矢量运算

$$\begin{cases}\nabla\times\dfrac{\vec{e_i}}{h_i}=0\\[2mm]\nabla\cdot\sum\limits_{k=1}\dfrac{\varepsilon_{ijk}\vec{e_k}}{h_ih_j}=0\end{cases}$$

$$\nabla=\begin{cases}柱 & \hat{\rho}\partial_r+\dfrac{\hat{\varphi}}{r}\partial_\phi+\hat{z}\partial_z\\[2mm] 球 & \hat{r}\partial_r+\dfrac{\hat{\theta}}{r}\hat{\theta}+\dfrac{\hat{\varphi}}{r\sin\theta}\partial_\varphi\end{cases}$$

$$\nabla^2\frac{1}{r}=-4\pi\delta^{(3)}(\vec{r})$$

电磁基本

$$\vec{J}\begin{cases}\vec{J}=\rho\vec{v}\\ Id\vec{l}=\vec{J}dV\\ \nabla\cdot\vec{J}+\dfrac{\partial\rho}{\partial t}=0\end{cases}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\int_V\frac{\vec{J}(\vec{x}')\times\vec{r}}{r^3}d^3x'=\frac{\mu_0}{4\pi}\oint_C\frac{Id\vec{l}\times\vec{r}}{r^3}$$

$$\vec{A}=\frac{\mu_0}{4\pi}\int_V\frac{\vec{J}(\vec{x}')}{r}d^3x'$$

参考

$$\vec{E}'=\vec{E}+\vec{v}\times\vec{B}\Longleftrightarrow\underbrace{\oint_C\vec{E}\cdot d\vec{l}}_{=\frac{d\Phi_B}{dt}}=\oint_S(\partial_t\vec{B}-\nabla\times(\vec{v}\times\vec{B}))\cdot d\vec{S}$$

$$\vec{f}=\rho\vec{E}+\vec{J}\times\vec{B}$$

真空
$$\begin{cases}\nabla\cdot\vec{E}=\dfrac{\rho}{\varepsilon_0}\\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\\ \nabla\cdot\vec{B}=0\\ \nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}\end{cases}$$

介质

$$\vec{P}\begin{cases}\nabla\cdot\vec{P}=-\rho_P\\ \vec{J}_P=\dfrac{\partial\vec{P}}{\partial t}\end{cases}$$

$$\vec{M}\begin{cases}\vec{J}_M=\nabla\times\vec{M}\\ \vec{M}=\dfrac{\sum\vec{m}_i}{\Delta V}=\chi_M\vec{H}\end{cases}$$

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$$

$$\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M}$$

$$\begin{cases}\nabla\cdot\vec{D}=\rho_f\\ \nabla\times\vec{E}+\dfrac{\partial\vec{B}}{\partial t}=0\\ \nabla\cdot\vec{B}=0\\ \nabla\times\vec{H}-\dfrac{\partial\vec{D}}{\partial t}=\vec{J}_f\end{cases}$$

边值关系
$$\begin{cases}\nabla\to\vec{n}_{12}\\ 矢量\to矢量_2-矢量_1\end{cases}$$

$$\sigma_f=\frac{\Delta I}{\Delta l}$$

面电流密度

$$p_\mu p_\mu=-m^2c^2=p^2-\frac{W^2}{c^2}$$

动能 $K=(\gamma-1)mc^2$

4-电磁力 $K_\mu=\dfrac{dp_\mu}{d\tau}=\gamma_u(\vec{F},\frac{i}{c}\vec{F}\cdot\vec{u})$

$$K_\mu p_\mu=0$$

能量
$$\begin{cases}\vec{S}=\vec{E}\times\vec{H}\\ \dfrac{\partial w}{\partial t}=\vec{E}\cdot\dfrac{\partial\vec{D}}{\partial t}+\vec{H}\cdot\dfrac{\partial\vec{B}}{\partial t}\end{cases}$$

守恒 $-\nabla\cdot\vec{S}=\vec{f}\cdot\vec{v}+\dfrac{\partial w}{\partial t}$

动量 $\vec{g}=\varepsilon_0\vec{E}\times\vec{B}$

$$\frac{\partial\vec{g}}{\partial t}=\underbrace{\vec{J}_D}_{\varepsilon_0\frac{\partial\vec{E}}{\partial t}}\times\vec{B}+\underbrace{\vec{J}_D^{(M)}}_{-\varepsilon_0\frac{\partial\vec{B}}{\partial t}}\times\vec{E}$$

守恒
$$\begin{cases}\vec{f}+\dfrac{\partial\vec{g}}{\partial t}=-\nabla\cdot\overleftrightarrow{T}\\ T_{ij}=\frac{1}{2}\delta_{ij}\left(\varepsilon_0E^2+\dfrac{B^2}{\mu_0}\right)-\varepsilon_0E_iE_j-\dfrac{1}{\mu_0}B_iB_j\end{cases}$$

应力 $\vec{F}=\oint_\Sigma d\vec{\sigma}\cdot\overleftrightarrow{T}$ (内对外)

沿 $\vec{E}$ “拉力”
垂直 $\vec{E}$ “压缩力”

粒子 $\vec{p_i}=m\vec{v}+Q\vec{A}$ (在外磁场中)

电势 $\varphi_P=\dfrac{1}{4\pi\varepsilon_0}\int_V\dfrac{\rho(\vec{x}')d^3V'}{r_P}$

$\nabla^2\varphi=-\dfrac{\rho}{\varepsilon}$　能量 $W=\frac{1}{2}\int_\Omega\rho_f\varphi\,dV$

边界条件
$$\begin{cases}\varphi_1|_s=\varphi_2|_s\\ \varepsilon_2\dfrac{\partial\varphi_2}{\partial n}\Big|_s-\varepsilon_1\dfrac{\partial\varphi_1}{\partial n}\Big|_s=-\sigma_f\end{cases}$$

轴对称、柱坐标、 $\varphi=(a_0+b_0\ln r)(c_0+d_0\varphi)+\sum\limits_{n=0}^{+\infty}(a_nr^n+b_nr^{-n})(c_n\cos n\varphi+d_n\sin n\varphi)$

球坐标、 $\varphi=\sum\limits_{n=0}^{+\infty}\sum\limits_{m=0}^{n}\left(A_nr^n+B_n\dfrac{1}{r^{n+1}}\right)P_n^m(\cos\theta)(C_{nm}\cos m\varphi+D_{nm}\sin m\varphi)$

$r\to\infty$, $r\to0$ + 边界条件

电多极. $\varphi(\vec{x})=\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{Q}{R}+p_i\partial_i\dfrac{1}{R}+\dfrac{1}{6}D_{ij}\partial_i\partial_j\dfrac{1}{R}+\cdots\right]$

$p_i=\int_Vx_i'\rho(\vec{x}')dV'$
↓
ρ 原点不对称

无迹

$D_{ij}=\int_V(3x_i'x_j'-r'^2\delta_{ij})\rho(\vec{x}')d^3V'$
↓
ρ 非球对称

$\varphi\doteq\dfrac{\vec{p}\cdot\vec{R}}{4\pi\varepsilon_0R^3}$　$\vec{F}=-\nabla W=\vec{p}\cdot\nabla\vec{E}_e$

$W=-\vec{p}\cdot\vec{E}_e(0)$　$\vec{L}=-\partial_\theta W=\vec{p}\times\vec{E}_e$

$W=\frac{1}{6}D_{ij}\partial_i\partial_j\varphi_e(0)$

矢势 $\Phi_B=\oint_C\vec{A}\cdot d\vec{l}$
$$\begin{cases}\vec{A}_1|_s=\vec{A}_2|_s\\ \vec{n}_{12}\times\left[\dfrac{1}{\mu_2}\nabla\times\vec{A}_2-\dfrac{1}{\mu_1}\nabla\times\vec{A}_1\right]=\vec{\sigma}_f\end{cases}$$

轴对称、 $\vec{A}=A(\rho,\theta)\hat{k}$
$$\begin{cases}A_1|_{\rho=R}=A_2|_{\rho=R}\\ \dfrac{1}{\mu_2}\dfrac{\partial A_2}{\partial\rho}\Big|_{\rho=R}-\dfrac{1}{\mu_1}\dfrac{\partial A_1}{\partial\rho}\Big|_{\rho=R}=-\sigma_f\end{cases}$$

$\vec{A}(\vec{x})=\dfrac{\mu}{4\pi}\int_V\dfrac{\vec{J}_f(\vec{x}')}{r}dV'$

源有限

无穷长直导线 $\vec{A}=-\dfrac{\mu_0I}{2\pi}\ln r\,\hat{k}+\vec{c}$

磁标势（无I_f单连通区域）

$$\begin{cases}\nabla\cdot\vec H=\frac{\rho_m}{\mu_0}=-\nabla\cdot\vec M\\ \nabla\times\vec H=0\end{cases}$$

$\mu\to\infty$ 物质界面 $\Rightarrow$ 等磁势面

磁多极 $\int_V[f\vec J\cdot\nabla' g+g\vec J\cdot\nabla' f]dV'=0$ （V包含所有$\vec J$）

$$\vec A^{(0)}=\frac{\mu_0}{4\pi R}\int_V\vec J(\vec x')dV'=0$$

$$\vec A^{(1)}=\frac{\mu_0}{4\pi R^3}x_i\int_V x_i'\vec J(\vec x')dV'=\frac{\mu_0}{4\pi}\frac{\vec m\times\vec x}{R^3}$$

$$\vec m=\frac12\int_V\vec x'\times\vec J(\vec x')dV'$$

$$\varphi_m^{(1)}=\frac{\vec m\cdot\vec x}{4\pi R^3}$$

磁单极: $\vec B=\frac{Q_m\vec r}{4\pi r^3}-Q_m\delta(x)\delta(y)H(z)\hat k$

磁偶极与外磁场相互作用能. $W_i^{(1)}=\vec m\cdot\vec B_e$

$$\begin{cases}F=-\nabla U\\ U=-\vec m\cdot\vec B_e(0)\ (\text{等效磁势能})\end{cases}\qquad \vec N=\vec m\times\vec B_e(0)$$

$W^{(1)}=W_m+W_{源}+U=-U$

电功　运动势能

电磁波

$$\begin{cases}\nabla\times\vec E=i\omega\mu\vec H\\ \nabla\times\vec H=-i\omega\varepsilon\vec E\\ \nabla\cdot\vec E=0\\ \nabla\cdot\vec H=0\end{cases}\Rightarrow\begin{cases}\nabla^2\vec E+k^2\vec E=0\\ k^2=\omega^2\mu\varepsilon\\ \nabla\cdot\vec E=0\\ \vec B=-\frac{i}{\omega}\nabla\times\vec E\end{cases}$$

$\nabla\to i\vec k$　　$\partial_t\to -i\omega$

反射折射（边界条件）

$$\begin{cases}k_x=k_x'=k_x'',\quad k_y=k_y'=k_y''\\ k=k'=\omega\sqrt{\mu_1\varepsilon_1}\quad k''=\omega\sqrt{\mu_2\varepsilon_2}\\ \theta=\theta'\quad\frac{\sin\theta}{\sin\theta''}=\frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}=n_{21}\end{cases}$$

平面电磁波 $\vec E$与$\vec B$同位相. $\vec B=-\frac{i}{\omega}\nabla\times\vec E=\frac1\omega\vec k\times\vec E$

$$\frac{E_0}{B_0}=\frac{\omega}{k}=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$\vec S=\frac1\mu\vec E\times\vec B\propto\vec k=\frac{1}{\sqrt{\mu\varepsilon}}w\vec e_k=v_pw\vec e_k$$

$$w=\frac12(\varepsilon E^2+\frac{B^2}{\mu})$$

$$\overline{FG}=\frac12\mathrm{Re}[F^*(t)G(t)]$$

菲涅耳公式

$\vec E$在入射面内
$$\begin{cases}\frac{E_0'}{E_0}=-\frac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}\\ \frac{E_0''}{E_0}=\frac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')}\end{cases}$$

$\vec E\perp$入射面 ↓ 半波损
$$\begin{cases}\frac{E_0'}{E_0}=-\frac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')}\\ \frac{E_0''}{E_0}=\frac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')}\end{cases}$$

全反射 $k_z''=\sqrt{k''^2-k_x''^2}=k\sqrt{n_{21}^2-\sin^2\theta}=i\kappa$

H_z''与E''同相位，H_x''有$\frac\pi2$位相差

前$\frac T2$储能，后$\frac T2\to$反射波能量.

导体. $\rho(t)=\rho_0e^{-\frac\sigma\varepsilon t}=\rho_0e^{-\frac t\tau}$　良导体　$\omega\tau<<1$　$\rho\approx0$

$$\begin{cases}\nabla^2\vec E+k^2\vec E=0\\ \vec k=\vec\beta+i\vec\alpha\\ \beta^2-\alpha^2=\omega^2\mu\varepsilon\\ \vec\alpha\cdot\vec\beta=\frac12\omega\mu\sigma\end{cases}$$

垂直入射 $\Rightarrow\alpha_z\approx\beta_z\approx\sqrt{\frac{\omega\mu\sigma}{2}}$

良导体　$\delta=\frac{1}{\alpha_z}$，$\vec H''=\sqrt{\frac{\sigma}{\mu\varepsilon}}\vec e_z\times\vec E''e^{i\frac\pi4}$

理想导体 — 体内无电磁场
$$\begin{cases}\vec n\times\vec E|_S=0\ (E_\tau=0)\\ \vec n\times\vec H|_S=\vec\alpha\\ \partial_nE_n|_S=0\ \boxed{(\nabla\cdot\vec E=0)}\end{cases}$$

谐振腔:
$$\begin{cases}E_x=A_1\cos(\frac{m\pi x}{L_1})\sin(\frac{n\pi y}{L_2})\sin(\frac{p\pi z}{L_3})\\ E_y=A_2\sin\quad\cos\quad\sin\\ E_z=A_3\sin\quad\sin\quad\cos\\ \omega_{mnp}=\frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{L_1^2}+\frac{n^2}{L_2^2}+\frac{p^2}{L_3^2}}\end{cases}$$

波导　双导线　同轴传输线　波导管

$\vec E=\vec E(x,y)e^{i(k_zz-\omega t)}$　　$\vec H=-\frac{i}{\omega\mu}\nabla\times\vec E$

$$\left.\begin{cases}E_x=A_1\cos(k_xx)\sin(k_yy)\\ E_y=A_2\sin\quad\cos\\ E_z=A_3\sin\quad\sin\end{cases}\right\}e^{i(k_zz-\omega t)}\to\sin+\cos$$

$\nabla\cdot\vec E=0\Rightarrow k_xA_1+k_yA_2-ik_zA_3=0$　　TE_{mn}　TM_{mn}.

截止频率　$\omega>\frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{(\frac{m\pi}{a})^2+(\frac{n\pi}{b})^2}=\omega_{c,mn}$

势　$-\nabla\varphi=\vec E+\frac{\partial\vec A}{\partial t}$

规范变换 $\begin{cases}\vec A'=\vec A+\nabla\psi\\ \varphi'=\varphi-\partial_t\psi\end{cases}$

$$\begin{cases}\nabla^2\varphi+\frac{\partial}{\partial t}\nabla\cdot\vec A=-\frac{\rho}{\varepsilon_0}\\ \nabla^2\vec A-\frac1{c^2}\frac{\partial^2\vec A}{\partial t^2}-\nabla(\nabla\cdot\vec A+\frac1{c^2}\frac{\partial\varphi}{\partial t})=-\mu_0\vec J\end{cases}$$

$$\nabla\cdot\vec A=0\begin{cases}\nabla^2\vec A-\frac1{c^2}\frac{\partial^2\vec A}{\partial t^2}-\frac1{c^2}\frac{\partial}{\partial t}\nabla\varphi=-\mu_0\vec J\\ \nabla^2\varphi=-\frac{\rho}{\varepsilon_0}\\ \nabla^2\vec A-\frac1{c^2}\frac{\partial^2\vec A}{\partial t^2}=-\mu_0\vec J_T\ (\text{横，无散分量})\end{cases}$$

$$\nabla\cdot\vec A+\frac1{c^2}\frac{\partial\varphi}{\partial t}=0$$
$$\Downarrow$$
$$\begin{cases}\nabla^2\vec A-\frac1{c^2}\frac{\partial^2\vec A}{\partial t^2}=-\mu_0\vec J\\ \nabla^2\varphi-\frac1{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon_0}\end{cases}$$

剩余变换 $\vec A'=\vec A+\gamma\vec k$

$$\varphi(t,\vec x)=\frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho(t-\frac rc,\vec x')}{r}dV'$$

$$\vec A(t,\vec x)=\frac{\mu_0}{4\pi}\int_V\frac{\vec J(t-\frac rc,\vec x')}{r}dV'$$

φ 超距
$\vec A$ 推迟势.

辐射. $\vec A(t,\vec x)=\left[\frac{\mu_0}{4\pi}\int_Vd^3x'\frac{\vec J(\vec x')e^{i\frac\omega cr}}{r}\right]e^{-i\omega t}\Rightarrow\begin{cases}\vec E=\frac{ic}{k}\nabla\times\vec B\\ \vec B=\nabla\times\vec A\end{cases}$

$\begin{cases}l<<\lambda\\ l<<r\end{cases}\Rightarrow\begin{cases}r<<\lambda\sim\text{近场区 静磁场}\\ r>>\lambda\sim\text{辐射区}\ \vec A^{(0)}(\vec x)=\frac{\mu_0e^{ikR}}{4\pi R}\dot{\vec p}\ (\dot{\vec p}=-i\omega\vec p)\end{cases}$

$P=\frac{1}{4\pi\varepsilon_0}\frac{|\ddot{\vec p}|^2}{3c^3}$　　$\vec S\propto\sin^2\theta$

$$\Rightarrow\begin{cases}\vec B=\frac{e^{ikR}}{4\pi\varepsilon_0c^3R}\ddot{\vec p}\times\hat r\\ \vec E=\frac{e^{ikR}}{4\pi\varepsilon_0c^2R}(\ddot{\vec p}\times\hat r)\times\hat r\end{cases}$$

$$\vec A^{(1)}(\vec x)=-i\frac{\mu_0ke^{ikR}}{4\pi R^2}(\vec m\times\vec x+\frac16x_i\vec e_RD_{ik})$$

相对论

$$\begin{cases}x'=\gamma(x-vt)\\ t'=\gamma(t-\frac{vx}{c^2})\\ \Delta t=\gamma\Delta\tau\quad L=\frac{L_0}{\gamma}\end{cases}$$

$a_{ij}=\delta_{ij}+\frac{\gamma^2}{\gamma+1}\beta_i\beta_j$　$a_{i4}=i\gamma\beta_i=-a_{4i}$　$a_{44}=\gamma$

$d\tau=\frac{ids}{c}$

$\begin{cases}x_\mu=(\vec r,ict)\\ dx_\mu=(d\vec r,icdt)\end{cases}$　$U_\mu=\frac{dx_\mu}{d\tau}=\gamma_u(\vec u,ic)$　$U_\mu U_\mu=-c^2$

$k_\mu=(\vec k,\frac{i\omega}{c})$　$\phi=k_\mu x_\mu$　　$J_\mu=(\vec J,ic\rho)=\rho_0U_\mu$　$\partial_\mu J_\mu=0$

$$a_\mu=\frac{dU_\mu}{d\tau}\Rightarrow\begin{cases}a_\mu U_\mu=0\\ a_\mu a_\mu=a^2\\ \vec u\parallel\dot{\vec u}\quad a=\gamma_u^3|\dot{\vec u}|\\ \vec u\perp\dot{\vec u}\quad a=\gamma_u^2|\dot{\vec u}|\end{cases}$$

$A_\mu=(\vec A,\frac ic\varphi)\Rightarrow\begin{cases}\partial_\mu\partial_\mu A_\nu=-\mu_0J_\nu\\ \partial_\mu A_\mu=0\leftarrow\text{规范}\end{cases}$　$(F_{\mu\mu}=0)$.

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\Rightarrow\begin{cases}F_{ij}=\varepsilon_{ijk}B_k\quad E_i=icF_{i4}\\ F_{i4}=-\frac{iE_i}{c}\quad B_i=\frac12\varepsilon_{ijk}F_{jk}\end{cases}$$

$$\begin{cases}\partial_\mu F_{\mu\nu}=-\mu_0J_\nu\leftarrow\text{源}\\ \partial_\mu F_{\nu\rho}+\partial_\rho F_{\mu\nu}+\partial_\nu F_{\rho\mu}=0\end{cases}$$

$\tilde F_{\mu\nu}=\frac12\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ $\begin{cases}\partial_\mu F_{\mu\nu}=-\mu_0J_\nu\\ \partial_\mu\tilde F_{\mu\nu}=0\end{cases}$　$\begin{cases}F_{\mu\nu}F_{\mu\nu}\Rightarrow(E^2-c^2B^2)\text{不变}\\ F_{\mu\nu}\tilde F_{\mu\nu}\Rightarrow\vec E\cdot\vec B\text{ 不变}\end{cases}$

$$\begin{cases}\vec E=\gamma(\vec E'-c\vec\beta\times\vec B')-\frac{\gamma^2}{\gamma+1}\vec\beta(\vec\beta\cdot\vec E')\\ \vec B=\gamma(\vec B'+\frac1c\vec\beta\times\vec E')-\frac{\gamma^2}{\gamma+1}\vec\beta(\vec\beta\cdot\vec B')\end{cases}$$

$\frac{dp_\mu}{d\tau}=qF_{\mu\nu}U_\nu$　$(p_\mu=mU_\mu=(\vec p,\frac icW))$