Project #3

Due on 6/9/2019

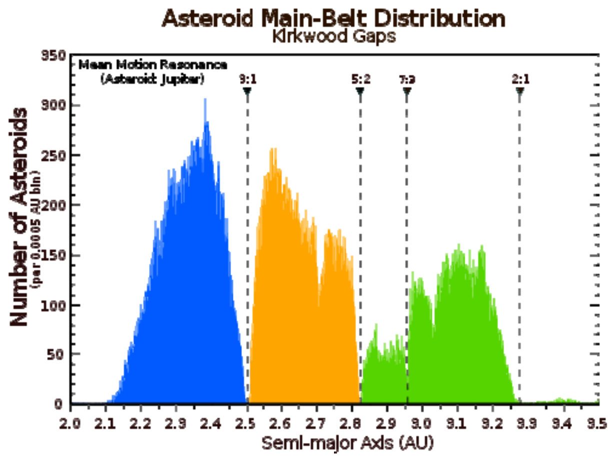
Problem A: The Asteroid Belt (小行星带)

A histogram of the number of asteroids versus their semi-major axes (or equivalently their orbital periods) shows some distinct gaps. These gaps, called the Kirkwood gaps, are due to the orbital resonances with Jupiter (木星). That is, if asteroids were in these gaps, their periods would be simple fractions of the period of Jupiter.

- Model the orbit of an asteroid around the Sun under the influence of Jupiter.

- Study the orbital resonances of asteroids with semi-major axes in the range from 2.0 to 3.5 AU, and gain some insights into the formation of the Kirkwood gaps.

Reference:
https://en.Wikipedia.org/wiki/Kirkwood_gap



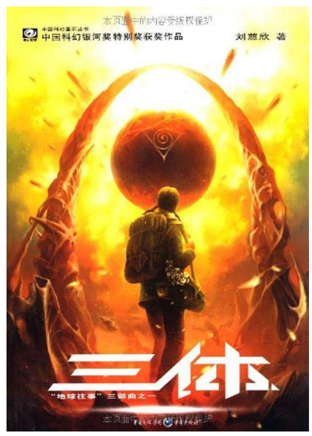
Some Thoughts on Problem A

- 这里我们研究一个简化的特殊三体问题：假设太阳固定不动，小行星与木星的轨道平面重合[?]，它们共同围绕太阳运动。

- 由于质量相对很小，可以忽略小行星的引力对木星的影响。

- 如果忽略木星的引力，只考虑小行星与太阳的两体问题，小行星的总能量E（包括动能和小行星与太阳之间的引力势能）与椭圆轨道的半长轴a有如下关系：$a = \frac{-GMm}{2E}$。

- 研究小行星在太阳和木星共同的引力作用下，其椭圆轨道半长轴a随时间的演化会帮助我们理解轨道共振的过程。在模拟过程中可能不方便直接测量半长轴a。利用前面a与E的关系，可以研究$\frac{-GMm}{2E}$随时间的演化，从而帮助我们理解轨道共振的过程。



Problem B: Fractional Quantum Oscillator (分数阶量子振子)

Fractional calculus may be described as an extension of the concept of a derivative operator from integer order n to arbitrary order α, where α is a real or complex value. The fractional Schrodinger equation includes the space derivative of fractional order α, instead of the second order ($\alpha = 2$) space derivative in the standard Schrodinger equation.

Here we study the 1-D fractional quantum oscillator with the Hamiltonian operator H_α defined as

$$H_\alpha = \frac{1}{2}\left((-\Delta)^{\alpha/2} + |x|^\alpha\right),$$

where the index α is called the Lévy index, and we consider $1 < \alpha \leq 4$. The operator $(-\Delta)^{\alpha/2}$ in the equation is the so-called 1-D fractional Riesz derivative.

Numerically solve the eigenvalues of the 1-D fractional quantum oscillator, and compare your results with the values obtained from semi-classical approximation.

Reference:

https://en.wikipedia.org/wiki/Fractional_calculus

https://en.wikipedia.org/wiki/Fractional_Schr%C3%B6dinger_equation

Problem C: The Levitron (Spin-stabilized Magnetic Levitation)

Earnshaw's theorem does not allow for a static configuration of permanent magnets to stably levitate another permanent magnet against gravity. This theorem does not apply to devices consisting of a properly configured magnetic base and corresponding magnetic top.

Spin-stabilized magnetic levitation is possible whereby a spinning magnet is levitated via magnetic forces above another magnet or array of magnets, and stabilized by gyroscopic effect due to a spin that is neither too fast, nor too slow to allow for a necessary precession. The Levitron top device displays the phenomenon of spin-stabilized magnetic levitation. The physics of the magnetic stability is similar to magnetic gradient traps.

Establish a model and set up the relevant equations that describes the motion of the Levitron top. Perform simulations of the top with different angular velocities and study the conditions for levitation.

Reference:

https://en.wikipedia.org/wiki/Levitron

https://en.wikipedia.org/wiki/Earnshaw%27s_theorem

https://en.wikipedia.org/wiki/Spin-stabilized_magnetic_levitation