# Maxima and Minima

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# Critical Points

- In 2D, maxima and minima are when the tangents of the curve are flat
- This is when the first derivative (slope of the tangent line) is equal to zero
- In 3D, this is similar, except that we need to use the gradient
- So, maxima and minima happen when the gradient is zero, but this can happen outside of maxima and minima
- We introduce a new term, a critical point, defined by the gradient being equal to zero

# Second Derivative test

- We can use the second derivative to help decide local maxima and minima in 2D
- If f" ¡ 0, we will have a local maxima
- If f" ¿ 0, we will have a local minima
- If f" $=$ 0, we don't know
- The equivalent in 3D is
  - Use the descriminant of f: $D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - f_{xy}^2$
  - D ¿ 0 and $f_{xx} > 0$ : f has a local minimum
  - D ¿ 0 and $f_{xx} < 0$ : f has a local maximum
  - D ¡ 0 : f has a saddle point
  - D $=$ 0 : we don't know

# Global Maxima and Minima

- 'Global' maxima and minima are always defined on an interval
- With two independent variables, may use the unit disc $(x^2 + y^2 \leq 1$
- may need to use a parameterization to figure out the behaviour of the equation