MATH 6605 Homework 1

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Question 1. *Let $\Omega = \mathbb{R}$ be the set of real numbers. Describe the σ-algebra generated by each of the following collections of subsets of $\mathbb{R}$.*

(i) $\mathcal{C}_1$ is the collection of all subsets of $[0, \infty)$.

(ii) $\mathcal{C}_2$ is the collection of all subsets of $\mathbb{R}$ that contain exactly three points.

Question 2. *Let $(\Omega, \mathcal{F}, P)$ be a probability triple. For sets A and B in $\mathcal{F}$, write $A \equiv B$ if $P(A \Delta B) = 0$. Prove*

(i) The relation "$\equiv$" is an equivalence relation on $\mathcal{F}$.

(ii) If $A \equiv B$, then $P(A) = P(B) = P(A \cap B)$

(iii) $\{C \in \mathcal{F} : C \equiv \emptyset \text{ or } C \equiv \Omega\}$ is a σ-algebra.

Question 3. *Let $(\Omega, \mathcal{F}, P)$ be a probability triple. Fix a set $D \in \mathcal{F}$ such that $P(D) > 0$. Define*

$$\mathcal{F}_D = \{A \in \mathcal{F} : A \subset D\} \qquad P_D(A) = \frac{P(A)}{P(D)}$$

for $A \in \mathcal{F}_D$. Prove that $(D, \mathcal{F}_D, P_D)$ is a probability triple.