(a) $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left\{x(x+y)^2\right\} = (x+y)^2 + x2(x+y)$

$= (x+y)(x+y+2x) = (x+y)(3x+y)$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x(x+y)^2\right) = x2(x+y) = 2x(x+y)$

(b) $\frac{\partial u}{\partial r} = +\cos\theta \cos(r\cos\theta)$

$\frac{\partial u}{\partial \theta} = -r\sin\theta \cos(r\cos\theta)$

(c) $\frac{\partial f}{\partial x} = y x^{y-1}$

$\frac{\partial f}{\partial y} = x^y \ln(x)$

Q4

(a) $f_x = 4x^3y - 6x^2y^4$ $\qquad f_y = x^4 - 8x^3y^3$

$f_{xx} = 12x^2y - 12xy^4$ $\qquad f_{yy} = -24x^3y^2$

$f_{xy} = 4x^3 - 24x^2y^3$ $\qquad f_{yx} = 4x^3 - 24x^2y^3$

$f_{xy} = f_{yx}$

(b) $T_t = -2e^{-2t}\cos\theta$ $\qquad T_\theta = -e^{-2t}\sin\theta$

$T_{tt} = 4e^{-2t}\cos\theta$ $\qquad T_{\theta\theta} = -e^{-2t}\cos\theta$

$T_{t\theta} = 2e^{-2t}\sin\theta$ $\qquad T_{\theta t} = 2e^{-2t}\sin\theta$

$T_{t\theta} = T_{\theta t}$

Q1

(a) $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$

$x^2 - 2x - 1 + 1 - y^2 + 2y + 1 - 1 + z^2 + 4z + 4 - 4 + 2 = 0$

$(x-1)^2 - 1 - (y-1)^2 + 1 + (z+2)^2 - 4 + 2 = 0$

$(x-1)^2 - (y-1)^2 + (z+2)^2 = 2$

This surface is two cones connected by a throat.

(b) xy-plane – hyperbolas

yz-plane – hyperbolas

xz-plane – Circles

Q2 We need the distance from a point $P = (x, y, z)$ to the point $(x_0, y_0, z_0) = (0, 0, 1)$ and the plane $z = -1$.

$D_1 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = \sqrt{x^2 + y^2 + (z-1)^2}$

$D_2 = \sqrt{(x-x)^2 + (y-y)^2 + (z+1)^2} = \sqrt{(z+1)^2}$

$D_1 = D_2 \Rightarrow \sqrt{x^2 + y^2 + (z-1)^2} = \sqrt{(z+1)^2}$

$x^2 + y^2 + (z-1)^2 = (z+1)^2$

$x^2 + y^2 + (z^2 - 2z + 1) - (z^2 + 2z + 1) = 0$

$x^2 + y^2 - 4z = 0$

This is a parabaloid surface opening to the z-direction.