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Lecture 7 - Revision

https://kahoot.it/

✓ Trajectory generation

- Axis to axis movement
- Simultaneous movement
- Coordinated path
- Continuous straight path

✓ Point to point trajectory

- Quintic polynomial
- Trapezoidal (LSPB)
- Bang-bang trajectory
- Trajectory with via points

✓ Manipulability

- Near singularities $\det J(\mathbf{q}) = 0$: Poorly conditioned

❑ $m = \sqrt{\det(J(\mathbf{q})J(\mathbf{q})^T)}$

MTRN4230
Robotics

Lecture 8

Robotic Path Planning

Hoang-Phuong **Phan** – T2 2023

Learning Objectives

- Introduction to path planning – key definitions
  - Workspace
  - Configuration space

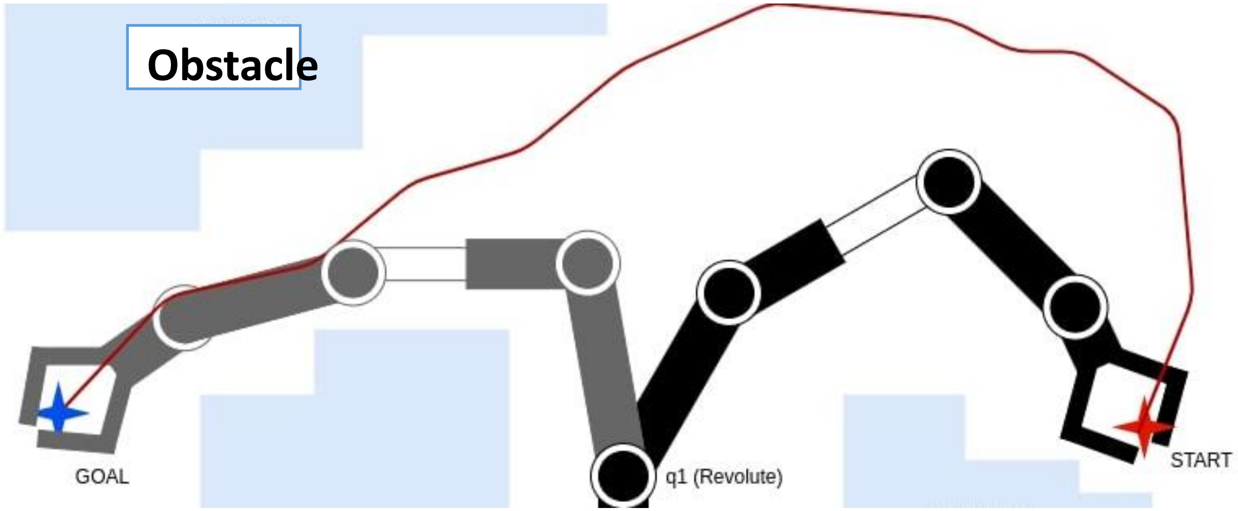
- Map based method
  - Reactive planning
  - D* Method
  - Probabilistic Roadmap method (PRM)
  - Others: Voronoi roadmap, Rapidly-exploring Random Tree (RRT)

- Artificial Potential Field (APF) method

I. Introduction to Path Planning

Introduction to Path Planning

- Last lecture: Trajectory Generation
  - How to control the end effector to pass through a sequence of configurations.
- Path Planning
  - How to generate that sequence of configurations.
  - Configuration space where obstacle exist



Configuration Space

- Configuration space is the set of positions reachable by an end-effector in 3D space, often denoted $SE(3)$.

- The set of joint parameter values is called the **joint space** (a convenient method to represent the Configuration Space - ***C***).

- The robot's forward & inverse kinematics equations define mappings between Workspace and Joint Space.

- Path planning uses these mappings to find a path in joint space that provides a desired path in the workspace of the end-effector.

Configuration Space

- If the robot is a fixed-base serial manipulator with N revolute joints, C is N-dimensional.

- The set of configurations that avoids collision with obstacles or itself is called the free space C_{free}.

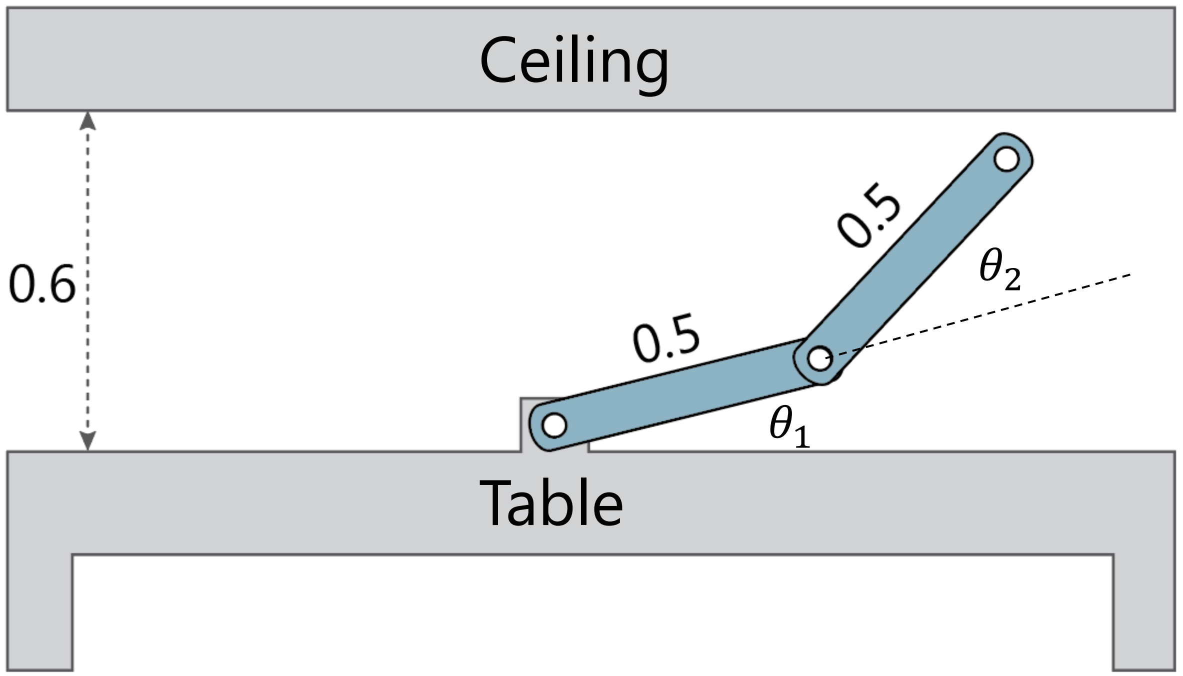
- The complement of C_{free} in C is called the obstacle region, C_{obst}.

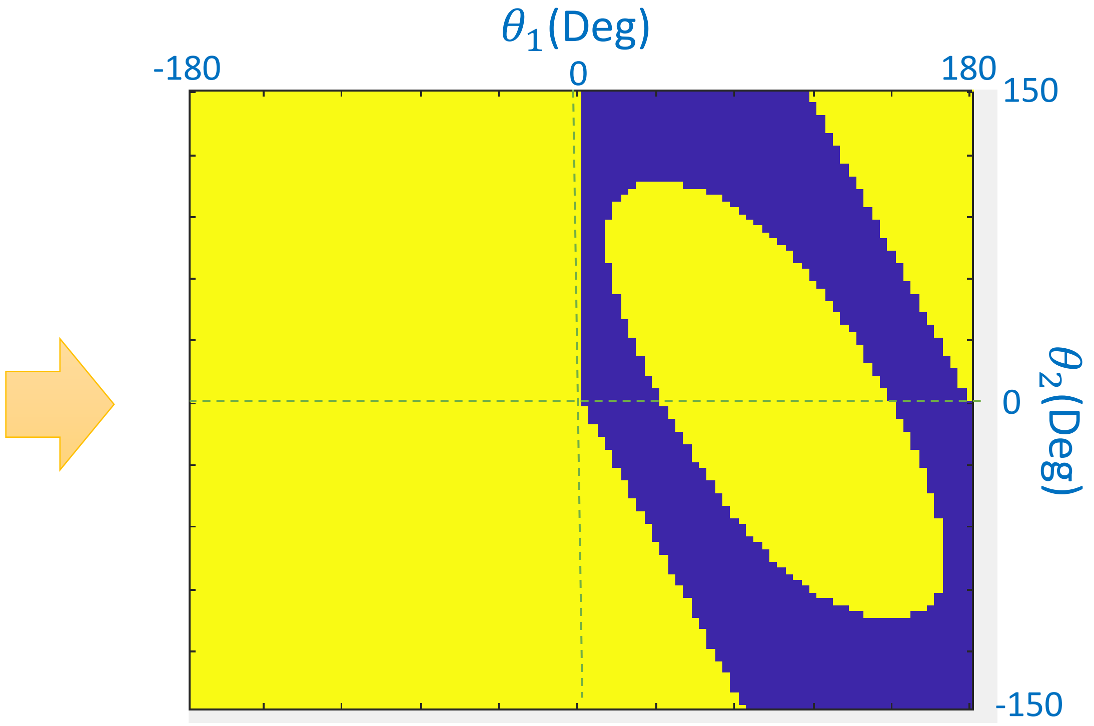
Configuration Space

- Often, it is prohibitively difficult to explicitly compute the shape of C_{free}.
  - However, testing whether a given configuration is in C_{free} is efficient.
  - First, forward kinematics determine the position of the robot's geometry. Then, collision detection tests if the robot's geometry collides with the environment's geometry.

Example: Configuration Space

- The robot is a two-link planar arm, and the workspace contains a table





<u>Workspace</u>

Workspace: the Cartesian space in which the robot moves

<u>Configuration space (See Lab 8)</u>

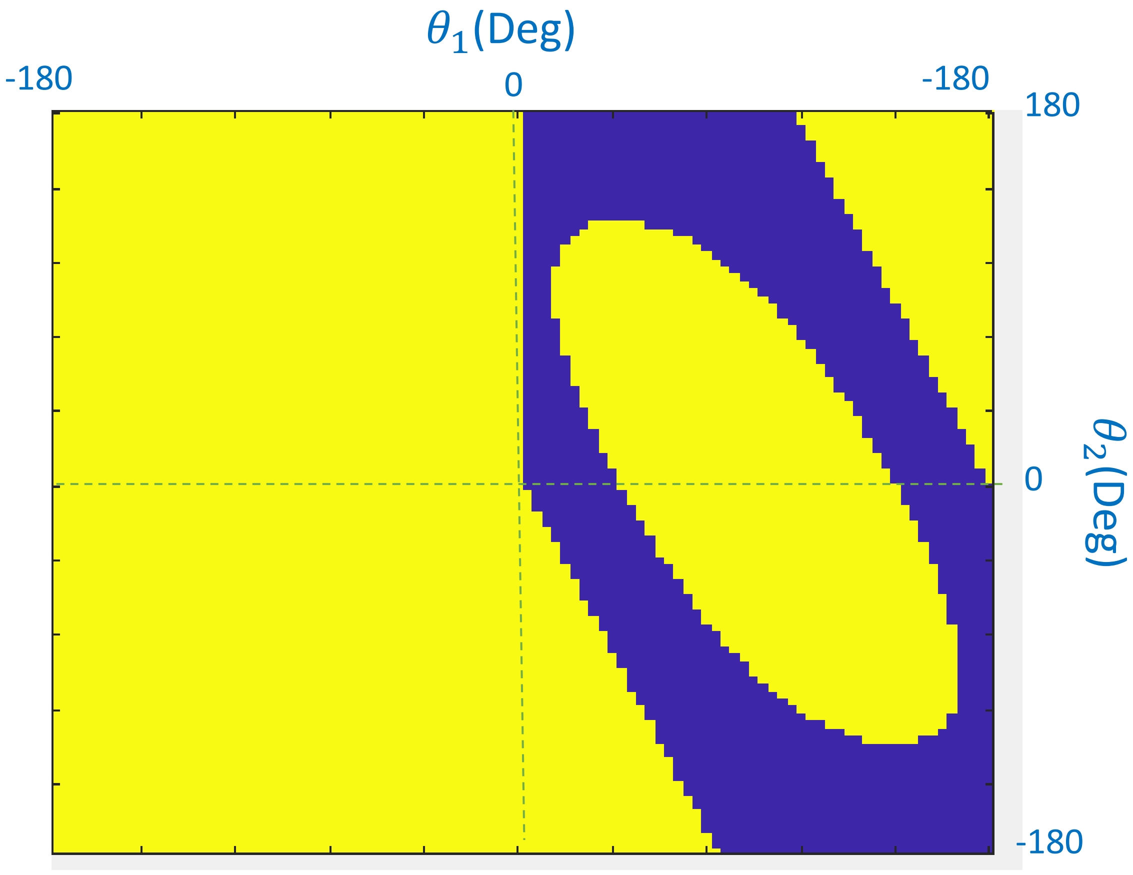
Vector of joint variables provides a convenient representation of a configuration

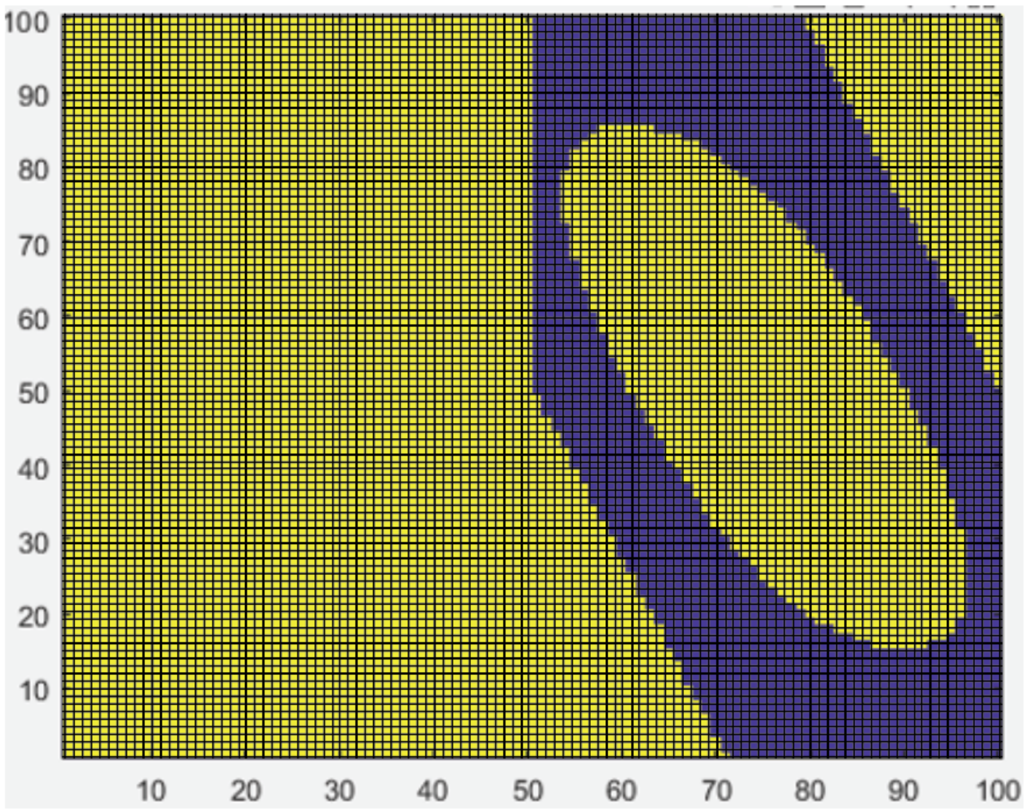
Source: Spong, Mark W., Seth Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley, 2005.

II. Grid Based Methods

Grid Based Methods

- Overlay a grid on configuration space, and assume each configuration is identified with a grid point.

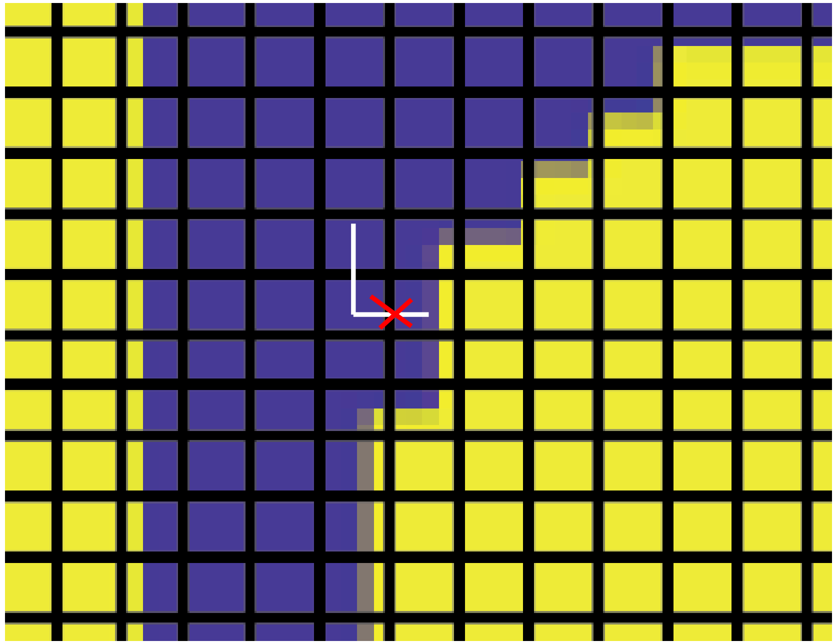


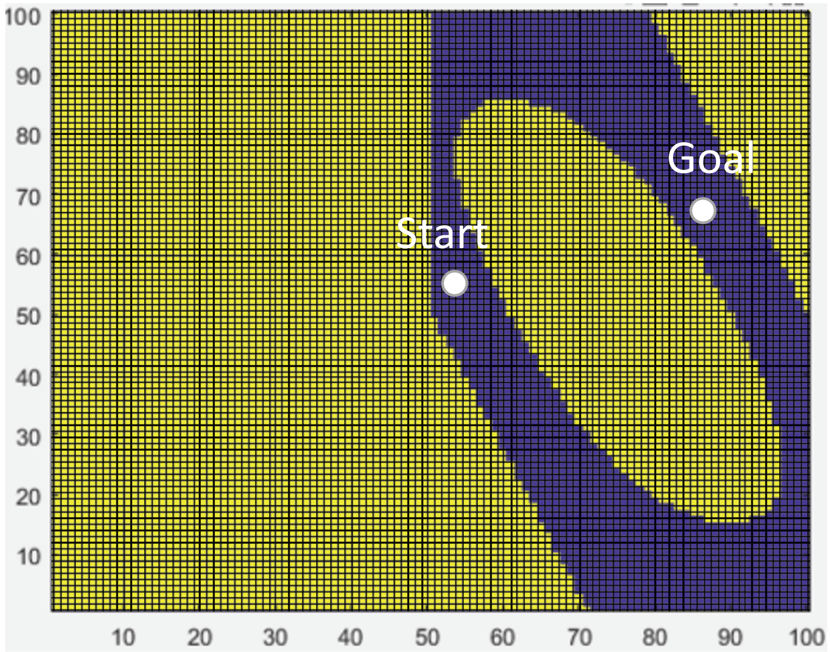


Grid Based Methods

- At each grid point, the robot is allowed to move to adjacent grid points as long as the line between them is completely contained within C_{free} (this is tested with collision detection).

- This discretizes the set of actions, and search algorithms (like D*) are used to find a path from the start to the goal.





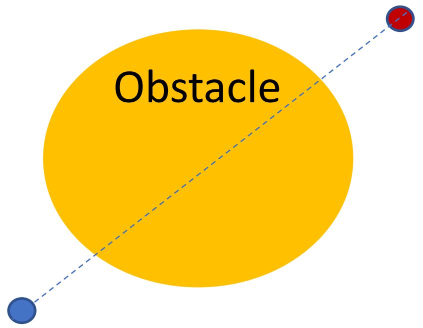
Grid Based Methods

- Reactive planning
- D* Method
- Probabilistic Roadmap method (PRM)

Reactive Planning

❑ Example: Bug2 Algorithm

- Moves in straight line towards GOAL
- Moves counter-clockwise around an obstacle
- Repeat until it encounter a point that lies along its original line that is closer to GOAL



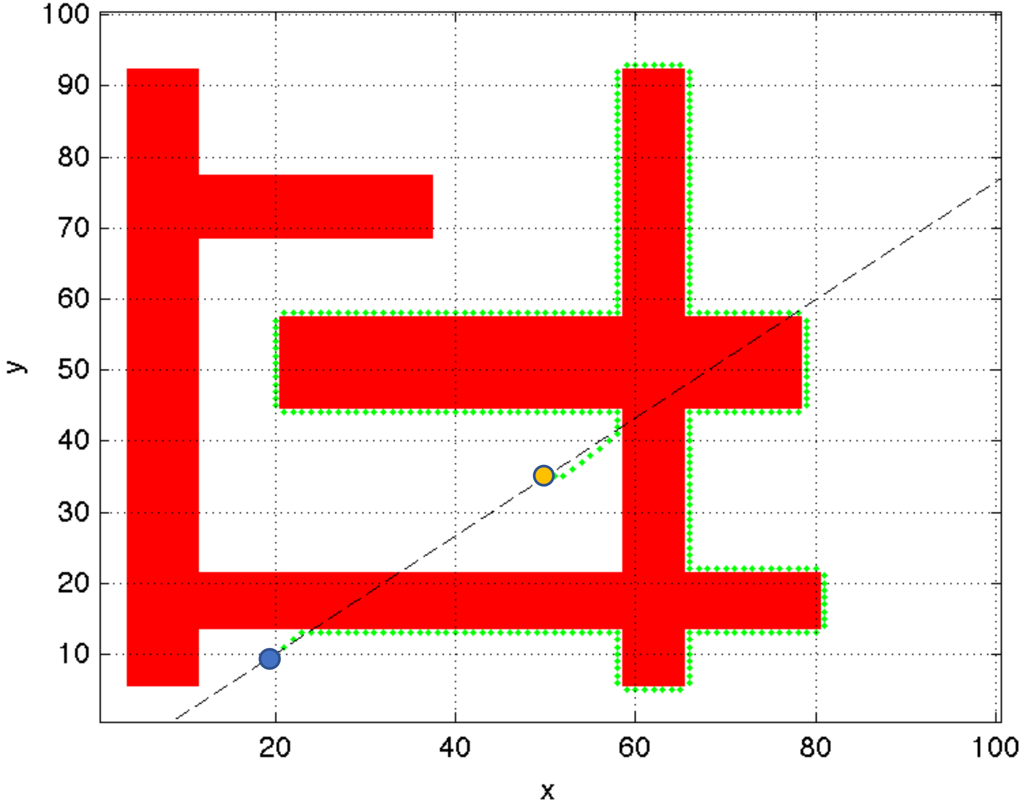
Reactive Planning – Bug2 Algorithm

- In the RVC toolbox

```
bug = Bug2(map) % planning phase

bug.goal = [x; y]; % set the GOAL

p = bug.query([START]); % query phase

dx.plot(p)  % Visualise the path
```

Use makemap(dimension) to create/edit the map

Reactive Planning – Bug2 Algorithm



Not globally optimal!

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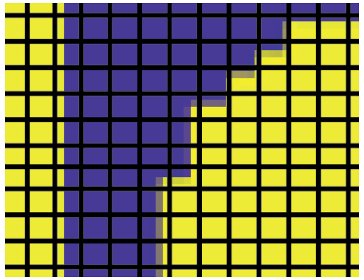
D* Algorithm

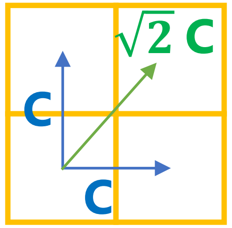
- ❑ A traversal cost is defined for each cell
  - In horizontal or vertical direction: C
  - In diagonal direction: $\sqrt{2}$ C
  - ∞ cost at obstacles

- ❑ This cost can be: distance (travel time or distance), the roughness of the terrain

- ❑ The graph search minimises the cost of traversal (optimal)



Grid



Cost

Search optimal path

Relatively analogous

Sources: P. Corke, *Robotics, Vision and Control: Fundamental Algorithms In MATLAB® Second, Completely Revised, Extended And Updated Edition*. Springer, 2017.
https://www.youtube.com/watch?v=-L-WgKMFuhE&ab_channel=SebastianLague

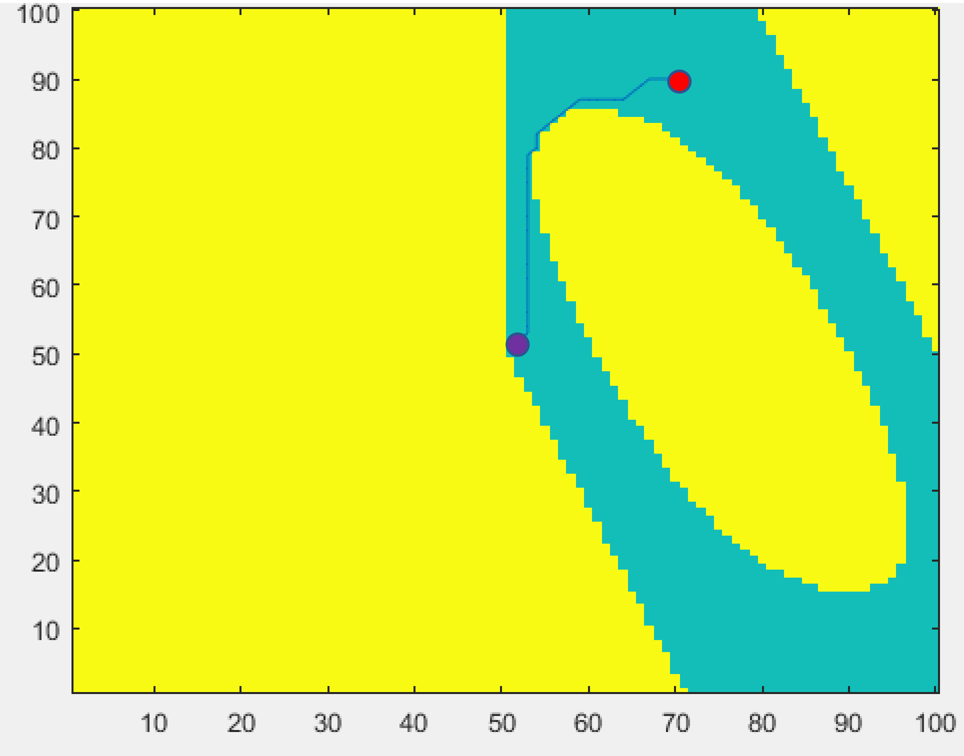
D* Algorithm

❑ In the toolbox

- ds = Dstar(map) % Create a navigation object
- c = ds.costmap() % Convert Grid map into a cost map
- ds.plan(goal) % Creates a very dense directed graph

Each graph vertex has a cost, a distance to GOAL, and a link to the neighbouring cell closest to GOAL

- ds.query(start) % Query phase



P. Corke, *Robotics, Vision and Control: Fundamental Algorithms In MATLAB® Second, Completely Revised, Extended And Updated Edition*. Springer, 2017.

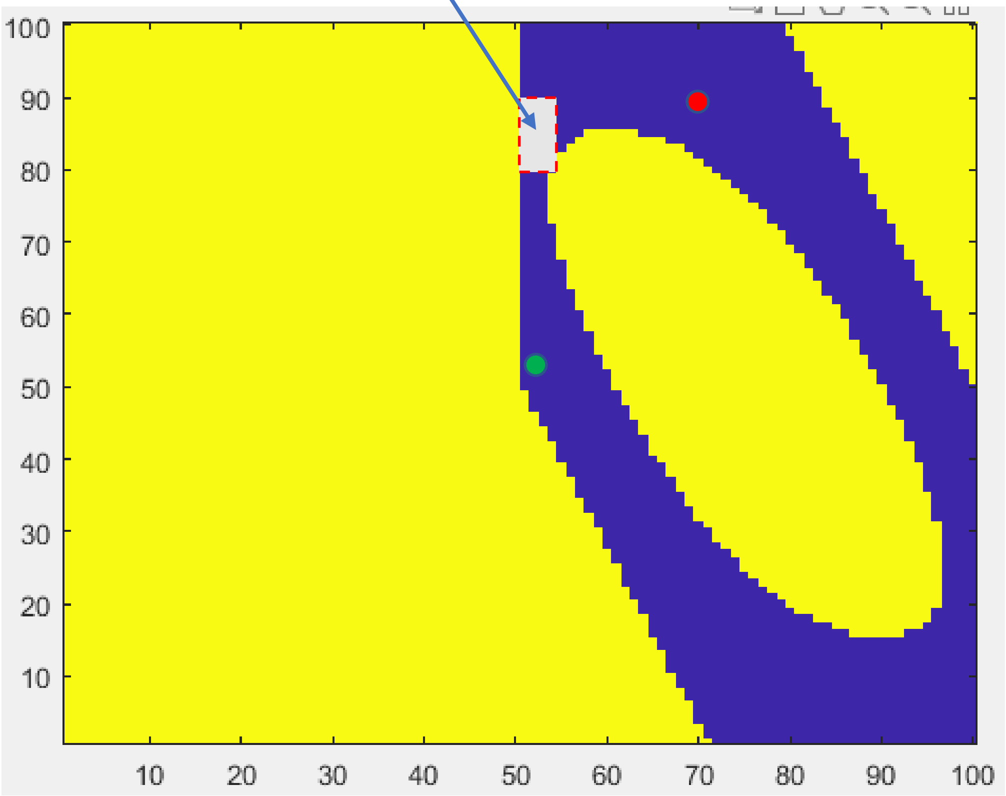
D* Algorithm

- The real power of D*: efficiently change the cost map during the mission.

```
ds.modify_cost( [50,55; 80,99], 100 );
% Modify cost
ds.plan();  % Replan
ds_path1 = ds.query(start_cell); % Search
```

- No need to change the whole graph (incremental replanning)

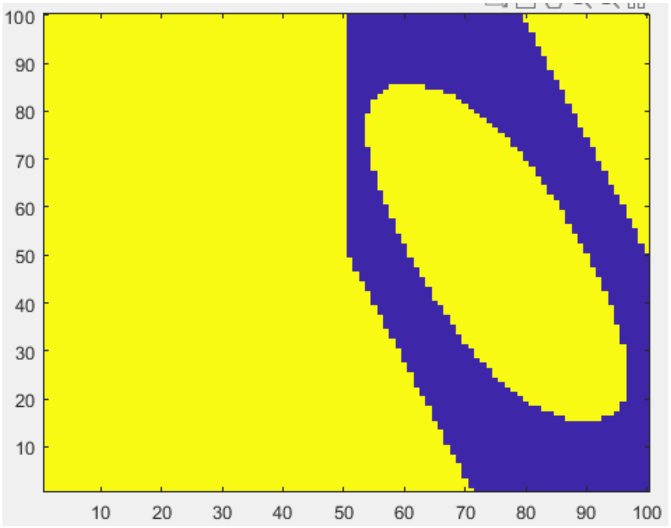
Modify cost in this area



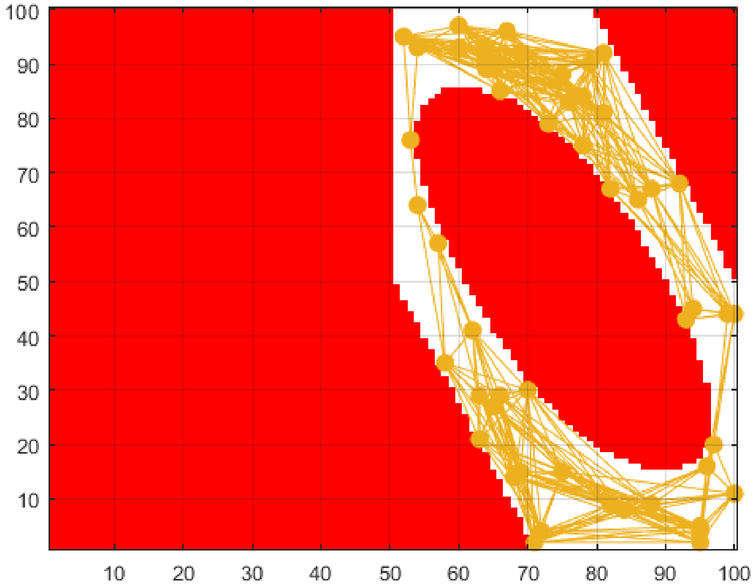
P. Corke, *Robotics, Vision and Control: Fundamental Algorithms In MATLAB® Second, Completely Revised, Extended And Updated Edition*. Springer, 2017.

Probabilistic Roadmap Method (PRM)

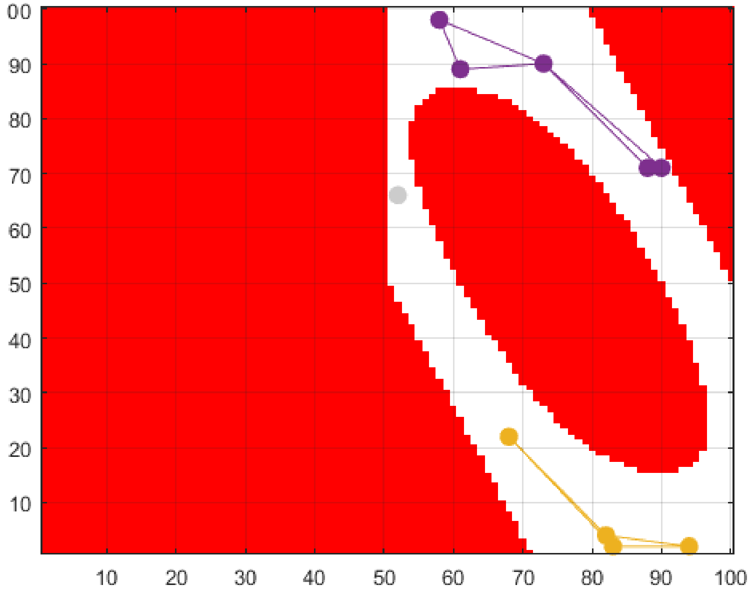
- ❑ The plan is independent of the goal
- ❑ Planning phase finds N random points
- ❑ Each point is connected to its nearest neighbours (distance threshold parameter of) by a straight-line path that does not cross any obstacles



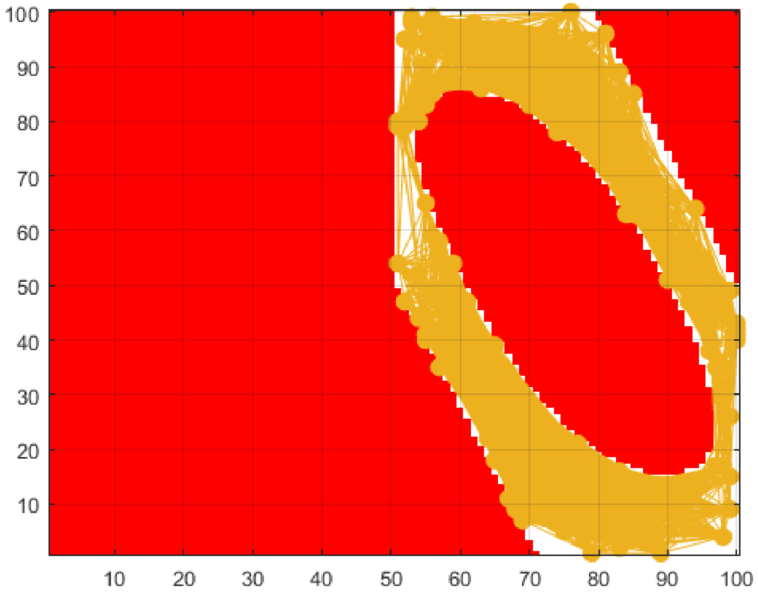
50 sampling points



10 sampling points



200 sampling points



Probabilistic Roadmap method (PRM)

- Toolbox PRM planner

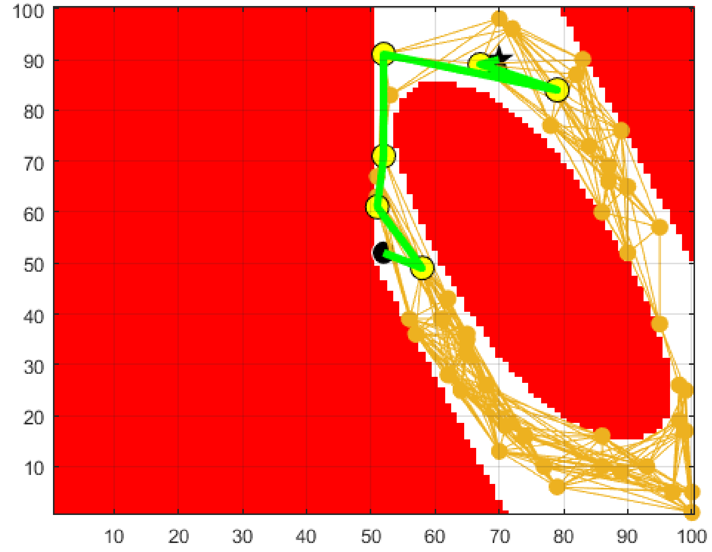
    prm = PRM(map) % Create PRM object

    prm.plan()   % Create Plan

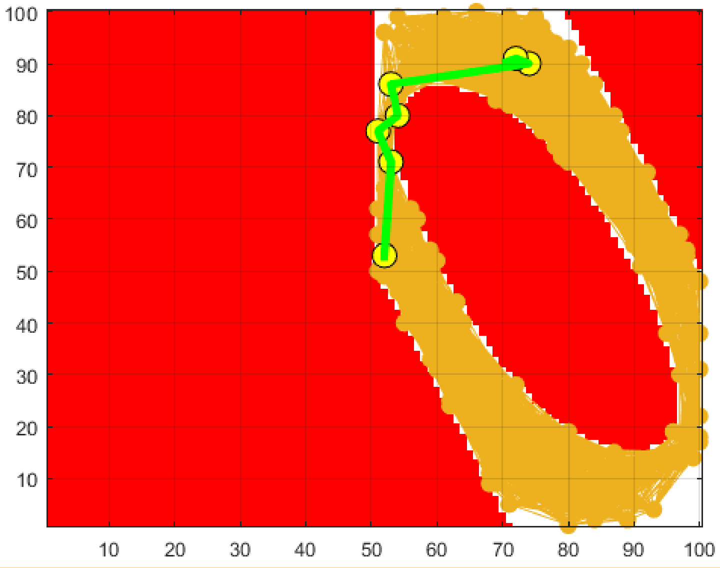
    prm.plot() % Visualize graph

    prm.query(start, goal) % Query phase:

Moving to the closest node in the network then follow the existing path



50 points



200 points

III. Artificial Potential Fields

Artificial Potential Fields (APFs)

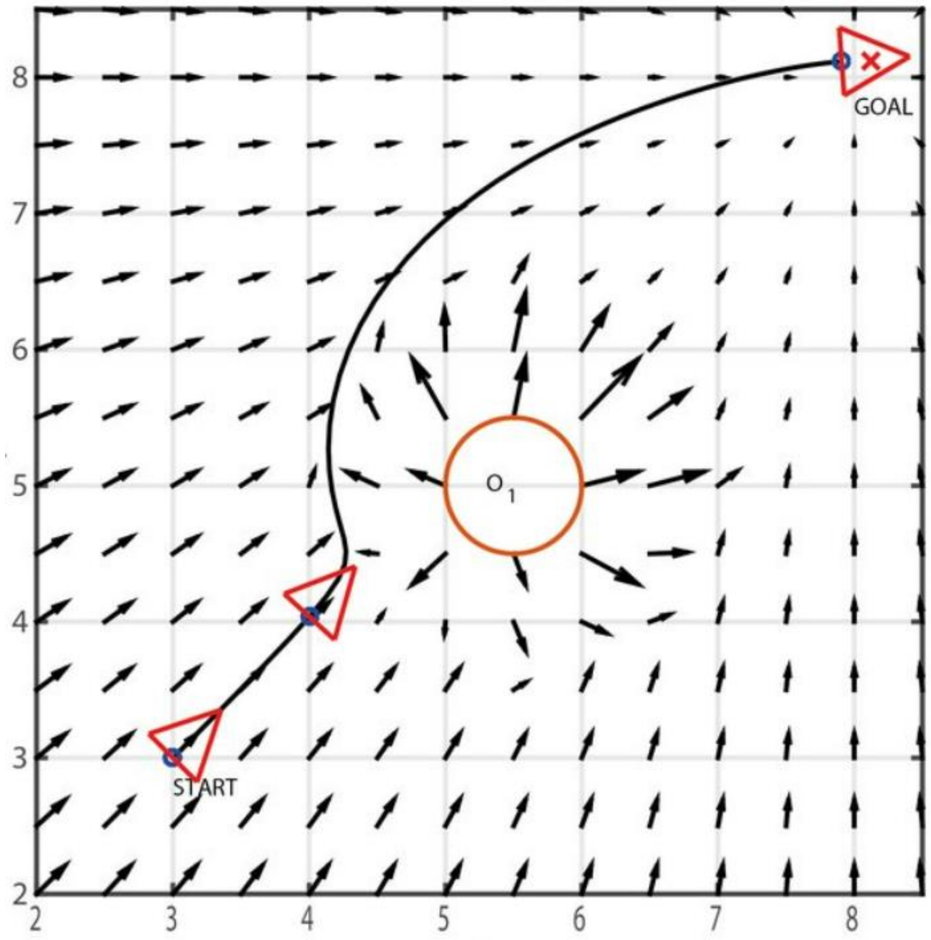
- Treat the robot's configuration as a point in a workspace potential field that combines attraction (to the goal) and repulsion (from obstacles) forces.

$$U(q) = U_{att}(q) + U_{rep}(q)$$

- Use **gradient descent** on the field to define a path (i.e., the negative gradient of *U* can be considered as a force acting on the robot)

$$F = -\nabla U(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q)$$

Artificial Potential Fields (APFs)



Gradient descent

Source: Obstacles avoidance based on switching potential functions. Journal of Intelligent & Robotic Systems, 90(3), pp.387-405.

Attractive Potential

- $U_{att,i}(q)$ should be monotonically increasing with distance from q_f
- The simplest choice is a field that grows linearly with the distance from q_f(i.e., **Conic Well Potential**)

$$\boxed{U_{att,i}(q) = \zeta_i \parallel o_i(q) - o_i(q_f) \parallel}$$

where ζ_i is used as a scaling factor. Denote the position of the origin of the i^{th} DH frame by $o_i(q)$ and $o_i(q_f)$ as the goal configuration.

- The gradient has unit magnitude everywhere but the origin, where it is zero. This can lead to stability problems

$$\boxed{F_{att}(q) = -\nabla U_{att}(q) = -\zeta_i}$$

Attractive Potential

- A quadratic field (so as to be continuously differentiable), called the **Parabolic Well Potential** is defined as,

$$U_{att,i}(q) = \frac{1}{2}\zeta_i \parallel o_i(q) - o_i(q_f) \parallel^2$$

ζ_i : a scaling factor.

- The attractive force for o_i is equal to its negative gradient,

$$F_{att,i}(q) = -\nabla U_{att,i}(q) = -\zeta_i(o_i(q) - o_i(q_f))$$

- At long distances, a conic potential can be used instead of the quadratic field to avoid a too large attractive force

Source: Spong et al. Robot Modeling and Control, 2005

Attractive Potential

- A combination of **Parabolic Well Potential** and **Conic Well Potential**

$$U_{\mathrm{att},i}(q) = \begin{cases} \frac{1}{2}\zeta_i \|o_i(q) - o_i(q_{\mathrm{f}})\|^2 & : \quad \|o_i(q) - o_i(q_{\mathrm{f}})\| \le d \\ d\zeta_i \|o_i(q) - o_i(q_{\mathrm{f}})\| - \dfrac{1}{2}\zeta_i d^2 & : \quad \|o_i(q) - o_i(q_{\mathrm{f}})\| > d \end{cases}$$

- The attractive force is

$$F_{\mathrm{att},i}(q) \quad = \quad \begin{cases} -\zeta_i (o_i(q) - o_i(q_{\mathrm{f}})) & : \quad \|o_i(q) - o_i(q_{\mathrm{f}})\| \le d \\ -d\zeta_i \dfrac{(o_i(q) - o_i(q_{\mathrm{f}}))}{\|o_i(q) - o_i(q_{\mathrm{f}})\|} & : \quad \|o_i(q) - o_i(q_{\mathrm{f}})\| > d \end{cases}$$

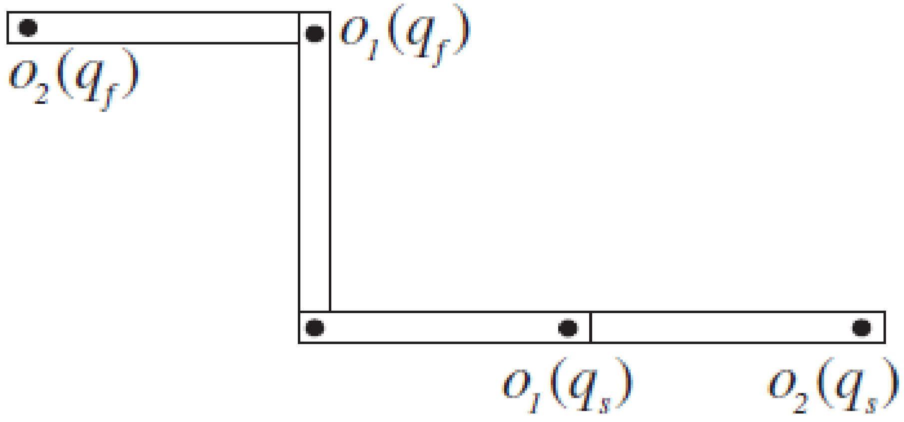
- The force is continuous at the boundary (*d*)

Source: Spong et al. Robot Modeling and Control, 2005

Example 1 - Attractive Potential for 2 Link Arm

Arm lengths are a1 = a2 = 1with initial and final configurations given by

$$q_s = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad q_f = \begin{bmatrix} \frac{\pi}{2} \\ \frac{\pi}{2} \end{bmatrix}$$



Calculate the attractive for if a parabolic well potential is used and the scaling factor ζ_1 and ζ_2 are 1 and 2 respectively.

Answer:

$$F_{att,i}(q) = -\nabla U_{att,i}(q) = -\zeta_i\big(o_i(q) - o_i(q_f)\big)$$

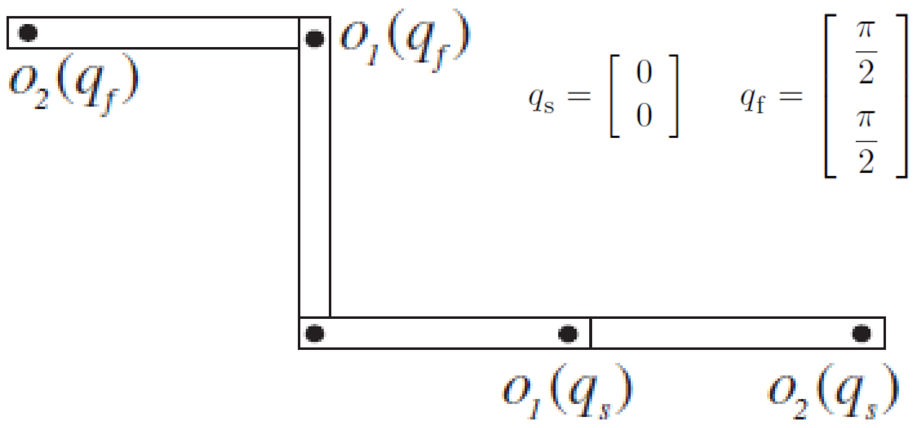
We need to find o_i(start) and o_i(final)

-> Use forward kinematics

Source: Spong et al. Robot Modeling and Control, 2005

Example 1 - Attractive Potential for 2 Link Arm

- Origin of the i^{th} DH frame by $o_i(q)$



$$o_1(q_s) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad o_1(q_f) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$o_2(q_s) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad o_2(q_f) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\boxed{F_{att,i}(q) = -\zeta_i\big(o_i(q) - o_i(q_f)\big)}$$

- Attractive force at O_1

$$F_{\text{att},1}(q_s) = -\zeta_1(o_1(q_s) - o_1(q_f)) = \zeta_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

- Attractive force at O_2

$$F_{\text{att},2}(q_s) = -\zeta_2(o_2(q_s) - o_2(q_f)) = \zeta_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Repulsive Potential

- ❑ Repel robot from nearby obstacle
- ❑ ρ_0 is the distance of influence of an obstacle
- ❑ $\rho(o_i(q))$ is the shortest distance between o_i and any workspace obstacle

$$U_{rep,i}(q) = \begin{cases} \frac{1}{2}\eta_i\left(\frac{1}{\rho\left(o_i(q)\right)} - \frac{1}{\rho_0}\right)^2 & ; \quad \rho\left(o_i(q)\right) \leq \rho_0 \\ 0 & ; \quad \rho\left(o_i(q)\right) > \rho_0 \end{cases}$$

- η_i is a scaling factor

Source: Spong et al. Robot Modeling and Control, 2005

Repulsive Potential

- Hence the repulsive force is,

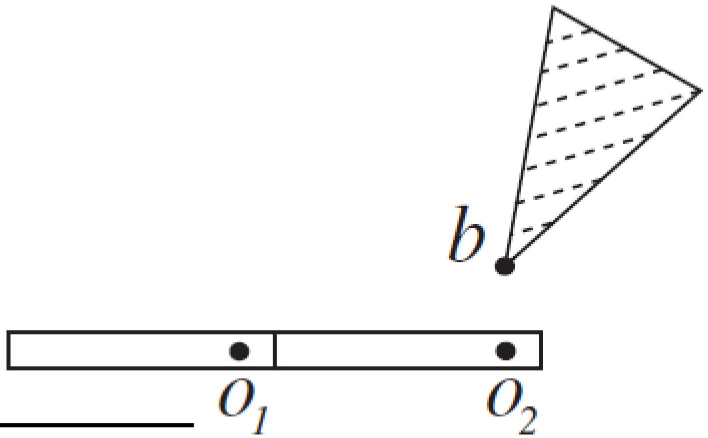
$$F_{rep,i}(q) = \eta_i \left(\frac{1}{\rho\big(o_i(q)\big)} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2\big(o_i(q)\big)} \nabla \rho\big(o_i(q)\big)$$

- Note the gradient is not always continuous midway between obstacles

- Add floating repulsive control points on each link at closest position to nearest obstacle to avoid any part of the link colliding

Example 2 - Repulsive Force of Two-Link Arm

The vertex b of the triangle obstacle has the coordinates (2; 0.5). The coordinate of O_2(start) is (2; 0).
If $\rho_0 = 1$ and the scaling factor is $\eta_2 = 3$, calculate the repulsive force at O_2(start).



Answer:

Let x and y be the coordinate of O_2 $\quad \rho(O_2) = \sqrt{(x-2)^2+(y-0.5)^2}$

Distance from O_2(start) to b is $\rho(o_2(q_s)) = 0.5$

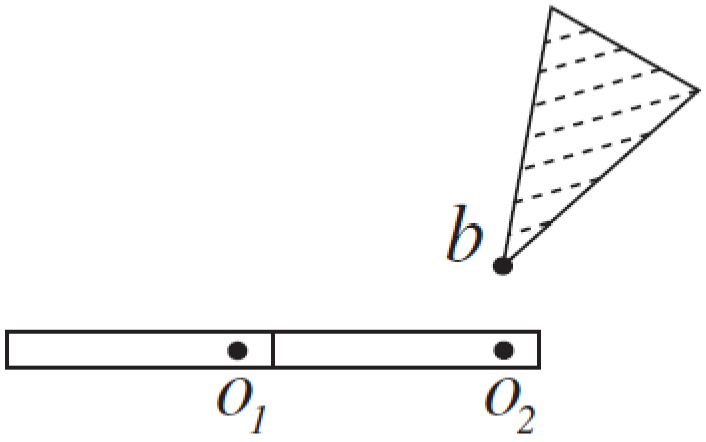
Gradient:

$$\nabla\rho(O_2) = \begin{pmatrix} \dfrac{x-2}{\sqrt{(x-2)^2+(y-0.5)^2}} \\ \dfrac{y-0.5}{\sqrt{(x-2)^2+(y-0.5)^2}} \end{pmatrix} \Rightarrow \nabla\rho(o_2(q_s)) = [0, -1]^T$$

Example 2 - Repulsive Force of Two-Link Arm

$$\rho(o_2(q_s)) = 0.5$$

$$\nabla\rho(o_2(q_s)) = [0, -1]^T$$



$$\boxed{F_{rep,i}(q) = \eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2(o_i(q))} \nabla\rho(o_i(q))}$$

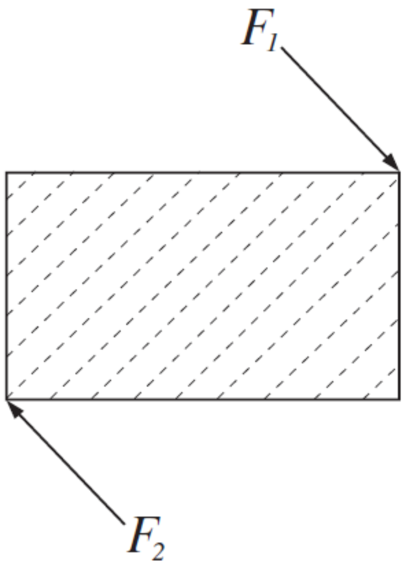
The repulsive force at $O_2(q_s)$ is

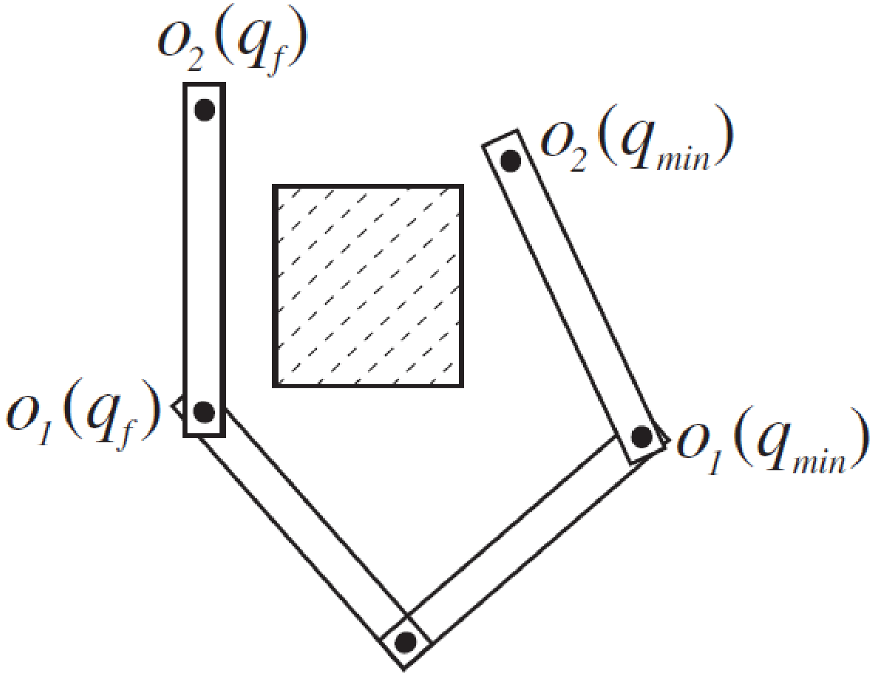
$$F_{\mathrm{rep},2}(q_s) = \eta_2 \left(\frac{1}{0.5} - 1 \right) \frac{1}{0.25} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \eta_2 \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Control with APFs – Using The Jacobian

- We **MUST use the joint torques** and not the workspace forces otherwise unexpected rigid body motion could occur, such as when two workspace forces induce a net moment

- Use the Jacobian to transform forces to joint torques





Local minima may also occur

Source: Spong, Mark W., Seth Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley, 2005.

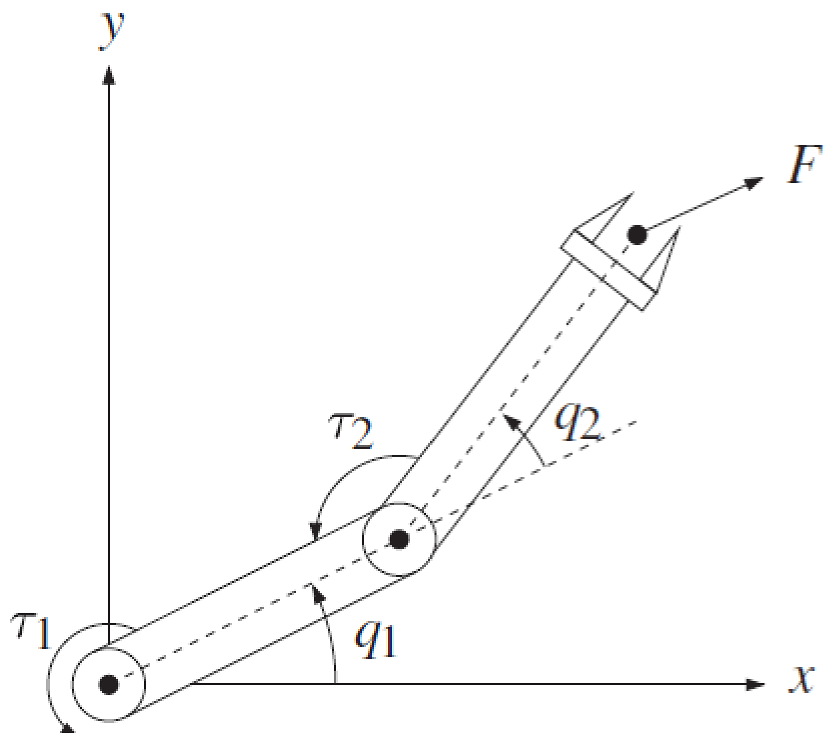
Control with APFs – Using The Jacobian

- The vector of forces and moments at the end effector

$$F = (F_x, F_y, F_z, n_x, n_y, n_z)$$

- The vector of joint torque

$$\tau = J^T(q)F$$



P. Corke, *Robotics, Vision and Control: Fundamental Algorithms In MATLAB® Second, Completely Revised, Extended And Updated Edition*. Springer, 2017.

Control with APFs – Using The Jacobian

- Can be applied directly to control.

$\boldsymbol{\tau}$: Vector of joint torques $$\boxed{\boldsymbol{\tau} = \boldsymbol{J}_v^T \boldsymbol{F}}$$ $\boldsymbol{F}$: workspace force at end effector

- **Total artificial joint torque** acting on the arm is the sum of the artificial joint torques that result from all attractive and repulsive potentials.

$$\boxed{\tau(q) = \sum_i J_{o_i}^T(q) F_{att,i}(q) + \sum_i J_{o_i}^T(q) F_{rep,i}(q)}$$

Source: Spong et al. Robot Modeling and Control, 2005

Example 3: Control with APFs

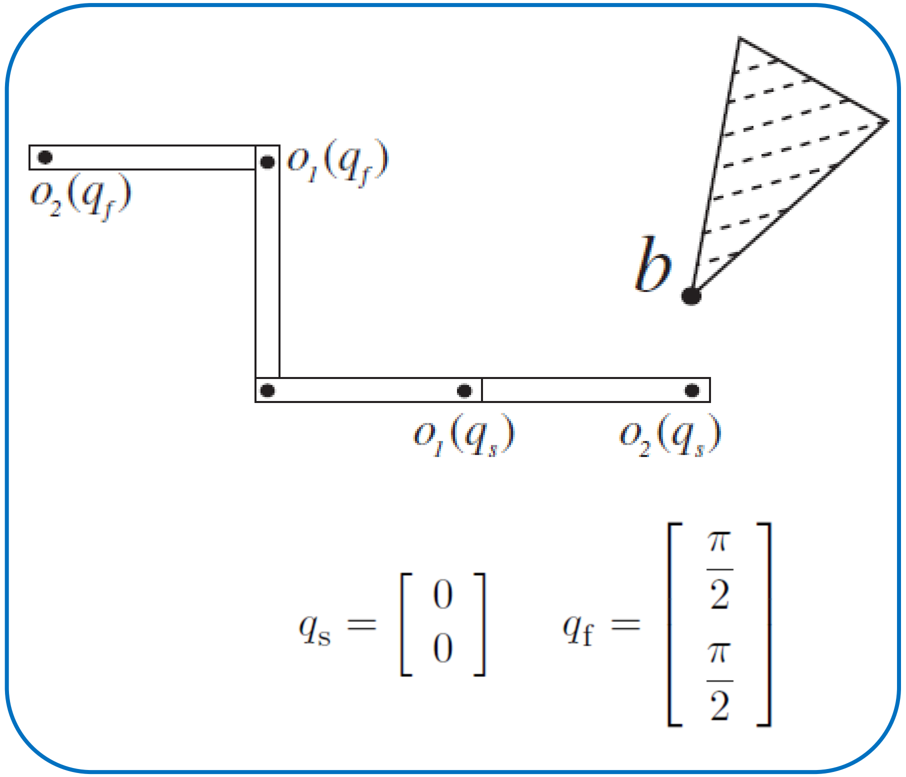
- A two-link arm with repulsive forces from an obstacle at (2, 0.5) and ρ_0 = 1 and q_f as per Example 1. Calculate the total artificial joint torque acting on the arm if $\eta_2 = \zeta_1 = \zeta_2 = 1$.

Answer

$$\tau(q) = \sum_i J_{o_i}^T(q) F_{att,i}(q) + \sum_i J_{o_i}^T(q) F_{rep,i}(q)$$

Step 1: Find the Jacobian for J_{o_1} and J_{o_2}

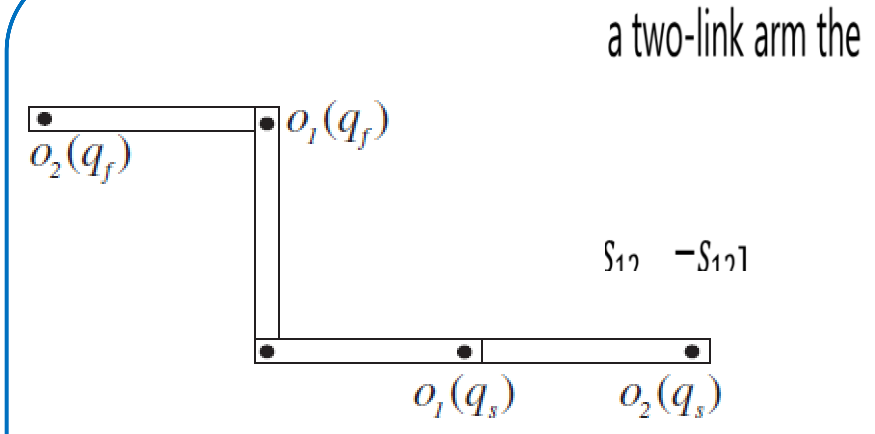
$$\dot{o}_i = J_{o_i}(q) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$



Example 3: Control with APFs

- From lecture 5 (*Slide 36*), For a two-link arm the Jacobian matrix for o_2 is,

$$J_{o_2}(q_1, q_2) = \begin{bmatrix} -s_1 - s_{12} & -s_{12} \\ c_1 + c_{12} & c_{12} \end{bmatrix}$$



$$q_s = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad q_f = \begin{bmatrix} \frac{\pi}{2} \\ \frac{\pi}{2} \end{bmatrix}$$

Example: Control with APFs

- Using the similar method, the Jacobian matrix for o_1 is

$$J_{o_1}(q_1, q_2) = \begin{bmatrix} -s_1 & 0 \\ c_1 & 0 \end{bmatrix}$$

At $q_s = (0,0)$ we have,

$$J_{o_1}^T(q_s) = \begin{bmatrix} -s_1 & c_1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

And

$$J_{o_2}^T(q_s) = \begin{bmatrix} -s_1 - s_{12} & c_1 + c_{12} \\ -s_{12} & c_{12} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$$

Example 3: Control with APFs

- The attractive potentials are (see example 1),

$$F_{att,1}(q_s) = -\zeta_1 \left(o_1(q_s) - o_1(q_f)\right) = \zeta_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$F_{att,2}(q_s) = -\zeta_2 \left(o_2(q_s) - o_2(q_f)\right) = \zeta_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

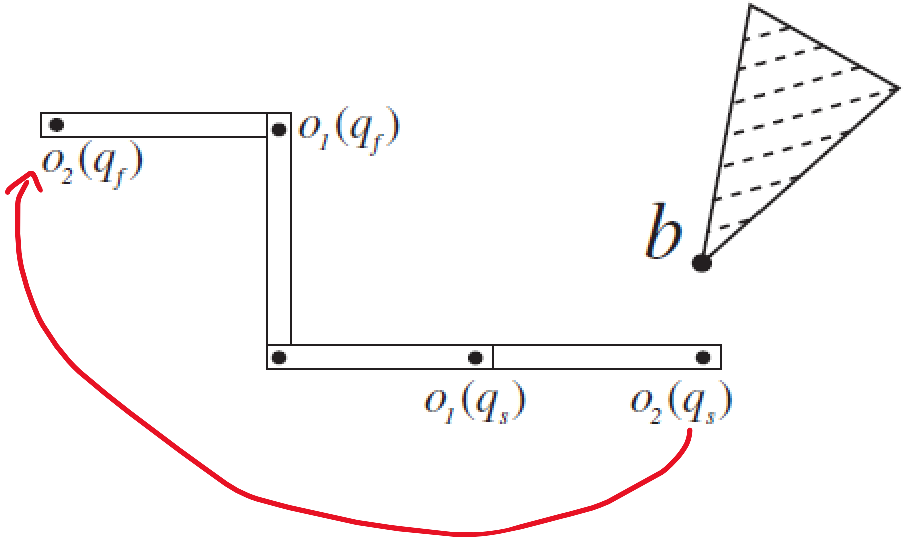
- The repulsive potential on link 2 is (see example 2),

$$F_{rep,2}(q_s) = \eta_2 \left(\frac{1}{0.5} - 1\right) \frac{1}{0.25} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \eta_2 \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Example 3: Control with APFs

- Substitute $\eta_2 = \zeta_1 = \zeta_2 = 1$, we can map the workspace attractive and repulsive forces to joint torques:

$$\tau_{att,1}(q_s) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$\tau_{att,2}(q_s) = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$
$$\tau_{rep,2}(q_s) = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$



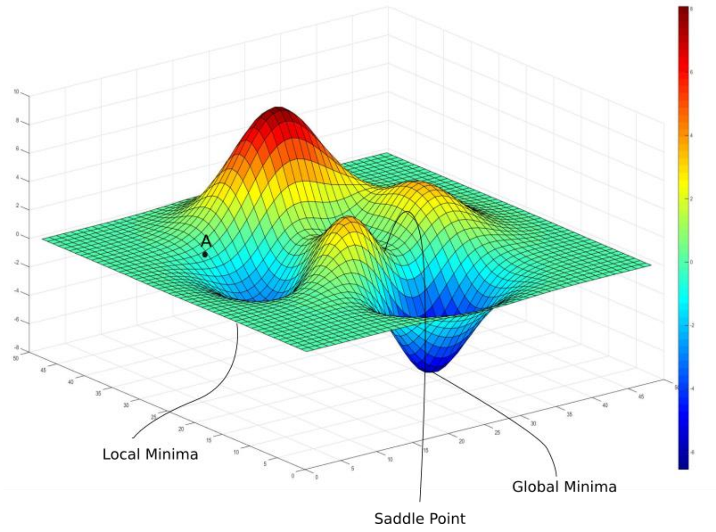
- The total joint torque induced given by,

$$\tau(q_s) = \tau_{att,1}(q_s) + \tau_{att,2}(q_s) + \tau_{rep,2}(q_s)$$

$$\tau(q_s) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -8 \\ -4 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$$

Hence both joints will rotate clockwise to avoid the obstacle

Critique of Artificial Potential Fields



- Efficient
- Easily adapted to the problem being solved
- Local minima
  - Harmonic Potential Fields can avoid this
  - Randomised Path Planner (RPP) includes random walks to avoid local minima (*Barraquand, 1991*)
  - See *Kavraki (1998)* and *Masoud (2013)* for a review of AFPs and Harmonic PFs respectively

https://bdtechtalks.com/2020/04/27/deep-learning-mode-connectivity-adversarial-attacks/gradient-descent-local-minima/

Next Week – Lecture 9 Robotics Dynamics

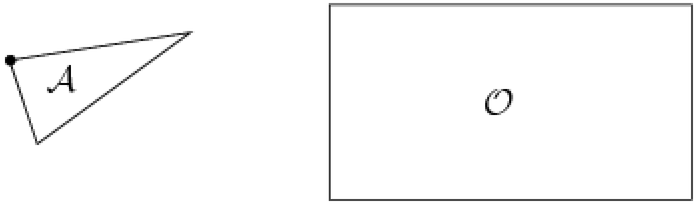
- The Lagrangian
- Euler-Lagrange Equations
- Applying Euler-Lagrange Method to an $\boldsymbol{n}$-link Manipulator

Appendix:
Configuration Space of non-point shaped robots

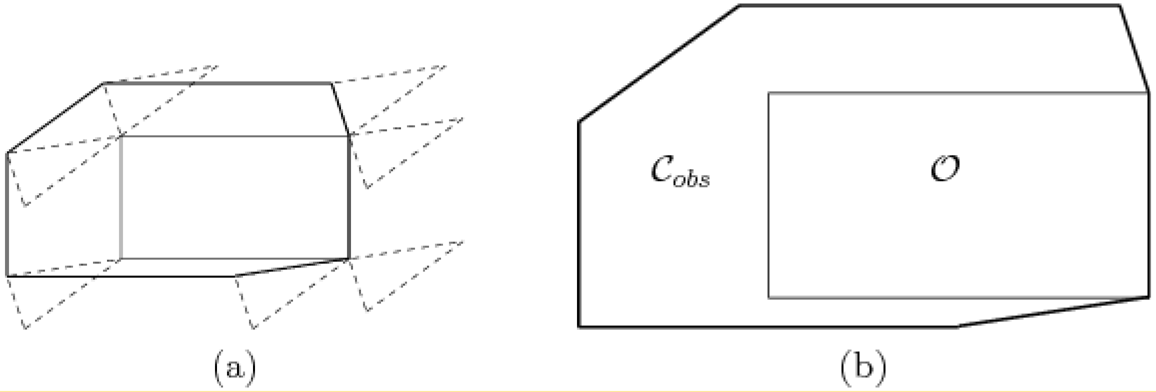
- Expanding $\mathcal{C}_{obst}$ for non-point shaped robots (translation only case shown below)
- This can be done in 3D for convex polygonal obstacles and robots but is cumbersome.

Consider a triangle-shaped mobile robot who possible motions include only translation in the plane

Robot: A
Obstacle: O



The boundary of $\mathcal{C}_{obst}$ can be obtained by computing the convex envelop of the configuration at which the robot makes vertex-to-vertex contact with obstacle



Source: Spong, Mark W., Seth Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley, 2005.