

# Lagrange Multipliers

Joey Bernard

University of New Brunswick

June 2, 2024

# Constrained Optimization Problem

- We have a problem of the form
  - ▶ Find the maximum and minimum values of the function  $f(x,y)$  for  $(x,y)$  on the curve  $g(x,y) = 0$
- where
  - ▶  $f(x,y)$  is the objective function
  - ▶  $g(x,y)$  is the constraint function

# Lagrange Multipliers

- Let  $f(x,y,z)$  and  $g(x,y,z)$  have continuous first partial derivatives in a region of  $R^3$  that contains the surface  $S$  given by the equation  $g(x,y,z) = 0$ . Further assume that  $\nabla g(x,y,z) \neq 0$  on  $S$ .
- If  $f$ , restricted to the surface  $S$ , has a local extreme value at the point  $(a,b,c)$  on  $S$ , then there is a real number  $\lambda$  such that
  - ▶  $\nabla f(a,b,c) = \lambda \nabla g(a,b,c)$
- that is
  - ▶  $f_x(a,b,c) = \lambda g_x(a,b,c)$
  - ▶  $f_y(a,b,c) = \lambda g_y(a,b,c)$
  - ▶  $f_z(a,b,c) = \lambda g_z(a,b,c)$
- The number  $\lambda$  is called a Lagrange Multiplier