Measuring the D^0 Meson Lifetime with the LHCb Detector

JOSEF WILLSHER * 26 JUNE 2017

Abstract

The lifetime of the D^0 meson is measured using data from the 2011 run of the LHCb detector. The run collected data with an integrated luminosity $573.019 \pm 1.564\,\text{fb}^{-1}$ and centre-of-mass energy 7 TeV. Decays of the Cabbibo-favoured decay mode $D^0 \to K^-\pi^+$ are observed. The D^0 meson is produced at a displaced vertex by the decay of a B meson, allowing better subtraction of background. To remove background cuts are performed on the data, and background subtraction is performed using an un-binned maximum likelihood fit; the D^0 lifetime is found to be $414.8 \pm 0.7_{\text{stat}} \pm 7.4_{\text{syst}}$ fs, close to the accepted value of 410.1 ± 1.5 fs.

1 Introduction

The LHCb is one of the 4 main experiments around the large hadron collider (LHC) in CERN. This experiment collides millions of high energy protons per second, and detectors like the LHCb track the many particles produced. The LHCb is specialised in measuring the decays of B mesons (particles with beauty b and anti-b quarks) [1]. These decays have important uses in testing the validity of the Standard Model of particle physics, particularly around CP violation.

An important question this experiment aims to answer is: *why is the universe made of matter, and not antimatter?*; we observe the universe is dominated by matter, but this implies an asymmetry in the production rate of antimatter at the beginning of the universe. Studying B decays has already led to results validating CP violation—that matter and antimatter do not behave the same when interchanged—and the LHCb hopes to further the experimental evidence for this with charmed D mesons.

This work investigates the $D^0 \to K^-\pi^+$ decay mode which, despite not being CP violating, its lifetime is needed to calculate the extent of violation in other modes, and test parameters the Standard Model predicts. The D^0 lifetime is calculated from decays at a displaced B vertex, a method which has not been done at the LHCb before.

2 Theory

2.1 The LHCb Detector

Figure 1 shows a cross section of the detector; its sub-detectors are located in the forward region, along the beam pipe. The VELO (vertex locator) is an array of silicon detectors located 8mm from the collision point, and is used to measure the location of B decays to 10μm.

The rest of the detector is used to identify particles, measure energy and momenta, and reconstruct particle tracks. There are 20 million collisions in the detector per second, but only a small subset of these are of scientific value; the detector has an inbuilt *trigger*, which immediately selects the data to pass to storage. The level 0 trigger is an FPGA (a hardware computer) which only accepts events with high energy deposits in the calorimeters and muon system, and lots of events in the particle trackers.

The HLT (high level trigger) software stages then reconstruct particle tracks, and performs *cuts* on the events, removing background and improving signal quality.

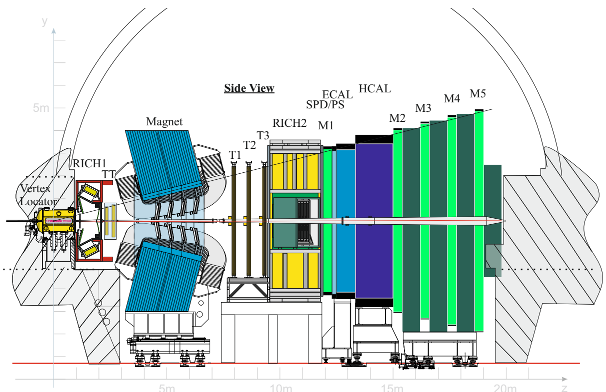

Figure 1: A cross section of the LHCb detector [2]. The vertex locator is a silicon detector for pinpointing particle decay positions; both the RICH and RICH2 are Cherenkov detectors used to identify particles; the sillicon and outer trackers measure the trajectories; the magnet is used to bend charged particles; and at the rear are the muon stations.

2.2 Particle Physics & Weak Decays

This work considers the decay mode of the D^0 meson shown in Figure 3: $D^0 \rightarrow K^- \pi^+$. Mesons are a bound state of a quark and antiquark, held together by the strong force. This decay occurs through the exchange of a W^+boson, the force carrier of the weak field. The decay is said to be hadronic as the W^+decays into hadrons (any bound states of quarks), here the kaon and pion.

There are 6 of flavours quarks which can be arranged into 3 generations: the up and down quarks, the charm and strange, and the top and bottom [3]. Each generation is heaver than the last, and only the first exists in most of the universe now; D mesons are called *charmed* as they contain a c quark.

The Standard Model underpins our understanding of these decays; it is a quantum field theory, meaning it extends quantum mechanics' particle quantisation picture to the fields that cause particle interactions. By requiring the theory to remain locally symmetrical under mathematical operations imposed by a group, a so called *local gauge symmetry*, the fields representing particles can be made to interact [4]. In the standard model the gauge group is $SU(3) \otimes SU(2) \otimes U(1)$; each of the three subgroups is responsible for an interaction: strong, weak, and electromagnetic (EM) respectively.

The EM force has an infinite range as its force carrier is massless. The weak bosons—the W^+, W^-, and Z—have a large mass of 80–90 GeV/c^2, so the range is limited because they decay quickly [3]. The strong force bosons, the 8 gluons, also have a limited range despite being massless. The strong force is so strong that all its field lines join with another strongly charged particle; any attempt to pull apart two quarks requires so much energy that more quarks are produced. This behaviour is called confinement.

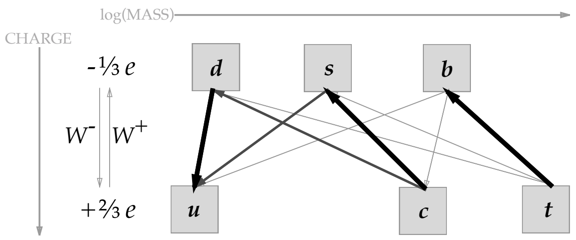

Figure 2: CKM weights of different allowed transitions. Boxes represent quark flavour, and arrows show the transitions between them, either by emission of W^+or W^-. Heavier arrows represent more favoured transitions, and particles further right are more masssive.

Just like with the EM field, particles only interact with a field if they have that field's charge—leptons do not have a strong colour charge, and photons do not have a weak charge (called isospin), so they never interact with these fields.

The weak interaction is the only one which can change the flavour of quarks. The probability of a quark changing flavour to any other lighter quark via the weak force is not constant. Each weak decay occurs at a *vertex* in a Feynman diagram. The likelihood of this vertex appearing in a decay is described by a vertex weight, and depends on the coupling strength between the two quarks and the weak field. Cabbibo theory is used to predict these coupling constants [5] (they are the elements of the CKM matrix), and predicts some decays are *Cabbibo-suppressed* and some are *Cabbibo-favoured*. Figure 2 shows how transitions in the same generation are favoured, and others are suppressed [6].

In the decays considered, the D^0 is produced from the decay of a $D^{\star+}$, which is another charmed meson with a positive charge and an angular momentum which is excited above the ground state. This quickly decays via the strong force in an exchange of a gluon—the strong force's force carrier—as shown in Figure 4. The decay occurs in only ~10^{-21} s, whereas the D^0 decay, shown in Figure 3, takes 4.101×10^{-13} s [7].

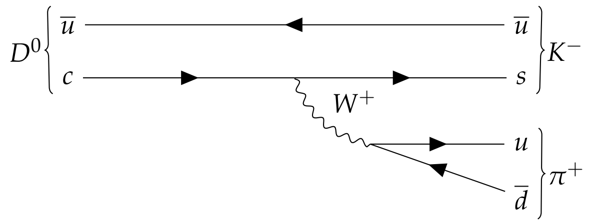

Figure 3: Feynman diagram of $D^0 \rightarrow K^- \pi^+$ weak hadronic decay.

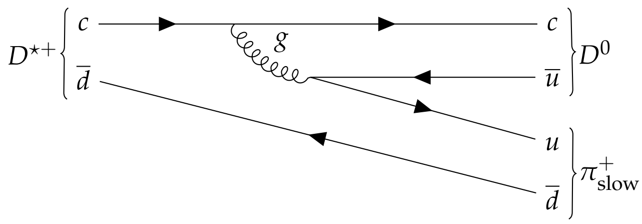

Figure 4: Feynman diagram of $D^{\star+} \rightarrow D^0 \pi^+_{\text{slow}}$ strong hadronic decay.

2.3 Decay Statistics

The probability density function $\mathcal{P}(t)$ (p.d.f.) is the probability the particle will decay between t and $d + \mathrm{d}t$. Because the particle is equally likely to decay at any time, the rate of change of population N is just the product of the population remaining and the decay rate R: $\frac{\mathrm{d}N}{\mathrm{d}t} = -RN$.

This has exponentially decaying solutions [8], so the decay p.d.f. is $1/\tau \exp(-t/\tau)$ where τ is the particle lifetime, or inverse rate. Note $1/\tau$ is a normalising constant, imposed because $\int_0^\infty \mathcal{P}\,\mathrm{d}t = 1$.

Although the D^0 meson decays exponentially, the detector has a gaussian error which will act to *spread* the readings at each point. The p.d.f. used in this analysis is the result of this spreading: the *convolution* of an exponential with a gaussian, as given by Equation 1 [9]. Given $\mathcal{P}_{\text{resolution}}$ is gaussian, Equation 2 gives the form of the convolved function [10], which depends on the decay time τ, gaussian width σ, and normalising constant A. Note erfc (t) is defined as $\frac{2}{\sqrt{\pi}}\int_t^\infty e^{-t^2}\mathrm{d}t$.

$$\mathcal{P}(t) = e^{-t/\tau} * \mathcal{P}_{\text{resolution}} \tag{1}$$

$$\mathcal{P}(t,\tau,\sigma) = A\exp\left(\frac{\sigma^2/2\tau - t}{\tau}\right)\operatorname{erfc}\left(\frac{\sigma^2/\tau - t}{\sigma\sqrt{2}}\right) \tag{2}$$

2.4 Relativistic Kinematics

The time taken for a decay depends on the strength of the mediating interaction; strong decays occur much faster than others, as this force is 7 orders of magnitude stronger than the weak interaction.

In the decays considered, the D^0 meson is not created at the primary vertex, but by the decay of a $D^{\star+}$ meson formed at the decay site of a B meson. As the $D^{\star+}$ decays strongly, the particle does not fly before decaying, so these events happen at the same place in the detector. This means the particle tracks from the B and $D^{\star+}$ decays can be used to form a vertex of the D^0 initial position, and the tracks of the daughter pion and kaon are used to locate the D^0 decay point. The collision and decay path is shown in Figure 5.

The detector measures the momentum of the pion and kaon, including the slow pion emitted by the $D^{\star+}$ decay. The relativistic energy of the pion and kaon (with masses $m = m_\pi$ and m_K, respectively) is given by $\sqrt{(pc)^2 + (mc^2)^2}$, for the particle of mass m and momentum p, and c is the speed of light. By conservation of relativistic energy and momentum in the lab frame, $E_{D^0} = E_\pi + E_K$ and $p_{D^0} = p_\pi + p_K$. Now the invariant rest energy $M_{D^0} = \sqrt{{E_{D^0}}^2 - (p_{D^0}c)^2}$, giving rest mass $m_{D^0} = \sqrt{(E_{D^0}/c^2)^2 - (p_{D^0}/c)^2}$ [3].

As the D^0 has a short and finite lifetime, there is an uncertainty in its mass implied by Heisenberg's energy-time uncertainty—a width Γ of masses will be measured by the detector, where Γ is proportional to $1/\tau$ [11].

The decay time in the D^0 frame t' is calculated from the mass, momentum, and distance travelled $\boldsymbol{x}$ by the D^0, given by

$$t' = \operatorname{sign}(\boldsymbol{x}\cdot\boldsymbol{p_{D^0}})\,\frac{x\,m_{D^0}}{p_{D^0}} \tag{3}$$

The sign operation simply returns the sign (-1 or $+1$) of the dot product of the D^0 momentum and its travel, if it is negative it shows the particle decayed in a negative time, as it travelled opposite to its velocity. This is an unphysical result but is important to consider as errors due to the detector resolution can cause this for some decays.

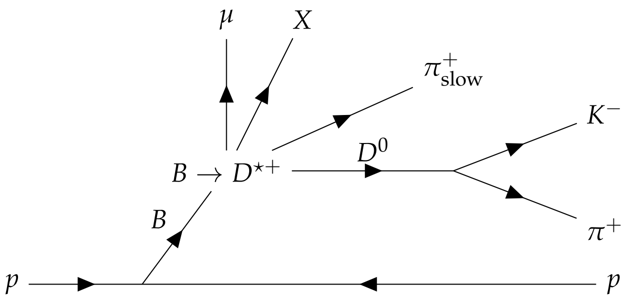

Figure 5: Overall decay process sampled. A B meson is produced at the interaction site, which decays semileptonically to $D^{\star+}$, a muon, and other particles including a undetected neutrino. The $D^{\star+}$ meson decays via the strong force to a slow pion and D^0. The decay studied is the $D^0 \to K^-\pi^+$.

2.5 *CP* Violation

CP symmetry says a system remains unchanged under the charge $\hat{C}$ and parity $\hat{P}$ operators, which respectively replace particles with their antiparticles (and thus invert the charge), and mirror space.

In the 1960s experiments showed radioactive cobalt only decayed to release neutrinos with a right handed spin [12], implying parity violation by the weak force. The weak force does not obey *CP* symmetry either: it causes different decay rates for particles and antiparticles, which is what the LHCb aims to detect.

Consider two decay modes of the D^0 meson: $D^0 \to K^-\pi^+$ and $D^0 \to K^+K^-$. The latter's products are a *CP eigenstate*; applying the $\hat{C}$ operator yields K^-K^+, and then $\hat{P}$ yields K^+K^-. The Standard Model predicts that these eigenstates can be *CP* violating, a parameter A_Γ can be defined to characterise the extent of this violation. If the partial width Γ is the inverse lifetime $1/\tau$, then Equation 4 gives the asymmetry parameter A_Γ [6].

$$A_\Gamma = \frac{\Gamma(D^0 \to K^+K^-) - \Gamma(\overline{D^0} \to K^+K^-)}{\Gamma(D^0 \to K^+K^-) + \Gamma(\overline{D^0} \to K^+K^-)} \tag{4}$$

To experimentally measure this parameter for D^0 mesons, the width of the non *CP* eigenstate $\Gamma(D^0 \to K^-\pi^+)$ is needed as a control [13].

The decay rate is also important in finding the value of another parameter: y_{CP}, the 'deviation from unity of the ratio of the widths of the CP eigenstate $D^0 \to K^+K^-$ and Cabbibo-favoured $D^0 \to K^-\pi^+$ modes' (Equation 5) [13, 14].

$$y_{CP} = \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to K^-\pi^+)} - 1 \tag{5}$$

There is now evidence for CP violation by B particles, and the Standard Model is able to explain this, even leading to exotic behaviour like neutral meson mixing, where mesons can swap between particle and antiparticle as they fly. One of the LHCb's main tasks is to investigate these violations in the charm sector; so far there have been no statistically significant results [14].

3 Method

3.1 Data Analysis & Cuts

Data are analysed from 941,039 candidate D^0 events originating from a displaced B vertex, as selected by the detector's trigger-line systems. The collider operated at a centre-of-mass energy $\sqrt{s}$ of 7 TeV, and the integrated luminosity (total events per unit cross sectional collision area) of the data analysed is $573.019 \pm 1.564\,\text{fb}^{-1}$, where a femtobarn (fb) is $10^{-43}\,\text{m}^2$. This corresponds to about half LHCb's running in 2011. The LHCb magnet can have its polarity inverted; the data analsed were taken when the magnetic field was pointing down.

The detector fits tracks of particles and forms vertices of possible decays early on in the trigger system. The data provided by the detector is an array of vertex position vectors (of the collision, and every particle's origin and end position) and momentum vectors (for the daughter pion and kaon and the slow pion).

Before storing data, the detector performs *cuts* on it based upon parameters like goodness of track fit and certainty of particle ID, where it discards data below a threshold. It also cuts on more variables which can act to bias data. The particle momentum must be above a threshold as this improves particle ID performance and lessens the risk of scattering in the detector material, and high transverse momentum indicates the presence of the a heavy particle decay. The *impact parameter* is the shortest perpendicular distance a particle comes to the collision point, and setting a lower accepted bound tends to select higher energy events and reduces noise as production is biased to be further from the detector.

The tendency of a detector to bias the data is understood as an acceptance effect [9]. If one considers D^0 production from the collision point this would be a significant effect [6], and work is required to extract lifetimes from the data. In this experiment, the D^0 is produced at a displaced B vertex, meaning any acceptance bias will only influence the B lifetime readings, and not the D^0. Secondly, the displaced vertex positions the decay vertices away from the collision, where there is the most noise, so there should be less background in the signal too.

Using this decay route does reduce the statistics available: fewer B mesons are produced than D^0 at the collision site, and fewer still decay $B \to D^{\star+} \to D^0$. Note that A_Γ and y_{CP} involve taking ratios of similar lifetimes, and are typically very small (i.e. $\mathcal{O}(10^{-3})$) [14]. The factor limiting resolution of these parameters is the statistical uncertainty in the data, and as this scales as $\sqrt{N}$ for N events [15], the method used cannot be expected to provide results which better define these parameters.

3.2 Maximum Likelihood Estimation & Background Subtraction

The lifetime of the D^0 decay is found using a maximum likelihood estimator. Given an observed decay distribution, a likelihood $\mathcal{L}$ that it is distributed by a p.d.f. $\mathcal{P}$ can be calculated.

Given a decay time at t_i, the likelihood that it is distributed by an exponential of lifetime τ is $\mathcal{L}_i = \mathcal{P}(t_i|\tau)$. For N events, the total likelihood is given by $\mathcal{L}(\tau) = \prod_N \mathcal{P}(t_i|\tau)$ [15].

The aim is to maximise this likelihood function; computationally this is done by minimising a negative log likelihood $-\log \mathcal{L}(\tau)$ using a Newton-Raphson minimisation on the derivative of this.

The data are comprised of signal and background events; the most common source of background is *combinatorial* background, where the detector combines unrelated tracks of particles and constructs a D^0 meson that was not produced.

To accurately calculate lifetime, this must be subtracted; done by identifying a background region (where there is no signal) and estimating the total number of background events. The method of identifying background and signal regions is performed in the analysis in Section 4; if there is a background region with N_b background events, and a signal region with signal events and N_a background events, the weight needed is $w_b = -N_a/N_b$. This negative weight acts to subtract the effect of the background events in the signal region.

The *weighted* maximum likelihood fit is then performed: $-\log \mathcal{L}(\tau) = -\log \prod_N \mathcal{P}(t_i|\tau)^{w_i} = -\sum_N w_i \log \mathcal{P}(t_i|\tau)$ [16] where $w_i = w_b$ if the event is identified as being in the background region, else $w_i = 1$.

4 Results & Analysis

The data from the detector has significant background present. Figure 6 is a 2D histogram of the number of events in a bin at a given D^0 reconstructed mass, and Δm (defined as the difference in reconstructed mass of the $D^{\star +}$ and D^0). Note there is a large signal peak at $m_{D^0} = 1.865\ \text{GeV}/c^2$, the central mass of the D^0 meson [7]; data with a reconstructed mass lying more than 30 MeV$/c^2$ from this central point are discarded, as this is likely entirely background. The same cut is done on data where the $D^{\star +}$ reconstructed mass is more than 30 MeV$/c^2$ from its central value, as given by the Particle Data Group.

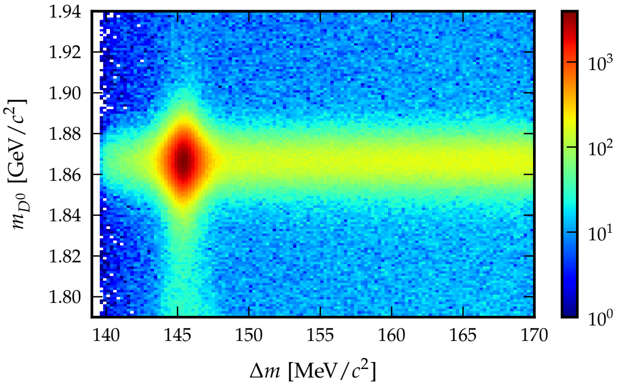

Figure 6: A histogram of number of particles against m_{D^0} and Δm, the invariant mass difference between the $D^{\star +}$ and D^0 mesons. The signal shows a clear peak around the central value of $m_{D^0} = 1.865\ \text{GeV}/c^2$.

Note how the peak is sharp along the horizontal axis, and no background events exist with $\Delta m < 139\ \text{MeV}/c^2$—this is the kinematic limit of the mass difference in the $D^{\star +} \to D^0 \pi^+_{\text{slow}}$ decay. The invariant mass difference will be $m_{\pi^+_{\text{slow}}} = 139.6\ \text{MeV}/c^2$ [7] if the pion is emitted with zero kinetic energy; larger mass differences can occur as most decays give the pion some kinetic energy.

The Δm variable is used to identify a background region. Figure 7 shows the clear signal peak centred at $\Delta m = 145.5\ \text{MeV}/c^2$. The signal is fit to a double gaussian—the sum of two gaussians centred at the same point with different widths. The background is fit with a function of the form $(\Delta m - m_\pi)^p$ where $p < 1$. Taking events 3σ away from the centre of the gaussian, we can say that 99.87% [15] of the signal events are in this region, allowing the signal region to be defined. The data gave a signal region as $142.3 < \Delta m < 148.6$. In this region there are 438,994 signal events, and it is 11.7% background.

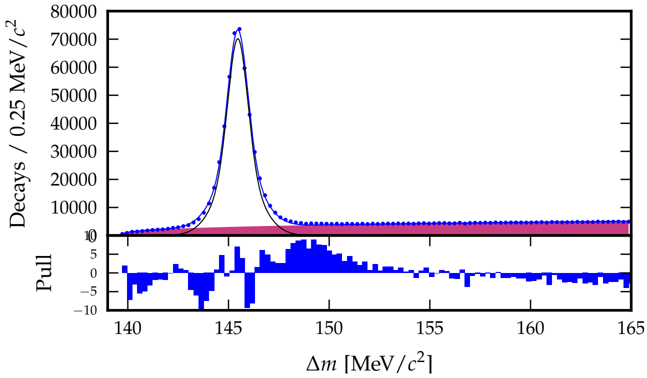

Figure 7: A histogram of the measured Δm distribution. Shown are the signal and background fits, in blue are the data and the combined fit. The signal is fit to a double gaussian (black), and background to a power function (purple). Underneath the pull is shown, defined as the difference between the number of events in the bin and the expected number, divided by the uncertainty.

By integrating the fitted background function in both the signal and background regions, the background weight w_b is found to be -0.263. Using this, the weighted maximum likelihood fit described in Section 3.2 is performed.

To estimate background numbers in this way requires it to follow the estimated distribution in the signal region, and that all background has the same properties. Firstly, the upper and lower sidebands decay distributions are compared and are found to have the same decay distribution. Secondly, to check that particle mis-ID is not a cause of background, the D^0 mass calculations are done with the hypothesis that the two daughters are K^+K^- or $\pi^+\pi^-$; the resulting distributions have no data which agree with the mass of the D^0, so this is not a source of background here.

Figure 8 shows the plot of $-\log(\mathcal{L})$ per event: the likelihood ratio.

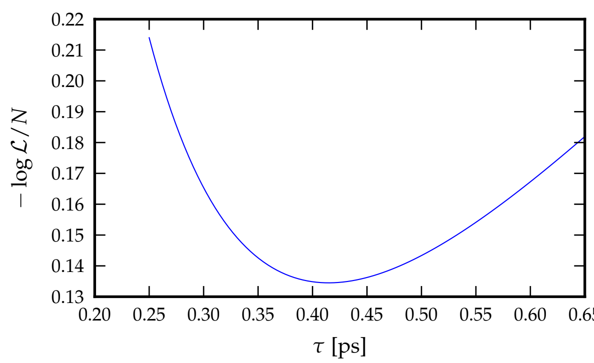

Figure 8: How $-\log(\mathcal{L})$ per event (for N events) varies with estimates of τ, a minimum of this graph represents the most likely value.

A Newton-Raphson algorithm is used to find the minimum of this likelihood ratio function (at the root of its derivative function, found numerically); the algorithm converges when successive iterations of τ are within 10^{-5} ps. To perform this estimation, the value of the resolution width σ was taken as 0.14 ps as this minimised the statistical uncertainty of the resulting fit (motivated by the fact that the best resolution the detector can achieve is ± 0.045 ps for B decays [17], and this decay will perform worse as the fitted tracks are further from the collision point).

The lifetime given by this analysis is $\tau = 414.8$ ps. Figure 9 shows a binned histogram of the decay, with weights applied to subtract background (note this is for illustration: the maximum likelihood method is un-binned).

The statistical uncertainty is estimated using the *method of likelihood ratios* [18, 19]: the statistical error of the maximum likelihood fit is given by reading the values of τ where the curve is 1/2 higher than its maximum [15].

This statistical error is found to be ± 0.7 ps, which is likely an underestimation; the background subtraction acts to remove the effect of background, meaning τ changes less for a change in $\mathcal{L}$ than the data implies.

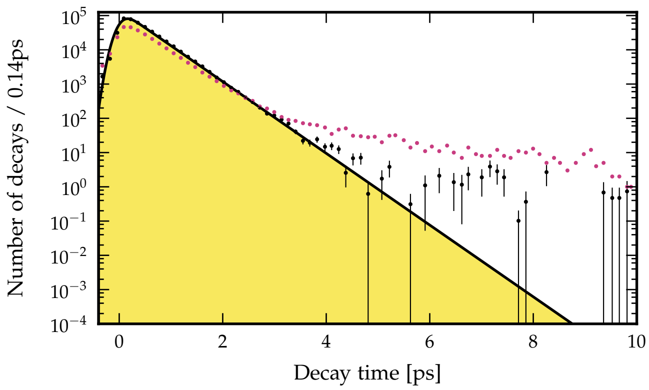

Figure 9: A weighted histogram of decay times of the D^0 meson, after background subtraction. The black points are the background subtracted decay time data, fit to a convolution of an exponential decay and gaussian resolution function. The purple data are the sideband background events, which have a different distribution to the signal.

There are many, and complex, causes of systematic error in particle physics experiments. Acceptance effects, combinatorial background estimation errors, fit errors, and an incorrect mass model are all possible causes of systematics [20]. The error in the background readings is considered; by looking at the sum of error of the background fit, the error in w_b is estimated to be ± 0.003. When this is propagated to the background subtraction (by re-running the maximum likelihood with upper and lower estimates) the systematic error on τ is found to be ± 7.4 fs.

5 Conclusion

The lifetime of the D^0 meson is found to be $414.8 \pm 0.7_{\text{stat}} \pm 7.4_{\text{syst}}$ fs, close to the Particle Data Group central value. It would likely be in agreement if a thorough systematic error analysis were performed, as currently these errors are not considered. This lifetime value is found by considering $B \to D^{\star +} \to D^0 \to K^- \pi^+$; the initial B travels, so the D^0's origin is a displaced vertex, reducing combinatorial background. The $D^0 \to K^- \pi^+$ path is non-CP violating and Cabbibo enhanced, used to find standard model parameters A_Γ and y_{CP}.

Momentum data of the daughter mesons (found from their curvature in the magnetic field of the detector) and decay vertices (measured from their intersecting paths) are used to reconstruct the mass of the D^0 and find its decay time t' in its frame. The decay lifetime is found by using a maximum likelihood fit on the data, assuming a p.d.f. $\mathcal{P}$ which is the convolution of a exponential and gaussian of width 0.14 fs. This process involves minimising the negative log likelihood $-\log \mathcal{L}(\tau)$ w.r.t. τ, using a Newton-Raphson algorithm. Cuts are performed to remove data with reconstructed D^0 and $D^{\star +}$ masses which deviate from what is possible, and a signal region is defined under the peak of the Δm plot. Background data is a significant fraction of the signal (at 11.7%) in the signal region, so background subtraction is performed by weighting the maximum likelihood fit.

This experiment could be improved by better accounting for statistical and systematic errors. A better analysis of the detector resolution (possibly by fitting this σ parameter to a decay curve of a known lifetime, like the $D^{\star +}$) would improve the accuracy of these results.

6 Bibliography

References

[1] CERN: European Organisation of Nuclear Research, *The LHCb Detector*. available from: https://lhcb-public.web.cern.ch/lhcb-public/en/Detector/Detector-en.html [accessed 18$^{\text{th}}$ june 2017].

[2] CERN: European Organisation of Nuclear Research, *Detector Geometry*. available from: http://lhcb-geom.web.cern.ch/lhcb-geom/ [accessed 18$^{\text{th}}$ june 2017].

[3] H. D. Young, R. A. Freedman, *University Physics with Modern Physics* 14$^{\text{th}}$ Edition, Harlow: Pearson Press; 2016

[4] University of Cambridge Part III Mathematical Tripos, *Quantum Field Theory* 2006

[5] Manchester University, *FPP: Interacting with the W*. available from: http://www.hep.man.ac.uk/u/roger/PHYS40521/lecture1.pdf [accessed 24$^{\text{th}}$ june 2017].

[6] M. Smith *Measurements of Indirect CP Violation in Charm at LHCb*, P.h.D. Thesis, University of Manchester, 2015

[7] C. Amsler et al. *Particle Data Group*, PL B667, 1 (2008) available from: http://pdg.lbl.gov.

[8] M. L. Boas, *Mathematical Methods in the Physical Sciences* 3$^{\text{rd}}$ Edition, Hoboken: John Wiley & Sons; 2006

[9] S. Ilieva, Measurement of *b*-flavoured hadron lifetimes at LHCb. available from: https://cds.cern.ch/record/2217946/files/Measurement%20of%20b-flavoured%20hadron%20lifetimes%20at%20LHCb.pdf.

[10] MIT, Appendix 3 — Convolution. available from: https://ocw.mit.edu/courses/chemistry/.

[11] O. Buchmueller, for the High Energy Physics Group of Imperial College London, *Particle Physics* (course notes)

[12] P. Kooijman, N. Tuning, *Lectures on CP violation*. available from: https://www.nikhef.nl/~h71/Lectures/2015/ppII-cpviolation-29012015.pdf [accessed 24$^{\text{th}}$ june 2017].

[13] The LHCb collaboration. Measurement of indirect *CP* asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays using semileptonic *B* decays, *Physics Letters B*. 2015; 767 (177–187) https://doi.org/10.1016/j.physletb.2017.01.061.

[14] The LHCb collaboration. Measurement of mixing and *CP* violation parameters in two-body charm decays, *Journal of High Energy Physics*. 2012; 2012,4 (129) 10.1007/JHEP04(2012)129.

[15] R. Barlow, *Statistics: a guide to the use of statistical methods in the physical sciences*, John Wiley and Sons, Chichester (1989)

[16] Wouter Verkerke (UCBC), *Parameter Estimation and χ^2 likelihood* available from: https://www.physik.hu-berlin.de/de/gk1504/block-courses/autumn-2010/program_and_talks/Verkerke_part3.

[17] CERN: European Organisation of Nuclear Research, *Standard set of performance numbers* available from: https://lhcb.web.cern.ch/lhcb/speakersbureau/html/PerformanceNumbers.html.

[18] Frank Porter (Caltech), *Statistics: Interval Estimation* available from: http://www.hep.caltech.edu/~fcp/statistics/sluo00/sluolec4.pdf.

[19] Caltech, *Statistics; Method of likelihood ratios*. available from: http://www.hep.caltech.edu/~fcp/statistics/sluo00/sluolec4.pdf [accessed 24$^{\text{th}}$ june 2017].

[20] Measurement of the effective $B^0 \to K^+K^-$ lifetime, *Phys.Lett.* B716 (2012) 393-400 http://dx.doi.org/10.1016/j.physletb.2012.08.033.