Investigation into Interference and Diffraction: Single and Double Slit Experiments

Joeun Yook
1010101462

Mitchell Baboolal
1010077970

Abstract

This experiment was conducted to study light interference and diffraction patterns through single-slits and double-slits. Using a green laser, the characteristics of the interference and diffraction patterns of light were verified, and the theoretical model for predicting the locations of the maxima and minima for these patterns was also verified. Finally, using the results of the tests, Heisenberg's Uncertainty Principle was experimentally confirmed.

1 Introduction

The wave properties of light are demonstrated through the phenomena of interference and diffraction, both of which are observable when light passes through single or double slits. In the single-slit setup, light diffracts as it passes through the slit, resulting in a pattern of alternating bright and dark regions. In the double-slit setup, interference occurs as overlapping light waves produce regions of constructive interference (bright) and destructive interference (dark).

For a single slit, the positions of the dark or minimum points follow the equation:

$$a \sin\theta = m'\lambda \tag{1}$$

where a represents the slit width, θ is the angle from the central axis to the m'-th minimum, and λ is the wavelength. In the double slit setup, the bright, or maximum, spots are given by:

$$d \sin\theta = m\lambda \tag{2}$$

where d is the distance between the two slits, and m is the order of the maxima.

The intensity distribution at different points on the screen follows specific mathematical expressions. For single slits, the intensity is given by:

$$I(\Phi) = I_0 \left(\frac{\sin(\Phi)}{\Phi}\right)^2 \tag{3}$$

where $\Phi = \frac{\pi a \sin\theta}{\lambda}$. In the case of double slits, the intensity pattern combines both diffraction and interference effects, and is represented by:

$$I(\Phi) = I_0 \left(\frac{\sin(\Phi)}{\Phi}\right)^2 \cos^2\left(\frac{\delta}{2}\right) \tag{4}$$

where $\delta = \frac{2\pi d \sin\theta}{\lambda}$. These patterns effectively demonstrate the wave nature of light, with the double slit setup displaying additional maxima due to interference between the slits.

This experiment also explores Heisenberg's uncertainty principle, which suggests that improving precision in one measurement, such as position, reduces the accuracy in a related variable, like momentum. By analyzing the diffraction patterns and the angular position of the first minimum, this principle is further examined. The angle for the first minimum, θ_1, can be calculated with:

$$\tan\theta_1 = \frac{l}{b} \tag{5}$$

where l is the measured distance from the central maximum to the first minimum, and b is the distance between the slit and the screen.

2 Materials and Methods

The experiment had two parts involving single-slit and double-slit setups.

2.1 Materials

Both parts of the experiment require a set of general materials. Both parts require:

- Light Sensor
- Motion Controller
- Laser

- Recording Software

The single-slit part of the experiment requires:

- Single-slit 0.04mm wide
- Single-slit 0.08mm wide
- Single-slit 0.16mm wide

The double-slit part of the experiment requires:

- Double-slit with slits 0.04mm wide and 0.25mm apart, centre to centre
- Double-slit with slits 0.04mm wide and 0.50mm apart, centre to centre
- Double-slit with slits 0.08mm wide and 0.25mm apart, centre to centre
- Double-slit with slits 0.08mm wide and 0.50mm apart, centre to centre

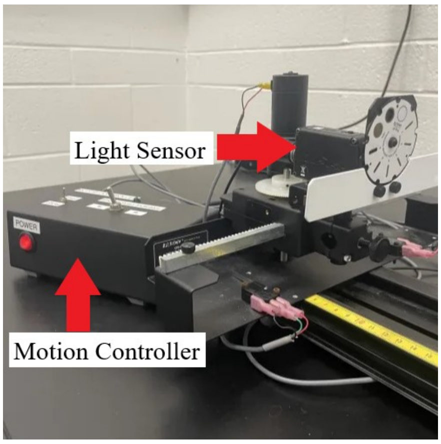

Figure 1: The image shows the experimental setup's light sensor and motion controller.

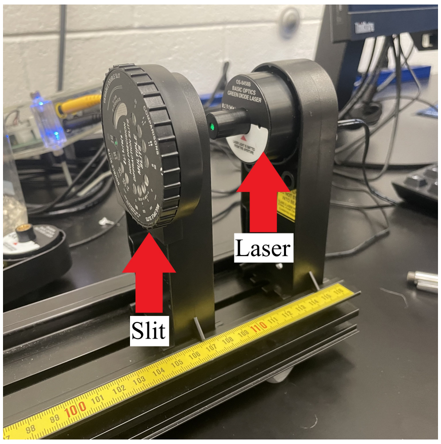

Figure 2: The image shows the experimental setup's laser and slit.

2.2 Methods

All parts of this experiment follow the same procedure several times until all the slits have been tested:

1. Set the laser so it is pointed perpendicular to the path of the light sensor.
2. Place the slit between the laser and record the distance between the slit and the light sensor.
3. Using the motion controller, set the light sensor 66mm to the left of the laser's path.
4. Begin collecting data with the Recording Software.
5. Using the motion controller, have the light sensor move from its current location to 66mm to the right of the laser's path.
6. Stop collecting and export the data.

This process was completed once for each slit. Additionally, to identify and account for random error, the process was completed a further 2 times for the single 0.04mm slit.

3 Data and Analysis

3.1 Light Intensity Curve Fitting

The following equations are used to fit the light intensity data as a function of position. For the single-slit experiment, the intensity distribution follows the form $\frac{\sin(x)}{x}$. For the double-slit experiment, the intensity distribution is modeled using a combination of $\sin(x)/x$ and $\cos^2(x)$ terms to account for both diffraction and interference effects. These functions are implemented in Python as part of the data analysis [1].

3.1.1 Single-Slit Intensity Patterns

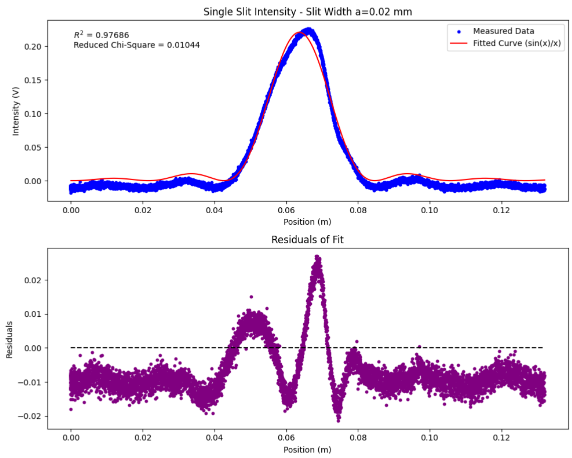

Figure 3: Single-slit pattern with slit width $a = 0.02$ mm, showing the diffraction pattern produced by the smallest slit in this experiment.

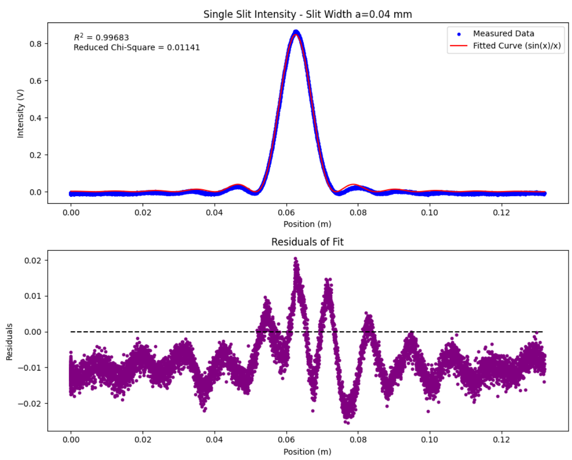

Figure 4: Single-slit pattern with slit width $a = 0.04$ mm, illustrating how the intensity distribution changes with a wider slit.

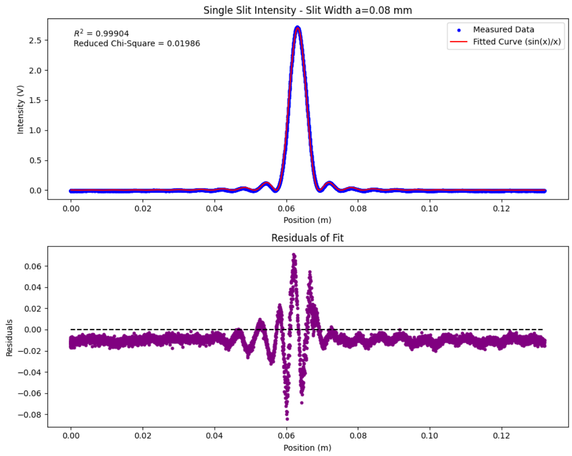

Figure 5: Single-slit pattern with slit width $a = 0.08$ mm, highlighting the diffraction effects as the slit width increases.

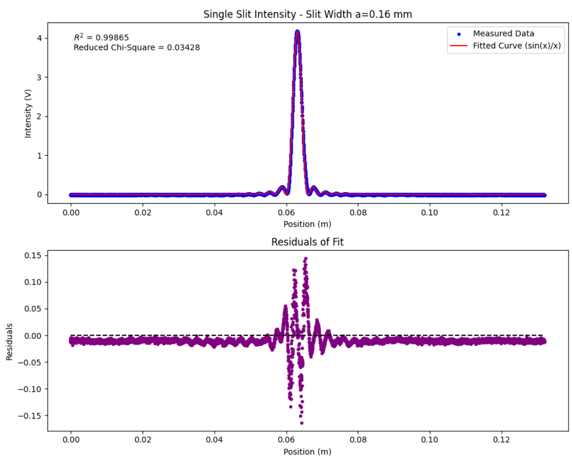

Figure 6: Single-slit pattern with slit width $a = 0.16$ mm, showing the broadest diffraction pattern in the single-slit setup.

3.1.2 Double-Slit Intensity Patterns

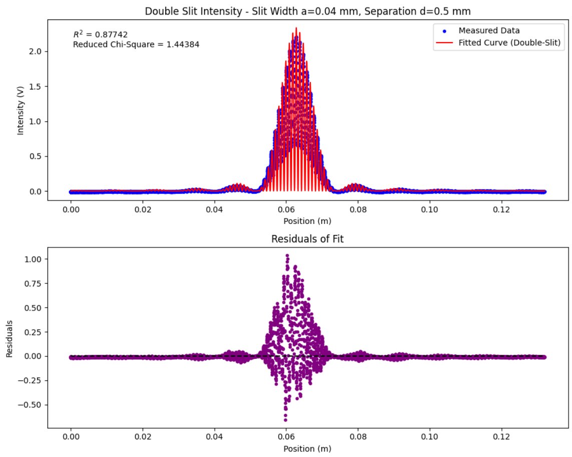

Figure 7: Double-slit pattern with slit width $a = 0.04$ mm and slit separation $d = 0.50$ mm, showing interference and diffraction effects.

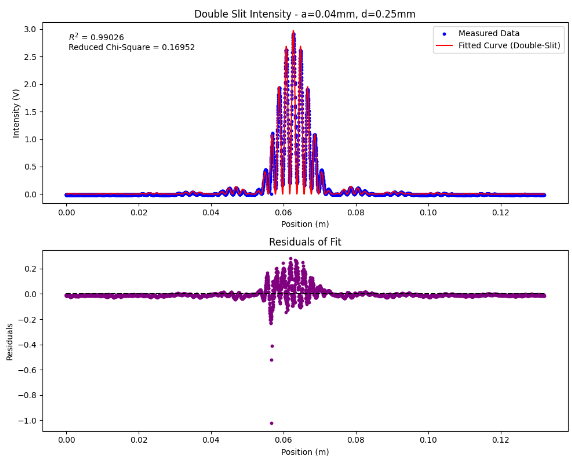

Figure 8: Double-slit pattern with slit width $a = 0.04$ mm and slit separation $d = 0.25$ mm, showing narrower separation and more closely spaced interference fringes.

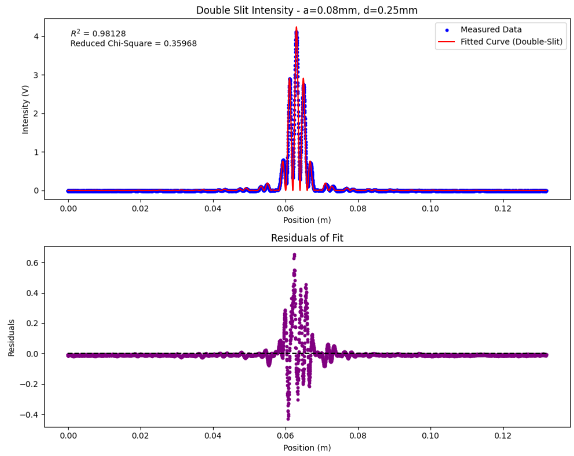

Figure 9: Double-slit pattern with slit width $a = 0.08$ mm and slit separation $d = 0.25$ mm, illustrating how increasing slit width affects the intensity distribution.

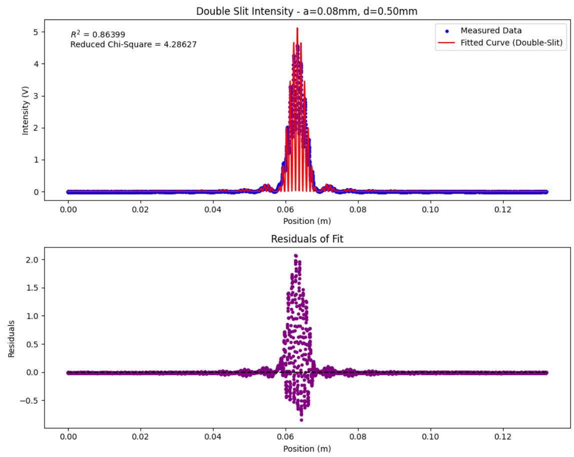

Figure 10: Double-slit pattern with slit width $a = 0.08$ mm and slit separation $d = 0.50$ mm, showing the widest slit separation in the double-slit experiment.

3.2 Goodness of Fit Analysis

3.2.1 Qualitative Justification of Light Intensity Patterns

Single-Slit Diffraction

The observed single-slit intensity patterns demonstrate diffraction, where light bends around the edges of a narrow slit, producing a distinct central maximum with side minima. The fitted curve using $\sin(x)/x$ clearly shows that a narrower slit (e.g., $a = 0.02$ mm) produces a broader central maximum, as given by the relationship between slit width and diffraction pattern:

$$a \sin\theta = m'\lambda \quad (1)$$

Double-Slit Diffraction and Interference

The double-slit intensity patterns demonstrate both interference and diffraction effects. When light passes through two closely spaced slits, it produces an interference pattern of bright and dark fringes. The fitted curves using a combined $\sin(x)/x \cdot \cos^2(x)$ function follow the intensity maxima position equation:

$$d \sin\theta = m\lambda \quad (2)$$

This pattern clearly shows that decreasing the slit separation (e.g., $d = 0.25$ mm) produces more closely spaced fringes, while increasing the slit width a affects the width of the diffraction envelope.

3.2.2 Statistical Measures for Goodness

Split Type	R^2	Reduced Chi-Square
SS0.02	0.97686	0.01044
SS0.04	0.99683	0.01141
SS0.16	0.99865	0.03428
DS0.04_0.5	0.87742	1.44384
DS0.08_0.25	0.98128	0.35968
DS0.08_0.5	0.86399	4.28627
DS0.04_0.25	0.99026	0.16952

Table 1: Summary of R^2 and Reduced Chi-Square values for each dataset. SS = single slit, DS = double slit. Numbers followed by slit type is a and d, recpectively

The R^2 values in the table indicate the fit quality, with values closer to 1 suggesting better fits. For datasets with R^2 being substantially less than 1 (e.g., SS0.02, SS0.04, SS0.16), experimental uncertainties were overestimated. On the other hand, for high R^2 values, (e.g., DS0.08_0.25) experimental uncertainties were underestimated.

The high R^2 values (ranging from 0.87742 to 0.99865) indicate a good fit, meaning the model captures the intensity pattern well. However, R^2 and reduced chi-square mainly reflect vertical deviations (deviations in intensity in this case), and may not fully account for horizontal (positional) shifts.

3.2.3 Slit Width Measurement Comparison and Uncertainty Analysis

The error bars were omitted due to the oscillatory nature of the data, as the residual plot effectively captures the error in the measurements. The percentage differences between the raw data and graph-calculated slit widths are all below 5%, with most differences closer to 2.5% or even as low as 1.25%. This slight deviation means consistency between the measured and calculated values, demonstrating that the experimental data aligns closely with the theoretical model.

Slit Type	**Raw Data (m)**	**Graph Calculated (m)**
SS0.02	2.00E-5 $\pm$ 1.00E-7	2.10E-5 $\pm$ 1.00E-7
SS0.04	4.00E-5 $\pm$ 2.00E-7	3.90E-5 $\pm$ 2.00E-7
SS0.08	8.00E-5 $\pm$ 4.00E-7	8.10E-5 $\pm$ 4.00E-7
SS0.16	1.60E-4 $\pm$ 8.00E-7	1.58E-4 $\pm$ 8.00E-7

Slit Type	**Percentage Difference (%)**
SS0.02	5.00 $\pm$ 0.03
SS0.04	2.50 $\pm$ 0.02
SS0.08	1.25 $\pm$ 0.01
SS0.16	1.25 $\pm$ 0.01

Table 2: Comparison of Raw and Graph-Calculated Slit Widths with Percentage Differences and Uncertainties for Single-Slit Configurations

Slit Type	**Raw Data (m)**	**Graph Calc. (m)**
DS0.04_0.25	4E-5 $\pm$ 2E-7	4.1E-5 $\pm$ 2E-7
DS0.04_0.50	4E-5 $\pm$ 2E-7	3.9E-5 $\pm$ 2E-7
DS0.08_0.25	8E-5 $\pm$ 4E-7	7.9E-5 $\pm$ 4E-7
DS0.08_0.50	8E-5 $\pm$ 4E-7	8.2E-5 $\pm$ 4E-7

Slit Type	**Pct. Diff. (%)**
DS0.04_0.25	2.5 $\pm$ 0.0175
DS0.04_0.50	2.5 $\pm$ 0.0179
DS0.08_0.25	1.25 $\pm$ 0.0089
DS0.08_0.50	2.5 $\pm$ 0.0175

Table 3: Comparison of Raw and Graph-Calculated Slit Widths with Percentage Differences and Uncertainties for Double-Slit Configurations

4 Discussion

4.1 Comparison of Single and Double-Slit

Between the single and double-slit experiments, the wavelength of the light remained constant. This choice allows for the comparison of the interference and diffraction patterns to identify similarities that reveal characteristics of each that may have been missed through systematic error or limitations in data collection. For some slit width, the distance between the maximum and the first minimum of the single-slit and the distance between the maximum and the first diffraction minimum of the double-slit, as confirmed in Figure pairs 4 & 7 or 8 and 5 & 9 or 10. If the intensity of the laser is held constant, then the physical quantity determines the amplitude of the absolute peak maximum. For a given double-slit setup with slit widths of 0.04mm and a separation of 0.25mm, using the properties outlined in Equations 1 and 2, we can predict that there would be a theoretical 12.5 maximumxs within the first central envelope. It is impossible to have 12.5 maximums within the central envelope; there will be some odd integer value of peaks. Therefore, the expected value is closest to 13. Figure 8 shows 11 identifiable peaks, but the remaining two may be indistinguishable from the noise in the data generated by the experimental conditions.

4.2 Quantum Mechanical Interpretation

The Heisenberg Uncertainty Principle states that there is a limit to how precise a measurement can be, which applies to the interference and diffraction patterns observed in this experiment. Trigonometrically, Equation 5 calculates the angle of the first minimum. Further, Equation 1 theoretically predicts the angle of the first minimum. Thus, to confirm the theoretical model of Equation 1, the angle from Equations 1 and 5 should be compared.

Slit Width	HWCM	Distance
0.04 mm	11.1 ± 0.1 mm	1 ± 0.01m
0.08 mm	6.1 ± 0.1 mm	1 ± 0.01m
0.16 mm	4.0 ± 0.1 mm	1 ± 0.01m

Table 4: The table records the width of the slit, the Half Width of the Central Maximum (HWCM) and the distance between the slit and the light sensor.

Slit Width	Equ.1 θ	Equ.5 θ (rads)
0.04 mm	0.012	0.011 ± 0.001
0.08 mm	0.006	0.006 ± 0.001
0.16 mm	0.003	0.004 ± 0.001

Table 5: The table records the results of the theoretical and experimental values for the angle of the first minimum.

The angles predicted from Equation 1 are within the uncertainty ranges of the experimental angles found through Equation 5; therefore, these experimental results can verify the efficacy of Equation 1. Given the confirmation of Equation 1, it is possible to rearrange the equation into a form of the Heisenberg Uncertainty [1]. Thus, with the confirmation of Equation 1, the results of this experiment uphold the principle.

5 Sources of Uncertainties

5.1 Random Error

The random error in the measurements was assessed by analyzing three separate trials for the SS0.04 slit. The calculated peak intensities had a standard deviation of 0.0262, with a mean percentage difference from the mean intensity around 2.78%. As shown in Table 6, this percentage error is less than the 5% error observed between raw data and graph-calculated slit width, which suggests that the random error is acceptably low.

Metric	**Value**
Peak Intensities (Trials 1, 2, 3)	0.87, 0.83, 0.81
Mean Intensity	0.83
Std. Deviation	0.026
Pct. Diff. from Mean (%)	4.2, 0.74, 3.4

Table 6: Summary of Peak Intensity Measurements for SS0.04 Trials

5.2 Experimental Sources of Uncertainty

The experiment was conducted in a lab environment with a mix of avoidable and unavoidable uncertainties. Major factors include:

- **Ambient Light Interference:** External light, such as hallway light entering when the lab door opened, and light from computer screens, could interfere with the monochromatic laser and disrupt the interference pattern on the sensor. Ideally, the experiment should be conducted in complete darkness to avoid these effects.
- **Measurement Alignment:** Positioning the light sensor precisely with the laser path was challenging due to the ruler's orientation, which could introduce minor misalignment errors in readings.
- **Software and Device Limitations:** The software connected to the light sensor may

contain precision limitations, introducing slight uncertainties. These may be inherent to the device and software design, resulting in minor but unavoidable discrepancies.

- **Environmental Movement:** People moving in and out of the lab could disturb the light path and cause variations in measurements, particularly in sensitive interference experiments.

6 Conclusion

In this experiment, a laser was shown through single and double slits to analyze the interference and diffraction patterns. Using the characteristic Equation 4 in a standard statistical fit, the regressive model's coefficient adequately approximated the ideal peak intensity value for all the experimental trials. Further, understanding the base, single-slit interference pattern was used to reasonably predict certain double-slit pattern behaviours. Finally, the experimental results confirmed the Heisenberg Uncertainty Principle through a method outlined by Serbanescu et al. [1].

References

[1] R. Serbanescu, C. Lee and T. Vahabi. 2019. *Interference and Diffraction of Light*. https://www.physics.utoronto.ca/~phy224_324/LabManuals/InterferenceAndDiffraction.pdf.