# Notes on Introduction to Condensed Matter Physics

# Xupeng Yang

#### Contents

1	Basic Introduction	1
2	Conventional Metal Physics: Electrons and Phonons	1
3	Transport	3
4	Metal Insulator Transition (MIT)	5
5	Low Dimensional Electron Systems	7
6	Others	8

#### 1 Basic Introduction

- 1. Condensed Matter:  $\sim 10^{23} / cm^3$
- 2. Major Study: Electrons, Phonons, The interactions between
- 3. Drives: New materials & New technologies

### 2 Conventional Metal Physics: Electrons and Phonons

- Basic Properties of Normal Metals: 

   Ductile
   Excellent electrical conductor
   Excellent thermal conductor
   Most are weak paramagnet, some ferromagnet
   Opaque
  - At low T:  $\rho$  increases with T  $\chi \sim Const.$   $c_V \propto T$
- 2. Drude Free Electron Model:
  - Assumptions: Free electrons (Ignore interaction with lattice)
     Independent electrons (Ignore interactions between electrons)
     Electrons were treated as independent classical particles

- Maxwell-Boltzmann distribution
- Successes: Electrical conductivity and thermal conductivity Wiedemann-Franz law (by luck!) The Hall effect and magnetoresistance AC conductivity and optical properties of metals
- Problems: Heat capacity puzzle:  $c_V = \frac{3}{2}nk_B = Const.$  The susceptibility puzzle:  $\chi$  does not change with temperature, non-Curie like  $\chi \sim 1/T$
- 3. The Sommerfeld Model: Free Electron Gas + Schrodinger Equation + Fermi Statistics
  - The Fermi Surface: In 3D  $E_F \sim n^{2/3}$  Typical Values:  $E_F \sim 7$  eV,  $k_F \sim 1 \times 10^8$  cm $^{-1}$ ,  $T_F \sim 8 \times 10^4$  K,  $v_F \sim 2 \times 10^8$  cm/s DOS In 3D:  $D(\varepsilon) \sim \varepsilon^{1/2}$  Linear T Heat Capacity at Low T
  - Pauli Paramagnetism (at low T):  $\chi = \mu_B^2 D(E_F) = const.$
- 4. Landau's Fermi Liquid Theory: •Quasi-particle: Same charge, spin, momentum as non-interacting electron Adiabatic Continuity, Only valid at low T and low energy Qualitatively explain the susceptibility, heat capacity Only require a Fermi sea Entropy, distribution function unchanged Energy modified by the effective mass & the Fermi interaction function Low energy excitation like single particle  $\varepsilon = \frac{\hbar^2 k^2}{2m^*}$   $\varepsilon_V = \frac{1}{3} \frac{m^* k_F}{\hbar^2} k_B^2 T$   $\chi = \frac{m^* k_F}{\hbar^2 \pi^2} \frac{1}{1 + F_0^a} \mu_B^2$  Landau parameter:  $m^*$ ,  $F_0^a$  Wilson ration:  $R_W = \frac{\pi^2 k_B^2}{3\mu_B^2} \frac{\chi}{\gamma}$  For non-interacting electron gas  $R_W = 1$   $T^2$  Law (Experimental Signature): qp-qp scattering Scattering rate  $\frac{1}{\tau} \sim k_B T \cdot k_B T \propto T^2$  But electron seems to be a wrong place to start for many novel phenomena Spinon, holon, fractional charge: Collective mode looks like a fraction of an electron
- 5. Bloch Theory: Bloch Theorem: For periodic potential,  $\psi_{nk}(r) = e^{ik \cdot r} u_{nk}(r)$ , where  $u_{nk}(r) = u_{nk}(R + r)$ 
  - Momentum is no longer a good quantum number Band index n
  - NFE model: Perturbation to the free electron plane wave states (maximum mixing) (highly delocalized)
  - First order:  $\varepsilon_k^1 = \overline{V}$  Second order (non-degenerate):  $\varepsilon_k^2 = \sum_{g \neq 0} \frac{|V_g|^2}{\varepsilon_k^0 \varepsilon_{k-g}^0}$  Second order (degenerate):  $\varepsilon_k^2 = \pm |V_n|$  Energy gap: Level repulsion, Explains metal or insulator
  - +2 Metals: Band overlap
  - Tight Binding Model: (nearly localized) such as transition metal and rare earth metal with partially filled d and f orbitals.  $\psi_k(r) = \sum_{R} e^{ik \cdot R} \psi_a(r R)$  Overlap integral:  $t_R = \int \psi_a^*(r + R) (\Delta V) \psi_a(r) dr \rightarrow$  Nearest neighbor approximation Band width:  $W \sim 2zt$ , where z is coordination number, t is overlap integral Useful starting point
- 6. Lattice Vibrations: Harmonic Approximation:  $V(a + \delta x) = V_0 + \frac{1}{2}\beta(\delta x)^2$   $\varepsilon = \sum_k (n_k + \frac{1}{2})\hbar\omega$  Phonons: The quantum of the lattice vibration,  $n_k = \frac{1}{e^{\hbar\omega/kT} 1}$  Mono-atomic 1D Chain:  $\omega = 2\sqrt{\frac{\beta}{m}} \left| \sin\left(\frac{aq}{2}\right) \right|$

- Di-atomic 1D Chain: Acoustic & Optical phonon
  Phonon specific heat: Debye model: Assume linear dispersion
  Define a cutoff in the integral: Debye frequency
- $T \rightarrow 0$   $c_V \sim T^3$   $\Leftarrow$ Blackbody radiation
- Specific Heat: 

   Directly related to internal energy
   To extract important microscopic parameters
   To study phase transition
   Calorimetry: What, How, Better resolution and accuracy, Flecibility
   Adiabatic

   Nernst Calorimeter: Slow, Heat leak problem, Need big sample
  - Relaxation time calorimeter:  $\Delta T = \Delta T_0 \mathrm{e}^{-t/\tau}$ , where  $\tau = c_V l/\kappa S$  (with addenda) Advantage: Accurate, Fast, Microgram crystals, Small, Work in extreme conditions Disadvantage: The addenda
  - Membrane calorimeter: Nano-gram crystals, Measure in-situ evaporated thin films, Extreme conditions
  - Heater: Resistance stable with T Thermometer: Resistance has Linear relationship with T
- 8. Anharmonic Potential: Universal  $\Leftarrow V = 0$  when  $r \to \infty$  Phonon-phonon interaction: No longer independent excitations  $\Rightarrow$  Phonon heat conduction (low T, high T) Thermal Expansion:  $\alpha = \frac{1}{l} \frac{\partial^2 l}{\partial T \partial p} = \frac{1}{3V} \frac{\partial^2 V}{\partial T \partial p}$  Provide similar information as specific heat Bad in engineering
  - XRD  $\rightarrow$  Measure l or V, High resolution, Hard to use in extreme physical conditions
  - Capacitive dilatometer: High resolution (capacitance bridge) (0.01 Å), ultra low T and large B (compact design)
    - Negative thermal expansion:  $ZrW_2O_8 \rightarrow Rigid Unit Modes$
- 9. Main Frame: Landau Fermi Liquid Theory + Band Theory

## 3 Transport

- 1. Basic Notions: Movement of Particles or Quantities Non-equilibrium steady state  $J = L \cdot F$ 
  - Very informative and instructive, esp. on Novel materials and in Extreme conditions
  - Normally the first to be carried out Close relations to device applications
- 2. Fractional Quantum Hall Effect: Ultra low *T*, Super strong *B*, Very clean Strong electron correlations
  - Most precise method to measure h
- 3. Cryogenic Technology: Dilution fridge method: He-3 rich & He-3 poor phase at T < 0.87 K
  - He-3 diffuse, absorb heat Down to  $\sim 10 \text{ mK}$
  - Superconducting magnet: up to 20 tesla ← critical field
  - Super high megnetic field: Florida-Bitter resistive magnet, Hybrid magnet

- 4. The Boltzmann Transport Equation:  $\bullet \frac{\partial f_k}{\partial t}\Big|_{\text{diffusion}} + \frac{\partial f_k}{\partial t}\Big|_{\text{field}} + \frac{\partial f_k}{\partial t}\Big|_{\text{scattering}} = 0 \quad \bullet \frac{\partial f_k}{\partial t}\Big|_{\text{diffusion}} = -\dot{\boldsymbol{r}} \cdot \nabla_{\boldsymbol{r}} f_k$   $\bullet \frac{\partial f_k}{\partial t}\Big|_{\text{field}} = -\dot{\boldsymbol{k}} \cdot \nabla_{\boldsymbol{k}} f_k \quad \bullet \frac{\partial f_k}{\partial t}\Big|_{\text{scattering}} = -\frac{f_k f_k^0}{\tau}$
- 5. Electrical Transport:  $\bullet J_e = \sigma E$   $\bullet$  Measurements: Four-probe, Low frequency ac lock-in method
  - Drude model:  $\sigma = \frac{ne^2\tau}{m}$  Semi classical:  $\delta k = \frac{e\tau E}{\hbar}$  Only the surface of the FS changed!
    - Ignore the diffusion effect, Complexity of the FS
  - The Boltzmann transport equation:  $\overleftrightarrow{\sigma} = \frac{1}{4\pi^3} \frac{e^2 \tau}{\hbar} \int \frac{v_k v_k dS_F}{v_k}$
  - Cubic symmetry:  $\sigma_{x,y,z} = \frac{e^2}{3} v_F lD(\varepsilon_F)$
  - Matthiessen's rule: Different scattering mechanisms don't interfere each other  $\Rightarrow \frac{1}{\tau} = \frac{1}{\tau_{imp}} + \frac{1}{\tau_{ph}} + \cdots$
  - Electron-electron scattering:  $\frac{1}{\tau} \sim T^2$
  - Electron-lattice scattering:  $\rho \sim T$  , at high T  $\rho \sim T^5$  , at low T
  - Electron-impurity scattering: Roughly, Temperature-independent Residual resistivity:  $\rho(T=0)$  Residual resistivity ratio (RRR):  $\frac{\rho_{300K}}{\rho_0}$  Higher the better
- 6. Thermal Transport:  $\bullet J_Q = \kappa(-\nabla T)$   $\bullet$  Measurement: One-heater, Two-thermometer
  - Drude:  $J_{Qe,x} = \frac{1}{2}nv_x[\varepsilon(T_{x-v\tau} T_{x+v\tau})] = \frac{1}{3}c_Vvl$   $J_Q = 2\int f_k(\varepsilon_k \mu)v_k dk$   $\kappa_e = \frac{\pi^2}{3}\frac{k_B^2}{e^2}T\sigma$
  - Phonon Thermal Conductivity: Good metals ~ 1%
- 7. Thermoelectric Power: Seebeck Coefficient:  $S = \frac{E}{\nabla T} = \frac{c_V}{3ne}$  The piece of heat carried by each charge e
  - Inversely proportional to  $\varepsilon_F$   $S = \frac{\pi^2}{3} \frac{k_B}{e} k_B T \left( \frac{\partial \ln \sigma(\varepsilon)}{\partial \varepsilon} \right) \Big|_{\varepsilon = \mu}$  Reveal abrupt change of electronic stucture
  - Study novel electronic phases and phase transitions, But poorly understood
  - Thermal couple:  $V = (S_B S_A)(T_x T_0)$  Thermoelectric power generation:  $\Pi$ -junction consisting of N type & P type material  $V = (|S_N| + |S_P|)(T_h T_c)$  No moving part, reliable Environmental friendly Arbitrary Shape & Size Radioisotope thermoelectric generator: For unmanned situations, Low power, Long durations Thermoelectric refrigeration:  $\Pi$ -junction consisting of N type & P type material  $J_Q = J_e(|\Pi_N| + |\Pi_P|)$
- 8. Peltier Effect:  $\bullet J_Q = \Pi J_e$
- 9. Onsager reciprocal relations:  $\bullet \begin{pmatrix} J_e \\ J_Q \end{pmatrix} = \begin{pmatrix} \sigma & \sigma ST \\ \sigma \Pi & -\kappa T \end{pmatrix} \begin{pmatrix} E \\ \frac{\nabla_r T}{T} \end{pmatrix} \bullet \Pi = ST$
- 10. The Thermoelectric Figure of Merit:  $\bullet ZT = \frac{\sigma S^2 T}{\kappa} \quad \bullet ZT \sim 3$ , for application, Now  $ZT \sim 1$ 
  - Now focusing on heavily doped narrow-band semiconductors,  $n \sim 10^{19} 10^{20}/cm^3$ , Not promising because of W-F law Minimize phonon thermal conductivity  $\Rightarrow$  Low Dimension, Amorphous, Nanomaterials  $Bi_2Se_3$  &  $Bi_2Te_3$ : Quasi-2D system (Quintuple-layer) Future focus: Considering Spin, Strong-correlation system

- 11. Magnetic Field: Free electron gas: No magnetoresistance Hall coefficient:  $R_H = \frac{1}{ne}$ 
  - Hall angle:  $\tan \theta = \frac{E_y}{E_x} = \frac{Be\tau}{m} = \omega_c \tau$  Quantum oscillations:  $\omega_c \tau \gg 1$ , Shubnikov-de Haas oscillations
- 12. Thermo-magnetic Transport: Thermal Hall effect: Heat current (x) produces  $\nabla T$  (y)
  - Nernst effect:  $v = \frac{E_y}{\nabla T_x B_z}$ , Powerful technique for novel metals & superconducting vortices in type-II superconductors

### 4 Metal Insulator Transition (MIT)

- 1. I-M Transition within Band Theory: Doping: Donors & Acceptors ⇒ impurity bands
  - Pressure: Structure change ⇒ Overlap ← Tight binding model Wilson transition
- 2. Mott Insulator: Mott's Gedanken Experiment: Increase the distance between atoms: Smaller hopping integral (t) and carrier density (n)
  - Thomas-Fermi Theory: A negative charge added to the Fermi Sea  $\Rightarrow \delta V \Rightarrow \delta n = -D(\varepsilon_F)\delta V \Rightarrow \nabla^2(\delta V) = k^2\delta V$  Yukawa Potential:  $\delta V = \frac{e^2}{r} \mathrm{e}^{-kr}$  Screening length:  $\lambda = \frac{1}{k}, \ k = \sqrt{\frac{4me^2k_F}{\pi\hbar^2}} \propto n^{1/6}$  for 3D FEG Good metals  $\Rightarrow$  Non-interacting FEG
  - Mott Insulator: Coulomb energy cost will exceed the kinetic energy gain Low dimensional materials with large lattice constant IMT: Tune the U/W ratio; Change band filling by doping. Pressure induced IMT: Lattice contraction at IMT ( $\sim 0.2\%$ )  $\Leftarrow$  Metallic bonds Increase of  $m^*$  due to strong electron interaction in doped Mott insulator Many transition metal oxides (TMO): Separated by O, small density; Inner d electrons, weak overlap
  - The Hubbard Model:  $H = -t \sum_{\langle i,j\rangle,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$ , where  $t = \int \varphi_{j}^{*} \left[ V(r) v_{i}(r) \right] \varphi_{i} dr$   $U = \int \left| \varphi_{i} \left( r_{1} \right) \right|^{2} \frac{e^{2}}{r_{12}} \left| \varphi_{i} \left( r_{2} \right) \right|^{2} dr \quad 1^{st} \text{ tern: Hopping tern, } 2^{nd} \text{ tern: on-site Coulomb repulsion tern}$ 
    - Band split: U > W (W is the width of the original band) Energy gap:  $E_g \sim (U W)$
  - Perovskite Structure:  $ABO_3$  A only donates electronic charge and stabilizes the structure For electronic properties, the  $BO_6$  octahedral is most relevant  $RNiO_3$  system: Charge transfer insulators: O's p orbits and Ni's d orbits strongly hybridized. gap  $\sim 10\text{-}30~meV$  Bonding angle: W is the largest for straight bond ( $Ni^{3+} O^{2-} Ni^{3+} 180^{\circ}$ ) and smaller in distorted case. Becomes better insulator with increasing R atomic number (smaller radius, more twisted bond of B O B) Different transition metal: Different d-electron configuration, Different p d hybridization, Different d Different carrier density Extra d or d deficiency: Different hole concentration Substitution of d: Different on site configuration Different dimensionality
  - Magnetic Structure: Most have Antiferromagnetically ordered ground state

- Typical Strongly Correlated Materials: Incompletely filled d or f electron shells with narrow bands
- Wigner Crystal: Crystal of electrons Potential  $\sim \frac{e^2}{r_0}$  Kinetic  $\sim \frac{\hbar^2}{mr_0^2}$  when  $r_0$  is large
- 3. Anderson Localization  $\bullet$  Dilute, Nonmagnetic Impurity, T=0, The distribution of the interaction
  - The Spin Diffusion Puzzle: The relaxation time of donor electron spin is way longer at low concentrations
  - Electron Spin Resonance (ESR): Unpaired electrons, Resonance frequency  $\rightarrow$  microwave (9GHz for 0.3T), challenging, Study electron spin dynamics. Phase sensitive (lock-in) detector, The first derivative of absorption line.
  - The Anderson Hamiltonian:  $H = \sum_{i} \varepsilon_{i} n_{i} + \sum_{i,j} t_{ij} c_{i}^{\dagger} c_{j}$  V = 0, as  $\varepsilon_{i}$  is const, Tight-binding model t = 0, Atomic orbitals at each site  $\frac{V}{W} \ll 1 \Rightarrow$  Impurity scattering of Bloch waves  $\frac{V}{W} > 1 \Rightarrow$  Localization
  - Mobility Edge:  $\pm \varepsilon_c$ :  $0 < \frac{V}{W} < 1$ , Separating localized and non-localized states Tuned by changing the level of disorder. MIT:  $\varepsilon_F$  tuned by doping level or pressure Trait: No energy gap in DOS near  $\varepsilon_F$  (Psesdogap) The electron number need not be integer Coulomb repulsion in unnecessary
  - ► Near the localization transition Still Controversy
  - Mott's Minimum Conductivity: Localization transition is discontinuous. • $lk_F > 1$ ,  $l \ge a$  • $\sigma_{min}^{3D} \approx \frac{1}{3\pi^2} \frac{e^2}{\hbar} \frac{1}{a}$
  - Scaling Theory:  $\frac{\hbar}{e^2}G(L) = g(L) \approx \frac{\Delta E}{\Delta V}$   $\beta(g) = \frac{d \ln g}{d \ln L}$  The transition is continuous  $\beta(g_c) = 0$ ,  $g_c \Rightarrow$  Unstable fixed point  $g_0 > g_c \Rightarrow$  Conductor;  $g_0 < g_c \Rightarrow$  Insulator No extended states for any degree of disorder in 1D and 2D. Doesn't consider other effects can destroy the localization (Magnetic field, S-O coupling, E-E interaction ...)
  - Weak Localization:  $l \ll l_i < L < \xi$ , where l is for elastic scattering,  $l_i$  is for inelastic scattering, L is the sample's scale,  $\xi$  is the localization length,  $k_F l \gg 1$  Self-crossing loops:  $P = |A_1 + A_2|^2 = 2 \times 2|A|^2$ , decreases the conductivity, Localization Dephasing length:  $\tau_{\phi}$ , Loss of phase coherence will destroy weak localization

$$\frac{\delta\sigma}{\sigma} = -\gamma_d \begin{cases} \left(\frac{\tau_\phi}{\tau}\right)^{1/2} &= T^{-\eta/2} &, d = 1\\ \hbar \ln\left(\frac{\tau_\phi}{\tau}\right) &= \frac{\eta\hbar}{2} \ln\frac{\hbar}{k_BT} &, d = 2\\ \hbar^2 \left(\frac{\tau_\phi}{\tau}\right)^{-1/2} &= \hbar^2 T^{\eta/2} &, d = 3 \end{cases}$$

- Wave Property: Reported for light waves, microwaves, sound waves, matter waves (BEC)
- Non-interacting theory, hard to ideally realize in Solid
- 4. Charge Density Wave (CDW): (Lattice distortion induced)
  - Peierls' Theorem: 1D materials are insulating The distortion of lattice opens an energy gap around the original  $E_F$  The lattice distortion induced a CDW with  $\lambda = \frac{\pi}{k_F}$ ,  $k = 2k_F$ ,  $\frac{\lambda}{a}$  can be irrational Mechanism: e-ph coupling an electron-hole pair is created by a phonon Kohn Anomaly: Electron-phonon interaction has a strong influence on the phonon spectrum near  $q = 2k_F \Rightarrow$  Phonon softening 1D, Phonon Frequency

- can drop to 0 Hamiltonian:  $H = \sum_{k,\sigma} \varepsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_q \hbar \omega_q b_q^\dagger b_q + \sum_{k,\sigma,q} g(k) c_{k+q,\sigma}^\dagger c_{k,\sigma} \left( b_q + b_{-q}^\dagger \right)$
- CDW gap:  $\Delta=2W\mathrm{e}^{-1/g}$  CDW transition T:  $T_P=\frac{\Delta(0)}{1.76k_B}$  , From BCS theory
- TTF (donor)-TCNQ (acceptor): MIT:  $\sigma$  drops at 55K and 38K (superstructure with 3.4a), has a thermal activation behavior  $\Rightarrow$  Energy gap Kohn anomaly: Neutron inelastic scattering Fermi Surface Nesting: two pieces of parallel FS Quasi-1D: FS as parallel planes  $\Rightarrow$  Warped, Still has FS nesting
- Pressure: Pressure  $\uparrow \rightarrow T_{CDW} \uparrow \Leftarrow$  Imperfect nesting
- NbS  $e_3$ : Two sharp increases of  $\rho$  at 144K and 59K, remains metallic down to T=0
- 3 types of chains, 2 successive Peierls trasitions
- NbS  $e_2$ : Layered structure, can be grown to extremely high quality A superstructure with 3a
- CDW state below 33K while being metallic, Superconductor below 7.2K Multiband material

#### 5. Jahn-Teller Theorem:

- Non-linear degenerate molecules cannot be stable. Change of electronic structure ⇒ MIT
- Energy Gain: Electronic Energy Cost: Elastic, Hund's rule coupling
- Octahedral complexes of the transition metals: Elongation along z, Lower  $d_{z^2}$ , Upper  $d_{x^2-y^2}$
- $K_xC_{60}$  Monolayers: MIT From  $K_3C_{60}$  to  $K_4C_{60}$  •Electrons in the LUMO orbital interact with certain vibrational mode of the  $C_{60}$  molecule and cause a permanent molecular distortion

## 5 Low Dimensional Electron Systems

- 1. The Motion of Microscopic Degrees-of-freedom
- 2. Examples:
  - ullet Quasi-2D: ullet Giant magnetoresistance in multilayer magnetic films ullet High  $T_c$  superconductivity in layered copper oxides
  - 2D: QHE in semiconductor MOSFET Fractional QHE in semiconductor heterostructures
  - Quasi-1D: CDW in TTF-TCNQ and  $NbSe_3$
  - 1D: Luttinger liquid in semiconducter nanowires Carbon nanotubes
  - 0D: Quantum dots Molecular electronics and magnetism
- 3. Affect Propagation of waves and Formation of ordered phases
- 4. Susceptible to defects and thermal fluctuations
- 5. Electron DOS shows quantized behavior

#### 6. Surface and Interface:

- Surface is a special type of interface
- Why? : Break the periodicity in one dimension Ideal for low-dimensional systems Information and electronics industries heavily rely on Surface catalysis can greatly increase the rate
- Surface structure: Techniques (Surface sensitive): LEED, RHEED, SXD Surface relaxation: May extend several layers Surface reconstruction: Si (100) -- dimer rows, Si (111) --  $7 \times 7$  superstructure, Au (111) --  $22 \times \sqrt{3}$  superlattice (hcp & fcc) Mechanism: Minimization of surface free energy Semiconductors: Surface healing process  $\rightarrow$  Reduce dangling bonds Metals: Formation of denser packing  $\rightarrow$  Maximize the surface metallic bonding
- Surface Electronic Structure: Parallel: Bloch waves Perpendicular: Extended Bloch wave within the crystal, Exponentially decaying tail outside the surface Surface state energy may lies within the bulk gap  $\Rightarrow$  Surface electronic state, for both semiconductors and conductors (specific directions) Energy:  $E_s = E_0 + \frac{\hbar^2 k_{\parallel}^2}{2m^*}$
- Applications: Surface catalysis
- Creation: Cleaving method: Materials with Van der Waals force between layers, In ultrahigh vacuum (UHV) environment

#### 6 Others

- 1. Topological Insulator: Single electron model Mainly spin-orbit interaction
- 2. Superfluidity: Liquid He-4 & He-3: Low boiling  $T \Leftarrow$  Weak van der Waals force & low atomic mass He-4 (Boson) < 2.17 K (BEC), He-3 (Fermion) < 2.49 mK (BCS)
- 3. Diamond: Indirect band gap:  $E_g = 5.5 \ eV$ , good insulator

