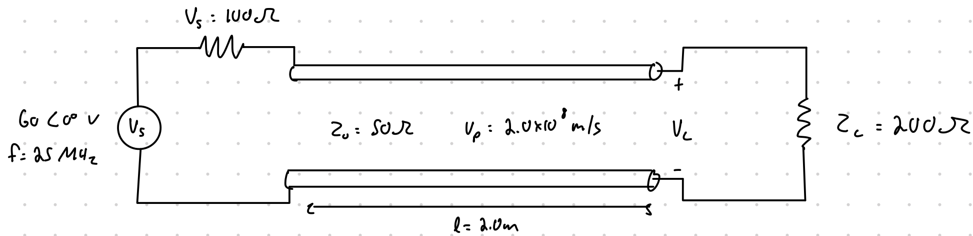
Exercise: for the lossless transmission line presented in figure 21.1, find the load voltage V_L. Write a MATLAB program to plot the load voltage V_L v.s. time. Is the magnitude of the load voltage V_L a function of the source frequency f? If it is, plot $|V_L|$ vs f; if not, explain.



At any point on the line the voltage is given by:

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} = V_0^+ \left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$$

@ the load, z = 0. Therefore:

$$V_L = V_0^+ (1 + \Gamma_L)$$

Then we need to find V_0^+ and Γ_L. We have:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200 - 50}{200 + 50} = \frac{150}{250} = \frac{3}{5} = 0.6$$

Finding V_0^+ requires we know the input impedance Zin. For a lossless transmission line:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)} = (50) \frac{200 + j50 \tan(\beta \ell)}{50 + j200 \tan(\beta \ell)}$$

$$; \quad \beta = \frac{\omega}{V_p} = \frac{2\pi f}{V_p} = \frac{2\pi (25 \times 10^6)}{2 \times 10^8}$$

$$\beta = \frac{\pi}{4} \text{ rad/m}, \quad \ell = 2$$

$$\beta \ell = \frac{\pi}{2}$$

$$Z_{in} = 12.5\ \Omega$$

Then we can calculate the input voltage Vin:

$$V_{in} = V_s \frac{Z_{in}}{Z_s + Z_{in}} = 60 \cdot \frac{12.5}{100 + 12.5} = 6.67 \angle 0°\ V$$

Now we have for the forward voltage:

$$V_0^+ = \frac{V_{in}}{e^{+\gamma \ell} + \Gamma_L e^{-\gamma \ell}} \quad ; \quad \gamma \ell = (\underbrace{\alpha}_{\alpha = 0 \text{ since line is lossless}} + j\beta)\ell = j\beta \ell$$

$$\gamma \ell = \frac{\pi}{2} j$$

$$= \frac{6.67}{e^{+j\frac{\pi}{2}} + 0.6 e^{-j\frac{\pi}{2}}} = \frac{6.67}{-j + 0.6j} = \frac{6.67}{-0.4j} = -16.675 j = 16.675 e^{-j\frac{\pi}{2}}\ V$$

Finally:

$$V_L = V_0^+ (1 + \Gamma_L) = 16.675 e^{-j\frac{\pi}{2}} (1 + 0.6) = 26.675 e^{-j\frac{\pi}{2}}\ V$$

Convert this phasor into its instantaneous form gives us the voltage across the load:

$$V_L = |V_L| \cos(\omega t + \angle V_L) \quad ; \quad \omega = 2\pi f = 2\pi(25\times10^6) \approx 1.57\times10^8 \text{ rad/s}$$

$$V_L = 26.675 \cos\left(1.57\times10^8 t - \frac{\pi}{2}\right)$$

Is the magnitude of the load voltage V_L a function of the source frequency f?

$\boxed{\text{Yes, } |V_L| \text{ is a function of the source frequency } f.}$

This is because Z_{in} and the factors $e^{\pm j\beta l}$ depend on $\beta = \frac{2\pi f}{v_p}$. Only at the given frequency $f = 25$ MHz does the line behave as a quarter-wave transformer with $Z_{in} = Z_0^2/Z_L$. Away from this frequency, both the magnitude and phase of V_L vary with f.

In the example we saw for this set, the $|V_L|$ was not a function of f because both the input Z_{in} and the source resistance Z_s are real, and the magnitude of V_{in} doesn't vary as f varies. The difference between this example and our exercise is in the example: $Z_L = Z_0 \Rightarrow \Gamma_L = 0$. This results in no reflections, no standing waves, and the line looking like a single resistor Z_0 at all frequencies $\Rightarrow |V_L|$ is independent of f.