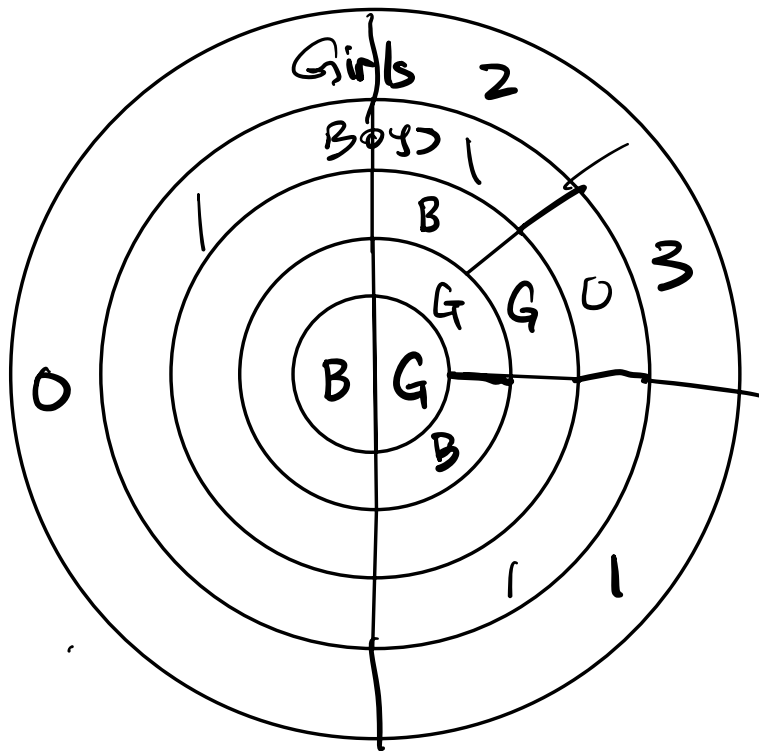


Couples in a given country want to have boys so they decide that if they have a boy or if they have three girls in a row then they will stop having children.

B GB GGB GGG

What is the expected number of boys and girls on average for 10,000 couples?:

- ☒ (a) more than half boys
- ☐ (b) more than half girls
- ☐ (c) same number of boys and girls
- ☐ (d) don't know/don't care



Q. What is expected # of Boys?

What is expected # of Girls?

$$E(\text{Boys}) = \frac{7}{8} \cdot 1 + \frac{1}{8} \cdot 0 = \frac{7}{8}$$

$$\begin{aligned} E(\text{Girls}) &= \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot 1 + \frac{1}{8} \cdot 2 + \frac{1}{8} \cdot 3 \\ &= \frac{2}{8} + \frac{2}{8} + \frac{3}{8} = \frac{7}{8} \end{aligned}$$


$$E(X) = 1 \cdot .4929 - 1 \cdot .507$$


$$= -.014 \dots$$

House
P(Win) P(Lose) Advantage

Name of Bet	Description	Payoff per \$1	P(Win)	P(Lose)	House Advantage
Pass Bet	2,3,12 -lose 7,11-win 4,5,6,8,9,10- this is the point shooter rolls again until either 7 or the point comes up if point is first then win if 7 is first then lose	\$1	.492929	.507070	1.4%
Don't Pass Bet	2,3- win 12-roll again 7,11- lose 4,5,6,7,9,10- this is the point shooter rolls again until either 7 or the point comes up if the point is first then lose if 7 is first then win	\$1	.492929	.507070	1.4%
Field Bet	The next roll is 2,3,4,9,10,11,12-win 5, 6, 7, 8-lose	\$2 for 2 or 12 \$1 otherwise	$\frac{2+3+4+9+10+11}{36}$	$\frac{4+5+6+7+8}{36}$	$1 \cdot (\frac{3}{36} + \frac{3}{36} + \frac{4}{36} + \frac{3}{36} + \frac{3}{36}) + 2 \cdot (\frac{1}{36} + \frac{2}{36}) - 1 \cdot (\frac{1}{36} + \frac{2}{36} + \frac{1}{36} + \frac{1}{36}) \approx 5.5\%$
Any Craps	The next roll is 2, 3, 12-win 4, 5, 6, 7, 8, 9, 10, 11- lose	\$7	$\frac{1+2+1}{36} = \frac{1}{9}$	$\frac{8}{9}$	$\frac{1}{9} \cdot 7 - \frac{8}{9} \cdot 1 = -\frac{1}{9}$ 11.1%
Any 7	The next roll is a 7- win 2,3,4,5,6,8,9,10,11,12- lose	\$4	$\frac{6}{36}$	$\frac{5}{6} = \frac{30}{36}$	$4 \cdot \frac{1}{6} - 1 \cdot \frac{5}{6} = -\frac{1}{6}$ 16.6%
Big 6	If a 6 is rolled before a 7-win If a 7 is rolled before a 6-lose	\$1	$\frac{5}{11}$	$\frac{6}{11}$	$1 \cdot \frac{5}{11} - 1 \cdot \frac{6}{11} = -\frac{1}{11}$ 9.09%
Big 8	If a 8 is rolled before a 7-win If a 7 is rolled before a 8-lose	\$1	$\frac{5}{11}$	$\frac{6}{11}$	$1 \cdot \frac{5}{11} - 1 \cdot \frac{6}{11} = -\frac{1}{11}$ 9.09%
4 Hardway	If a pair of twos is rolled before a 7 or before a 1 and 3- win otherwise lose	\$7	$\frac{1}{9}$	$\frac{8}{9}$	$7 \cdot \frac{1}{9} - 1 \cdot \frac{8}{9} = -\frac{1}{9}$ 11.1%
10 Hardway	If a pair of fives is rolled before a 7 or before a 4 and a 6- win otherwise lose	\$7	$\frac{1}{9}$	$\frac{8}{9}$	$7 \cdot \frac{1}{9} - 1 \cdot \frac{8}{9} = -\frac{1}{9}$ 11.1%
6 Hardway	If a pair of threes is rolled before a 7 or before 2&4 or 1&5-win otherwise lose	\$9	$\frac{1}{11}$	$\frac{10}{11}$	$9 \cdot \frac{1}{11} - 1 \cdot \frac{10}{11} = -\frac{1}{11}$ 9.09%
8 Hardway	If a pair of fours is rolled before a 7 or before 2&6 or 3&5-win otherwise lose	\$9	$\frac{1}{11}$	$\frac{10}{11}$	$9 \cdot \frac{1}{11} - 1 \cdot \frac{10}{11} = -\frac{1}{11}$ 9.09%

Pass Line

Don't pass Bar 

Don't come bar  10 **NINE** 8 **SIX** 5 4


COME

3 4 9 10 11

pays double **2** pays double **12**



FIELD



8 6



Don't pass Bar 



Pass Line

5 for 1 **SEVEN**

 8 for 1 


 10 for 1 


 31 for 1 

 16 for 1 

CRAPS
8 for 1

Pass Line

Don't pass Bar 

Don't come bar  4 5 **SIX** 8 **NINE** 10


COME

3 4 9 10 11

pays double **2** pays double **12**

FIELD

8 6

Don't pass Bar 

Pass Line

Some Monkey Words

The letters of the following words were chosen at random with equal probability.

zotwz kjptg uudrx mtagh gaqvl uhuaf oymcu iljaa jancm bxpbi cvcdo oceza wrgzg
wweuk phqvq rqtwq axqqm yxngh rsxch gsocp sssda jmjhl ykym l kdefv tmafc ncsiu
anunv hcwyu ueikr ifzta abjrq kiaez abjka hufwa fjenl ptpqm exjhx vzmeg uayxu
dkxid agsst gutif qzmyc lvood sobrh tkjoq vkghf rgxad frywd bkwql uvcpk qjdck
mugak krwcb gpgsf dwsxu vafik sxiev lejjq pznet fjtqc jhnic vojrv ucfib nuzgb
xahvu lvwyq spvtf zdxob aigfa wgjeq wcqoe bbmbm fdrvx znhbo ovxqu lzznh vmdjj
kfkig jnugp xxgnf hhliw oveje mvujj hqucs plcuo aijqt hlgdj pvhhi jwyqo wbhbo
qcwte ahbyt xbvrc kvcbv etllo bgouc lwver ewzpj yhysb grtba betir hekem mhjwl
zflze jtalcl gxvhz rkaxp qastv ibuzn jvqvt vtxgw nvnlj muqwx nddwh gggrk jqgrp
xxahd tbvuw hgjya bnoqg yfwap gohru huluh xmhtu kbbxv toywf imblw shacr taaqv
xyamm ggbcv ykial aipgv osyqq iwwrw ivhpj ybugr rscvx zbdgy ndesm endwt tbgmd
yroti vvqas xbixy qzckh ayzaz bmsns macws skbqb orcva mxaah wdjax rpfwa tgepf
ckjgv msyiv nkemr lksbd fyrtv sgxlb ryqbd zkjed uornf fgcft harsx xonas pttlr
qrraj hweru oanup pqkww wmtog sjblk hdgac etadi ljlql tfkla bfxug dwfow corfo
qnwjd tnaib sicil bukam jxhix uyygr yhfxx goxor vgxsb ngka pojfr mgoni rntkk
yxxua llggz yyuzf bijhf pfctu hnipg oczle glfli lozhs qcvmw ueuyl jnuxp xzpaz
otkyt jtaws ejigu xiqin txded zghgb jtovo oiaqa moiar yklzv kxmpe rtxwz qpvkz
chqxl mixcz kchjw xeljr hryqr kdbyp tirep whomf eoxwh iblae gbary qqhmi wcaam
vkiyp zlkjx fwium pwske xfchv xxcap tauyz jqodr lehsc ssvos avjoh pbqif kpuch
bhmvv yojmr dhfvw gwruc xjywu fmlqe joqbi pwhci hcnjq xwktm sslpc ktwwd biwbl
hqtzx yismt tpovm vbdxk nibab ukxqo ahyzk vrwhb fgezq tkves veqlr pjiww dheql
zgyyt gjbjp npobo rcncd pxaqj rzolr ibtfe ddfri ahglo cjfcf kjqrp cacwt aipoe
jcndr nhnhi efugo blkhi ahvjj rxoxq mbcnw wlvqd ryael optj grotg gnien jvruf
atedb piprx bywxl wluwv shxfr xmxxm aodlg mbfxk otpix rcfgo ylffg jyzvx wyvbe
exovx fgrqz umawo uyfua mzruo muols rpfyf fwkto tgdpr erwbi zqsva eyyxf mleas
trwhd nmosb kknqo nfiwm jaxhq woxeq sqipi xemfj cpxcq yhkvb zyelo mnaky rovg
yzrek grdzk qvkog pgxpx gghhz lclgc osdrd biedw euidz onvuz ducoc vohj rtodx
dygqw ezhkf vghih vzyzy aswum ulwcd jxsgf aibfp iwmwq qepaw npazz pwbzj aqpqo
jrfys ccegi anrge yrzzw hkwws csrmo kpcov bvsqk cmhlv kapoo mszzc zzoql vpzxp
wsixy oiljc joriz nmrfr prifj nnkaa zmdym kahue nicek zlxbo kelin wtvxd hekse

Some English Monkey Words

A monkey types on a keyboard but this time the frequencies of letters that he types are the same as one would find in average English text.

ccgai teeir euerd ommal iltom eeoye uneep risrf ocoee ifspo maxpu eerci upomr
samiu eeitr eeopo cuoic uelnr eoeew usleo mestr etfok thiee ocart tssss rseaa
anior oisda fteoa doals oiatl uyvre tiiit hifhq loean pvoom nuala appes aicha
capts ttwnh rsios fohln efsmh hnfee wtdwo ptnip toipi eretn jrpsh suodo rtuee
eosrf ueats tsedt aalet tplcf eanin tetdt evaos lefee dffsm eotfu seber keega
ehafe raaes uhtan nnspr dogoh esncn oooed hrrnu umrky hrhbe rttsn daofo otiqi
deepe fnaso atnnr heaes sinfa ogoeh oxint tdehr wsest reknd hcpte iaogg tdfro
ttsiu nrrle jpeof xyatt asdis aopon aoyrh ahsos dytic unswa efdrt eodrd toiro
efeni seief hnvaio itieu ffdht pottr exewo darnh teeos yilem taros rclof octns
ltdte liatr altsi eescr tsiru oavre dtiay esait kttha fpdqt iteas ooocf rtmee
rwtez isaru ptask uneea cbsoi sedsr tocre nitda utous ehtia dnifi ratla defls
yuiib aldcn neyed dqsse ogtsi wiexu erpas rsoyr ocpdi oitdu opsju neesh sneea
htnol smrtf ionfd ntcoo himam arhyv olodp eanei roaie clued fecrm haeei opnar
hilau tprrt etaen htnfs naahi tiavf dorat idcmp nlyic eeir nmgi h cuuie sopeo
lifdg rowni lftif eodfe ceese haeei pfnrp yenin aaeoi drnll nawtl anite eoeor
oreel ehtrt wrree ortcd rnnan seii j yeodf qetwl instb ynrcx rdct h ftrne oearth
eospr qclea ntyep emifo raotx eaodx eklai rn timer rcaiw euvao caiio udlvt eghte
swvel ltase axpth ypati hotfn xoevt hotor oomcr yhlof eoss c maego sahdt waopo
reeza elrdn itgen sauwu otdpe meeor wtnot dnvtt leuen eeeod tsdml nendf lnhta
chdeo rrrli kmolx nfwul ptste ntdwe ptehn pnosn dnoet hnols adtes asnen heoey
dofsr oyprg foina sosie nceoe suxlp eirpe tarac nyaot oedfa oaivf edgfn orgrt
nmrer noaes xeisi dttex mteiw ptwot afkwi deiee nludn iiinn nitlr tripb elret
eedrl ngsan tacse foudh oinnu xiiep footn rdlep ryvtn nlstn fneit ewemr eryae
pssnu ctmye ncenm aosnm ounnn ouswn tmref nnnol msplt meptc ebyke smeni inecl
edoen rnctr airwn mbere oanty sefcp eekdo steat eahir ootaf tfdel oslor oerop
enmst nssda gesrf nmtie staem uhfil erpun retel pdoea evwmr dnuvi aetme fwdeo
avlag syera nsnth eidns mitcc refst tfede lpsrw nswtn snere iloaa ioesy ispr
yuacm eoene rdrum rrtmy ubmol taemr ansst xeccs rrltn osnne nnern onsir mnite
stwss utrst iudeo dnnau tyllt etnoo ereho nslyl elpee sryen yctsm camee uheuw

Some Bi-letter English Monkey Words

A monkey types on a keyboard, the first letter appears with frequency in an English word, the second through fourth letters are chosen with a probability using a bi-letter frequency table given that the previous letter has been chosen.

thint oucon dingi these ryere veega manen elers dengi hedda hesnc rmiss isonk
ticic utasu thede thore ighab benan dilet sinya nthea rived there xched tenor
omobo diome emeth esast nyazz nscec rikir asthe everr blari disul hengi rinea
denth dmewo lralt rapol ldbre dities menti hesce osece oriom thear lloas ponoo
thesi ntiai beare veaki seere awhes suert theco ldent lenda sizal ingai irove
searm ereye atere antrn sunga endea plthe overu amese cyeb1 nynds hsere rtopl
pathe oares rstht stsut fennt enthe vangh ounen asile ninti xpress thegh speni
cigio tsore rendd halla erise siath erere sstra stsou theta ffant ingam onsty
reria costi denoa ezogr eewar gicau indma erofe tosth hetye terror uleem nicad
checo presa ilila derer fontr lasst ibuta atsth lirru maled doror expus grahe
nlsew rlune unche tourk shedr idens istis cenge sofic ruthe errgh rerga nftst
odata taler anthe renim lymer eulld cende ngsta piopo ctour sholl ndowa sengs
didos ndyin nthed toffi sega aveal hesee engom nenhi lesiv rksti susel wante
ofrra enero edemi mpast armop herel evede theys theco epotr lempi nsted rsper
omema ngeat satin pulli orunt erang peese ntiso istit ended resed rteem lldas
cathe evest nssll raicu entth errys ceeca theny tiong hecle seske inogh tlufo
pstav itofo giems feden eeaya erous cousc ikeng sulde scers derth renio penth
stuan shend calic whiso ndics eroro mprer spapo erint tluge tithe narda orkre
ecrad eromi ordst erofi iomim atofe trmin whall famer xcost stale sldev edowa
ourio dedut enerm narot nvily ndeys ndise sanmm egati ychis thamb dansh lomem
rmoni praut sesiv oumut erthe ethod enepa erkan poida lentl rorch fresi thess
inthe ctans ppomi halye owero site oneth oryei oulor whali deist eathe inicy
esfon gesti rleme ndesw wheit olalo eedit titha tanat isona haned ngrer tonmo
tryst ulyor tewed conce herrd lexar isoma savio dyent ainow senior cirsc utest
finin acewe weryo redee sofor nstta sousi kstco emese delin hedle demev ppain
nyesh sithe eatea witue ededi eathe wicel nther kesin latex foken ecoup londi
thofo esive arkeg ngrer deply anthe sermo peour rontl ayerc redul ncoti ullor
overt ttror tipom abyri aslld orient eyoch mersa heaye ameat ronef sede1 utidc
usper rares issth ysand amast nicec ereth ithan stson aysoi damia nsais shani
dsoth trket sstrb rolyl onthe retec egron stomy thena ooti rvesi esagh putro

A	->	73
B	->	9
C	->	30
D	->	44
E	->	130
F	->	28
G	->	16
H	->	35
I	->	74
J	->	2
K	->	3
L	->	35
M	->	25
N	->	78
O	->	74
P	->	27
Q	->	3
R	->	77
S	->	63
T	->	93
U	->	27
V	->	13
W	->	16
X	->	5
Y	->	19
Z	->	1

E	->	130
T	->	93
N	->	78
R	->	77
I	->	74
O	->	74
A	->	73
S	->	63
D	->	44
H	->	35
L	->	35
C	->	30
F	->	28
P	->	27
U	->	27
M	->	25
Y	->	19
G	->	16
W	->	16
V	->	13
B	->	9
X	->	5
K	->	3
Q	->	3
J	->	2
Z	->	1

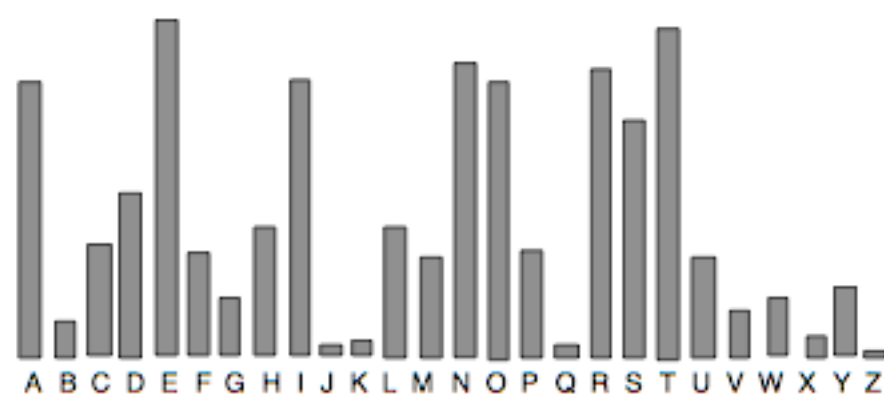
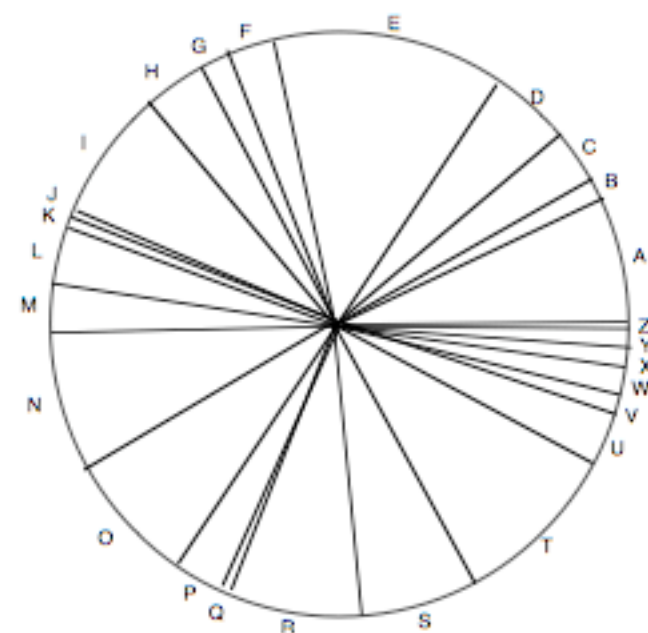


TABLE OF ENGLISH BILETTER CONDITIONAL PROBABILITIES

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	11	193	388	469	20	100	233	20	480	20	103	1052	281	1878	8	222	0	1180	1001	1574	137	212	57	26	312	23
B	932	57	16	8	3220	0	0	0	605	57	0	1243	49	0	965	0	0	662	229	49	727	16	0	0	1165	0
C	1202	0	196	4	1707	0	0	1277	761	0	324	369	15	11	2283	0	4	426	87	893	347	0	0	0	94	0
D	1044	20	26	218	3778	7	132	7	1803	33	0	125	178	53	733	0	7	324	495	13	601	99	40	0	264	0
E	660	36	433	1195	438	142	125	21	158	5	36	456	340	1382	40	192	34	1927	1231	404	48	215	205	152	121	4
F	838	0	0	0	1283	924	0	0	1608	0	0	299	9	9	2788	0	0	1215	26	496	462	0	0	0	43	0
G	1078	0	0	18	2393	0	177	1281	839	0	0	203	27	451	1140	0	0	1325	256	247	512	0	0	0	53	0
H	1770	5	14	8	5624	0	0	5	1168	0	0	16	16	38	786	0	0	153	27	233	85	0	11	0	41	0
I	380	82	767	459	437	129	280	2	16	0	50	567	297	2497	893	100	8	342	1194	1135	11	250	0	23	2	79
J	1259	0	0	0	1818	0	0	0	350	0	0	0	0	0	3147	0	0	70	0	0	3356	0	0	0	0	0
K	395	28	0	28	5283	28	0	198	1582	0	113	198	28	565	198	0	0	85	1102	28	28	0	0	0	113	0
L	1342	19	22	736	1918	105	108	0	1521	0	79	1413	82	4	778	41	0	34	389	254	269	56	11	0	819	0
M	1823	337	26	0	2976	10	0	0	1345	0	0	10	654	42	1246	722	0	26	244	5	337	5	0	0	192	0
N	550	4	621	1681	1212	102	1391	13	665	9	66	73	104	194	528	4	7	11	751	1641	124	68	18	2	157	4
O	85	101	162	231	37	1299	82	25	92	14	78	416	706	2191	222	292	0	1531	357	396	947	334	345	12	41	4
P	1358	0	6	0	1747	0	0	237	423	0	0	812	73	6	1511	581	0	2305	180	287	457	0	0	0	17	0
Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10000	0	0	0	0	0
R	1026	33	172	282	2795	31	175	17	1181	0	205	164	303	325	1114	55	0	212	655	596	192	142	17	2	306	0
S	604	12	284	27	1795	24	0	561	1177	0	91	145	112	21	706	386	9	27	836	2484	579	0	39	0	81	0
T	619	3	36	2	1417	7	2	3511	1406	0	0	101	44	15	1228	3	0	479	418	213	195	5	88	0	203	5
U	344	415	491	243	434	52	382	10	258	0	14	1097	329	1518	19	386	0	1460	1221	1255	29	14	0	10	14	5
V	749	0	0	23	6013	0	0	0	2568	0	0	0	12	0	530	0	0	0	23	0	12	12	0	0	58	0
W	2290	8	0	32	1942	0	0	1422	2104	0	0	41	0	357	1292	0	0	106	366	16	0	0	0	0	24	0
X	672	0	1119	0	1269	0	0	75	1119	0	0	0	75	0	75	3507	0	0	0	1716	0	0	0	373	0	0
Y	586	34	103	69	2898	0	0	0	691	0	34	172	379	172	2208	310	0	310	1518	172	138	0	103	0	69	34
Z	2278	0	0	0	4557	0	0	0	2152	0	0	127	0	0	506	0	0	0	0	0	127	0	0	0	0	253

To compute this table a typical example of english text was chosen. Then the first row of the table was obtained by recording, for each of 10.000 occurrences of the letter A, the letter that immediately followed it. Thus the entry 469 in the column indexed by D means that in this sample of 10.000 occurrences of the letter A the letter D was observed to immediately follow A exactly 469 times. The same procedure was repeated for each of the letters of the alphabet. We see that in 10.000 occurrences of Q, the letter U followed it all 10.000 times (not surprising!).

To get the conditional probability $P[\text{next lett}=\text{Y} \mid \text{prec lett} = \text{X}]$ you simply look at the entry in row X and column Y and divide by 10.000.

Thus

$$P[\text{next letter}=\text{E} \mid \text{preceeding letter} = \text{R}] = 2795/10.000= .27$$