Einstein on a Computer

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What we'll cover

A look at computational science

- What is General Relativity?
- Why is it hard?
- Finding a solution - techniques
- Using Einstein Toolkit

What is General Relativity?

- Developed by Einstein between 1907 - 1915
- Accounted for problems in Newtonian theory
    - the precession of the orbit of Mercury
    - bending of star light around the sun
    - progression of time differing in different gravitational fields

General Relativity - 2

A couple of the key ideas of General Relativity

- time is just another dimension, treated the same as the three spatial dimensions
- the speed of light, c, is the absolute speed limit locally
- acceleration and gravitation are indistinguishable
- gravitational force comes from bent spacetime
- spacetime bends due to mass and energy

Why is it hard?

$$\mathbf{G} = \frac{8\pi G}{c^4}\mathbf{T}$$

- **G** - tensor describing spacetime geometry
- **T** - tensor describing mass-energy
- G - gravitational constant
- c - speed of light

Hard - 2

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Λ is the cosmological constant (Einstein's biggest mistake that likely wasn't a mistake)
- The μ and ν are indices for the tensors
- Indices run through 1 for each dimension (e.g. 3+1)
- This means that for regular 4-dimensional spacetime, we end up with a system of 10 coupled, nonlinear, hyperbolic-elliptic PDEs

Hard - 3

Only the simplest solutions have an exact solution

- Schwarzschild (1915-1916) - Completely empty space with a single point source of mass (static, spherically symmetric)
- Reissner-Nordstrom (1916-1918) - static, spherically symmetric, charged
- Kerr (1963) - rotating, spherically symmetric
- Kerr-Newman (1965) - rotating, spherically symmetric, charged

These lead to various black hole solutions.
Anything more realistic needs a numeric solution.

Finding a solution - techniques

Basic technique breaks down into two separate problems

- initial value problem
- evolution problem

Useful for

- cosmological models
- critical phenomena
- perturbed black holes / neutron stars
- coalescence

Solutions 2

Break down spacetime back into 3+1 dimensions

- a set of 3-dimensional hypersurfaces, separated across time
- a lapse function describing how to go from one hypersurface to another
- a shift function describing how points on the hypersurfaces move around, from one hypersurface to another

Solutions 3

- Begin with a snapshot of the gravitational fields on some hypersurface (initial data)
- Evolve this data to the next hypersurface
- Like all numerical analysis, need to pay attention to stability and convergence
- Need to pay attention to the following to produce accurate solutions
  - gauge conditions
  - coordinates
  - actual formulation of the Einstein equations

Solutions 4

- Richard Arnowitt, Stanley Deser, Charles W. Misner - first published in the late 1950's
- The ADM formalism, decomposes spacetime into 3 space and 1 time dimensions
- Almost nobody uses the original ADM formulation, but most modern formulations are based on this
- First recorded attempt was Hahn and Lindquist in 1964
- Limited to 2+1 dimensions (cylindrical symmetry)

Solutions 5

Timeline of computational work:

- Early 1980's - gravitational waveforms from formation of a rotating black hole
- Late 1990's - head-on binary black hole collision
- 1995 - first 3D solution of a Schwarzschild black hole
- 1990's - introduction of excision and puncture methods to deal with singularities
- 1990's - adaptive mesh refinement introduced from computational fluid dynamics
- 2005 - first publication of merger of two black holes through excision

Using Einstein Toolkit

- Cactus Code is a software framework for high-performance computing
- Structured as a core portion (called the flesh), and plugins (called thorns)
- Plugins for Cactus Code include things like IO, scientific file formats, evolution engines, etc.
- Einstein Toolkit spun off as a separate project to focus on solving Einstein's equations

Einstein 2

- 1995 - Cactus Code developed at the Max Planck Institute, version 1 released
- 1999 - Version 4.0 Beta 1 released
- 2003 - Several members of the group left Germany to help found the Center for Computation and Technology at Louisiana State University. Development now happening at both sites
- Feb. 2009 - Version 4.0 Beta 16 released
- 2010 - Einstein Toolkit has first release

Einstein 3

- The flesh is independent of the thorns and provides the main program which parses the parameters and starts up the appropriate thorns
- No actual work is done by the flesh
- All user-supplied code goes into thorns
- Thorns are essentially independent, they communicate through the flesh API

Einstein 4

- Connections between thorns and flesh are handled through configuration files
- These are parsed at compile time
- Glue code is generated to encapsulate the external appearance of the thorn
- Mostly calls to registration routines in the flesh
- At runtime, the executable reads a parameter file that says which thorns should be activated, with what parameters

Einstein 5

Thorns specific to the Einstein toolkit

- time evolution methods
- initial data generators
- file readers

This provides a common set of tools for numerical relativity that you can extend and build on.

Einstein 6

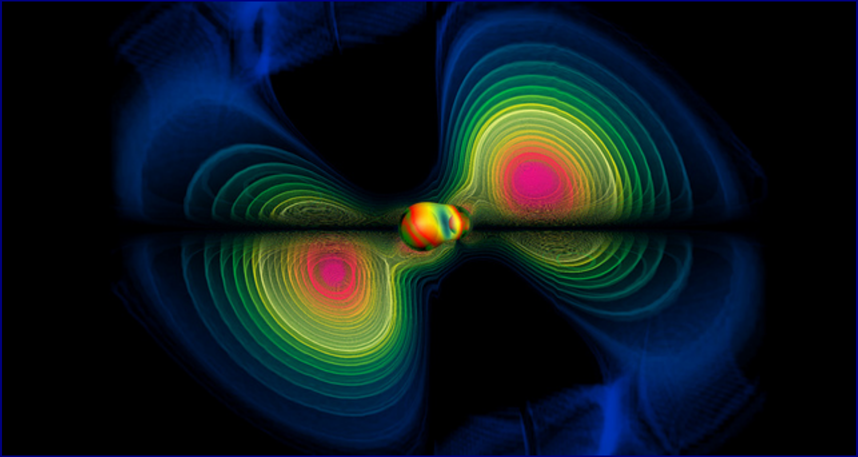
- The central thorn is ADMBase, provides
  - standard set of variables (metric, lapse, shift, curvature)
  - these are used for data import and export
  - not necessarily good for evolution
- Other thorns have the responsibility to translate these variables into their required form
- This allows different thorns to have access to the same data

Einstein 7

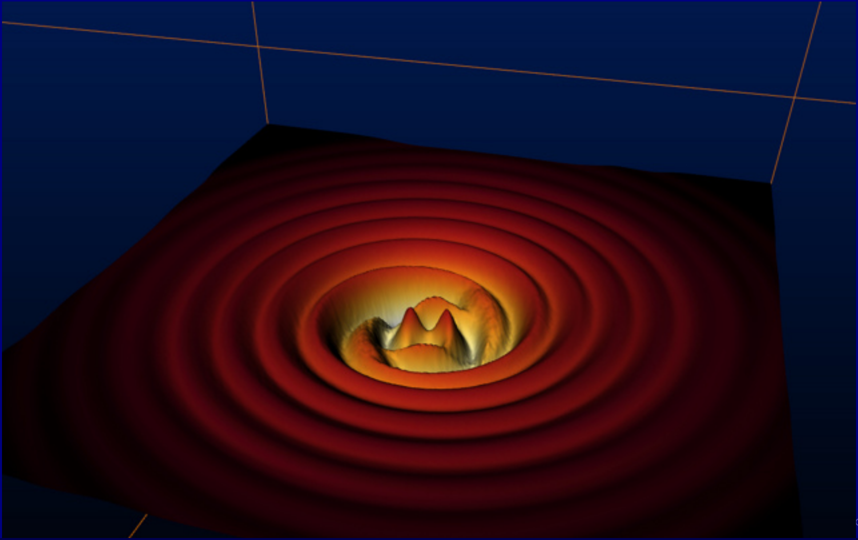
Einstein-Hydrodynamics Coupling

- Everything up till now has been vacuum
- There are thorns to provide matter fields
- TmunuBase has a standard set of variables and a set of schedule groups orchestrating when $T_{\mu\nu}$ is calculated

Example - Grazing black holes



Example - Binary waves



What's the point to me?

- Fundamental physics
- Electron-positron collision
- The resulting gamma rays
- Do they mesh? If not, what's wrong?