Christian-Albrechts-Universität zu Kiel **SS 2024**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Henrik Valett, Fan Yu, Ivo Richert, Anton Schellin Sheet 04

Computational Finance

Exercises for all participants

T-Exercise 13 (Exchange rates) (4 points)
Assume that the exchange rate $D(t)$ of the US-Dollar in Euro at time $t > 0$ follows the equation

$$dD(t) = D(t)\mu \mathrm{d}t + D(t)\sigma \mathrm{d}W(t)$$

with $D(0) > 0$ and μ, $\sigma \in \mathbb{R}$. Hence, the exchange rate of the Euro in US-Dollar at time $t > 0$ is given by $E(t) := \frac{1}{D(t)}$. Represent the process E as Itō process, i.e. in the form

$$dE(t) = \ldots \mathrm{d}t + \ldots \mathrm{d}W(t).$$

Interpret your result economically in the case $\mu = \frac{1}{2}\sigma^2$.

T-Exercise 14 (Vasiček model for interest rates) (4 points)
Let W be a standard Brownian motion and let x, κ, λ and σ real numbers. Show as in the lecture that the process X with

$$dX(t) := (\kappa - \lambda X(t))dt + \sigma dW(t)$$

and $X(0) = x$ solves the equation

$$X(t) = xe^{-\lambda t} + \frac{\kappa}{\lambda}(1 - e^{-\lambda t}) + \int_0^t e^{-\lambda(t-s)} \sigma dW(s).$$

T-Exercise 15 (4 points)
Let W be a standard Brownian motion. Show that the process

$$X(t) := \mathscr{E}(W)(t) \left(1 + \int_0^t \frac{1}{\mathscr{E}(W)(s)} \mathrm{d}s\right), \quad t \in \mathbb{R}_+,$$

solves the stochastic differential equation

$$\mathrm{d}X(t) = 1\mathrm{d}t + X(t)\mathrm{d}W(t), \quad X(0) = 1.$$

T-Exercise 16 (for math only) (4 points)
Let W be a standard Brownian motion and $T > 0$. Assume that the underlying filtration $(\mathscr{F}_t)_{t\geq 0}$ is generated by W. Let μ be an adapted process and Y an $\mathscr{F}_T$-measurable random variable. Show that there exist $x \in \mathbb{R}$ and a process H such that the process

$$X = x + \int_0^{\cdot} \mu(s)ds + \int_0^{\cdot} H(s)dW(s)$$

fulfills

$$X(T) = Y.$$

Determine x and H explicitly for $\mu = 0$ and

(a) $Y = (W(T))^2$,

(b) $Y = \int_0^T W(s)\mathrm{d}s$ and

(c) $Y = (W(T))^3$,

respectively.
Hint: Martingale representation theorem.

Please include your name(s) as comment in the beginning of the file.
Do not forget to include comments in your Python-programs.
Submit until: Thu, 30.05.2023, 12:00