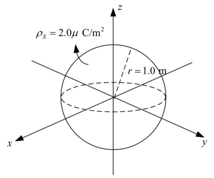
EE-2FH4 - MATLAB - SET 8

Exercise: Given the surface charge density $\rho_s = 2.0\,\mu C/m^2$ existing in the region $r = 1.0m$, $0 \le \phi \le 2\pi$, and $0 \le \theta \le \pi$ is zero elsewhere. Find analytically the energy stored in the region bounded by $2.0m < r < 3.0m$, $0 < \phi < 2\pi$, and $0 < \theta < \pi$.



$$Q_{enc} = \iint \rho_s\, ds = \int_0^{2\pi}\int_0^{\pi} (2\cdot 10^{-6})\, r^2 \sin\theta\, d\theta\, d\varphi$$

$$= 2\cdot 10^{-6} \int_0^{2\pi} d\varphi \cdot \int_0^{\pi} \sin\theta$$

$$= 2\cdot 10^{-6}\left(2\pi - 0\right)\left(-\cos(\pi) + \cos(0)\right)$$

$$Q_{enc} = 8\pi \cdot 10^{-6}$$

$$Q_{enc} = \oiint D\cdot ds$$

$$8\pi\cdot 10^{-6} = D_r \cdot 4\pi r^2$$

$$D_r = \frac{8\pi\cdot 10^{-6}}{4\pi r^2}$$

$$D_r = \frac{2\cdot 10^{-6}}{r^2}\ C/m$$

$$D = \varepsilon E$$

$$E = \frac{D}{\varepsilon} = \frac{2\cdot 10^{-6}}{\varepsilon r^2}$$

$$W_E = \frac{1}{2}\iiint_V \varepsilon_0 |E|^2 dv$$

$$= \frac{1}{2}\iiint D\cdot E\, dv$$

$$= \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_2^{3} \frac{2\cdot 10^{-6}}{r^2}\cdot\frac{2\cdot 10^{-6}}{\varepsilon r^2}\cdot r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$= \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_2^{3} \frac{4\cdot 10^{-12}}{r^4 \varepsilon}\, r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$= \frac{4\cdot 10^{-12}}{2\varepsilon}\cdot \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \int_2^{3} \frac{1}{r^2}\, dr$$

$$= \frac{4\cdot 10^{-12}}{2\varepsilon}\cdot (2\pi - 0)\left(-\cos(\pi) + \cos(0)\right)\cdot\left(-\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{4\cdot 10^{-12}}{2\varepsilon}\cdot (2\pi)(2)\left(\frac{1}{6}\right)$$

$$\boxed{W_E = 0.47\,\mu J}$$