Image Compression through Singular Value Decomposition

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BACKGROUND

WHAT IS SVD?

SVD, which stands for singular value decomposition, is a technique for representing some matrix A as the result of matrix multiplication between three matrices U, S, and V^T. The columns of U are the orthonormal eigenvectors of AA^T, the rows of V^T are the transposed orthonormal eigenvectors of A^TA, and S is a diagonal matrix of the singular values of A.

ORTHONORMAL?

Yes, the eigenvectors that make up the matrices U and V^T must be orthonormal. Since "orthonormal" is a portmanteau of "orthogonal" and "normal", it makes sense that orthonormal vectors are both orthogonal (at right angles, dot products of 0) and normal (unit length, magnitude of 1). MATLAB automatically outputs its eigenvectors as orthonormal, but it's good to know how to do it by hand; for this purpose, we learned the Gram-Schmidt process.

SINGULAR VALUES?

We mentioned earlier that S was composed of the singular values of A; any singular value is just the square root of a nonzero eigenvalue. The eigenvalues that we use are those of A^TA and AA^T, which are the same. If A is not square, A^TA and AA^T will have a different number of eigenvalues, but all of the non-overlapping ones should be 0.

DIMENSIONS?

If A is sized MxN, then U is MxM, S is MxN, and V^T is NxN.

LOGIC?

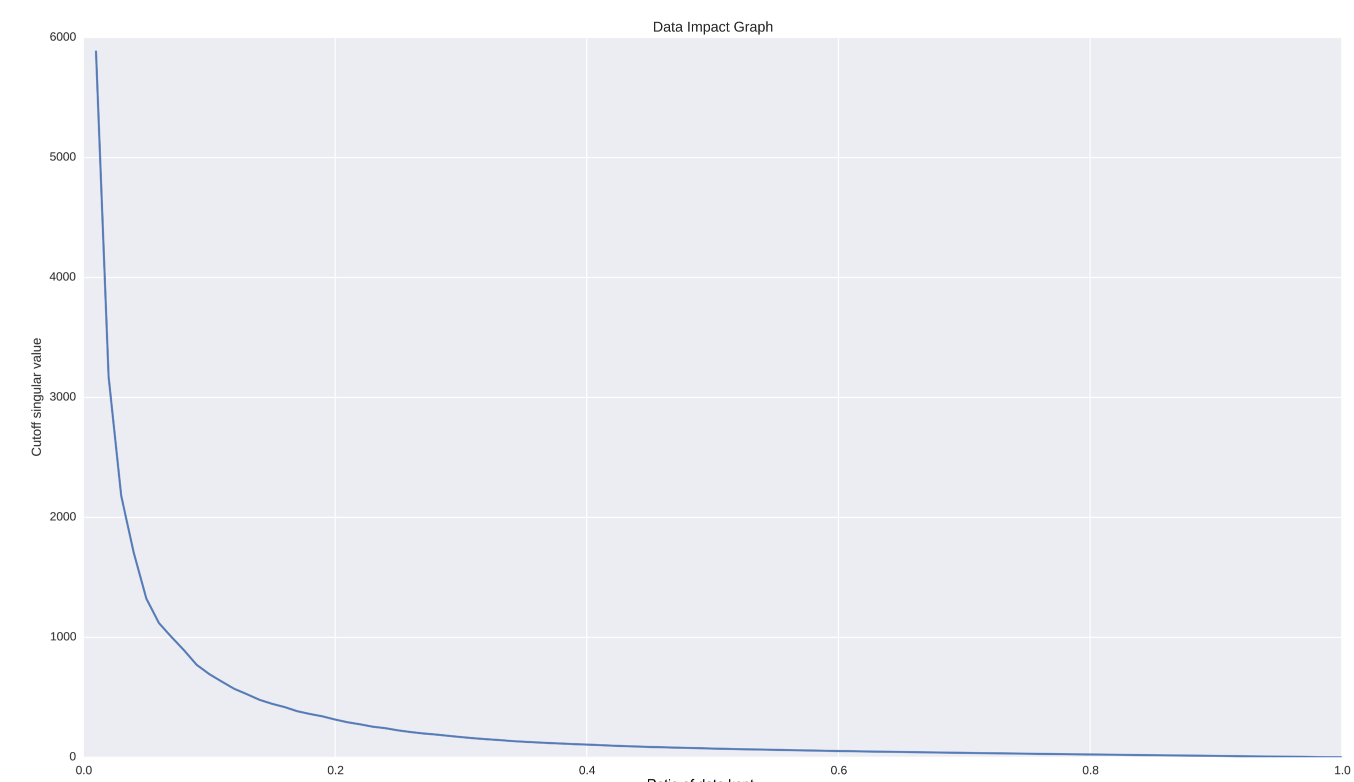
We can recall a few facts we know about matrices:

- A^TA and AA^T must be square matrices
- we can diagonalize any square matrix as PDP^{-1}
- for any orthonormal matrix M, $M^{-1} = M^T$

so we can say that $AA^T = US_1U^T$ and $A^TA = VS_2V^T$, where S_1 and S_2 (as stated earlier) contain the same singular values but may be padded with zeroes for proper sizing. <u>(explain how that leads into $A = USV^T$)</u>

HOW IT WORKS

Singular value decomposition can be used to represent any matrix A as the sum of n rank-1 matrices, with n being equal to the smaller dimension of A. These matrices are <u>(show calculation based on u,s,v)</u>



Original Image

1% 2% 20% 100%

Approximations

<u>(add visualization of discarded data for different levels of reconstruction)</u>