Mathematical Finance Winter term 2023/2024
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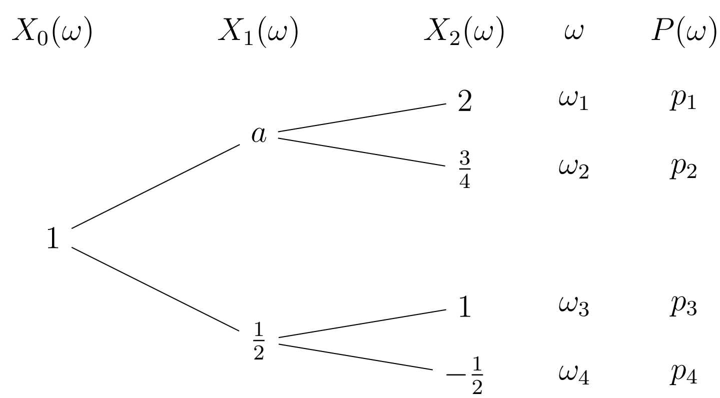
Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 21/11/2023)

In-Tutorial Exercise 1. We consider a stochastic process $X = (X_0, X_1, X_2)$ on the probability space $(\Omega, \mathcal{P}(\Omega), P)$ with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and $p_1, p_2 > 0$. The process X is given by the following tree.

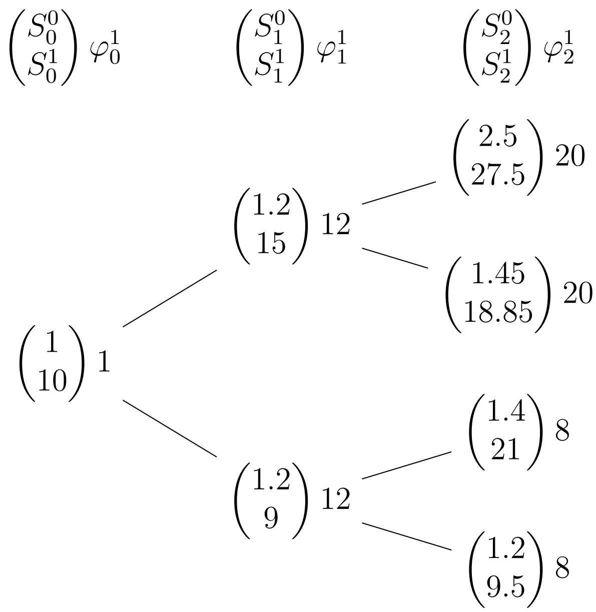


Let $\mathcal{F} = (\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2)$ be the filtration generated by X.

a) Assume $a = 3$. Is there a probability measure P such that X is a martingale w.r.t. $\mathcal{F}$ (i.e. are there reasonable values for p_1 and p_2)?

b) Assume $a = \frac{5}{4}$. Determine a probability measure P (i.e. find values for $p_1, \ldots, p_4$) such that X is a martingale w.r.t $\mathcal{F}$.

In-Tutorial Exercise 2. We consider a price process $S = (S^0, S^1)$ with time horizon $n = 2$. The process S and a predictable process φ^1 are given by the following tree.



a) How can you tell by just looking at the tree representation that φ^1 is predictable?

b) Determine a predictable process φ^0 such that $\varphi = (\varphi^0, \varphi^1)$ is a self-financing trading strategy with initial capital $V_0(\varphi) = 10$.

c) Determine the associated value process $V(\varphi)$.