

## T-Exercise 17

Josef Fella, Marvin Lohd

→ Combine  $S_1$  and  $S_2$

$$\begin{aligned} d(S_1(t) + S_2(t)) &= S_1(t)(4dt + dW_1(t) - dW_2(t)) \\ &\quad + S_2(t)(1dt - dW_1(t) + dW_2(t)) \\ &= S_1(t)4dt + S_2(t)1dt + S_1(t)(dW_1(t) - dW_2(t)) \\ &\quad + S_2(t)(-dW_1(t) + dW_2(t)) \end{aligned}$$

→ Set  $S_1(t) = S_2(t) \Rightarrow$  terms with  $dW_1(t)$  and  $dW_2(t)$  cancel out

$$d(S_1(t) + S_2(t)) = S_1(t)4dt + S_2(t)1dt$$

$\Rightarrow$  change of value is deterministic

→ Arbitrage-Strategy: Let  $S_1(0) = S_2(0) = S$

$\Rightarrow$  portfolio at time  $t=0$ :

$$\varphi = \begin{cases} -2S \\ S \\ S \end{cases} \Rightarrow V_0 = 0$$

→ at time  $t > 0$

$$\varphi = \begin{cases} -2S \\ S \\ S \end{cases} \Rightarrow V_{t>0} > 0, \text{ because the "debt" from } S_0 \text{ stays constant because } S_0 \text{ is fixed and the value of holding } S_1 \text{ and } S_2 \text{ increase deterministically as shown above.}$$

$\Rightarrow$  We start with 0 capital ( $V_0=0$ ) and are guaranteed to make money ( $V_{t>0} > 0$ , with  $P(V_{t>0} > 0) > 0$ ) because the value of the stocks increase deterministically, which constitutes arbitrage