Christian-Albrechts-Universität zu Kiel

SS 2024

Mathematisches Seminar

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Sheet 04





T-Exercise 13 (Exchange rates) (4 points)

Assume that the exchange rate D(t) of the US-Dollar in Euro at time t > 0 follows the equation

$$dD(t) = D(t)\mu dt + D(t)\sigma dW(t)$$

with D(0) > 0 and μ , $\sigma \in \mathbb{R}$. Hence, the exchange rate of the Euro in US-Dollar at time t > 0 is given by $E(t) \coloneqq \frac{1}{D(t)}$. Represent the process E as Itō process, i.e. in the form

$$dE(t) = \dots dt + \dots dW(t).$$

Interpret your result economically in the case $\mu = \frac{1}{2}\sigma^2$.

T-Exercise 14 (Vasiček model for interest rates) (4 points)

Let W be a standard Brownian motion and let x, κ, λ and σ real numbers. Show as in the lecture that the process X with

$$dX(t) := (\kappa - \lambda X(t))dt + \sigma dW(t)$$

and X(0) = x solves the equation

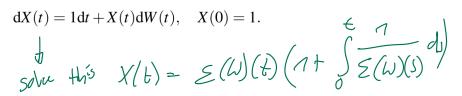
$$X(t) = xe^{-\lambda t} + \frac{\kappa}{\lambda}(1 - e^{-\lambda t}) + \int_0^t e^{-\lambda(t-s)} \sigma dW(s).$$

T-Exercise 15 (4 points) shuhs Burn

Let W be a standard Brownian motion. Show that the process

$$X(t) := \mathscr{E}(W)(t) \left(1 + \int_0^t \frac{1}{\mathscr{E}(W)(s)} ds\right), \quad t \in \mathbb{R}_+,$$

solves the stochastic differential equation





4. Strp: Ply empty into 160 famb $dE(t) = \frac{1}{D(t)^2} \left(ND(t) dt + \sigma D(t) dW(t) \right) + \frac{1}{2} \frac{2}{D(t)^3} \cdot \left(\sigma^2 D(t)^2 dt \right)$ $= \frac{1}{D(t)^2} \cdot \sigma(t) dW(t) \cdot D(t)$ (2) $\Rightarrow 2$ $\Rightarrow 2$ 3. Sep: Pet ength togeth $d \in (t) = - \underbrace{\mathcal{U}}_{D(t)} \left(\underbrace{dt} \right) - \underbrace{\sigma}_{D(t)} \left(\underbrace{ll}_{D(t)} \right) + \underbrace{\sigma}_{D(t)}^{2} \underbrace{\sigma}_{D(t)} \left(\underbrace{dl}_{D(t)} \right) - \underbrace{\sigma}_{D(t)}^{2} \left(\underbrace{dl}_{D(t)} \right) - \underbrace{\sigma}_{D(t)}^{2} \left(\underbrace{dl}_{D(t)} \right)$ $= \frac{\sigma^2 - N}{D(t)} \left(\frac{1}{dt} \right) - \frac{\sigma}{D(t)} dW(t) - \frac{1}{D(t)^2} \sigma(t) dW(t) / \frac{1}{D(t)} \frac{1}{2(t)} \frac{1}{2(t)}$ $d\xi(\xi) = (\sigma^2 \mu) \xi(\xi) d\xi - \sigma \xi(\xi) dW(\xi) - \xi(\xi)^2 \sigma(\xi) dW(\xi)$ 6 Stp: Interpetation: with $\mu = \frac{1}{2}\sigma^2$ the determination per of our Ho former would raid out. This mans on avery on the lig run are don't eacher. Only short her (rulinearess)