Math 2513 - Assignment 2

A1

$$f(x,y) = \frac{x^4}{y^3} \qquad x_0 = 1 \qquad \Delta x = -0.1$$

$$y_0 = 5 \qquad \Delta y = 0.01$$

$$f(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y$$

$$\approx \frac{1^4}{5^3} + \frac{4\cdot(1)^3}{5^3}\cdot(-0.1) + \frac{-3\cdot(1)^4}{5^4}\cdot 0.01$$

$$\approx \frac{1}{5^4} - \frac{0.4}{5^3} - \frac{0.03}{5^4}$$

$$\approx \frac{5-2-0.03}{5^4}$$

$$\approx \frac{2.97}{5^4}$$

A2

$$f(x) = \sin(x) \qquad x_0 = 90^\circ \qquad \Delta x = 1.1^\circ$$

$$f(x) \approx f(x_0) + f_x(x_0)\Delta x$$

$$\approx \sin(90^\circ) + \cos(90^\circ)(1.1^\circ)$$

$$\approx 1 + 0\cdot 1.1$$

$$\approx 1$$

A3

$$\vec{\nabla} f(x,y) = f_x \hat{\imath} + f_y \hat{\jmath}$$

$$f_x = ye^{xy} \sin(x+y) + e^{xy} \cos(x+y)$$

$$f_y = xe^{xy} \sin(x+y) + e^{xy} \cos(x+y)$$

$$\therefore \ \vec{\nabla} f = \left[ye^{xy} \sin(x+y) + e^{xy} \cos(x+y)\right] \hat{\imath} + \left[xe^{xy} \sin(x+y) + e^{xy} \cos(x+y)\right] \hat{\jmath}$$

A4

$$f(x,y,z) = 2x^3 + 5yz$$

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

$$f_r = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$$

$$= 6x^2 \cdot \sin\theta \cos\phi + 5z \sin\theta \sin\phi + 5y \cdot \cos\theta$$

$$= 6r^2 \sin^2\theta \cos^2\phi \sin\theta \cos\phi + 5r\cos\theta \sin\theta \sin\phi + 5r \sin\theta \sin\phi \cos\theta$$

$$= 6r^2 \sin^3\theta \cos^3\phi + 10r \sin\theta \cos\theta \sin\phi$$

$$f_\theta = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \theta}$$

$$= 6x^2 \cdot r\cos\theta \cos\phi + 5z \cdot r\cos\theta \sin\phi + 5y \cdot (-r\sin\theta)$$

$$= 6r^2 \sin^2\theta \cos^2\phi \, r\cos\theta \cos\phi + 5r\cos\theta \cos\theta \sin\phi - 5r\sin\theta \sin\phi \, r\sin\theta$$

$$= 6r^3 \cos\theta \sin^2\theta \cos^3\phi + 5r\cos^2\theta \sin\phi - 5r^2 \sin^2\theta \sin\phi$$

$$f_\phi = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \phi} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \phi}$$

$$= 6x^2 \cdot (-r\sin\theta \sin\phi) + 5z \cdot r\sin\theta \cos\phi + 5y \cdot 0$$

$$= -6r^2 \sin^2\theta \cos\phi \, r\sin\theta \sin\phi + 5r\cos\theta \, r\sin\theta \cos\phi$$

$$= -6r^3 \sin^3\theta \sin\phi \cos\phi + 5r^2 \sin\theta \cos\theta \cos\phi$$

A5

$f(x,y) = 5x^2 + e^y \qquad (x_0, y_0) = (0,0) \qquad \Delta x = x - x_0$

$\Delta y = y - y_0$

$f_x = 10x \qquad f_y = e^y$

The only non-zero terms are

$f_{xy} = 0 \qquad f_{yx} = 0$

$f_{xx} = 10 \qquad f_{yy} = e^y$

$f_{yxx} = 0$

$f_{xyy} = 0 \qquad f_{yxy} = 0$

$f_{xyx} = 0 \qquad f_{yyx} = 0$

$f_{xxy} = 0 \qquad f_{yyy} = e^x$

$f_{xxx} = 0$

l	m
0	0
1	0
2	0
0	1
0	2
0	3

$$f(x_0 + \Delta x, y_0 + \Delta y) \approx \sum \frac{1}{l!\,m!} \frac{\partial^{l+m}}{\partial x^l \partial y^m} f(x_0, y_0)\, \Delta x^l \Delta y^m$$

$$\approx \frac{1}{0!\,0!} f(x_0, y_0) + \frac{1}{1!\,0!} f_x(x_0,y_0)\,\Delta x + \frac{1}{2!\,0!} f_{xx}(x_0, y_0)(\Delta x)^2$$

$$+ \frac{1}{0!\,1!} f_y(x_0,y_0)\,\Delta y + \frac{1}{0!\,2!} f_{yy}(x_0,y_0)(\Delta y)^2 + \frac{1}{0!\,3!} f_{yyy}(x_0,y_0)(\Delta y)^3$$

$$\approx 5\cdot 0^2 + e^0 + 10\cdot 0\cdot(x-0) + \tfrac{1}{2}\cdot 10\cdot(x-0)^2 + e^0(y-0) + \tfrac{1}{2} e^0\cdot(y-0)^2 + \tfrac{1}{6} e^0 (y-0)^3$$

$$\approx 5x^2 + y + \tfrac{1}{2}y^2 + \tfrac{1}{6}y^3$$

A6

$f(x,y) = 3x^2 - 12y$ $\qquad$ $(x_0, y_0) = (4, 5)$

$\vec{u} = [-1, -1]$

$|\vec{u}| = \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{2}$

$\vec{\nabla} f = 6x\,\hat{i} - 12\,\hat{j}$

$D_{\vec{u}} = \vec{\nabla} f \cdot \vec{u}$

$= (6x\,\hat{i} - 12\,\hat{j}) \cdot (-1\,\hat{i} - 1\,\hat{j})$

$= -6x + 12$

$D_{\frac{\vec{u}}{|\vec{u}|}} = \vec{\nabla} f \cdot \frac{\vec{u}}{|\vec{u}|}$

$= \frac{1}{\sqrt{2}}(-6x + 12)$