

Notes on Introduction to Condensed Matter Physics

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1 Basic Introduction

1. Condensed Matter: $\sim 10^{23} /cm^3$
2. Major Study: Electrons, Phonons, The interactions between
3. Drives: New materials & New technologies

2 Conventional Metal Physics: Electrons and Phonons

1. Basic Properties of Normal Metals: • Ductile • Excellent electrical conductor • Excellent thermal conductor • Most are weak paramagnet, some ferromagnet • Opaque
• At low T : ρ increases with T $\chi \sim Const.$ $c_V \propto T$
2. Drude Free Electron Model:
 - Assumptions: • Free electrons (Ignore interaction with lattice) • Independent electrons (Ignore interactions between electrons) • Electrons were treated as independent classical particles
 - Maxwell-Boltzmann distribution

- Successes: • Electrical conductivity and thermal conductivity • Wiedemann-Franz law (by luck!)
 - The Hall effect and magnetoresistance • AC conductivity and optical properties of metals
 - Problems: • Heat capacity puzzle: $c_V = \frac{3}{2}nk_B = \text{Const.}$ • The susceptibility puzzle: χ does not change with temperature, non-Curie like $\chi \sim 1/T$
3. The Sommerfeld Model: • Free Electron Gas + Schrodinger Equation + Fermi Statistics
- The Fermi Surface: In 3D $E_F \sim n^{2/3}$ • Typical Values: $E_F \sim 7 \text{ eV}$, $k_F \sim 1 \times 10^8 \text{ cm}^{-1}$, $T_F \sim 8 \times 10^4 \text{ K}$, $v_F \sim 2 \times 10^8 \text{ cm/s}$ • DOS In 3D: $D(\epsilon) \sim \epsilon^{1/2}$ • Linear T Heat Capacity at Low T
 - Pauli Paramagnetism (at low T): $\chi = \mu_B^2 D(E_F) = \text{const.}$
 - Successes: • Realized the importance of Fermi distribution • Established the k-space language for electrons • Introduced Fermiology • Resolved the Pauli susceptibility puzzle • Resolved the heat capacity puzzle • Resolved the thermopower puzzle • Explained the Wiedemann-Franz Law
4. Landau's Fermi Liquid Theory: • Quasi-particle: Same charge, spin, momentum as non-interacting electron • Adiabatic Continuity, Only valid at low T and low energy • Qualitatively explain the susceptibility, heat capacity Only require a Fermi sea • Entropy, distribution function unchanged
- Energy modified by the effective mass & the Fermi interaction function • Low energy excitation like single particle • $\epsilon = \frac{\hbar^2 k^2}{2m^*}$ • $c_V = \frac{1}{3} \frac{m^* k_F}{\hbar^2} k_B^2 T$ • $\chi = \frac{m^* k_F}{\hbar^2 \pi^2} \frac{1}{1+F_0^a} \mu_B^2$ • Landau parameter: m^*, F_0^a
 - Wilson ration: $R_W = \frac{\pi^2 k_B^2 \chi}{3 \mu_B^2 \gamma}$ • For non-interacting electron gas $R_W = 1$ • T^2 Law (Experimental Signature): qp-qp scattering • Scattering rate $\frac{1}{\tau} \sim k_B T \cdot k_B T \propto T^2$ • But electron seems to be a wrong place to start for many novel phenomena • Spinon, holon, fractional charge: Collective mode looks like a fraction of an electron
5. Bloch Theory: • Bloch Theorem: For periodic potential, $\psi_{nk}(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} u_{nk}(\mathbf{r})$, where $u_{nk}(\mathbf{r}) = u_{nk}(\mathbf{R} + \mathbf{r})$ • Momentum is no longer a good quantum number • Band index n
- NFE model: Perturbation to the free electron plane wave states (maximum mixing) (highly delocalized) • First order: $\epsilon_k^1 = \bar{V}$ • Second order (non-degenerate): $\epsilon_k^2 = \sum_{g \neq 0} \frac{|V_g|^2}{\epsilon_k^0 - \epsilon_{k-g}^0}$ • Second order (degenerate): $\epsilon_k^2 = \pm |V_n|$ • Energy gap: Level repulsion, Explains metal or insulator
 - +2 Metals: Band overlap
 - Tight Binding Model: (nearly localized) such as transition metal and rare earth metal with partially filled d and f orbitals. • $\psi_k(\mathbf{r}) = \sum_{\mathbf{R}} e^{i\mathbf{k} \cdot \mathbf{R}} \psi_a(\mathbf{r} - \mathbf{R})$ • Overlap integral: $t_{\mathbf{R}} = \int \psi_a^*(\mathbf{r} + \mathbf{R})(\Delta V) \psi_a(\mathbf{r}) d\mathbf{r} \rightarrow$ Nearest neighbor approximation • Band width: $W \sim 2zt$, where z is coordination number, t is overlap integral • Useful starting point
6. Lattice Vibrations: • Harmonic Approximation: $V(a + \delta x) = V_0 + \frac{1}{2}\beta(\delta x)^2$ • $\epsilon = \sum_k (n_k + \frac{1}{2}) \hbar \omega$

- Phonons: The quantum of the lattice vibration, $n_k = \frac{1}{e^{\hbar\omega/kT} - 1}$
 - Mono-atomic 1D Chain: $\omega = 2\sqrt{\frac{\beta}{m}} \left| \sin\left(\frac{aq}{2}\right) \right|$
 - Di-atomic 1D Chain: Acoustic & Optical phonon
 - Phonon specific heat: Debye model:
 - Assume linear dispersion
 - Define a cutoff in the integral: Debye frequency
 - $T \rightarrow 0$ $c_V \sim T^3 \Leftarrow$ Blackbody radiation
7. Specific Heat:
 - Directly related to internal energy
 - To extract important microscopic parameters
 - To study phase transition
 - **Calorimetry**: What, How, Better resolution and accuracy, Flecibility
 - **Adiabatic Nernst Calorimeter**: Slow, Heat leak problem, Need big sample
 - **Relaxation time calorimeter**: $\Delta T = \Delta T_0 e^{-t/\tau}$, where $\tau = c_V l / \kappa S$ (with addenda)
 - Advantage: Accurate, Fast, Microgram crystals, Small, Work in extreme conditions
 - Disadvantage: The addenda
 - **Membrane calorimeter**: Nano-gram crystals, Measure in-situ evaporated thin films, Extreme conditions
 - **Heater**: Resistance stable with T
 - **Thermometer**: Resistance has Linear relationship with T
8. Anharmonic Potential:
 - Universal $\Leftarrow V = 0$ when $r \rightarrow \infty$
 - Phonon-phonon interaction: No longer independent excitations \Rightarrow Phonon heat conduction (low T , high T)
 - Thermal Expansion:
 - $\alpha = \frac{1}{l} \frac{\partial^2 l}{\partial T \partial p} = \frac{1}{3V} \frac{\partial^2 V}{\partial T \partial p}$
 - Provide similar information as specific heat
 - Bad in engineering
 - **XRD** \rightarrow Measure l or V , High resolution, Hard to use in extreme physical conditions
 - **Capacitive dilatometer**: High resolution (capacitance bridge) (0.01 Å), ultra low T and large B (compact design)
 - Negative thermal expansion: $ZrW_2O_8 \rightarrow$ Rigid Unit Modes
9. Main Frame: Landau Fermi Liquid Theory + Band Theory

3 Transport

1. Basic Notions:
 - Movement of Particles or Quantities
 - Non-equilibrium steady state
 - $\mathbf{J} = L \cdot \mathbf{F}$
 - Very informative and instructive, esp. on Novel materials and in Extreme conditions
 - Normally the first to be carried out
 - Close relations to device applications
2. Fractional Quantum Hall Effect:
 - Ultra low T , Super strong B , Very clean
 - Strong electron correlations
 - Most precise method to measure h
3. **Cryogenic Technology**:
 - **Dilution fridge method**: He-3 rich & He-3 poor phase at $T < 0.87 K$
 - He-3 diffuse, absorb heat
 - Down to ~ 10 mK

- **Superconducting magnet**: up to 20 tesla \Leftarrow critical field
 - **Super high magnetic field**: Florida-Bitter resistive magnet, Hybrid magnet
4. The Boltzmann Transport Equation: • $\left. \frac{\partial f_k}{\partial t} \right|_{\text{diffusion}} + \left. \frac{\partial f_k}{\partial t} \right|_{\text{field}} + \left. \frac{\partial f_k}{\partial t} \right|_{\text{scattering}} = 0$ • $\left. \frac{\partial f_k}{\partial t} \right|_{\text{diffusion}} = -\dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f_k$
- $\left. \frac{\partial f_k}{\partial t} \right|_{\text{field}} = -\dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f_k$ • $\left. \frac{\partial f_k}{\partial t} \right|_{\text{scattering}} = -\frac{f_k - f_k^0}{\tau}$
5. Electrical Transport: • $\mathbf{J}_e = \sigma \mathbf{E}$ • Measurements: Four-probe, Low frequency ac lock-in method
- Drude model: $\sigma = \frac{ne^2\tau}{m}$ • Semi classical: $\delta \mathbf{k} = \frac{e\tau \mathbf{E}}{\hbar}$ • Only the surface of the FS changed !
 - Ignore the diffusion effect, Complexity of the FS
 - The Boltzmann transport equation: $\overleftrightarrow{\sigma} = \frac{1}{4\pi^3} \frac{e^2\tau}{\hbar} \int \frac{\mathbf{v}_k \mathbf{v}_k dS_F}{v_k}$
 - Cubic symmetry: $\sigma_{x,y,z} = \frac{e^2}{3} v_F l D(\epsilon_F)$
 - Matthiessen's rule: Different scattering mechanisms don't interfere each other $\Rightarrow \frac{1}{\tau} = \frac{1}{\tau_{imp}} + \frac{1}{\tau_{ph}} + \dots$
 - Electron-electron scattering: $\frac{1}{\tau} \sim T^2$
 - Electron-lattice scattering: $\rho \sim T$, at high T $\rho \sim T^5$, at low T
 - Electron-impurity scattering: • Roughly, Temperature-independent • Residual resistivity: $\rho(T=0)$ • Residual resistivity ratio (RRR): $\frac{\rho_{300K}}{\rho_0}$ Higher the better
6. Thermal Transport: • $\mathbf{J}_Q = \kappa(-\nabla T)$ • Measurement: One-heater, Two-thermometer
- Drude: $J_{Qe,x} = \frac{1}{2} n v_x [\epsilon(T_{x-v\tau} - T_{x+v\tau})] = \frac{1}{3} c_V v l$ • $\mathbf{J}_Q = 2 \int f_k(\epsilon_k - \mu) \mathbf{v}_k d\mathbf{k}$ • $\kappa_e = \frac{\pi^2}{3} \frac{k_B^2}{e^2} T \sigma$
 - Phonon Thermal Conductivity: Good metals $\sim 1\%$
7. Thermoelectric Power: • Seebeck Coefficient: $S = \frac{E}{\nabla T} = \frac{c_V}{3ne}$ • The piece of heat carried by each charge e • Inversely proportional to ϵ_F • $S = \frac{\pi^2}{3} \frac{k_B}{e} k_B T \left(\frac{\partial \ln \sigma(\epsilon)}{\partial \epsilon} \right) \Big|_{\epsilon=\mu}$ • Reveal abrupt change of electronic structure • Study novel electronic phases and phase transitions, But poorly understood
- **Thermal couple**: $V = (S_B - S_A)(T_x - T_0)$ • **Thermoelectric power generation**: Π -junction consisting of N type & P type material • $V = (|S_N| + |S_P|)(T_h - T_c)$ • No moving part, reliable • Environmental friendly • Arbitrary Shape & Size • **Radioisotope thermoelectric generator**: For unmanned situations, Low power, Long durations • **Thermoelectric refrigeration**: Π -junction consisting of N type & P type material • $J_Q = J_e(|\Pi_N| + |\Pi_P|)$
8. Peltier Effect: • $\mathbf{J}_Q = \Pi \mathbf{J}_e$
9. Onsager reciprocal relations: • $\begin{pmatrix} J_e \\ J_Q \end{pmatrix} = \begin{pmatrix} \sigma & \sigma S T \\ \sigma \Pi & -\kappa T \end{pmatrix} \begin{pmatrix} E \\ \frac{\nabla_r T}{T} \end{pmatrix}$ • $\Pi = S T$
10. The Thermoelectric Figure of Merit: • $ZT = \frac{\sigma S^2 T}{\kappa}$ • $ZT \sim 3$, for application, Now $ZT \sim 1$
- Now focusing on heavily doped narrow-band semiconductors, $n \sim 10^{19} - 10^{20}/\text{cm}^3$, Not promising because of W-F law • Minimize phonon thermal conductivity \Rightarrow Low Dimension, Amorphous,

Nanomaterials • Bi_2Se_3 & Bi_2Te_3 : Quasi-2D system (Quintuple-layer) • Future focus: Considering Spin, Strong-correlation system

11. Magnetic Field: • Free electron gas: • No magnetoresistance • Hall coefficient: $R_H = \frac{1}{ne}$
 • Hall angle: $\tan \theta = \frac{E_y}{E_x} = \frac{B e \tau}{m} = \omega_c \tau$ • Quantum oscillations: $\omega_c \tau \gg 1$, Shubnikov-de Haas oscillations
12. Thermo-magnetic Transport: • Thermal Hall effect: Heat current (x) produces ∇T (y)
 • Nernst effect: $\nu = \frac{E_y}{\nabla T_x B_z}$, Powerful technique for novel metals & superconducting vortices in type-II superconductors

4 Metal Insulator Transition (MIT)

1. I-M Transition within Band Theory: • Doping: Donors & Acceptors \Rightarrow impurity bands
 • Pressure: Structure change \Rightarrow Overlap \Leftarrow Tight binding model • Wilson transition
2. Mott Insulator: •

5 Others

1. Topological Insulator: • Single electron model • Mainly spin-orbit interaction
2. Superfluidity: • Liquid He-4 & He-3: Low boiling $T \Leftarrow$ Weak van der Waals force & low atomic mass • He-4 (Boson) $< 2.17 K$ (BEC), He-3 (Fermion) $< 2.49 mK$ (BCS)
3. Diamond: Indirect band gap: $E_g = 5.5 eV$, good insulator

