Institute of Statistics and Econometrics Prof. Dr. Kai Carstensen

Home Assignment in Bayesian Econometrics

(Winter Term 2024/25)

Deadline: 26th of January

Preliminary remarks:

1. Please read these instructions carefully!
2. This is a **voluntary** exercise.
3. You can earn up to **10 bonus points** that are carried over to the written exam and are valid for both examination periods of the WS2024/25.
4. You are allowed to work solo or in groups of 2 participants.
5. Use MATLAB as the software package.
6. You are allowed to use the programs discussed in the PC Tutorial, built-in functions provided by MATLAB and create your own functions (with proper documentation).
7. Please electronically submit:
   (a) A pdf document which contains answers to Pen & Paper questions. This document should be readable but does not need any fancy formatting.
   (b) The MATLAB function that simulates posterior moments of interest.
   (c) The MATLAB script that generates the required results to computer-based questions. Make sure the script can be executed without any errors. Explain every step in your code by making comments so that a reader understands why and how you proceed. Also answer open questions as comments in your file.
8. **Be aware that you might be asked to orally explain and defend your answers and your code in class!**
9. Please submit your m-file and pdf document via email to richard.schn@stat-econ.uni-kiel.de until the **26th of January**.
10. Write your name and matriculation number as a comment at the top of your script and pdf document.
11. Please name your script "HA_EconIII_Name1_Name2.m".

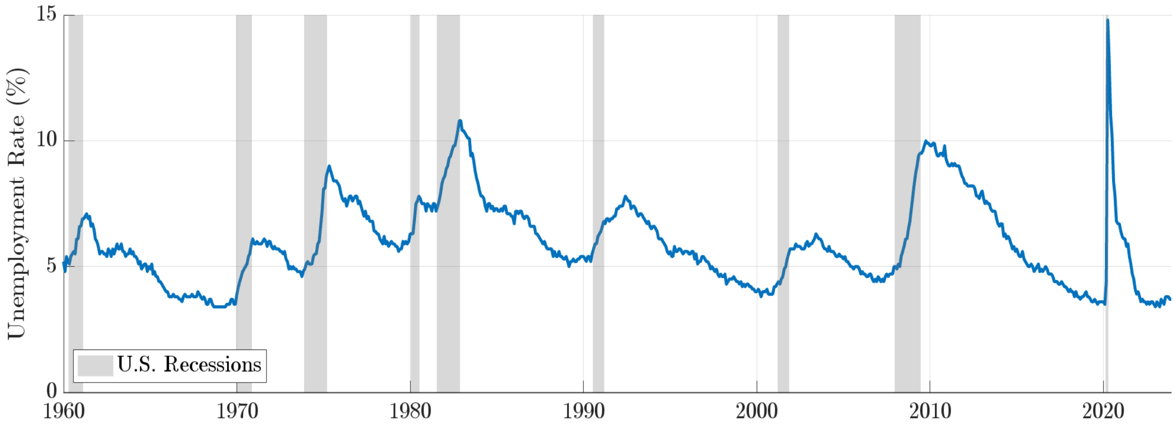
Good luck!

Christian–Albrechts–Universität zu Kiel
Institute of Statistics and Econometrics

Home Assignment Econometrics III
Winter term 2024/25

U.S. unemployment dynamics: recession vs. stable periods

The unemployment rate is a key indicator shaping U.S. monetary policy, especially during financial crises when labor markets become more volatile. Understanding how the unemployment rate behaves over time is essential for guiding Federal Reserve actions. Structural factors in U.S. labor markets create asymmetric cycles in the unemployment rate with sudden jumps during economic downturns and more gradual declines during recoveries, as shown below:



This asymmetry also results in heightened volatility (lower precision) during recessions compared to more stable periods. A Bayesian approach, which allows for differing error precision between recession and non-recession periods, offers a first step towards flexibility and provides a clearer picture of how the unemployment rate evolves across economic conditions. To this end, consider the following dynamic linear regression model:

$$\begin{aligned}\text{UNEMP}_t = {}& \mu + \alpha_1 \text{UNEMP}_{t-1} + \beta_1 \text{INPRO}_{t-1} + \ldots + \beta_q \text{INPRO}_{t-q} + \gamma_1 \text{CPI}_{t-1} + \ldots \\ & + \gamma_q \text{CPI}_{t-q} + \phi_1 \text{BCONF}_{t-1} + \ldots + \phi_q \text{BCONF}_{t-q} + \lambda \text{COVID}_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, h^{-1}), \end{aligned} \tag{1}$$

where $q = 4$ denotes the distributed lags while the monthly variables are the unemployment rate (UNEMP_t, first-difference), industrial production (INPRO_t, log first-difference), inflation (CPI_t, log first-difference), business confidence indicator (BCONF_t, first-difference), and COVID_t controls for the irregular behavior after the pandemic shock (April to June 2020). Assume that exogeneity between regressors in (1) and ε_t holds. This generalized regression framework can be written in matrix form:

$$y = X\beta + \varepsilon, \qquad\qquad \varepsilon \sim \mathcal{N}(0, \Sigma). \tag{2}$$

Moreover, let us assume two different error precisions h_1 and h_2 for calm and recession periods specified via the precision matrix such that

$$H = \Sigma^{-1} = \begin{bmatrix} h_i & 0 & \ldots & 0 \\ 0 & h_i & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \ldots & 0 & h_i \end{bmatrix} \quad \text{with} \quad h_i = \begin{cases} h_1 & \text{if } t \text{ is a } \textbf{calm} \text{ period} \\ h_2 & \text{if } t \text{ is a } \textbf{recession} \text{ period} \end{cases} \tag{3}$$

Finally, suppose prior beliefs concerning the unknown parameters of the model $\theta = (\beta', h_1, h_2)'$ are represented by the independent Normal-Gamma prior densities:

$$\beta \sim \mathcal{N}(\underline{\beta}, \underline{V}) \tag{4}$$

$$h_1 \sim G(\underline{s}_1^{-2}, \underline{\nu}_1) \tag{5}$$

$$h_2 \sim G(\underline{s}_2^{-2}, \underline{\nu}_2) \tag{6}$$

Questions

1. **(1P)** State the joint prior $p(\beta, h_1, h_2)$, the likelihood $p(y|\beta, h_1, h_2)$ and the joint posterior $p(\beta, h_1, h_2|y)$.
   Hint: define $d = \text{diag}(d_1, \ldots, d_T)$ as the diagonal matrix with recession indicators $d_t = 1$, for $t = 1, \ldots, T$, and thus note that (3) can be simplified to

   $$H = h_1(I_T - d) + h_2 d \quad \text{with} \quad |H| = h_1^{T_1} h_2^{T_2}, \tag{7}$$

   where $T_2 = \sum_{i=1}^{T} d_t$ is the number of recession periods and $T_1 = T - T_2$ is the number of calm periods in the sample.

2. **(2P)** Find the conditional posteriors $p(\beta|y, h_1, h_2)$, $p(h_1|y, \beta, h_2)$ and $p(h_2|y, \beta, h_1)$ and show in pseudo code how the Gibbs sampler can be used to estimate this model.

3. **(2P)** Write a MATLAB function `indnormgam_posterior.m` that computes the posterior moments of the multiple linear regression model with independent normal-gamma prior following (4)-(6). Specifically, the user should supply data y and X, along with the vector of dummies d, as well as prior parameters $\underline{\beta}$, $\underline{V}$, $\underline{s}_1^{-2}$, $\underline{\nu}_1$, $\underline{s}_2^{-2}$, and $\underline{\nu}_2$. Moreover, let the function's user choose the S_0 burn-in and S_1 MC posterior replications for the Gibbs sampler. From there, the function should compute and hand back posterior parameters, along with posterior means and variances of β, h_1 and h_2.

4. **(2P)** In your Main script, simulate posterior moments of β, h_1 and h_2 using the dataset `US_macro.mat` from 1960 to 2023. For that, choose an informative prior based on OLS estimates (hereby assuming homoskedasticity with $h_1 = h_2$). This means that the prior hyperparameters for β, h_1 and h_2 can be based on the latter OLS quantities. Find reasonable numbers for the S_0 and S_1 iterations. Finally, report the posterior means and standard deviations of β, h_1 and h_2.

5. **(1P)** Check for MCMC convergence either by examining trace plots or CD statistics. How reliable are posterior estimates based on these Markov chains?

6. **(1P)** Compare your posterior estimates for h_1 and h_2 and answer whether there is empirical evidence for lower precision of the unemployment rate in recession periods.

7. **(1P)** Extend your script to compute a one-step-ahead forecast UNEMP^*_{t+1} for 2024M01 based on past values $X^*_{t-q+1:t}$. Plot the predictive density of UNEMP^*_{t+1} and illustrate the conditional posterior mean along with percentiles of your choice.