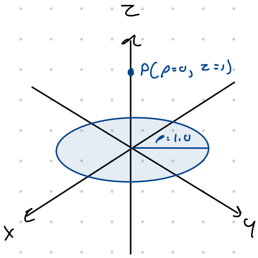
EE 2FH4 - Electromagnetics I - MATLAB - Set 4

Exercise: Given the surface charge density, $\rho_s = 2.0\,\mu C/m^2$, existing in the region $\rho < 1.0\,m$, $z = 0$, and zero elsewhere, find $\vec{E}$ at $P\ (\rho = 0,\ z = 1.0)$.



$dq = \rho_s\, dA$

$dA = \rho\, d\rho\, d\phi$

$dq = \rho_s\, \rho\, d\rho\, d\phi$

$d\vec{E} = \dfrac{dq}{4\pi \varepsilon_0 r^2}$, $r = \sqrt{z^2 + \rho^2}$

$d\vec{E}_z = d\vec{E} \cos\theta$, $\cos\theta = \dfrac{z}{\sqrt{z^2+\rho^2}}$

$$d\vec{E}_z = \frac{\rho_s \rho z\, d\rho\, d\phi}{4\pi\varepsilon_0 (z^2+\rho^2)^{3/2}}$$

$$\vec{E}_z = \frac{\rho_s z}{2\varepsilon_0} \int_{\rho=0}^{\rho=1} \frac{\rho\, d\rho}{(z^2+\rho^2)^{3/2}}$$

, let $u^2 = z^2 + \rho^2 \Rightarrow \rho\, d\rho = u\, du$

$$\int \frac{\rho\, d\rho}{(z^2+\rho^2)^{3/2}} = \int \frac{u\, du}{(u^2)^{3/2}} = \int \frac{1}{u^2}\, du = \int u^{-2}\, du = \left(\frac{-1}{u}\right)$$

$$\int_0^1 \frac{\rho\, d\rho}{(z^2+\rho^2)^{3/2}} = -\frac{1}{u}\Big|_z^{\sqrt{z^2+1}} = \frac{1}{z} - \frac{1}{\sqrt{z^2-1}}$$

$$\vec{E}_z = \frac{\rho_s z}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+1}}\right)$$

$$\vec{E}_z = \frac{\rho_s}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+1}}\right)$$

$$\vec{E}_z(\rho=0, z=1) = \frac{2\cdot 10^{-6}}{2(8.85\times 10^{-12})}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\boxed{\vec{E} = 3.31\times 10^{4}\ \vec{z}}$$