Assignment 3

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Assignment 3

we have a matric A $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$. We are goijng to reduce the matrix to echelon form R2 <- R2 + R1 R4 <- R4 - 5R1

that will give us this matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 1 & -2 & 1 \\ 0 & -6 & -17 & -23 \end{bmatrix}$

we continue to break down the matrix so we can get the non zero rows

R2 <- 1/2R2

R4 <- R4 - 6R3

That will give us $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 7/2 \\ 0 & 0 & -4 & 5/2 \\ 0 & 0 & -5 & -17 \end{bmatrix}$

R3 <- R3 + R4

R4 <- R4 + R4

Final matrix is $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 7/2 \\ 0 & 0 & 1 & 24/2 \\ 0 & 0 & 1 & -39/2 \end{bmatrix}$

Therefore, n,r(A) = n . we imply that rank is 4

```r
# initialize matrix
A <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3),nrow=4,byrow= TRUE)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
```

```
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```r
# use matrix library
library(Matrix)
rankMatrix(A)[1][1]
```

```
## [1] 4
```

##2 Given an m x n matrix where m > n, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

Thee maximum rank a matrix m x n can have the maximum rank of n because it is not possible to have more than n linearly independent columns. The minimum rank assuming the matrix is non-zero would be 1

#3 the rank for matrix

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

We are going to reduce the matrix to echelon form

R2 <- R2 -3R1

R3 <- R3 - 2R1

We will have this matrix:

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, n,r(A) = n . we imply that rank is 1

```r
B <- matrix(c(1,2,1,3,6,3,2,4,2), nrow = 3, ncol = 3, byrow = TRUE)
B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```r
rankMatrix(B)[1][1]
```

```
## [1] 1
```

Q3

Lets a matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

find the eigenvalues and eigenvectors:

Step 1

$$|A - \lambda I| = 0$$

so we have : $B =

-

  $

$$B = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix}$$

Step 2

$$1-\lambda \begin{bmatrix} 1-\lambda & 5 \\ 0 & 6-\lambda \end{bmatrix} - 2 \begin{bmatrix} 0 & 5 \\ 0 & 6-\lambda \end{bmatrix} + 3 \begin{bmatrix} 0 & 4-\lambda \\ 0 & 0 \end{bmatrix}$$
$$(1-\lambda)(4-\lambda)(6-\lambda)$$

Eigenvalue will be 6 4 1

$(1-\lambda)(4-\lambda)(6-\lambda)$ $(\lambda^3 - 5\lambda^2 + 4\lambda) - (6\lambda^2 - 30\lambda + 24)$ $\lambda^3 - 11\lambda^2 - 34\lambda - 24$

```
A <- matrix(c(1,2,3,0,4,5,0,0,6), nrow = 3, ncol = 3, byrow = TRUE)
eigen(A)

## eigen() decomposition
## $values
## [1] 6 4 1
##
## $vectors
##           [,1]      [,2] [,3]
## [1,] 0.5108407 0.5547002    1
## [2,] 0.7981886 0.8320503    0
## [3,] 0.3192754 0.0000000    0
```

Calculate the eigenvector. we find the eigenvectors corresponding to each eigenvalue by solving the equation $(A-)v =0 $

for $(\lambda - 6)$:

$$B = \begin{bmatrix} 1-6 & 2 & 3 \\ 0 & 4-6 & 5 \\ 0 & 0 & 6-6 \end{bmatrix}$$

$R1 <-- 1/5R1$ $R2 <-- 1/5R2$

$$B = \begin{bmatrix} 1 & -2/5 & -3/5 \\ 0 & 1 & 5/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$v2 - 5/2v3 = 0 \; v1 - 2/5v2 - 3/5v3 = 0 \; v1 = t$

for $(\lambda - 6) = [t \quad 5/2 \quad t]$

for $(\lambda - 4)$:

$$A = \begin{bmatrix} 1-4 & 2 & 3 \\ 0 & 4-4 & 5 \\ 0 & 0 & 6-4 \end{bmatrix}$$

$$R1 < --1/3R1$$

$$B = \begin{bmatrix} 1 & -2/53 & -1 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

$5v3 = 0 \; v1 - 2/3v2 - v3 = 0 \; v1 = -2/3t$

for $(\lambda - 4) = [-2/3 \quad 1 \quad 0]$

```r
# calculate the eigenvector
A <- matrix(c(1,2,3,0,4,5,0,0,6), nrow = 3, ncol = 3, byrow = TRUE)

eigen(A)[2]

## $vectors
##           [,1]      [,2] [,3]
## [1,] 0.5108407 0.5547002    1
## [2,] 0.7981886 0.8320503    0
## [3,] 0.3192754 0.0000000    0
```