2CJ4 – Lab 5 – Set 5

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Introduction

The purpose of this experiment is to study the behavior of a second-order Butterworth low-pass filter using the Sallen-Key topology. Active filters like this one are essential components in many applications of analog signal processing, such as in power electronics, communication systems, and control systems. This lab involves analyzing the theoretical transfer function, verifying the cutoff frequency, and comparing experimental results with simulations and calculations.

Operational Principle of the Experimental Circuit

The experimental circuit implemented in this lab is a Sallen-Key second-order low-pass filter, utilizing an LM358 operational amplifier and passive components (resistors and capacitors). This topology allows the design of filters with predictable frequency response characteristics, notably a maximally flat response (Butterworth) in the passband.

In this configuration, the op-amp operates in the linear region, maintaining the virtual short condition $V_+ = V_-$. Also, the filter's transfer function is derived using impedance analysis and Kirchhoff's Current Law (KCL). For the Butterworth configuration, component values are selected so that $R_1 = R_2$, $C_1 = C_2$, and k is approximately 1, which yields a cutoff frequency:

$$f_c = \frac{1}{2\pi RC}$$

This circuit attenuates frequencies above the cutoff while allowing lower frequencies to pass with minimal loss. The frequency response is validated by observing the gain decrease beyond the cutoff point using the Analog Discovery 3.

Experiment Results

a. Derive an expression for the transfer function o f the filter.

$$V_0 = V^+ = V^-$$

$$V^+ sC_2 + \frac{(V^+ - V_a)}{R_2} = 0V$$

$$V_0\left(\frac{1}{R_2} + sC_2\right) = \frac{V_a}{R_2}$$

$$V_a = V_0(1 + sC_2R_2)$$

$$k = (1 + sC_2R_2)$$

$$V_a = V_0 k$$

KCL at node V_a:

$$\frac{V_a - V_0}{R_2} + sC_1(V_a - V_0) + \frac{V_a - V_i}{R_1} = 0V$$

$$\frac{V_0 k - V_0}{R_2} + sC_1(V_0 k - V_0) + \frac{V_0 k - V_i}{R_1} = 0V$$

$$\frac{V_0(k-1)}{R_2} + sC_1(V_0(k-1)) + \frac{V_0 k - V_i}{R_1} = 0V$$

$$V_0\left(\frac{k-1}{R_2} + sC_1(k-1) + \frac{k}{R_1}\right) = \left(\frac{V_i}{R_1}\right)$$

$$\frac{V_0}{V_i} = \left(\frac{1}{R_1\left(\frac{k-1}{R_2} + sC_1(k-1) + \frac{k}{R_1}\right)}\right)$$

Sub in $k = (1 + sC_2R_2)$:

$$\frac{V_0}{V_i} = \left(\frac{1}{(sC_2R_1 + sC_1sC_2R_2R_1 + 1 + sC_2R_2\ )}\right)$$

$$\frac{V_0}{V_i} = \left(\frac{1}{(s^2C_1C_2R_2R_1 + sC_2(R_1 + R_2) + 1)}\right)$$

$$H(s) = \frac{1}{(s^2C_1C_2R_2R_1 + sC_2(R_1 + R_2) + 1)}$$

Sub in given values:

$$H(s) = \frac{1}{((1 * 10^{-6})s^2 + (2 * 10^{-3})s + 1)}$$

b. Evaluate the filter transfer function abs(Vo/Vi) using the transfer function derived in part (a) for the frequencies shown in the table

$$H(jw) = \frac{1}{\sqrt{(1 - R^2C^2W^2)^2 + (2RWC)^2}}$$

$$H(jw) = \frac{1}{\sqrt{(-4\pi f C^2R^2 + 1)^2 + (4\pi f CR)^2}}$$

Using the above formula, we sub in the frequency, capacitance, and resistance to fill out the table below.

c. Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Use a sine wave with an amplitude of 2V and offset of 0V (Vcc = ±5V).

Frequency	abs(Vo/Vi) (analytical)	abs(Vo/Vi) (measured)
50 Hz	0.91	0.902
100 Hz	0.72	0.708
200 Hz	0.39	0.391
500 Hz	0.09	0.101
1 kHz	0.0247	0.04
1.1 kHz	0.021	0.0385
1.2 kHz	0.0174	0.035
1.3 kHz	0.0147	0.034
1.4 kHz	0.013	0.0313
1.5 kHz	0.011	0.302
1.6 kHz	0.0098	0.0293
1.7 kHz	0.0089	0.0262
1.8 kHz	0.0078	0.0254
1.9 kHz	0.0070	0.0246
2 kHz	0.0062	0.0231
5 kHz	0.001	0.0216

Table 1: Theoretical Gain VS Measured Gain over different input frequency

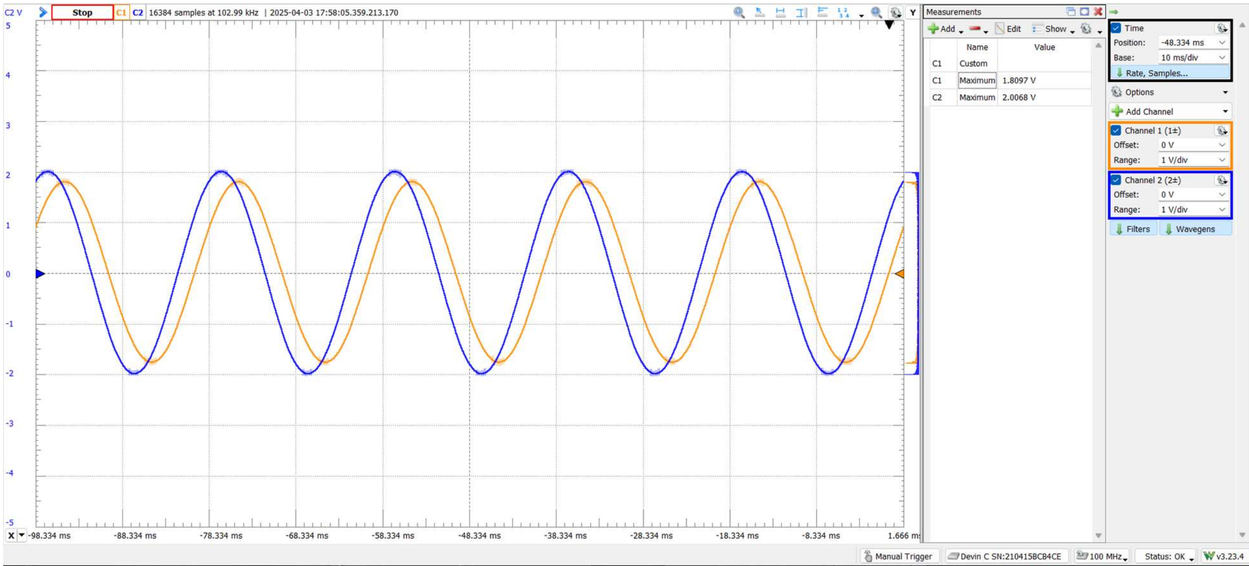

Figure 1. Vin and Vout with Input Frequency of 50 Hz

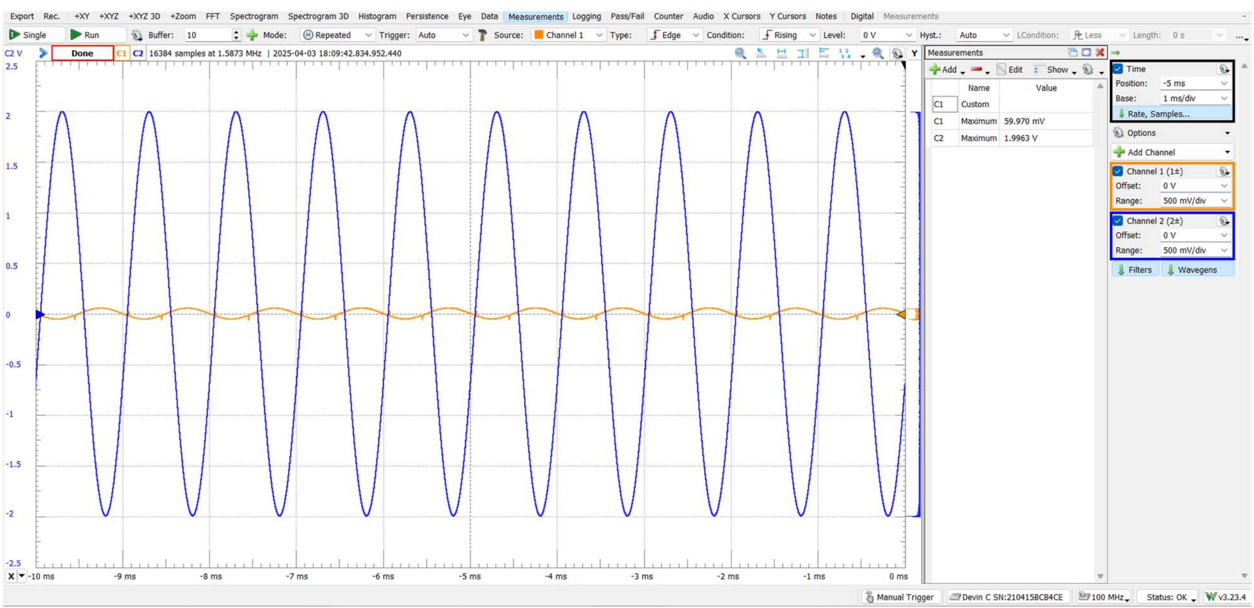

Figure 2. Vin and Vout with Input Frequency of 1 kHz

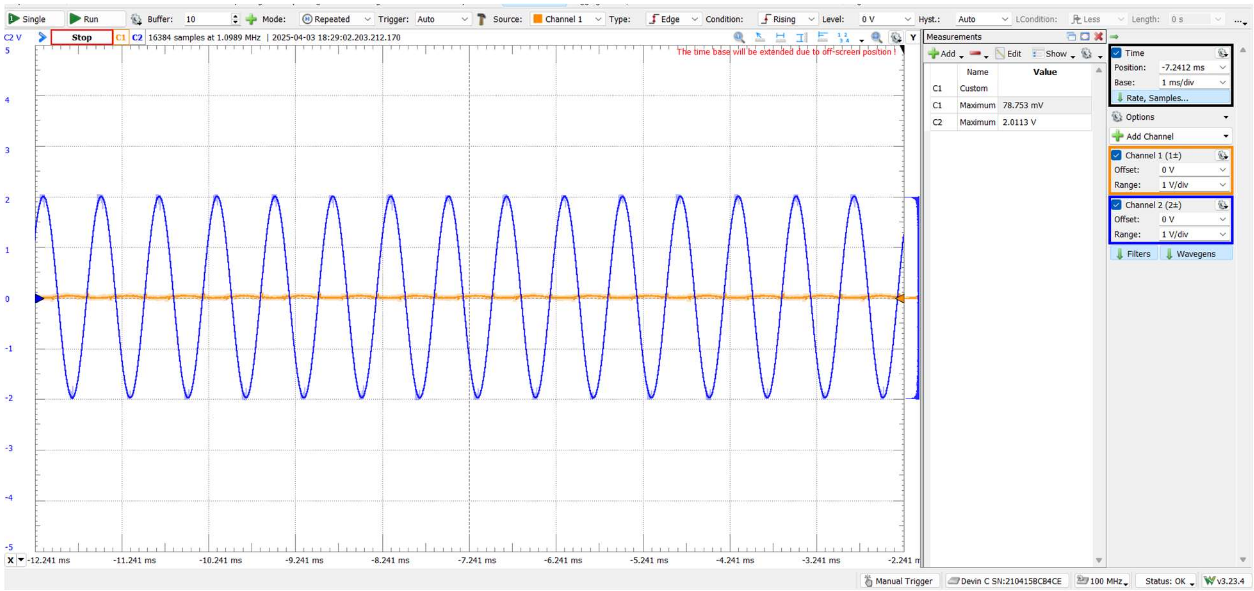

Figure 3. Vin and Vout with Input Frequency of 1.5 kHz

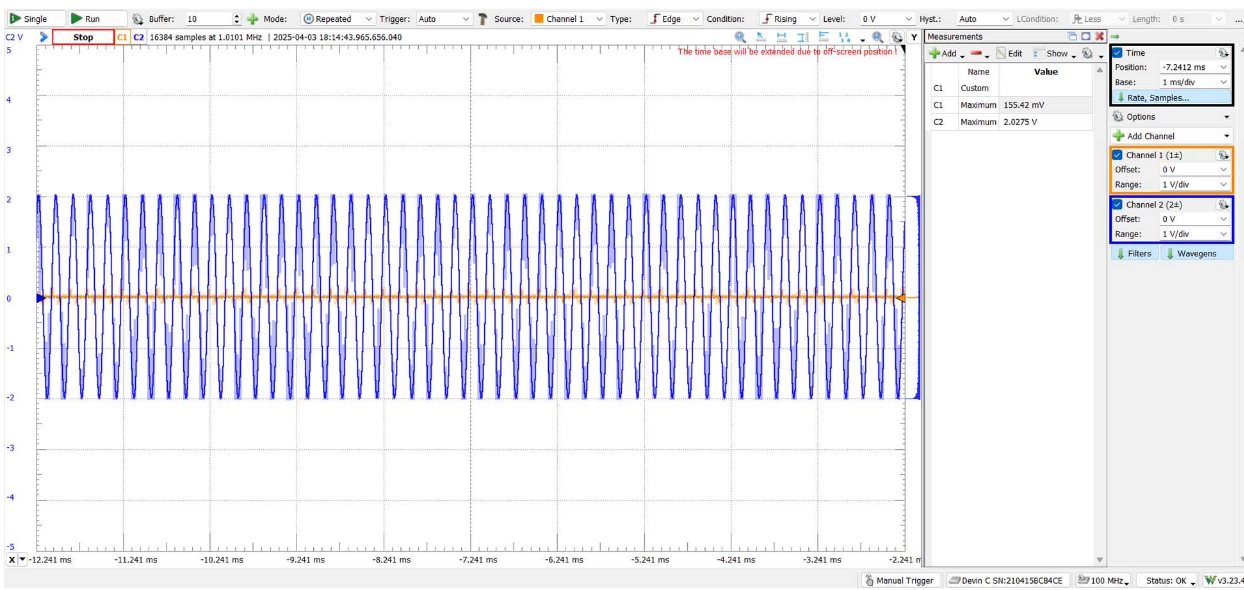

Figure 4. Vin and Vout with Input Frequency of 5 kHz

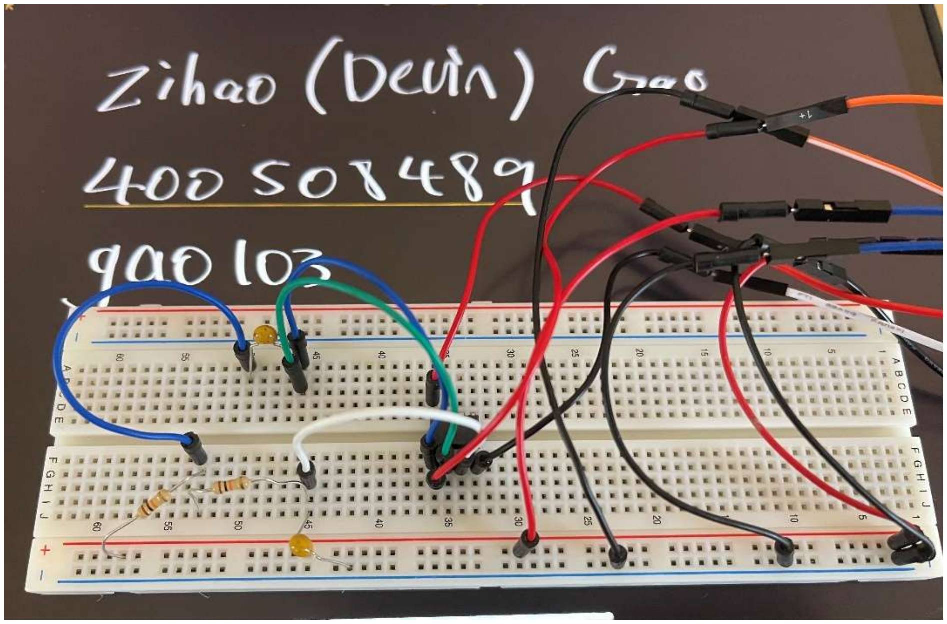

Figure 5. Physical Circuit Implementation

d. What is the cut-off frequency of this filter?

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(10 \cdot 10^{-3})(100 \cdot 10^{-9})} = 159.15\ Hz$$

e. How do the theoretical and measured results compare? Comment on your results.

The table above shows that our theoretical and experimental values were very consistent for 50Hz to 500Hz. As the frequency increases and jump to higher frequencies, the experimental values became less accurate compared to theoretical. This is expected to occur because the testing frequency has a larger difference to the cutoff frequency which we calculated to be approximately 160 HZ. So, this would explain why as we increase the frequency past the lower frequency and further away from 160 Hz, the experimental error is getting larger and larger.

Discussion

This experiment successfully demonstrated the behavior of a second-order Butterworth low-pass filter. The goal of this lab was to derive the theoretical transfer function, calculate the cutoff frequency, and compare the analytical and experimental values of the filter's gain over a range of frequencies.

The theoretical cutoff frequency was calculated to be approximately 159 Hz, which matched well with the experimental data. From 50 Hz to around 500 Hz, the measured gain was very consistent with the calculated values. However, as the input frequency increased further and further beyond the cutoff frequency, the gain started to drop significantly, as expected for a low-pass filter.

We noticed some differences between the theoretical and measured values at higher frequencies. This can be explained by real-world factors such as component tolerances and measurement errors from the AD3 scope. These errors are way more noticeable at higher frequencies, where the op-amp and components may also not behave ideally.

Overall, the filter performed as expected, with the measured results closely matching the theoretical predictions, especially around and below the cutoff frequency. The experiment confirmed the accuracy of the Sallen-Key design, and we gained insight into the frequency response of active filters.