Mathematical Finance
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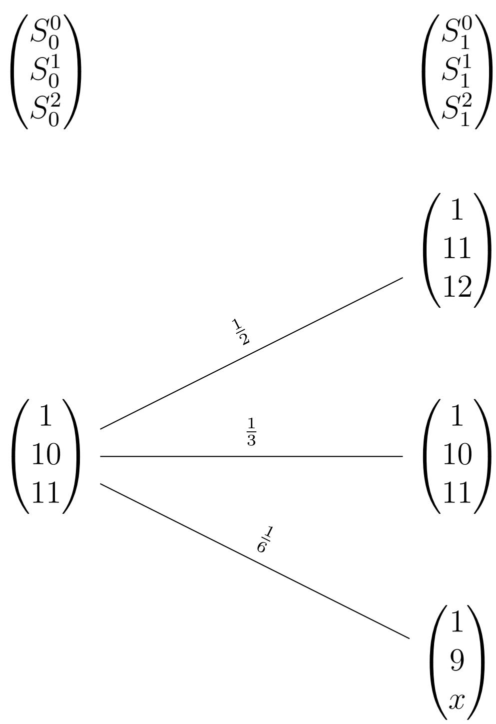
Winter term 2023/2024

Sheet QF05

Mathematical Finance: QF

Exercises (for discussion on Monday, 04.12.2023)

Exercise 1. Consider the market of three assets given by the following tree:



(a) For $x = 11$ construct an arbitrage strategy.

(b) Find all values x, for which the market is arbitrage-free. Prove your answer.

Exercise 2. We consider the model from sheet 4 exercise 4.

(a) Find transition probabilities such that $\hat{S}$ is a martingale.

(b) Find other transition probabilities such that the market is still arbitrage free. Prove your answer.

(c) Give transition probabilities such that there is an arbitrage opportunity.

Exercise 3. In an arbitrage-free market (S^0, S^1, S^2, S^3) with time horizon $N \in \mathbb{N}$, assume that $S^0_n = 1$ for all $n = 0, \dots, N$, and let S^2 and S^3 be price processes of European call and put options on S^1 with maturity N, i.e. $S^2_N = \max\{S^1_N - K, 0\}$, $S^3_N = \max\{K - S^1_N, 0\}$ for some $K \geq 0$. Using the law of one price, show that

$$S^2_n - S^3_n = S^1_n - K \quad \text{for all } n = 0, \dots, N.$$

Exercise 4. Let (S^0, S^1, S^2) be an arbitrage-free market with end-time $N \in \mathbb{N}$, S^0 deterministic and $S^1 \geq 0$. Assume that S^2 is the price process of a European-call option on S_1 with maturity N and strike $K \in [0, \infty)$, i.e. $S^2_N = (S^1_N - K)^+$. With the law of one price, show

$$\left(S^1 - K\frac{S^0}{S^0_N}\right)^+ \leq S^2 \leq S^1.$$

Submission of the homework until: Thursday, 30.11.2023, 10.00 a.m. via OLAT.