Fréchet derivatives under various parameterizations using chain rule

for LEGO module m_parameterization.f90

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Overview

$\boxed{\text{WaveEq lm} \Rightarrow \text{optim x,g}}$

- WaveEq uses lm and produces gkpa, glda, gmu and grho under moduli-density parameterization.
- m_parameterization.f90 converts them to user-specified parameterization, which can be
  - moduli-density (kpa, lda, mu, rho)
  - velocities-density (vp, vs, rho)
  - velocities-impedance (vp, vs, Ip)
  - slowness-density (sp, sps, rho)
- Also we may consider "passive parameters" for hard constraints (e.g. Gardner) between these parameters.

PARAMETERIZATION velocities-density

isotropic ACoustic

$$\begin{cases} \kappa = \rho V_P^2 \\ \rho_0 = \rho \end{cases}$$

$$\begin{aligned}
\nabla_{V_P} &= \nabla_\kappa \frac{\partial \kappa}{\partial V_P} + \nabla_{\rho_0} \frac{\partial \rho_0}{\partial V_P} \\
&= \nabla_\kappa 2\rho V_P \\
\nabla_\rho &= \nabla_\kappa \frac{\partial \kappa}{\partial \rho} + \nabla_{\rho_0} \frac{\partial \rho_0}{\partial \rho} \\
&= \nabla_\kappa V_P^2 + \nabla_{\rho_0}
\end{aligned}$$

$$\begin{cases} V_P = \sqrt{\kappa/\rho_0} \\ \rho = \rho_0 \end{cases}$$

$$\begin{aligned}
\nabla_\kappa &= \nabla_{V_P} \frac{\partial V_P}{\partial \kappa} + \nabla_\rho \frac{\partial \rho}{\partial \kappa} \\
&= \nabla_{V_P} 0.5(\kappa/\rho)^{0.5}/\rho \\
\nabla_{\rho_0} &= \nabla_{V_P} \frac{\partial V_P}{\partial \rho_0} + \nabla_\rho \frac{\partial \rho}{\partial \rho_0} \\
&= \nabla_{V_P} (-0.5)(\kappa/\rho)^{0.5} \rho^{-2} + \nabla_\rho
\end{aligned}$$

PARAMETERIZATION velocities-density + PASSIVE gardner

isotropic ACoustic

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$$\begin{aligned}
\nabla_{V_P} &= \nabla_\kappa \frac{\partial \kappa}{\partial V_P} + \nabla_{\rho_0} \frac{\partial \rho_0}{\partial V_P} \\
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&= \nabla_{V_P}(-0.5)(\kappa/\rho)^{0.5}\rho^{-2} + \nabla_\rho
\end{aligned}$$

PARAMETERIZATION velocities-density

P-SV

$$\begin{cases} \lambda = \rho(V_P^2 - 2V_S^2) \\ \mu = \rho V_S^2 \\ \rho_0 = \rho \end{cases}$$

$$\begin{aligned} \nabla_{V_P} &= \nabla_{\kappa} \frac{\partial \kappa}{\partial V_P} + \nabla_{\rho_0} \frac{\partial \rho_0}{\partial V_P} \\ &= \nabla_{\kappa} 2\rho V_P \\ \nabla_{V_S} &= \nabla_{\kappa} \frac{\partial \kappa}{\partial V_P} + \nabla_{\rho_0} \frac{\partial \rho_0}{\partial V_P} \\ &= \nabla_{\kappa} 2\rho V_P \\ \nabla_{\rho} &= \nabla_{\kappa} \frac{\partial \kappa}{\partial \rho} + \nabla_{\rho_0} \frac{\partial \rho_0}{\partial \rho} \\ &= \nabla_{\kappa} V_P^2 + \nabla_{\rho_0} \end{aligned}$$

$$\begin{cases} V_P = \sqrt{\frac{\lambda + 2\mu}{\rho_0}} \\ V_S = \sqrt{\mu/\rho_0} \\ \rho = \rho_0 \end{cases}$$

$$\begin{aligned} \nabla_{\kappa} &= \nabla_{V_P} \frac{\partial V_P}{\partial \kappa} + \nabla_{\rho} \frac{\partial \rho}{\partial \kappa} \\ &= \nabla_{V_P} 0.5(\kappa/\rho)^{0.5}/\rho \\ \nabla_{\rho_0} &= \nabla_{V_P} \frac{\partial V_P}{\partial \rho_0} + \nabla_{\rho} \frac{\partial \rho}{\partial \rho_0} \\ &= \nabla_{V_P}(-0.5)(\kappa/\rho)^{0.5}\rho^{-2} + \nabla_{\rho} \end{aligned}$$