Mathematical Finance: QF

Exercises (for discussion on Monday, 27.11.2023)

Exercise 1. Let $U_1, U_2, \ldots$ be independent and identically distributed random variables on a probability space $(\Omega, \mathfrak{P}(\Omega), P)$ with

$$P(U_1 = 2,4\%) = \frac{1}{2} = P(U_1 = -2,34475\%).$$

Moreover, we consider the stochastic process $X = (X_0, X_1, \ldots)$ with

$$X_n := \sum_{k=1}^{n} U_k \quad \text{for } n = 0, 1, 2, \ldots$$

(a) We model the evolution of the exchange rate of the Dollar, quoted in Euro, by the process

$$S^{\frac{\$}{€}} := \mathcal{E}(X).$$

Represent the process $S^{\frac{\$}{€}}$ at times $n = 0, 1, 2$ by a tree.

(b) Let the filtration $\mathcal{F}$ be generated by the process X. Examine whether the process $S^{\frac{\$}{€}}$ is a martingale, submartingale or supermartingale.

Exercise 2. Consider the process given in Exercise 1.

(a) From an American investor's point of view the evolution of the exchange rate of the Euro, quoted in Dollars, is given by

$$S^{\frac{€}{\$}} := \frac{1}{\mathcal{E}(X)}.$$

Represent $S^{\frac{€}{\$}}$ at times $n = 0, 1, 2$ by a tree.

(b) Find $Y = (Y_0, Y_1, Y_2)$ such that

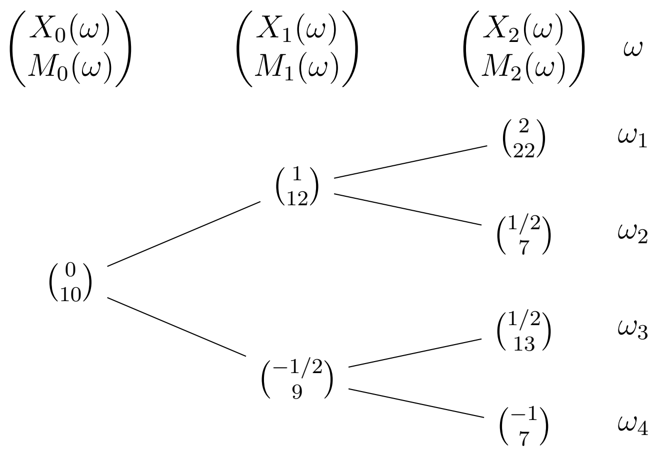
$$S^{\frac{€}{\$}} = \mathcal{E}(Y)$$

at times $n = 0, 1, 2$ and add Y to the tree from (a).

(c) Let the filtration $\mathcal{F}$ be generated by the process X. Examine whether the process $S^{\frac{€}{\$}}$ is a martingale, submartingale or supermartingale.

(d) Interpret the results from (c) and Exercise 1 (b) economically.

Exercise 3. We consider a stochastic process $X = (X_0, X_1, X_2)$ and a martingale $M = (M_0, M_1, M_2)$ relative to the filtration represented by the tree below.

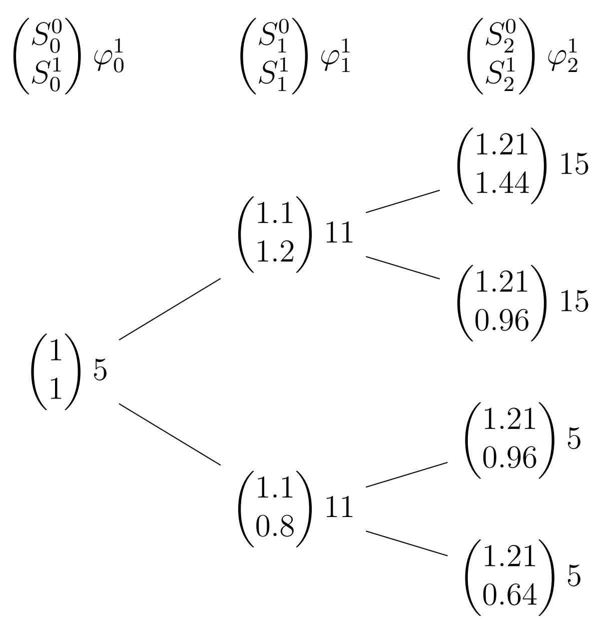


a) Determine the transition probabilites (i.e. the conditional probabilities) on the edges.

b) Determine a predictable process $H = (H_0, H_1, H_2)$ such that

$$M = M_0 + H \bullet X.$$

Exercise 4. We consider a price process $S = (S^0, S^1)$ with time horizon $n = 2$. The process S and a predictable process φ^1 are given by the following tree.



1. Determine a predictable process φ^0 such that $\varphi = (\varphi^0, \varphi^1)$ is a self-finacing trading strategy with initial capital $V_0(\varphi) = 10$.
2. Determine the associated value process $V(\varphi)$.

Submission of the homework until: Thursday, 23.11.2023, 10.00 a.m. via OLAT.