

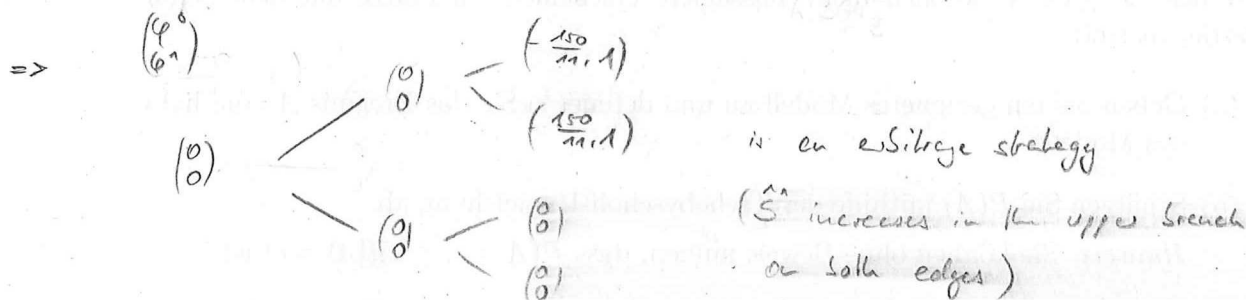
# Pos. 1

a) - enough to find an arbitrage in one step

- find a branch where both edges are going up in one component

→ put your money in this component & short the other one to get  $V_n = 0$  at the another node.

→ this holds only when considering the discounted process!



alt: solve  $\varphi_u^0(w)S_{u,n}^0(w) + \varphi_u^1(w)S_{u,n}^1(w) = 0$   
 $\varphi_u^0(w)S_{u,n}^0(w) + \varphi_u^1(w)S_{u,n}^1(w) \geq 0$  for  $w \in F$  where  $F$  denotes the  $w$  of one subbranch.

b) - 1FTAP: market is arbitrage-free  $\Leftrightarrow$  EMM  $Q$  exists

⇒ we only need to the martingale conditions on the branches which allow for arbitrage (why?)

- we obtain the system of equations:  $\begin{cases} xq_1 + 17q_2 = 15 \\ q_1 + q_2 = 1 \end{cases}$  ( $q_1, q_2$  denote the transition probabilities on the upper branch)

$$\Leftrightarrow \begin{cases} q_1 = -\frac{2}{x-17} \\ q_2 = \frac{x-15}{x-17} \end{cases}$$

- since  $Q$  has to be equivalent to  $P$  this implies  $q_1, q_2 \in (0,1)$ .

$$\Rightarrow 0 < \frac{-2}{x-17} < 1 \Rightarrow (x < 17) \text{ and } (x > 15)$$

$$\Rightarrow 0 < \frac{x-15}{x-17} < 1 \Rightarrow x < 17$$

$$x \in (15, 17)$$

otherwise there exists no EMM.

## POS. 2

a) Main idea: Find two replicating strategies  $\varphi$  and  $\psi$  such that  $V_N(\varphi)$  and  $V_N(\psi)$  are l.h.s and r.h.s of the assertion at time  $N$

$\Rightarrow$  law of one price then implies that the assertion is true for every  $u \leq N$ .

- define  $\varphi = (0, 0, 1)$  and  $\psi = (-\frac{O_0}{S_0^1}, 1, 0)$ . These strategies are self-financing (why?)

$$\Rightarrow V_N(\varphi) = S_N^2 \text{ and } V_N(\psi) = S_N^1 - O_0$$

- The assertion follows with law of one price.

b) -  $S_0^2 = \underset{\substack{\uparrow \\ \text{value of } S^1 \text{ at time } 0}}{S_0^1} - \frac{O_0}{(1+r)^N} \leftarrow \text{value of } O_0 \text{ at time } N$

- choose  $O_0 = (1+r)^N S_0^1 \leftarrow$  no profit at maturity of the future  $S^2$ .