

Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 12/12/2023)

In-Tutorial Exercise 1.

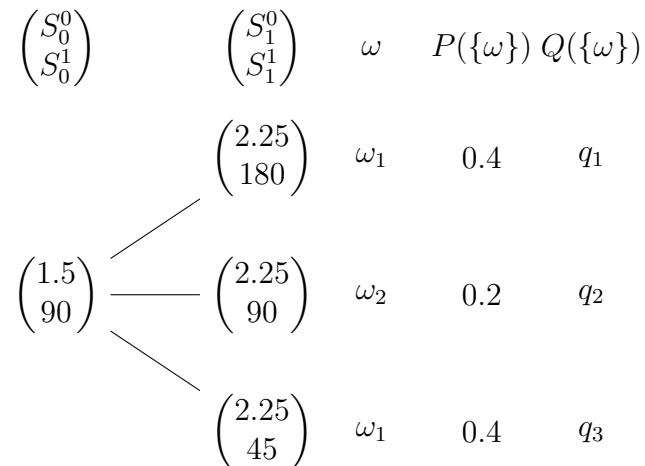
- a) Let $\Omega = \{\omega_1, \dots, \omega_4\}$ and $\mathcal{F} = \mathcal{P}(\Omega)$. Further, let Q be a probability measure on (Ω, \mathcal{F}) for which we know

$$Q(\{\omega_1\}) = 0.2, Q(\{\omega_2\}) = 0.6 \text{ and } Q(\{\omega_3\}) = Q(\{\omega_4\}) = 0.1.$$

Calculate $Q(A)$ of every element A in the domain of Q .

- b) What can you infer from the previous exercise about the relationship between a probability measure on a discrete space and its probability mass function?

Now consider an one period market with price processes (S^0, S^1) given by the following tree.



- c) Give a characterisation of the set $\mathcal{M} = \{Q \mid Q \text{ EMM for } \hat{S}\}$ in \mathbb{R}^3 using part (b). Are there elements in \mathcal{M} with $q_3 = 0, q_3 = 1$ or $q_3 = 0.5$?
- d) Compute the upper and lower price of a put option $S_1^2 = (S_1^1 - 67.5)^+$.
- e) Find a cheapest superhedge for this option.