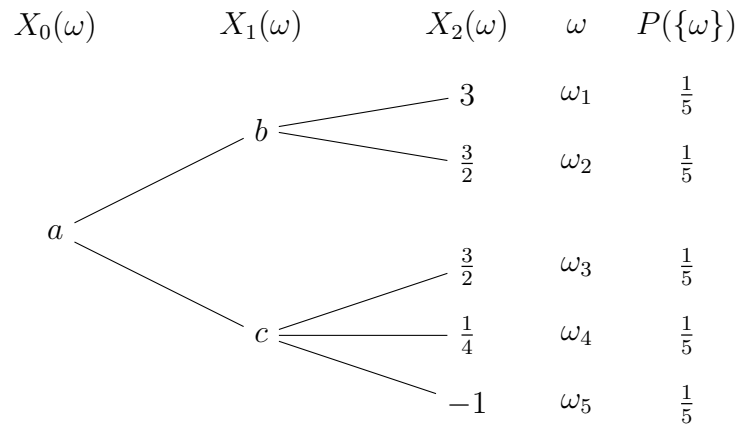## *Mathematical Finance: QF*
In-Tutorial exercises (for discussion on Tuesday, 14/11/2023)

In-Tutorial Exercise 1. We consider a stochastic process $X = (X_0, X_1, X_2)$ on the probability space $(\Omega, \mathcal{P}(\Omega), P)$ with $\Omega = \{\omega_1, \ldots, \omega_5\}$. The process X and the probability measure P are given by the following tree.

a) Determine the filtration $\mathcal{F} = (\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2)$ generated by X.

b) Determine the values a, b, c such that X is a martingale with respect to $\mathcal{F}$.

c) Now assume the values $a = 1, b = 2, c = 1$ and compute the Doob decomposition of X. To this end, determine the values of A_n and M_n for each node and each $n \in \{0, 1, 2\}$.

Now assume that a, b and c have the values calculated in b).

d) Show that $\tau = \inf\{n \in \mathbb{N} \mid X_n > 1.1\}$ is a stopping time w.r.t. to $\mathcal{F}$.

e) Find a filtration $\mathcal{G} = \{\mathcal{G}_0, \mathcal{G}_1, \mathcal{G}_2\}$ such that τ is not a stopping time w.r.t $\mathcal{G}$.