| C \| A \| U Christian-Albrechts-Universität zu Kiel | Wirtschafts- und Sozialwissenschaftliche Fakultät |
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Exam for the lecture

# "Econometrics III“

for students in the M.Sc. programmes

winter term 2022/2023

14.02.2023

*Please use block letters*:

| | | | |
|---|---|---|---|
| Name:<br>*Surname* | | Vorname:<br>*First Name* | |
| Studiengang:<br>*Course of study* | | Name der Universität (Bachelor):<br>*Name of university (Bachelor's degree)* | |
| Matrikelnummer:<br>*Student ID* | | Hochschulstandort (Bachelor):<br>*Place of university (Bachelor's degree)* | |

**Declaration:**

| PLEASE SIGN!!! |
|---|
| I hereby declare that I am able to be examined.<br><br>______________________________<br>Signature: |

**Preliminary remarks:**

- Write down your name and enrolment/matriculation number on all paper sheets provided for answers by the examiner.
- To write down your answers, use only the paper provided by the examiner.

**Result: (TO BE FILLED IN ONLY BY THE EXAMINER!)**

| Problem: | 1 | 2 | 3 | HA | S |
|---|---|---|---|---|---|
| Points earned: | | | | | |
| Grade: | | | | | |

Kiel,

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(Prof. Dr. Kai Carstensen)

Prof. Dr. Kai Carstensen
Chair of Econometrics

# Examination in Econometrics III

## (Winter Term 2022/23)

February 14, 2023 , 12:00 - 13:00

Preliminary remarks:

1. Write your name and enrolment (matriculation) number on every sheet of paper!
2. Don't use a pencil!
3. The exam is composed by 3 problems. Check your exam for completeness!
4. You have 60 minutes in total to answer the exam questions.

Good luck!

## <u>Problem 1</u> (22 credits)

Suppose you would like to model the distribution of grades among master's students at CAU Kiel in WiSe 22/23. For that, you assume that a random sample $y = (y_1, \ldots, y_N)'$ of exam grades comes from a Pareto distribution with **unknown shape parameter** $\theta > 0$ and known scale $k > 0$ (lower bound of $y_i$):

$$p(y_i|\theta, k) = \begin{cases} \theta k^{\theta} y_i^{-(\theta+1)} & \text{if } y_i \geq k \\ 0 & \text{if } y_i < k \end{cases} \tag{1}$$

Suppose prior beliefs concerning the **unknown parameter** $\theta$ are represented by a Gamma distribution with $p(\theta) \sim G(\underline{\mu}, \underline{\nu})$:

$$p(\theta) = \left(\frac{\underline{\nu}}{2\underline{\mu}}\right)^{\frac{\underline{\nu}}{2}} \Gamma\left(\frac{\underline{\nu}}{2}\right)^{-1} \theta^{\frac{\underline{\nu}-2}{2}} \exp\left(-\frac{\theta\underline{\nu}}{2\underline{\mu}}\right)$$

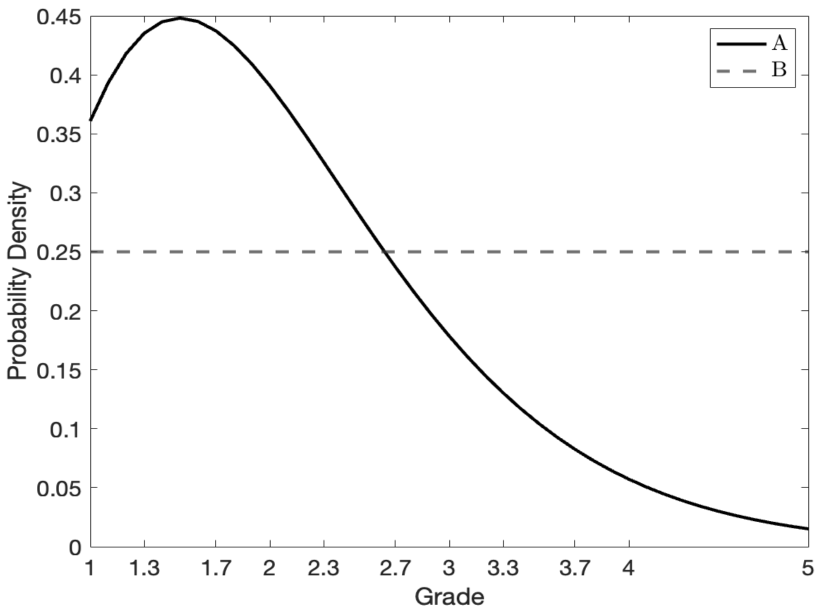
where $\Gamma(\cdot)$ denotes the Gamma function.

(a) **(6P)** State the likelihood.
*Hint*: to simplify notation, assume that $\left(\prod_{i=1}^{N} y_i\right) = \tilde{y}$.

(b) **(8P)** Derive the posterior distribution of $\theta$ (including posterior parameters).
*Hint*: remember the logarithm property $x = \exp(\ln(x))$.

(c) **(2P)** Is the Gamma distribution a conjugate prior? Briefly explain!

(d) **(6P)** Grades are given on the usual 1-5 scale (from best to worst). Consider two prior distributions A and B for $\theta$ which are characterized by the following densities:




Assume you have some information that a large share of students performed very well in the past but you are unsure whether you can trust your source. Which of the above prior specifications (A or B) would you choose? Briefly explain.

## Problem 2 (27 credits)

Suppose an estimation problem yields the following bivariate posterior distribution for the two parameters of interest, $h_1$ and $h_2$:

$$p(h_1, h_2|y) \propto h_1^{\frac{\underline{\nu}-2}{2}} h_2^{\frac{\underline{\nu}-2}{2}} \exp\left[-\frac{h_1\underline{\nu}}{2\,\lambda\,\underline{\mu}_1} - \frac{h_2\underline{\nu}}{2\,\underline{\mu}_2} - \frac{h_1 h_2 \underline{\nu}}{2}\right],$$

where $h_1 > 0$ and $h_2 > 0$ while $(\underline{\mu}_1, \underline{\mu}_2, \underline{\nu})$ are prior hyperparameters and $\lambda$ is a known parameter that reflects a prior relationship between $h_1$ and $h_2$.

(a) **(12P)** Derive the conditional distributions $p(h_1|h_2, y)$ and $p(h_2|h_1, y)$ and show that they are Gamma distributed.
*Hint*: Remember that the kernel of a Gamma distributed random variable $\theta$ follows:
$p(\theta) \propto \theta^{\frac{\nu-2}{2}} \exp\left[-\frac{\theta\nu}{2\mu}\right]$, whereas $\theta \sim G(\mu, \nu)$ and $\theta > 0$.

(b) **(8P)** Write a pseudo code that applies the Gibbs sampler to find the posterior means of $h_1$ and $h_2$.

(c) **(7P)** Assume that $\lambda$ is now unknown and a nonlinear function such that $\lambda = f(h_1, h_2)$. Which simulation procedure would you use to estimate $\mathrm{E}(h_1, h_2|y)$ in this scenario? Explain your choice.

## <u>Problem 3</u> (11 credits)

Consider the M-dimensional VAR(p) model

$$y_t = a_0 + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \varepsilon_t, \qquad t = 1, \cdots, T. \tag{2}$$

with $\varepsilon_t \sim N(0, \Sigma)$ and $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$.

(a) **(5P)** Why is shrinkage particularly important for large VAR models?

(b) **(6P)** Explain the key elements of the Minnesota prior that makes it appealing for Big Data applications and still allowing for analytical posterior results.