Notes on Introduction to Condensed Matter Physics

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1 Basic Introduction

1. Condensed Matter: $\sim 10^{23}\ /cm^3$
2. Major Study: Electrons, Phonons, The interactions between
3. Drives: New materials & New technologies

2 Conventional Metal Physics: Electrons and Phonons

1. Basic Properties of Normal Metals: • Ductile • Excellent electrical conductor • Excellent thermal conductor • Most are weak paramagnet, some ferromagnet • Opaque
   • At low T: ρ increases with T $\quad \chi \sim Const.$ $\quad c_V \propto T$
2. Drude Free Electron Model:
   • Assumptions: • Free electrons (Ignore interaction with lattice) • Independent electrons (Ignore interactions between electrons • Electrons were treated as independent classical particles
   • Maxwell-Boltzmann distribution
   • Successes: • Electrical conductivity and thermal conductivity • Wiedemann-Franz law (by luck!)
   • The Hall effect and magnetoresistance • AC conductivity and optical properties of metals

• Problems: • Heat capacity puzzle: $c_V = \frac{3}{2} n k_B = Const.$ • The susceptibility puzzle: χ does not change with temperature, non-Curie like $\chi \sim 1/T$

3. The Sommerfeld Model: • Free Electron Gas + Schrodinger Equation + Fermi Statistics

   • The Fermi Surface: In 3D $E_F \sim n^{2/3}$ • Typical Values: $E_F \sim 7\ eV$, $k_F \sim 1 \times 10^8\ cm^{-1}$, $T_F \sim 8 \times 10^4\ K$, $v_F \sim 2 \times 10^8\ cm/s$ • DOS In 3D: $D(\varepsilon) \sim \varepsilon^{1/2}$ • Linear T Heat Capacity at Low T

   • Pauli Paramagnetism (at low T): $\chi = \mu_B^2 D(E_F) = const.$

   • Successes: • Realized the importance of Fermi distribution • Established the k-space language for electrons • Introduced Fermiology • Resolved the Pauli susceptability puzzle • Resolved the heat capacity puzzle • Resolved the thermopower puzzle • Explained the Wiedemann-Franz Law

4. Landau's Fermi Liquid Theory: •Quasi-particle: Same charge, spin, momentum as non-interacting electron • Adiabatic Continuity, Only valid at low T and low energy • Qualitatively explain the susceptibility, heat capacity Only require a Fermi sea • Entropy, distribution function unchanged • Energy modified by the effective mass & the Fermi interaction function • Low energy excitation like single particle • $\varepsilon = \frac{\hbar^2 k^2}{2m^*}$ • $c_V = \frac{1}{3}\frac{m^* k_F}{\hbar^2} k_B^2 T$ • $\chi = \frac{m^* k_F}{\hbar^2 \pi^2} \frac{1}{1+F_0^a} \mu_B^2$ • Landau parameter: m^*, F_0^a • Wilson ration: $R_W = \frac{\pi^2 k_B^2}{3\mu_B^2} \frac{\chi}{\gamma}$ • For non-interacting electron gas $R_W = 1$ • T^2 Law (Experimental Signature): qp-qp scattering • Scattering rate $\frac{1}{\tau} \sim k_B T \cdot k_B T \propto T^2$ • <u>But</u> electron seems to be a wrong place to start for many novel phenomena • Spinon, holon, fractional charge: Collective mode looks like a fraction of an electron

5. Bloch Theory: • Bloch Theorem: For periodic potential, $\psi_{nk}(\boldsymbol{r}) = \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} u_{nk}(\boldsymbol{r})$, where $u_{nk}(\boldsymbol{r}) = u_{nk}(\boldsymbol{R}+\boldsymbol{r})$ • Momentum is no longer a good quantum number • Band index n

   • NFE model: Perturbation to the free electron plane wave states (maximum mixing) (highly delocalized) • First order: $\varepsilon_k^1 = \overline{V}$ • Second order (non-degenerate): $\varepsilon_k^2 = \sum_{g\neq 0} \frac{|V_g|^2}{\varepsilon_K^0 - \varepsilon_{k-g}^0}$ • Second order (degenerate): $\varepsilon_k^2 = \pm|V_n|$ • Energy gap: Level repulsion, Explains metal or insulator

   • +2 Metals: Band overlap

   • Tight Binding Model: (nearly localized) such as transition metal and rare earth metal with partially filled d and f orbitals. • $\psi_k(\boldsymbol{r}) = \sum_{\boldsymbol{R}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}} \psi_a(\boldsymbol{r} - \boldsymbol{R})$ • Overlap integral: $t_{\boldsymbol{R}} = \int \psi_a^*(\boldsymbol{r} + \boldsymbol{R})(\Delta V)\psi_a(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$ $\rightarrow$ Nearest neighbor approximation • Useful starting point

6. Lattice Vibrations: • Harmonic Approximation: $V(a + \delta x) = V_0 + \frac{1}{2}\beta(\delta x)^2$ • $\varepsilon = \sum_k (n_k + \frac{1}{2})\hbar\omega$ • Phonons: The quantum of the lattice vibration, $n_k = \frac{1}{\mathrm{e}^{\hbar\omega/kT} - 1}$ • Mono-atomic 1D Chain: $\omega = 2\sqrt{\frac{\beta}{m}}\left|\sin\left(\frac{aq}{2}\right)\right|$ • Di-atomic 1D Chain: Acoustic & Optical phonon • Phonon specific heat: Debye model: • Assume linear dispersion • Define a cutoff in the integral: Debye frequency

• $T \to 0$ $c_V \sim T^3$ ⇐Blackbody radiation

7. Specific Heat: • Directly related to internal energy • To extract important microscopic parameters
   • To study phase transition • Calorimetry: What, How, Better resolution and accuracy, Flecibility
   • Adiabatic Nernst Calorimeter: Slow, Heat leak problem, Need big sample
   • Relaxation time calorimeter: $\Delta T = \Delta T_0 e^{-t/\tau}$, where $\tau = c_V l/\kappa S$ (with addenda)
   Advantage: Accurate, Fast, Microgram crystals, Small, Work in extreme conditions
   Disadvantage: The addenda
   • Membrane calorimeter: Nano-gram crystals, Measure in-situ evaporated thin films, Extreme conditions
   • Heater: Resistance stable with T • Thermometer: Resistance has Linear relationship with T

8. Anharmonic Potential: • Universal ⇐ $V = 0$ when $r \to \infty$ • Phonon-phonon interaction: No longer independent excitations ⇒ Phonon heat conduction (low$T$, high$T$) • Thermal Expansion:
   • $\alpha = \frac{1}{l}\frac{\partial^2 l}{\partial T \partial p} = \frac{1}{3V}\frac{\partial^2 V}{\partial T \partial p}$ • Provide similar information as specific heat • Bad in engineering
   • XRD → Measure l or V, High resolution, Hard to use in extreme physical conditions
   • Capacitive dilatometer: High resolution (capacitance bridge) (0.01 Å), ultra low T and large $\boldsymbol{B}$ (compact design)
   • Negative thermal expansion: ZrW_2O_8 → Rigid Unit Modes

9. Main Frame: Landau Fermi Liquid Theory + Band Theory

3 Transport

1. Basic Notions: • Movement of Particles or Quantities • Non-equilibrium steady state • $\boldsymbol{J} = \boldsymbol{L} \cdot \boldsymbol{F}$
   • Very informative and instructive, esp. on Novel materials and in Extreme conditions
   • Normally the first to be carried out

2. Fractional Quantum Hall Effect: • Ultra low T, Super strong $\boldsymbol{B}$, Very clean • Strong electron correlations • Most precise method to measure h

3. Cryogenic technology: • Dilution fridge method: He-3 rich & He-3 poor phase at $T < 0.87\ K$
   • He-3 diffuse, absorb heat • Down to ~ 10 mK
   • Superconducting magnet: up to 20 tesla ⇐ critical field
   • Super High megnetic field: Florida-Bitter resistive magnet, Hybrid magnet

4. The Boltzmann transport equation: • $\left.\frac{\partial f_k}{\partial t}\right|_{\text{diffusion}} + \left.\frac{\partial f_k}{\partial t}\right|_{\text{field}} + \left.\frac{\partial f_k}{\partial t}\right|_{\text{scattering}} = 0$ • $\left.\frac{\partial f_k}{\partial t}\right|_{\text{diffusion}} = -\dot{\boldsymbol{r}} \cdot \nabla_{\boldsymbol{r}} f_k$ • $\left.\frac{\partial f_k}{\partial t}\right|_{\text{field}} = -\dot{\boldsymbol{k}} \cdot \nabla_{\boldsymbol{k}} f_k$ • $\left.\frac{\partial f_k}{\partial t}\right|_{\text{scattering}} = -\frac{f_k - f_k^0}{\tau}$

5. Electrical transport: •$\boldsymbol{J}_e = \sigma \boldsymbol{E}$ • Measurements: Four-probe, Low frequency ac lock-in method

   • Drude model: $\sigma = \frac{ne^2\tau}{m}$ • Semi classical: $\delta \boldsymbol{k} = \frac{e\tau \boldsymbol{E}}{\hbar}$ • Only the surface of the FS changed !

   • Ignore the diffusion effect, Complexity of the FS

   • The Boltzmann transport equation: $\overleftrightarrow{\sigma} = \frac{1}{4\pi^3}\frac{e^2\tau}{\hbar}\int \frac{\boldsymbol{v}_k \boldsymbol{v}_k \mathrm{d}S_F}{v_k}$

   • Cubic symmetry: $\sigma_{x,y,z} = \frac{e^2}{3} v_F l D(\varepsilon_F)$

   • Matthiessen's rule: Different scattering mechanisms don't interfere each other $\Rightarrow$ $\frac{1}{\tau} = \frac{1}{\tau_{imp}} + \frac{1}{\tau_{ph}} + \cdots$

   • Electron-electron scattering: $\frac{1}{\tau} \sim T^2$

   • Electron-lattice scattering: $\rho \sim T$, at high T $\rho \sim T^5$, at low T

   • Electron-impurity scattering: • Roughly, Temperature-independent • Residual resistivity: $\rho(T = 0)$ • Residual resistivity ratio (RRR): $\frac{\rho_{300K}}{\rho_0}$ Higher the better

6. Thermal transport: •$\boldsymbol{J}_Q = \kappa(-\nabla T)$ • Measurement: One-heater two-thermometer

   • Drude: $J_{Qe,x} = \frac{1}{2} n v_x [\varepsilon(T_{x-v\tau} - T_{x+v\tau})] = \frac{1}{3} c_V v l$ • $\kappa_e = \frac{\pi^2}{3}\frac{k_B^2}{e^2} T\sigma$ • Phonon Thermal Conductivity: Good metals $\sim 1\%$

7. Thermoelectric power: • Seebeck Coefficient: $S = \frac{E}{\nabla T} = \frac{c_V}{3ne}$

4 Others

1. Topological Insulator: • Single electron model • Mainly spin-orbit interaction

2. Superfluidity: • Liquid He-4 & He-3: Low boiling T $\Leftarrow$ Weak van der Waals force & low atomic mass • He-4 (Boson) $< 2.17\ K$ (BEC), He-3 (Fermion) $< 2.49\ mK$ (BCS)

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