T-Exercise 22 QF

→ Laplace Transform $\tilde{f}(z) = \int_{-\infty}^{\infty} f(x) e^{-zx}\, dx$

Stock $S(T) = e^{X(t)}$ (Script p 49)

Payoff: $f(x) = (e^x - K)^+ e^x$ (like Script p 57)

$$\Rightarrow \tilde{f}(z) = \int_{-\infty}^{\infty} (e^x - K)^+ e^x \cdot e^{-zx}\, dx = \int_{\log(K)}^{\infty} (e^x - K) e^{x(1-z)}\, dx$$

$$= \int_{\log(K)}^{\infty} e^{x(2-z)} - K e^{x(1-z)}\, dx$$

$$= \left[\frac{1}{2-z} e^{x(2-z)} - \frac{1}{1-z} K e^{x(1-z)}\right]_{\log(K)}^{\infty}$$

→ for all z with $R = Re(z) > 2$

$$= 0 - 0 - \frac{1}{2-z} e^{\log(K)(2-z)} + \frac{1}{1-z} K e^{\log(K)(1-z)}$$

$$= -\frac{1}{2-z} K^{(2-z)} + \frac{1}{1-z} K^{2-z}$$

$$= K^{2-z}\left(\frac{1}{1-z} - \frac{1}{2-z}\right)$$

→ insert $\tilde{f}(z)$ in equation 4.7 from script

$$V(t) = \frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} Re\Big(K^{2-(R+iu)}\left(\frac{1}{1-(R+iu)} - \frac{1}{2-(R+iu)}\right)$$

$$\cdot \exp\big(i(u-iR)(\log(e^{X(t)}) + r(T-t)) - (i(u-iR) + (u-iR)^2)\frac{\sigma^2}{2}(T-t)\big)\, du$$