PC Tutorial 3: Numerical Methods for Bayesian Linear Regression Models

1. Write a Matlab script that uses importance sampling to simulate the mean and variance of a truncated standard normal distribution with truncation bounds $a = -1$ and $b = 1.5$ using only normal random numbers. Compare to the analytically available results shown here.

2. Suppose you have a posterior distribution of the scalar parameter θ which is logistic. Assume the parameters are $\bar{\alpha} = 4$ and $\bar{\beta} = 2$. Apply Monte Carlo integration to find (i) the mean of θ, (ii) the variance of θ, and (iii) the expected value of $g(\theta) = \exp(\sqrt{|\theta|} - 1)$. Suppose you have access to random numbers from the uniform distribution and the standard normal distribution only. Compare your simulation results for (i) and (ii) to the analytic results available, e.g., here.

   (a) Write a Matlab script that applies the probability integral transform.

   (b) Write a Matlab script that applies the acceptance-rejection method using the t-distribution as proposal distribution. Report the scale factor M for different degrees of freedom parameter of the t-distribution. Plot the two pdfs. Which degrees of freedom do you choose?

   (c) Write a Matlab script that applies importance sampling. Choose the t-distribution as importance function. Report the average and variance of the weights for different degrees of freedom.

3. Consider the textbook example of Canadian house prices with independent normal-gamma prior, $\beta \sim \mathcal{N}(\underline{\beta}, \underline{V})$ and $h \sim Gamma(\underline{s}^{-2}, \underline{\nu})$. Choose the prior parameters as follows: $\underline{s}^{-2} = 4.0 \times 10^{-8}$, $\underline{\nu} = 5$, $\underline{\beta} = (0, 10, 5000, 10000, 10000)'$, and $\underline{V}$ is a diagonal matrix with main diagonal $(10000^2, 5^2, 2500^2, 5000^2, 5000^2)$.

   (a) Write a Matlab script that includes the following steps: (i) load the data from the file hprice.txt, (ii) compute OLS estimates, (iii) set

the priors, (iv) perform Gibbs sampling with 1000 burn-in replications and 10000 included replications, and (v) compute posterior means and standard deviations for β.

(b) Extend your script to report numerical standard deviations for β based on Newey-West long-run variances (the function NeweyWest will be supplied in the tutorial).

(c) Extend your script to report CD statistics for convergence based on subsamples $A = 10\%$, $B = 50\%$, and $C = 40\%$.

(d) (*) Extend your script to report estimated potential scale reductions for β based on $m = 20$ parallel Markov chains.

(e) (*) Extend your script to report posterior mean and standard deviation of the prediction y^* based on $X^* = (1, 5000, 2, 2, 1)$. Plot the predictive density as a histogram with 100 bins.