COMSW4731_001_2021_3

Computer Vision I: First Principles

Homework #4

计算机视觉

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Problem 1

The chain of transformations can be expressed as:

$$T : \text{Scaling(Stretching)} \to \text{Rotation} \to \text{Translation}$$

Now, we give the transformation matrix for each step. Since there is translation happening, we set the matrices to be 3×3.

Scaling(Stretching)

Since the side length of the square changes from d to $2d$, the transformation matrix is:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

since both x and y are stretched to 2 times of themselves.

Rotation

Since the square is rotated counterclockwise for θ (with the rotation center being the lower right corner), the transformation matrix is:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translation

We first need to know where the square has moved to.

Since C_1 was at (x_1, y_1) at the beginning, we know its position after stretching is:

$$(x_1, y_1) \to (2x_1, 2y_1)$$

The position after stretching is:

$$(2x_1, 2y_1) \to (x_1', y_1') = (2x_1\cos\theta - 2y_1\sin\theta, 2x_1\sin\theta + 2y_1\cos\theta)$$

Since the final position is at (x_2, y_2), we have

$$x_2 = x_1' + t_x, y_2 = y_1' + t_y$$

and

$$t_x = x_2 - x_1' = x_2 - (2x_1\cos\theta - 2y_1\sin\theta)$$
$$t_y = y_2 - y_1' = y_2 - (2x_1\sin\theta + 2y_1\cos\theta)$$

So, the transformation matrix for this step is:

$$\begin{bmatrix} 1 & 0 & x_2 - (2x_1\cos\theta - 2y_1\sin\theta) \\ 0 & 1 & y_2 - (2x_1\sin\theta + 2y_1\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}$$