Mathematical Finance
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Sheet QF07P

Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 12/12/2023)

In-Tutorial Exercise 1.

a) Let $\Omega = \{\omega_1, \dots, \omega_4\}$ and $\mathscr{F} = \mathcal{P}(\Omega)$. Further, let Q be a probability measure on $(\Omega, \mathscr{F})$ for which we know

$$Q(\{\omega_1\}) = 0.2, Q(\{\omega_2\}) = 0.6 \text{ and } Q(\{\omega_3\}) = Q(\{\omega_4\}) = 0.1.$$

Calculate $Q(A)$ of every element A in the domain of Q.

b) What can you infer from the previous exercise about the relationship between a probability measure on a discrete space and its probability mass function?

Now consider an one period market with price processes (S^0, S^1) given by the following tree.

$\begin{pmatrix} S_0^0 \\ S_0^1 \end{pmatrix}$	$\begin{pmatrix} S_1^0 \\ S_1^1 \end{pmatrix}$	ω	$P(\{\omega\})$	$Q(\{\omega\})$
	$\begin{pmatrix} 2.25 \\ 180 \end{pmatrix}$	ω_1	0.4	q_1
$\begin{pmatrix} 1.5 \\ 90 \end{pmatrix}$	$\begin{pmatrix} 2.25 \\ 90 \end{pmatrix}$	ω_2	0.2	q_2
	$\begin{pmatrix} 2.25 \\ 45 \end{pmatrix}$	ω_1	0.4	q_3

c) Give a characterisation of the set $\mathcal{M} = \{Q \mid Q \text{ EMM for } \hat{S}\}$ in $\mathbb{R}^3$ using part (b). Are there elements in $\mathcal{M}$ with $q_3 = 0, q_3 = 1$ or $q_3 = 0.5$?

d) Compute the upper and lower price of a put option $S_1^2 = (S_1^1 - 67.5)^+$.

e) Find a cheapest superhedge for this option.