Tutorial 4: The Metropolis-Hastings Algorithm

Review the Concepts and Proofs

1. What is a finite Markov chain? What is its transition matrix?

2. What is the stationary distribution of a finite Markov chain? How can you find it?

3. Define a continuous state Markov chain. What is different to a finite Markov chain?

4. Show that an autoregressive process of order 1 is a continuous state Markov chain.

5. What is the stationary distribution of a continuous state Markov chain? How can you find it?

6. Intuitively explain the detailed balance property.

7. For an autoregressive process of order 1, find the stationary distribution. Which condition do you have to impose?

8. Show that the Metropolis-Hastings algorithm gives rise to a Markov chain $\theta^{(s)}$ that is (i) reversible with respect to the posterior density $p(\theta^{(s)}|y)$, and (ii) stationary with stationary distribution $p(\theta^{(s)}|y)$.

9. How can the previous result be used to generate draws from the posterior distribution?

10. Explain the difference between the independence and random walk chain MH algorithms.

11. What is the posterior predictive p-value?

12. What is the Gelfand-Dey method?

Paper-pen and computer-based exercises

1. Consider estimation of the CES production function as outlined in the textbook. In particular, assume normality of the regression disturbances. Use the improper prior $p(\gamma, h) = 1/h$, $h > 0$.

   (a) Show that the marginal posterior pdf of γ is

   $$p(\gamma|y) \propto [(y - f(X,\gamma))'(y - f(X,\gamma))]^{-\frac{N}{2}} .$$

   (b) Write a pseudo code that uses an independence chain MH algorithm to estimate γ as the mean of the posterior distribution. Assume that the candidate distribution is a normal distribution with given mean $\hat{\gamma}$ and variance $\hat{\Sigma}$.

   (c) If the sample size is large, a good candidate distribution for the independence chain MH algorithm should be the asymptotic normal distribution based on a classical estimator. Find this distribution for the NLS estimator of the CES parameters. Then suppose for simplicity you estimate the CES function under the restriction $\gamma_4 = 1$ by OLS. Derive the mean vector $\hat{\gamma}$ and variance matrix $\hat{\Sigma}$ based on this estimator.

   (d) Write a Matlab script that performs the independence chain MH algorithm to estimate the CES parameters. Use the asymptotic normal distribution derived above as proposal distribution.

   (e) Write a Matlab script that performs the independence chain MH algorithm to estimate the CES parameters and h. Use the same proposal distribution for γ as in part (d). What might be a good proposal distribution for h?

   (f) Add to your script a part that computes predictive p-values for the skewness and kurtosis of the regression disturbances.

2. Consider estimation of the CES production function as outlined in the textbook. In particular, assume normality of the regression disturbances. Use the (mutually independent) informative priors $\gamma \sim \mathcal{N}(\underline{\gamma}, \underline{V})$ and $h \sim Gamma(\underline{s}^{-2}, \underline{\nu})$, where $\underline{\gamma} = [1, 1, 1, 1]'$, $\underline{V} = 0.25I_4$, $\underline{\nu} = 12$, and $\underline{s}^{-2} = 10$.

   (a) Show that the conditional posterior for h is

   $$h|\gamma, y \sim Gamma(\bar{s}^{-2}, \bar{\nu}),$$

   where $\bar{\nu} = N + \underline{\nu}$ and $\bar{s}^2\bar{\nu} = \underline{s}^2\underline{\nu} + [y - f(X, \gamma)]'[y - f(X, \gamma)]$.

   (b) Show that the conditional posterior for γ is proportional to

   $$\gamma|h, y \propto \exp\left\{-\frac{h}{2}[y - f(X, \gamma)]'[y - f(X, \gamma)] - \frac{1}{2}(\gamma - \underline{\gamma})'\underline{V}^{-1}(\gamma - \underline{\gamma})\right\}.$$

   (c) Write a Matlab script that uses the random walk chain MH algorithm to estimate γ and h.

   (d) Extend your script to report numerical standard deviations for the γ's and for h based on Newey-West long-run variances (the function NeweyWest will be supplied in the tutorial).

   (e) Extend your script to report CD statistics for convergence based on subsamples $A = 10\%$, $B = 50\%$, and $C = 40\%$.

   (f) Write a Matlab script that uses the Gelfand-Dey method to compute the posterior odds ratio for models $M_1 : \gamma_4 = 1$ and $M_2 : \gamma_4$ is unrestricted. Suppose prior model probabilities are $p(M_1) = p(M_2) = 0.5$. Use a truncated normal pdf with truncation parameters $p = 0.01, 0.05, 0.1$.