COMSW4731_001_2021_3

Computer Vision I: First Principles

Homework #1

计算机视觉

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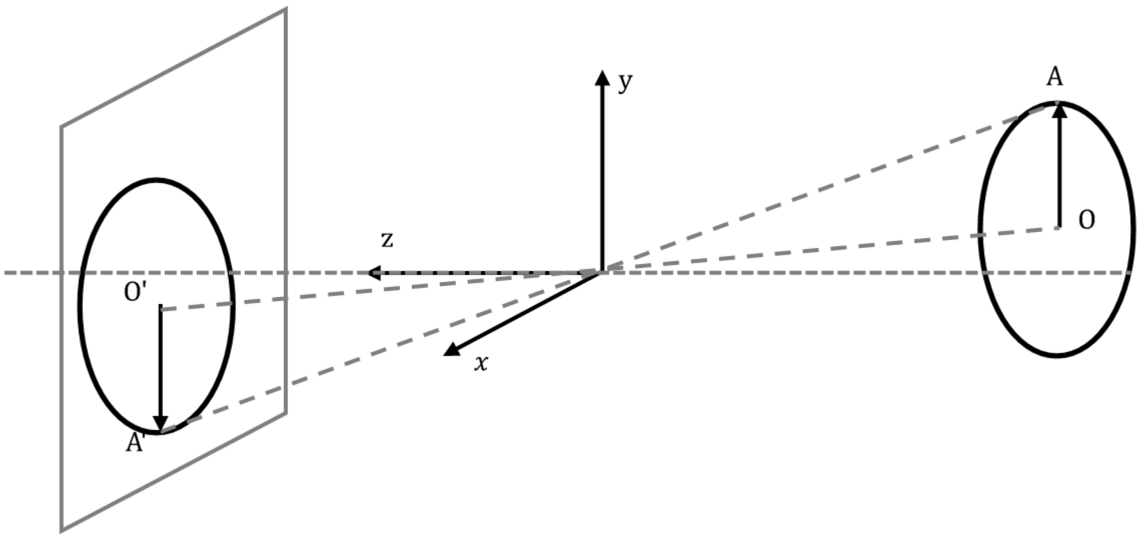
Problem 1

(a)



Suppose the center of the disk is O and an arbitrary point on the side of the disk is A. The image of OA on the image plane is $O'A'$.

Since the circular disk is on a plane that is parallel to the image plane, $OA \parallel O'A'$. By constructing similar triangles, we can know that $O'A' = kOA$ where k is a real number.

Since for any other point on the side of the disk, the corresponding radius and its image also hold the same equivalence above, we know that all the images of such points are of the same distance from the image of the center, O'.

This means that the image is also a circular disk, centered at O'. Its radius equals to $|O'A'|$.

Therefore, the image of the disk is a circular disk.

(b)

The formula of magnification m is

$$m = \frac{f}{z_0}$$

When f keeps unchanged, suppose that $z_1 = 1\text{m}$ and $z_2 = 2\text{m}$. In this way:

$$m_2 = \frac{1}{2}m_1 = \frac{f}{2z_1}$$

Suppose the area of image is $Area_i$ and the area of the object is $Area_o$. We have the following formula:

$$\frac{Area_i}{Area_o} = m^2$$

Thus we have:

$$\frac{Area_1}{Area_o} = m_1^2$$

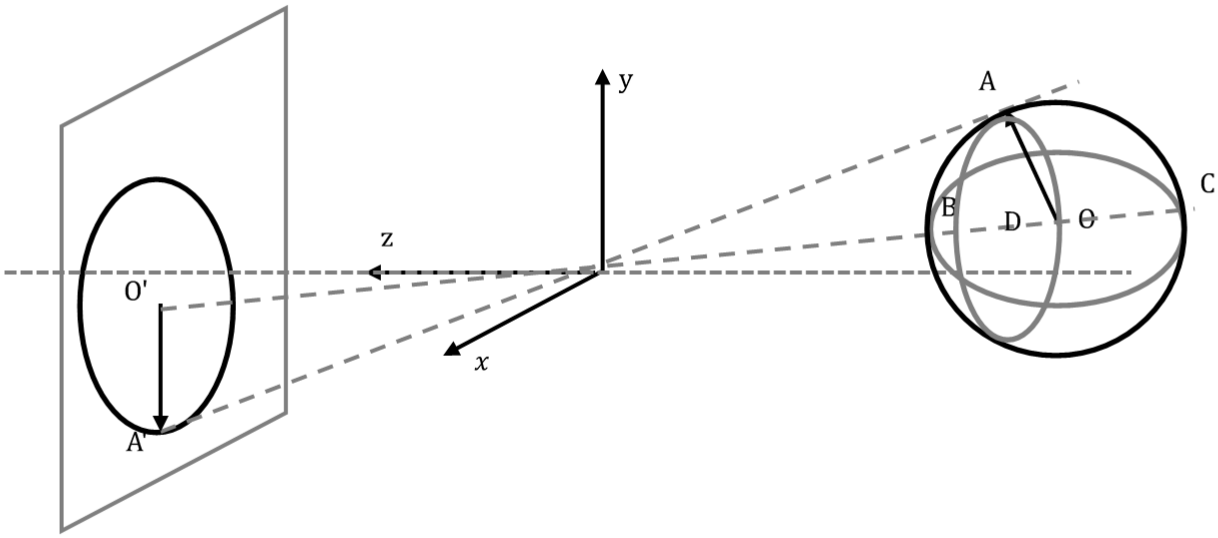
$$\frac{Area_2}{Area_o} = m_2^2$$

Since $m_2 = 2m_1$ and $Area_1 = 1\text{mm}^2$, we now have:

$$Area_2 = m_2^2 \cdot Area_o = \frac{1}{4}m_1^2 \cdot \frac{Area_1}{m_1^2} = \frac{1}{4} \cdot Area_1 = 0.25\text{mm}^2$$

Hence, the area of the image is now 0.25mm^2.

(c)



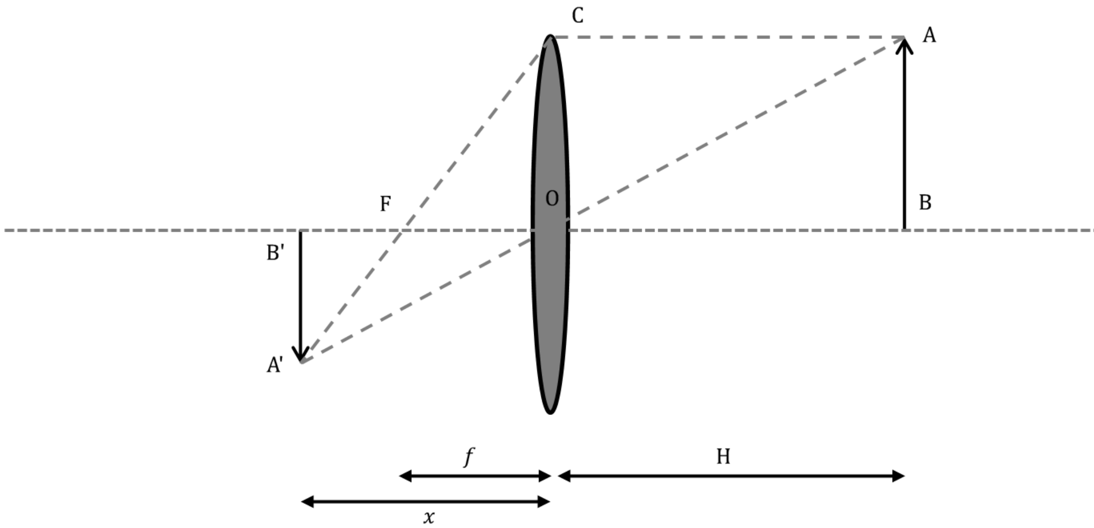
Suppose the center of the sphere is O. There is a line that passes through the center of the pinhole camera and is tangent to the sphere with the tangent point A. The image of A on the image plane is A'. Line OO' passes through the sphere shell twice. The intersection points are B and C.

According to (a), we already know that the image of the circle AD (it has point A and it is on a plane that is parallel to the image plane) is also a circle.

Point B and C are on the same line passing through the pinhole camera, so they share the same image O' on the image plane. For all other lines that pass through both the pinhole camera and the sphere, we know that the line will intersect with the sphere shell twice and these two intersection points will share the same image point on the image plane. The image points are all inside circle $O'A'$ because all such lines pass through disk AD.

Hence, the image of the sphere is a circular disk.

Problem 2



In the above graph, AB is the object and $A'B'$ is the image (and they are all perpendicular to line BB'), O is the center of the lens, F is the focus point, $AC \parallel BB'$.

Also, $AC = H$, $OF = f$, $A'B' = \frac{c}{2}$. Let $OB' = x$ and $AB = \frac{D}{2}$ (D is the aperture).

Since triangles $A'B'F$ and OCF are similar, we have

$$\frac{\frac{c}{2}}{x - f} = \frac{\frac{D}{2}}{f}$$

and

$$x = f + \frac{cf}{D}$$

Since triangles $A'B'O$ and OAB are similar, we have

$$\frac{\frac{D}{2}}{H} = \frac{\frac{c}{2}}{x}$$

and

$$H = \frac{Dx}{c}$$

Hence

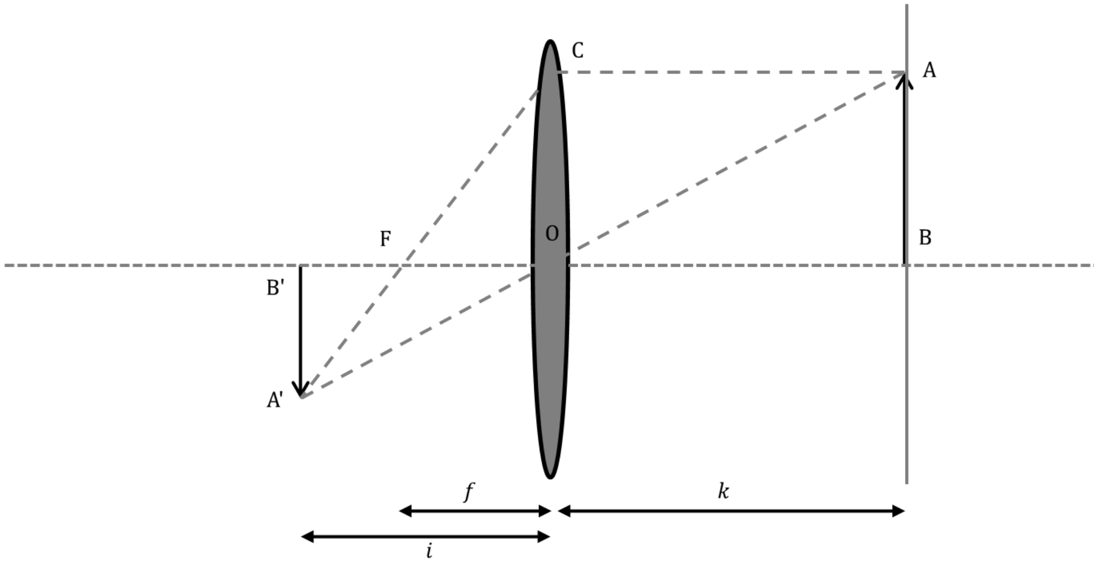
$$H = \frac{D}{c}(f + \frac{cf}{D}) = \frac{Df}{c} + f$$

Since f-number$= N = \frac{f}{D}$, we have $D = \frac{f}{N}$, and:

$$H = \frac{f}{N}\frac{f}{c} + f = \frac{f^2}{Nc} + f$$

Problem 3

(a)



In the above graph, AB is the object and $A'B'$ is the image (and they are all perpendicular to line BB'), O is the center of the lens, F is the focus point, $AC \parallel BB'$.

Also, $OB = k$, $OB' = i$, $OF = f$.

By Gaussian Lens Law, we have:

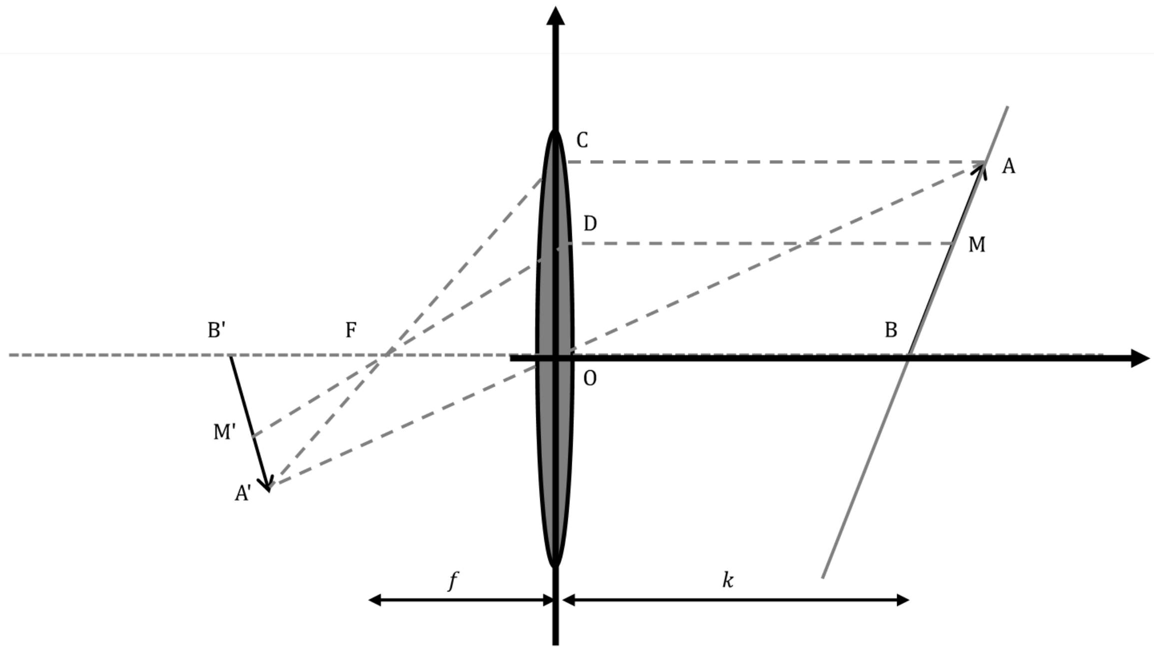
$$\frac{1}{i} + \frac{1}{k} = \frac{1}{f}$$

and

$$i = \frac{kf}{k - f}$$

Hence, the distance of a focused image of the scene line be formed is $\frac{kf}{k-f}$.

(b)



In the above graph, AB is the object and $A'B'$ is the image (AB is on the scene plane), M is a point on AB and M' is the image of M, O is the center of the lens, F is the focus point, $AC \parallel MD \parallel BB'$.

Also, $OB = k$, $OB' = i$, $OF = f$. A system with x-axis and y-axis has also been set up.

The scene line has the parametric function of

$$\begin{cases} x = & k + t\sin\theta \\ y = & \quad\quad t\cos\theta \end{cases}$$

Point M can be denoted as

$$M(k + t_M \sin\theta, t_M \cos\theta)$$

By Gaussian Lens Law, we have:

$$\frac{1}{i_M} + \frac{1}{x_M} = \frac{1}{f}$$

and

$$i_M = \frac{x_M f}{x_M - f} = \frac{f(k + t_M \sin\theta)}{k + t_M \sin\theta - f}$$

Also (h_M is the distance from M' to line BB'):

$$\frac{x_M}{y_M} = \frac{i_M}{h_M}$$

Then

$$h_M = \frac{y_M i_M}{x_M} = \frac{f(t_M \cos\theta)}{k + t_M \sin\theta - f}$$

Hence, point M' can be denoted as

$$M'(\frac{f(k+t_M\sin\theta)}{k+t_M\sin\theta-f},\frac{f(t_M\cos\theta)}{k+t_M\sin\theta-f})$$

M' is on the curve with the following parametric function:

$$\begin{cases} x = & \dfrac{f}{k+t\sin\theta-f}(k+t\sin\theta) \\ y = & \dfrac{f}{k+t\sin\theta-f}(t\cos\theta) \end{cases}$$

Now, we calculate $\frac{dy}{dx}$:

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\cos\theta\frac{f}{k+t\sin\theta-f}-(t\cos\theta)f(k+t\sin\theta-f)^{-2}\sin\theta}{\sin\theta\frac{f}{k+t\sin\theta-f}-(k+t\sin\theta)f(k+t\sin\theta-f)^{-2}\sin\theta}$$

$$=\frac{\cos\theta(k+t\sin\theta-f)-(t\cos\theta)\sin\theta}{\sin\theta(k+t\sin\theta-f)-(k+t\sin\theta)\sin\theta}=\frac{(k-f)\cos\theta}{-f\sin\theta}=-\frac{k-f}{f}\cot\theta$$

Since $\frac{dy}{dx}$ is a constant, we have verified that the image of the scene line has the equation form of $y=\frac{dy}{dx}x+C$, which means that it is still a line. The slope ($\frac{dy}{dx}$) shows that it is tilted (to the other direction of the scene line because of the negative sign).

(c)

In part (b), we have calculated that the slope of the image line is

$$\frac{dy}{dx}=-\frac{k-f}{f}\cot\theta$$

The angle ϕ has the following relationship with $\frac{dy}{dx}$ (there is a negative sign because the image plane is leaning opposite of the scene line):

$$\tan\phi=-\frac{1}{\frac{dy}{dx}}$$

Hence, we have

$$\tan\phi=-\frac{1}{-\frac{k-f}{f}\cot\theta}=\frac{f}{k-f}\tan\theta$$