计算机视觉

COMSW4731_001_2021_3

Computer Vision I: First Principles

Homework #3

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Problem 1

Sinusoid

The line equation is

$$x \sin\theta - y\cos\theta + \rho = 0$$

Currently, the independent and dependent variables are x and y. If we regard θ and ρ as the independent and dependent variables instead, we can rewrite the expression into

$$\rho = y\cos\theta - x\sin\theta$$

The right hand side of this expression can be written into the form of

$$R\cos(\theta - \alpha)$$

where

$$R = \sqrt{x^2 + y^2}, \tan\alpha = -\frac{y}{x}$$

Therefore, for each point (for example, (x_0, y_0)) in the (x, y)-image space, since it's on the line $x\sin\theta - y\cos\theta + \rho = 0$, and when regarding θ and ρ as the variables (by rewriting $x_0\sin\theta - y_0\cos\theta + \rho = 0$), we have the function:

$$\rho = R\cos(\theta - \alpha), R = \sqrt{x_0^2 + y_0^2}, \tan\alpha = -\frac{y_0}{x_0}$$

Also, since $\cos(x) = \sin(\frac{\pi}{2} - x)$ and $\sin(-x) = -\sin(x)$,

$$\rho = R\cos(\theta - \alpha) = R\sin(\frac{\pi}{2} - \theta + \alpha) = -R\sin(\theta - (\frac{\pi}{2} + \alpha))$$

Since this function is in the form of $y = A\sin(\omega x + \phi)$, we get a sinusoid in the (ρ, θ)-Hough space.

Amplitude & Phase

For the function

$$y = A\sin(\omega x + \phi)$$

Its amplitude is $|A|$ and its phase at x is $\omega x + \phi$ (so the "phase" when $x = 0$ is ϕ).

In the case of the problem, we have

$$\rho = -R\sin(\theta - (\frac{\pi}{2} + \alpha))$$

where

$$R = \sqrt{x^2 + y^2}, \tan\alpha = -\frac{y}{x}$$

Therefore, the amplitude is:

$$\sqrt{x^2 + y^2}$$

and the phase (at $\theta = 0$) is (since $\arctan(-x) = -\arctan(x)$):

$$-(\frac{\pi}{2} + \alpha) = -(\frac{\pi}{2} + \arctan(-\frac{y}{x})) = \arctan(\frac{y}{x}) - \frac{\pi}{2}$$

https://www.overleaf.com/project/616b9cdf56e3c938772d4089

Period

For the function

$$y = A\sin(\omega x + \phi)$$

Its period is $\frac{2\pi}{\omega}$.

In the case of the problem, we have

$$\rho = -R\sin(\theta - (\frac{\pi}{2} + \alpha))$$

where

$$R = \sqrt{x^2 + y^2}, \tan\alpha = -\frac{y}{x}$$

Therefore, the period is:

$$\frac{2\pi}{1} = 2\pi$$

Therefore, the period of the sinusoid does NOT vary with the image point (x, y) since the period is a constant (2π) here.