1. Exercise Sheet
Statistical Methods for Data Analysis A
Submission: 26.04.2022 23:59

Summer Term 2022
Prof. W. Rhode, Dr. M. Nöthe

Time	Group	Submission in Moodle; Mails with subject: [SMD2022/23]
Th.12:15–13:00	A	leonora.kardum@udo.edu and karolin.hymon@udo.edu
Fr. 10:15–11:00	B	lukas.nickel@udo.edu and noah.biederbeck@udo.edu
Fr. 12:15–13:00	C	rune.dominik@udo.edu and felix.geyer@udo.edu

Exercise 1 *Numerical stability* **4 p.**

Consider the functions

(a) $f(x) = (x^3 + 1/3) - (x^3 - 1/3)$ and

(b) $g(x) = ((3 + x^3/3) - (3 - x^3/3))/x^3$.

Determine empirically for which ranges of x (roughly) the numerical result

- deviates from the algebraic by more than 1 %,
- is equal to zero.

(c) Graphically represent the result in a suitable form (for exaple, logarithmic x-scale)!

```
x = np.logspace(start, stop, num)
```

(d) How does the representation change if you create the data point with the float32 or float64 data type?

```
x_32 = np.logspace(start, stop, num, dtype='float32')
x_64 = np.logspace(start, stop, num, dtype='float64')
```

Exercise 2 *Numerical stability and condition* **6 p.**

The expression $f(E, \Theta)$ represents a summand of the differential cross section for the reaction $e^- e^+ \to \gamma\gamma$

$$f(E, \Theta) = \frac{2 + \sin^2\Theta}{1 - \beta^2 \cos^2\Theta}.$$

and is given by

$$\beta = \sqrt{1 - \gamma^{-2}},$$
$$\gamma = \frac{E}{m} \quad (m = 511\,\text{keV}).$$

(a) Is $f(E, \Theta)$ numerically stable? If yes, which range for Θ is the equation for $E = 50\,\text{GeV}$ numerically unstable?

(b) Resolve the numerical instability by a suitable analytic transformation. (Hint: Make use of $1 - \beta^2 = 1/\gamma^2$ and $1 = \sin^2\Theta + \cos^2\Theta$)

(c) Show that you have fixed the stability problems by plotting both equations in the critical interval.

(d) Calculate the condition number. How does this depend on Θ?

(e) Plot the value of the condition number as a function of Θ ($0 \leq \Theta \leq \pi$). In which area is the problem well or poorly conditioned?

(f) What is the difference between stability and condition?