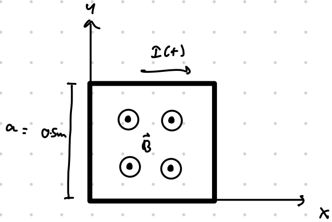
EE 3FK4 – Electromagnetics II – MATLAB Assignment 1 – Set 3

Exercise: A perfectly conducting filament containing a small $500\,\Omega$ resistor is formed into a square, as illustrated in figure 14.3. Find $I(t)$ if $\vec{B} = 0.4\cos[\pi(ct-y)]\,\hat{a}_z\ \mu T$, where $c = 3\times10^8$ m/s. Solve for the current $I(t)$ from $t=0$ to $t = 3.0\times10^{-8}$ s.



$a = 0.5$ m
$R = 500\,\Omega$
$c = 3\times10^8$ m/s

$\vec{B} = 0.4\cos[\pi(ct-y)]\,\hat{a}_z\ \mu T$
$B_z = 0.4\times10^{-6}\cos[\pi(ct-y)]$ T

$$\phi(t) = \iint_{\text{loop}} \vec{B}\cdot d\vec{S} = \int_{x=0}^{0.5}\int_{y=0}^{0.5} B_z(t,y)\,dy\,dx$$
$$= \int_0^{0.5} x \int_0^{0.5} B_z(t,y)\,dy$$
$$= 0.5\int_0^{0.5} 0.4\times10^{-6}\cos[\pi(ct-y)]\,dy$$
$$\phi(t) = 0.2\times10^{-6}\int_0^{0.5}\cos[\pi(ct-y)]\,dy$$

Evaluate integral:
let $p = \pi c t$
$$\int_0^{0.5}\cos(p-\pi y)\,dy$$

let $u = p - \pi y$, $du = -\pi\,dy$ $\therefore$ $dy = -\frac{du}{\pi}$
$$\int_0^{0.5}\cos(p-\pi y)\,dy = -\frac{1}{\pi}\int_p^{p-0.5\pi}\cos u\,du = -\frac{1}{\pi}\Big[\sin u\Big]_p^{p-0.5\pi}$$
$$= -\frac{1}{\pi}\left(\sin(p-0.5\pi) - \sin(p)\right)$$

Using $\sin(p-\frac{\pi}{2}) = -\cos p$:
$$\int_0^{0.5}\cos(p-\pi y)\,dy = -\frac{1}{\pi}(-\cos p - \sin p)$$
$$= \frac{1}{\pi}(\cos p + \sin p)$$

$$\phi(t) = 0.2\times10^{-6}\cdot\frac{1}{\pi}\left[\cos(\pi c t) + \sin(\pi c t)\right]$$

Let $\omega = \pi c = 3\pi\times10^8$ m/s
$$\Phi(t) = \frac{0.2\times10^{-6}}{\pi}\left(\cos(\omega t) + \sin(\omega t)\right)$$
Faradays Law:
$$\mathcal{E}(t) = -\frac{d\Phi}{dt}$$
$$= -\frac{0.2\times10^{-6}}{\pi}\left(-\omega\sin(\omega t) + \omega\cos(\omega t)\right)$$
$$= \frac{0.2\times10^{-6}\cdot\omega}{\pi}\left(\sin(\omega t) - \cos(\omega t)\right)$$
$$= \frac{0.2\times10^{-6}\cdot 3\pi\cdot10^8}{\pi}\left(\sin(\omega t) - \cos(\omega t)\right)$$
$$\mathcal{E}(t) = 60\left(\sin(\omega t) - \cos(\omega t)\right)$$

$$I(t) = \frac{\mathcal{E}(t)}{R} = \frac{60}{500}\left[\sin(\omega t) - \cos(\omega t)\right]$$
$$I(t) = 0.12\left(\sin(\omega t) - \cos(\omega t)\right)\ A$$
Sub in $\omega = 3\pi\times10^8$

$$\boxed{I(t) = 0.12\left(\sin\left(3\pi\times10^8\right) - \cos\left(3\pi\times10^8\right)\right)\ A}$$