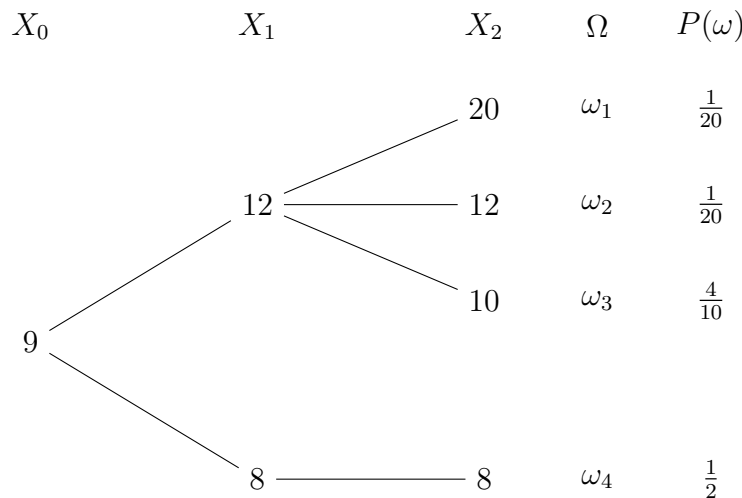


*Mathematical Finance: QF*

In-Tutorial exercises (for discussion on Tuesday, 19/12/2023)

**In-Tutorial Exercise 1.** Consider the process  $X = (X_0, X_1, X_2)$  given by the following tree:

Further, define the random variables

$$\tau_1 = 1 \quad \text{and} \quad \tau_2(\omega) = \begin{cases} 1, & \omega = \omega_1 \\ 2, & \omega = \omega_2 \\ 0, & \omega = \omega_3 \\ 1, & \omega = \omega_4 \end{cases}.$$

- Determine the natural filtration  $\mathcal{F} = (\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2)$  of the process  $X$ .
- Are  $\tau_1$  and  $\tau_2$  stopping times w.r.t. to  $\mathcal{F}$ ? Prove your claims.
- Determine the values of  $X_{\tau_1}$  and  $X_{\tau_2}$ .
- Calculate the stopped processes  $X^{\tau_1}$  and  $X^{\tau_2}$  and represent each in a separate tree.
- Calculate the Snell envelope of the process  $X$ .
- Determine the random variables  $\tau_f$  and  $\tau_s$  from Thm. 6.2. for  $n = 0$ . Further, calculate the condition expectations  $E(T_{\tau_f}|\mathcal{F}_i)$  and  $E(X_{\tau_s}|\mathcal{F}_i)$  for  $i = 1, 2$ .