

List of questions for the course
Mathematical Finance
Winter Term 23/24

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The following questions are for self-study. The list will be expanded and updated throughout the semester. Some of the questions will be part of the exam.

1. Explain the basic idea of how the principle of no arbitrage can be used for pricing in an example of your choice.
2. Explain the intuitive meaning of the following mathematical terms: filtration, adaptedness, conditional expectation, stopping time.
3. You roll a die twice and someone tells you whether both results are > 3 or not. Describe the σ -algebra \mathcal{F} that models this information and find $E(X|\mathcal{F})$.
4. Decide which of the following rules are stopping times when you receive the information from an asset price process:
 - (a) Stop when the process reaches its (overall) maximum value (for the first time).
 - (b) Stop the first time the asset price is above 30 (or at terminal time).
 - (c) Stop two days after the asset price was below 20 for the first time (or at terminal time).
 - (d) Stop two days before the asset price goes below 20 (or at terminal time).
5. In the previous question, specify a filtration such that all times are stopping times.
6. Is the time of the annual high of the DAX stock index a stopping time?
7. What is a stochastic integral and where is it used in mathematical finance?
8. Give an example of a process X with the following property (e.g., by drawing a tree) or argue why such a process does not exist:
 - (a) X is a martingale.
 - (b) X is a supermartingale but not a martingale.
 - (c) X is a submartingale but not a martingale.
 - (d) X is a martingale, but not a submartingale.

- (e) X is a martingale, but not a supermartingale.
 - (f) X is both a submartingale and a supermartingale.
 - (g) X is neither a submartingale nor a supermartingale.
9. What is a stochastic exponential?
 10. Why is the stochastic exponential relevant to finance?
 11. Which equation describes the change in value of a self-financing trading strategy?
 12. What is the intuitive meaning of this equation?
 13. What are discounted price processes and why are they used in mathematical finance?
 14. What is arbitrage?
 15. What is the role of arbitrage in mathematical finance?
 16. State the first fundamental theorem of asset pricing. Explain its relevance.
 17. What can be said about the price process of a liquidly traded European option in an arbitrage-free market?
 18. In a complete market with the trivial numeraire $S^0 = 1$, how can the value $Q(A)$, $A \subseteq \Omega$, for the EMM Q be interpreted?
 19. What is a complete market?
 20. How can one check whether a given market model is complete?
 21. What are the reasons why real financial markets are not (always) complete?
 22. What are the upper and lower prices of an option? Explain their economic meaning for a bank that wants to trade the option over the counter.
 23. What is the cheapest superhedge, and what is its price?
 24. What is the perfect hedging strategy?
 25. Is it possible for perfect hedging strategies to exist with different prices for an option in an arbitrage-free market?
 26. Is it possible that, for a given call option, the two initial option prices 10 and 12 do not lead to arbitrage, but 11 does?
 27. What are the similarities and differences between futures and forwards from a mathematical finance perspective?

28. A bank trades a forward or an option over the counter. What are the risks of loss? Which are reflected in the mathematical model from the lecture?
29. A bank uses the binomial model to calculate the unique fair price of an option sold to a client. The bank hedges the market risk by using a replication strategy. Discuss the risk of loss that still exists in real markets.
30. Why does the fair forward price not depend on the valuations and preferences of market participants?
31. Why is it necessary to consider markets with short selling constraints in the treatment of American options?
32. What can be said about the price process of a liquidly traded American option in an arbitrage-free market?
33. In what sense can the price of an American option be interpreted as the solution to an optimal stopping problem?
34. What is the value of the stopping problem with maturity N

$$\sup_{\tau} E(X_{\tau}),$$

when X is a supermartingale? Give an optimal stopping time.

35. How are the price processes of an American and a European call option (with same strike and maturity) related?
36. Explain the concept of calibration.
37. For a market where plain vanilla options are liquidly traded, a bank wants to describe the dynamics of the price process relative to the equivalent martingale measure from the first fundamental theorem of asset pricing. To this end, it uses a model calibrated to option prices. Which evidence could indicate that the model is not appropriate to describe the real market?
38. Why is it problematic, from a theoretical perspective, to recalibrate a given model?
39. Give some definitions that cannot directly be taken over from discrete to continuous time.
40. What are the essential assumptions on the stock price process made in the Black-Scholes model?
41. Discuss strengths and weaknesses of (a) the binomial model and (b) the Black-Scholes model for option pricing.