Math 2513

Final Exam – Due: August 6, 2024

Q1 – What are the three equations for the line going through the points (0,1,2) and (-1,3,2)? Label each equation.

Q2 – What are the three equations of the plane going through the point (9,2,3), with the normal vector [1,2,0]? Label each equation.

Q3 – Show that Clairault's Theorem holds for the equation $e^{xy}\sin(x^2)$.

Q4 – Given the surface $z=x^2y + xy^2$, find the tangent plane at the point (0,0,0).

Q5 – Given the equation $f(x,y) = e^x\sin(y)$, we know that the value at (0,0) is 0. What is the constant approximation of the value at (0.5,0.5)? What is the linear approximation at the same point? What is the actual value?

Q6 – What is the 3rd order Taylor polynomial of the function $f(x,y,z) = 5x + e^{yz}$, about the point (0,0,0)?

Q7 -   What is the directional derivative of the function $f(x,y) = xy^2$ in the direction of the x-axis unit vector?

Q8 – What are the critical points of the equation $f(x,y) = x^2y$? Are any minimums, maximums or saddle points?

Q9 – Using Lagrange multipliers, find the maxima and minima of the surface $(xy+5y^2)$ along the curve $x^2+y^2=2$.

Q10 – Is the function $f(x,y) = \sin(x) + 7y$ separable? What is the integral over the domain $0\leq x\leq 3$, $0\leq y\leq x$?

Q11 – Convert the following points from Cartesian to both cylindrical and spherical.

1. (2,3,-9)
2. (0,0,42)
3. (-3,4,-2)

Q12 – What is the integral of the equation f(x,y,z) = 5xy, over the cylinder with a diameter of 8 and a height of 3?

Q13 – What is the integral of the equation f(x,y,z) = 7yz, over the sphere of radius 6?

Q14 – Is the vector field F = (e^x)i + (2xy)j + (z-x)k conservative? If so, what is the potential?

Q15 – What is the potential function for Q14?

Q16 – Given the potential function $5xe^{y+z}$, what is the associated conservative vector function?

Q17 – What is the integral of the vector function from Q16, along the curve r(t) = $[t,t^2,t]$, from the point (0,0,0) to the point (4,16,4)?

Q18 - Use Green's theorem to evaluate the line integral $\int_C -y/(x^2+y^2)\,dx + \int_C x/(x^2 + y^2)\,dy$ where C is the arc of the parabola $y = 0.25\,x^2 + 1$ from (-2, 2) to (2, 2).

Q19 – Let S be the unit sphere, centered on the origin oriented by the outward pointing normal. What is the flux of the vector function F = (x)i + (y)j + (z^2)k?

Q20 – Find the surface area of the part of the paraboloid $z = a^2 - x^2 - y^2$ which lies above the xy–plane.

Bonus

BQ1 – What is the fundamental theorem of line integrals?

BQ2 – Find the tangent plane to the unit sphere at the point (1,0,0).

BQ3 – Using the chain rule, what is the second derivative of $f(x,y) = e^{xy} + y$ with respect to t, where $x = \sin(t)$ and $y = \cos(t)$.

BQ4 – What is the shape of the object defined by the equation $f(x,y,z) = x^2 + 2x – y^2 – 5z + z^2$? Where is the center of this object?

BQ5 – What are the tangent vectors for the surface x^3-12y^2?