Mathematical Finance Winter term 2023/2024
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Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 24.10.2023)

In-Tutorial Exercise 1. Decide whether the following propositions are true or false:

1. There exists a real number x such that $x^2 = 1$.
2. There exists a unique real number x such that $x^2 = 1$.
3. For each positive real number x it holds that: $x^2 = 1$ if and only if $x = 1$.
4. For each non-negative integer x it holds that: if $x < 1$, then $x = 0$.
5. For each integer n there exists an integer M such that: $n < M$.
6. There exists an integer M such that for each integer n: $n < M$.
7. The following statements are equivalent: 'If rains the floor gets wet'; 'If the floor is wet it has rained'
8. The following statements are equivalent: 'If I don't do it, someone else will do it'; 'If no one does it, I'll do it'
9. The statements 'It rains' and 'If it rains, all bikers get wet' implies 'A biker gets wet'

In-Tutorial Exercise 2.

1. Solve the following set of equations:

$$\begin{aligned} 2x + y - z &= 1 \\ x - y - z &= 3 \\ 2x + 2y + z &= 1 \end{aligned}$$

2. Determine all values for $c \in \mathbb{R}$ such that the following set of equations and inequalities has (1) one, (2) no or (3) infinitely many solutions:

$$\begin{aligned} 8x + 3y &= 6 \\ 14x + 6y &\geq 3 \\ x - 9y &\geq c \end{aligned}$$

In-Tutorial Exercise 3. Give a short definition or explanation of the following fundamental notions from probability theory. *Hint: the corresponding Wikipedia entries might serve as useful references.*

1. real-valued mapping, integer-valued mapping
2. probability space, probability measure
3. probability density function, cumulative distribution function
4. random variable, distribution of a random variable
5. independent events, independent random variables
6. Bernoulli and binomial distribution, Poisson distribution
7. uniform distribution, normal distribution, exponential distribution

In-Tutorial Exercise 4.

1. Consider a real-valued random variable X on the probability space $(\Omega, \mathcal{P}(\Omega), P)$ with $\Omega = \{\omega_1, \ldots, \omega_6\}$. Which of the following statements do make or do not make sense, respectively?
   (a) $P(\omega_2) = \frac{1}{6}$.
   (b) $X(\{\omega_2\}) = 2$.
   (c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for $A, B \subset \mathcal{P}(\Omega)$.
   (d) $\{X \leq 3\} = \{\omega \in \Omega \mid X(\omega) \leq 3\}$.
   (e) $\{X \leq 3\} = \{A \subset \Omega \mid X(A) \leq 3\}$.
2. Let Y be an additional real-valued random variable, and let $A, B \subset \mathbb{R}$. Find other formally equivalent representations of $P(X \in A, Y \in B)$.