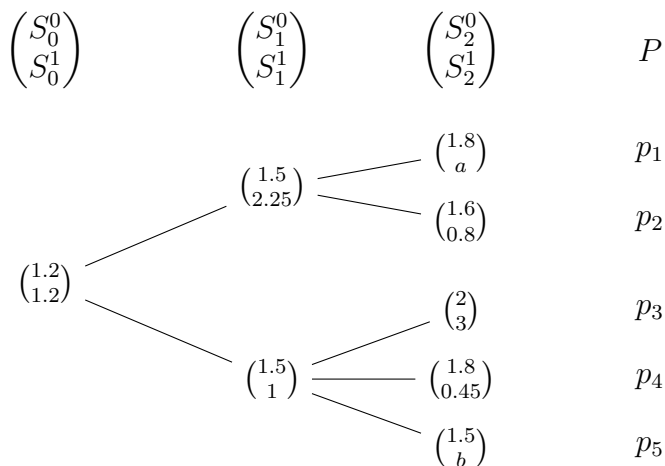


*Mathematical Finance: QF*

In-Tutorial exercises (for discussion on Tuesday, 05/12/2023)

**In-Tutorial Exercise 1.** We consider a market model with price process  $S = (S^0, S^1)$  and time horizon  $N = 2$  given by the following tree. We assume  $p_1, \dots, p_4 > 0$ . Further, consider  $S^0$  as the numeraire.



- Find conditions for  $a$  and  $b$  such that the market doesn't allow for arbitrage.
- What are possible conditions for  $p_1, \dots, p_5$  such that the market is complete assuming it is arbitrage-free?

Now assume  $a = 3.6$  and  $p_5 = 0$ . Further, let  $K = 1$  and consider the contingent claim  $X = (K - S_2^1)^+ \mathbb{1}_{\{\max_{n \leq N} S_n^1 \leq B\}}$ .

- Find an EMM  $Q$  for  $\hat{S}$ .
- Find the values for  $X$  for several boundaries  $B \in \{2, 2.5, 3\}$ . Options of this type are called *barrier options*. Why?
- Assume  $B = 2.5$ . Find the fair price process for  $X$  given by  $S_n^2 = S_n^0 E_Q(\hat{X} | \mathcal{F}_n)$  for  $n \in \{0, 1, 2\}$ .
- Explain, how you would construct a self-financing trading strategy  $\varphi = (\varphi^0, \varphi^1)$  such that  $V(\varphi)_2 = X$ .