Exam for the lecture

"Econometrics III / Bayesian Econometrics"

for students in the M.Sc. programmes
winter term 2023/2024

13.02.2024

Please use block letters:

Name: *Surname*		Vorname: *First Name*	
Studiengang: *Course of study*		Name der Universität (Bachelor): *Name of university (Bachelor's degree)*	
Matrikelnummer: *Student ID*		Hochschulstandort (Bachelor): *Place of university (Bachelor's degree)*	

Declaration:

PLEASE SIGN!!!
I hereby declare that I am able to be examined. ______________________________ Signature:

Preliminary remarks:

- Write down your name and enrolment/matriculation number on all paper sheets provided for answers by the examiner.
- To write down your answers, use only the paper provided by the examiner.

Result: (TO BE FILLED IN ONLY BY THE EXAMINER!)

Problem:	1	2	3	HA	S
Points earned:					
Grade:					

Kiel,

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(Prof. Dr. Kai Carstensen)

Prof. Dr. Kai Carstensen
Chair of Econometrics

Examination in Econometrics III / Bayesian Econometrics

(Winter Term 2023/24)

February 13, 2024 , 10:00 - 12:00

Preliminary remarks:

1. Write your name and enrollment (matriculation) number on every sheet of paper!
2. Don't use a pencil!
3. The exam is composed by 3 problems. Check your exam for completeness!
4. You have 60 minutes in total to answer the exam questions.

Good luck!

Problem 1 (21 credits)

Suppose you would like to model extreme events to better inform risk management strategies in the stock market. To this end, you assume that a random sample of extreme asset returns $y = (y_1, \ldots, y_N)'$ are characterized by a Weibull density with **unknown scale** $\theta > 0$ and known shape $k > 0$,

$$p(y_i|\theta, k) = \begin{cases} \frac{k}{\theta} y_i^{k-1} \exp\left(-\frac{y_i^k}{\theta}\right) & \text{if } y_i \geq 0 \\ 0 & \text{if } y_i < 0. \end{cases} \tag{1}$$

Suppose prior beliefs concerning the **unknown parameter** θ are represented by an inverse Gamma distribution with shape $\underline{\alpha} > 0$ and scale $\underline{\beta} > 0$:

$$p(\theta|\underline{\alpha}, \underline{\beta}) = \frac{\underline{\beta}^{\underline{\alpha}}}{\Gamma(\underline{\alpha})} \theta^{(-\underline{\alpha}-1)} \exp\left(-\frac{\underline{\beta}}{\theta}\right), \tag{2}$$

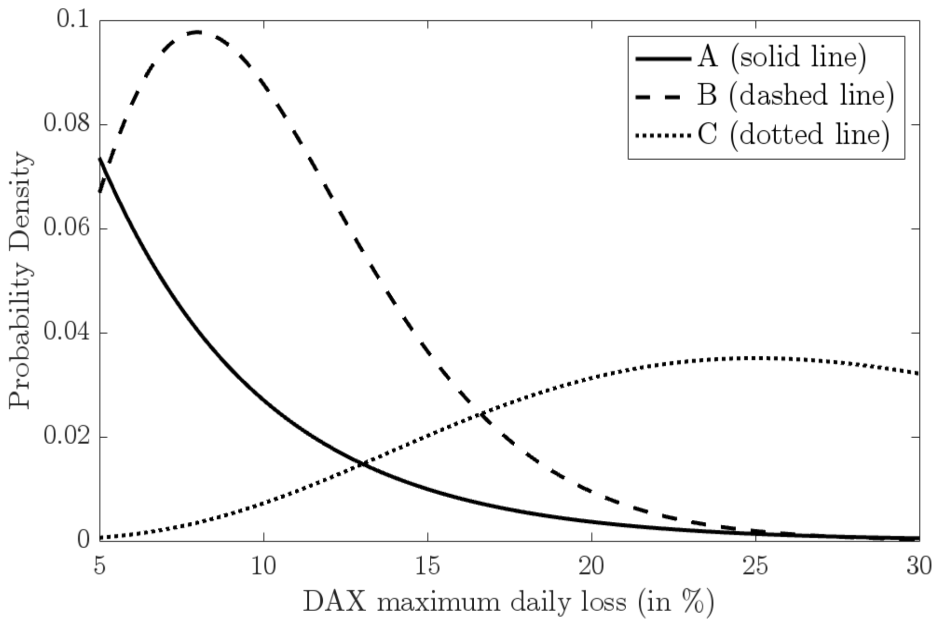
where $\Gamma(\cdot)$ denotes the Gamma function.

(a) **(5P)** State the kernel of the prior and the likelihood.

(b) **(5P)** Find the posterior distribution.

(c) **(4P)** Is the inverse Gamma distribution a conjugate prior? Briefly explain! Also answer whether you can apply the "fictitious prior sample interpretation" in this case.

(d) **(7P)** Based on your prior beliefs about θ (what you have already experienced from past crisis episodes), which of the following prior specifications would you choose? Defend your choice!
Hint: the mean of the Weibull distribution is hereby directly proportional to the scale parameter θ, and hence is associated with the maximum loss within a period.



Problem 2 (24 credits)

Consider the multiple linear regression model with k regressors and N observations:

$$y = X\beta + \varepsilon \qquad\qquad \varepsilon \overset{iid}{\sim} \mathcal{N}(0, h^{-1}I)$$

To impose a strong and flexible variable selection in a setting where k is large relative to N, assume a non-conjugate independent Laplace-Gamma prior with

$$\beta \sim \text{Laplace}(\underline{\beta}, \underline{\tau}) \quad \Rightarrow \quad p(\beta) \propto \exp\left[-\frac{|\beta - \underline{\beta}|}{\underline{\tau}}\right]$$
$$h \sim \text{Gamma}(\underline{s}^{-2}, \underline{\nu}) \quad \Rightarrow \quad p(h) \propto h^{\frac{\underline{\nu}-2}{2}} \exp\left[-\frac{h\,\underline{\nu}\underline{s}^2}{2}\right]$$

The joint posterior kernel then yields

$$p(\beta, h|y) \;\;\propto\;\; h^{\frac{N+\underline{\nu}-2}{2}} \exp\left[-\frac{h}{2}(y - X\beta)'(y - X\beta) - \frac{|\beta - \underline{\beta}|}{\underline{\tau}} - \frac{h\,\underline{\nu}\underline{s}^2}{2}\right]$$

(a) **(6P)** Derive the conditional posterior of h by explicitly stating its posterior parameters.

(b) **(10P)** Write a pseudo code that applies the Metropolis-within-Gibbs algorithm to find the posterior means of h and β. This implies setting up a Gibbs sampler to draw from $p(h|y, \beta)$ and a Metropolis-Hastings block (within the Gibbs) to draw from $p(\beta|y, h)$.
Note: you don't need to find a proposal density for $p(\beta|y, h)$; just assume that a good proposal density $q^*(\beta|y, h)$ is available.

(c) **(8P)** How would you check for efficiency of the Metropolis-Hastings algorithm implemented in (b)? Which strategy one can use for tuning the Metropolis-Hastings algorithm to improve its efficiency and convergence properties?

Problem 3 (15 credits)

Consider the M-dimensional VAR(p) model

$$y_t = a_0 + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \varepsilon_t, \qquad t = 1, \cdots, T. \tag{3}$$

with $\varepsilon_t \sim N(0, \Sigma)$ and $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$.

1. **(7P)** Explain the key elements of the Minnesota prior that makes it appealing for Big Data applications and still allowing for analytical posterior results.
2. **(8P)** Suppose you would like to estimate a VAR(2) model for the German inflation rate and unemployment rate. Based on the Minnesota prior choice with $\Sigma = \hat{\Sigma}$ and $\alpha \sim N(\underline{\alpha}; \underline{V}_M)$, explain how you would set up the prior mean of A_1 and A_2. Finally, explain which elements of the VAR model determine the amount of shrinkage imposed to the diagonal and off-diagonal elements of A_1 and A_2.