P05.1

a) - enough to find an arbitrage in one step

- find a branch where both edges are going up in one component

  → put your money in this component & short the other one to get $V_n = 0$ at the mother node

  → this holds only when considering the discounted process!

⇒ $\begin{pmatrix}\varphi^0 \\ \varphi^1\end{pmatrix}$

$\begin{pmatrix}0\\0\end{pmatrix}$ — $\begin{pmatrix}0\\0\end{pmatrix}$ — $\left(-\frac{150}{11}, 1\right)$, $\left(\frac{150}{11}, 1\right)$ is an arbitrage strategy

$\begin{pmatrix}0\\0\end{pmatrix}$ — $\begin{pmatrix}0\\0\end{pmatrix}$, $\begin{pmatrix}0\\0\end{pmatrix}$

($\hat{S}^1$ increases in the upper branch on both edges)

alt: solve
$$\varphi^0_n(\omega) S^0_{n-1}(\omega) + \varphi^1_n(\omega) S^1_{n-1}(\omega) = 0$$
$$\varphi^0_n(\omega) S^0_n(\omega) + \varphi^1_n(\omega) + S^1_n(\omega) \geq 0$$
for $\omega \in F$ where F denotes the ω of one subbranch.

b) - 1FTAP: market is arbitrage-free ⇔ EMM Q exists

⇒ we only need to the martingale conditions on the branches which allow for arbitrage (why?)

- we obtain the system of equations: $\begin{cases} x q_1 + 17 q_2 = 15 \\ q_1 + q_2 = 1 \end{cases}$ (q_1, q_2 denote the transition probabilities on the upper branch)

$$\Leftrightarrow \begin{cases} q_1 = -\dfrac{2}{x-17} \\ q_2 = \dfrac{x-15}{x-17} \end{cases}$$

- since Q has to be equivalent to P this implies $q_1, q_2 \in (0,1)$.

$$\Rightarrow \quad 0 < \underbrace{\frac{-2}{x-17}}_{=q_1} < 1 \quad \Rightarrow \quad (x < 17) \text{ and } (x > 15)$$

$$\Rightarrow \quad 0 < \frac{x-15}{x-17} < 1 \quad \Rightarrow \quad x < 17$$

$\Bigg\}$ $x \in (15, 17)$

otherwise there exists no EMM.

P05.2

a) main Idea: Find two replicating strategies φ and ψ such that $V_N(\varphi)$ and $V_N(\psi)$ are lhs and rhs of the assertion at time N

$\Rightarrow$ law of one price then implies that the assertion is true for every $n \leq N$.

- define $\varphi = (0,0,1)$ and $\psi = (-\frac{O_0}{S_N^0}, 1, 0)$. These strategies are self-financing (why?)

$\Rightarrow V_N(\varphi) = S_N^2$ and $V_N(\psi) = S_N^1 - O_0$

- the assertion follows with law of one price.

b) - $S_0^2 = S_0^1 - \frac{O_0}{(1+r)^N}$ ← value of O_0 at time N

value of S^1 at time 0

- choose $O_0 = (1+r)^N S_0^1$ ← no profit at maturity of the future S^2.