Mathematical Finance Winter term 2023/2024
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Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 11/07/2023)

In-Tutorial Exercise 1. Let $\Omega = \{1, 2, 3, a\}$ and let $\mathcal{A}, \mathcal{B} \subseteq \Omega$ be σ-algebrae defined via

$$\mathcal{A} = \sigma(\{1, 2\}, \{3, a\}), \mathcal{B} = \sigma(\{1, a\}, \{2, 3\}) \text{ and } \mathcal{C} = \sigma(\{1, 2\}, \{3\}, \{a\}).$$

a) Give all the elements of $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$.

b) Give all the elements of $\mathcal{A} \cup \mathcal{B}$ and $\mathcal{A} \cap \mathcal{B}$.

In-Tutorial Exercise 2. Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ and define a stochastic process $X = (X_0, X_1, X_2)$ on Ω by the following values:

ω	$X_0(\omega)$	$X_1(\omega)$	$X_2(\omega)$
ω_1	1	0.5	0.25
ω_2	1	0.5	1
ω_3	1	2	1
ω_4	1	2	1.5
ω_5	1	2	3.5

a) Represent X in the tree and determine the filtration $\mathcal{F} = (\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2)$ generated by X.

b) Let P be the uniform distribution on Ω. Calculate the conditional expectations $E(X_2|\mathcal{F}_1), E(X_1|\mathcal{F}_0)$ and $E(X_2|\mathcal{F}_0)$.

c) Determine whether X is a martingale, a submartingale or a supermartingale?