Lab 5 – Active Filter Circuits

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ELECENG 3EJ4 – Electronic Devices and Circuits II

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Part 1: First-order Low-pass Filter

A. Spice Simulation

1.2.

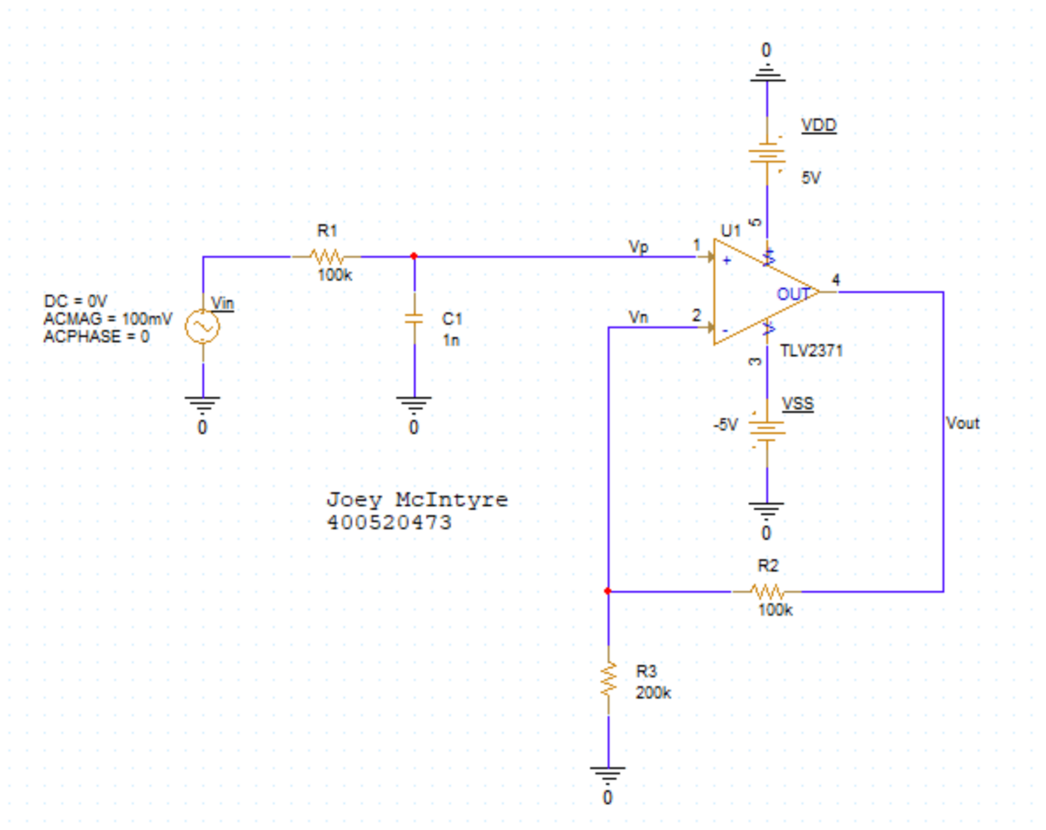


Figure 1: Circuit used for PSpice simulation - First-order low-pass filter

1.3

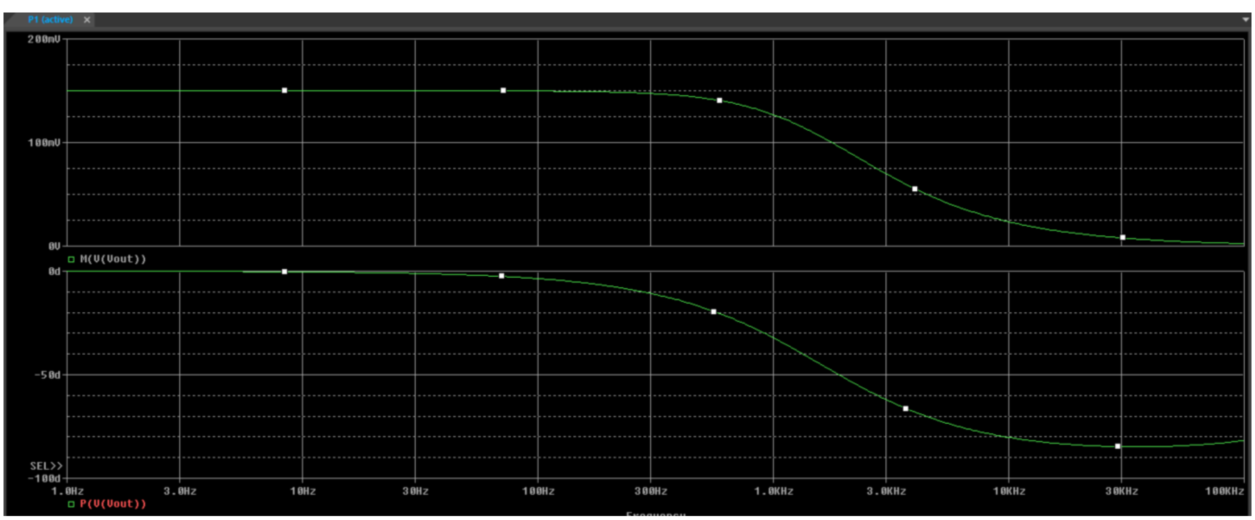


Figure 2: PSpice simulation results - First-order low-pass filter

B. AD3 Measurement

1.5

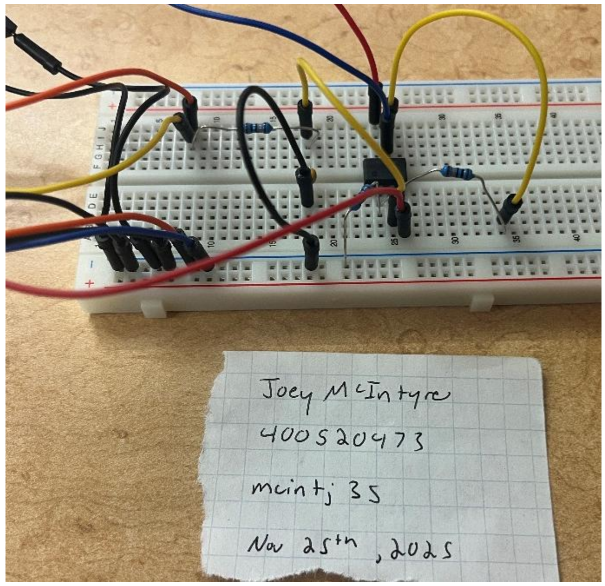


Figure 3: Physical circuit used for AD3 measurements - First-order low-pass filter

C. Questions for Part 1

Q1. (20 Points) (1) Find the transfer function of the first-order LPF, its low-frequency gain, and its - 3dB frequency f_c. **(2)** Compare the calculated low-frequency gain and the -3dB frequency f_c with the simulated data from Step 1.3 and the measured data from Step 1.8, respectively. Justify/discuss the observation and comparison.

(1)

The transfer function of the first-order low-pass filter can be derived as follows:

$$T(s) = \frac{V_o(s)}{V_{in}(s)}$$

$$For\ an\ ideal\ op-amp: V_+ = V_- \ and\ I_+ = I_- = 0$$

$$V_- = V_o(s)\frac{R_3}{R_2 + R_3}$$

$$\frac{V_{in} - V_+}{R_1} = (sC_1)V_+$$

$$\frac{V_{in} - V_o(s)\frac{R_3}{R_2 + R_3}}{R_1} = (sC_1)\left(V_o(s)\frac{R_3}{R_2 + R_3}\right)$$

$$T(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_2 + R_3}{R_3(R_1 sC_1 + 1)}$$

The low frequency gain can be found by evaluating $T(s)$ when $s = 0$:

$$T(0) = \frac{V_o(0)}{V_{in}(0)} = \frac{R_2 + R_3}{R_3} = \frac{100k\Omega + 200k\Omega}{200k\Omega} = 1.5$$

$$20log_{10}(1.5) = 3.522\ dB$$

The -3dB frequency can be calculated as follows

$$f_c = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(100k\Omega)(1nF)} = 1591\ Hz$$

(2)

From the simulated data obtained in Step 1.3, we can calculate the low-frequency gain and the -3dB frequency as follows:

- Low-Frequency Gain:

$$A_v = T(0) = \frac{V_o}{V_{in}} = \frac{0.150V}{0.100V} = 1.5$$

$$20\log(1.5) = 3.522\ dB$$

- -3dB Frequency:

$$20\log\left(\frac{V_o}{0.1V}\right) = 3.522 - 3 = 0.522\ V$$

$$V_o = 0.1062V$$

Frequency	M(V(Vout))	P(V(Vout))
Hz	Volts	Degrees
1577.683	0.106319	-44.7459

When $V_o = 0.1062V$, the simulated -3dB frequency is $f_c = 1577.683\ Hz$.

From the measured data obtained in Step 1.8, we can calculate the low-frequency gain and the -3dB frequency using the same methods:

- Low-Frequency Gain:

$$A_v = T(0) = \frac{V_o}{V_{in}} = \frac{0.1509V}{0.100V} = 1.509$$

$$20\log(1.509) = 3.574\ dB$$

- -3dB Frequency:

$$20\log\left(\frac{V_o}{0.1V}\right) = 3.574 - 3 = 0.574\ V$$

$$V_o = 0.1068V$$

Frequency	M(V(Vout))	P(V(Vout))
Hz	Volts	Degrees
1659.587	0.107141	-43.521

When $V_o = 0.1068V$, the simulated -3dB frequency is $f_c = 1659.587\ Hz$.

When comparing the calculated low-frequency gain and the -3dB frequency with the simulated data obtained in Step 1.3 and the measured data obtained in Step 1.8, it can be observed that the results from our experiments match the expected results.

Starting with the simulated results obtained in Step 1.3, the low-frequency gain is exactly the same as the calculated low-frequency gain, resulting in a zero percent difference. The -3dB frequency was found to be 1577.683 dB, when the calculated value was 1591 Hz, meaning there was a 0.84% difference between the simulated and expected value. For the measured results obtained in Step 1.8, the low-frequency gain was found to be 3.574 dB, when the calculated/expected value was 3.522 dB, which means there was a 1.47% difference between these values. The -3dB frequency was measured to be 1659.587 Hz, when the calculated value was 1591 Hz, which means there was a 4.22% difference between the measured and expected value.

Overall, when comparing the calculated low-frequency gain and the -3dB frequency to the simulated and measured results, there is a very low percent difference between all the values obtained in this question. These small percent differences can be attributed to inaccuracies with the AD3's measurements, component tolerances, and small internal resistances present in physical circuits. This means that the simulated and measured data align with the calculated/expected results very closely, therefore reinforcing the accuracy of the simulated and measured data obtained in Step 1.3 and 1.8.

Part 2: Second-order Low-pass Filter

A. SPICE Simulation

2.1

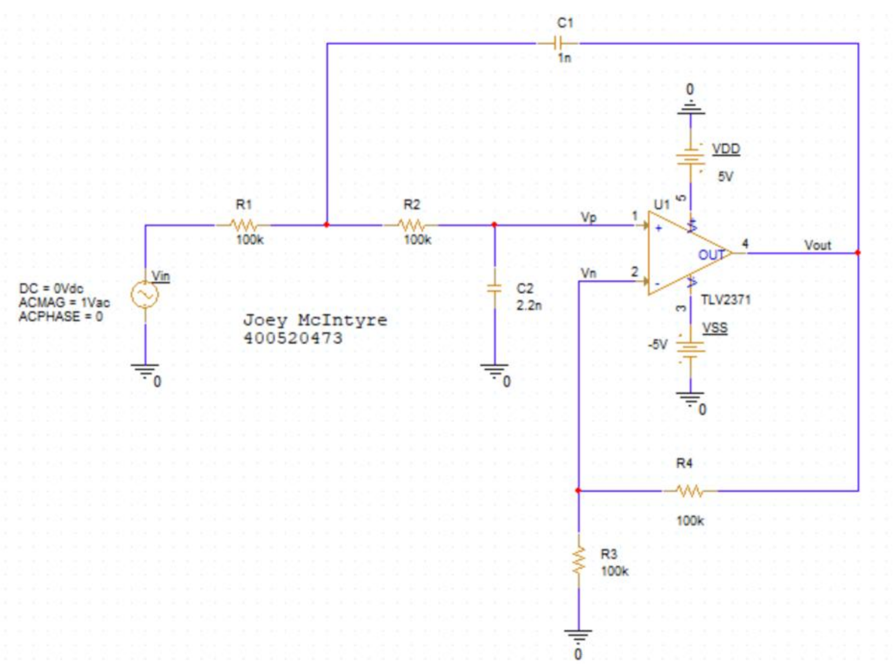


Figure 4: Circuit used for PSpice simulation - Second-order low-pass filter

2.2

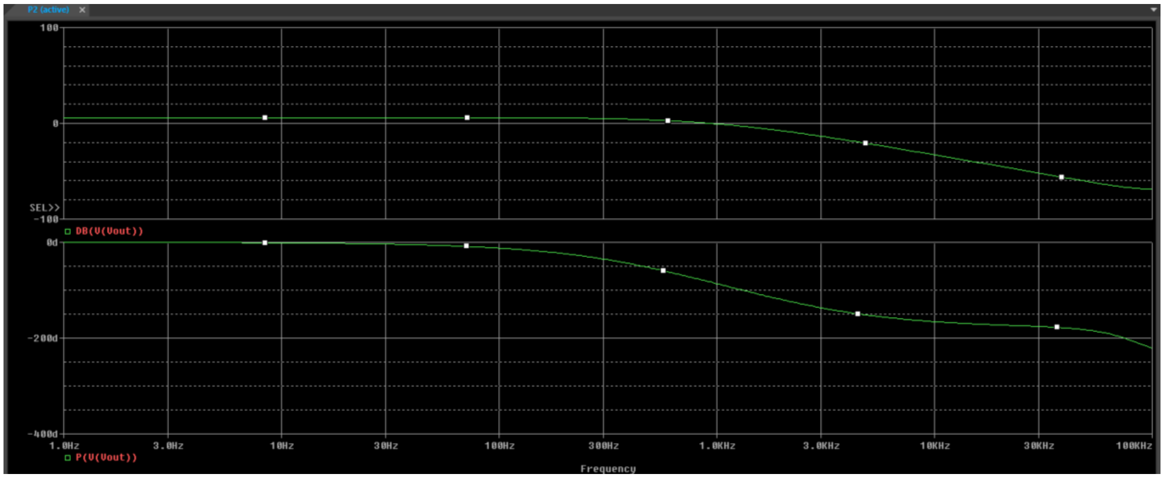


Figure 5: PSpice simulation results - Second-order low-pass filter

B. AD3 Measurement

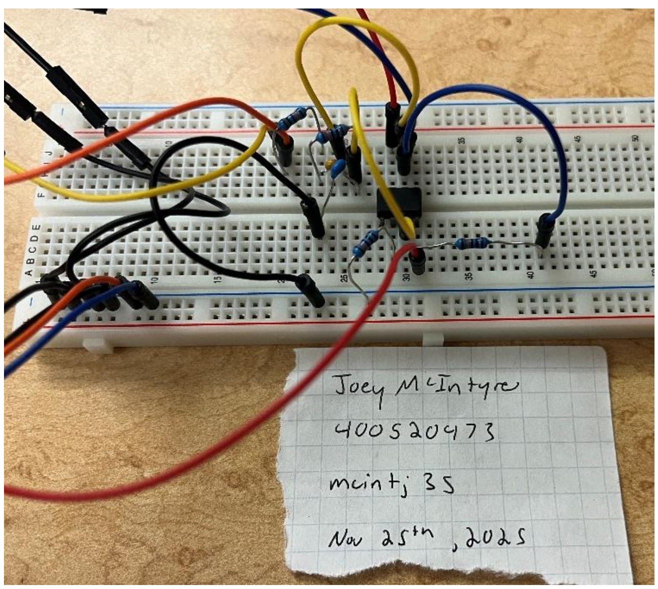


Figure 6: Physical circuit used for AD3 measurement - Second-order low-pass filter

C. Questions for Part 2

Q2. (20 points) Derive the transfer function and calculate the low-frequency gain. Verify the calculated gain using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

- The transfer function can be derived the same way as in Question 1:

$$T(s) = \frac{V_o(s)}{V_{in}(s)}$$

$$For\ an\ ideal\ op-amp: V_+ = V_-\ and\ I_+ = I_- = 0$$

$$Let\ V_1\ be\ the\ node\ between\ R_1, C_1, and\ R_2: Apply\ KCL\ at\ V_1:\ \frac{V_1 - V_{in}}{R_1} + \frac{V_1 - V_{out}}{sC_1} + \frac{V_1}{R_2 + sC_2} = 0$$

$$V_+ = V_- = V_{out}\frac{R_3}{R_3 + R_4}$$

$$T(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_3 + R_4}{s^2(C_1C_2R_1R_2R_3) + s(C_2R_2R_3 + C_2R_1R_3 - C_1R_1R_4) + R_3}$$

$$Plug\ in\ component\ values:$$

$$R_1 = R_2 = R_3 = R_4 = 100k\Omega = 1 * 10^5\Omega,\ C_1 = 1nF = 1 * 10^{-9}F,\ C_2\ = 2.2nF = 2.2 * 10^{-9}F$$

$$T(s) = \frac{90.9 * 10^6}{s^2 + (15.45 * 10^3)s + 45.45 * 10^6}$$

- The low-frequency gain can be found by evaluating T(s) when s = 0, just like in question 1:

$$T(0) = \frac{R_3 + R_4}{R_3} = \frac{100k\Omega + 100k\Omega}{100k\Omega} = 2$$

$$20\log(2) = 6.021\ dB$$

We can use the simulated and measured values from Step 2.2 and 2.6 respectively to verify the calculated gain of 6.021 dB. The simulated frequency in Step 2.2 was found to be 6.021 dB, and the measured frequency in step 2.6 was found to be 6.008 dB. These values are extremely close to the calculated value, which confirms that each step was done correctly.

Q3. (20 points) Calculate **(1)** the pole frequency f_o **(2)** the cut-off frequency (or -3dB frequency) f_c **(3)** the pole quality factor Q, **(4)** the peak value of the magnitude of the transfer function $|T(s)|_{max}$, and **(5)** the frequency f_{max} where the peak value of the magnitude of the transfer function happens. Verify the calculated f_c using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

(1)

The pole frequency can be calculated using the formula:

$$\omega_o\ = 2\pi f_o$$

$$f_o = \frac{\omega_o}{2\pi}, where\ \omega_o^2\ = 45.45 * 10^6\ (from\ Q2)$$

$$f_o = \frac{\sqrt{45.45 * 10^6}}{2\pi} = 1072.97\ Hz$$

This value for the pole frequency f_o can be verified by observing the graphs obtained from the simulated data in Step 2.2 and the measured data in Step 2.6. Both the simulated and measured data shows that the passband of the filter ends at approximately 1000Hz, which confirms that the calculated pole frequency is a reasonable result.

(2)

The cut-off frequency (or -3dB frequency) can be calculated using the formula:

$$|T(j\omega_c)| = \frac{90.9 * 10^6}{\sqrt{(45.45 * 10^6 - \omega_c^2)^2 + (15.45 * 10^3)^2\omega_c^2}} = \sqrt{2}$$

$$Solving\ for\ \omega_c\ we\ get:$$

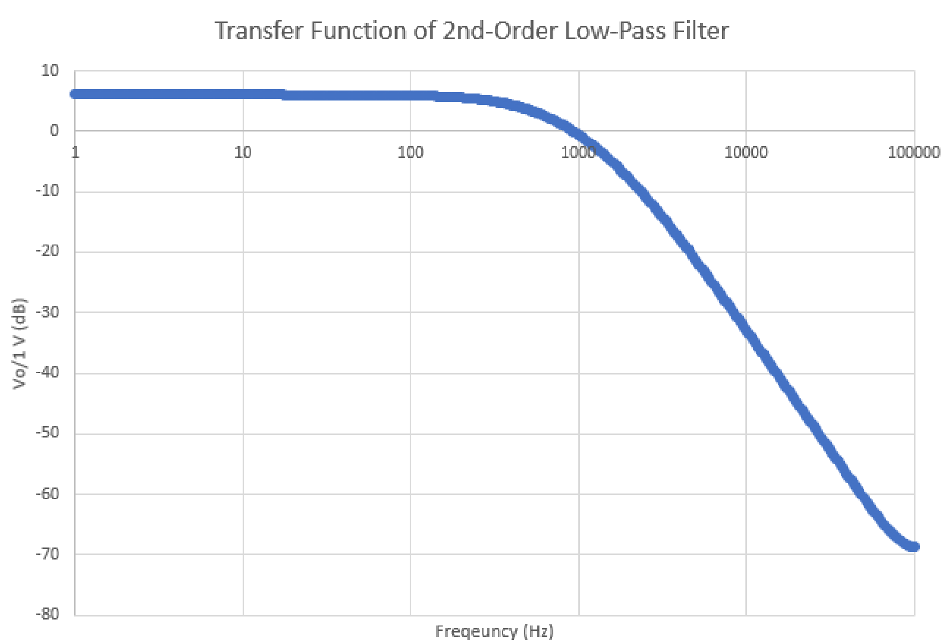
$$\omega_c = 3585.7\frac{rad}{m}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{3585.7\frac{rad}{m}}{2\pi} = 570.68\ Hz$$

This calculated f_c can be verified using the simulated data obtained in Step 2.2 and the measured data in Step 2.6. We can observe in both graphs below that a cut-off occurs around 570Hz in both graphs, meaning out calculated value matches the experimental results. We can also verify this further by observing in Step 2.2, when the magnitude of the output is 3.037 dB, the frequency is equal to 565.56 Hz. In Step 2.6, the magnitude of the output is 3.029 dB when the frequency is equal to 575.44 dB.

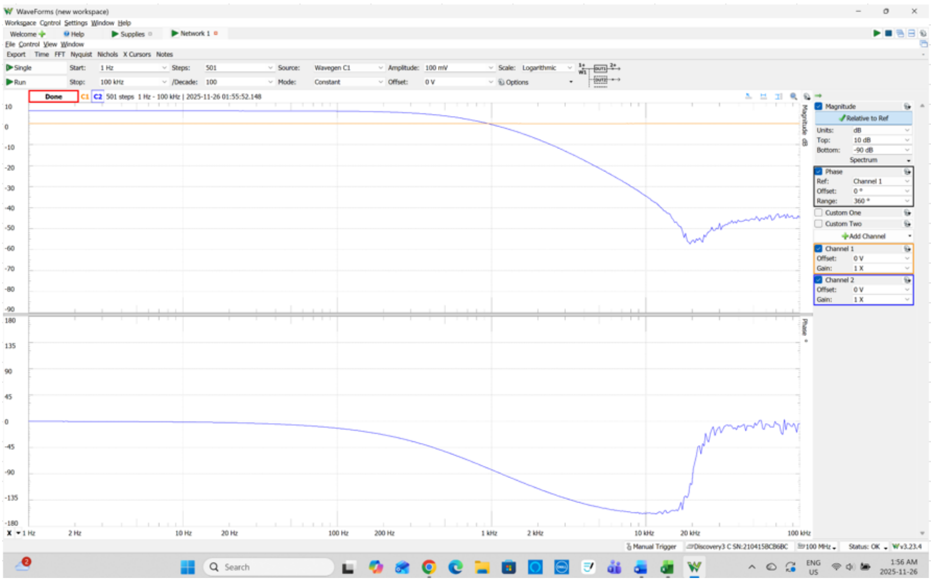
- Step 2.2:

Frequency	DB(V(Vout))	P(V(Vout))
Hz	dBV	Degrees
565.5555	3.036727	-59.1725



- Step 2.6:

Frequency	Manitude	Phase
Hz	dB	Degrees
575.43994	3.0293366	-57.49206



(3)

The pole quality factor (Q), can be found using the equation:

$$\frac{\omega_c}{Q} = 15.45 * 10^3$$

$$Q = \frac{\omega_c}{15.45 * 10^3} = \frac{\sqrt{45.45 * 10^6}}{15.45 * 10^3} = 0.436$$

(4)

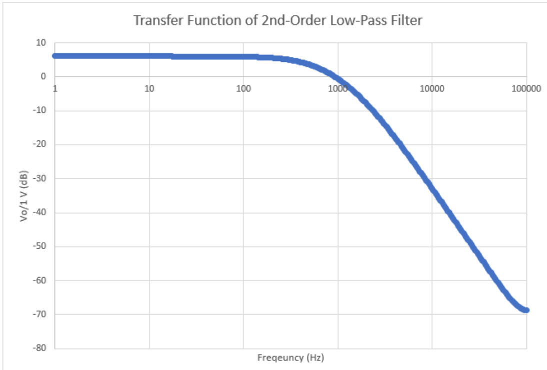
Since the Q value calculated in the previous part of the question is less than $\frac{1}{\sqrt{2}} = 0.707$, we know that the system is overdamped. This means the peak value of the magnitude can be found using the following equation:

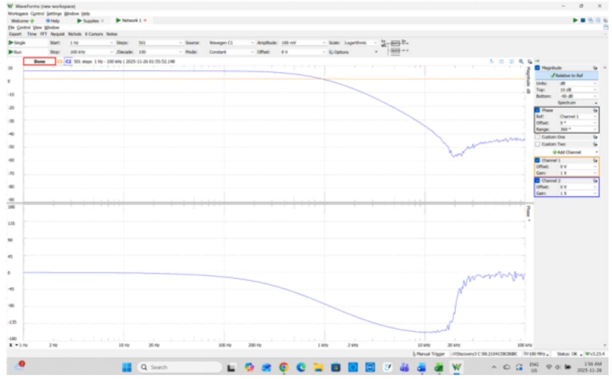
$$|T(s)|_{max} = 1 + \frac{R_3}{R_4} = 1 + \frac{100k\Omega}{100k\Omega} = 1 + 1 = 2$$

$$20\log(2) = 6.021\ dB$$

(5)

The peak value of the magnitude of the transfer function f_{max} first occurs at 1Hz (the first sampled frequency), and is held until the pole frequency which we found to be 1072.93 Hz in part 1 of this question. This range of f_{max} can be verified by observing the simulated results in Step 2.2 and the measured results in Step 2.6. It is clear that the peak value of the magnitude of the transfer function occurs for this same range of frequencies.





Part 3: Second-order Bandpass Filter

A. SPICE Simulation

3.1

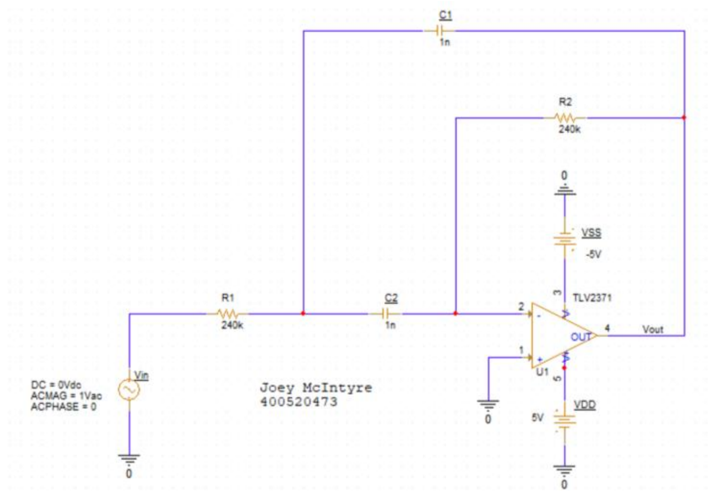


Figure 7: Circuit used for PSpice simulation - Second-order bandpass filter

3.2

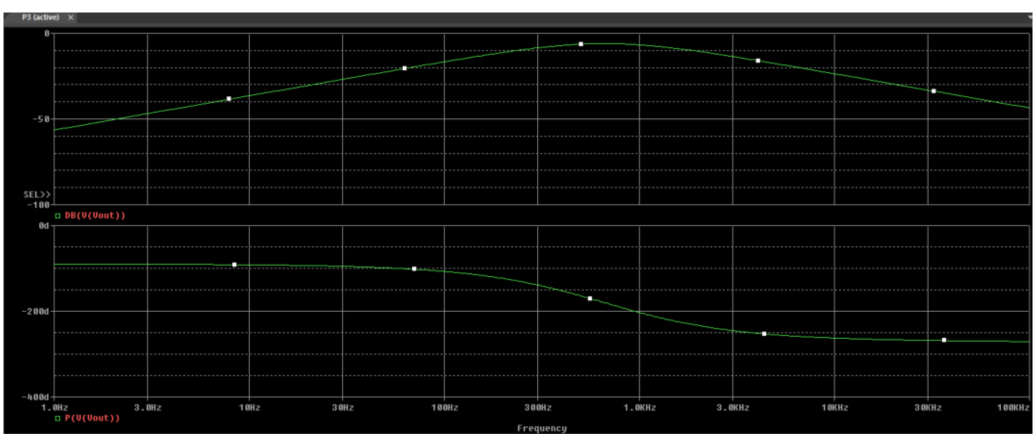


Figure 8: PSpice simulation results - Second-order bandpass filter

B. AD3 Measurement

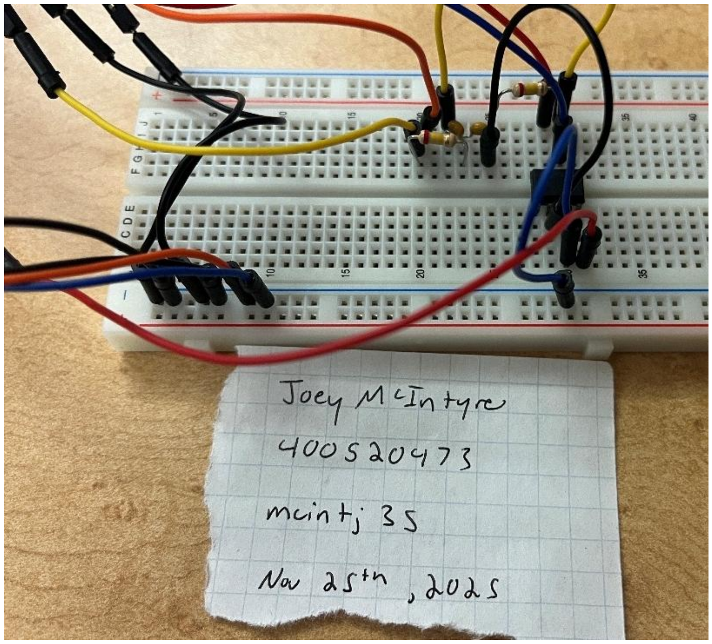


Figure 9: Physical circuit used for AD3 measurement - Second-order bandpass filter

C. Questions for Part 3

Q4. (20 points) Derive the transfer function and calculate the center frequency gain. Verify the calculated gain using the simulated data obtained in Step 3.2 and the measured data obtained in Step 3.6, respectively.

- The transfer function can be derived the same way as in Question 1 and Question 2:

$$T(s) = \frac{V_o(s)}{V_{in}(s)}$$

$$For\ an\ ideal\ op-amp: V_+ = V_-\ and\ I_+ = I_- = 0$$

$$Let\ V_1\ be\ the\ node\ between\ R_1, C_1, and\ C_2: Apply\ KCL\ at\ V_1:\ \frac{V_1 - V_{in}}{R_1} + \frac{V_1 - V_{out}}{\frac{1}{sC_1}} + \frac{V_1}{\frac{1}{sC_2}} = 0$$

$$From\ this\ KCL\ equation, we\ can\ derive:\ T(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{-s}{R_1C_1}}{s^2 + s\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right) + \frac{1}{\frac{1}{C_1C_2R_1R_2}}}$$

- To calculate the center frequency gain, we first must find the center frequency and then the gain:
  - The center frequency can be found using the equation:

$$\omega_o = 2\pi f_o$$

$$f_o = \frac{\omega_o}{2\pi}, where\ \omega_o^2 = \frac{1}{C_1C_2R_1R_2}, => \ \omega_o = \frac{1}{\sqrt{(1nF)(1nF)(240k\Omega)(240k\Omega)}} = 4166.67\frac{rad}{s}$$

$$f_o = \frac{4166.67}{2\pi} = 663.15\ Hz$$

- Now we can calculate the center frequency gain by plugging ω_o into our derived transfer function:
  - To do this, we sub in $s = j\omega_o$, simplify component values, and solve:

$$T(j\omega_o) = \frac{\frac{-j\omega_o}{R_1C_1}}{(j\omega_o)^2 + (j\omega_o)\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right) + \frac{1}{\frac{1}{C_1C_2R_1R_2}}} = \frac{\frac{-j\omega_o}{R_1C_1}}{(j\omega_o)\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right)}$$

$$T(j\omega_o) = \frac{-1}{R_1C_1\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right)} = -\frac{1}{2} = -0.5$$

$$20\log(0.5) = -6.021\ dB$$

The calculated center frequency gain of -6.021 dB can be verified using the simulated data obtained in Step 3.2 and the measured data obtained in step 3.6. In step 3.2, the center frequency gain was found to be -6.021 dB at the center frequency of 663.41 Hz. In step 3.6, the center frequency gain was found to be -6.378 dB at the center frequency of 660.69 Hz. These values align very closely to the calculated center frequency gain of -6.021 dB, verifying the validity of the calculations and experiments. The small percent difference present between the calculated value and the measured value (5.76%), can be explained by slight inaccuracies with the AD3's measurements, component tolerances, and small internal resistances present in physical circuits.

Q5. (20 points) Calculate **(1)** the center frequency ω_0, **(2)** the pole quality factor Q, **(3)** the two -3dB frequencies ω_1 and ω_2, and **(4)** the 3-dB bandwidth BW = $\omega_2 - \omega_1$. Verify the calculated results using the simulated data obtained in Step 3.2 and the measured data obtained in Step 3.6 respectively.

(1)

We found the center frequency in Question 4:

$$\omega_o = 2\pi f_o$$

$$f_o = \frac{\omega_o}{2\pi}, where\ \omega_o^2 = \frac{1}{C_1C_2R_1R_2}, => \ \omega_o = \frac{1}{\sqrt{(1nF)(1nF)(240k\Omega)(240k\Omega)}} = 4166.67\frac{rad}{s}$$

$$f_o = \frac{4166.67}{2\pi} = 663.15\ Hz$$

(2)

The pole quality factor (Q) can be found from the following formula from part of the transfer function derived in Question 4:

$$\frac{\omega_o}{Q} = \frac{1}{C_1 R_2} + \frac{1}{C_2 R_2}$$

$$Q = 0.5$$

(3)

The two -3dB frequencies ω_1 and ω_2 can be calculated by the following formula (using values solved for in part 1 and 2 of this question):

$$\omega_o^2 = \omega_1 \omega_2$$

$$BW = \omega_2 - \omega_1 = \frac{\omega_o}{Q} = \frac{4166.67}{0.5} = 8.33 * 10^3 \frac{rad}{s}$$

$$\omega_1^2 + BW\omega_1 - \omega_o^2 = 0$$

$$\omega_1 = \frac{-BW + \sqrt{BW^2 + 4\omega_o^2}}{2} = 1726.38 \frac{rad}{s}$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1726.38}{2\pi} = 274.76\ Hz$$

$$\omega_2 = \omega_1 + BW = 10056.38 \frac{rad}{s}$$

$$f_2 = \frac{\omega_2}{2\pi} = \frac{10056.38}{2\pi} = 1600.52\ Hz$$

(4)

The 3-dB bandwidth was already found in the previous part of this question:

$$BW = \frac{\omega_o}{Q} = \omega_2 - \omega_1 = 10056.38 \frac{rad}{s} - 1726.38 \frac{rad}{s} = 8330 \frac{rad}{s}$$

$$8330 \frac{rad}{s} = 1325.76\ Hz$$

These results can be compared to the simulated data obtained in Step 3.2 and the measured data obtained in Step 3.6 to verify the calculated results. The data from the simulation and measurements closely match each other and the calculated results, which helps confirm the validity of the calculations and the simulated and physical experiments.

- Two -3dB frequencies from Step 3.2 (simulation):

Frequency	)B(V(Vout)	P(V(Vout))
Hz	dBV	Degrees
272.6739	-9.075977	-134.7081

Frequency	)B(V(Vout)	P(V(Vout))
Hz	dBV	Degrees
1614.064	-9.081884	-225.3579

- Two -3dB frequencies from Step 3.6 (measurement):

Frequency	Manitude	Phase
Hz	dB	Degrees
275.42287	-9.846569	-131.5513

Frequency	Manitude	Phase
Hz	dB	Degrees
1621.8101	-9.032138	137.06286