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## **Lecture 3 - Revision**

https://kahoot.it/

#### ✓ Pose

$${}^{A}\xi_{B}$$

- Transformation Operator
- $\bullet \quad {}^{A}\boldsymbol{p} = {}^{A}R_{B} \, {}^{B}\boldsymbol{p} + {}^{A}\boldsymbol{p}_{Bo}$

#### **✓** Rotation

- Euler angle, Roll-Pitch-Yaw
- Rotation about current axes and fixed axes
- rotx(),roty(),rotz() in RVC Toolbox

#### ✓ Homogeneous Transformations

$${}^{A}T_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}\boldsymbol{p}_{Bo} \\ 0 \ 0 \ 0 & 1 \end{bmatrix}$$

$$SE(3) = \mathbb{R}^3 \times SO(3)$$



# MTRN4230 Robotics



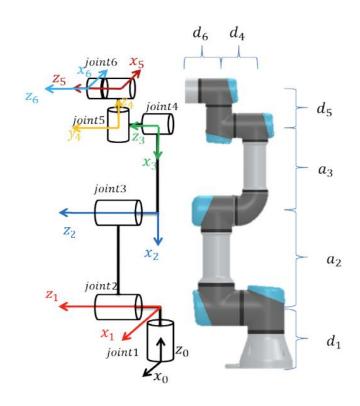
# Lecture 4

# **Denavit-Hartenberg Convention**

Hoang-Phuong **Phan** – T2 2023

## **Learning objectives**

- ☐ Understand the DH Convention
- ☐ Define the DH parameters
- ☐ Solve the forward kinematics of serial manipulators

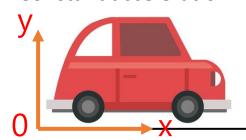


UR5e						
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]		
Joint 1	0	0	0.1625	π/2		
Joint 2	0	-0.425	0	0		
Joint 3	0	-0.3922	0	0		
Joint 4	0	0	0.1333	π/2		
Joint 5	0	0	0.0997	-π/2		
Joint 6	0	0	0.0996	0		

## Why D-H Convention?

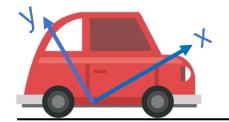
☐ In many kinematic problems, by properly defining coordinate frames, we can significantly simplify the solutions

#### Constant acceleration



$$s_x = v_o t + \frac{at^2}{2}$$
$$s_y = 0$$

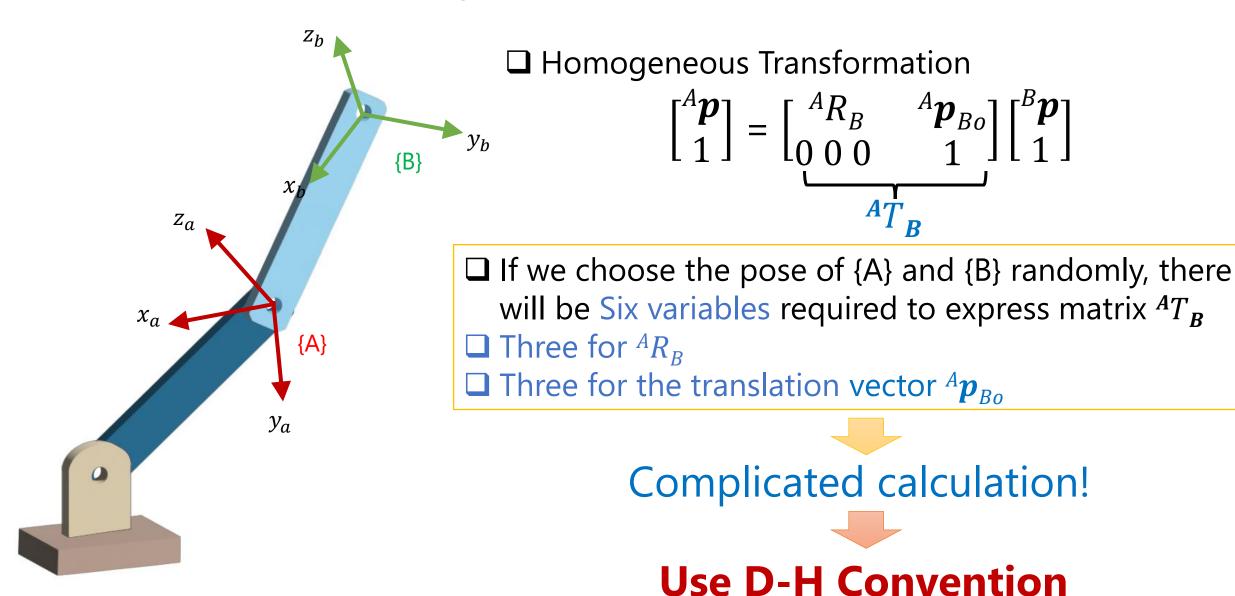
$$s_y = 0$$



$$s_{x} = ?$$

$$s_y = ?$$

## **Why D-H Convention?**





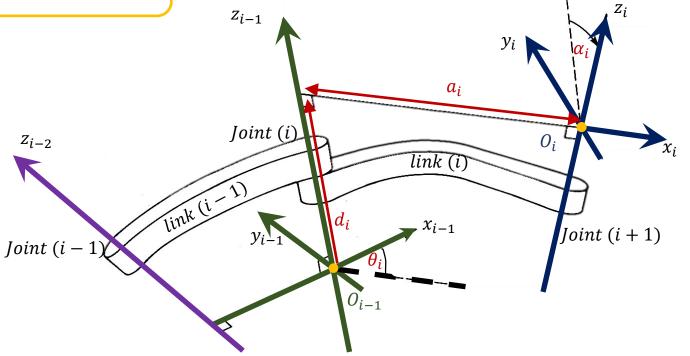
## **D-H Convention**

- ☐ A common convention used to attach coordinate frames to the links of a robot manipulator
- ☐ Its aim was to standardize coordinate frames for spatial linkages
- ☐ Parameters from the DH Convention can be used to transform one coordinate frame to another in a robot arm manipulator

## **D-H Convention**

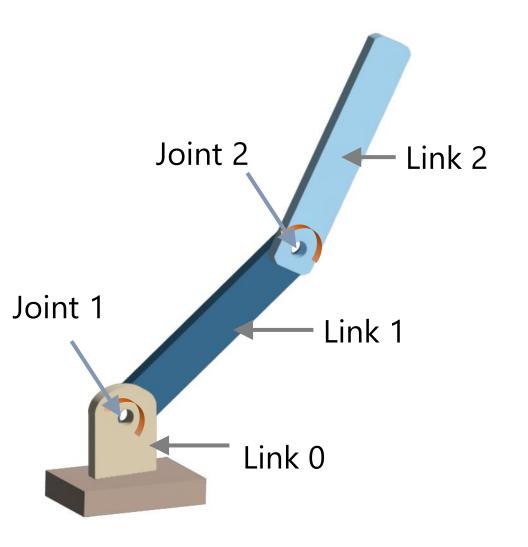
$$^{i-1}T_i = R_{(i-1)}(\theta_i).Q_{(i-1)}(d_i).Q_i(a_i).R_i(\alpha_i)$$

- ☐ First two terms: the previous axis
- ☐ Last two terms: the new axis



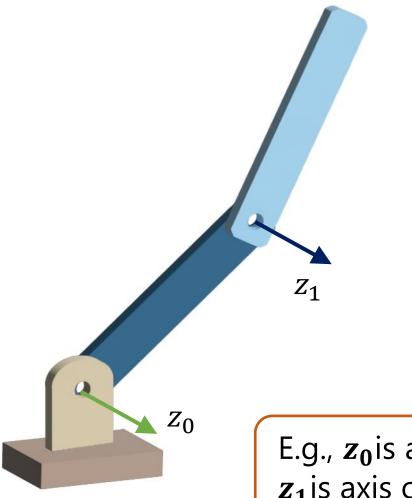


## **Transformation Rules for Robot Arms**



- $\square$  A serial robot with n joints will has n+1 links since each joint connects 2 links
- □ Number the **link** from **0** to **n**, starting from the base (link 0)
- ☐ Number the joint from 1 to n
- $\square$  Joint variable  $q_i$ :
  - If revolute,  $q_i = \theta_i$  (angle of rotation)
  - If prismatic,  $q_i = d_i$ , (joint displacement)

## **Step 1 – Setup Z axis for all frames**

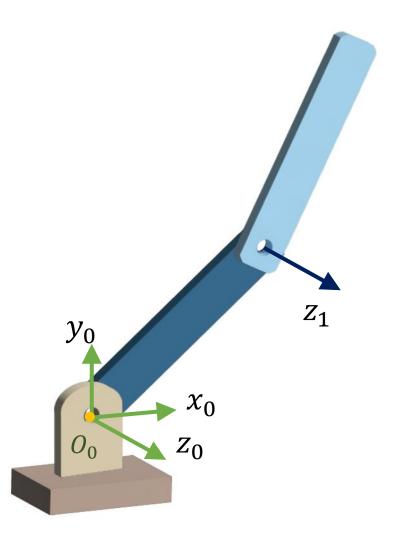


- $\square$  Identify all the joint axes, and label them as,  $z_0, z_1 \dots, z_{n-1}$
- $\Box z_i$  is the actuation axis of joint i+1
- ☐ Rotation axis for revolute joint, or axis of translation for prismatic joint

E.g.,  $z_0$  is axis of actuation of joint 1,  $z_1$  is axis of actuation for joint 2

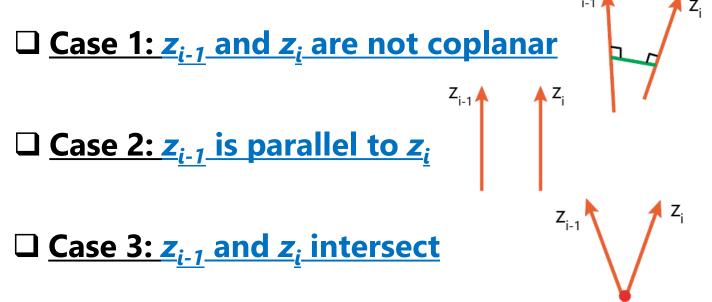


# **Step 2 – Define Frame Z**<sub>0</sub>



- $\Box$  The origin  $O_0$  of the base frame can be anywhere along the  $z_0$  axis
- $\square$  Choose  $x_0$  and  $y_0$  to satisfy the right-hand rule

- $\square z_i$  are defined in step 1  $\rightarrow$  only need to define  $x_i$ ,  $y_i$  and the origin  $O_i$
- $\square$  Since  $y_i$  can be defined from  $x_i$  and  $z_i$  using the right-hand frame  $\rightarrow$  only need to define  $x_i$  and  $O_i$
- $\Box$  The construction of  $x_i$  and  $O_i$  depends on the relative position between  $z_{i-1}$  and  $z_i$
- $\Box$  Case 3:  $z_{i-1}$  and  $z_i$  intersect





## $\square$ Case 1: $z_{i-1}$ and $z_i$ are not coplanar

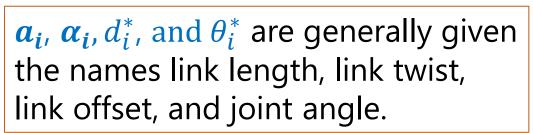
Let consider the home position first (i.e.,  $\theta_i$  =0), then we will add joint variables ( $\theta_i$ ) later

• The <u>common normal</u> between  $z_{i-1}$  and  $z_i$  defines  $x_i$ 

• The point where  $x_i$  and  $z_i$  intersect is the origin  $O_i$  of frame  $\{i\}$ 

Transformation from {i-1} to {i} has 4 steps:

Rotate by  $\theta_i^*$  about  $z_{i-1}$ ; Translate by  $d_i$  along  $z_{i-1}$ ; Translate by  $a_i$  along  $x_i$ ; Rotate by  $\alpha_i$  along  $x_i$ 





## $\square$ Case 1: $z_{i-1}$ and $z_i$ are not coplanar

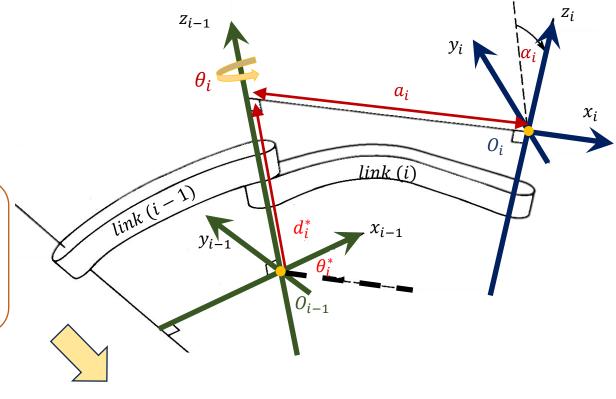
Link	Joint angle	Link offset	Link length	Link twist
	(degree)	(cm)	(cm)	(degree)
i	$ heta_i^*$	$d_i^*$	$a_i$	$\alpha_i$

Robot actuation: Joint (i-1)<sup>th</sup> rotates an angle of  $\theta_i$  (for revolute joint) or translate a distance of  $d_i$  (for prismatic joint). We need to add this joint variable to the DH table



#### i-1 is a revolute joint

Link	Joint angle (degree)	Link offset (cm)	Link length (cm)	Link twist (degree)
i	$\theta_i + \theta_i^*$	$d_i^*$	$a_i$	$\alpha_i$



#### i-1 is a prismatic joint

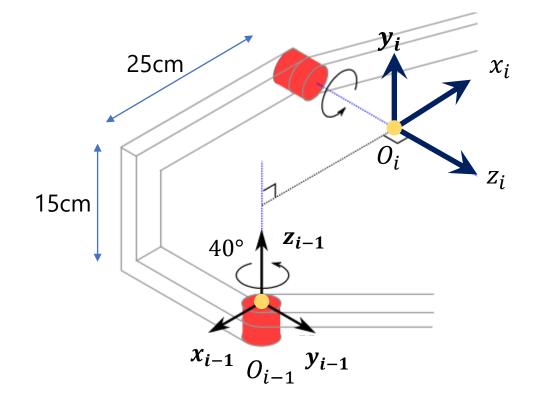
Link	Joint angle (degree)	Link offset (cm)	Link length (cm)	Link twist (degree)
i	$ heta_i^*$	$d_i + d_i^*$	$a_i$	$\alpha_i$



### $\square$ Case 1: $z_{i-1}$ and $z_i$ are not coplanar

Example

Link	$\theta_i$ (degree)	$d_i$ (cm)	$a_i$ (cm)	$\alpha_i$ (degree)
i	<del>40+</del> 180	15	25	90



## $\square$ Case 2: $z_{i-1}$ is parallel to $z_i$

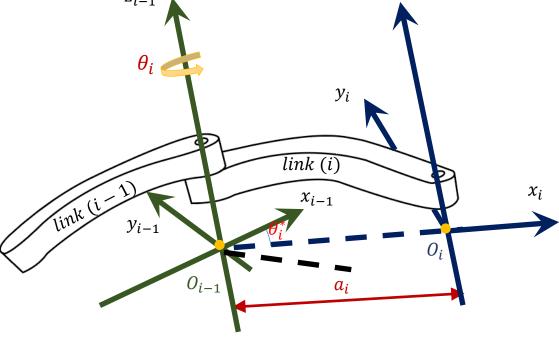
• There are infinitive common normals between  $z_{i-1}$  and  $z_i$ 

• A <u>common method</u> to choose  $x_i$  is the common normal that **pass through**  $O_{i-1}$ 

•  $O_i$  is the intersect between the selected  $x_i$  and  $z_i$ 

Link	$\theta_i$ (degree)	$d_i$ (cm)	$a_i$ (cm)	$\alpha_i$ (degree)
i	$oldsymbol{ heta}_i^* + oldsymbol{ heta}_i$	0	$a_i$	0

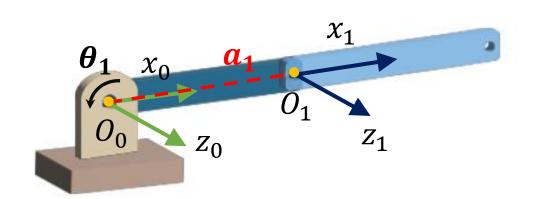
$$H = R_{z_{i-1}}(\theta_i^* + \theta_i). T_{x_i}(a_i)$$



(or 180 if you choose the opposite direction)



 $\square$  Case 2:  $z_{i-1}$  is parallel to  $z_i$ 



**□** Example

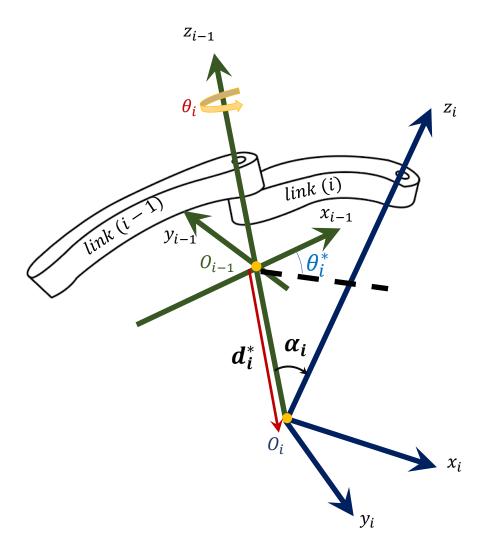
Link	$\theta_i$ (degree)	$d_i$ (cm)	$a_i$ (cm)	$\alpha_i$ (degree)
1	$oldsymbol{ heta}_1$	0	$a_1$	0

In this example (before the robot move an angle of  $\theta_1$ ), we have  $\theta_1^*=0$ 

## $\square$ Case 3: $z_{i-1}$ intersects $z_i$

- $x_i$  is chosen normal to the plane formed by  $z_{i-1}$  and  $z_i$ . The direction of  $x_i$  is arbitrary
- Generally,  $O_i$  is the intersect between  $z_{i-1}$  and  $z_i$

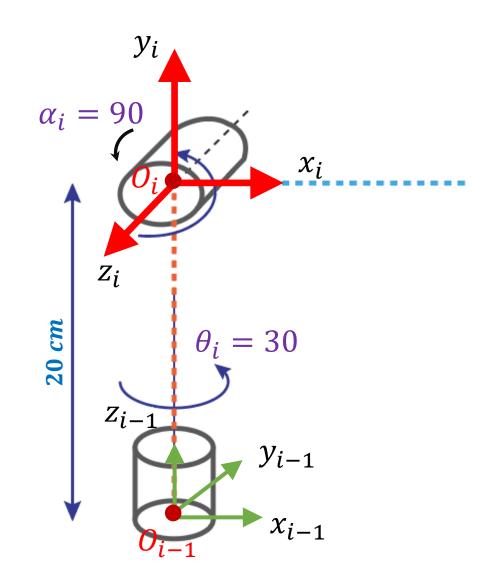
Link	$\theta_i$ (degree)	$d_i$ (cm)	$a_i$ (cm)	$\alpha_i$ (degree)
i	$\theta_i^* + \boldsymbol{\theta_i}$	$\boldsymbol{d_i^*}$	0	$\alpha_i$





- $\square$  Case 3:  $z_{i-1}$  intersects  $z_i$
- Example

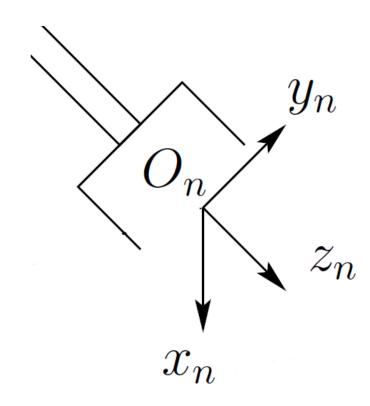
Link	$ heta_i$ (degree)	$d_i$ (cm)	$a_i$ (cm)	$\alpha_i$ (degree)
i	30	20	0	90





## **Step 4 – Construct the end-effector frame**

- ☐ The final coordinate system {n} is the end-effector or tool frame
  - There are several approaches to define the tool frame (in some cases, can be similar to frame {n-1})
  - In Spong' book,  $z_n$ : approach direction (the gripper approaches an object)
  - $y_n$ : sliding direction (open and close robot finger)
  - $x_n$ : right-hand rule



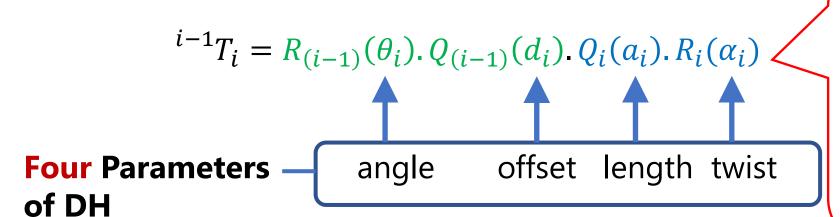


## **Step 5 Complete the DH table & apply the chain rule**

Link	$\theta_i$	$d_i$	$a_i$	$\alpha_i$
0	0	0	0	0
1	$oldsymbol{ heta_1}$	$d_1$	$a_1$	$\alpha_1$
2	$ heta_2$	$d_2$	$a_2$	$\alpha_2$
n	$\boldsymbol{\theta_n}$	$d_n$	$a_n$	$\alpha_n$

$$^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

•  $^{i-1}T_i$  represents the product of four basic transformations.



**Note:** In many textbooks, for simplification,  $\theta_i^* + \theta_i$  is written as  $\theta_i$ , which includes offset angle inside.

Please remember to check if there is any offset angle at the home position



## **Step 5 Complete the DH table & apply the chain rule**

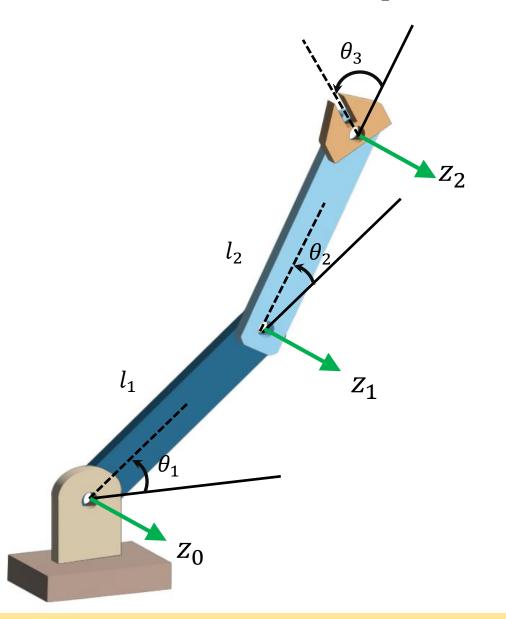
☐ Transformation matrix from Frame {0} to Frame {n}:

$${}^{0}T_{n} = {}^{0}T_{1} {}^{1}T_{2} .... {}^{n-2}T_{n-1} {}^{n-1}T_{n}$$

as the position and orientation of the tool frame w.r.t the base frame.

☐ A vector relative to the tool frame can be expressed in base coordinates as:

$$^{\mathbf{0}}\mathbf{p} = {}^{0}T_{n}{}^{n}\mathbf{p}$$

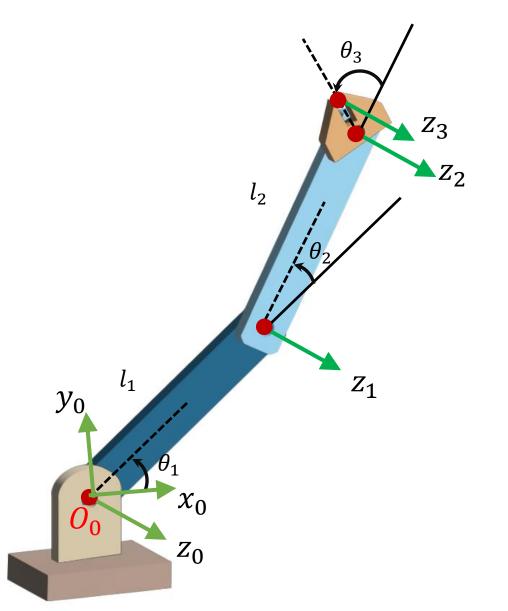


If  $l_1 = 3$ ,  $l_2 = 4$ ,  $\theta_1 = 25$ ,  $\theta_2 = 40$ ,  $\theta_3 = 0$ , and the end effector tool point is considered to be at (0,0,0) of frame 2 (i.e,  $l_3 = 0$ ).

What is the position of the end effector in frame 0 coordinates?

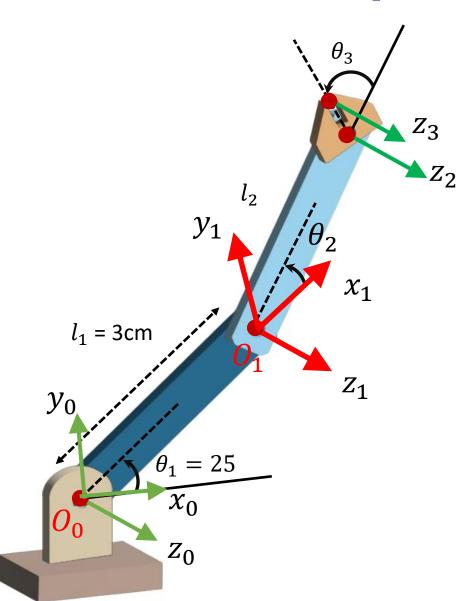
$${}^{0}p = {}^{0}T_{n}{}^{n}p$$
, where  ${}^{n}p = (0\ 0\ 0)$ 





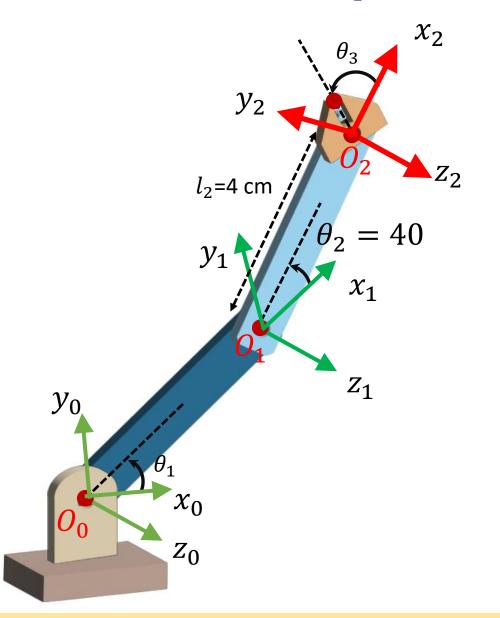
- $\square$  Identify all the joint axes,  $z_0, z_1, z_2$  ( $z_{i-1}$  is the axis of revolution or translation of joint i)
- $\square$  Select the origin  $O_0$  of the base frame
- $\square x_0$  and  $y_0$  to satisfy the right-hand rule





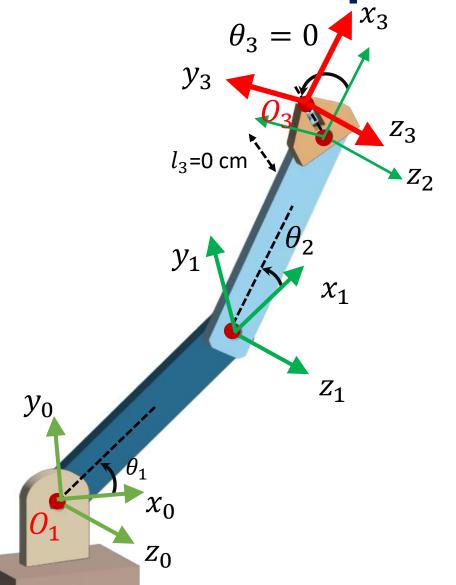
- $\square$  Frame {1}: As  $z_1$  is parallel to  $z_0$ ,  $x_1$  is a common normal between  $z_0$  and  $z_1$ ,
- $x_1$  in the direction from  $z_0$  to  $z_1$ that passthrough  $O_0$
- $O_1$  is the intersect between  $x_1$  and  $z_1$

i	$\boldsymbol{\theta_i}$	$d_i$	$a_i$	$\alpha_i$
1	25	0	3	0



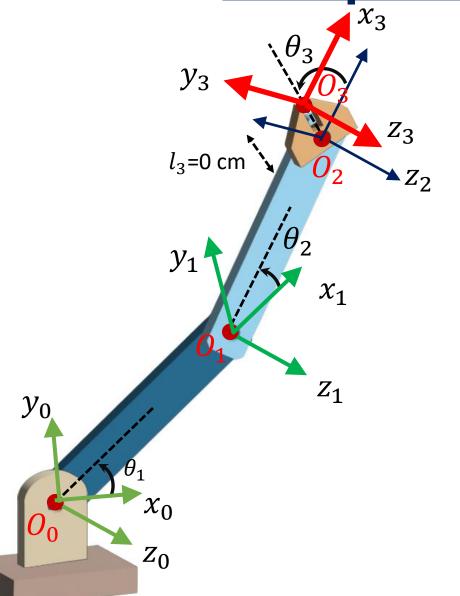
- ☐ Apply the same procedure for frame {2}
- $\square$   $z_1$  is parallel to  $z_2$ ; hence,  $x_2$  is a common normal between  $z_1$  and  $z_2$

i	$oldsymbol{ heta}_i$	$d_i$	$a_i$	$\alpha_i$
2	40	0	4	0



- $\square$  Establish the end-effector frame  $O_3x_3y_3z_3$
- $\square$   $z_3$  is parallel to  $z_2$ , and and  $x_3$  is a common normal between  $z_2$  and  $z_3$ .

i	$oldsymbol{ heta_i}$	$d_i$	$a_i$	$\alpha_i$
3	$\theta_3 = 0$	0	$l_3 = 0$	0



#### **DH** Parameter Table

i	$oldsymbol{ heta}_i$	$d_i$	$a_i$	$\alpha_i$
1	25	0	3	0
2	40	0	4	0
3	0	0	0	0

$${}^{0}p = \begin{pmatrix} 0.906 & -0.423 & 0 & 2.718 \\ 0.423 & 0.906 & 0 & 1.268 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0.766 & -0.643 & 0 & 3.064 \\ 0.643 & 0.766 & 0 & 2.571 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$${}^{0}p = \begin{pmatrix} 4.409 \\ 4.893 \\ 0 \\ 1 \end{pmatrix}$$



## **Using RVC toolbox for transformation matrix**

$$^{i-1}T_i = R_{(i-1)}(\theta_i).Q_{(i-1)}(d_i).Q_i(a_i).R_i(\alpha_i)$$

- Rotation matrix in 3D: rotx(), roty(), and rotz() for x, y z axis respectively
- Homogeneous transformation (HT) matrix for rotation: trotz() for z axis, and trotx() for x axis
- HT matrix for translation: transl([x y z])



## **Quiz: Using RVC toolbox for transformation matrix**

What is the transformation matrix  $\binom{i-1}{i}$  between frame  $\{i-1\}$  and  $\{i\}$ ?

$$^{i-1}T_i = R_{(i-1)}(\theta_i).Q_{(i-1)}(d_i).Q_i(a_i).R_i(\alpha_i)$$

Hint: Trotx, transl

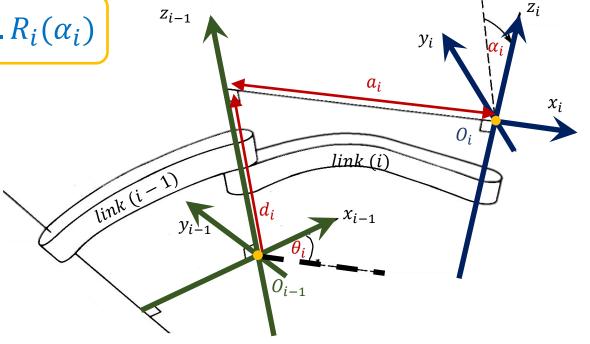
#### **Answer**

startup\_rvc

%%% Convert degree to radian %%%

theta\_i = deg2rad(theta\_degree); alpha\_i = deg2rad(alpha\_degree);

$$^{i-1}T_i = \text{trotz}(\text{theta_i})*\text{transl}(\text{d_i})*\text{transl}(\text{a_i})*\text{trotx}(\text{alpha_i})$$





## **Using RVC toolbox in Example 1**

```
startup_rvc
%%%% Convert degree to radian %%%%
theta = deg2rad([25 40 0]);

%%%% Create three link of the robot %%%

L(1) = Link('revolute', 'd', d1, 'a', a1, 'alpha', alpha1, 'offset', 0); % Link 1.
L(2) = Link('revolute', 'd', d2, 'a', a2, 'alpha', alpha2, 'offset', 0); % Link 2.
L(3) = Link('revolute', 'd', d3, 'a', a3, 'alpha', alpha3, 'offset', 0); % Link 3.
```

i	$\boldsymbol{\theta_i}$	$d_i$	$a_i$	$\alpha_i$
1	25	0	3	0
2	40	0	4	0
3	0	0	0	0

%%%%% Connect all links in a series %%%%

robot= SerialLink(L, 'name', ' three link');

%%%%% Forward kinematic %%%%

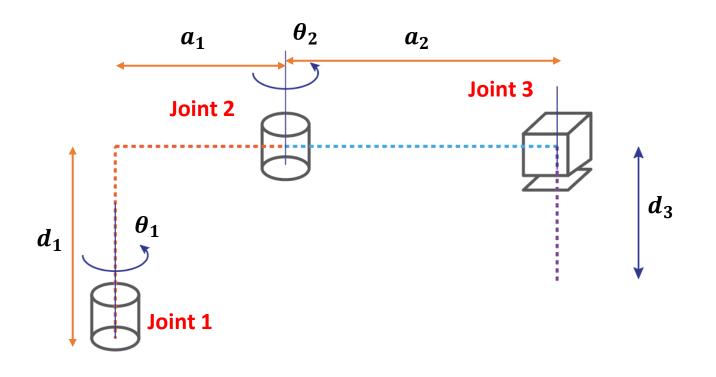
Matrix= robot.fkine([theta]);

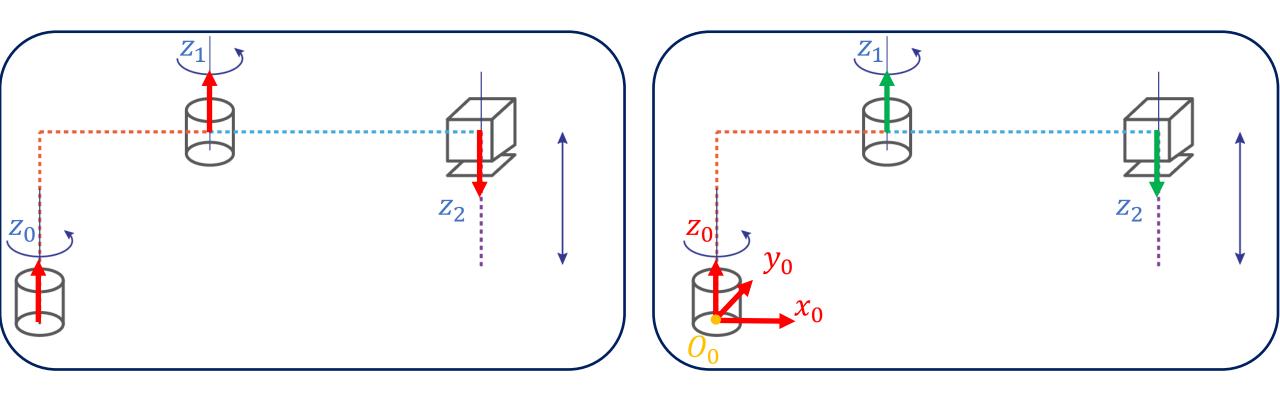


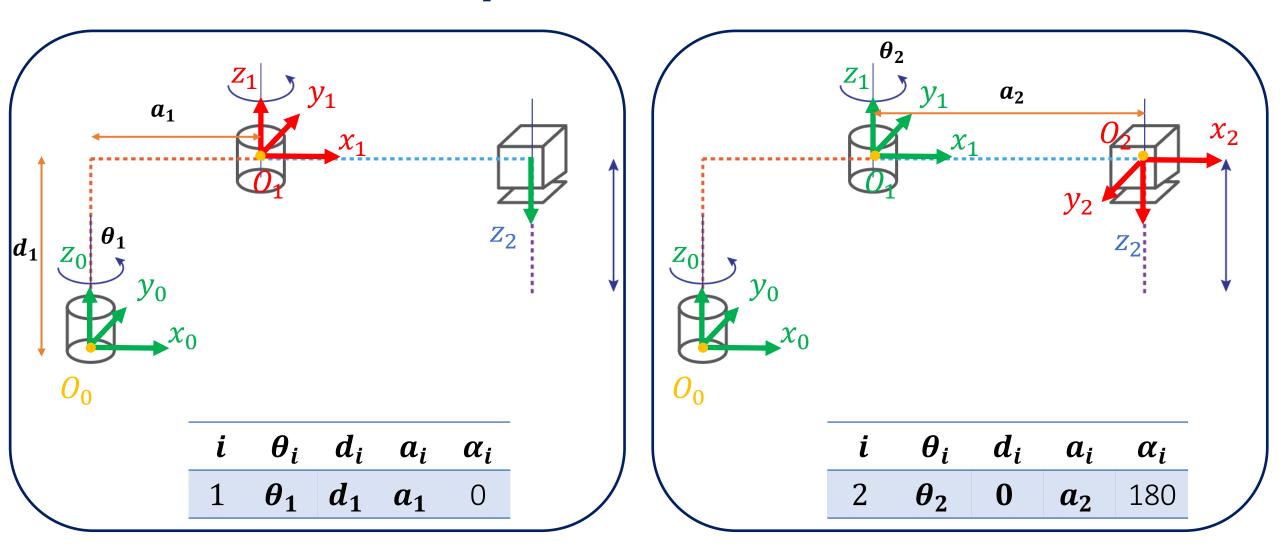
(set  $\theta_i$ =0 if there is no offset angle at the home position)

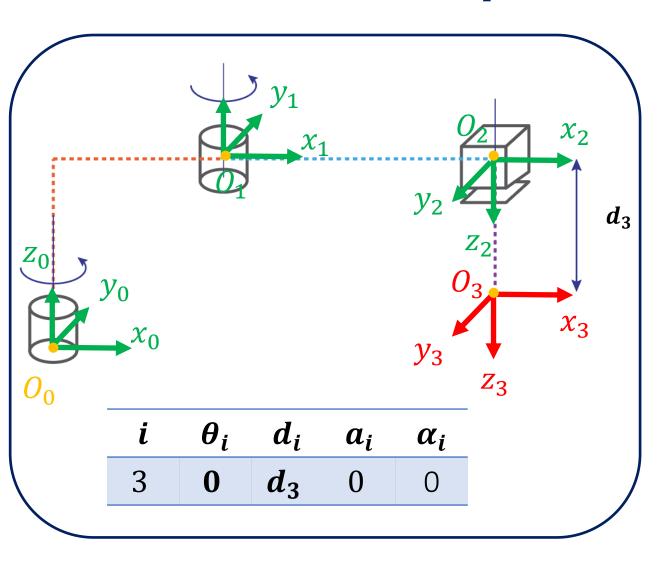












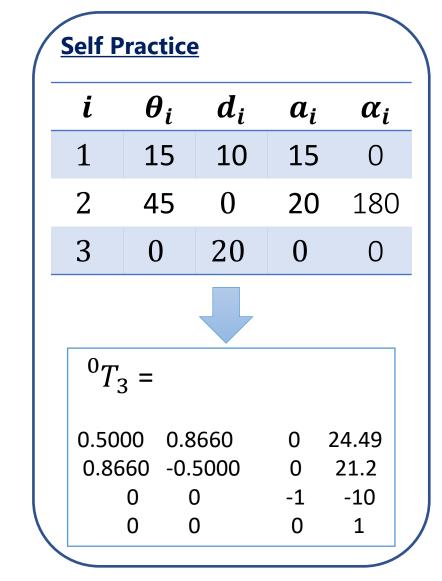
i	$oldsymbol{ heta_i}$	$d_i$	$a_i$	$\alpha_i$
1	$oldsymbol{ heta}_1$	$d_1$	$a_1$	0
2	$oldsymbol{ heta}_2$	0	$a_2$	180
3	0	$d_3$	0	0

#### **Example 2: SCARA Robot**

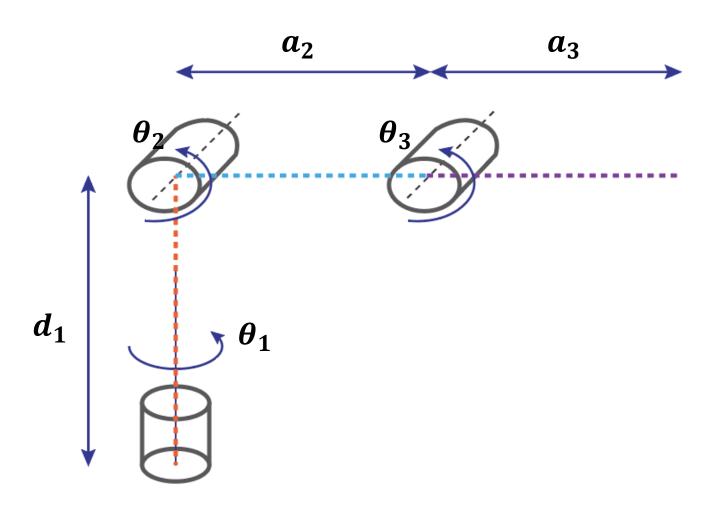
i	$\boldsymbol{\theta_i}$	$d_i$	$a_i$	$\alpha_i$
1	$oldsymbol{ heta_1}$	$d_1$	$a_1$	0
2	$oldsymbol{ heta}_2$	0	$a_2$	180
3	0	$d_3$	0	0

#### ☐ Hint using RVC

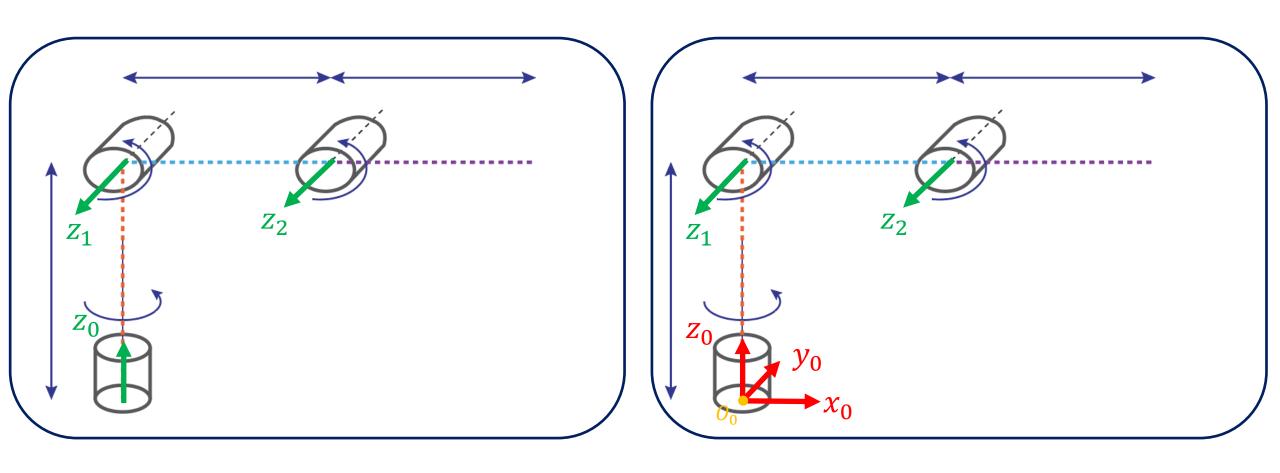
- (i) Use the 'link' function as the previous example
- (ii) robot= SerialLink(L, 'name', 'SCARA');
- (iii) f= robot.fkine([theta1 theta2 0]); % theta3=0 for revolute joint

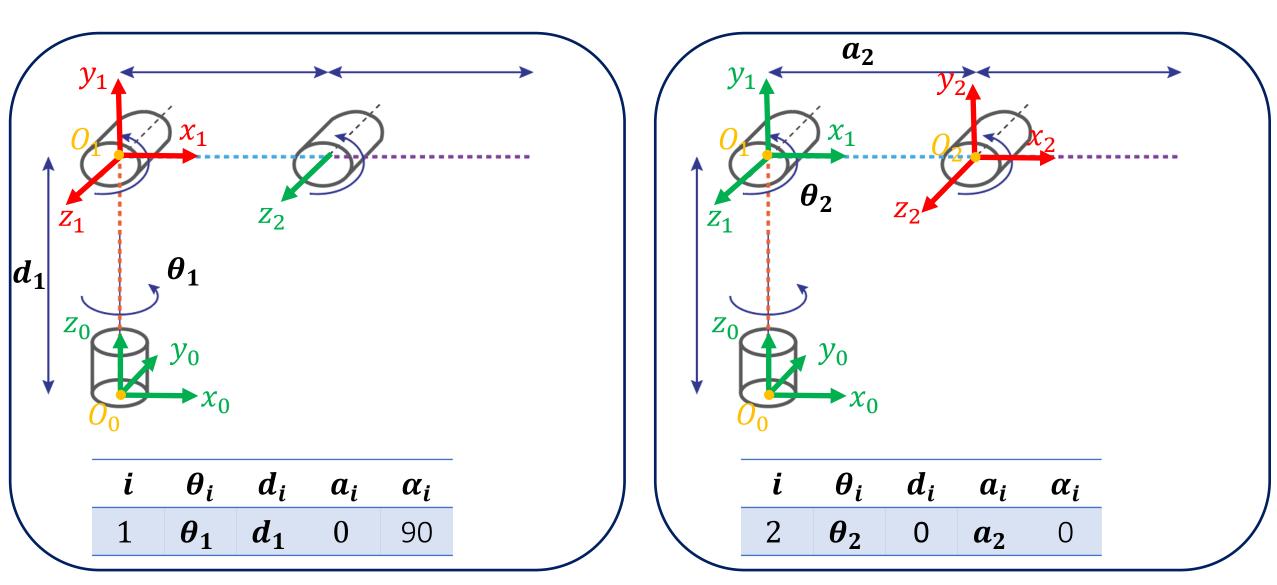


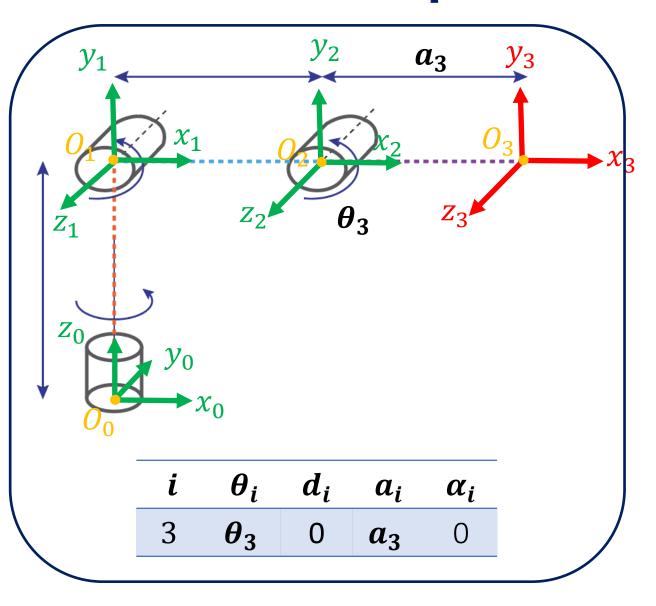










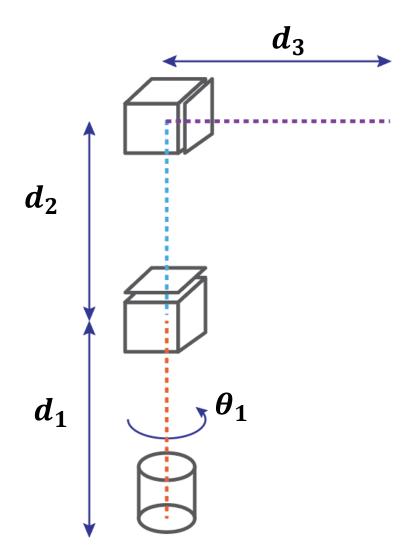


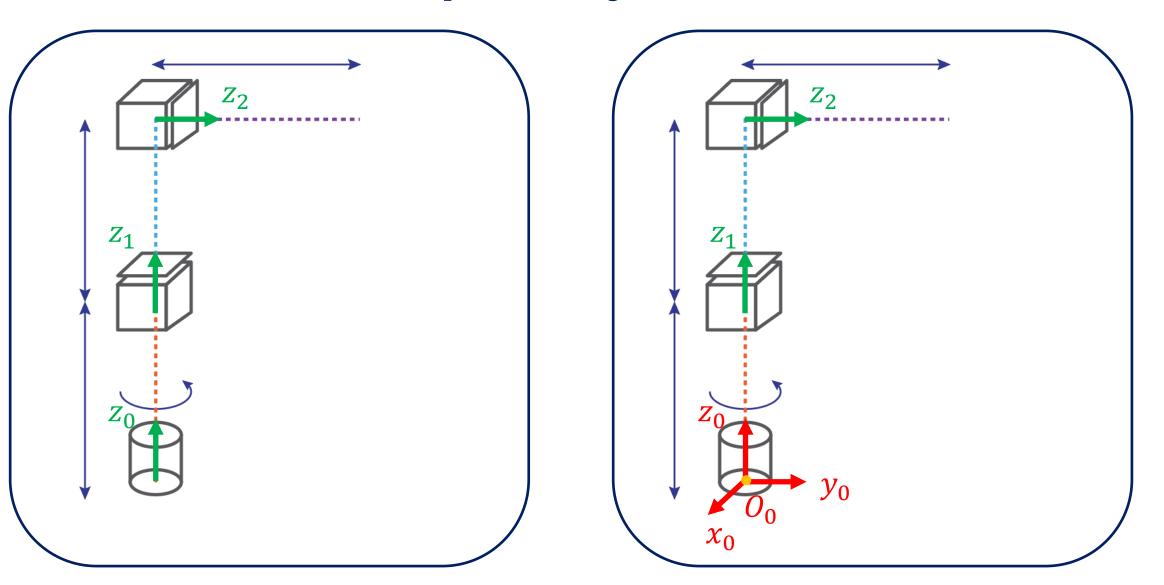
i	$oldsymbol{ heta_i}$	$d_i$	$a_i$	$\alpha_i$
1	$oldsymbol{ heta}_1$	$d_1$	0	90
2	$oldsymbol{ heta}_2$	0	$a_2$	0
3	$\boldsymbol{\theta}_3$	0	$a_3$	0

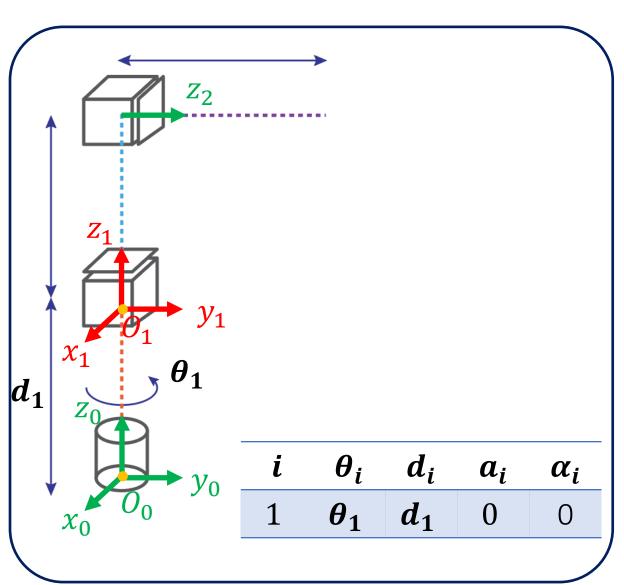
- ☐ Hint using RVC
- (i) Use the 'link' function as the previous example
- (ii) robot= SerialLink(L, 'name', 'Articulated');
- (iii) f= robot.fkine([theta1 theta2 theta3]);

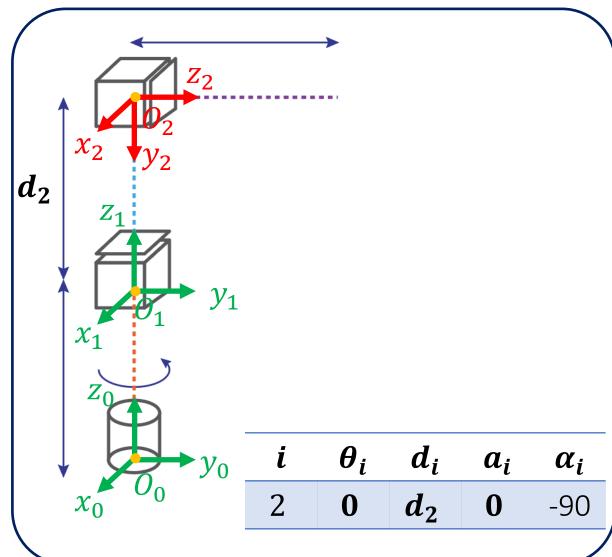
i	$oldsymbol{ heta_i}$	$d_i$	$a_i$	$\alpha_i$	${}^{0}T_{3} =$	
	10	32	0	90	-0.4162 -0.8925	0.1736 0.30
	50	0	11	0		0.9848 0.05
	65	0	16	0	0.9063 -0.4226 0 0	0 54.9 0 1

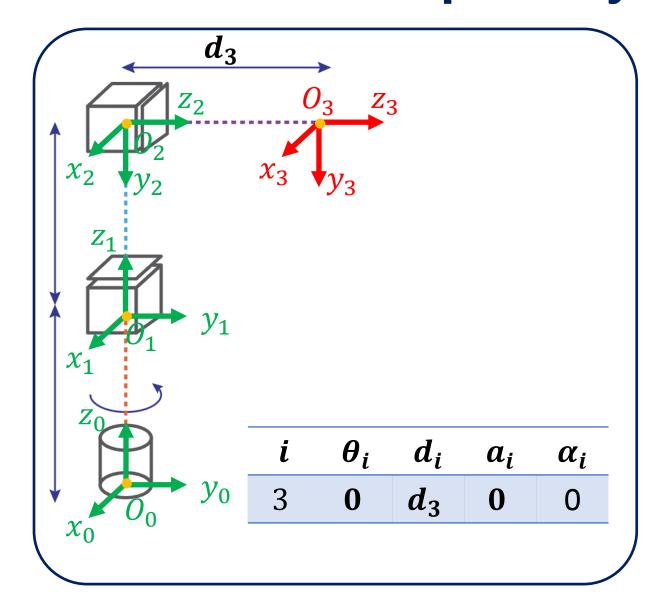












#### All combined

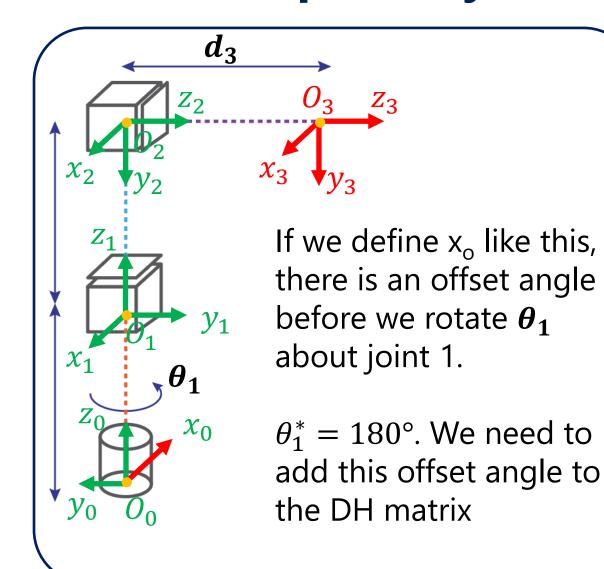
i	$oldsymbol{ heta_i}$	$d_i$	$a_i$	$\alpha_i$
1	$\theta_1$	$d_1$	0	0
2	0	$d_2$	0	-90
3	0	$d_3$	0	0

$${}^{0}T_{3} = {}^{0}T_{1}{}^{1}T_{2}{}^{2}T_{3}$$

- ☐ Hint using RVC
- (i) Use the 'link' function as the previous example
- (ii) robot= SerialLink(L, 'name', 'cylindrical');
- (iii) f= robot.fkine([theta1 theta2 theta3]);

Exar	nple $ heta$	<sub>1</sub> = 6	8°		0			
i	$oldsymbol{ heta_i}$	$d_i$	$a_i$	$\alpha_i$	$^{0}T_{3} =$			
1	68	8	0	0	0.3746		-0.9272	
2	0	6	0	-90	0.9272	0 -1	0.3746 0	2.997 14
3	0	8	0	0	0	0	0	1

#### **Example 5: Cylindrical robot-offset angle**



i	$oldsymbol{ heta}_i$	$d_i$	$a_i$	$lpha_i$
1	$\theta_1 + 180$	$d_1$	0	0
2	0	$d_2$	0	-90
3	0	$d_3$	0	0

Example  $\theta_1 = 68^{\circ}$ 

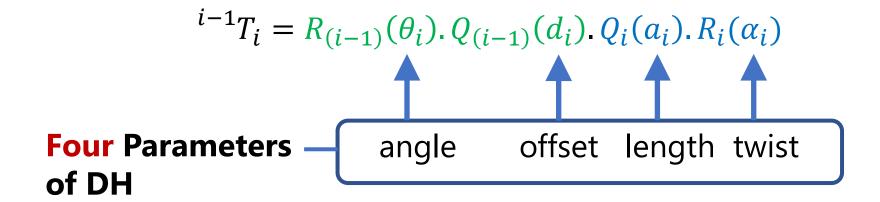
$${}^{0}T_{3} = {}^{-0.3746} {}^{0} {}^{0.9272} {}^{7.417} {}^{0}$$
 ${}^{0}T_{3} = {}^{-0.9272} {}^{0} {}^{-0.3746} {}^{-2.997} {}^{-1} {}^{0} {}^{0} {}^{14} {}^{0}$ 



#### **Lecture 3: Summary**

i	$oldsymbol{ heta}_i$	$d_i$	$a_i$	$\alpha_i$
0	0	0	0	0
1	$ heta_1$	$d_1$	$a_1$	$lpha_1$
2	$\theta_2$	$d_2$	$a_2$	$\alpha_2$
3	$ heta_3$	$d_3$	$a_3$	$\alpha_3$

RVC Toolbox is very useful to build the robot & calculate the matrix!





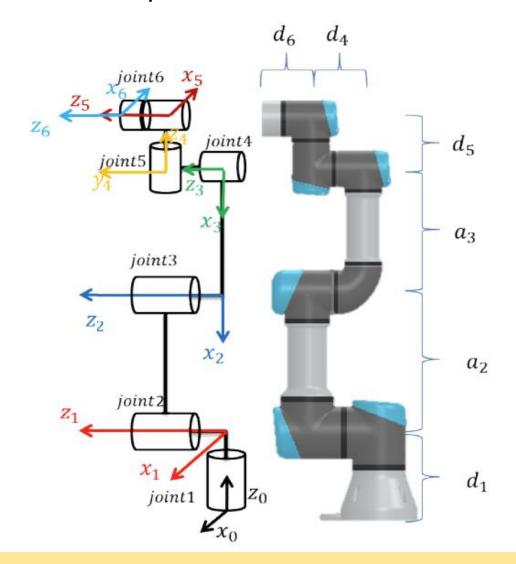
#### **Next week**

- ☐ Lecture 5: Inverse Kinematics and The Jacobian
- ☐ Marking ROBOT2 during lab sessions
- □ Quiz1 (70 minutes on Moodle Monday 26<sup>th</sup> from 16pm)
  - Time: 70 minutes on Moodle. Please start before 16:15pm
  - From weeks 1 to 4 (except slides 52-59 in Lecture 3)
  - Seven questions, including multiple choice and calculation questions
  - Can use Matlab for your calculation (e.g., RVC Toolbox)
  - Write your answer with 4 decimal places



## **Self practice**

Find the DH parameters for UR5e (forward kinematics of MTRN4230's robots)



UR5e							
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]			
Joint 1	0	0	0.1625	π/2			
Joint 2	0	-0.425	0	0			
Joint 3	0	-0.3922	0	0			
Joint 4	0	0	0.1333	π/2			
Joint 5	0	0	0.0997	-π/2			
Joint 6	0	0	0.0996	0			

