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Sheet QF05P

Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 28/11/2023)

In-Tutorial Exercise 1. Consider the following market with $p_1, p_2 > 0$.

$$\begin{pmatrix} S_0^0(\omega) \\ S_1^0(\omega) \end{pmatrix} \qquad \begin{pmatrix} S_1^0(\omega) \\ S_1^1(\omega) \end{pmatrix} \qquad \begin{pmatrix} S_2^0(\omega) \\ S_0^1(\omega) \end{pmatrix} \qquad \omega \qquad P(\{\omega\})$$

$$\begin{pmatrix} 1.1 \\ 15 \end{pmatrix} \qquad \begin{pmatrix} 1.2 \\ 17 \end{pmatrix} \qquad \omega_2 \qquad p_2$$

$$\begin{pmatrix} 1 \\ 10 \end{pmatrix} \qquad \begin{pmatrix} 1.2 \\ 17 \end{pmatrix} \qquad \omega_3 \qquad p_3$$

$$\begin{pmatrix} 1.1 \\ 7 \end{pmatrix} \qquad \begin{pmatrix} 1.2 \\ 8 \end{pmatrix} \qquad \omega_4 \qquad p_4$$

- a) Assume x = 20. Find an arbitrage strategy.
- b) Find all values $x \in \mathbb{R}$ for which the market is arbitrage-free.

In-Tutorial Exercise 2. Let $S = (S^0, S^1, S^2)$ be an arbitrage-free market with $S_n^0 = (1+r)^n$, $r \ge 0$, and $S_N^2 = S_N^1 - O_0$ where O_0 is constant.

a) Show that

$$S_n^2 = S_n^1 - \frac{O_0}{(1+r)^{N-n}}, \quad n = 0, 1, \dots, N.$$

b) What can you conclude about the fair price of a forward contract sold at time n = 0 with forward price O_0 ? What is a reasonable choice for O_0 ?