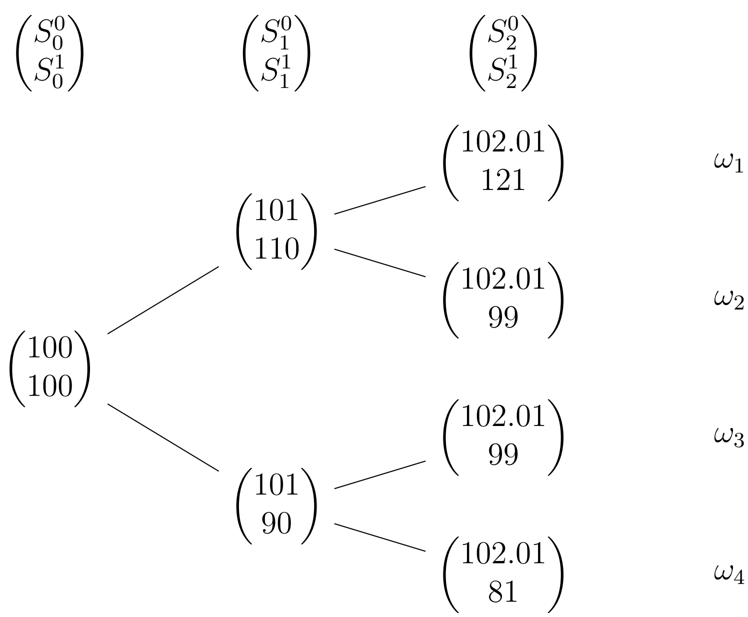
Mathematical Finance: QF

Exercises (for discussion on Monday, 22.01.2024)

Exercise 1. We consider the binomial model with time horizon $N = 2$ and parameters $S_0^0 = S_0^1 = 100$, $\widetilde{r} = 0.01$, $u = 1.1$, $d = 0.9$ and $p = 0.7$. Hence, the price process $S = (S^0, S^1)$ is given by the following tree:



(a) Determine the fair price processes S^2 und S^3 of a European and an American put option on S^1 with strike $K = 100$ and maturity $N = 2$.

(b) Indicate the nodes in the tree where it is reasonable for the holder of the American option to exercise it. Give economic reasons why they should do so.

Exercise 2. Consider a binomial model with time horizon $N = 2$ and parameters $S_0^0 = 1$, $S_0^1 = 100$, $r = 5\%$, $u = 1.1$, $d = 0.8$ and $p = 0.65$.

(a) Represent the processes S^0 and S^1 in a tree.

(b) We consider an American put option on S^1 with strike $K = 100$ and maturity $N = 2$. Compute the value process S^2 of the American put option and add it to the tree from part (a).

Exercise 3. In the setting of Exercise 2:

(a) Suppose, a bank sold the American option from 1 (b) to a customer. Assuming that the customer acts rationally find the perfect hedging strategy that allows the bank to eliminate the risk from this trade.

(b) Assume that the customer fails to exercise the American option early when this is optimal. How will the bank react? How much money does the customer lose by missing this opportunity?

Exercise 4. Suppose, you are playing the following game. You are allowed to throw a die, that has the face values $1, 2, 3, 4, 5, 6$, five times. After each of the first four throws you can either choose to receive the number on the die in Euros or you can choose to throw the die again. After the fifth throw you receive the number on the die of the fifth throw in Euros. Suppose you are playing this game optimally. What is your expected pay-off from playing this game? What is the optimal strategy?

Hint: Model the dice throws by independent random variables $X_1, \ldots, X_5$ and use the concept of the Snell-Envelope

Submission of the homework until: Thursday, 18.01.2024, 10.00 a.m. via OLAT.