

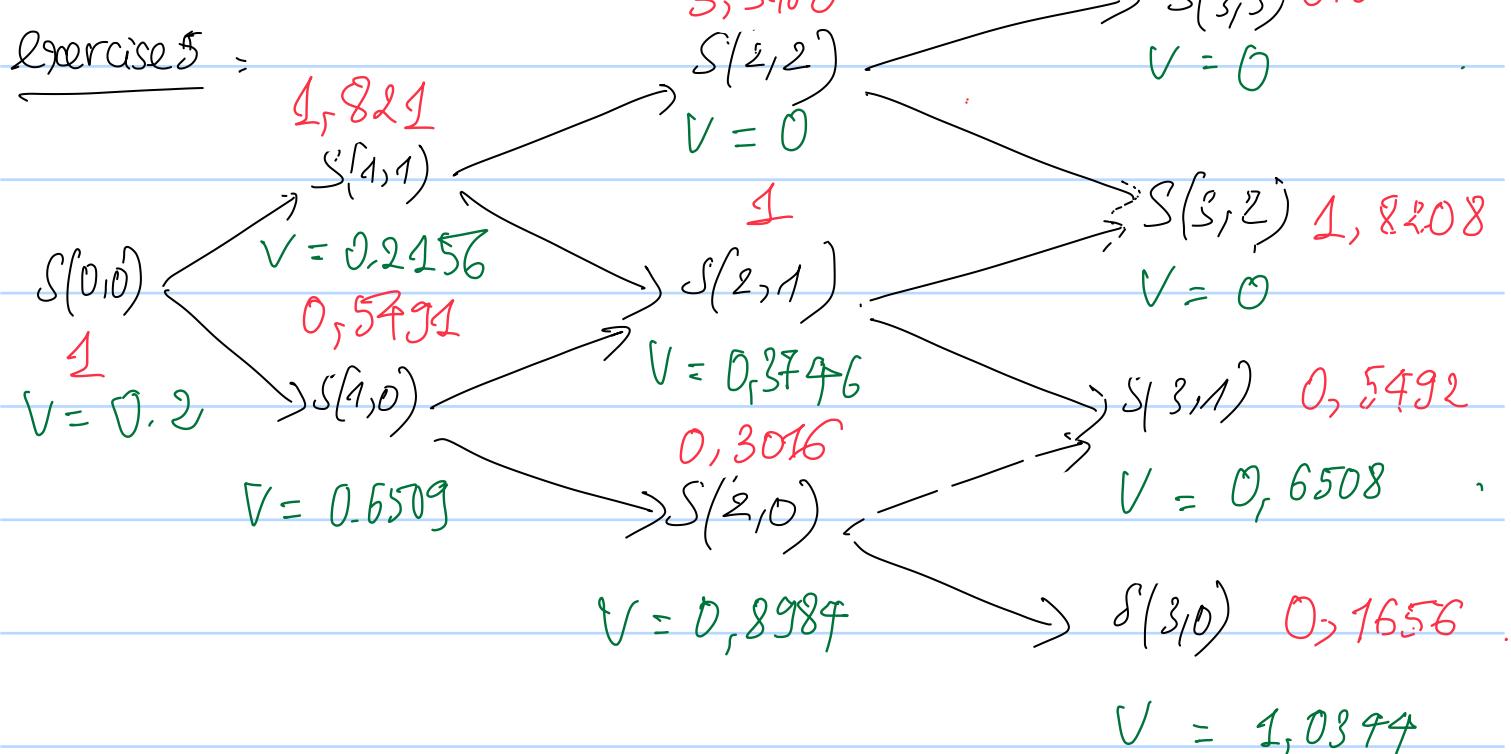
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$$\Delta t = T/n = \frac{1}{3} = 1$$

$$\alpha = e^{r \cdot \Delta t} = e^{0.05} = 1.0512$$

$$\beta = \frac{1}{2} \left( \frac{1}{\alpha} + e^{(r+\alpha^2) \cdot 1} \right)^{1.0512} = 1.1851$$

$$u = \beta + \sqrt{\beta^2 - 1} = 1.821 ; \quad d = \frac{1}{\beta} = 0.5491$$

$$q = \left( e^{r \cdot \Delta t} - d \right) / (u - d) = 0.5948$$

$$S(1,1) = 1,821$$

$$S(2,0) = 0,8987$$

$$S(1,0) = 0,5491$$

$$S(3,2) = 1,8208$$

$$S(2,1) = 1$$

$$S(3,1) = 0,5492$$

$$S(2,0) = 1$$

$$S(3,0) = 0,1656$$

$$V(3,3) = \max \left\{ (k - S(3,3)), 0 \right\} = 0$$

$$V(3,2) = \max \left\{ (k - S(3,2)), 0 \right\}$$

$$= \max \{ 1,2 - 1,8208, 0 \} = 0.$$

$$\begin{aligned} V(3,1) &= \max \{ K - S(3,1), 0 \} \\ &= \max \{ 1,2 - 0,5492, 0 \} = 0,6508. \end{aligned}$$

$$\begin{aligned} V(3,0) &= \max \{ K - S(3,0), 0 \} \\ &= \max \{ 1,2 - 0,1656, 0 \} = 1,0344. \end{aligned}$$

$$\begin{aligned} V(2,2) &= \max \left\{ \max [K - S_{j,i}]^+, e^{-r \cdot \Delta t} (q \cdot V_{j+1,i+1} + (1-q) V_{j,i+1}) \right\} \\ &= \max \left\{ [1,2 - 3,8160, 0]; e^{-0.05} \cdot 0 \right\} = 0 \end{aligned}$$

$$\begin{aligned} V(2,1) &= \max \left\{ [K - S(2,1)]^+, e^{-0.05} (q \cdot V(3,2) + (1-q) V(3,1)) \right\} \\ &= \max \left\{ [1,2 - 1,0]; e^{-0.05} \cdot (0.3998 \cdot 0 + 0.6052 \cdot 0.6508) \right\} \\ &= \max \{ 0,2 ; 0,3746 \} = 0,3746 \end{aligned}$$

$$\begin{aligned} V(2,0) &= \max \left\{ [K - S(2,0)]^+, e^{-r \cdot \Delta t} (q \cdot V(3,1) + (1-q) V(3,0)) \right\} \\ &= \max \left\{ \max (1,2 - 0,3016, 0); 0.9512 \left( 0.3998 \times 0.6508 + 0.6052 \times 1,0344 \right) \right\} \\ &= \max \{ 0,8984; 0,8398 \} \\ &= 0,8984 \end{aligned}$$

$$\begin{aligned} V(1,1) &= \max \left\{ (K - S(1,1))^+, e^{-r \cdot \Delta t} (q \cdot V(2,2) + (1-q) V(2,1)) \right\} \\ &= \max \left\{ \max (1,2 - 1,821, 0); 0.9512 \times (0.3998 \times 0 + 0.6052 \times 0.3746) \right\} \\ &= \max \{ -0,621; 0,2156 \} \\ &= 0,2156 \end{aligned}$$

$$\begin{aligned}
 V(1,0) &= \max \left\{ (K - S(1,0)) ; e^{-r\Delta t} (q \cdot V(2,1) + (1-q) \cdot V(2,0)) \right\} \\
 &= \max \left\{ (1,2 - 0,5492, 0) ; 0,9512 \times (0,3848 + 0,8776 \right. \\
 &\quad \left. + 0,6052 \times 0,3076) \right\} \\
 &= \max \left\{ 0,6509 ; 0,3143 \right\} \\
 &= 0,6509
 \end{aligned}$$

$$\begin{aligned}
 V(0,0) &= \max \left\{ (K - S(0,0)) ; e^{-r\Delta t} (q \cdot V(1,1) + (1-q) \cdot V(1,0)) \right\} \\
 &= \max \left\{ \max (1,2 - 1,0) ; 0,9512 (0,3848 \times 0,2156 \right. \\
 &\quad \left. + 0,6052 \times 0,6509) \right\} \\
 &= \max \left\{ 0,2 ; 0,0808 \right\} = 0,2
 \end{aligned}$$

$$b) Q_1(t_i) = \frac{V(S(t_{i-1})u, t_i) - V(S(t_{i-1})d, t_i)}{S(t_{i-1})(u-d)}$$

$$Q_0(t_i) = \frac{V(S(t_{i+1}, t_{i-1}) - Q_1 \times S(t_{i-1}))}{e^{rt}}$$

$$\begin{aligned}
 t_i &= 3 \quad Q_1(1) = \frac{0 - 0}{0(u-d)} = 0 \\
 Q_1(1) &= 0 - 0 = 0
 \end{aligned}$$

$$\varphi_1(2) = \frac{0 - 0,6508}{e^{r \cdot 2}} = -0,4024$$

$u - d$

$$\varphi_0(2) = \frac{0,3746 - (-0,4024 \times 1)}{e^{0,05 \cdot 2}}$$

$$= 0,2858$$

$$\varphi_0(3) = \frac{0,6508 - 1,0244}{0,3016 (u - d)} = -1$$

$$\varphi_1(3) = \frac{0,8984 - (-0,3016 \times 0,3016)}{e^{r \cdot T}}$$

$$= 0,8952$$

$$t = 2 : \quad \varphi_1(1) = \frac{0 - 0,3746}{1,1821 (u - d)} = -0,2646$$

$$\varphi_0(1) = \frac{0,2156 - (-0,8616 \times 1,821)}{e^{0,05 \cdot 1}}$$

$$= 0,6582$$

$$\varphi_1(2) = \frac{0,3746 - 0,8984}{0,54951 (u - d)} = -0,75$$

$$\varphi_0(2) = \frac{0.6509 - (-0.75 \times 0.5931)}{e^{0.05}}$$

$$= 2,011$$

$t = 1$

$$\varphi_1(1) = \frac{0.2156 - 0.6509}{1.821 - 0.5931} = -0.3422$$

$$\varphi_0(1) = \frac{0.2 - (-0.3422 \cdot 1)}{e^0}$$

$$= 0,5422$$

$t = 0$

$$\varphi_1(t_1) = \varphi_1(t_0)$$

$$\varphi_0(t_1) = \varphi_0(t_0)$$

