Final Project
Due 12/13/24 at 11:59PM *

For this semester's final exam, I would like you to record and submit a video no more than 30 minutes long, following one of the project options listed below. You should submit your video, notes/slides, and references through Canvas. To aid in your presentation, you will create lecture notes or slides to share on your screen as you present. You will submit the video and a PDF of your notes/slides to Canvas.

Target audience for video: A student like yourself who is taking this class and has learned the relevant material leading up to the concept of your choice.

I recommend creating the video by recording yourself on Zoom. During the video, you should share your screen with prepared slides/notes at the same time as you are speaking. You must submit your slides/notes as a PDF file and video.
You must also submit a list of references you consulted while planning your video. This includes websites you visited, books/notes referenced, and people or AIs you had discussions with. You should include this list in your slides or notes.

Topics to Choose From

Choose **one** of the following project options below. If you want to cover a different concept that is not on this list, you must get my approval first.

*grace period until 5AM the next day.

A. Google's PageRank Algorithm

Project Overview: This project explores the mathematics of a Google search. It has no coding component.

Lesson Plan

Create a lecture following the notes at https://pi.math.cornell.edu/~mec/Winter2009/RalucaRemus/Lecture3/lecture3.html. The presentation should cover the following points:

1. Background. Don't read a script for this. It should be short and move quickly to the example.
2. Describe the motivating example. Make sure to define the transition matrix (the previous lecture does this in more detail).
3. Explain the dynamical systems point of view. Using what you learned about Markov matrices, be sure to clarify why the sequence tends to an equilibrium value. What is the eigenvalue/eigenvector for this equilibrium direction?
4. Explain the linear algebra point of view.
5. (skip the probabilistic point of view)
6. Talk about how we deal with dangling nodes and disconnected components and explain their meaning in this context.
7. Discuss the solution of Page and Brin.
8. State what the Perron–Frobenius Theorem and Power Method Convergence Theorems are and how they apply to this discussion (no need to go into a lot of detail, just point out how they are being used).
9. Solve either problem 3 or 4.

Project Deliverables

- Video presentation (at most 30 minutes) covering all points of the lesson plan.
- Notes/slides including a list of references. (No coding requirement.)

B. Computer Graphics

Project Overview:. This project explores how images are moved and scaled in 3D using linear algebra.

Lesson Plan

Create a lecture following Chapter 10.6 of Gilbert Strang's *Introduction to Linear Algebra*. (see Canvas for a PDF). The presentation should cover the following points:

1. Explain what homogeneous coordinates are and why we need to use 4 by 4 matrices rather than 3 by 3.
2. Discuss translation. With MATLAB/Geogebra/or the software of your choice, create an illustration showing how the translation matrix acts on a given vector.
3. Discuss scaling. Similarly, with the software of your choice, create an illustration showing how the translation matrix acts on a given vector. Show some illustrations.
4. Quickly define an affine (pronounced: "aff-eye-n") transformation.
5. Discuss Rotation. Make sure to explain how to rotate about any given point. As above, create an illustration using MATLAB/Geogebra/etc.
6. Discuss Projection and create an illustration.
7. Present your solutions to problems 9, 10, and 11 of the problem set.

Project Deliverables

- Video presentation (at most 30 minutes) covering all points of the lesson plan.
- Code used to create illustrations.
- Notes/slides including a list of references.

C. Iterative Methods for Solving Linear Systems: the Gauss–Seidel method and the Jacobi method

If you did not choose this option for the midterm project, you can choose to present on it now. If you already presented on this for the midterm, please select between options A and B.

Lesson Plan

1. **Gauss-Seidel method**. Go to the Wikipedia page for the Gauss-Seidel method and read about how it works. An outline of the points you should explain in your presentation include:
   - Start with a square system of n linear equations.
   - Matrix-based formula for Gauss-Seidel.
   - Explanation of why the matrix-based formula works.
2. **Examples of Gauss-Seidel:** Give two examples of the matrix version of the Gauss-Seidel method. The first should show an example where the algorithm converges. The second should show an example when the algorithm does NOT converge. *Your examples should be different from those on the Wikipedia page.* You don't need to work these out live, and this would be a good thing to prep ahead of time.
3. Go to the Wikipedia page for the Jacobi method and read about how it works. An outline of the points you should explain in your presentation include:
   - Start with a square system of n linear equations.
   - Matrix-based formula for the Jacobi method.
   - Explanation of why the matrix-based formula works.
4. **Examples of Jacobi method:** Give two examples of the matrix version of the Jacobi method. The first should show an example where the algorithm converges. The second should show an example when the algorithm does NOT converge. *Your examples should be different from those on the Wikipedia page.* You don't need to work these out live, and this would be a good thing to prep ahead of time.
5. **Implementation:** The Wikipedia pages give a sample code Python for the Gauss-Seidel and Jacobi methods.
   - Copy these codes into your own editor.
   - Briefly what each line of the code is doing in your own words.
   - Run the codes for examples different from those on the Wikipedia page.
6. **Applications:** The Gauss-Seidel method is used in solving sparse systems, which arise in finite element methods, numerical solutions of PDEs, and circuit analysis. Do some googling to explain why this is the case.
7. **Convergence Analysis:** Investigate the conditions under which the Gauss-Seidel/Jacobi methods converge. This will require you to learn and explain new terms like "positive definite" matrices, "diagonally dominant" matrices, and the "spectral radius" of a matrix. You can find information about these on the Wiki pages.

Project Deliverables

- A presentation covering all 7 points in the lesson plan (at most 30 minutes).
- A written set of notes/slides containing references for your presentation.
- An analysis and demonstration of the code for Gauss-Seidel and Jacobi's method as outlined in point 5.

Rubric for Grading

NOTE: The project should be *at most* 30 minutes. If you submit a longer video, only the first 30 minutes will be graded. I will not consider content after the 30-minute mark in your grade.

	Meets Expectations (2)	Room For Improvement (1)	Failed to meet expectations (0)	
Structure: Did the presentation... • Address each point in the lesson plan? • Have coherent structure?				
Clarity: Was the presentation... • Clear? • Concise? • Conversational? (e.g., did not read from a script.)				
Accuracy: Did the presentation... • Provide rigorous and correct explanations of the math? • Correctly explain each point in the lesson plan?				
Quality of Content: Did the presentation... • Submit all project deliverables? • Cover an appropriate amount of material at an appropriate pace? • Was the code clearly explained and working? (if required)				
Quality of Slides/Notes/Code: • Was the code clearly explained and working? (if required) • Did the notes/slides include a list of all references/resources consulted during preparation? • Were the slides/notes used to support the presentation effectively, and were **not** used as a script during the presentation? • Was the speaker's face visible for almost all of the presentation?				