

DATA 605 Assignment 9

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A tourist in Las Vegas was attracted by a certain gambling game in which the customer stakes 1 dollar on each play; a win then pays the customer 2 dollars plus the return of her stake, although a loss costs her only her stake. Las Vegas insiders, and alert students of probability theory, know that the probability of winning at this game is $1/4$. When driven from the tables by hunger, the tourist had played this game 240 times. Assuming that no near miracles happened, about how much poorer was the tourist upon leaving the casino? What is the probability that she lost no money?

Solution

Lets $X_n = Y_n + 1$ Y_n is an independent random variables having a mean $\mu = 0$

```
sd <- sqrt(365*.25) #will use this for sd for a, b, & c
q <- 100-100
pr <- pnorm(q, 0, sd, lower.tail = FALSE)
cat("the probability that Y365 is ≥ 100: ", pr, "\n")
```

```
## the probability that Y365 is ≥ 100: 0.5
```

```
# calculating x as : (value - mean)/sqrt(n)
mean <- 0
variance <- 1/4
sd <- sqrt(variance)
n <- 364

q2 <- (110 - 100)/sqrt(n)
cat("the probability that Y365 is ≥ 110: ",pnorm(q2, mean = mean, sd = sd, lower.tail = FALSE),"\n")
```

```
## the probability that Y365 is ≥ 110: 0.1472537
```

```
## calculating x as : (value - mean)/sqrt(n)
q3 <- (120 - 100)/sqrt(n)
cat("the probability that Y365 is ≥ 120: ",pnorm(q3, mean = mean, sd = sd, lower.tail = FALSE),"\n")
```

```
## the probability that Y365 is ≥ 120: 0.01801584
```

Question 2

Moment generating function $Mz(t) = Ee^{tx} f(x)$ Binomial distribution function: $P(x) = \sum_{j=0}^n \binom{n}{x} p^x q^{(n-x)}$

$$P(x) = \sum_{j=0}^n \binom{n}{x} (pe^t)q^{(n-x)} \quad P(x) = (pe^t)q^n \quad g'(t) = n(pe^t + q)^{n-1}pe^t \quad g'(0) = n(p + q)^{n-1}p$$

$$g'(0) = np$$

$$\text{then, } g''(0) = n(n-1)p^2 + np \quad Var = n(n-1)p^2 + np - (np)^2 = (n^2 - n)(p^2) + np - n^2p^2$$

$$= np(1 - p)$$

Question 3

Calculate the expected value and variance of the exponential distribution using the moment generating function.

$$G(t) = \int_0^\infty e^{tx} \lambda e^{-\lambda x} dx \quad \frac{\lambda(e^{(t-\lambda)})}{t-\lambda} \Big|_0^\infty \quad \frac{\lambda}{\lambda-t} \quad g'(t) = \frac{\lambda}{(\lambda-t)^2} \quad g'(0) = \frac{\lambda}{(\lambda)^2} \quad g'(0) = \frac{1}{\lambda}$$

The second derivative evaluated at 0, and the square of the first derivative evaluated at 0. $g''(t) = \frac{2\lambda}{(\lambda-t)^3}$

$$g'(0) = \frac{2\lambda}{(\lambda)^3} \quad g'(0) = \frac{2}{(\lambda)^2} \quad Var = \frac{2}{(\lambda)^2} - \left(\frac{1}{\lambda}\right)^2 \quad Var = \frac{2}{\lambda^2} - \frac{1}{\lambda} \quad Var = \frac{1}{\lambda^2}$$

he variance of an exponential distribution is $\frac{1}{\lambda^2}$