

# Math 2513

Final Exam – Due: August 6, 2024

Q1 – What are the three equations for the line going through the points  $(0,1,2)$  and  $(-1,3,2)$ ? Label each equation.

Q2 – What are the three equations of the plane going through the point  $(9,2,3)$ , with the normal vector  $[1,2,0]$ ? Label each equation.

Q3 – Show that Clairaut's Theorem holds for the equation  $e^{xy}\sin(x^2)$ .

Q4 – Given the surface  $z=x^2y + xy^2$ , find the tangent plane at the point  $(0,0,0)$ .

Q5 – Given the equation  $f(x,y) = e^x\sin(y)$ , we know that the value at  $(0,0)$  is 0. What is the constant approximation of the value at  $(0.5,0.5)$ ? What is the linear approximation at the same point? What is the actual value?

Q6 – What is the 3<sup>rd</sup> order Taylor polynomial of the function  $f(x,y,z) = 5x + e^{yz}$ , about the point  $(0,0,0)$ ?

Q7 - What is the directional derivative of the function  $f(x,y) = xy^2$  in the direction of the x-axis unit vector?

Q8 – What are the critical points of the equation  $f(x,y) = x^2y$ ? Are any minimums, maximums or saddle points?

Q9 – Using Lagrange multipliers, find the maxima and minima of the surface  $(xy+5y^2)$  along the curve  $x^2+y^2=2$ .

Q10 – Is the function  $f(x,y) = \sin(x) + 7y$  separable? What is the integral over the domain  $0 \leq x \leq 3$ ,  $0 \leq y \leq x$ ?

Q11 – Convert the following points from Cartesian to both cylindrical and spherical.

1. (2,3,-9)
2. (0,0,42)
3. (-3,4,-2)

Q12 – What is the integral of the equation  $f(x,y,z) = 5xy$ , over the cylinder with a diameter of 8 and a height of 3?

Q13 – What is the integral of the equation  $f(x,y,z) = 7yz$ , over the sphere of radius 6?

Q14 – Is the vector field  $F = (e^x)i + (2xy)j + (z-x)k$  conservative?

Q15 – What is the potential function for Q14?

Q16 – Given the potential function  $5xe^{y+z}$ , what is the associated conservative vector function?

Q17 – What is the integral of the vector function from Q16, along the curve  $r(t) = [t, t^2, t]$ , from the point (0,0,0) to the point (4,16,4)?

Q18 - Use Green's theorem to evaluate the line integral  $\int_C -y/(x^2+y^2) dx + \int_C x/(x^2 + y^2) dy$  where C is the arc of the parabola  $y = 0.25 x^2 + 1$  from (-2, 2) to (2, 2).

Q19 – Let S be the unit sphere, centered on the origin oriented by the outward pointing normal. What is the flux of the vector function  $F = (x)i + (y)j + (z^2)k$ ?

Q20 – Find the surface area of the part of the paraboloid  $z = a^2 - x^2 - y^2$  which lies above the xy-plane.

## Bonus

BQ1 – What is the fundamental theorem of line integrals?

BQ2 – Find the tangent plane to the unit sphere at the point (1,0,0).

BQ3 – Using the chain rule, what is the second derivative of  $f(x,y) = e^{xy} + y$  with respect to  $t$ , where  $x = \sin(t)$  and  $y = \cos(t)$ .

BQ4 – What is the shape of the object defined by the equation  $f(x,y,z) = x^2 + 2x - y^2 - 5z + z^2$  ?  
Where is the center of this object?

BQ5 – What are the tangent vectors for the surface  $x^3 - 12y^2$ ?