Mathematical Finance Winter term 2023/2024

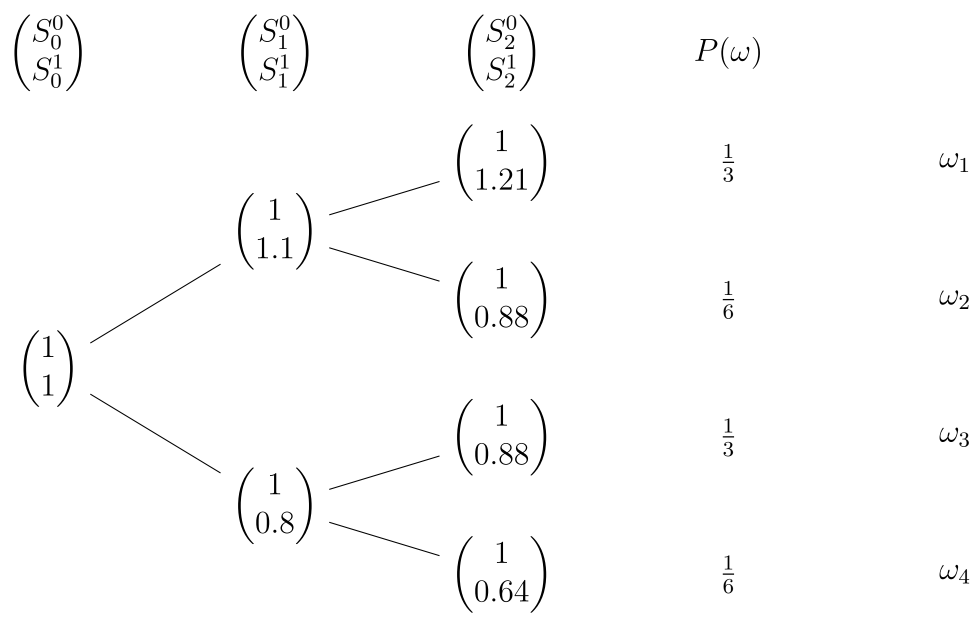
S. Christensen

P. Le Borne, B. Schroeter, B. Schultz **Sheet QF08**

Mathematical Finance: QF

Exercises (for discussion on Monday, 08.01.2024)

Exercise 1. Consider the market



Find all stopping times (with respect to the filtration given by the tree), with $\tau \leq 2$.

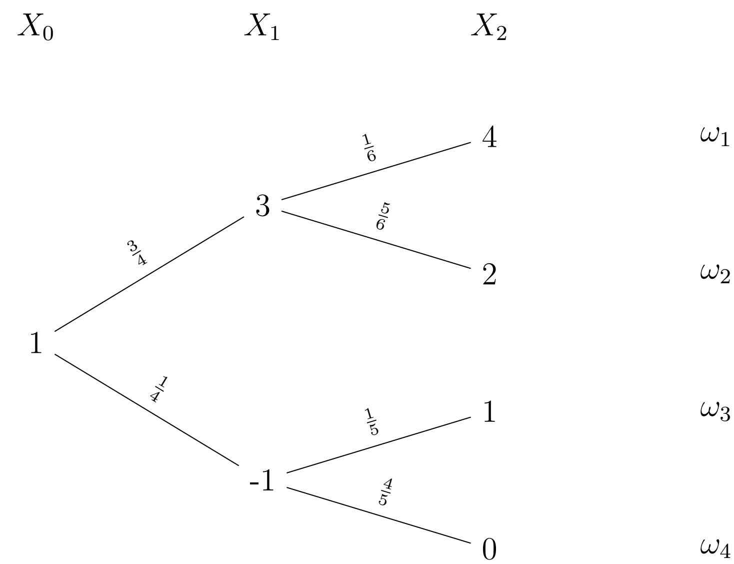
Hint: There are 5.

Exercise 2. In the setting of the previous exercise, answer the following questions:

(a) For which of these stopping times does $\hat{S}_\tau^1$ have the highest expectation? We call this an optimal stopping time and name it τ^*. Calculate $E(\hat{S}_{\tau^*}^1)$.

(b) For each equivalent martingale measure Q and all stopping times τ find $E_Q(\hat{S}_\tau^1)$.

Exercise 3. The process X is given by the tree.



1. Compute the Snell envelope U of X.

2. Calculate $Y := 1_{\{X_- \neq U_-\}} \bullet U$.

Exercise 4. Let $X = (X_n)_{n\in\{0,1,2,3,4\}}$ be a stochastic process with $X_0 = 0$. Assume that $X_1, \dots, X_4$ are independent and uniformly distributed on $[0,1]$. Let $(\mathcal{F}_n)_{n\in\{0,1,2,3,4\}}$ be the filtration generated by X and let $\mathcal{T}$ denote the set of $\{0,1,2,3,4\}$-valued stopping times associated to the filtration. Find a $\tau \in \mathcal{T}$ such that $E(X_\tau) = \sup_{\tau\in\mathcal{T}} E(X_\tau)$.

Submission of the homework until: Thursday, 21.12.2023, 10.00 a.m. via OLAT.