Exam for the lecture

"Econometrics III"

for students in the M.Sc. programmes
winter term 2022/2023

28.03.2023

Please use block letters:

Name: *Surname*		Vorname: *First Name*	
Studiengang: *Course of study*		Name der Universität (Bachelor): *Name of university (Bachelor's degree)*	
Matrikelnummer: *Student ID*		Hochschulstandort (Bachelor): *Place of university (Bachelor's degree)*	

Declaration:

PLEASE SIGN!!!
I hereby declare that I am able to be examined. __________________________________ Signature:

Preliminary remarks:

- Write down your name and enrolment/matriculation number on all paper sheets provided for answers by the examiner.
- To write down your answers, use only the paper provided by the examiner.

Result: (TO BE FILLED IN ONLY BY THE EXAMINER!)

Problem:	1	2	3	HA	S
Points earned:					
Grade:					

Kiel,

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(Prof. Dr. Kai Carstensen)

Prof. Dr. Kai Carstensen
Chair of Econometrics

Examination in Econometrics III

(Winter Term 2022/23)

March 28, 2023 , 12:00 - 13:00

Preliminary remarks:

1. Write your name and enrolment (matriculation) number on every sheet of paper!
2. Don't use a pencil!
3. The exam is composed by 2 problems. Check your exam for completeness!
4. You have 60 minutes in total to answer the exam questions.

Good luck!

Problem 1 (38 credits)

It is often the case that posterior distributions are analytically intractable and their moments thus not available in closed form. In this scenario, you may resort to Monte Carlo integration.

1. **(6P)** Consider the general integration problem with no analytical solution

$$\mathrm{E}(g(\theta)|y) = \int g(\theta)p(\theta|y)d\theta, \tag{1}$$

where $g(\theta)$ is a function of interest. Briefly explain the general principle by which Monte Carlo integration can be used to replace analytical solutions.

2. Suppose your estimation problem of three parameters $\beta \geq 0$, $h > 0$ and $\gamma \geq 0$ yields the following posterior distribution:

$$p(\beta, h, \gamma|y) \propto h^{\frac{N+\underline{\nu}}{2}-1} \exp\left[-\frac{1}{2}(\underline{\kappa}h + \underline{\kappa}\beta + h\beta f(y,\gamma))\right],$$

where $\underline{\kappa} > 0$ and $\underline{\nu} > 0$ are some known hyperparameters, N is the sample size and $f(y, \gamma)$ is a nonlinear function of data y and parameter γ.

   a. **(11P)** Derive the conditional distributions $p(\beta|h, \gamma, y)$ and $p(h|\beta, \gamma, y)$ along with the posterior parameters.

   Hints:

   - The Exponential density for $x \geq 0$ is $f(x) = \lambda\ \exp(-\lambda x)$ with known rate $\lambda > 0$ such that we write $x \sim \mathrm{Exp}(\lambda)$.
   - The Gamma kernel for $x > 0$ is $f(x) \propto x^{\frac{\nu-2}{2}} \exp\left[-\frac{x\nu}{2\mu}\right]$ with known mean $\mu > 0$ and degrees of freedom $\nu > 0$ such that we write $x \sim G(\mu, \nu)$.

   b. **(11P)** Write a pseudo code that applies the Metropolis-within-Gibbs algorithm to find the posterior means of β, h and γ.
   Note: you don't need to find a proposal density for $p(\gamma|\beta, h, y)$; just assume that a good proposal density $q^*(\gamma|\beta, h, y)$ is available.

3. **(4P)** Explain the importance to check for convergence of the Markov chains generated previously.

4. **(6P)** Explain how you would check for convergence of the Markov chain for γ. Briefly describe in words the chosen method.

Problem 2 (22 credits)

Consider the M-dimensional VAR(p) model

$$y_t = a_0 + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \varepsilon_t, \qquad t = 1, \cdots, T$$

with $\varepsilon_t \sim N(0, \Sigma)$ and $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$. The vectorized likelihood can be written as

$$f(y|\alpha, \Sigma) \propto |\Sigma|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}(y - \mathbf{X}\alpha)'(\Sigma^{-1} \otimes I_T)(y - \mathbf{X}\alpha)\right\},$$

where $\alpha = \text{vec}(A)$ and $\mathbf{X} = I_M \otimes X$.

1. **(15P)** The Minnesota prior with $\alpha \sim \mathcal{N}(\underline{\alpha}, \underline{V})$ and $\Sigma = \hat{\Sigma}$ has appealing features that make estimation of the VAR(p) feasible even in the big data setting with large M. Show that under the Minnesota prior the posterior distribution $\alpha|y \sim \mathcal{N}(\bar{\alpha}, \bar{V})$ such that

$$f(\alpha|y) \propto |\bar{V}|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\alpha - \bar{\alpha})'\bar{V}^{-1}(\alpha - \bar{\alpha})\right]$$

   with $\bar{\alpha} = \bar{V}[(\hat{\Sigma}^{-1} \otimes X')y + \underline{V}^{-1}\underline{\alpha}]$ and $\bar{V} = (\hat{\Sigma}^{-1} \otimes X'X + \underline{V}^{-1})^{-1}$.

   Hints:

   - Simplify both the prior distribution and the likelihood as far as possible before you compute their product.
   - Mixed-product property of the Kronecker product: $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$ for suitable dimensions of A, B, C, and D.

2. **(7P)** Explain why the BVAR with Minnesota prior is particularly appealing for forecasting under a big data setting.