Mathematical Finance Winter term 2023/2024
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Mathematical Finance: QF

Exercises (for discussion on Monday, 05.02.2024)

Exercise 1. (*8 points*)
Let W be a standard Brownian motion. Compute the Itō process representation of the following processes, i.e. write them in the form

$$X = X_0 + \dots \bullet I + \dots \bullet W \quad \text{or} \quad X = X_0 + \int \dots ds + \int \dots dW_s.$$

1. $X_t = \sin(W_t)$
2. $X_t = e^{W_t}$
3. $X_t = e^{-\frac{t}{2}+W_t}$
4. $X_t = tW_t$
5. $X_t = \sin(t)W_t$
6. $X_t = \sin(-t - W_t)e^{-W_t}$

Hint: Use Itō's formula.

Exercise 2. Compute the quadratic variation $[X]$ for

1. $X_t = W_t^2$
2. $X_t = tW_t$

Exercise 3. Prove that a Brownian motion with drift

$$X_t = \mu t + \sigma W_t, \quad \mu \in \mathbb{R}, \sigma \geq 0$$

is a martingale with respect to the natural filtration $\mathcal{F}_t = \sigma(X_s : s \leq t)$ if and only if $\mu = 0$.

Remark: A stochastic process X_t, $t \geq 0$, is called a (continuous time) martingale with respect to the filtration $\mathcal{F}_t$, $t \geq 0$, if and only if

$$\mathbb{E}[X_t|\mathcal{F}_s] = X_s$$

for any $s, t \geq 0$ with $t \geq s$.

Submission of the homework until: Thursday, 01.02.2024, 10.00 a.m. via OLAT.