

# Mathematical Finance

## Information on the exam for participants of the programme 'Quantitative Finance'

### 1 Allowed auxiliary materials

You are allowed to use in the exam

- writing utensils,
- a calculator,
- one handwritten A4 sheet (you can use front and back page) with your personal notes,

but nothing else.

### 2 Exercises in the exam

#### 2.1 Types of exercises

In the exam, there will be two types of exercises:

1. questions about the course, which will be very similar to the questions in the list from Section 5 below, and
2. exercises that are strongly related to the exercises of the tutorial.

Approximately 25% of the points can be attained in exercises of type 1, and the rest in those of type 2.

#### 2.2 How to provide your solutions

Please answer the questions in exercises of type 1 in complete sentences. Stick to the question and do not wander off the point.

Your answers to questions of exercise type 2 should meet the same requirements as for the homework assignments. In particular:

- If not explicitly stated differently, please give reasons for your answer.
- When you compute certain quantities, e.g. a price process in a tree, please explain first in abstract terms (formulas) what you compute and why. Then do the computation with the explicit numbers of the exercise.
- If a certain type of computation is required several times in an exercise (e.g. the computation of conditional expectations in a tree), you may provide your computation for one example in detail and state only the result for the remaining cases.

### 3 Language

The exercises of the exam will be provided in English. You may give your answers in English or German.

### 4 Duration of the exam

The exam is designed such that it is doable within 120 minutes if you are well prepared. However, we do not want you to feel rushed if you want to think about your answers. Therefore you will have 180 minutes of time.

### 5 List of questions about the course

1. Explain the intuitive meaning of the following mathematical notions: filtration, adapt-  
edness, conditional expectation, stopping time.
2. What is a stochastic integral, and where is it used in mathematical finance?
3. What role do martingales play in mathematical finance?
4. Is the time of the all-year-high of the DAX stock index a stopping time?
5. What is the stochastic exponential?
6. According to which equation does the value of a self-financing trading strategy change?  
What is the intuitive meaning of this equation?
7. What are discounted price processes, and why are they used in mathematical finance?
8. What is an arbitrage?
9. What role does the concept of arbitrage play in mathematical finance?
10. State the first fundamental theorem of asset pricing with dividends.
11. Given the dividend process  $D$ , why is the discounted dividend process defined as  $\frac{1}{S_0} \cdot D$   
and not as  $\frac{D}{S_0}$ ?
12. What can be said about the price process of a liquidly traded European option in an  
arbitrage-free market?
13. What is a complete market?
14. How can one check whether a given market model is complete?
15. What is the upper and the lower price of an option? Explain their economic meaning  
for a bank that wants to trade the option *over the counter*.
16. What is a cheapest superhedge, and what is its price?
17. What is a perfect hedging strategy?
18. Is it possible that there exist perfect hedging strategies with different prices for an  
option in an arbitrage-free market?

19. What are the similarities and differences of futures and forwards from the viewpoint of mathematical finance?
20. Why is it necessary to consider markets with shortselling constraints when treating American options?
21. What can be said about the price process of a liquidly traded American option in an arbitrage-free market?
22. In which sense can the price of an American option be interpreted as the solution to an optimal stopping problem?
23. How are the price processes of an American and a European call option (with same strike and maturity) related?
24. Why does the fair forward price not depend on valuations and preferences of the market participants?
25. A bank uses the binomial model to compute the unique fair price of an option sold to a customer. The bank hedges the market risk by the corresponding replicating strategy. Discuss the risks of loss that exist nevertheless in real markets.
26. Explain the concept of calibration.
27. A bank trades a forward or an option *over the counter*. Which risks of loss does the bank face? Which of them are reflected by the mathematical model from the lecture?
28. What are the essential assumptions on the stock price process made in the Black-Scholes model?
29. Discuss strengths and weaknesses of
  - (a) the binomial model and
  - (b) the Black-Scholes modelfor option pricing.
30. For a market where plain vanilla options are liquidly traded, a bank wants to describe the dynamics of the price process relative to the equivalent martingale measure from the first fundamental theorem of asset pricing. To this end, it uses a model calibrated to option prices. Which evidence could indicate that the model is not appropriate to describe the real market?