Bayesian Econometrics
PC Tutorial 04

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Exercise 1 (Computer-Based Exercises)

Consider estimation of the CES production function as outlined in the textbook. In particular, assume normality of the regression disturbances. Use the improper prior $p(\gamma, h) = 1/h$, $h > 0$.

- (d) Write a Matlab script that performs the independence chain MH algorithm to estimate the CES parameters. Use the asymptotic normal distribution derived above as proposal distribution.
- (e) Write a Matlab script that performs the independence chain MH algorithm to estimate the CES parameters and h (brute force MH). Use the same proposal distribution for γ as in part (d). What might be a good proposal distribution for h?
- (*) Add to your script a part that computes predictive p-values for the skewness and kurtosis of the regression disturbances.

Exercise 1 (d): Pseudo-Code

- Initial value: draw $\gamma^{(1)}$ from candidate distribution $\mathcal{N}(\hat{\gamma}, \hat{\Sigma})$
- Start iteration: $s = 2, \ldots, S_0 + S_1$
  - Proposal: draw γ^* from candidate distribution $\mathcal{N}(\hat{\gamma}, \hat{\Sigma})$
  - Acceptance probability: compute

    $$\alpha(\gamma^{(s-1)}, \gamma^*) = \min \left\{ \frac{p(\gamma^*|y)q^*(\gamma^{(s-1)})}{p(\gamma^{(s-1)}|y)q^*(\gamma^*)}, 1 \right\},$$

    where

    $$p(\gamma|y) \propto \{u(\gamma)'u(\gamma)]\}^{-\frac{N}{2}} \quad \text{with} \quad u(\gamma) = y - f(X, \gamma)$$

    is the posterior kernel derived above and

    $$q^*(\gamma) \propto \exp\left\{-\frac{1}{2}(\gamma - \hat{\gamma})'\hat{\Sigma}^{-1}(\gamma - \hat{\gamma})\right\}$$

    is the kernel of the proposal distribution.
  - Draw a uniform random number u
  - If $u \leq \alpha(\gamma^{(s-1)}, \gamma^*)$ then
    $\gamma^{(s)} = \gamma^*$
    else
    $\gamma^{(s)} = \gamma^{(s-1)}$
    end if
- End iteration
- Take the average of $\gamma^{(S_0+1)}, \ldots, \gamma^{(S_0+S_1)}$.

Solution to Exercise 1 (e)

Remember that the joint posterior is given by

$$p(\gamma, h|y) \propto h^{\frac{N}{2}-1} \exp\left[-\frac{h}{2}u(\gamma)'u(\gamma)\right].$$

Let us work with two relatively simple proposal distributions for h, both based on the previous finding that $h|y,\gamma \sim \mathcal{G}(\mu = N/[u(\gamma)'u(\gamma)], \nu = N)$.

(i) The first suggestion is to make the proposal distributions of γ and h mutually independent such that

$$q^*(\gamma, h) = q^*_\gamma(\gamma)q^*_h(h)$$

To achieve this, we may use the Gamma distribution above and the OLS estimator $\hat{\gamma}$ to sample proposal draws h from $\mathcal{G}(\hat{\mu}, N)$ with $\hat{\mu} = N/[u(\hat{\gamma})'u(\hat{\gamma})]$.

The joint proposal distribution is then given by

$$q^*(\gamma, h) = q^*_\gamma(\gamma)q^*_h(h) \propto \exp\left\{-\frac{1}{2}(\gamma - \hat{\gamma})'\hat{\Sigma}^{-1}(\gamma - \hat{\gamma})\right\} h^{\frac{N}{2}-1} \exp\left[-\frac{hN}{2\hat{\mu}}\right]$$

Solution to Exercise 1 (e)

(ii) Alternatively, we may use the conditional-marginal factorization:

$$q^*(\gamma, h) = q^*_{h|\gamma}(h|\gamma) q^*_\gamma(\gamma),$$

which could lead to increased efficiency since we incorporate the fact that the conditional posterior of h (given γ) is known in closed form. The conditional proposal pdf of h can then be derived from the joint posterior distribution by treating γ as fixed:

$$\begin{aligned} q^*_{h|\gamma}(h|\gamma) &= c_G^{-1} h^{\frac{N}{2}-1} \exp\left[-\frac{h}{2} u(\gamma)' u(\gamma)\right] \\ &= \left(\frac{1}{2}\right)^{\frac{N}{2}} \Gamma\left(\frac{N}{2}\right)^{-1} \left\{u(\gamma)' u(\gamma)\right\}^{\frac{N}{2}} h^{\frac{N}{2}-1} \exp\left[-\frac{h}{2} u(\gamma)' u(\gamma)\right]. \end{aligned}$$

The joint pdf is thus proportional to

$$q^*(\gamma, h) = \exp\left\{-\frac{1}{2}(\gamma - \hat{\gamma})' \hat{\Sigma}^{-1} (\gamma - \hat{\gamma})\right\} \left\{u(\gamma)' u(\gamma)\right\}^{\frac{N}{2}} h^{\frac{N}{2}-1} \exp\left[-\frac{h}{2} u(\gamma)' u(\gamma)\right].$$

Solution to Exercise 1 (e)

A nice feature for this choice (ii) of proposal pdf is that $p(\gamma, h|y)/q^*(\gamma, h)$ simplify considerably to

$$p(\gamma, h|y)/q^*(\gamma, h) = \exp\left\{\frac{1}{2}(\gamma - \hat{\gamma})'\hat{\Sigma}^{-1}(\gamma - \hat{\gamma})\right\}\{u(\gamma)'u(\gamma)\}^{-\frac{N}{2}}$$

Hence, the acceptance probability reduces to:

$$\begin{aligned}\alpha(\theta^{(s-1)}, \theta^*) &= \min\left\{\frac{p(\theta^*|y)/q(\theta^*)}{p(\theta^{(s-1)}|y)/q^*(\theta^{(s-1)})}, 1\right\} \\ &= \min\left\{\frac{[u(\gamma^*)'u(\gamma^*)]^{-\frac{N}{2}}\exp\left\{\frac{1}{2}(\gamma^* - \hat{\gamma})'\hat{\Sigma}^{-1}(\gamma^* - \hat{\gamma})\right\}}{\left[u(\gamma^{(s-1)})'u(\gamma^{(s-1)})\right]^{-\frac{N}{2}}\exp\left\{\frac{1}{2}(\gamma^{(s-1)} - \hat{\gamma})'\hat{\Sigma}^{-1}(\gamma^{(s-1)} - \hat{\gamma})\right\}}, 1\right\}\end{aligned}$$

which does not depend on h^* or $h^{(s-1)}$.

Exercise 2 (Computer-Based Exercises)

Consider estimation of the CES production function as outlined in the textbook. In particular, assume normality of the regression disturbances. Use the (mutually independent) informative priors $\gamma \sim \mathcal{N}(\underline{\gamma}, \underline{V})$ **and** $h \sim Gamma(\underline{s}^{-2}, \underline{\nu})$**, where** $\underline{\gamma} = [1, 1, 1, 1]'$**,** $\underline{V} = 0.25 I_4$**,** $\underline{\nu} = 12$**, and** $\underline{s}^{-2} = 10$**.**

- (c) Write a Matlab script that uses the random walk chain MH algorithm to estimate γ and h.
- (d) Extend your script to report numerical standard deviations for the γ's and for h based on Newey-West long-run variances (the function NeweyWest.m will be supplied in the tutorial).
- (e) Extend your script to report *CD* statistics for convergence based on subsamples $A = 10\%$, $B = 50\%$, and $C = 40\%$.
- (*) Write a Matlab script that uses the Gelfand-Dey method to compute the posterior odds ratio for models $M_1 : \gamma_4 = 1$ and $M_2 : \gamma_4$ is unrestricted. Suppose prior model probabilities are $p(M_1) = p(M_2) = 0.5$. Use a truncated normal pdf with truncation parameters $p = 0.01, 0.05, 0.1$.