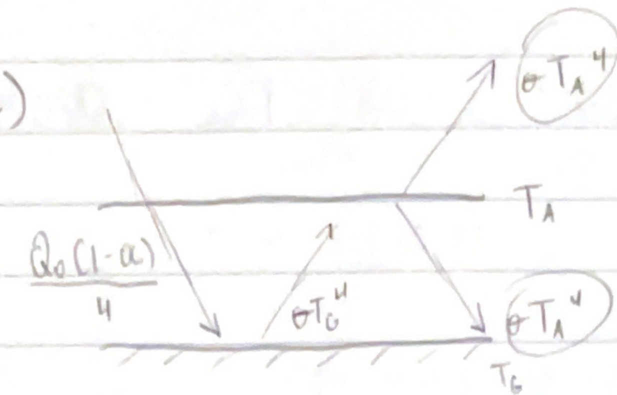
Joey Huang ESS201 p-set 2

1. a)



b) Given: $\sigma T_G^4 = \sigma T_A^4 + \sigma T_A^4 \rightarrow \sigma T_G^4 = 2\sigma T_A^4$

and: $\dfrac{Q_0(1-\alpha)}{4} + \sigma T_A^4 = \sigma T_G^4$

subsitute σT_G^4: $\dfrac{Q_0(1-\alpha)}{4} + \sigma T_A^4 = \underline{2\sigma T_A^4}$ was σT_G^4

$$\frac{Q_0(1-\alpha)}{4} = \sigma T_A^4$$

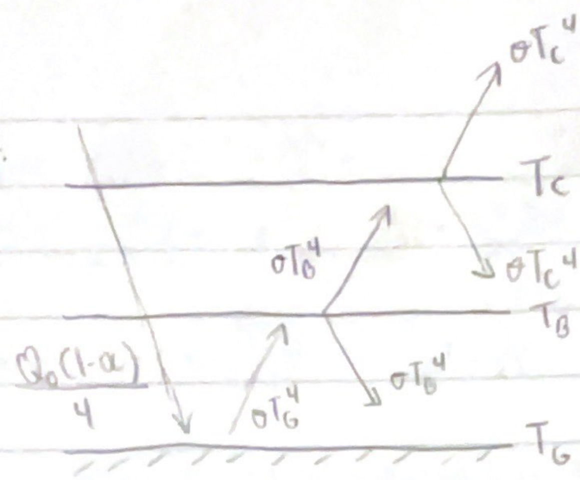
rearrange: $T_A = \left(\dfrac{Q_0(1-\alpha)}{4\sigma}\right)^{1/4}$

c) Given: $Q_0 = 1368\,\mathrm{Wm^{-2}}$, $\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$, $\alpha = 0$

subsitute: $T_A = \left(\dfrac{1370\,\mathrm{Wm^{-2}}\,(1-0)}{4(5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}})}\right)^{1/4} = \left(\dfrac{1370\,\mathrm{Wm^{-2}}}{2.27 \times 10^{-7}\,\mathrm{Wm^{-2}K^{-4}}}\right)^{1/4}$

$T_A = (6.03 \times 10^{9}\,\mathrm{K^4})^{1/4} = 279\,\mathrm{K}$

2. Given:



a) layer B: sources = sinks

$$\underline{\sigma T_G^4 + \sigma T_C^4 = \sigma T_B^4 + \sigma T_B^4}$$

b) layer C: sources = sinks

$$\underline{\sigma T_B^4 = \sigma T_C^4 + \sigma T_C^4}$$

c) Given: $\underline{\sigma T_B^4 = 2\sigma T_C^4}$ and $\frac{Q_0}{4}(1-\alpha) + \sigma T_B^4 = \sigma T_G^4$

subsitute σT_B^4: $\sigma T_G^4 + \sigma T_C^4 = 2\sigma T_C^4 + 2\sigma T_C^4 \rightarrow \underline{\sigma T_G = 3\sigma T_C^4}$

subsitute σT_B^4 and σT_G^4: $\frac{Q_0}{4}(1-\alpha) + 3\sigma T_C^4 = 4\sigma T_C^4$

rearrange: $\sigma T_C^4 = \frac{Q_0(1-\alpha)}{4} \rightarrow T_C = \left(\frac{Q_0(1-\alpha)}{4\sigma}\right)^{1/4}$

subsitute: $T_C = \left(\frac{1370\,Wm^{-2}(1-0)}{4(5.67\times10^{-8}\,Wm^{-2})}\right)^{1/4} = 279K$

same calculation as 1c

d) Given: $\sigma T_B^4 = \sigma T_C^4 + \sigma T_C^4$, so $\sigma T_C^4 = \frac{1}{2}\sigma T_B^4$

$\sigma T_G^4 + \frac{1}{2}\sigma T_B^4 = \sigma T_B^4 + \sigma T_B^4$, so $\sigma T_G^4 = \frac{3}{2}\sigma T_B^4$

subsitute $\frac{Q_0(1-\alpha)}{4} + \sigma T_B^4 = \frac{3}{2}\sigma T_B^4 \rightarrow \frac{Q_0(1-\alpha)}{4} = \frac{1}{2}\sigma T_B^4$

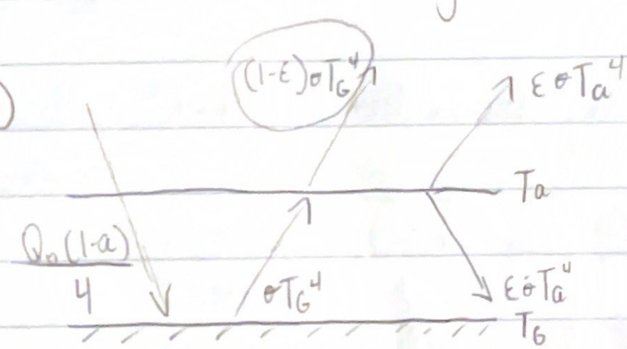
rearrange: $T_B = \left(\frac{2Q_0(1-\alpha)}{4\sigma}\right)^{1/4} = \left(\frac{2 \cdot 1370\,Wm^{-2}(1-0)}{4(5.67\times10^{-8}\,Wm^{-2}K^{-4})}\right)^{1/4}$

$T_B = \left(\frac{2740\,Wm^{-2}}{2.27\times10^{-7}\,Wm^{-2}K^{-4}}\right)^{1/4} = (1.21\times10^{10}\,K^4)^{1/4} = 331\,K$

e) The answer from 1c is the same as 2c, since they absorbed the same amount of radiation.

f) T_B is warmer than T_C, since that layer is closer to the surface so the gases have a greater density.

3. a)



b) atmos layer: sources = sinks

$$\sigma T_G^4 = (1-\varepsilon)\sigma T_G^4 + \varepsilon\sigma T_a^4 + \varepsilon\sigma T_a^4$$

c) ground layer: sources = sinks

$$\frac{Q_0(1-\alpha)}{4} + \varepsilon\sigma T_a^4 = \sigma T_G^4$$

d) Given: $\sigma T_G^4 = (1-\varepsilon)\sigma T_G^4 + 2\varepsilon\sigma T_a^4 \rightarrow \varepsilon\sigma T_a^4 = \frac{1}{2}\sigma T_G^4(1-(1-\varepsilon))$

$\hookrightarrow \varepsilon\sigma T_a^4 = \frac{1}{2}\sigma T_G^4(\varepsilon)$

subsitute $\varepsilon\sigma T_a^4$: $\frac{Q_0(1-\alpha)}{4} + \frac{1}{2}\sigma T_G^4\varepsilon = \sigma T_G^4 \rightarrow \varepsilon = \frac{\sigma T_G^4 - Q_0(1-\alpha)/4}{\frac{1}{2}\sigma T_G^4}$

Subsitute values: $\varepsilon = \frac{(5.67\times10^{-8}\,Wm^{-2}\cancel{K^{-4}})(288\cancel{K})^4 - (1370\,Wm^{-2})(1-0)/4}{\frac{1}{2}(5.67\times10^{-8}\,Wm^{-2}\cancel{K^{-4}})(288\cancel{K})^4}$

$\varepsilon = \frac{(390.\,Wm^{-2}) - (342\,Wm^{-2})}{195\,Wm^{-2}}$

$\varepsilon = \frac{48.0\,Wm^{-2}}{195\,Wm^{-2}} = 0.246$