

# Question concerning the course

## *Risk Management*

Jan Kallsen

December 6, 2024

### Abstract

The questions below may be helpful for understanding the lecture notes of the course *Risk Management* and in particular in preparing for the oral exam. Some have a clear simple answer, some do not. Some have a short answer, others a longer one. Not all of them are equally relevant. As a rough guide, red questions are more important than blue ones, which in turn are more relevant than green questions. As a rough rule, technical proofs are more relevant for (financial) mathematicians; I omitted questions of this kind in the list below.

1. What are market, credit, and operational risk?
2. What is a loss, the loss distribution, and the loss operator in the context of risk management theory?
3. What are linearised losses and why do we consider them?
4. What are natural risk factors and what are the corresponding loss operator resp. linearised loss operator in the context of a stock portfolio?
5. What are the main steps in applying risk management?
6. What can go wrong in these steps?
7. What is the goal and the meaning of a risk measure?
8. What are examples of risk measures? What are their advantages and disadvantages?
9. What is the Value-at-Risk (VaR)? Can you compute the VaR at level 99% of a loss that is exponentially distributed with parameter  $\lambda$ ?
10. What are quantile functions and generalized inverses? How are they related to the Value-at-Risk?
11. Do we always have  $F^-(F(x)) = x$  resp.  $F(F^-(x)) = x$  for all  $x$ ? Under what conditions does either of these properties hold? Can you draw a picture illustrating your claim?
12. Is the VaR convex? Why does this matter?

13. Can you compute the VaR of a Gaussian or a lognormal random variable?
14. What does backtesting mean? How can you backtest whether your VaR predictions are appropriate?
15. What is the expected shortfall (ES)? Can you compute the ES at level 99% of a loss that is exponentially distributed with parameter  $\lambda$ ?
16. Which representations are useful for computing the ES of concrete distributions?
17. Do you know how to compute the VaR of a Gaussian or a lognormal random variable?
18. Why does the definition of the ES for general (possibly discontinuous) distributions differ from the simpler one for continuous laws?
19. What are convex and coherent risk measures? Why do we care?
20. What are examples and counterexamples of convex or coherent risk measures?
21. What does *generalized scenarios* mean in the context of risk measures?
22. Give examples of standard basic models in risk management.
23. What are standard approaches for computing VaR and ES based on given data? What are their advantages and disadvantages?
24. How does the variance-covariance method work? What are its advantages and disadvantages?
25. How does historical simulation work? What are its advantages and disadvantages?
26. What are the empirical law, the empirical distribution function, the empirical quantile, and the empirical expected shortfall?
27. How does one compute the empirical quantile?
28. How are the empirical distribution function, the empirical quantile, the empirical expected shortfall related to the cumulative distribution function, the quantile function, and the empirical expected shortfall of a random variable?
29. How does maximum likelihood estimation work? Why is it needed in the context of risk management?
30. How does one compute the maximum likelihood estimator of the parameter of an exponential distribution based on  $n$  iid observations from this law?
31. What does the log-likelihood in the case of (possibly) dependent data look like?
32. Why and how do we apply Monte Carlo methods in risk management?
33. What is extreme value theory about? Why does it matter in risk management?

34. What does heavy tails mean? How can we get an idea of whether heavy tails are present in given data?
35. What is a qq plot? How does it work? What does the plot tell us?
36. What does it mean if the qq plot shows a straight line which does not coincide with the main diagonal?
37. What are regularly varying functions and random variables?
38. Are exponentially distributed random variables regularly varying? Why?
39. Why does it matter whether losses in risk management are regularly varying or not?
40. What is the Hill estimator? How does it work? Why do we consider it in risk management?
41. What is the Hill plot? What is it needed for?
42. How do we choose the threshold parameter in the Hill estimator?
43. How do we estimate the extremal cdf based on the Hill estimator?
44. How do we estimate VaR and ES under the assumption of regularly varying losses?
45. What is the peaks over threshold (POT) method? How does it work? Why does it matter in risk management?
46. What is the excess distribution function? Why do we consider it?
47. Where and why do generalised Pareto distributions occur in risk management? What are they? How can one estimate their parameters from data?
48. What are the mean excess function, the empirical mean excess function, the mean excess plot? Why do we care?
49. How do we choose the threshold parameter in the peaks over threshold method?
50. How do we estimate the extremal cdf based on the POT method?
51. How do we estimate VaR and ES based on the peaks over threshold method?
52. Why do we consider multivariate distributions in risk management?
53. What are joint and marginal distribution functions?
54. How do we compute the pdf from the distribution function in the multivariate case?
55. How can we check for independence based on the joint cdf, the joint pdf, or the joint characteristic function?
56. What is the definition of the multivariate normal distributions? What are their properties? What are the advantages and disadvantages of applying the multivariate normal distribution to risk factor changes?

57. What does comonotone and countermonotone mean?
58. Which notions of correlation do you know? What are their properties?
59. Why does one consider Kendall's  $\tau$  and Spearman's  $\rho$  in risk management?
60. How can one estimate the various correlation coefficients from iid observations? How can the estimators be motivated?
61. What does tail dependence mean? How can it be measured? Why does it matter in risk management?
62. What are multivariate normal mixtures? Can you give an example?
63. What is the multivariate  $t$ -distribution? Is it a multivariate normal mixture? Is it spherical? Is it elliptic?
64. What are spherical distributions? Do you know examples?
65. What are elliptical distributions? Do you know examples? What are their properties? Why do we care about them in risk management?
66. Do elliptical distributions,  $t$ -distributions, Gaussian distributions allow for tail dependence?
67. What are copulas? What are they needed for? Why do we care about them in risk management?
68. What does Sklar's theorem state?
69. What is the copula of a random vector? Is it unique?
70. Do you know examples of copulas?
71. What are Gaussian,  $t$ -, elliptical, Gumbel-, Clayton-, resp. Archimedean copulas?
72. How are correlation coefficients resp. tail dependence coefficients related to copulas? Are they determined by the copula?
73. What are properties, advantages, and disadvantages of Gaussian,  $t$ -, elliptical, Gumbel-, Clayton-, Archimedean copulas from the point of view of risk management?
74. How can one simulate Gaussian and  $t$ -copulas?
75. What are completely monotone functions? Why do we care?
76. How can I decide which copula family to choose in risk management?
77. How do we estimate its parameters based on an iid sample of random vectors having the copula?
78. How could I apply the notions and methods of multivariate distributions in the context of risk management?

79. How does plug-in estimation work? Can you give examples?
80. Can you justify why the estimation with historical simulation should give reasonable results?
81. Can you justify why the Monte-Carlo method should give reasonable results if expected values are to be computed?