Exam for the lecture

“Econometrics III / Bayesian Econometrics”

for students in the M.Sc. programmes
winter term 2023/2024

05.04.2024

Please use block letters:

Name: *Surname*		Vorname: *First Name*	
Studiengang: *Course of study*		Name der Universität (Bachelor): *Name of university (Bachelor's degree)*	
Matrikelnummer: *Student ID*		Hochschulstandort (Bachelor): *Place of university (Bachelor's degree)*	

Declaration:

PLEASE SIGN!!!
I hereby declare that I am able to be examined. ________________________________ Signature:

Preliminary remarks:

- Write down your name and enrolment/matriculation number on all paper sheets provided for answers by the examiner.
- To write down your answers, use only the paper provided by the examiner.

Result: (TO BE FILLED IN ONLY BY THE EXAMINER!)

Problem:	1	2	3	HA	S
Points earned:					
Grade:					

Kiel,

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(Prof. Dr. Kai Carstensen)

Prof. Dr. Kai Carstensen
Chair of Econometrics

Examination in Econometrics III / Bayesian Econometrics

(Winter Term 2023/24)

April 05, 2024 , 10:00 - 12:00

Preliminary remarks:

1. Write your name and enrollment (matriculation) number on every sheet of paper!
2. Don't use a pencil!
3. The exam is composed by 3 problems. Check your exam for completeness!
4. You have 60 minutes in total to answer the exam questions.

Good luck!

Problem 1 (19 credits)

Suppose you are analyzing the daily number of visits to the sales page of a website, which you believe can be modeled using a Negative Binomial distribution. The number of daily visits y_i for day i can be modeled as $y_i \sim \text{NegBin}(r, \theta)$:

$$p(y_i|r,\theta) = \begin{cases} \binom{y_i+r-1}{y_i}(1-\theta)^{y_i}\theta^r & \text{if } y_i \in \{0,1,2,3,\ldots\} \\ 0 & \text{otherwise.} \end{cases}$$

where $r > 0$ is the **known** number of failures until a predefined number of successful visits occurs and the experiment is stopped. The **unknown** parameter θ represents the probability of success on each trial.

Given a set of N daily observations $y = (y_1, \ldots, y_N)$, you decide to use a Beta prior for θ of the form:

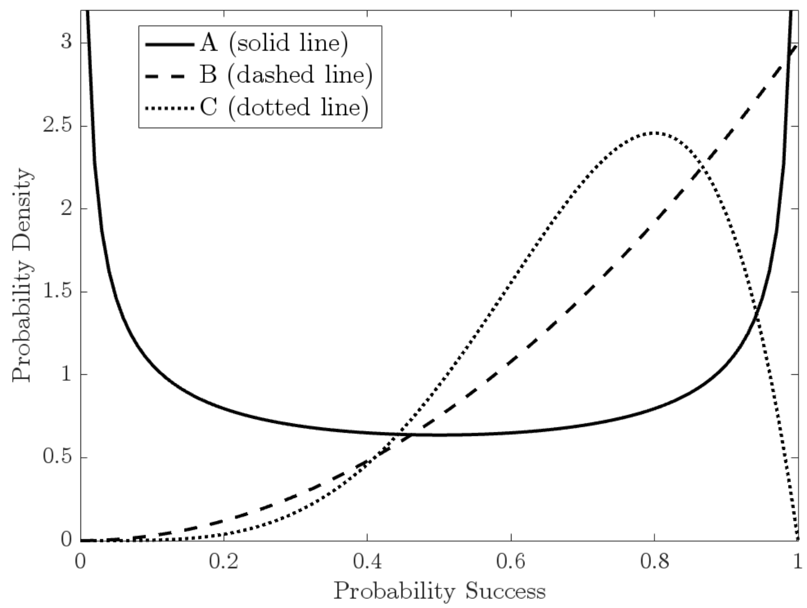
$$p(\theta|\underline{\alpha},\underline{\beta}) = \begin{cases} B(\underline{\alpha},\underline{\beta})^{-1}\theta^{\underline{\alpha}-1}(1-\theta)^{\underline{\beta}-1} & \text{if } 0 \leq \theta \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $B(\underline{\alpha},\underline{\beta})$ is the Beta function and $\text{E}(\theta) = \frac{\underline{\alpha}}{\underline{\alpha}+\underline{\beta}}$.

(a) **(6P)** Find the posterior distribution (including posterior parameters).

(b) **(7P)** What happens to the posterior mean as the sample size N increases? Demonstrate it mathematically and interpret your results.

(c) **(6P)** Based on a data analysis using a previous sales campaign, you find that the success rate of visits is relatively high but it never reaches 100% due to the attractiveness of other products that are not on sale. Which of the following Beta prior specifications would you choose? Defend your choice!



<u>Problem 2</u> (17 credits)

Consider a multiple linear regression model with many regressors and N observations that yields the following joint posterior kernel:

$$p(\beta, h|y) \quad \propto \quad h^{\frac{N+\underline{\nu}-2}{2}} \exp\left[-\frac{h}{2}(y - X\beta)'(y - X\beta) - \frac{|\beta - \underline{\beta}|}{\underline{\tau}} - \frac{h\,\underline{\nu}\underline{s}^2}{2}\right]$$

(a) **(10P)** Propose one method for obtaining posterior mean estimates for β and h, and briefly explain in pseudo-code how its key steps would be implemented.
<u>*Hint*</u>: $p(h|\beta, y)$ is available in closed form solution but $p(\beta|h, y)$ is not available analytically.

(b) **(7P)** Discuss one strategy for assessing the convergence of any MCMC algorithm and briefly explain its importance.

Problem 3 (24 credits)

Suppose you are studying the interplay between consumer confidence and unemployment rates over time, aiming to understand how changes in consumer confidence influence unemployment rates and vice versa. To this end, you fit monthly German data for the consumer confidence index (CCI) and unemployment rates (UR) over the last 20 years using the bivariate VAR(1) model:

$$\begin{aligned} CCI_t &= a_{10} + a_{11}CCI_{t-1} + a_{12}UR_{t-1} + \varepsilon_{1t} \\ UR_t &= a_{20} + a_{21}CCI_{t-1} + a_{22}UR_{t-1} + \varepsilon_{2t} \end{aligned}$$

This model can be simplified to:

$$y_t = a_0 + A_1 y_{t-1} + \varepsilon_t, \qquad t = 1, \cdots, T$$

with $\varepsilon_t \sim N(0, \Sigma)$ and $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$, whereas the vectorized likelihood can be written as

$$f(y|\alpha, \Sigma) \propto |\Sigma|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}(y - \mathbf{X}\alpha)'(\Sigma^{-1} \otimes I_T)(y - \mathbf{X}\alpha)\right\},$$

where $\alpha = \text{vec}(A)$ and $\mathbf{X} = I_2 \otimes X$.

1. **(18P)** Show that under the improper prior, with $f(\alpha, \Sigma) = |\Sigma|^{-\frac{3}{2}}$, the posterior distribution of VAR coefficients follows a conditional normal distribution, $\alpha|\Sigma, y \sim \mathcal{N}(\bar{\alpha}, \Sigma \otimes \bar{V})$, such that
$$f(\alpha|\Sigma, y) \propto \exp\left[-\frac{1}{2}(\alpha - \bar{\alpha})'(\Sigma^{-1} \otimes \bar{V}^{-1})(\alpha - \bar{\alpha})\right]$$
with $\bar{\alpha} = (I_2 \otimes (X'X)^{-1}X')y$ and $\bar{V} = (X'X)^{-1}$.

   Hints:

   - Simplify the relevant part of the likelihood as far as possible for later comparison with the conjectured conditional posterior for α.
   - Mixed-product property of the Kronecker product: $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

2. **(6P)** Explain why the improper prior is a good prior choice for this bivariate VAR(1) model.