

Mathematical Finance: QF

In-Tutorial exercises (for discussion on Tuesday, 06/02/2024)

In-Tutorial Exercise 1. We consider a market with a bond and an asset given by a CRR model with the parameters $S_0^0 = 1, S_0^1 = 100, \tilde{r} = 0.1, p = 0.6, u = 1.1, d = 0.9$ and $N = 2$. Within this market we want to maximize the expected utility of the terminal wealth with the utility function $u(x) = \log(x)$.

1. Look up the definition of a concave function and show (graphically) that u is concave.
2. Draw the tree representation of the market model.
3. Determine the quantity $\Delta \hat{X}_1$ from Theorem 9.3, i.e. calculate

$$\Delta \hat{X}_1 = \frac{1 + \Delta \tilde{X}_1}{1 + \tilde{r}} - 1,$$

where \tilde{X}_1 denote the increment variable of the CRR model (see p.63 in the lecture notes).

4. Show that $\gamma = \frac{101}{99}$ satisfies Eq. (9.2) from the lecture notes.
5. Calculate the discounted wealth process \hat{V} where we assume $\hat{V}_0 = c \in \mathbb{R}_{\geq 0}$. Also determine the undiscounted wealth process V .
6. Calculate the optimal strategy $\varphi = (\varphi^0, \varphi^1)$ for u and initial wealth c