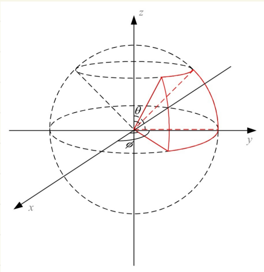
EE2FH4 - Electromagnetics I - MATLAB - Set 2

Exercise: The surfaces $r=0$ and $r=2$, $\phi=45°$, $\phi=90°$, $\theta=45°$ and $\theta=90°$ define a closed surface. Find the enclosed volume and the area of the closed surface S.



$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$V = \iiint_V dv = \iiint_V r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \int_{r=0}^{r=2} \int_{\theta=45°}^{\theta=90°} \int_{\phi=45°}^{\phi=90°} r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \int_{r=0}^{r=2} r^2 dr \int_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{2}} \sin\theta\, d\theta \int_{\phi=\frac{\pi}{4}}^{\phi=\frac{\pi}{2}} d\phi$$

$$= \frac{r^3}{3}\Big|_{r=0}^{r=2} \cdot (-\cos\theta)\Big|_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{2}} \cdot \phi\Big|_{\phi=\frac{\pi}{4}}^{\phi=\frac{\pi}{2}}$$

$$= \frac{1}{3}\left(2^3 - 0^3\right)\cdot\left(-\cos\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{4}\right)\right)\cdot\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$= \frac{8}{3}\cdot\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}$$

$$\boxed{V = \frac{\pi\sqrt{2}}{3} \doteq 1.481 \text{ m}^3}$$

$$S_1 = \iint_S dS_1$$

$$= \iint_S r^2 \sin\theta\, d\theta\, d\phi \Big|_{r=0}$$

$$S_1 = 0 \text{ m}^2$$

$$S_2 = \iint_S dS_2 = \iint_S r^2 \sin\theta\, d\theta\, d\phi$$

$$= 2^2 \int_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{2}} \sin\theta\, d\theta \int_{\phi=\frac{\pi}{4}}^{\phi=\frac{\pi}{2}} d\phi$$

$$= 4\cdot(-\cos\theta)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cdot \phi\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 4\cdot\left(-\cos\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{4}\right)\right)\cdot\left(\frac{\pi}{4}\right)$$

$$= 4\cdot\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}$$

$$S_2 = \frac{\pi}{\sqrt{2}} \text{ m}^2$$

$$S_3 = \iint_S dS_3 = \iint_S r\sin\theta\, dr\, d\phi$$

$$= \sin\left(\frac{\pi}{4}\right)\int_{\phi=\frac{\pi}{4}}^{\phi=\frac{\pi}{2}} d\phi \int_{r=0}^{r=2} r\, dr$$

$$= \frac{1}{\sqrt{2}}\cdot\phi\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cdot\frac{r^2}{2}\Big|_0^2$$

$$= \frac{1}{\sqrt{2}}\cdot\left(\frac{\pi}{2}-\frac{\pi}{4}\right)\cdot\left(\frac{2^2}{2}-\frac{0^2}{2}\right)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}\cdot 2$$

$$S_3 = \frac{\pi}{2\sqrt{2}} \text{ m}^2$$

$$S_4 = \iint_S dS_4 = \iint_S r\sin\theta\, dr\, d\phi$$

$$= \sin\left(\frac{\pi}{2}\right)\int_{\phi=\frac{\pi}{4}}^{\phi=\frac{\pi}{2}} d\phi \int_{r=0}^{r=2} r\, dr$$

$$= \phi\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cdot\frac{r^2}{2}\Big|_0^2$$

$$= \left(\frac{\pi}{2}-\frac{\pi}{4}\right)\cdot\left(\frac{2^2}{2}-\frac{0^2}{2}\right)$$

$$= \frac{\pi}{4}\cdot 2$$

$$S_4 = \frac{\pi}{2} \text{ m}^2$$

$$S_5 = \iint_S dS_5 = \iint_S r\, dr\, d\theta$$

$$= \int_{r=0}^{r=2} r\, dr \int_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{2}} d\theta$$

$$= \frac{r^2}{2}\Big|_0^2\cdot\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$= \left(\frac{2^2}{2}-\frac{0^2}{2}\right)\cdot\frac{\pi}{4}$$

$$= 2\cdot\frac{\pi}{4}$$

$$S_5 = \frac{\pi}{2} \text{ m}^2$$

$$S_6 = S_5 = \iint_S dS_6 = \iint_S r\, dr\, d\theta$$

...

$$S_6 = \frac{\pi}{2} \text{ m}^2$$

Total Surface Area:

$$S_{closed} = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$

$$= 0 + \frac{\pi}{\sqrt{2}} + \frac{\pi}{2\sqrt{2}} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$$

$$\boxed{S_{closed} = \frac{3\pi}{2}\left(1+\frac{1}{\sqrt{2}}\right) \qquad S_{closed} \doteq 8.045 \text{ m}^2}$$

∴ The enclosed volume is $\frac{\pi\sqrt{2}}{3}$ m^3 or aprox. 1.481 m^3. The area of the closed surface S is $\frac{3\pi}{2}\left(1+\frac{1}{\sqrt{2}}\right)$ m^2 or aprox. 8.045 m^2.