DATA 605 Assignment 8

Warner Alexis

2024-03-10

Week Assignment 8

1. 1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y .

Solution

The probabiblity that any single X_i is greater that y $\frac{k-y}{k}$, so $k-y$ is greater than y out of total of k.

$P(Y = y) = ()^n - ()^n $

for $y = 1, 2, \ldots, k$.

Y is the minimum of $X_1, X_2 \ldots X_n$ where X_i is uniformly distributed.

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

```
# Probability of Success
p <- 9/10

# Probability the mission will fail after 8 years
fail_prob <- 1 - p^8
cat("Probability the machine will fail after 8 years", fail_prob,"\n")
```

```
## Probability the machine will fail after 8 years 0.5695328
```

```
#Expected value
cat("The expected value is ", 1/p,"\n")
```

```
## The expected value is  1.111111
```

```
# Standard Deviation
cat("The Standard deviation is ", sqrt((1-p)/(p^2)),"\n")
```

```
## The Standard deviation is  0.3513642
```

b. b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```
# rate of lambda
lambda <- 1/10

cat("The probability that the machine will fail " , exp(-lambda * 8), "\n")
```

```
## The probability that the machine will fail  0.449329
```

```
cat("The expected value", 1/ lambda,"\n")
```

```
## The expected value 10
```

```
cat("The standar deviation is: ", 1/ lambda,"\n")
```

```
## The standar deviation is:  10
```

C. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

The probability that there is 0 failure after 8 years. $P(X = 8) = \sum_0^n \binom{n}{k}(1-p)^n - k$

Asumming $p = 1/10$ and $n = 8$

```
# probability of success
p <- 1/10
#number of years
n <- 8

# the number of failure
# number of failures in first 8 years
failures <- 0

cat("Probaility that machine will face after 8 years",pbinom(failures,n,p),"\n")
```

```
## Probaility that machine will face after 8 years 0.4304672
```

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

Let calculate the probability that the machine will fail after 8 years. $P(X = 8) = \sum_0^n \binom{8}{0} \frac{\lambda_e^x - \lambda}{x!}$

```
lambda <- 1

cat("The probability that the machine will fail after 8 years: ", 1 - ppois(7, lambda*8),"\n")
```

```
## The probability that the machine will fail after 8 years:  0.5470392
```

```
# Expected Value
cat("The Expected Value is: ", lambda * 8, "\n")
```

```
## The Expected Value is:  8
```

```
# Standard Deviation
cat("The Standard Deviation is ", sqrt(lambda * 8),"\n")
```

```
## The Standard Deviation is  2.828427
```