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Numerical simulation of fifth order KdV equations occurring in magneto-acoustic waves

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ABSTRACT

In this work, we aim to apply a numerical approach based on Homotopy perturbation transform method (HPTM) for derive the exact and approximate solutions of nonlinear fifth order KdV equations for study magneto-acoustic waves in plasma. The approach is a mixed form of the standard Laplace transform with the classical Homotopy perturbation technique. Nonlinear term can be handled with the aid of He's Polynomials. In this technique, the solution is calculated in the form of a convergent series and convergence of the HPTM solutions to the exact solutions is shown. The Homotopy perturbation transform method presents a wide applicability to handling nonlinear wave equations in science and engineering. As a novel application of HPTM, the present work shows some essential difference with the existing similar method. Several examples are provided for illustrate the simplicity and reliability of the method and highlighted the significance of this work.

1. Introduction

Nonlinear phenomena appear in many areas of scientific and engineering field such as plasma physics, fluid dynamics, nonlinear optics, quantum mechanics, solid state physics, mathematical biology and chemical kinetics etc. These phenomena are modeled in terms of nonlinear partial differential equations with different higher order. Partial differential equations are widely used to describe physical systems. Most of the important phenomena of physical systems are hidden in their nonlinear nature. The exact solution of these nonlinear phenomena may not be available. These phenomena can only be studied with the appropriate methods to solve these nonlinear systems [1–4].

In 1895, the Kortweg and De-Vries developed KdV equation to model the Russell's phenomena of Solitons such as shallow water waves with small and finite amplitudes. Solitons are stable solitary waves and suggests that these solitary waves behave as a particle. KdV equations are the mathematical model for study the dispersive wave phenomena in various field of science such as plasma physics, fluid dynamics, optics and quantum mechanics etc. The fifth order KdV type equations used to study various nonlinear phenomena in plasma physics. It has a significant role in the wave propagation [5]. The KdV type equation have third and fifth order dispersive term in their study related to the problem of magneto-acoustic wave in cold collision free plasma and dispersive term appear propagation near critical angle [6].

Plasma is a complex, quasi-neutral and electrically conductive fluid. It consists of electrons, ions and neutral particles. Due to electrical conductive behavior of plasma, it consists of electric and magnetic fields. Interaction between particles and field's, plasma support different type's waves. Wave phenomena are important for heating plasmas, instabilities and diagnostics etc. A magneto-acoustic wave is a dispersion less and longitudinal wave of ions in a magnetized plasma propagating perpendicular to the stationary magnetic field. In the range of low magnetic field, the magneto-acoustic wave behave as ion acoustic wave and in the low temperature range, the magneto-acoustic wave behave as Alfvén wave. Magneto-acoustic waves finds in solar corona [7].

The general form of fifth order KdV equation is:

$$u_t - u_{5x} = F(x, t, u, u^2, u_x, u_{2x}, u_{3x}) \tag{1}$$

This fifth order KdV equations the generic model for the study of magneto-acoustic waves in plasma and shallow water waves with surface tension. Recently, researcher has been investigated

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that the travelling wave solutions of this equation do not vanish at infinity [8,9].

Consider the four well known forms of the fifth order KdV equation as [10–12]:

(a) $$u_t + u_x + u^2 u_{2x} + u_x u_{2x} - 20u^2 u_{3x} + u_{5x} = 0 \tag{2}$$

with initial condition $u(x,0) = \frac{1}{x}$

(b) $$u_t + uu_x - uu_{3x} + u_{5x} = 0 \tag{3}$$

with initial condition u(x,0) $= e^x$

(c) $$u_t + uu_x + u_{3x} - u_{5x} = 0 \tag{4}$$

with initial condition $u(x,0) = \frac{105}{169} sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right)$

(d) $$u_t - uu_x + u_{5x} = cosx + 2tsinx + \frac{t^2}{2} sin2x \tag{5}$$

with initial condition $u(x,0) = 0$

Eqs. (2) and (3) called fifth order KdV equation, Eq. (4) called Kawahara equation and Eq. (5) called non-homogeneous fifth order KdV equation respectively.

It is very difficult to find the analytical solutions of these physical problems when these are highly nonlinear. In recent decade, many researchers have paid attention to study the behavior of these physical problems by using various analytical and numerical schemes which are not described by the common observations. Some important approaches are the Adomian decomposition technique [13], He's semi-inverse scheme [14], Differential transform technique [15], Inverse scattering algorithm [16], Hirota's bilinear techniques [17], Laplace decomposition approach [18], Tanh scheme [19], Fractional homotopy analysis transform algorithm [20], Variational iteration technique [21], Homotopy analysis technique [22,23], Homotopy perturbation method [24,25], Modified homotopy perturbation technique [26], Group analysis method [27] and many more.

He proposed the Homotopy perturbation method for solving different nonlinear physical problems. This method is combination of the homotopy in topology and well known perturbation method. This provides us with an easy way to found numerical solutions to diverse variety of physical problems occurring in different fields of science, engineering and finance [28–31]. The Laplace transform is inadequate of solving the physical systems because of the difficulties occurring due to the nonlinear terms. Moreover, Homotopy perturbation method is also merged with the Laplace transformation to give a highly effective technique for handling many nonlinear physical problems. This scheme is known as Homotopy perturbation transform method (HPTM). It is an analytical method for analysis the nonlinear behavior of physical systems [32,33].

In a recent paper Khan and Wu [34] suggested the Homotopy perturbation transform method (HPTM) for solving the nonlinear partial differential equations. This method is a simple combination of the Laplace transformation, the Homotopy perturbation method and He's polynomials [35] and is mainly due to Ghorbani [36,37]. The Homotopy perturbation transform method keeps the solution in a rapid convergent series which may lead to the solution in a closed form. The supremacy of this approach is its capability of merging two powerful techniques for obtaining exact solutions for nonlinear equations. In this article, we apply the HPTM for solving the nonlinear partial differential equations such as fifth order KdV equation, Kawahara equation and non-homogeneous fifth order KdV equation to show the simplicity and straight forwardness of the method.

This paper is organized as follows: in Section 2, we describe the Homotopy perturbation transform method (HPTM). In Section 3, contains the examples of physical systems to show the efficiency of method. In Section 4, results of physical systems discussed. In Section 5, conclusion is given. At the end of conclusion, references are given.

2. Homotopy perturbation transform method (HPTM)

To provide basic idea of this scheme, we have taken up a general nonlinear partial differential equation with the initial conditions of the form:

$$\begin{aligned} &Du(x,t) + Ru(x,t) + Nu(x,t) = g(x,t) \\ &u(x,0) = h(x) \\ &u_t(x,0) = f(x) \end{aligned} \tag{6}$$

In the above equation D is the second order linear differential operator, R is denoting the linear differential operator of less order than D, N is representing the general nonlinear differential operator and $g(x, t)$ is denoting the source term.

Using the same process as presented in a series of papers [32–34], we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = G(x,t) - p\left(L^{-1}\left[\frac{1}{s^2} L\left[R\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(u)\right]\right]\right) \tag{7}$$

where $G(x, t)$ represents the term arising from the source term and the prescribed initial conditions and $H_n(u)$ represents the He's polynomials.

This is the combination of the Laplace transform and the Homotopy perturbation method using He's polynomials. Comparing the like powers of p, the following approximations are obtained:

$$p^0 : u_0(x,t) = G(x,t) \tag{8}$$

$$p^1 : u_1(x,t) = -L^{-1}\left[\frac{1}{s^2} L[Ru_0(x,t) + H_0(x,t)]\right] \tag{9}$$

$$p^2 : u_2(x,t) = -L^{-1}\left[\frac{1}{s^2} L[Ru_1(x,t) + H_1(x,t)]\right] \tag{10}$$

$$p^3 : u_3(x,t) = -L^{-1}\left[\frac{1}{s^2} L[Ru_2(x,t) + H_2(x,t)]\right] \tag{11}$$

$$\vdots$$

$$p^n : u_n(x,t) = -L^{-1}\left[\frac{1}{s^2} L[Ru_{n-1}(x,t) + H_{n-1}(x,t)]\right] \tag{12}$$

Finally we get the following series solution

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{13}$$

3. Numerical applications

In this section, we employ the Homotopy perturbation transform method (HPTM) for solving the generic model of nonlinear fifth order KdV equations for the study of magneto-acoustic waves in plasma.

Example 3.1. Firstly, we consider the following nonlinear fifth order KdV equation

$$u_t + u_x + u^2 u_{2x} + u_x u_{2x} - 20u^2 u_{3x} + u_{5x} = 0 \tag{14}$$

with initial condition

$$u(x,0) = \frac{1}{x} \tag{15}$$

Applying the Laplace transform on Eq. (14) and using the initial conditions (15), we have

$$L[u(x,t)] = \frac{1}{x}\frac{1}{s} - \frac{1}{s}L[u_x + u^2u_{2x} + u_xu_{2x} - 20u^2u_{3x} + u_{5x}] \tag{16}$$

Next, making use of the inverse of Laplace transform, it implies

$$u(x,t) = \frac{1}{x} - L^{-1}\left[\frac{1}{s}L[u_x + u^2u_{2x} + u_xu_{2x} - 20u^2u_{3x} + u_{5x}]\right] \tag{17}$$

Now, applying the HPM, we get

$$\sum_{n=0}^{\infty}p^nu_n(x,t) = \frac{1}{x} - p\left(L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}p^nH_n(u)\right) + \left(\sum_{n=0}^{\infty}p^nu_n(x,t)\right)_x + \left(\sum_{n=0}^{\infty}p^nu_n(x,t)\right)_{5x}\right]\right]\right) \tag{18}$$

In the above expression (18) $H_n(u)$ are He's polynomial [35,36] that represents the nonlinear terms. The first few components of He's polynomials, are given by

$$\begin{aligned}
H_0(u) &= u_0^2(u_0)_{2x} + (u_0)_x(u_0)_{2x} - 20u_0^2(u_0)_{3x}\\
H_1(u) &= u_0^2(u_1)_{2x} + 2u_0u_1(u_0)_{2x}\\
&\quad + (u_0)_x(u_1)_{2x} + (u_0)_{2x}(u_1)_x - 20u_0^2(u_1)_{3x} - 40u_0u_1(u_0)_{3x}\\
H_2(u) &= u_0^2(u_2)_{2x} + 2u_0u_1(u_1)_{2x} + 2u_0u_2(u_0)_{2x}\\
&\quad + u_1^2(u_0)_{2x} + (u_0)_x(u_2)_{2x} + (u_1)_x(u_1)_{2x} + (u_0)_{2x}(u_2)_x\\
&\quad - 20u_0^2(u_2)_{3x} - 40u_0u_1(u_1)_{3x} - 40u_0u_2(u_0)_{3x}\\
&\quad - 20u_1^2(u_0)_{3x}\\
H_3(u) &= u_0^2(u_3)_{2x} + 2u_0u_1(u_2)_{2x} + 2u_0u_2(u_1)_{2x}\\
&\quad + 2u_0u_3(u_0)_{2x} + u_1^2(u_1)_{2x} + 2u_1u_2(u_0)_{2x}\\
&\quad + (u_0)_x(u_3)_{2x} + (u_1)_x(u_2)_{2x}\\
&\quad + (u_1)_{2x}(u_2)_x + (u_0)_{2x}(u_3)_x\\
&\quad - 20u_0^2(u_3)_{3x} - 40u_0u_1(u_2)_{3x} - 40u_0u_2(u_1)_{3x}\\
&\quad - 40u_0u_3(u_0)_{3x} - 20u_1^2(u_1)_{3x}\\
&\quad - 40u_1u_2(u_0)_{3x}\\
H_4(u) &= u_0^2(u_4)_{2x} + 2u_0u_1(u_3)_{2x} + 2u_0u_2(u_2)_{2x}\\
&\quad + 2u_0u_3(u_1)_{2x} + 2u_0u_4(u_0)_{2x} + u_1^2(u_2)_{2x}\\
&\quad + 2u_1u_2(u_1)_{2x} + 2u_1u_3(u_0)_{2x} + u_2^2(u_0)_{2x}\\
&\quad + (u_0)_x(u_4)_{2x} + (u_1)_x(u_3)_{2x} + (u_2)_{2x}(u_2)_x\\
&\quad + (u_1)_{2x}(u_3)_x + (u_0)_{2x}(u_4)_x - 20u_0^2(u_4)_{3x}\\
&\quad - 40u_0u_1(u_3)_{3x} - 40u_0u_2(u_2)_{3x} - 40u_0u_3(u_1)_{3x}\\
&\quad - 40u_0u_4(u_0)_{3x} - 20u_1^2(u_2)_{3x} - 40u_1u_2(u_1)_{3x}\\
&\quad - 40u_1u_3(u_0)_{3x} - 20u_2^2(u_0)_{2x}\\
&\ \ \vdots
\end{aligned} \tag{19}$$

Comparing the coefficients of like powers of p, we have

$$\begin{aligned}
p^0 &: u_0(x,t) = \frac{1}{x}\\
p^1 &: u_1(x,t) = -L^{-1}\left[\frac{1}{s}L[H_0(u) + (u_0)_x + (u_0)_{5x}]\right] = \frac{t}{x^2}\\
p^2 &: u_2(x,t) = -L^{-1}\left[\frac{1}{s}L[H_1(u) + (u_1)_x + (u_1)_{5x}]\right] = \frac{t^2}{x^3}\\
p^3 &: u_3(x,t) = -L^{-1}\left[\frac{1}{s}L[H_2(u) + (u_2)_x + (u_2)_{5x}]\right] = \frac{t^3}{x^4}\\
p^4 &: u_4(x,t) = -L^{-1}\left[\frac{1}{s}L[H_3(u) + (u_3)_x + (u_3)_{5x}]\right] = \frac{t^4}{x^5}\\
p^5 &: u_5(x,t) = -L^{-1}\left[\frac{1}{s}L[H_4(u) + (u_4)_x + (u_4)_{5x}]\right] = \frac{t^5}{x^6}\\
&\ \ \vdots
\end{aligned} \tag{20}$$

Therefore the solution $u(x,t)$ is given by

$$\begin{aligned}
u(x,t) &= \sum_{i=0}^{\infty}u_i(x,t)\\
&= \frac{1}{x} + \frac{t}{x^2} + \frac{t^2}{x^3} + \frac{t^3}{x^4} + \frac{t^4}{x^5} + \frac{t^5}{x^6} + \cdots
\end{aligned} \tag{21}$$

It can be written in closed form as

$$u(x,t) = \frac{1}{x-t} \tag{22}$$

Example 3.2. Next, consider the following nonlinear fifth order KdV equation

$$u_t + uu_x - uu_{3x} + u_{5x} = 0 \tag{23}$$

with initial condition

$$u(x,0) = e^x \tag{24}$$

Appealing to the Laplace transform on both sides of Eq. (23) subject to the initial conditions (24), we have

$$L[u(x,t)] = \frac{e^x}{s} + \frac{1}{s}L[uu_{3x} - uu_x - u_{5x}] \tag{25}$$

The inversion of Laplace transform results the following equation

$$u(x,t) = e^x + L^{-1}\left[\frac{1}{s}L[uu_{3x} - uu_x - u_{5x}]\right] \tag{26}$$

Now, applying the HPM, we get

$$\sum_{n=0}^{\infty}p^nu_n(x,t) = e^x + p\left(L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}p^nH_n(u)\right) - \left(\sum_{n=0}^{\infty}p^nu_n(x,t)\right)_{5x}\right]\right]\right) \tag{27}$$

In the above Eq. (27) $H_n(u)$ are He's polynomial [35–36] that indicates the nonlinear terms. The first few components of He's polynomials, are given by

$$\begin{aligned}
H_0(u) &= u_0(u_0)_{3x} - u_0(u_0)_x\\
H_1(u) &= u_1(u_0)_{3x} + u_0(u_1)_{3x} - u_1(u_0)_x - u_0(u_1)_x\\
H_2(u) &= u_2(u_0)_{3x} + u_1(u_1)_{3x} + u_0(u_2)_{3x}\\
&\quad - u_2(u_0)_x - u_1(u_1)_x - u_0(u_2)_x\\
H_3(u) &= u_3(u_0)_{3x} + u_2(u_1)_{3x} + u_1(u_2)_{3x} + u_0(u_3)_{3x}\\
&\quad - u_3(u_0)_x - u_2(u_1)_x - u_1(u_2)_x - u_0(u_3)_x\\
H_4(u) &= u_4(u_0)_{3x} + u_3(u_1)_{3x} + u_2(u_2)_{3x}\\
&\quad + u_1(u_3)_{3x} + u_0(u_4)_{3x}\\
&\quad - u_4(u_0)_x - u_3(u_1)_x - u_2(u_2)_x\\
&\quad - u_1(u_3)_x - u_0(u_4)_x\\
&\ \ \vdots
\end{aligned} \tag{28}$$

Comparing the coefficients of like powers of p, we have

$$\begin{aligned}
p^0 &: u_0(x,t) = e^x\\
p^1 &: u_1(x,t) = L^{-1}\left[\frac{1}{s}L[H_0(u) - (u_0)_{5x}]\right] = -te^x\\
p^2 &: u_2(x,t) = L^{-1}\left[\frac{1}{s}L[H_1(u) - (u_1)_{5x}]\right] = \frac{t^2}{2}e^x = \frac{t^2}{2!}e^x\\
p^3 &: u_3(x,t) = L^{-1}\left[\frac{1}{s}L[H_2(u) - (u_2)_{5x}]\right] = -\frac{t^3}{6}e^x = -\frac{t^3}{3!}e^x\\
p^4 &: u_4(x,t) = L^{-1}\left[\frac{1}{s}L[H_3(u) - (u_3)_{5x}]\right] = \frac{t^4}{24}e^x = \frac{t^4}{4!}e^x
\end{aligned}$$

$$p^5: u_5(x,t) = L^{-1}\left[\frac{1}{s}L[H_4(u) - (u_4)_{5x}]\right] = -\frac{t^5}{120}e^x = -\frac{t^5}{5!}e^x$$
$$\vdots \tag{29}$$

Therefore the solution $u(x,t)$ is given by

$$u(x,t) = \sum_{i=0}^{\infty} u_i(x,t) = e^x - te^x + \frac{t^2}{2!}e^x - \frac{t^3}{3!}e^x + \frac{t^4}{4!}e^x - \frac{t^5}{5!}e^x + \cdots \tag{30}$$

It can be written in closed form as

$$u(x,t) = e^{x-t} \tag{31}$$

Example 3.3. Consider the Kawahara equation

$$u_t + uu_x + u_{3x} - u_{5x} = 0 \tag{32}$$

with initial condition

$$u(x,0) = \frac{105}{169}sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right) \tag{33}$$

Using the Laplace transform on both sides of Eq. (32) and subject to the initial conditions (33), we have

$$L[u(x,t)] = \frac{105}{169}sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right).\frac{1}{s} + \frac{1}{s}L[u_{5x} - u_{3x} - uu_x] \tag{34}$$

The inverse of Laplace transform implies that

$$u(x,t) = \frac{105}{169}sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right) + L^{-1}\left[\frac{1}{s}L[u_{5x} - u_{3x} - uu_x]\right] \tag{35}$$

Now, applying the HPM, we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = \frac{105}{169}sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right) + p\left(L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right)_{5x} .- \left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right)_{3x} - \left(\sum_{n=0}^{\infty} p^n H_n(u)\right)\right]\right]\right) \tag{36}$$

here $H_n(u)$ are He's polynomial [35–36] that indicates the nonlinear terms. The first few components of He's polynomials, are given by

$$\begin{aligned}
H_0(u) &= u_0(u_0)_x\\
H_1(u) &= u_0(u_1)_x + u_1(u_0)_x\\
H_2(u) &= u_0(u_2)_x + u_1(u_1)_x + u_2(u_0)_x\\
H_3(u) &= u_0(u_3)_x + u_1(u_2)_x + u_2(u_1)_x + u_3(u_0)_x\\
H_4(u) &= u_0(u_4)_x + u_1(u_3)_x + u_2(u_2)_x + u_3(u_1)_x + u_4(u_0)_x\\
&\vdots
\end{aligned} \tag{37}$$

Comparing the coefficients of like powers of p, we have

$$p^0: u_0(x,t) = \frac{105}{169}sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right)$$

$$p^1: u_1(x,t) = L^{-1}\left[\frac{1}{s}L[(u_0)_{5x} - (u_0)_{3x} - H_0(u)]\right] = -\frac{7560}{28561\sqrt{13}}tsech^4\left(\frac{x-x_0}{2\sqrt{13}}\right)tanh\left(\frac{x-x_0}{2\sqrt{13}}\right)$$

$$p^2: u_2(x,t) = L^{-1}\left[\frac{1}{s}L[(u_1)_{5x} - (u_1)_{3x} - H_1(u)]\right] = -\frac{68040}{62748517}t^2sech^6\left(\frac{x-x_0}{2\sqrt{13}}\right)\left[-3 + 2cosh\left(\frac{x-x_0}{2\sqrt{13}}\right)\right]$$

$$\begin{aligned}p^3: u_3(x,t) &= L^{-1}\left[\frac{1}{s}L[(u_2)_{5x} - (u_2)_{3x} - H_2(u)]\right]\\
&= -\frac{816480}{10604499373\sqrt{13}}t^3sech^7\left(\frac{x-x_0}{2\sqrt{13}}\right)\\
&\times\left[-13sinh\left(\frac{x-x_0}{2\sqrt{13}}\right) + 2sinh\left(\frac{3(x-x_0)}{2\sqrt{13}}\right)\right]\end{aligned}$$

$$\begin{aligned}p^4: u_4(x,t) &= L^{-1}\left[\frac{1}{s}L[(u_3)_{5x} - (u_3)_{3x} - H_3(u)]\right]\\
&= -\frac{3674160}{23298085122481}t^4sech^8\left(\frac{x-x_0}{2\sqrt{13}}\right)\\
&\times\left[-49cosh\left(\frac{x-x_0}{2\sqrt{13}}\right) + 4cosh\left(\frac{2(x-x_0)}{2\sqrt{13}}\right) + 52\right]\end{aligned}$$

$$\begin{aligned}p^5: u_5(x,t) &= L^{-1}\left[\frac{1}{s}L[(u_4)_{5x} - (u_4)_{3x} - H_4(u)]\right]\\
&= -\frac{13226976}{3937376385699289\sqrt{13}}t^5sech^9\left(\frac{x-x_0}{2\sqrt{13}}\right)\\
&\times\left[171sinh\left(\frac{3(x-x_0)}{2\sqrt{13}}\right) - 8sinh\left(\frac{5(x-x_0)}{2\sqrt{13}}\right) - 661sinh\left(\frac{5(x-x_0)}{2\sqrt{13}}\right)\right]\\
&\vdots\end{aligned} \tag{38}$$

Therefore the solution $u(x,t)$ is given by

$$u(x,t) = \sum_{i=0}^{\infty} u_i(x,t)$$

$$\begin{aligned}u(x,t) &= \frac{105}{169}sech^4\left(\frac{x-x_0}{2\sqrt{13}}\right) - \frac{7560}{2856\sqrt{13}}tsech^4\left(\frac{x-x_0}{2\sqrt{13}}\right)tanh^4\left(\frac{x-x_0}{2\sqrt{13}}\right)\\
&- \frac{68040}{62748517}t^2sech^6\left(\frac{x-x_0}{2\sqrt{13}}\right)\left[-3 + 2cosh\left(\frac{x-x_0}{2\sqrt{13}}\right)\right]\\
&- \frac{816480}{10604499373\sqrt{13}}t^3sech^7\left(\frac{x-x_0}{2\sqrt{13}}\right)\left[-13sinh\left(\frac{x-x_0}{2\sqrt{13}}\right)\right.\\
&\left.+2sinh\left(\frac{x-x_0}{2\sqrt{13}}\right)\right] - \frac{3674160}{23298085122481}t^4sech^8\left(\frac{x-x_0}{2\sqrt{13}}\right)\\
&\times\left[-49cosh\left(\frac{x-x_0}{2\sqrt{13}}\right) + 4cosh\left(\frac{2(x-x_0)}{2\sqrt{13}}\right) + 52\right]\\
&- \frac{13226976}{3937376385699289\sqrt{13}}t^5sech^9\left(\frac{x-x_0}{2\sqrt{13}}\right)\\
&\times\left[171sinh\left(\frac{3(x-x_0)}{2\sqrt{13}}\right) - 8sinh\left(\frac{5(x-x_0)}{2\sqrt{13}}\right)\right.\\
&\left.-661sinh\left(\frac{5(x-x_0)}{2\sqrt{13}}\right)\right]\cdots\end{aligned} \tag{39}$$

It can be written in closed form as

$$u(x,t) = \frac{105}{169}sech^4\left[\frac{1}{2\sqrt{13}}(x + \frac{36t}{169} - x_0)\right] \tag{40}$$

Table 1
Comparison of Absolute errors between exact solution and HPTM solution at $x = 8.0$.

T	E_2	E_3	E_4	E_5	E_6
0.0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.1	2.47250000E−07	3.10937500E−09	5.76171880E−11	1.94702153E−11	1.89933781E−12
0.2	2.00320000E−06	5.00750000E−08	1.24687500E−09	2.61718750E−11	4.34135567E−11
0.3	6.84865000E−06	2.56853125E−07	9.66074220E−09	3.91027845E−10	4.34570312E−11
0.4	1.64474000E−05	8.22400000E−07	4.11500000E−08	2.08750000E−09	1.34375000E−10
0.5	3.25520500E−05	2.03447188E−06	1.27123247E−07	7.91395740E−09	4.63376803E−10

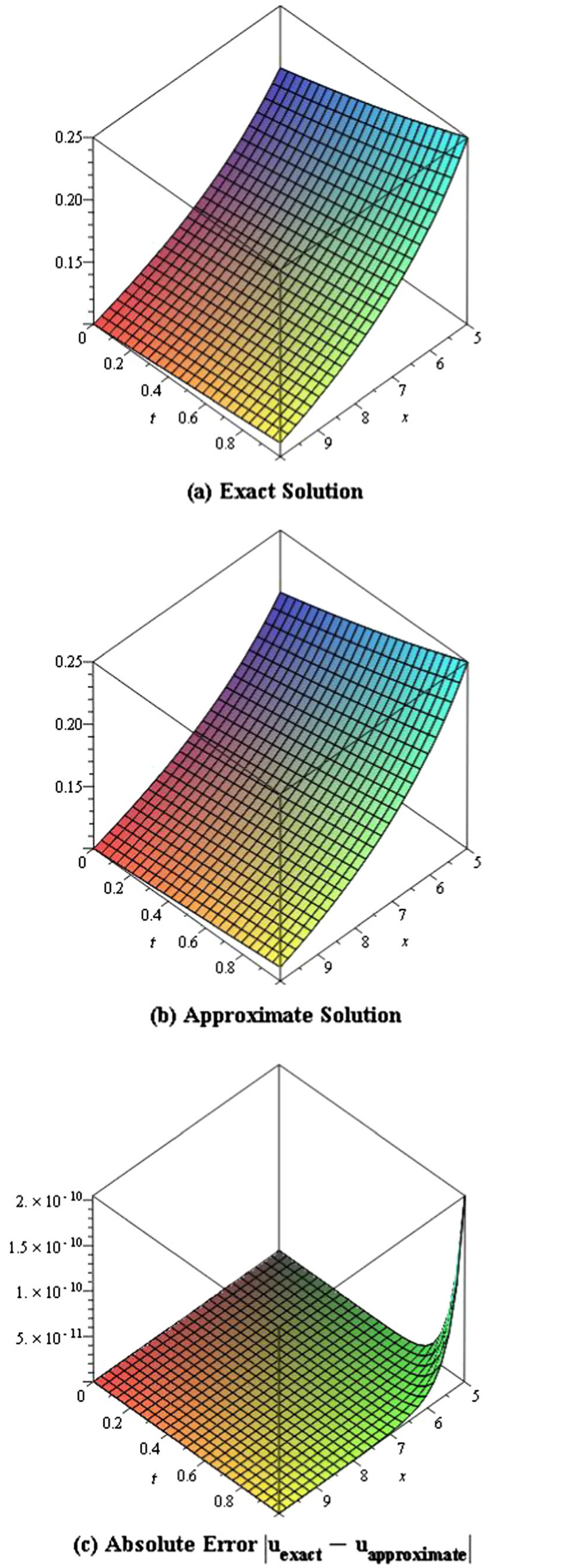

Fig. 1. The surface shows the solution $u(x,\,t)$ for Eq. (14): (a) exact solution, (b) approximate solution and (c) absolute error $|u_{exact} - u_{approximate}|$ for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$.

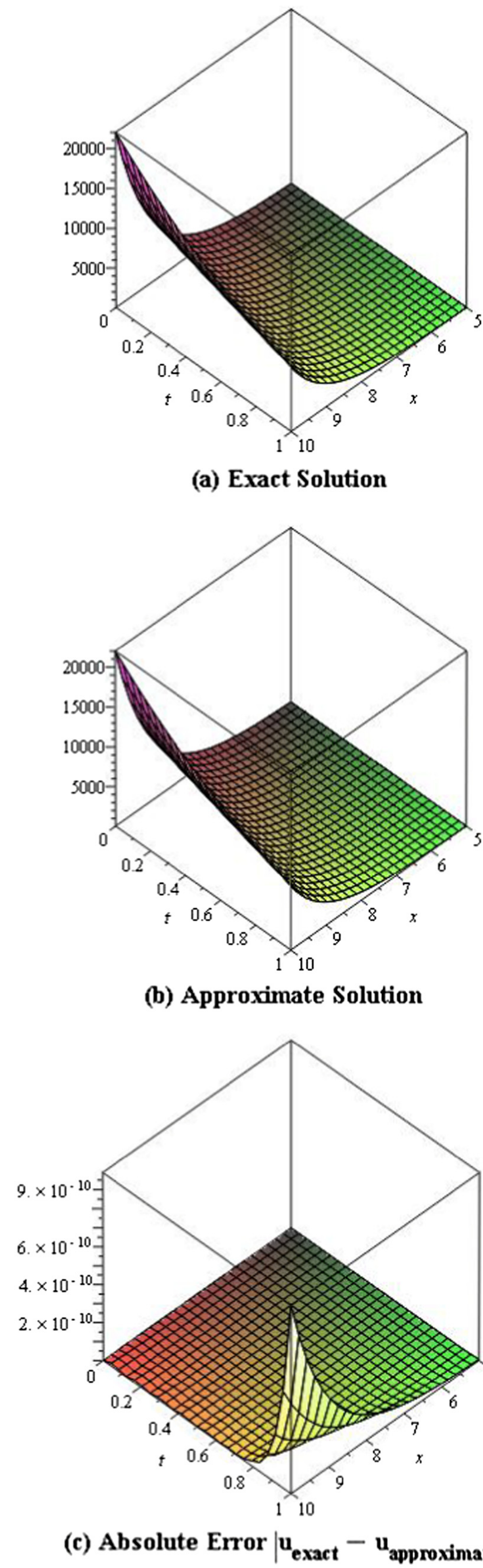

Fig. 2. The surface shows the solution u(x, t) for Eq. (23): (a) exact solution, (b) approximate solution and (c) absolute error $|u_{exact} - u_{approximate}|$ for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$.

Table 2
Comparison of Absolute errors between exact solution and HPTM solution at $x = -6.0$.

t	E_2	E_3	E_4	E_5	E_6
0.0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.1	4.03001000E−07	1.01250000E−08	2.04000000E−10	4.00000000E−12	2.00000000E−12
0.2	3.14614900E−06	1.58853000E−07	6.39800000E−09	2.13000000E−10	8.00000000E−12
0.3	1.03655950E−05	7.88790000E−07	4.77890000E−08	2.40600000E−09	1.03000000E−10
0.4	2.39942070E−05	2.44581700E−06	1.98186000E−07	1.33350000E−08	7.66000000E−10
0.5	4.57809180E−05	5.85975200E−06	5.95333000E−07	5.01760000E−08	3.61500000E−09

Table 3
Comparison of Absolute errors between exact solution and HPTM solution at $x = 6.0$.

t	E_2	E_3	E_4	E_5	E_6
0.0	0.00000000	0.00000000	0.0000000000	0.00000000	0.00000000
0.1	1.64943800E−06	1.43061600E−06	1.41912600E−06	1.41911800E−06	1.41910800E−06
0.2	6.67437400E−06	5.64584100E−06	5.55392100E−06	5.55362600E−06	5.55361200E−06
0.3	1.51864130E−05	1.25340720E−05	1.22238400E−05	1.22223490E−05	1.22222300E−05
0.4	2.72976520E−05	2.19832100E−05	2.12478430E−05	2.12431310E−05	2.12426300E−05
0.5	4.31168900E−05	3.38844600E−05	3.24482000E−05	3.24367000E−05	3.24351600E−05

Example 3.4. Finally consider the non–homogeneous fifth order KdV equation as an example of the application of the self-canceling phenomena [38,39]

$$u_t - uu_x + u_{5x} = cosx + 2tsinx + \frac{t^2}{2}sin2x \tag{41}$$

with initial condition

$$u(x,0) = 0 \tag{42}$$

Applying the Laplace transform on both sides of Eq. (41) and using subject to the initial condition (42), we have

$$L[u(x,t)] = \frac{u(x,0)}{s} + \frac{1}{s}L\left[cosx + 2tsinx + \frac{t^2}{2}sin2x - u_{5x} + uu_x\right] \tag{43}$$

The inversion of Laplace transform gives the following result

$$u(x,t) = L^{-1}\left[\frac{1}{s}L\left[cosx + 2tsinx + \frac{t^2}{2}sin2x - u_{5x} + uu_x\right]\right] \tag{44}$$

Now, applying the HPM, we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = p\left(L^{-1}\left[\frac{1}{s}L\left[cosx + 2tsinx + \frac{t^2}{2}sin2x - \left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right)_{5x} + \left(\sum_{n=0}^{\infty} p^n H_n(u)\right)\right]\right]\right) \tag{45}$$

Where $H_n(u)$ are He's polynomial [35,36] that represents the non-linear terms.

Recently, Wazwaz suggested that the construction of zeroth component of the series can be defined in a slightly different way. He supposed that if the zeroth component is $u_0(x,t) = G(x,t)$, the function $G(x,t)$ is possible to divide into two parts such as $G_0(x,t)$ and $G_1(x,t)$. Then one can construct the recursive algorithm $u_0(x,t)$. The same idea we can use in HPTM. The Eq. (13) general term in a form of modified recursive scheme as follows:

$$u_0(x,t) = G_0(x,t)$$

$$u_1(x,t) = G_1(x,t) - L^{-1}\left[\frac{1}{s^2}L[Ru_0(x,t) + H_0(u)]\right]$$

$$p^2 : u_{n+1}(x,t) = -L^{-1}\left[\frac{1}{s^2}L[Ru_n(x,t) + H_n(u)]\right],\ n \geqslant 1$$

This type of modification is giving more flexibility to HPTM in order to solve complicate nonlinear partial differential equations. In many case the modification avoids unnecessary computations such as calculation of the He's Polynomials. In addition, sometimes we do not need to evaluate He's Polynomials or if we need to evaluate He's Polynomials the computation will be reduced very considerably by using modified recursive scheme.

Comparing the coefficients of like powers of p, we have

$$\begin{aligned} &p^0 : u_0(x,t) = tcosx \\ &p^1 : u_1(x,t) = t^2sinx + \frac{t^3}{6}sin2x - t^2sinx - \frac{t^3}{6}sin2x \\ &\vdots \\ &p^n : u_{n+1}(x,t) = 0, n \geqslant 1 \end{aligned} \tag{46}$$

It is evident that the noise terms appear between the components of $u_1(x,t)$, and these are all cancelled. As seen Eq. (46), the closed form of the solution can be find very simply by proper selection of $G_0(x,t)$ and $G_1(x,t)$ .In the case of right choice of these functions, the modification scheme accelerates the convergence of the series solution by computing just $u_0(x,t)$ and $u_1(x,t)$ terms of the series. The term $u_0(x,t)$ provides the exact solution. This can be justifies through substitution and this has been justified by A. M. Wazwaz [40] and M. Hussain [41].

Therefore the solution $u(x,t)$ is given by

$$\begin{aligned} u(x,t) &= u_0(x,t) \\ &= tcosx \end{aligned} \tag{47}$$

4. Result and discussion

Here, we calculate the numerical results and absolute errors from second to sixth term approximations. The Absolute error is defined as:

$$E_n = |u_{exact} - \sum_{i=0}^{n} u_i| \tag{48}$$

From Table 1, it can also be observed that the accuracy increases as the order of approximation increases. Fig. 1 shows the comparison of the exact solution with approximate solution and presents the absolute error $|u_{exact} - u_{approximate}|$ for fifth order KdV equation for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$ for Eq. (14). From Fig. 1, it is observed that the values of the approximate solution of different grid points

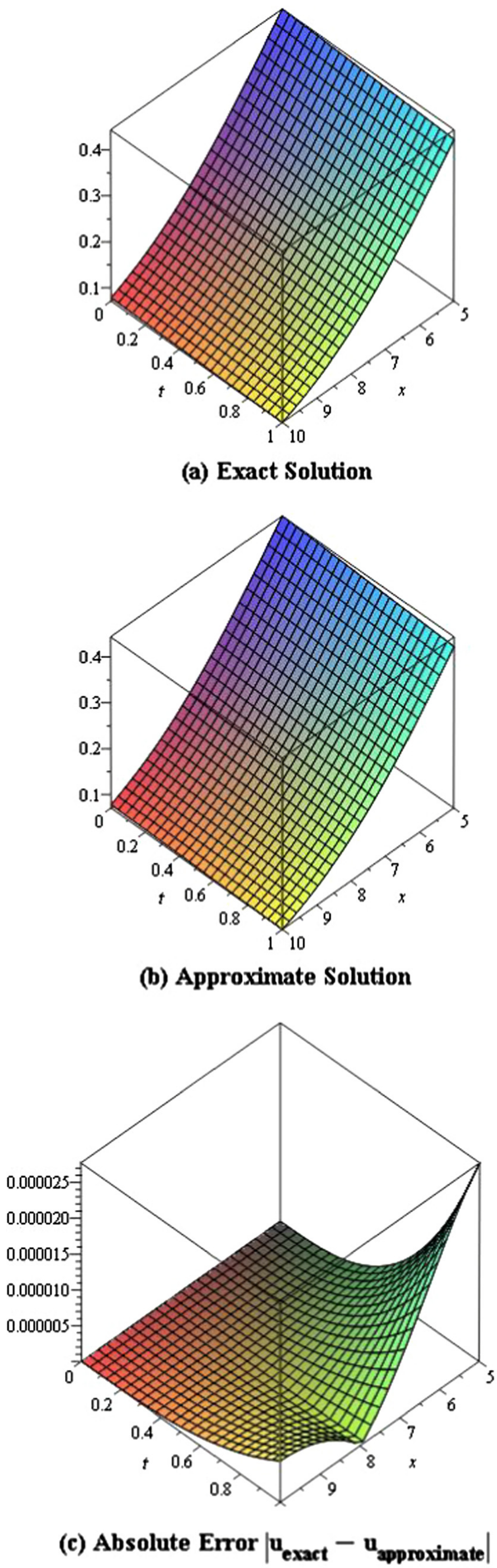

Fig. 3. The surface shows the solution $u(x, t)$ for Eq. (32): (a) exact solution, (b) approximate solution and (c) absolute error $|u_{exact} - u_{approximate}|$ for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$ with $x_0 = 2.0$.

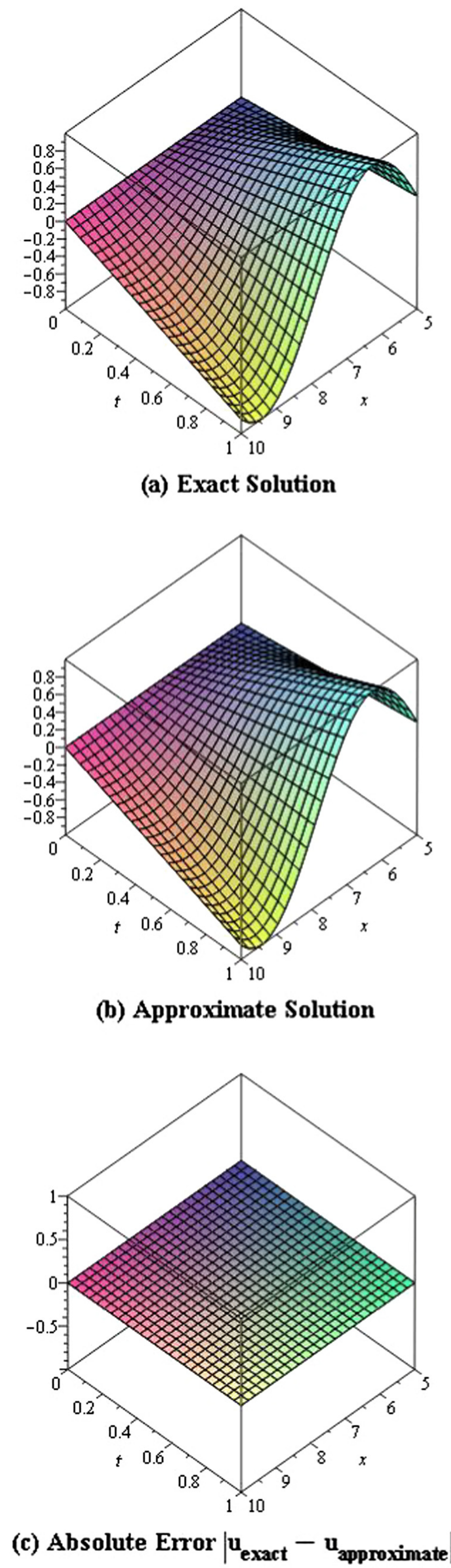

Fig. 4. The surface shows the solution u(x, t) for Eq. (41): (a) exact solution, (b) approximate solution and (c) absolute error $|u_{exact} - u_{approximate}|$ for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$.

obtained by the HPTM are very close to the values of the exact solution with high accuracy at the sixth-term approximation and revealing a high level of agreement between the two results. It is evident from Table 1 and Fig. 1(a)–(c) that the approximate solution converges very rapidly with the exact solution and the level of convergence between the two results is excellent.

From Table 2, it can be seen that the accuracy increases as the order of approximation increases. Fig. 2 indicates the correlation of the exact solution with approximate solution and absolute error $|u_{exact} - u_{approximate}|$ for fifth order KdV equation for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$ for Eq. (23). From Fig. 2, it is seen that the values of the approximate solution of different grid points obtained by the homotopy perturbation transform method are close to the values of the exact solution with high accuracy at the sixth-term approximation and admitting a high level of compliance between the two results. It is manifest from Table 2 and Fig. 2(a)–(c) that the approximate solution converges very rapidly with the exact solution and the level of convergence between the two results is good argument.

From Table 3, it can be noticed that the accuracy increases as the order of approximation increases. Fig. 3 displays the analysis of the exact solution with approximate solution and absolute error $|u_{exact} - u_{approximate}|$ for Kawahara equation for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$ with $x_0 = 2.0$ for Eq. (32). From Fig. 3, it is noticed that the values of the approximate solution of different grid points gained by the homotopy perturbation transform method are close to the values of the exact solution with high accuracy at the sixth-term approximation and acknowledging a high level of arrangement between the two results. It is noticeable from Table 3 and Fig. 3(a)–(c) that the approximate solution converges very rapidly with the exact solution and the level of convergence between the two results is excellent.

Fig. 4 shows the comparison of the exact solution with approximate solution and absolute error $|u_{exact} - u_{approximate}|$ for non-homogeneous fifth order KdV equation for $0 \leqslant t \leqslant 1$ and $5 \leqslant x \leqslant 10$ for Eq. (41). From Fig. 4, it is observed that the values of the approximate solution of different grid points obtained by the HPTM are close to the values of the exact solution with high accuracy and revealing a high level of agreement between the two results.

5. Conclusions

In this paper, we have suggested a combination of the Laplace transform with homotopy perturbation method for solving fifth order KdV equation, non-homogeneous fifth order KdV equation and Kawahara equation. This combination develops a strong method called homotopy perturbation transform method (HPTM). This method has been applied for numerical simulation of nonlinear behavior of magneto-acoustic waves occurring in plasma.

The Homotopy perturbation transform method (HPTM) has been applied for finding the exact and approximate solutions of nonlinear fifth order KdV equations with initial conditions to show the significance of this method. Thus, it can be used to solve other higher order nonlinear integer and fractional order equations. An important advantage of this new method is its low computational work.

It may be concluded that the homotopy perturbation transform method is powerful and efficient technique in finding exact and approximate solutions for wide classes of nonlinear problems. The method is capable of reducing the volume of the computational work and maintaining the high accuracy of the numerical result. The fact that the HPTM solves nonlinear equations without using Adomian's polynomials is a clear advantage of this technique over the decomposition method. It is predicted that the proposed algorithm can be widely applied to other nonlinear problems in science and engineering.

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