

附錄 A

參考圖 A-1 至 A-6，吾人推導之離散化後統御方程式書寫如下：

A.1 方程式之離散化

孔質區動量方程式

$$A_j \mathbf{d}_{j-1} + B_j \mathbf{d}_j + C_j \mathbf{d}_{j+1} = D_j \quad <A.1>$$

其中

$$\begin{aligned} j &= 1 \\ B_1 &= \frac{2}{h_1} + h_1(E_1 + 2E_2 v_1) \\ \hat{C}_1 &= \frac{-2}{h_1} = A_1 + C_1 \\ D_1 &= \frac{2}{h_1} v_2 - \frac{2}{h_1} v_1 + 2\mathbf{m}_{eff} \cdot \mathbf{s} + [E_3 - (E_1 + E_2 v_1) v_1] h_1 \end{aligned} \quad <A.2>$$

$$\begin{aligned} j &= J-1 \\ A_{J-1} &= \frac{-1}{h_{J-2}} \\ B_{J-1} &= \frac{1}{h_{J-2}} + \frac{1}{h_{J-1}} + \frac{h_{J-2} + h_{J-1}}{2} (E_1 + 2E_2 v_{J-1}) \\ C_{J-1} &= 0 \\ D_{J-1} &= -\left(\frac{1}{h_{J-2}} + \frac{1}{h_{J-1}} \right) v_{J-1} + \frac{1}{h_{J-2}} v_{J-2} + [E_3 - (E_1 + E_2 v_{J-1}) v_{J-1}] \frac{h_{J-1} + h_{J-2}}{2} \end{aligned} \quad <A.3>$$

管壁能量方程式

$$A_j \bar{\mathbf{q}}_{j-1}^{n+1} + B_j \bar{\mathbf{q}}_j^{n+1} + C_j \bar{\mathbf{q}}_{j+1}^{n+1} = D \quad <A.4>$$

$$3 \leq j \leq J-2$$

$$A_j = 1 = C_j, B_j = -\left[2 + \frac{(\Delta y)^2}{\mathbf{a}' \Delta \mathbf{t}}\right], D_j = \frac{(\Delta y)^2}{\mathbf{a}' \Delta \mathbf{t}} \bar{\mathbf{q}}_j^n \quad \text{<A.5>}$$

在內部介面 $y = 1$

$$j = 1 \quad \bar{\mathbf{q}}_1 = \mathbf{q}_{w1} \quad \text{<A.6>}$$

$$j = 2 \quad \bar{\mathbf{q}}_1 = \mathbf{q}_{w1} \quad B_2 = -\left[2 + \frac{(\Delta y)^2}{\mathbf{a}'_w \Delta \mathbf{t}}\right] \quad \text{其中 } D_2 - A_2 \mathbf{q}_{w1} = D_2^* \quad \text{<A.7>}$$

$$C_2 = 1 \quad D_2^* = -\frac{(\Delta y)^2}{\mathbf{a}'_w \Delta \mathbf{t}} \bar{\mathbf{q}}_2^n - 1 \cdot \mathbf{q}_{w1}$$

在外部邊界 $y = 1 + \frac{H_w}{H} \quad \text{<A.8>}$

$$j = J-1 \quad A_{J-1} = 1, B_{J-1} = -\left[2 + \frac{(\Delta y)^2}{\mathbf{a}'_w \Delta \mathbf{t}}\right], C_{J-1} = 0, D_{J-1} = \frac{(\Delta y)^2}{\mathbf{a}'_w \Delta \mathbf{t}} \bar{\mathbf{q}}_{J-1}^n \quad \text{<A.9>}$$

流體相能量方程式

$$A_{i,j} \langle \mathbf{q} \rangle'_{f(i,j-1)} + B_{i,j} \langle \mathbf{q} \rangle'_{f(i,j)} + C_{i,j} \langle \mathbf{q} \rangle'_{f(i,j+1)} = D_{i,j} \quad \text{<A.9>}$$

其中

$$i = 2, 3, \dots, I-1$$

$$j = 2, 3, \dots, J-1$$

$$A_{i,j} = \frac{k_{fej-\frac{1}{2}}}{h_{j-1}}, \quad C_{i,j} = \frac{k_{fej+\frac{1}{2}}}{h_j} \quad \text{<A.10>}$$

$$(A) = k_{fex} \frac{\langle \mathbf{q} \rangle'_{f(i-1,j)} - 2\langle \mathbf{q} \rangle'_{f(i,j)} + \langle \mathbf{q} \rangle'_{f(i+1,j)}}{(\Delta x)^2}$$

$$\begin{aligned}
B_{i,j} &= -(A_{i,j} + C_{i,j}) - \text{Pr}_f \text{Re}_h \langle u \rangle_{i,j} \frac{h_{j-1} + h_j}{2\Delta x} - \frac{\mathbf{e}}{\Delta t} \frac{h_{j-1} + h_j}{2} \\
D_{i,j} &= \left[-\text{Bik}_s (\langle \mathbf{q} \rangle_{s(i,j)} - \langle \mathbf{q} \rangle_{f(i,j)}) - \frac{\text{Pr}_f \text{Re}_h}{\Delta x} \langle u \rangle_{i,j} \langle \mathbf{q} \rangle_{f(i-1,j)}' - \frac{\mathbf{e}}{\Delta t} - k_{f(i,j)} - (A) \right] \frac{h_{j-1} + h_j}{2}
\end{aligned}$$

<A.11>

$$i = 2, 3, \dots, I-1$$

$$j = 1$$

$$B_{i,j} \langle \mathbf{q} \rangle_{f(i,1)}' + (A_{i,1} + C_{i,1}) \langle \mathbf{q} \rangle_{f(i,2)}' = D_{i,j} \quad \text{<A.12>}$$

$$\hat{C}_{i,1} = 2 \frac{k_{fe, \frac{3}{2}}}{h_1}$$

$$\begin{aligned}
B_{i,1} &= -\frac{2}{h_1} k_{fe, \frac{3}{2}} - \text{Pr}_f \text{Re}_h \langle u \rangle_{i,1} \frac{h_1}{\Delta x} - \frac{\mathbf{e} h_1}{\Delta t} \\
D_{i,1} &= h_1 \left[-\text{Bik}_s \langle \mathbf{q} \rangle_{s(i,1)} - \frac{\text{Pr}_f \text{Re}_h}{\Delta x} \langle u \rangle_{(i,1)} \langle \mathbf{q} \rangle_{f(i,1)} - \frac{\mathbf{e}}{\Delta t} \langle \mathbf{q} \rangle_{f(i,1)} - (A) \right]
\end{aligned}$$

<A.13>

固相能量方程式

離散化與液相類似，所以不加詳述。