
Deformation & Skinning

形变与蒙皮

北京大学 前沿计算研究中心

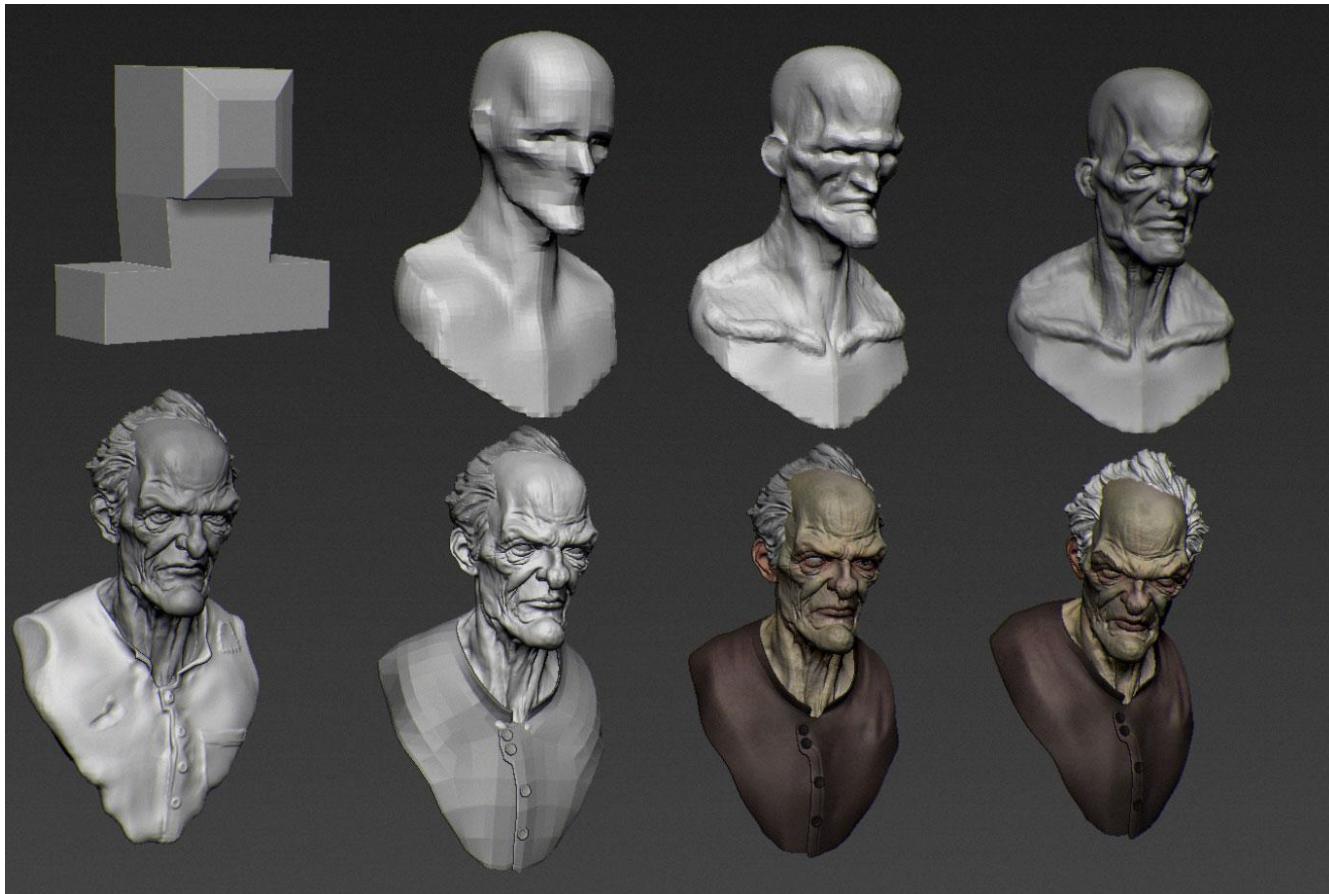
刘利斌

本节内容

- 几何形状的编辑
 - 自由形变 (Free-form Deformation)
 - 基于优化的形变
- 蒙皮形变 (Skinning Deformation)
 - 线性蒙皮
 - Linear-Blend Skinning (LBS)
 - Multi-linear Skinning
 - 非线性蒙皮
 - Dual-quaternion Skinning (DQS)
 - 基于样例的蒙皮
 - Blendshapes
 - 人脸动画

几何形变与编辑

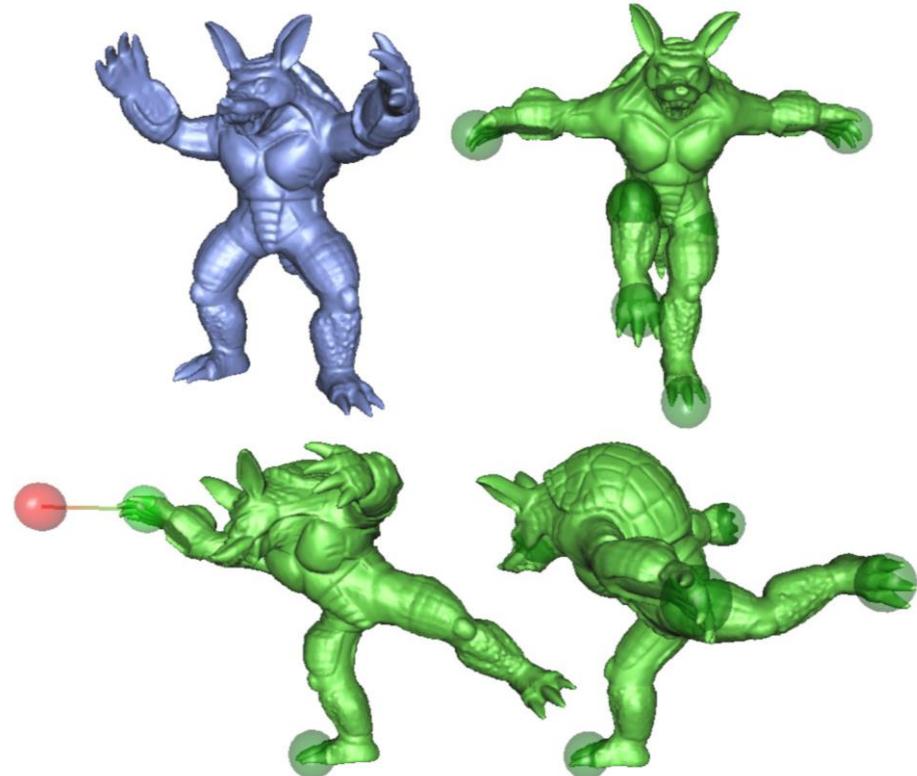
- 直接编辑几何模型顶点/控制点



<https://khanhha.github.io/portfolio/skeleton-to-mesh-conversion/>

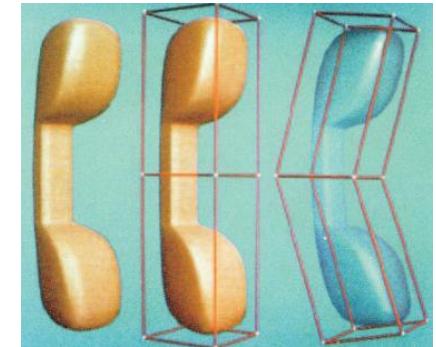
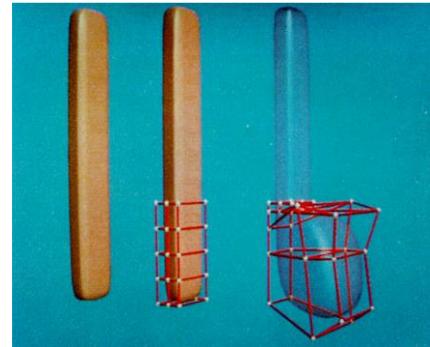
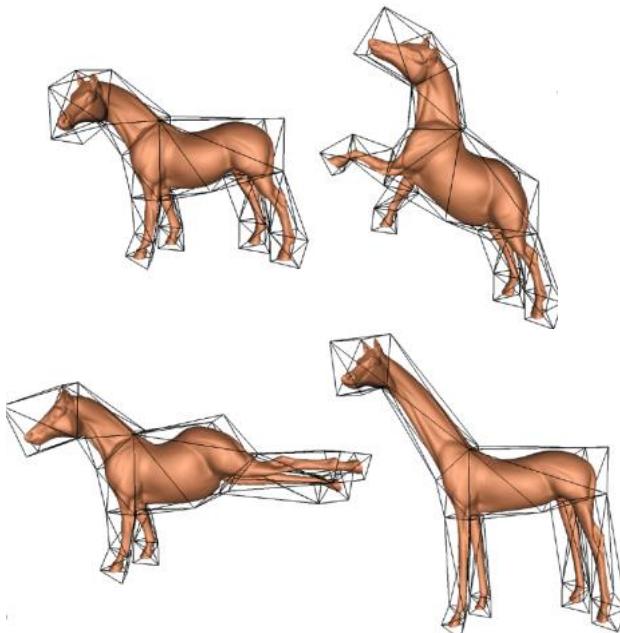
几何形变与编辑

- 编辑修改几何形状，同时保留几何模型细节
- 一般通过整体性的改变几何模型的参数来实现
 - 网格 -> 顶点
 - NURBS -> 控制点
 -



自由变形与Cage-based Deformation

- “扭曲”模型附近的空间来实现变形
 - 将几何模型嵌入到简单网格中
 - 改变简单网格的形状间接的改变模型的形状



Thomas W. Sederberg and Scott R. Parry. 1986. *Free-form deformation of solid geometric models*. In Proceedings of the 13th annual conference on Computer graphics and interactive techniques (SIGGRAPH '86).

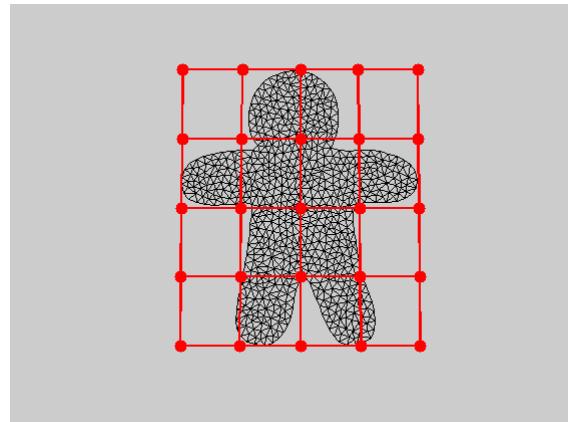
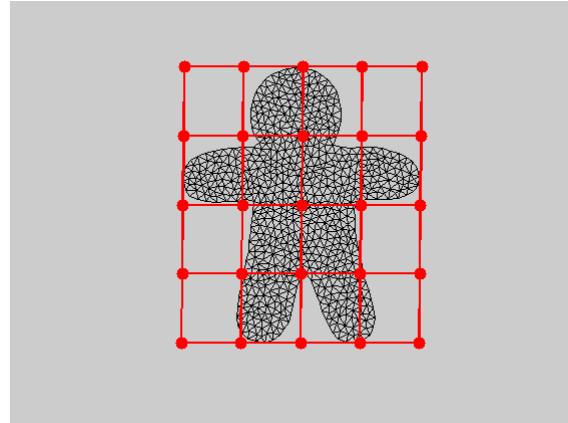
自由变形

- 如何定义嵌入关系?
 - 嵌入的精细网格 $\{x_i\}$
 - 简单网格 $\{p_j\}$

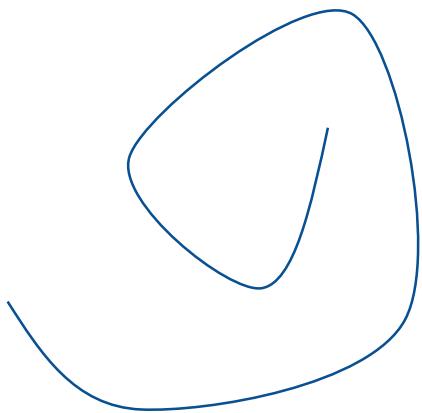
嵌入关系：

$$x_i = \sum_j \omega_{ij} p_j$$

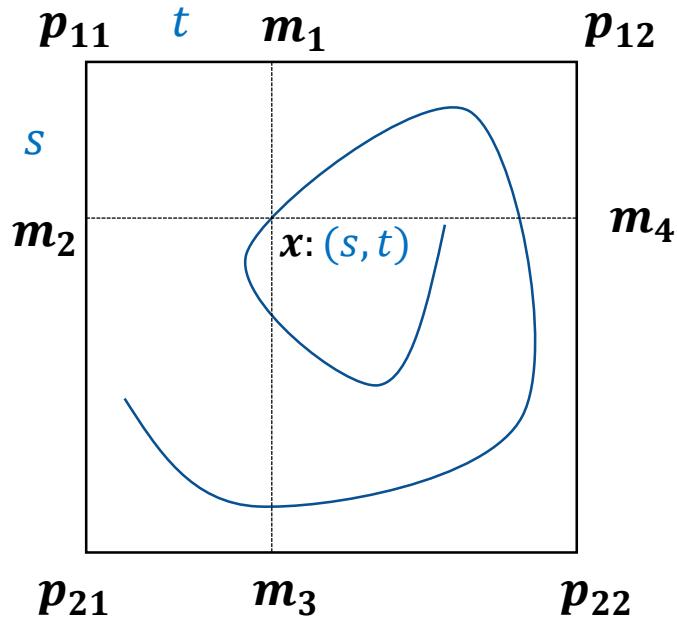
- 如何确定 ω_{ij} ??



线性插值

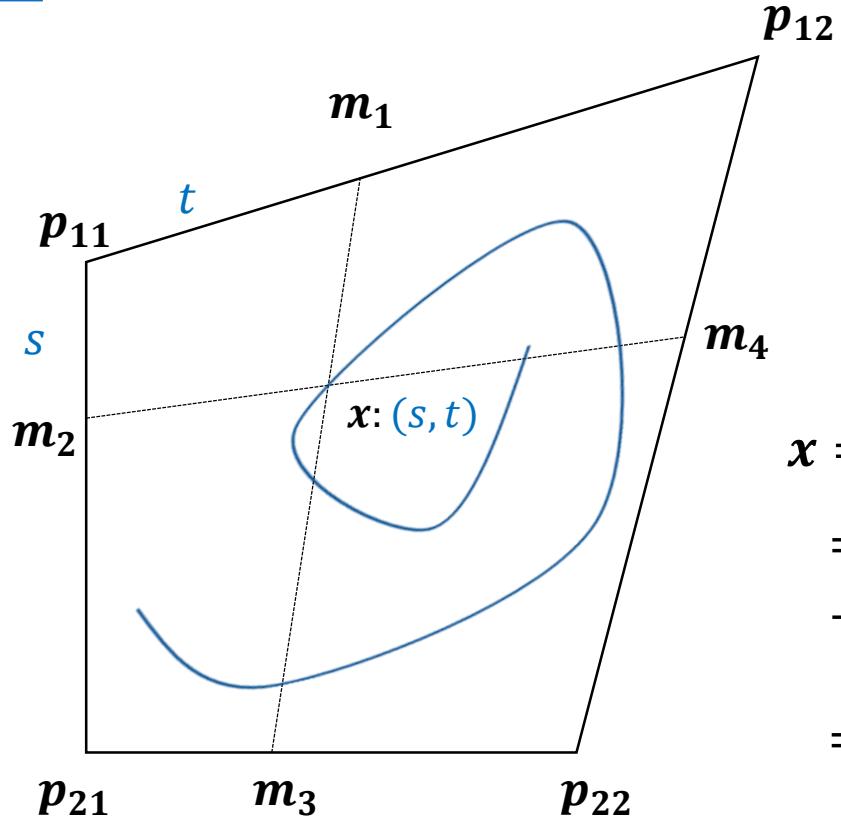


线性插值



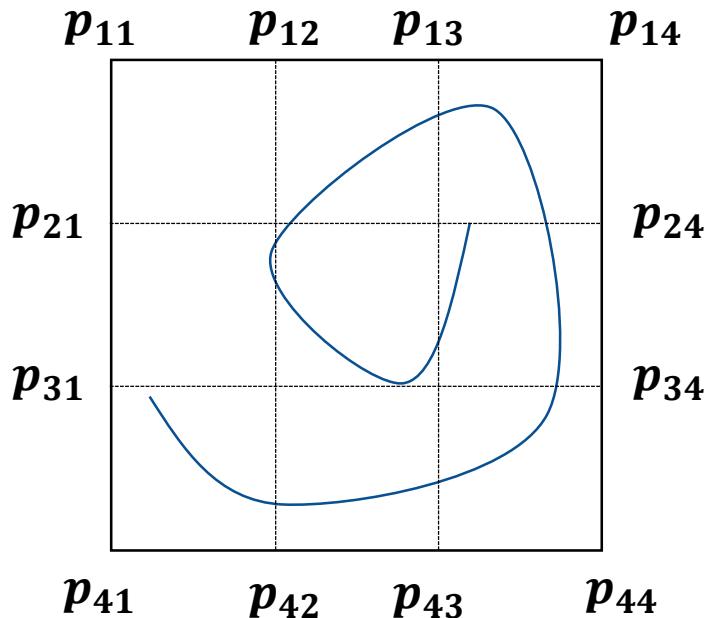
$$\begin{aligned}x &= (1 - t)m_2 + tm_4 \\&= (1 - t)((1 - s)p_{11} + sp_{21}) \\&\quad + t((1 - s)p_{12} + sp_{22}) \\&= \sum_i \sum_j B_i(s)B_j(t)\mathbf{p}_{ij}\end{aligned}$$

线性插值



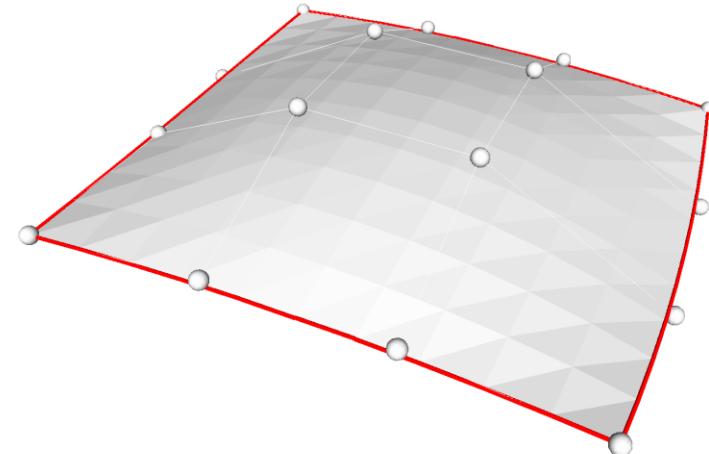
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曲线插值



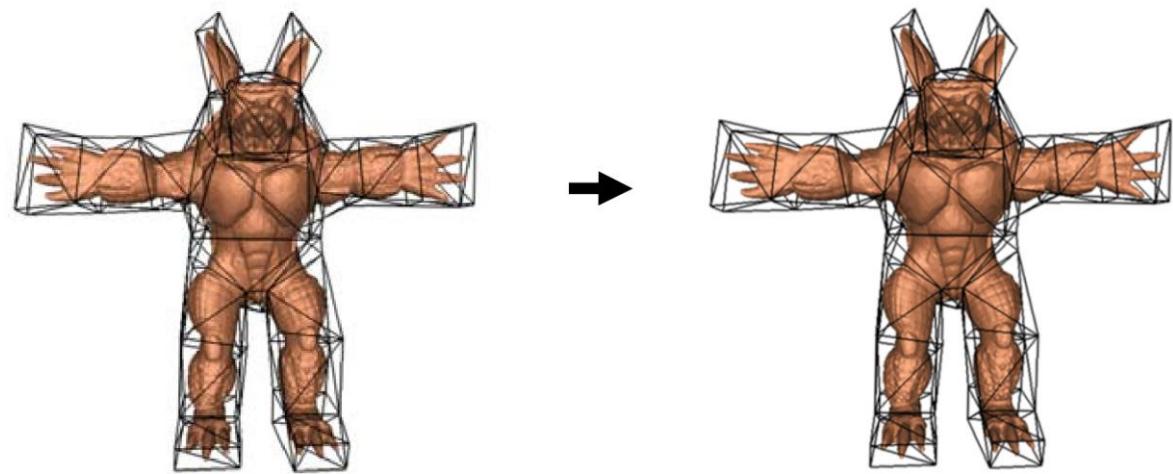
$$x = \sum_{i=0}^n \sum_{j=0}^m w_{i,j}(u, v) p_{i,j}$$

Bézier Surface, NURBS Surface...

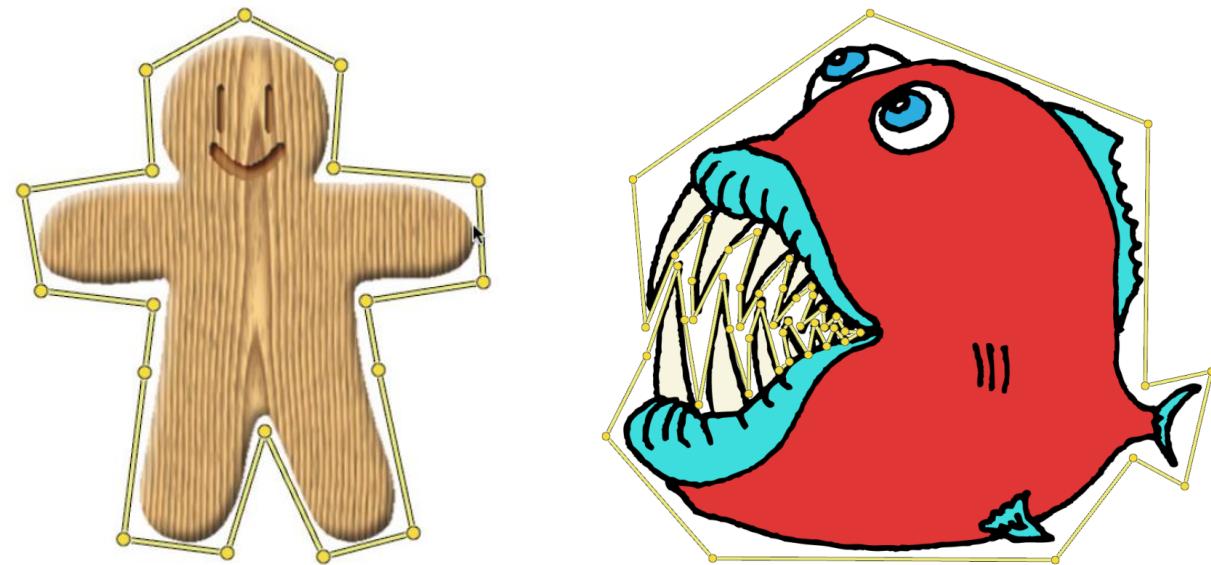


Cage-based Deformation

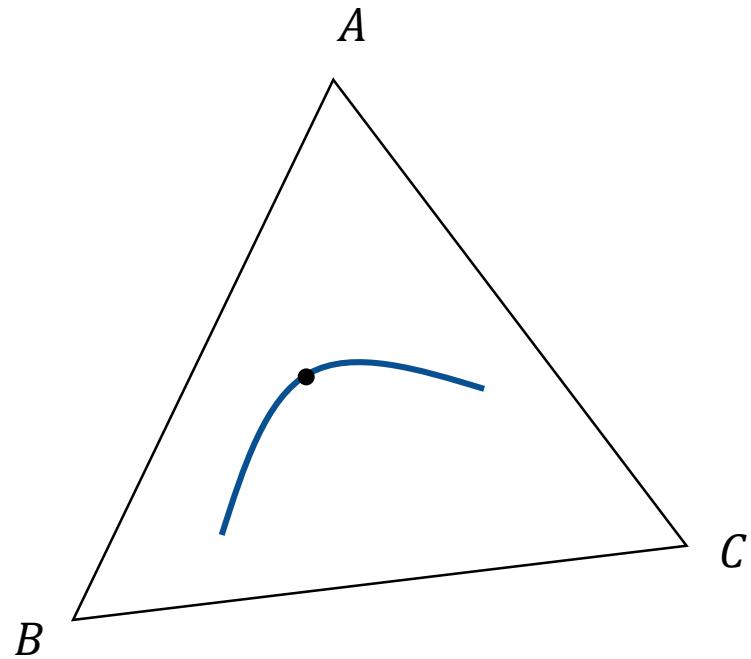
$$x_i = \sum_j \omega_{ij} p_j$$



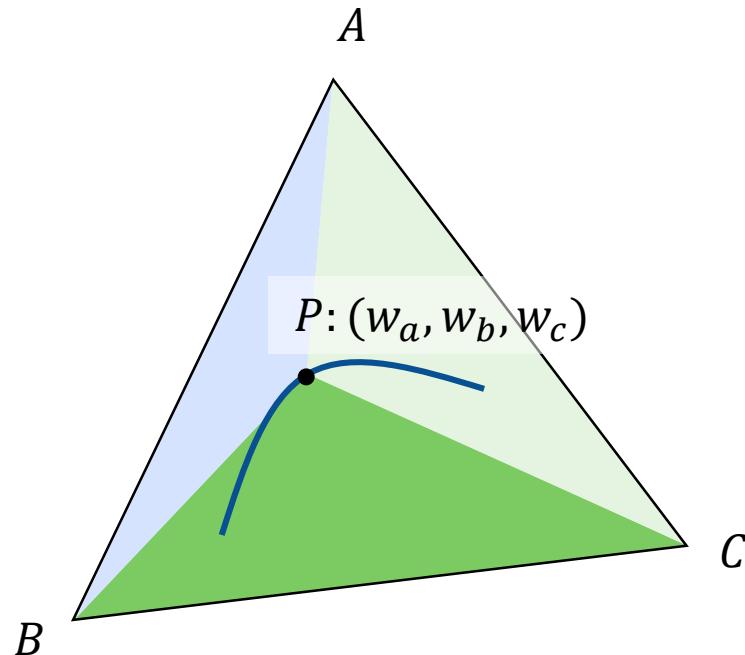
基于Cage计算权重



重心坐标



重心坐标



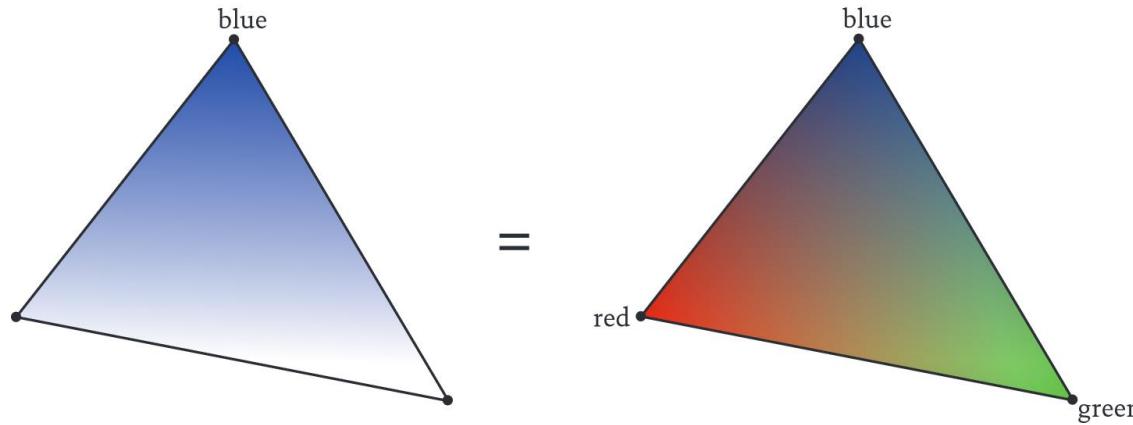
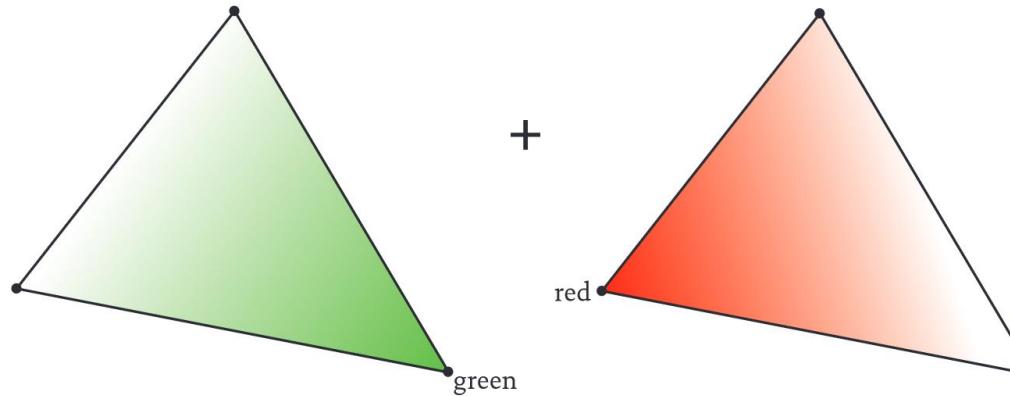
$$P = w_a A + w_b B + w_c C$$

$$w_a = \frac{\Delta PBC}{\Delta ABC}$$

$$w_b = \frac{\Delta PCA}{\Delta ABC}$$

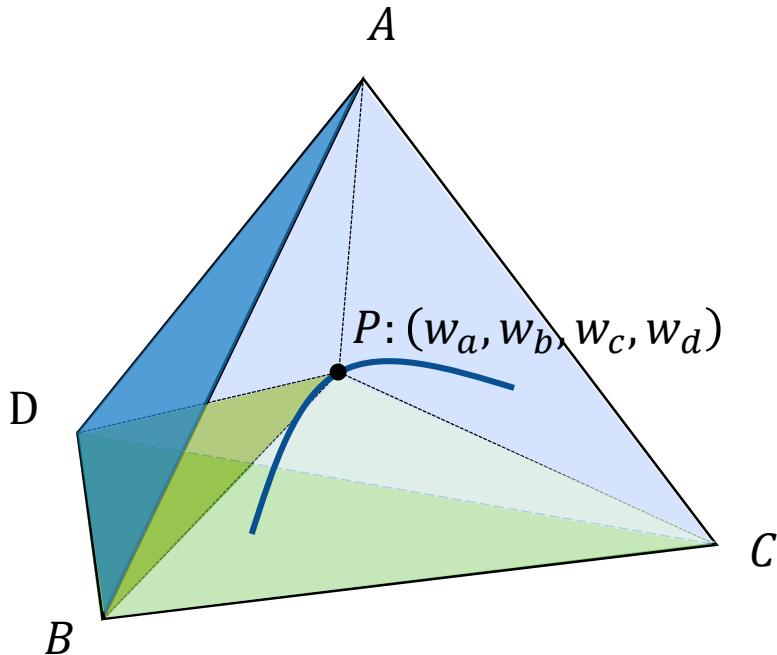
$$w_c = \frac{\Delta PAC}{\Delta ABC}$$

重心坐标



Lidberg, Petter. "Barycentric and Wachspress coordinates in two dimensions: theory and implementation for shape transformations." (2011).

重心坐标



$$P = w_a A + w_b B + w_c C + w_d D$$

$$w_a = \frac{\Delta PBCD}{\Delta ABCD}$$

$$w_b = \frac{\Delta PCDA}{\Delta ABCD}$$

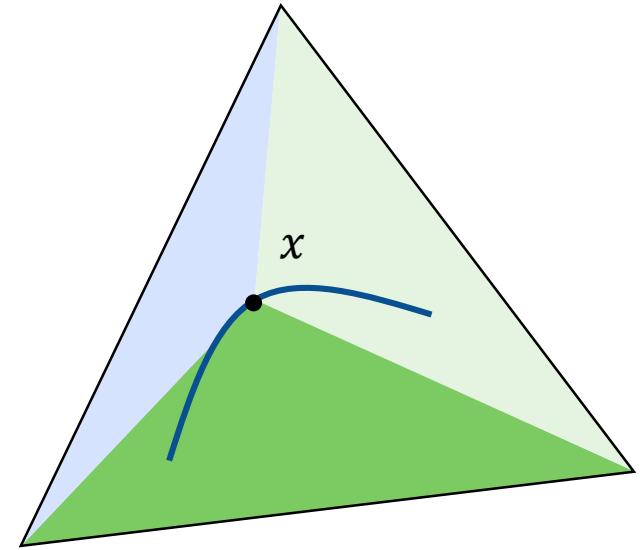
$$w_c = \frac{\Delta PDAB}{\Delta ABCD}$$

$$w_d = \frac{\Delta PABC}{\Delta ABCD}$$

三角形的重心坐标

$$x = \sum_i w_i p_i, \quad \sum_i w_i = 1, \text{且 } w_i \geq 0$$

$$x = \frac{\sum_i w'_i p_i}{\sum_i w'_i}$$



- 可写作 $x = (w_0, w_1, \dots, w_n)$
- 三角形中的点有唯一重心坐标 (why?)
 - 不同坐标组合仅差常系数
 - Homogeneous coordinates
 - 可扩展到N-维单纯形

任意多边形的情况

$$x = \sum_i w_i p_i / \sum_i w_i$$

- 任意多边形重心坐标不唯一

- Wachspress (WP) coordinates

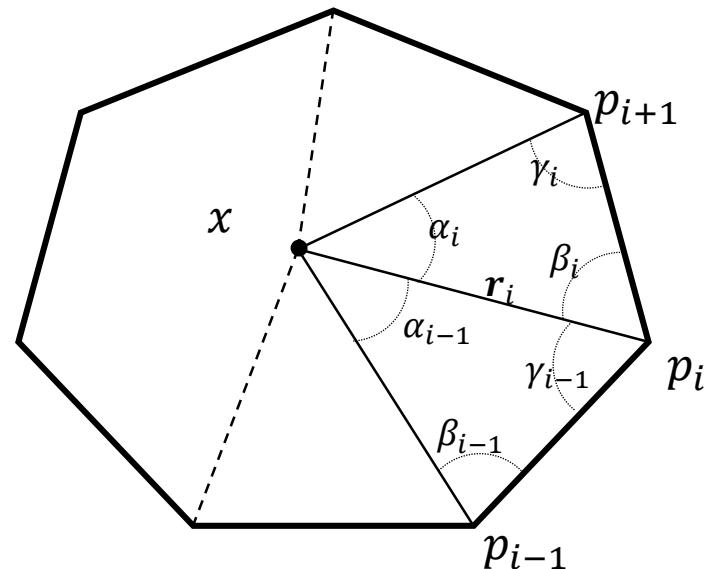
$$w_i = \frac{\cot \gamma_{i-1} + \cot \beta_i}{r_i^2}$$

- mean value (MV) coordinates

$$w_i = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{r_i}$$

- discrete harmonic (DH) coordinates

$$w_i = \cot \beta_{i-1} + \cot \gamma_i$$



任意多边形的情况

$$x = \sum_i w_i p_i / \sum_i w_i$$

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- Wachspress (WP) coordinates

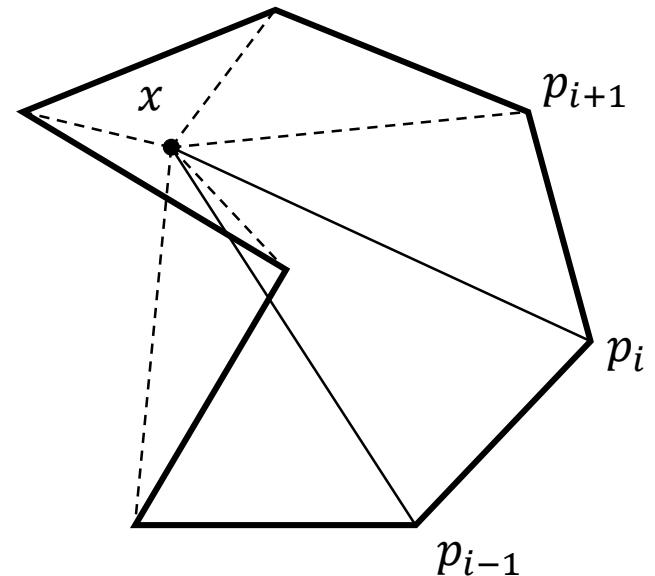
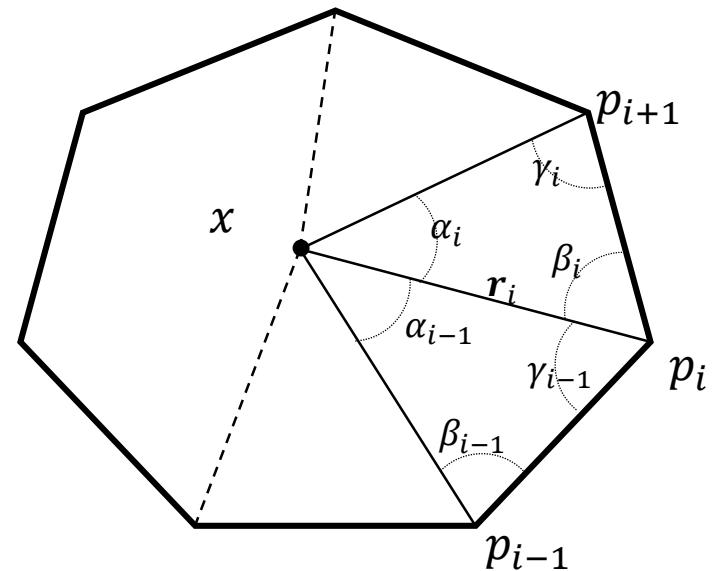
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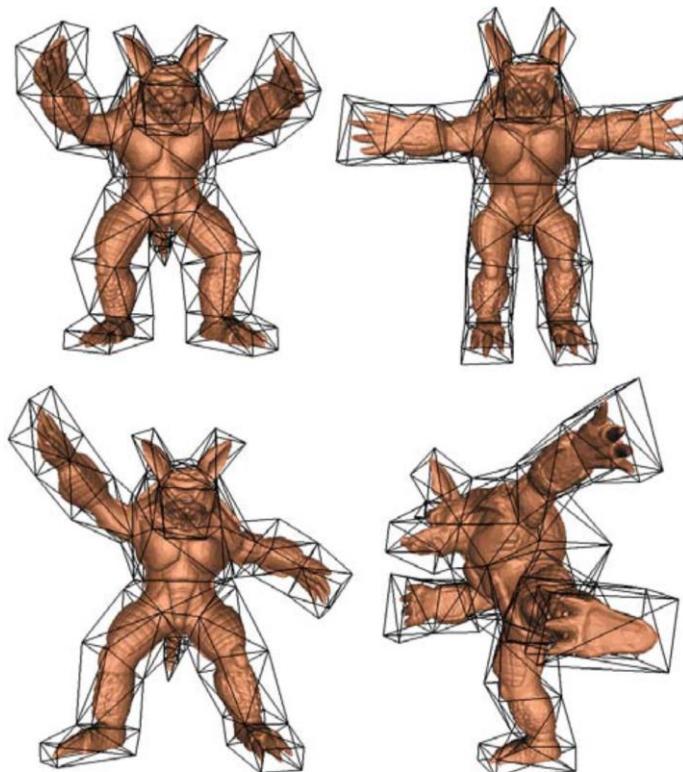
- discrete harmonic (DH) coordinates

$$w_i = \cot \beta_{i-1} + \cot \gamma_i$$



任意多边形的情况

- Generalize to 3D Polyhedron is non-trivial
 - See <https://www.inf.usi.ch/hormann/barycentric/>

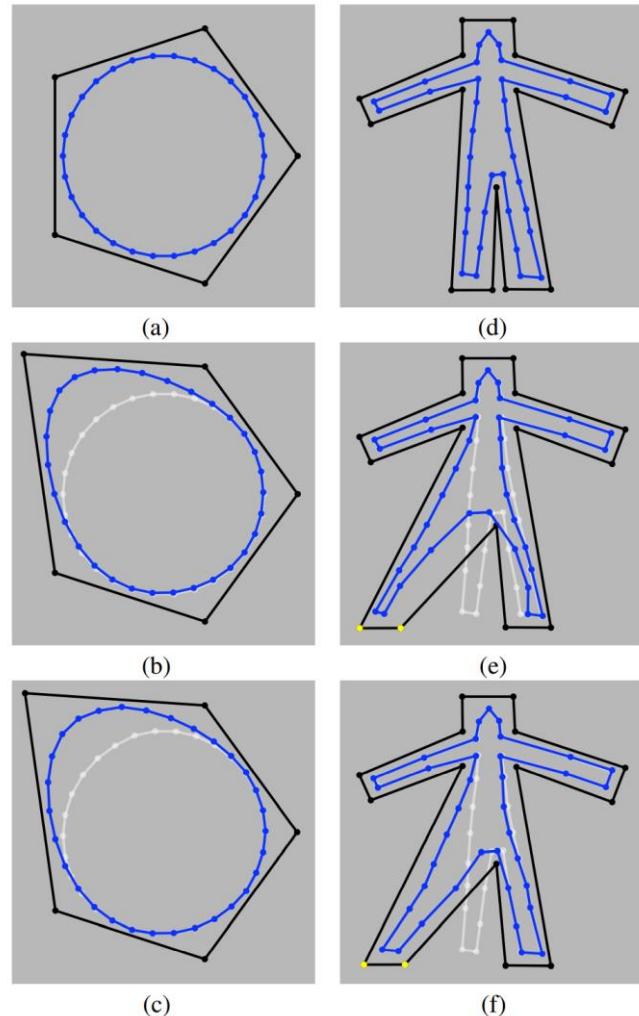


Ju, T., Schaefer, S., Warren, J.: ***Mean value coordinates for closed triangular meshes***. In: ACM SIGGRAPH 2005 Papers, SIGGRAPH '05, pp. 561–566.

更一般的情况

- Cage权重的一些性质：
 - 控制点本身的权重为1
 - 每个点的权重和为1
 - Cage内部权重连续/光滑
 - 权重非负
 - 支持线性插值

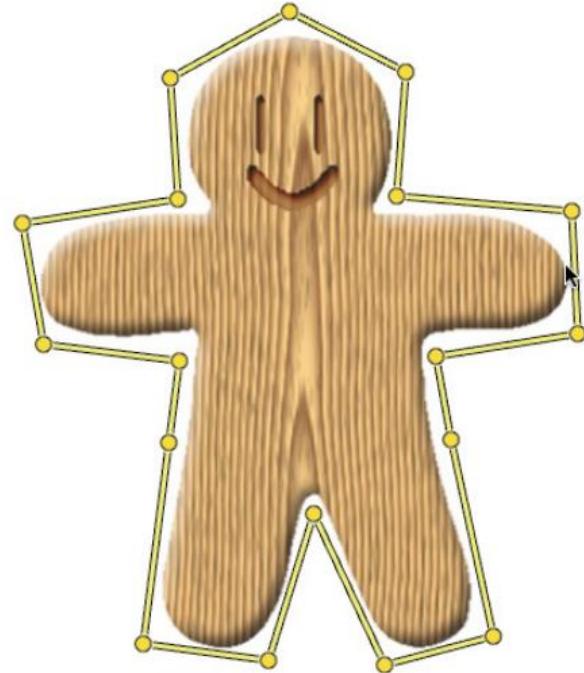
$$x_i = \sum_j \omega_{ij} p_j$$



Pushkar Joshi, Mark Meyer, Tony DeRose, Brian Green, and Tom Sanocki. 2007. *Harmonic coordinates for character articulation*. ACM Trans. Graph. 26, 3 (July 2007), 71–es.

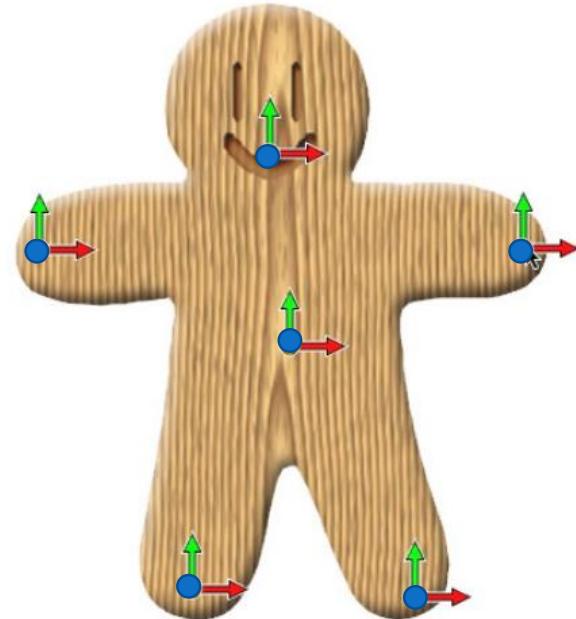
另一种形式的cage

$$x_i = \sum_j \omega_{ij} p_j$$



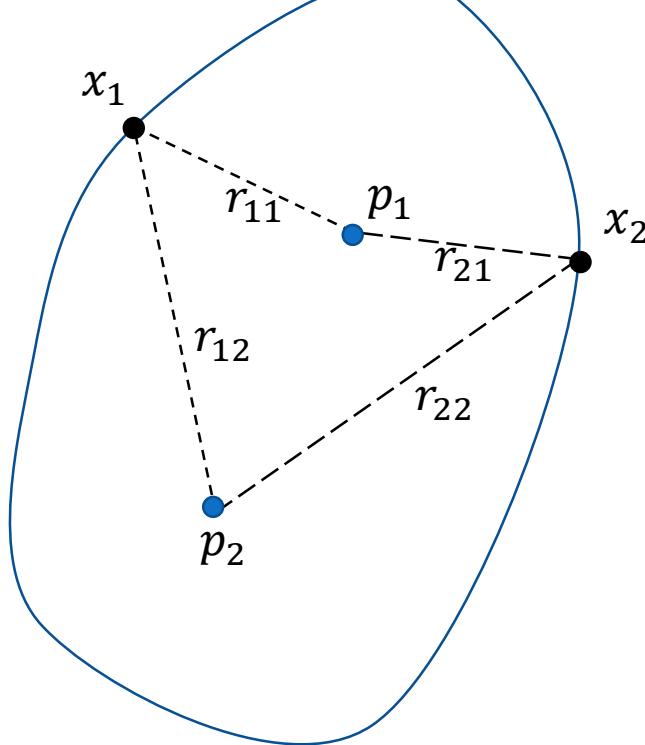
Cage

$$x_i = \sum_j \omega_{ij} p_j \quad ??$$



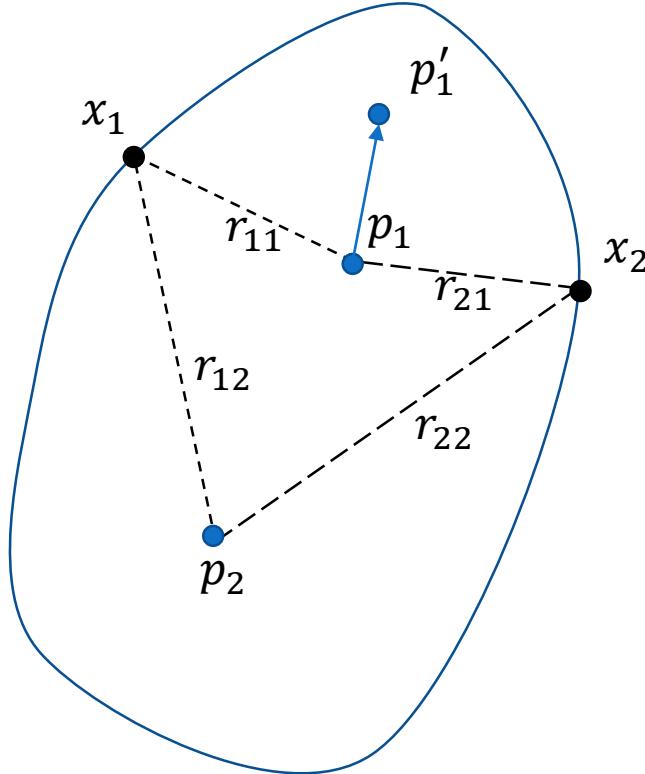
控制点

基于控制点的变形



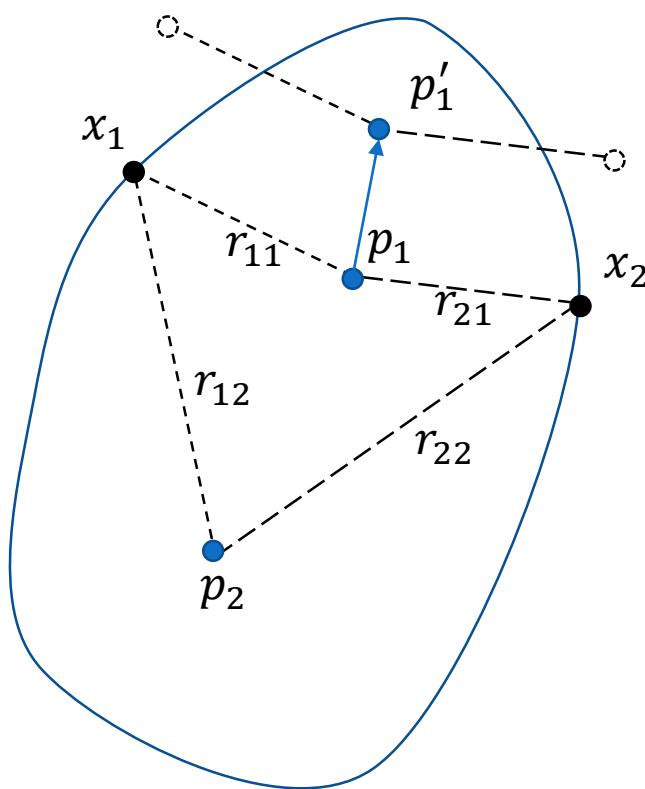
$$\begin{aligned}x_i &= \sum_j \omega_{ij} (x_i - p_j + p_j) \\&= \sum_j \omega_{ij} (r_{ij} + p_j)\end{aligned}$$

基于控制点的变形



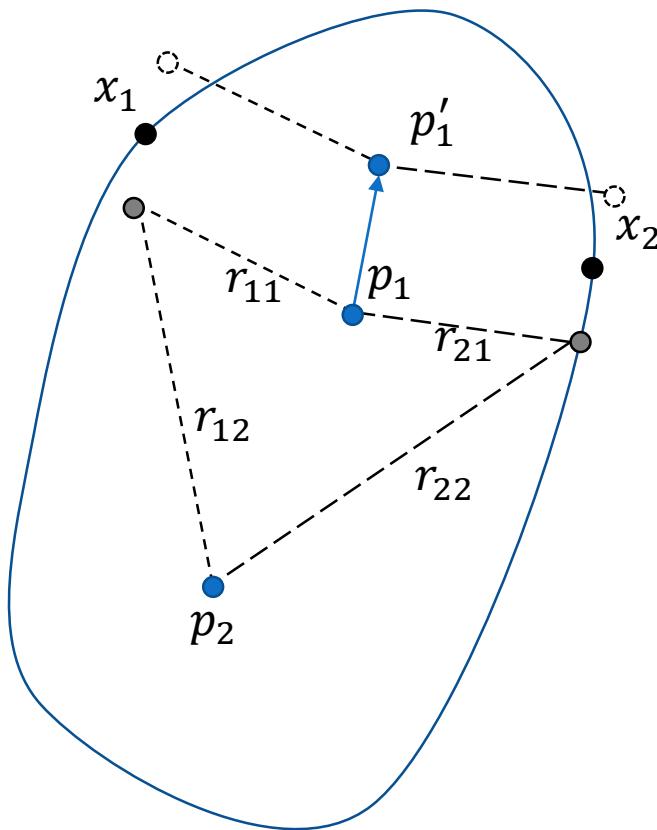
$$x'_i = \sum_j \omega_{ij} (\textcolor{red}{r}_{ij} + p'_j)$$

基于控制点的变形



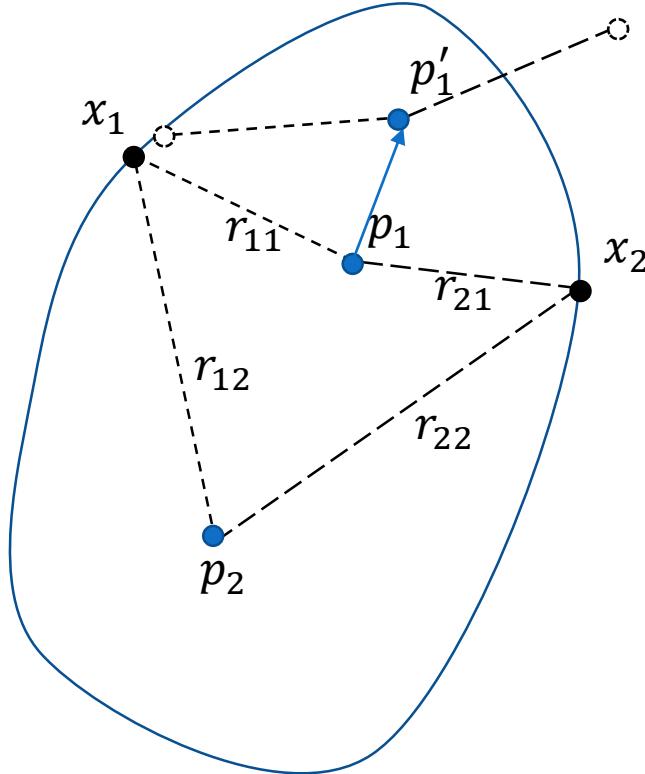
$$x'_i = \sum_j \omega_{ij} (\textcolor{red}{r}_{ij} + p'_j)$$

基于控制点的变形



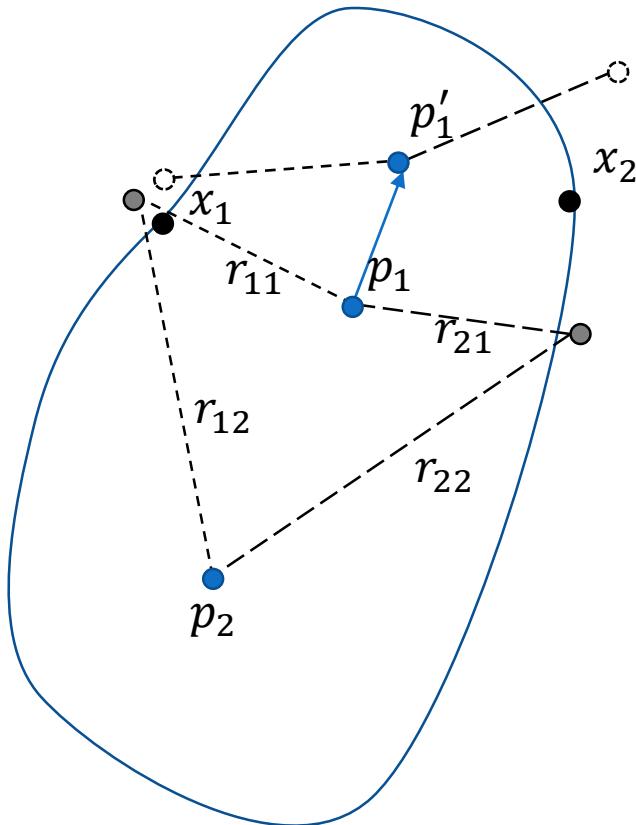
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基于控制点的变形



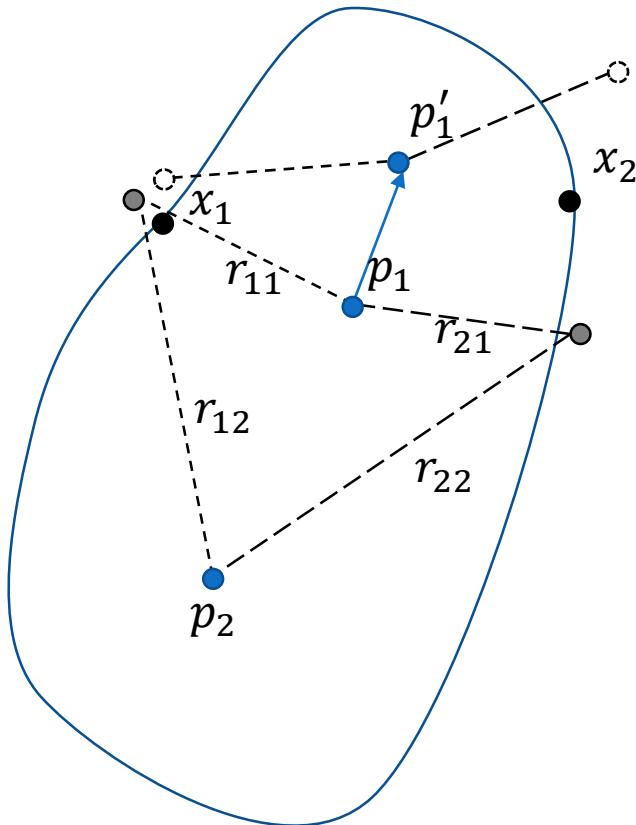
$$x'_i = \sum_j \omega_{ij} (R_j r_{ij} + p'_j)$$

基于控制点的变形



$$x'_i = \sum_j \omega_{ij} (R_j r_{ij} + p'_j)$$

基于控制点的变形



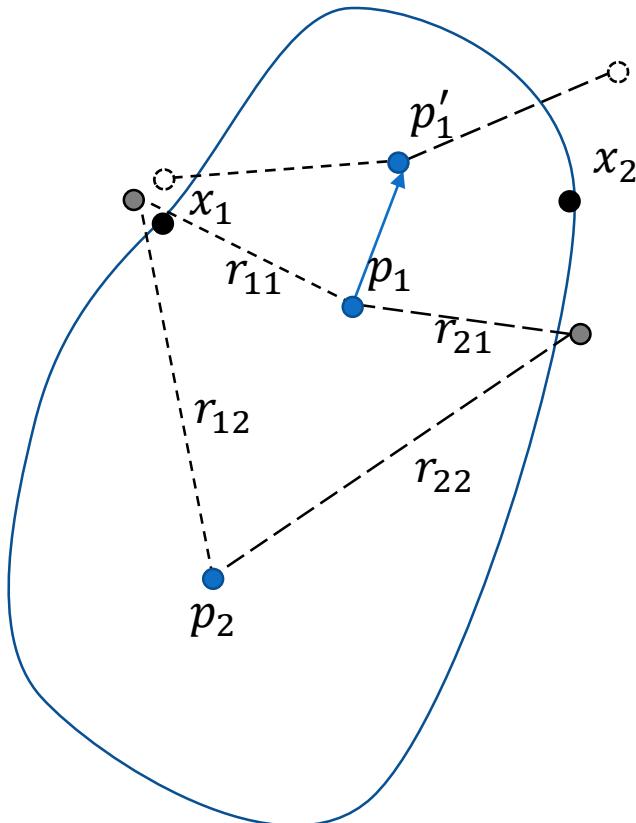
$$x'_i = \sum_j \omega_{ij} (R_j r_{ij} + p'_j)$$



$$r_{ij} = x_i - p_j$$

$$x'_i = \sum_j \omega_{ij} (R_j x_i + t_j)$$

基于控制点的变形



$$x'_i = \sum_j \omega_{ij} (R_j r_{ij} + p'_j)$$



$$r_{ij} = x_i - p_j$$

$$x'_i = \sum_j \omega_{ij} (R_j x_i + t_j)$$



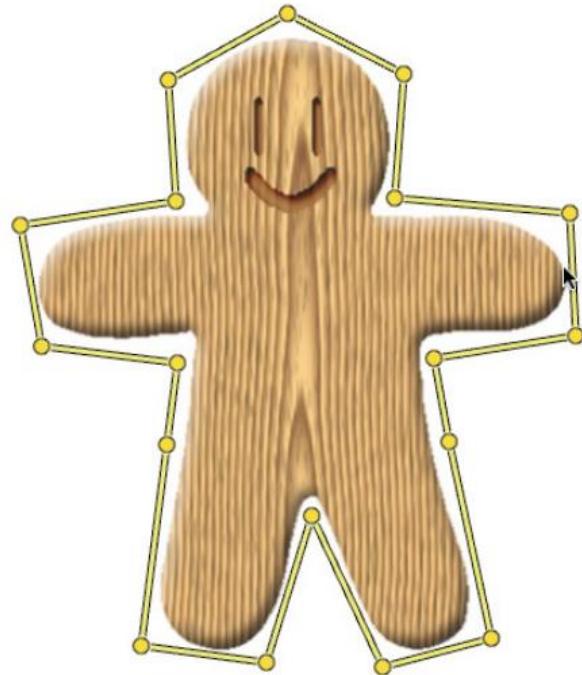
$$T = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x'_i \\ 1 \end{pmatrix} = \sum_j \omega_{ij} T_j \begin{pmatrix} x_i \\ 1 \end{pmatrix}$$

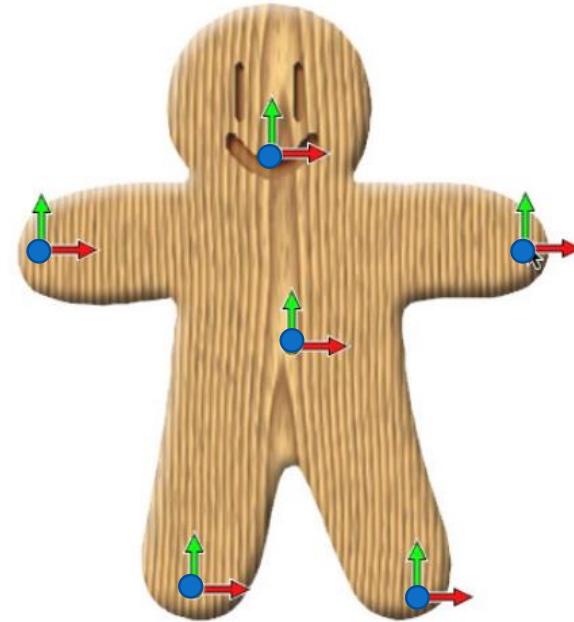
另一种形式的cage

$$x_i = \sum_j \omega_{ij} p_j$$

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



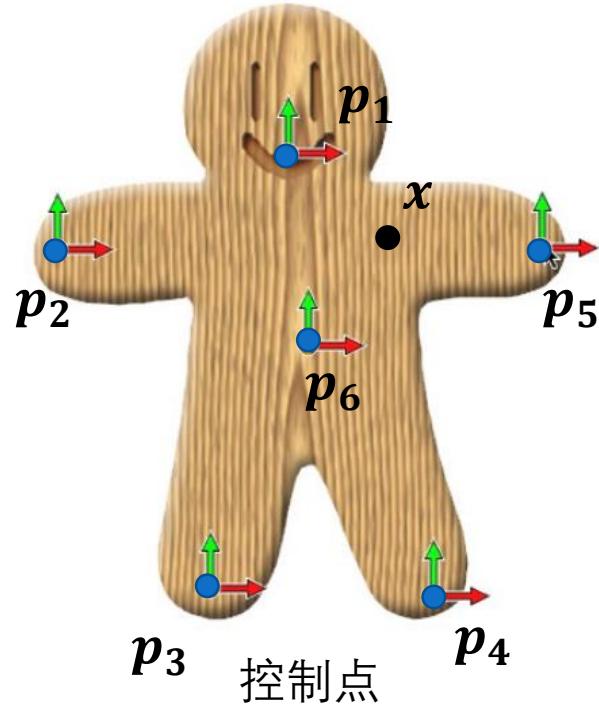
Cage



控制点

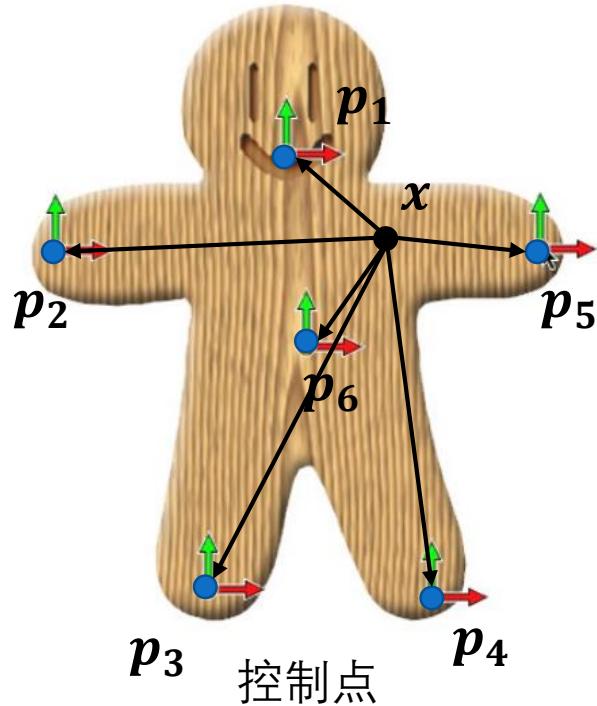
控制点权重的计算

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



控制点权重的计算

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



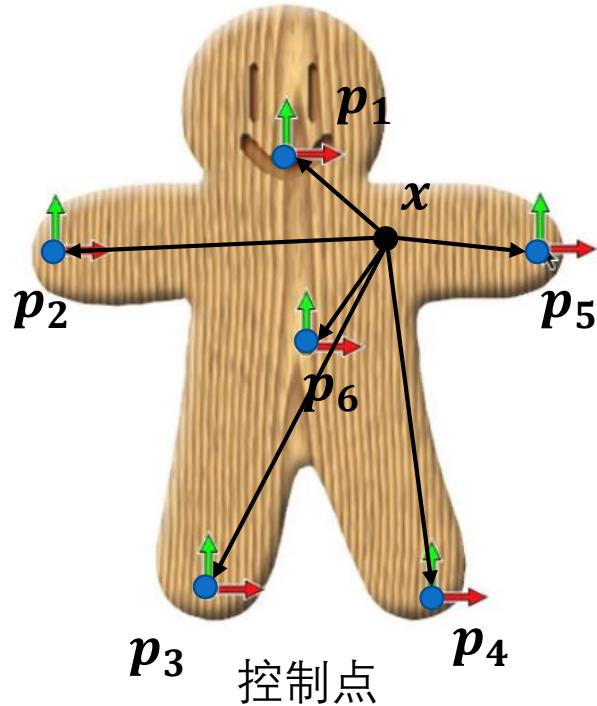
基于距离的权重

欧氏距离: $d_j = \|x - p_j\|$

权重: $w_{ij} = \begin{cases} 1, & d_j < d_k, \forall k \neq j \\ 0, & \text{otherwise} \end{cases}$

控制点权重的计算

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



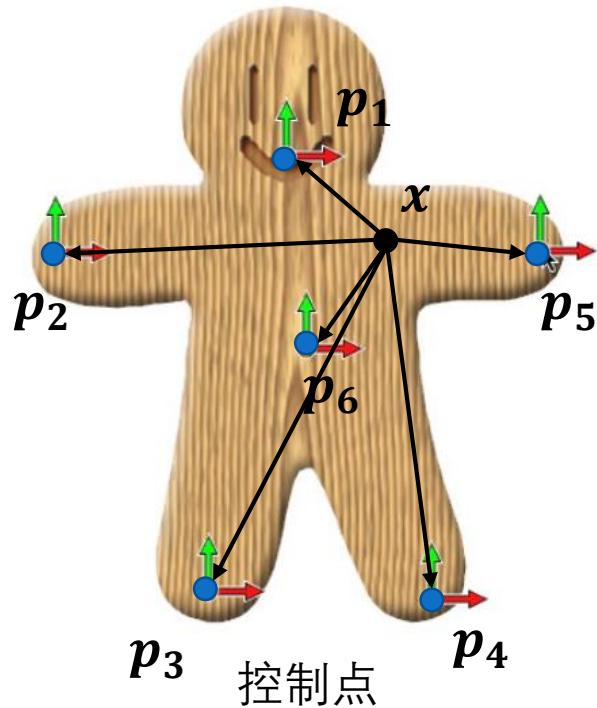
基于距离的权重

欧氏距离: $d_j = \|x - p_j\|$

权重: $w_{ij} = \frac{1}{d_j^p}$

控制点权重的计算

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



基于距离的权重

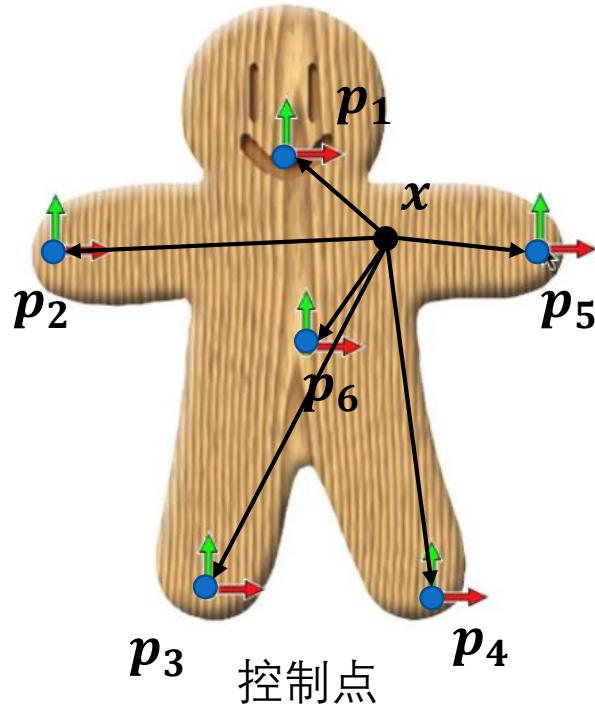
欧氏距离: $d_j = \|x - p_j\|$

权重: $\hat{w}_{ij} = \frac{1}{d_j^p}$

$$w_{ij} = \frac{\hat{w}_{ij}}{\sum_j \hat{w}_{ij}}$$

控制点权重的计算

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



基于距离的权重

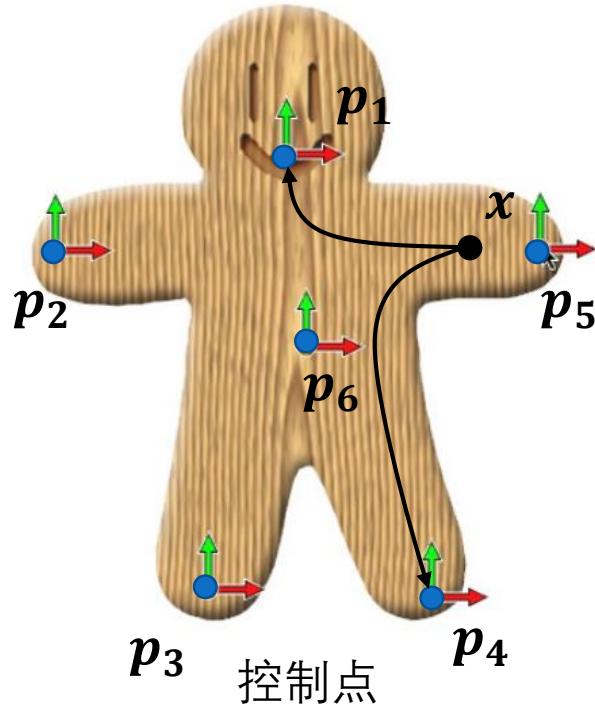
欧氏距离: $d_j = \|x - p_j\|$

权重: $\hat{w}_{ij} = \frac{1}{d_j^p + \epsilon}$

$$w_{ij} = \frac{\hat{w}_{ij}}{\sum_j \hat{w}_{ij}}$$

控制点权重的计算

$$x'_i = \sum_j \omega_{ij} T_j x_i$$



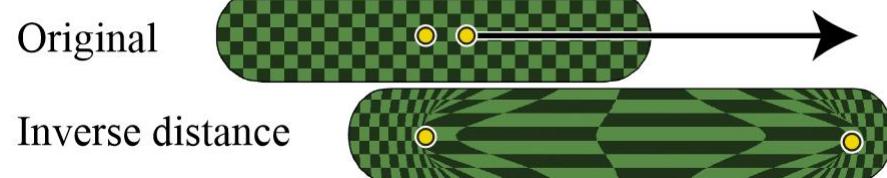
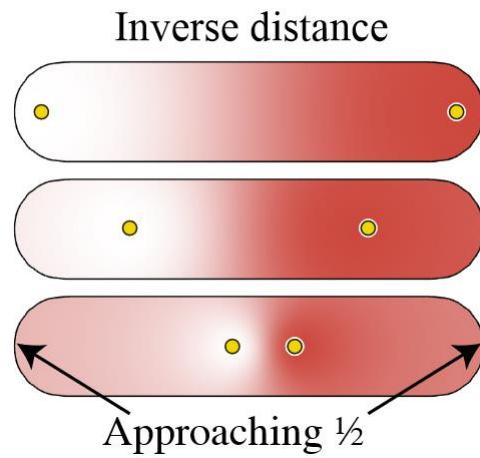
基于距离的权重

几何距离: $d_j = d(x, p_i)$

权重: $\hat{w}_{ij} = \frac{1}{d_j^p + \varepsilon}$

$$w_{ij} = \frac{\hat{w}_{ij}}{\sum_j \hat{w}_{ij}}$$

控制点权重的计算



控制点权重的计算

- 基于能量优化的控制点权重
 - 权重在控制点处为1，平滑过渡到整个空间
 - 平滑能量函数

Common name	Least squares description	Energy	PDE
Dirichlet	f should be as constant as possible	$\int_{\Omega} \ \nabla f\ ^2 dV$	$\Delta f = 0$
Laplacian	f should be as harmonic as possible	$\int_{\Omega} (\Delta f)^2 dV$	$\Delta^2 f = 0$
Laplacian gradient	Δf should be as constant as possible	$\int_{\Omega} \ \nabla \Delta f\ ^2 dV$	$\Delta^3 f = 0$
...			

控制点权重的计算

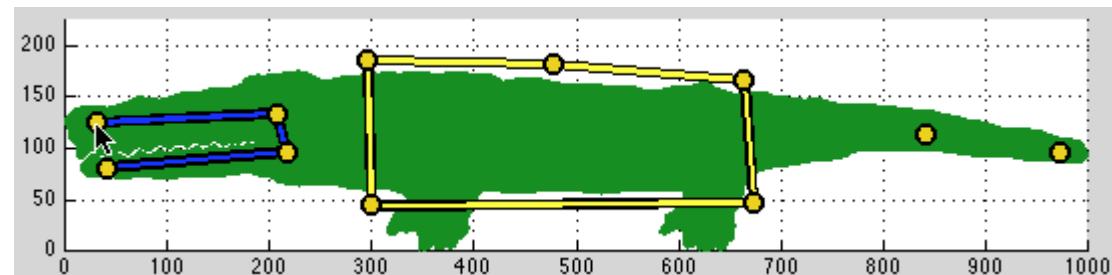
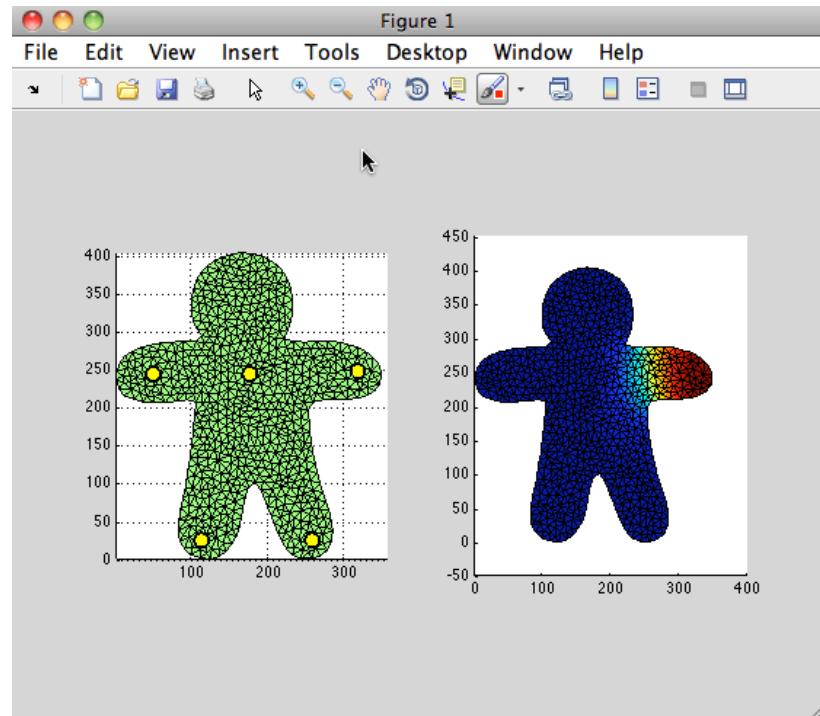
Bounded Biharmonic Weights

$$\operatorname{argmin}_{w_1, \dots, w_m} \sum_{j=1}^m \frac{1}{2} \int_{\Omega} (\Delta w_j(\mathbf{v}))^2 dV,$$

subject to $w_j(\mathbf{v}) = \varphi_j(\mathbf{v}) \quad \forall \mathbf{v} \in H,$

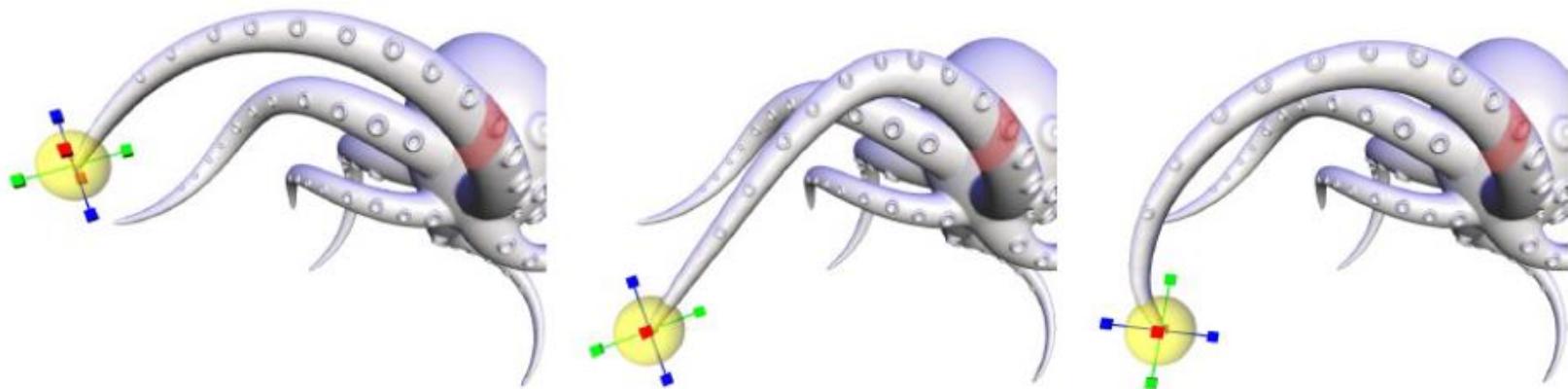
$$w_j(\mathbf{v}) > 0,$$

$$\sum_{j=1}^m w_j(\mathbf{v}) = 1.$$



作为优化问题的几何形变

- 更加直接的形变编辑
 - 直接操作几何模型上的点
 - 将修改平滑地施加在整个几何模型上
 - 构造优化问题求解

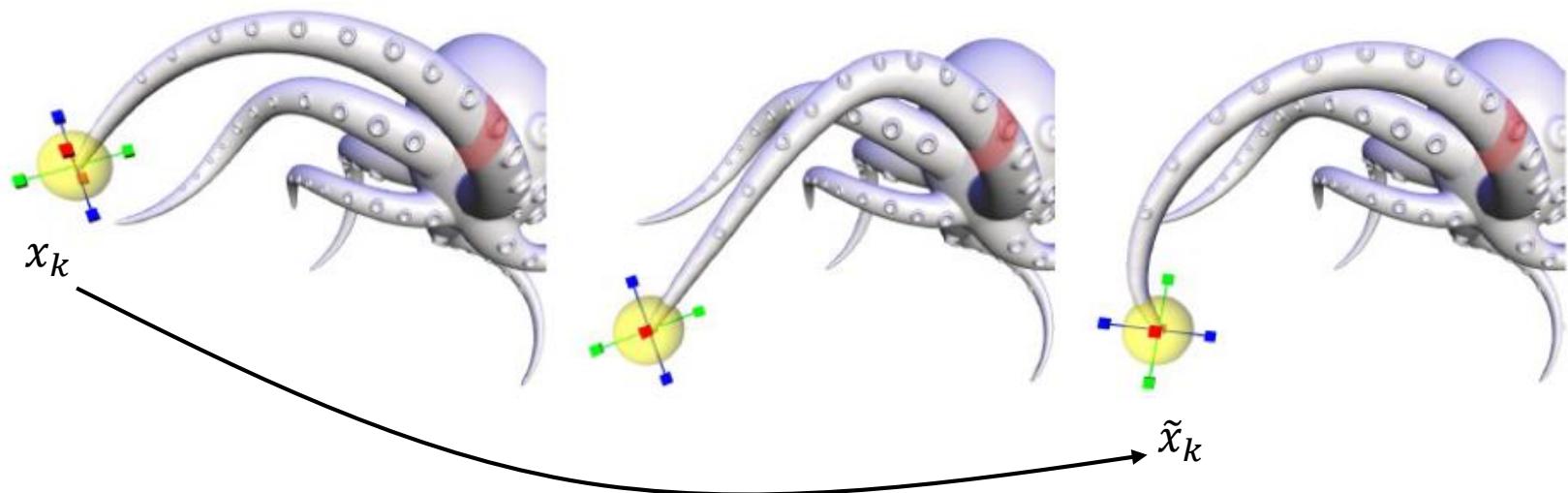


作为优化问题的几何形变

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + E(X, X')$$

编辑约束

平滑能量



作为优化问题的几何形变

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + E(X, X')$$

编辑约束

平滑能量

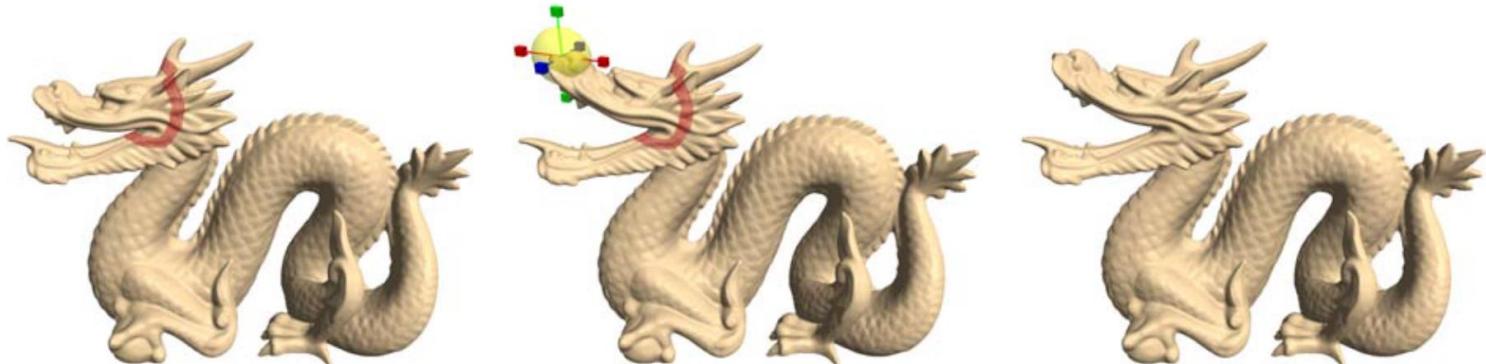
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...			

Laplacian Mesh Editing

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + \sum_i \|T_i(X')\mathcal{L}(x_i) - \mathcal{L}(x'_i)\|^2$$

编辑约束

平滑能量



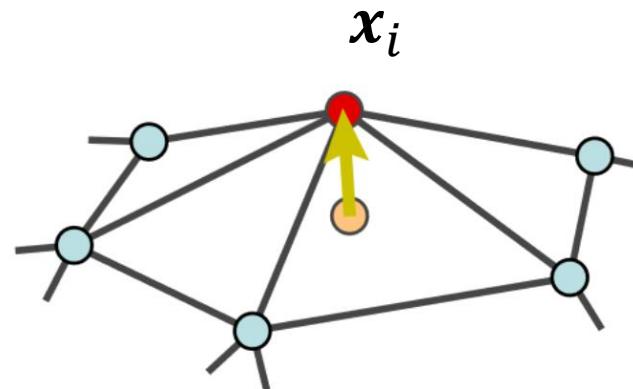
O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rössl, and H.-P. Seidel. 2004. *Laplacian surface editing*. In *Proceedings of the 2004 Eurographics/ACM SIGGRAPH symposium on Geometry processing (SGP '04)*, Association for Computing Machinery, New York, NY, USA, 175–184.

Laplacian坐标

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + \sum_i \|T_i(X')\mathcal{L}(x_i) - \mathcal{L}(x'_i)\|^2$$

$$\mathcal{L}(x) = \Delta f(x)$$

Laplace-Beltrami operator



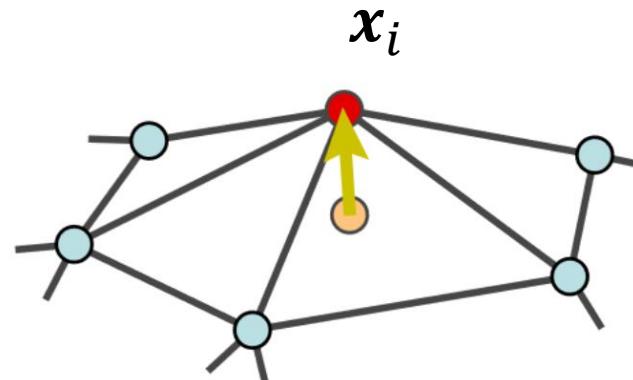
\mathcal{N}_i : 1-neighbors

Laplacian坐标

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + \sum_i \|T_i(X')\mathcal{L}(x_i) - \mathcal{L}(x'_i)\|^2$$

$$\mathcal{L}(x_i) = x_i - \frac{1}{d_i} \sum_{j \in \mathcal{N}_i} x_j$$

“Umbrella” operator

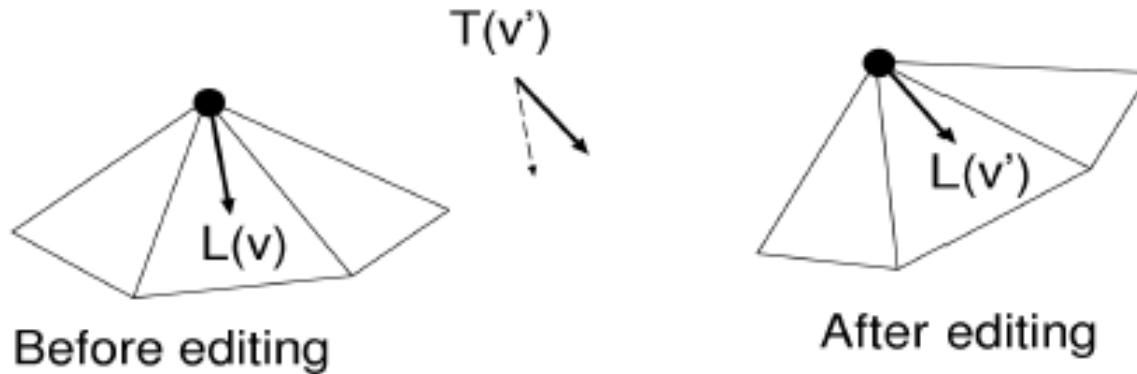


\mathcal{N}_i : 1-neighbors

Laplacian坐标

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + \sum_i \|T_i(X')\mathcal{L}(x_i) - \mathcal{L}(x'_i)\|^2$$

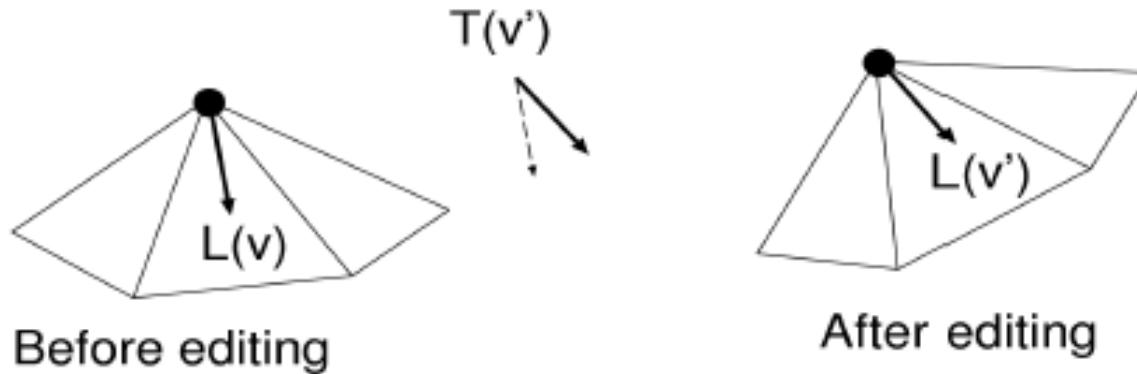
- 我们希望变换过后每个点的Laplacian坐标不变



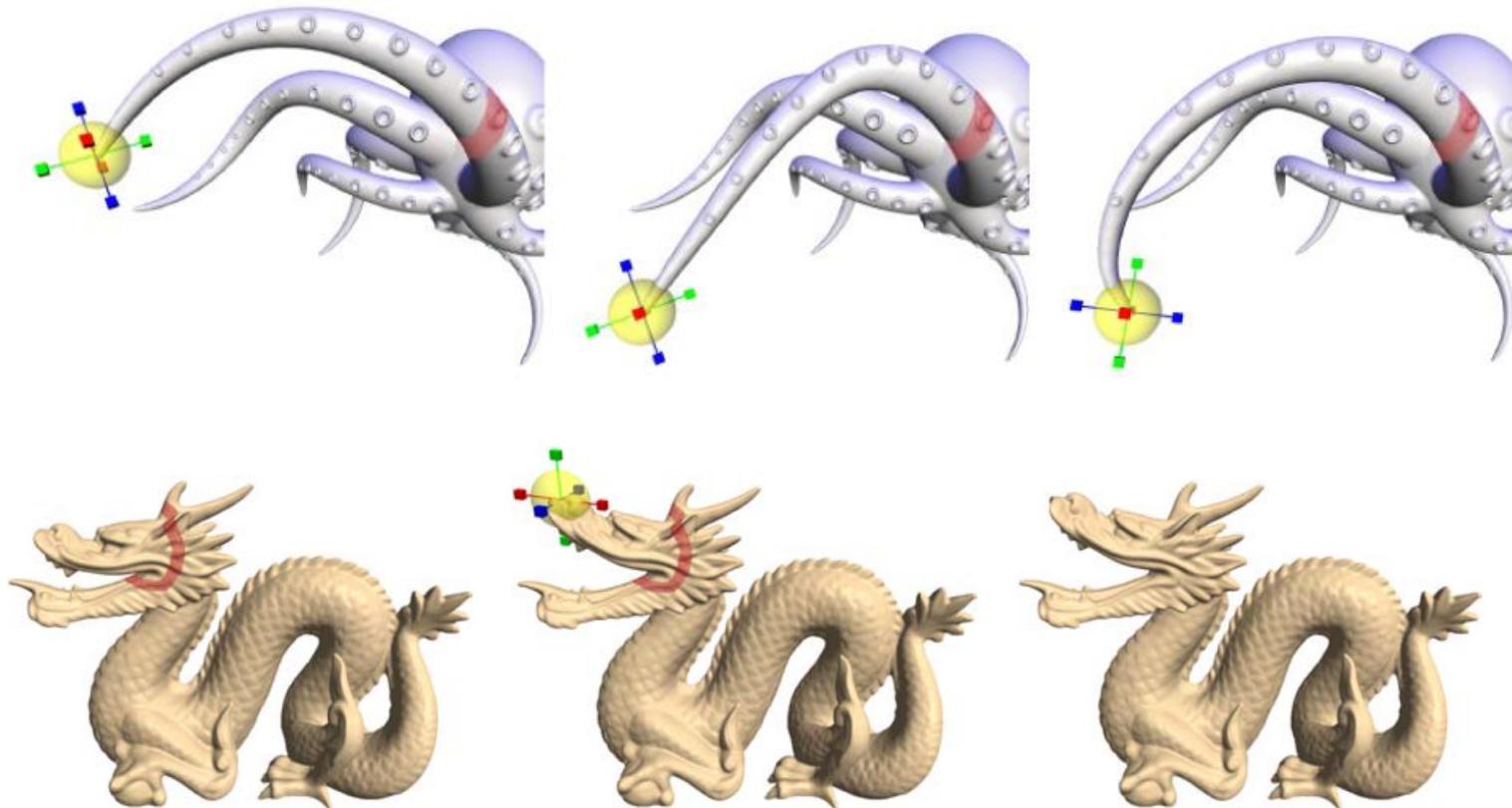
Laplacian坐标

$$\min_{x', \textcolor{red}{T_i}} \sum_k \|x'_k - \tilde{x}_k\|^2 + \sum_i \|T_i(x') \mathcal{L}(x_i) - \mathcal{L}(x'_i)\|^2$$

- 我们希望变换过后每个点的Laplacian坐标不变



Laplacian Mesh Editing

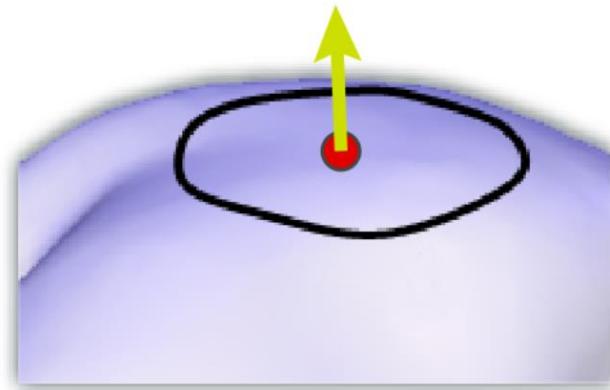
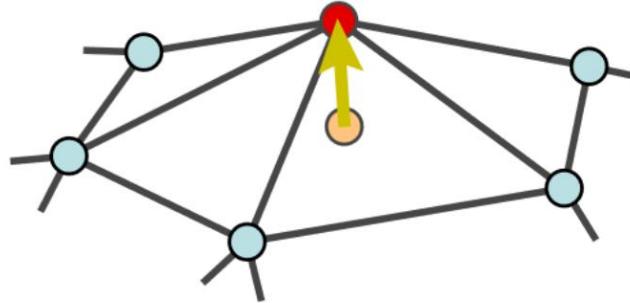


O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rössl, and H.-P. Seidel. 2004. *Laplacian surface editing*. In *Proceedings of the 2004 Eurographics/ACM SIGGRAPH symposium on Geometry processing (SGP '04)*, Association for Computing Machinery, New York, NY, USA, 175–184.

Laplacian坐标的意义

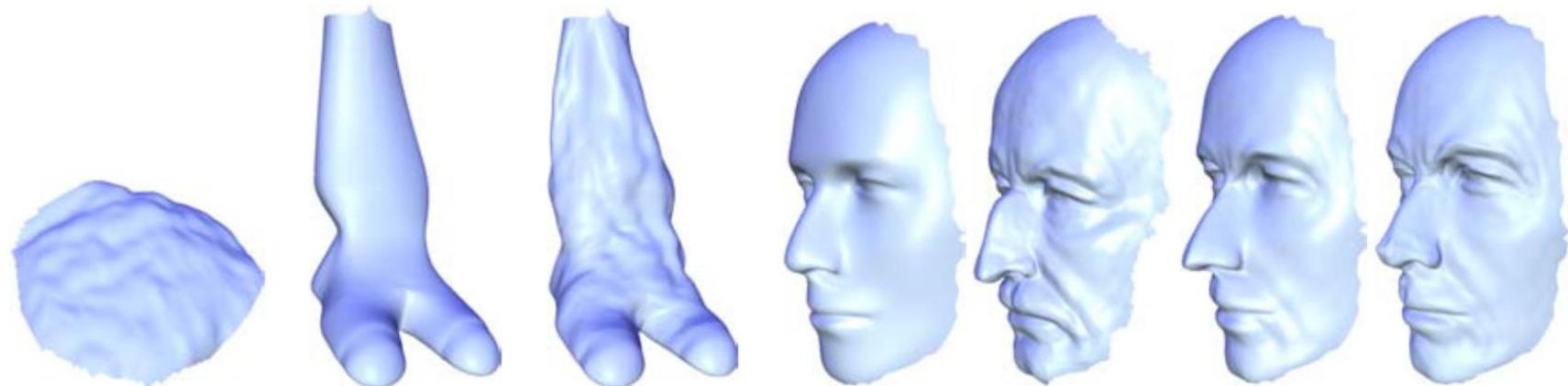
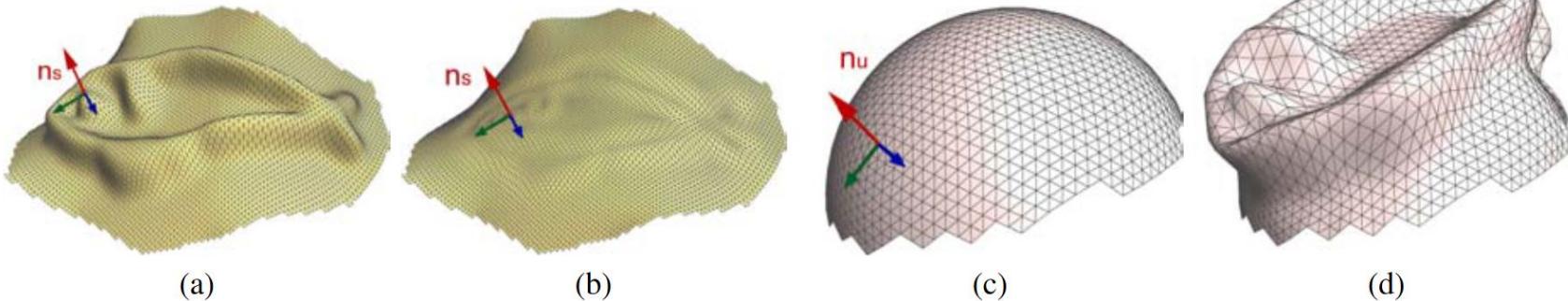
- 几何表面局部的细节

- 方向 \approx 法向
- 大小 \approx 曲率



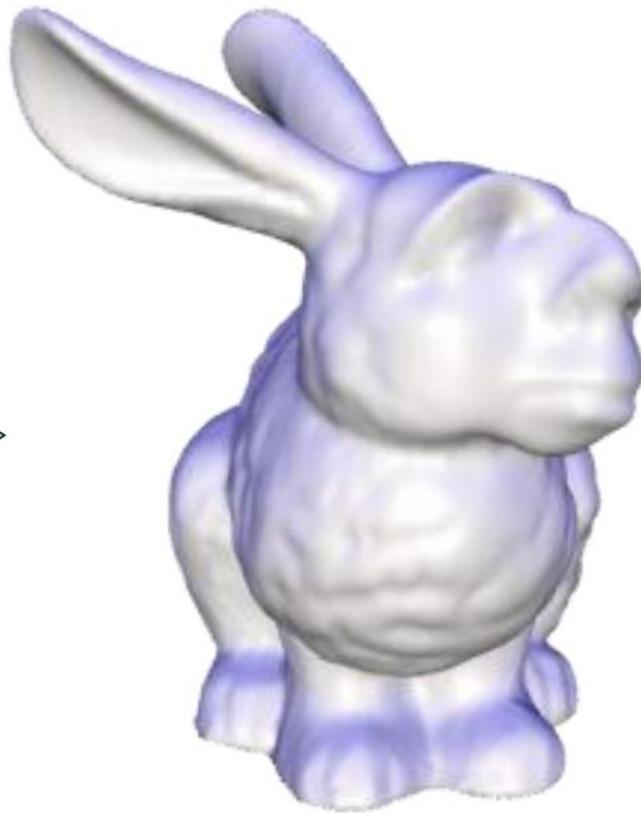
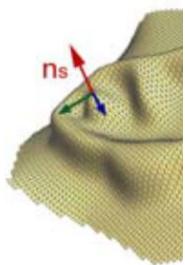
Coating Transfer

$$\delta_i = \mathcal{L}(x_i) - \widehat{\delta}_i + \widehat{\delta}'_i = \delta'_i$$



Coating Transfer

δ_i



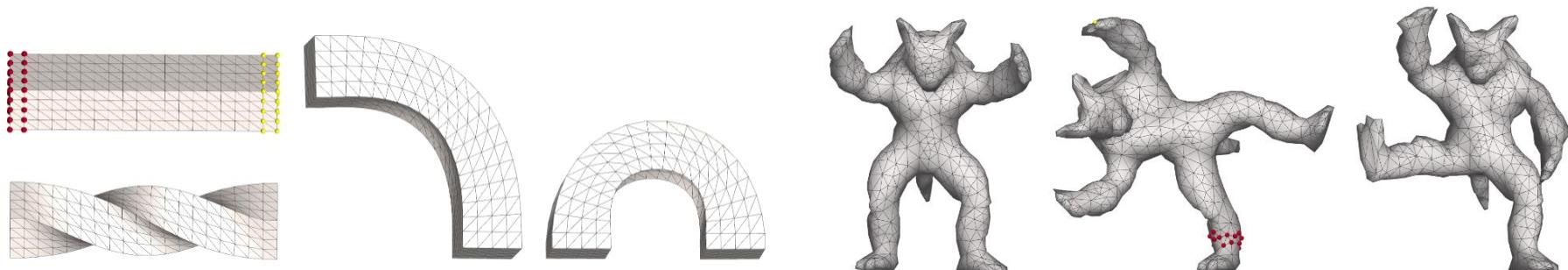
As Rigid As Possible Deformation

$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + E(X, X')$$

编辑约束

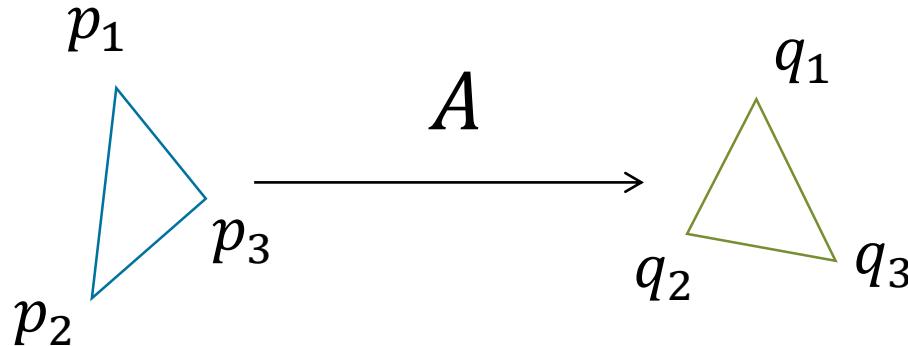
平滑能量项

尽可能减少非“刚性”形变 → 减少不必要的缩放



Olga Sorkine and Marc Alexa. 2007. *As-rigid-as-possible surface modeling*. In *Proceedings of the fifth Eurographics symposium on Geometry processing (SGP '07)*, Eurographics Association, Goslar, DEU, 109–116.

As Rigid As Possible Deformation

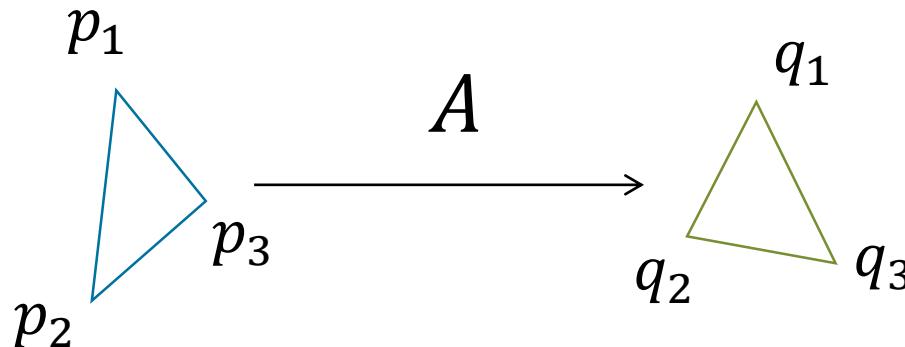


$$\mathbf{q}_i = A\mathbf{p}_i + \mathbf{t}$$

$$A = U\Sigma V^T = (UV^T)(VDV^T) = RS$$

How to find the “closest” rotation between two point sets?
See: https://igl.ethz.ch/projects/ARAP/svd_rot.pdf

As Rigid As Possible Shape Interpolation



Interpolation between two shapes

$$\mathbf{q}_i(t) = \mathbf{A}(t)\mathbf{p}_i + \mathbf{t}$$

$$\mathbf{A}(t) = \mathbf{R}(t)((1-t)\mathbf{I} + t\mathbf{S})$$

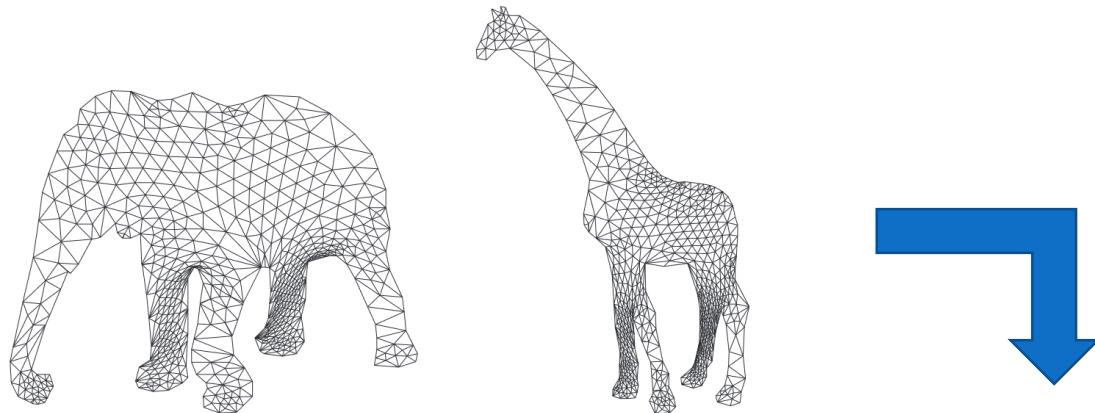
顶点线性插值



刚性变换插值



As Rigid As Possible Shape Interpolation



Marc Alexa, Daniel Cohen-Or, and David Levin. 2000. ***As-rigid-as-possible shape interpolation***. In *Proceedings of the 27th annual conference on Computer graphics and interactive techniques* (SIGGRAPH '00)

As Rigid As Possible Deformation

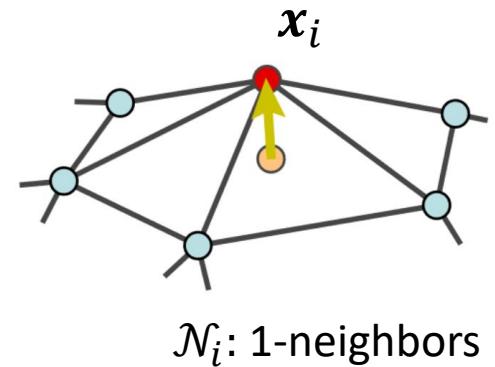
$$\min_{X'} \sum_k \|x'_k - \tilde{x}_k\|^2 + E(X, X')$$

编辑约束

平滑能量项

尽可能减少非“刚性”形变 → 减少不必要的缩放

$$E(X, X') = \sum_{j \in \mathcal{N}_i} w_{ij} \|(x'_i - x'_j) - R_i(x_i - x_j)\|^2$$



As Rigid As Possible Deformation

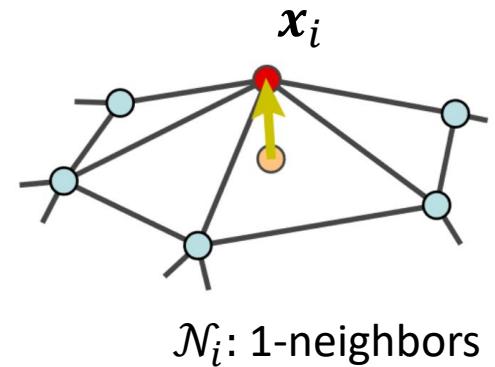
$$\min_{X', \mathbf{R}_i} \sum_k \|x'_k - \tilde{x}_k\|^2 + E(X, X')$$

编辑约束

平滑能量项

尽可能减少非“刚性”形变 → 减少不必要的缩放

$$E(X, X') = \sum_{j \in \mathcal{N}_i} w_{ij} \|(x'_i - x'_j) - R_i(x_i - x_j)\|^2$$



As Rigid As Possible Deformation



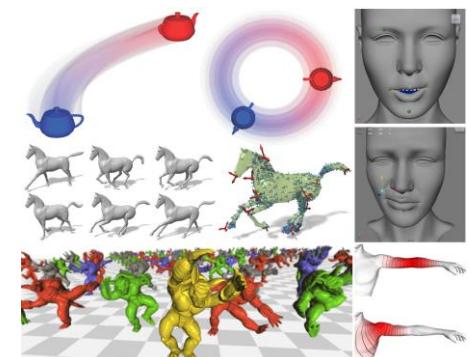
Olga Sorkine and Marc Alexa. 2007. **As-rigid-as-possible surface modeling**. In *Proceedings of the fifth Eurographics symposium on Geometry processing* (SGP '07), Eurographics Association, Goslar, DEU, 109–116.

蒙皮变形

Skinning

What is covered

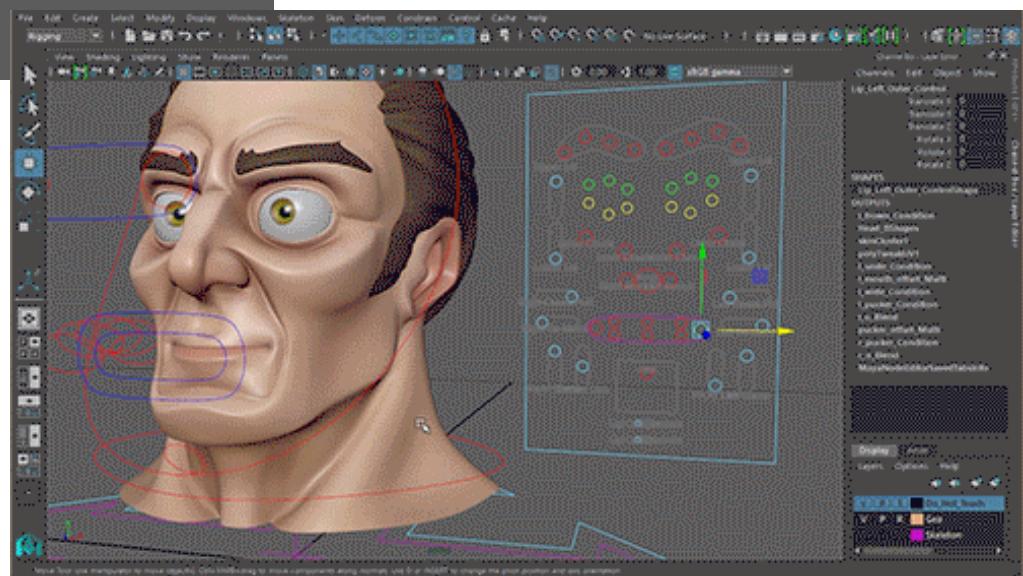
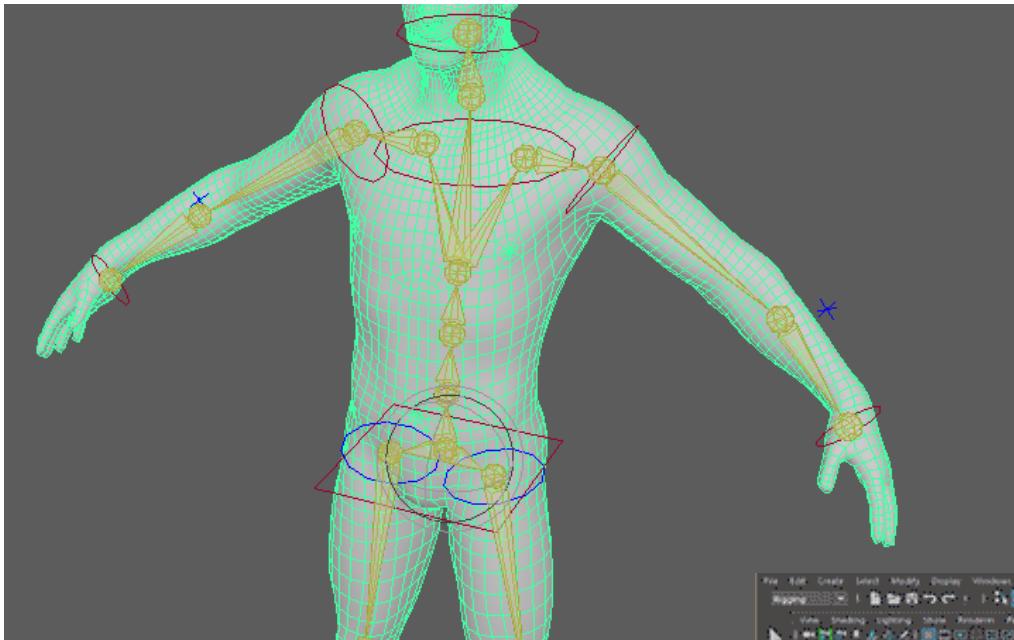
- How to deform a mesh according to skeletal animation?
 - Linear Blend Skinning (LBS)
 - Multi-linear Skinning
 - Non-linear Skinning
 - Dual-Quaternion Skinning (DQS)
- How to compute weights?
- Example-based methods?



Many images are from: <https://skinning.org/>

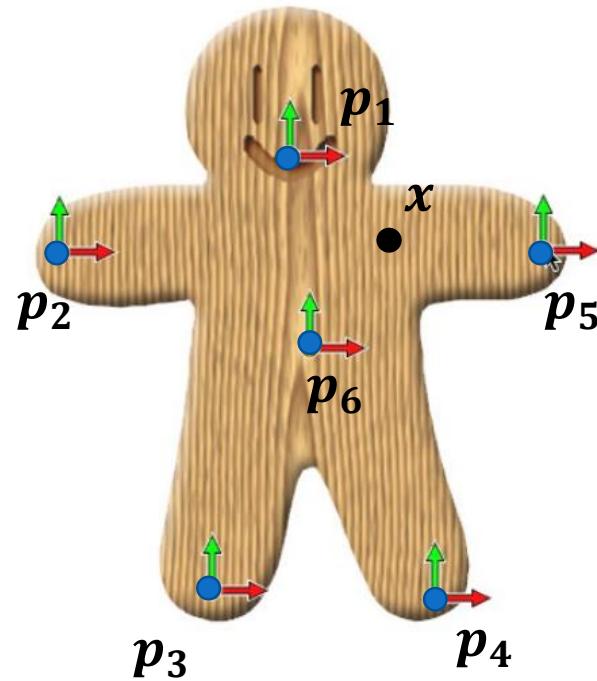
Alec Jacobson, Zhigang Deng, Ladislav Kavan, and J. P. Lewis. 2014. *Skinning: real-time shape deformation*. In ACM SIGGRAPH 2014 Courses (SIGGRAPH '14)

Skinning



Recall: deformation with control points

$$x'_i = \sum_j \omega_{ij} T_j x_i$$

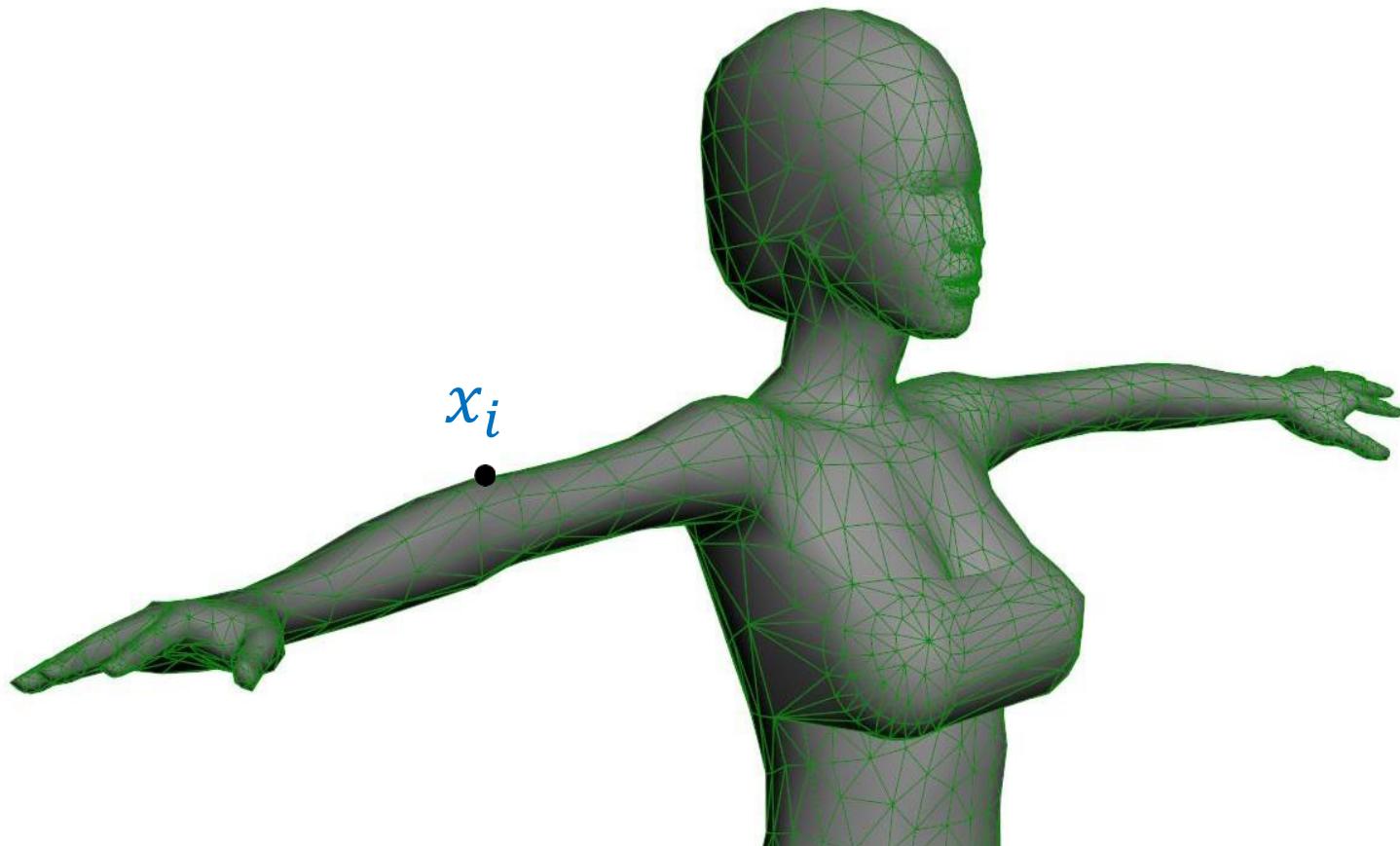


Basic Setup

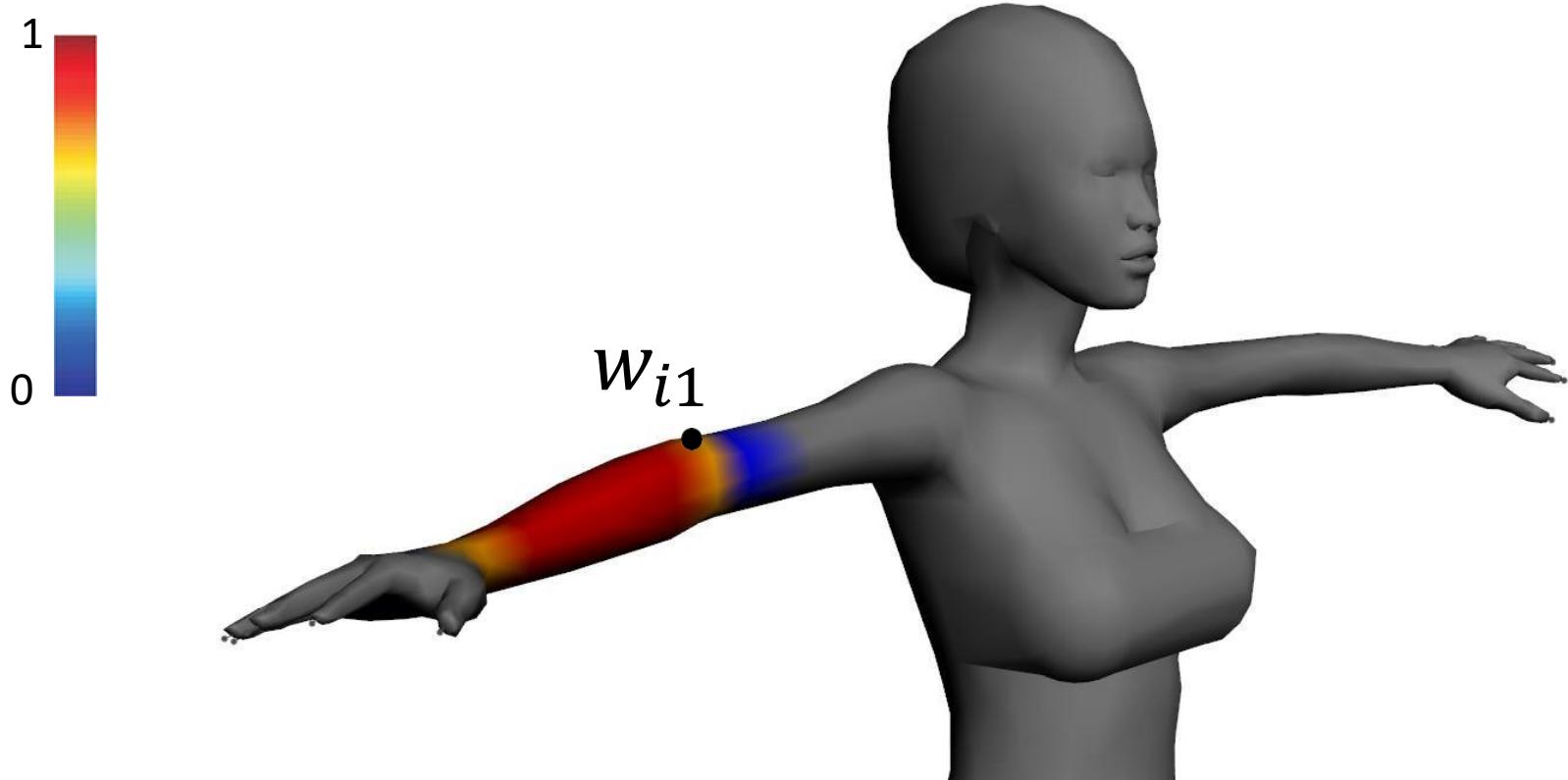
- Rest pose
- Skinning weights
- Skinning transformation

$$x'_i = \sum_{j=1}^m \omega_{ij} T_j x_i$$

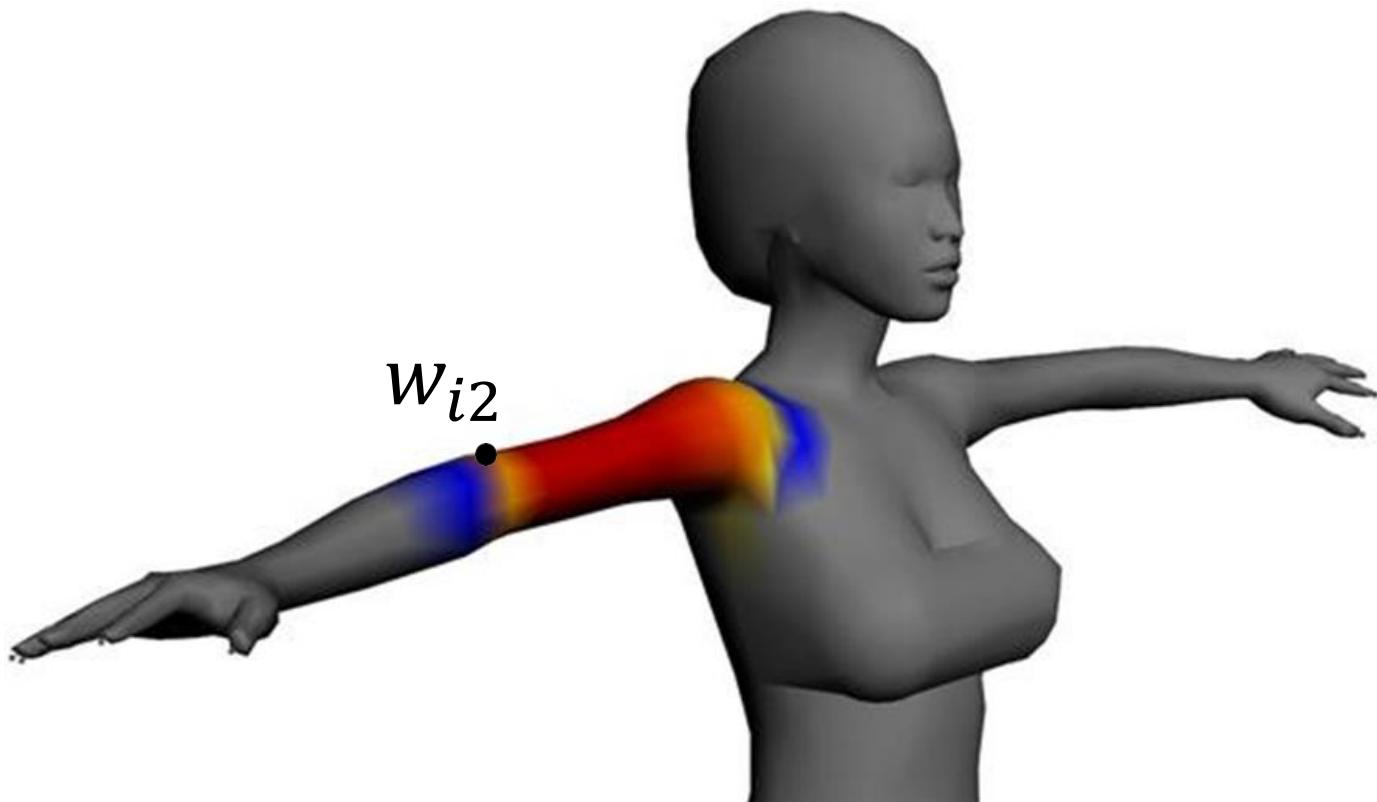
Basic Setup: Rest Pose/Bind Pose



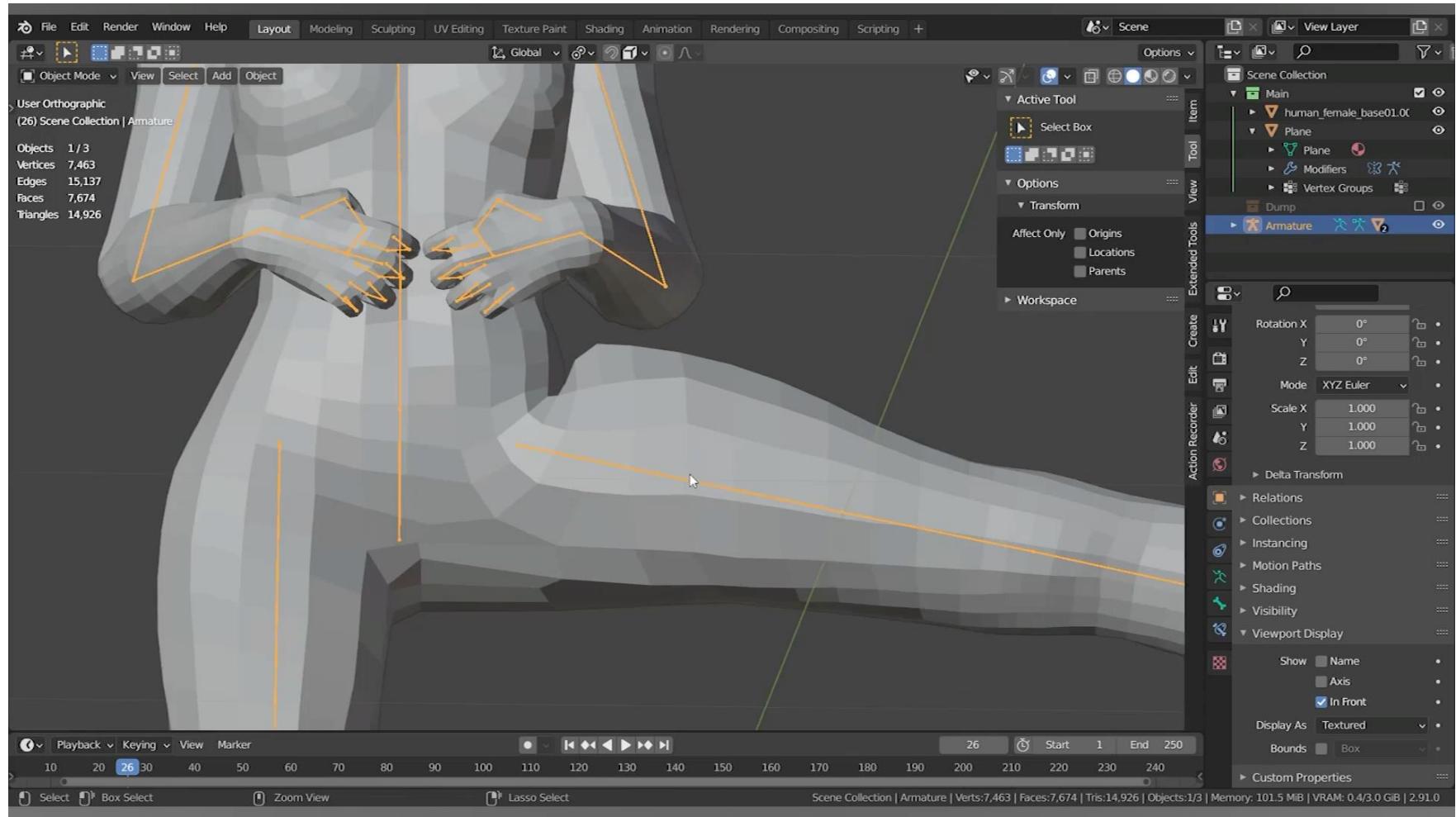
Basic Setup: Skinning Weights



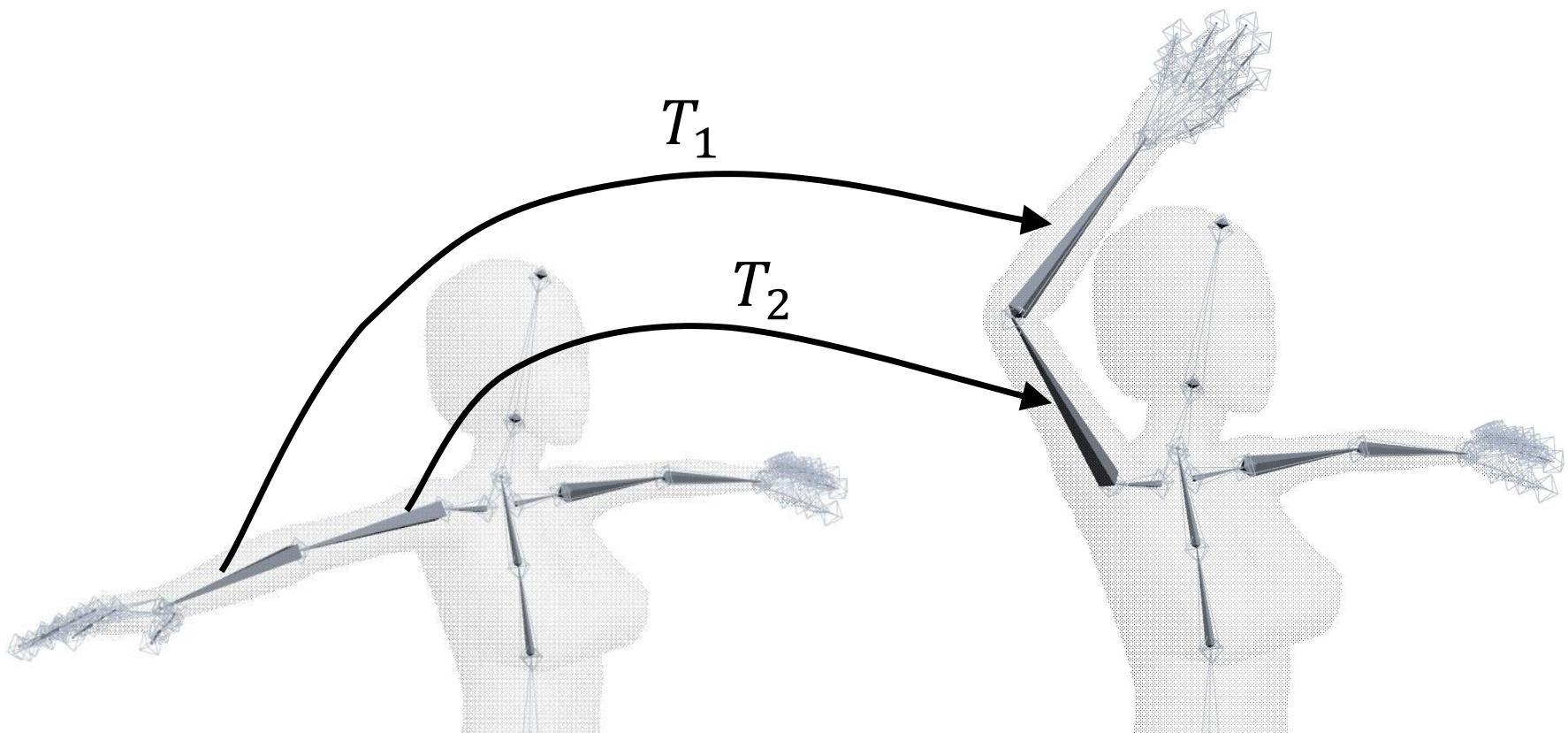
Basic Setup: Skinning Weights



Basic Setup: Skinning Weights

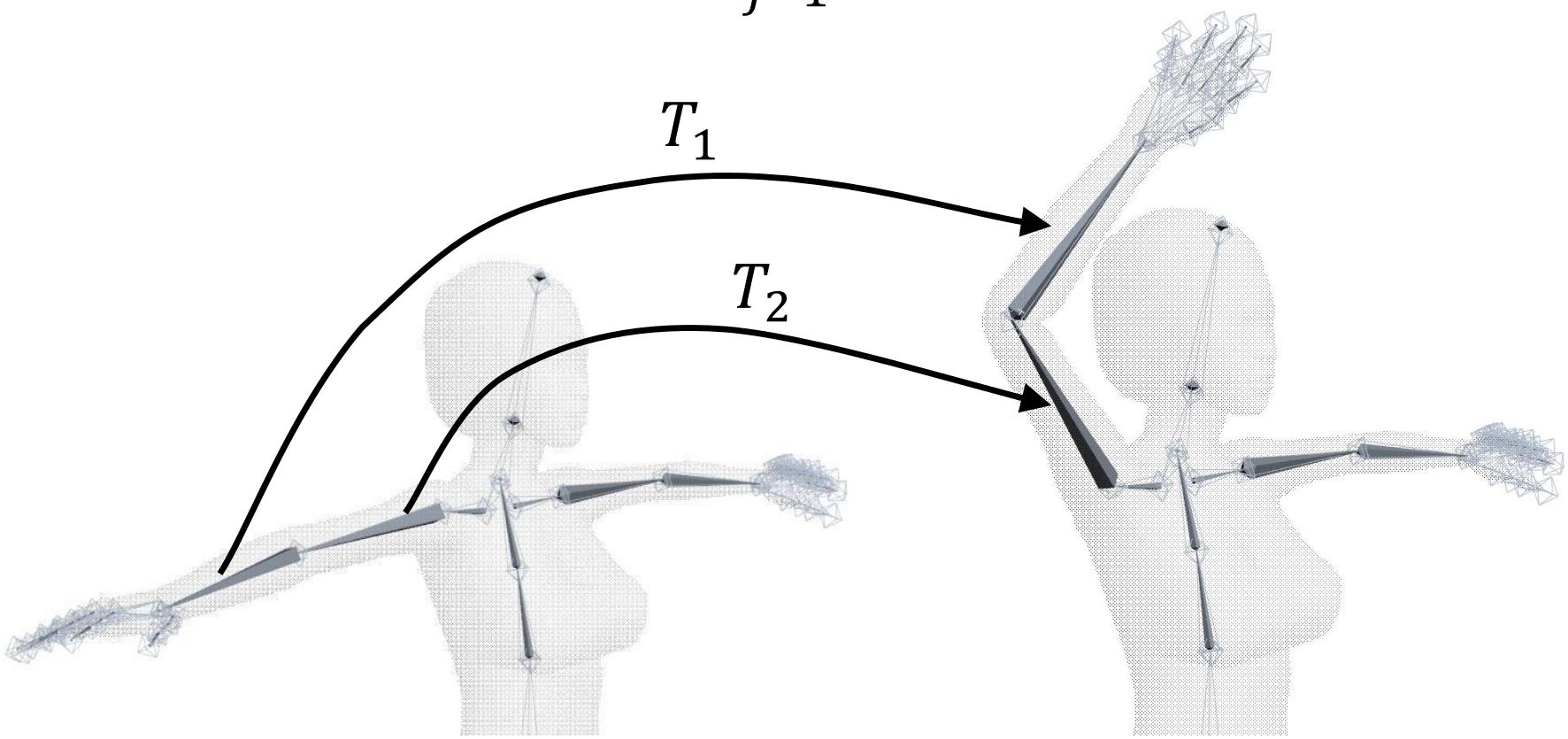


Basic Setup: Skinning Transformation



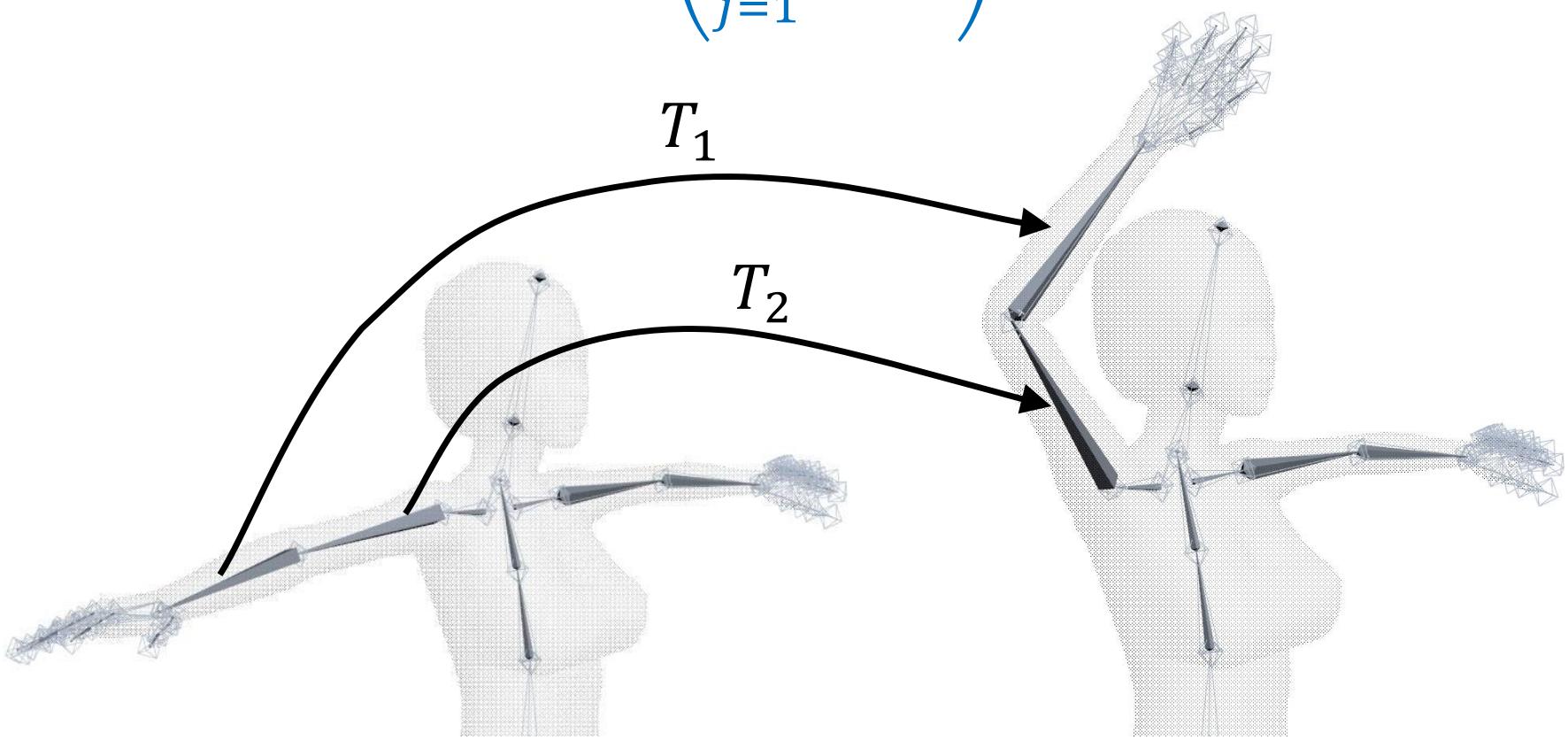
Linear Blend Skinning (LBS)

$$x'_i = \sum_{j=1}^m \omega_{ij} T_j x_i$$



Linear Blend Skinning (LBS)

$$x'_i = \left(\sum_{j=1}^m \omega_{ij} T_j \right) x_i$$



Linear Blend Skinning (LBS)

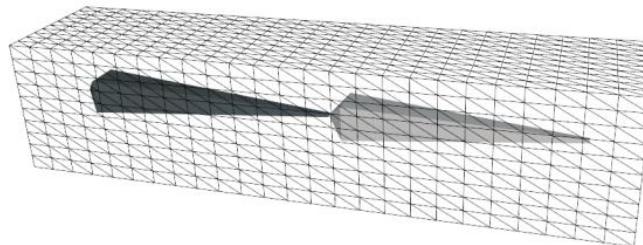
$$x'_i = \left(\sum_{j=1}^m \omega_{ij} T_j \right) x_i$$

- Used widely in the industry
- Efficient and GPU-friendly
 - Games like it

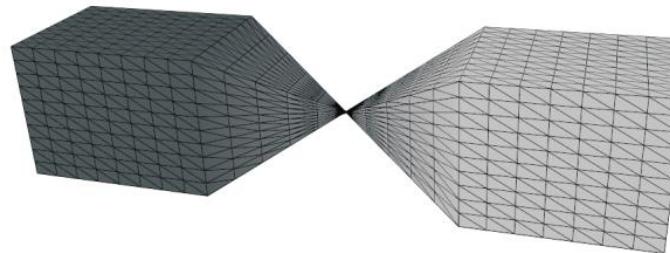


Halo 3

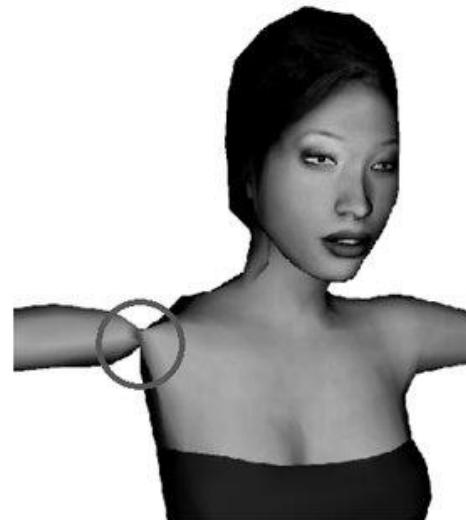
Candy-Wrapper Artifact



Rest pose



Linear blend skinning



Candy-Wrapper Artifact



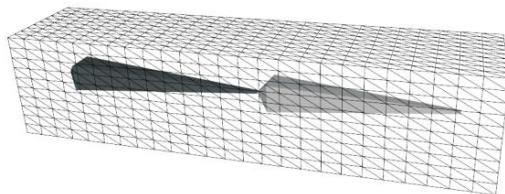
$$x'_i = \left(\sum_{j=1}^m \omega_{ij} T_j \right) x_i$$

Consider $T = (R \quad t)$ and

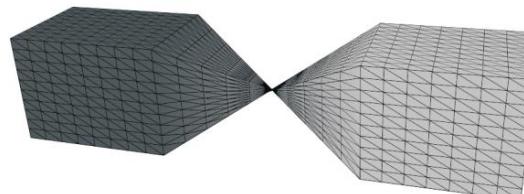
$$\mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Advanced skinning methods

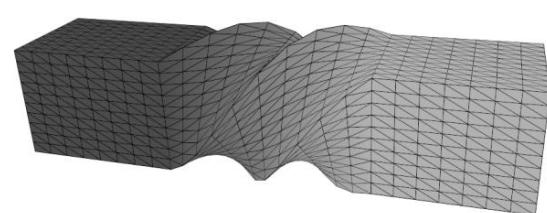
- Multi-linear Skinning
- Nonlinear Skinning
- Dual-quaternion Skinning (DQS)



Rest pose



Linear blend skinning



Dual quaternion skinning

Multi-linear Skinning

- Multi-weight enveloping [Wang and Phillips 2002]
- Animation Space [Merry et al. 2006]

Multi-linear Skinning

$$x' = Tx$$

$$T = [R \mid u] \in \mathbb{R}^{3 \times 4}$$



$$v' = Xt$$

$v' \in \mathbb{R}^{3n}$: stack of all vertices x'

$t \in \mathbb{R}^{12m}$: stack of all transformation T

$X \in \mathbb{R}^{3n \times 12m}$: skinning matrix

Multi-linear Skinning

The structure of matrix X

$$X \in \mathbb{R}^{3n \times 12m} \quad X = \begin{bmatrix} X_{11} & \cdots & X_{1m} \\ \vdots & \ddots & \vdots \\ X_{n1} & \cdots & X_{nm} \end{bmatrix}$$

$$X_{ij} \in \mathbb{R}^{3 \times 12}$$

Multi-linear Skinning

Matrix vectorization

$$\text{vec}(\mathbf{A}) = (a_{11}, a_{12}, a_{13}, \dots, a_{mn})^T$$

Kronecker product

$$\mathbf{A} \in \mathbb{R}^{p \times q}, \quad \mathbf{B} \in \mathbb{R}^{r \times s}$$

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}B & \cdots & a_{1q}B \\ \vdots & \ddots & \vdots \\ a_{p1}B & \cdots & a_{pq}B \end{bmatrix} \in \mathbb{R}^{pr \times qs}$$

$$\text{vec}(\mathbf{AB}) = (\mathbf{I}_s \otimes \mathbf{A}) \text{vec}(\mathbf{B}) = (\mathbf{B}^T \otimes \mathbf{I}_p) \text{vec}(\mathbf{A})$$

Multi-linear Skinning

$$x'_i = \sum_{j=1}^m \omega_{ij} T_j x_i$$



$$x'_i = \text{vec}(x'_i) = \sum_{j=1}^m \text{vec}(T_j \omega_{ij} x_i) = \sum_{j=1}^m (\omega_{ij} x_i \otimes I) \text{vec}(T_j)$$

$$= \sum_{j=1}^m X_{ij}^{LBS} t_j = X_i^{LBS} t$$

LBS in matrix form

$$X_{ij}^{LBS} = [w_{ij}x_{i1}\mathbf{I}_3 \quad w_{ij}x_{i2}\mathbf{I}_3 \quad w_{ij}x_{i3}\mathbf{I}_3 \quad w_{ij}\mathbf{I}_3]$$

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix}$$

$$\begin{bmatrix} w_{ij}x_{i1} & 0 & 0 & w_{ij}x_{i2} & 0 & 0 & w_{ij}x_{i3} & 0 & 0 & w_{ij} & 0 & 0 \\ 0 & w_{ij}x_{i1} & 0 & 0 & w_{ij}x_{i2} & 0 & 0 & w_{ij}x_{i3} & 0 & 0 & w_{ij} & 0 \\ 0 & 0 & w_{ij}x_{i1} & 0 & 0 & w_{ij}x_{i2} & 0 & 0 & w_{ij}x_{i3} & 0 & 0 & w_{ij} \end{bmatrix}$$

LBS has one weight per vertex/bone pair

Multi-linear Skinning

$$X_{ij} = \begin{bmatrix} * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \end{bmatrix}$$
$$\in \mathbb{R}^{3 \times 12}$$

We can have 36 weights per vertex/bone pair in the most general form

Multi-weight Enveloping [Wang and Phillips 2002]

$X_{ij}^{MWE} =$

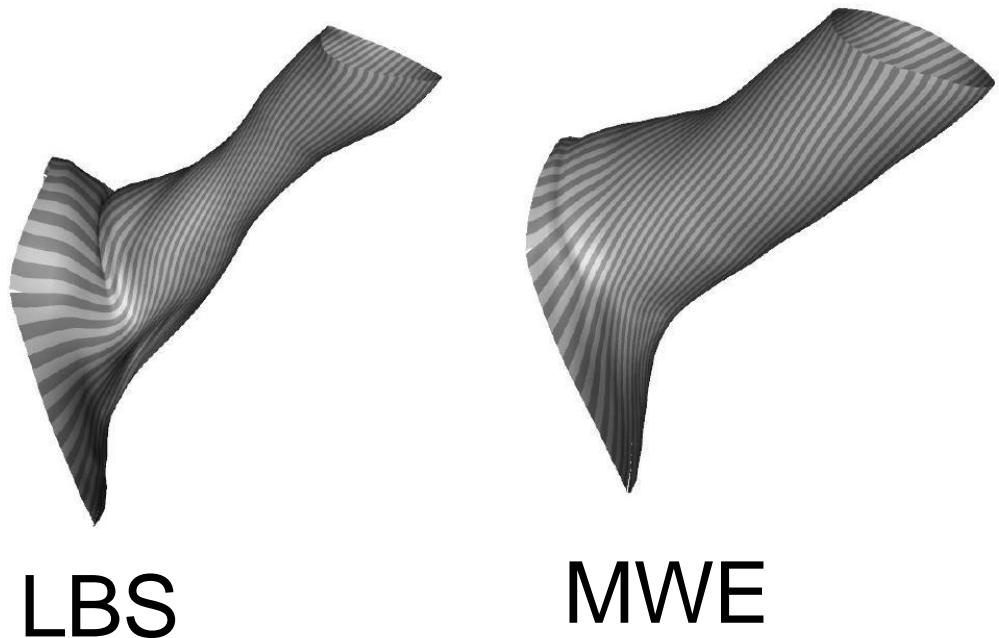
$$\begin{bmatrix} w_{ij}^1 x_{i1} & 0 & 0 & w_{ij}^4 x_{i2} & 0 & 0 & w_{ij}^7 x_{i3} & 0 & 0 & w_{ij}^{10} & 0 & 0 \\ 0 & w_{ij}^2 x_{i1} & 0 & 0 & w_{ij}^5 x_{i2} & 0 & 0 & w_{ij}^8 x_{i3} & 0 & 0 & w_{ij}^{11} & 0 \\ 0 & 0 & w_{ij}^3 x_{i1} & 0 & 0 & w_{ij}^6 x_{i2} & 0 & 0 & w_{ij}^9 x_{i3} & 0 & 0 & w_{ij}^{12} \end{bmatrix}$$

12 weights per vertex/bone

Multi-weight Enveloping [Wang and Phillips 2002]

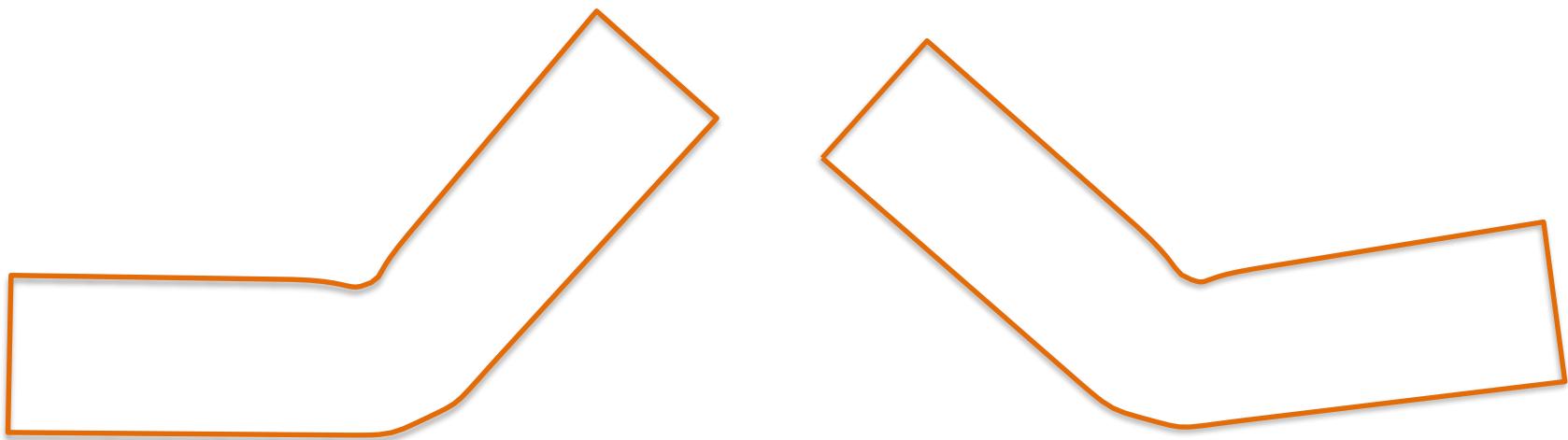
More powerful

Weights trained
from examples



Animation Space [Merry et al. 2006]

World-space rotation invariance.



Animation Space [Merry et al. 2006]

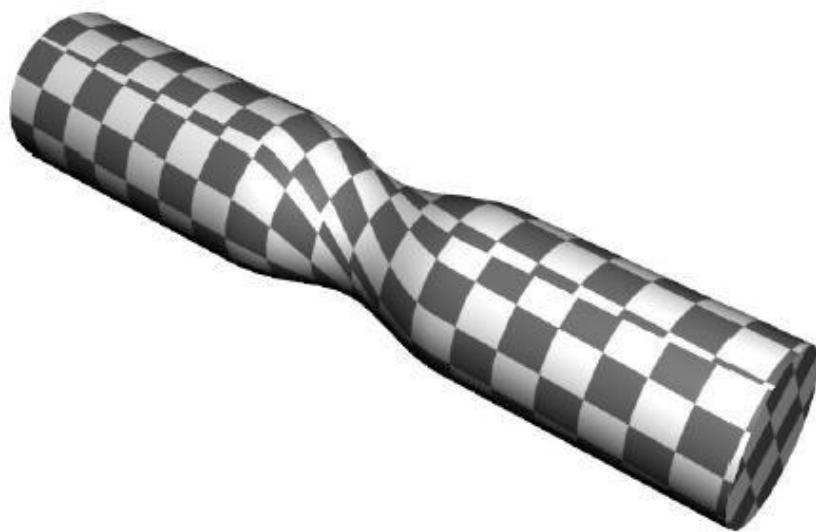
Rotation-invariant results in 4 weights per vertex/bone pair

$$X_{ij}^{LBS} = [y_{ij1}\mathbf{I}_3 \quad y_{ij2}\mathbf{I}_3 \quad y_{ij3}\mathbf{I}_3 \quad y_{ij4}\mathbf{I}_3]$$

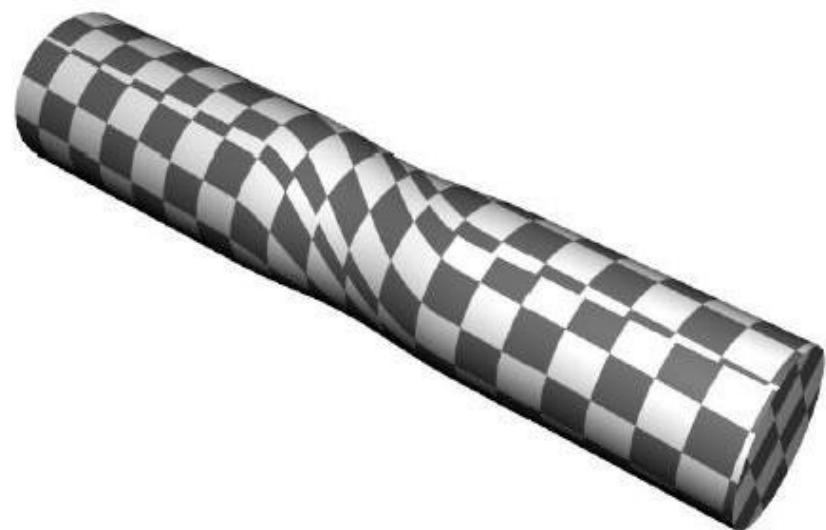
And translation invariance requires:

$$\sum_{j=1}^m y_{ij4} = 1$$

Animation Space [Merry et al. 2006]



LBS



Animation space

Non-linear Skinning

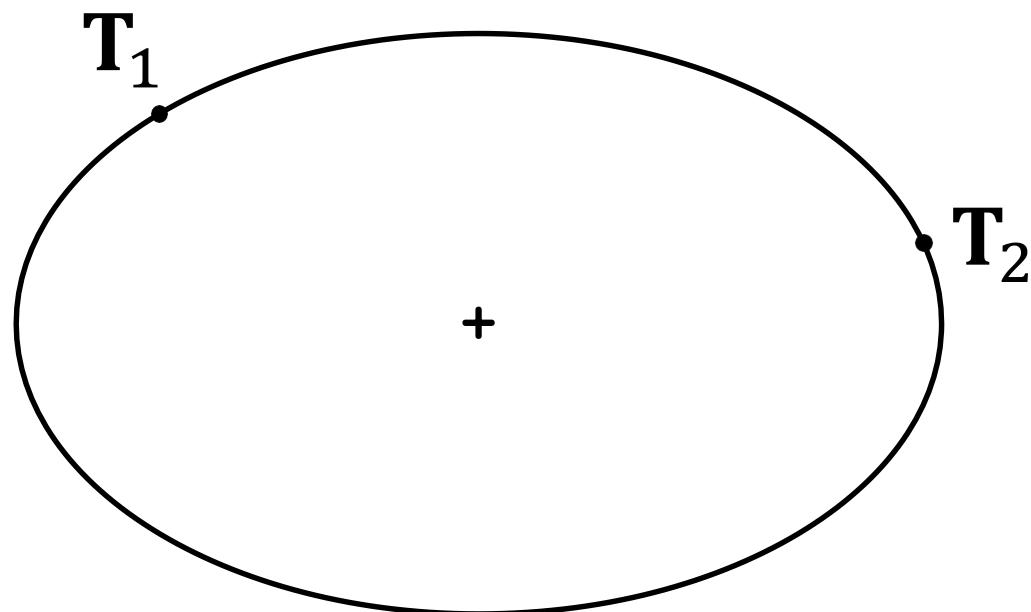
$$x'_i = \left(\sum_{j=1}^m \omega_{ij} T_j \right) x_i$$

$T_j \in SE(3)$: 刚性变换群

$R \in SO(3)$: 旋转变换群

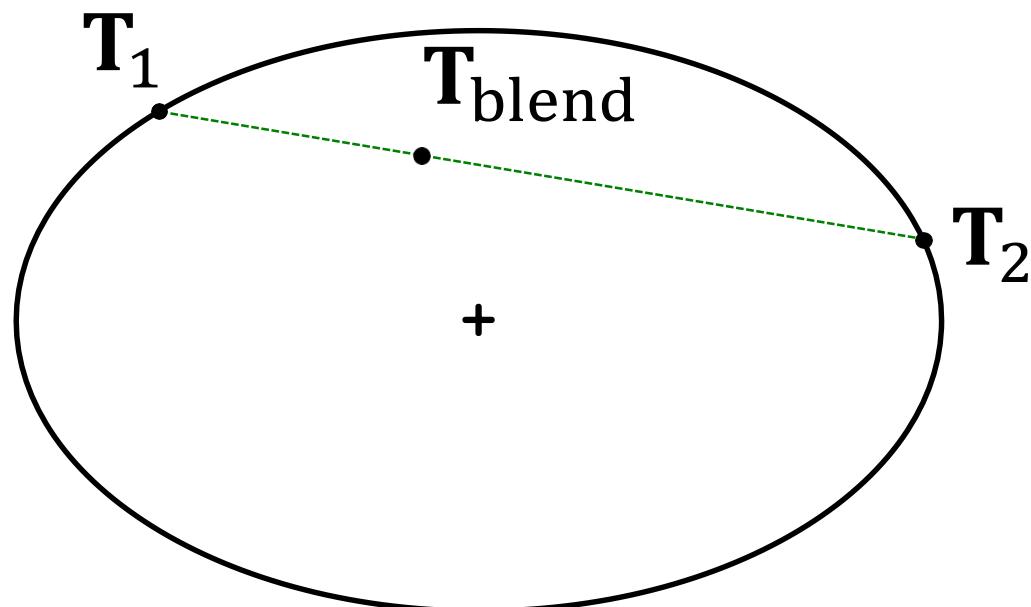
Interpolation in SE(3)

SE(3)



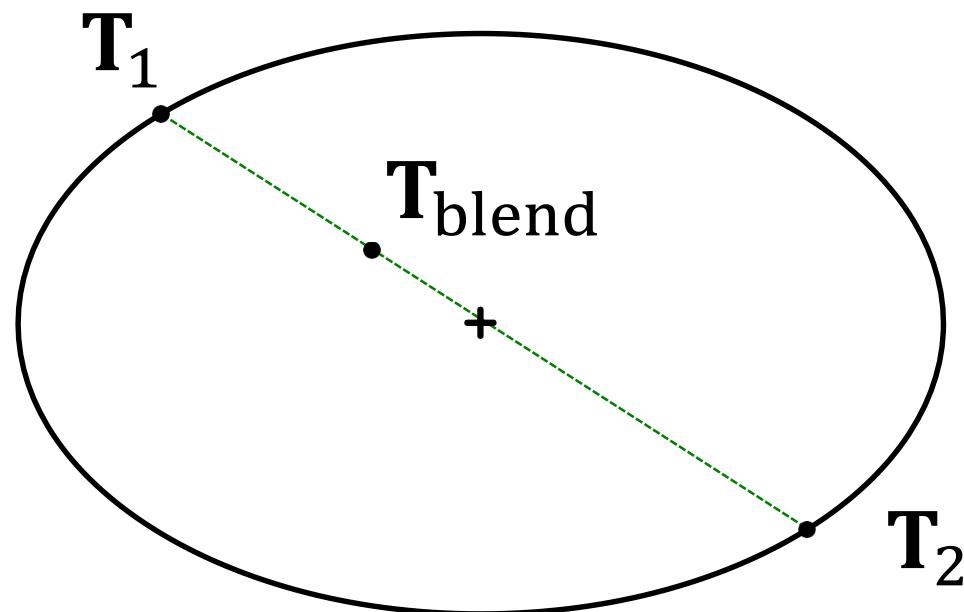
Interpolation in SE(3)

SE(3)

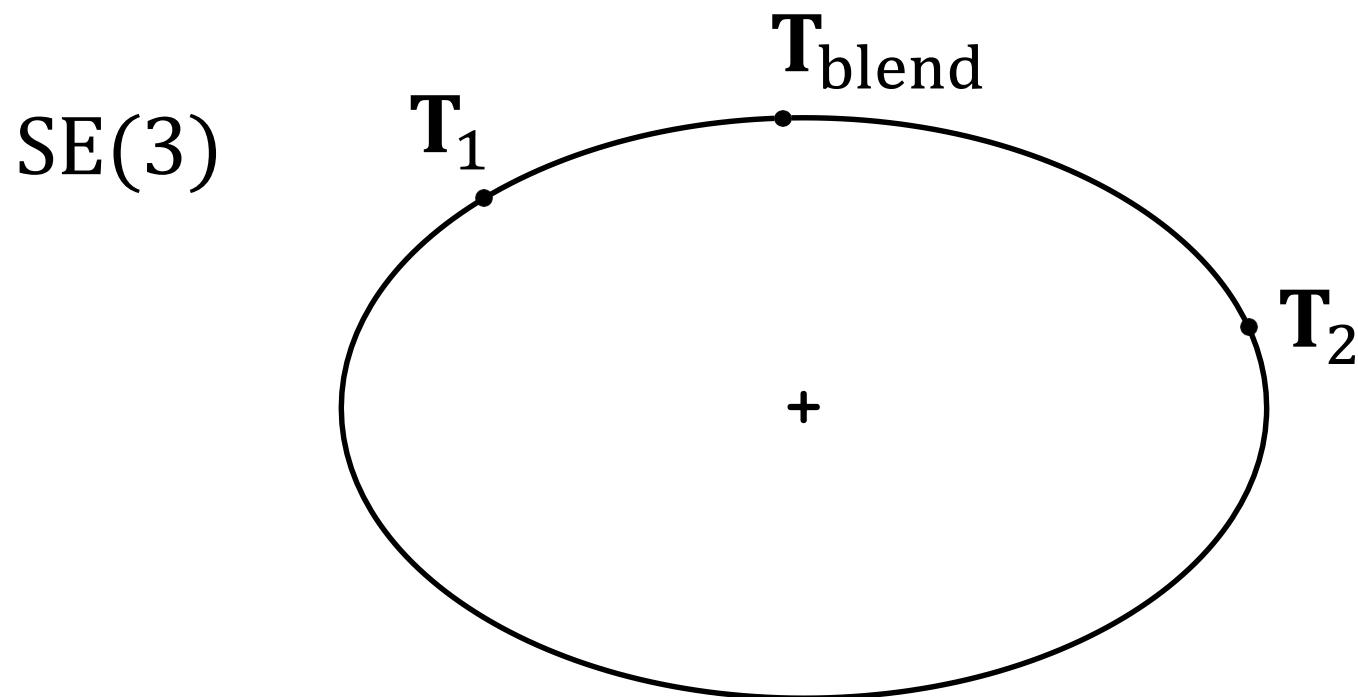


Interpolation in SE(3)

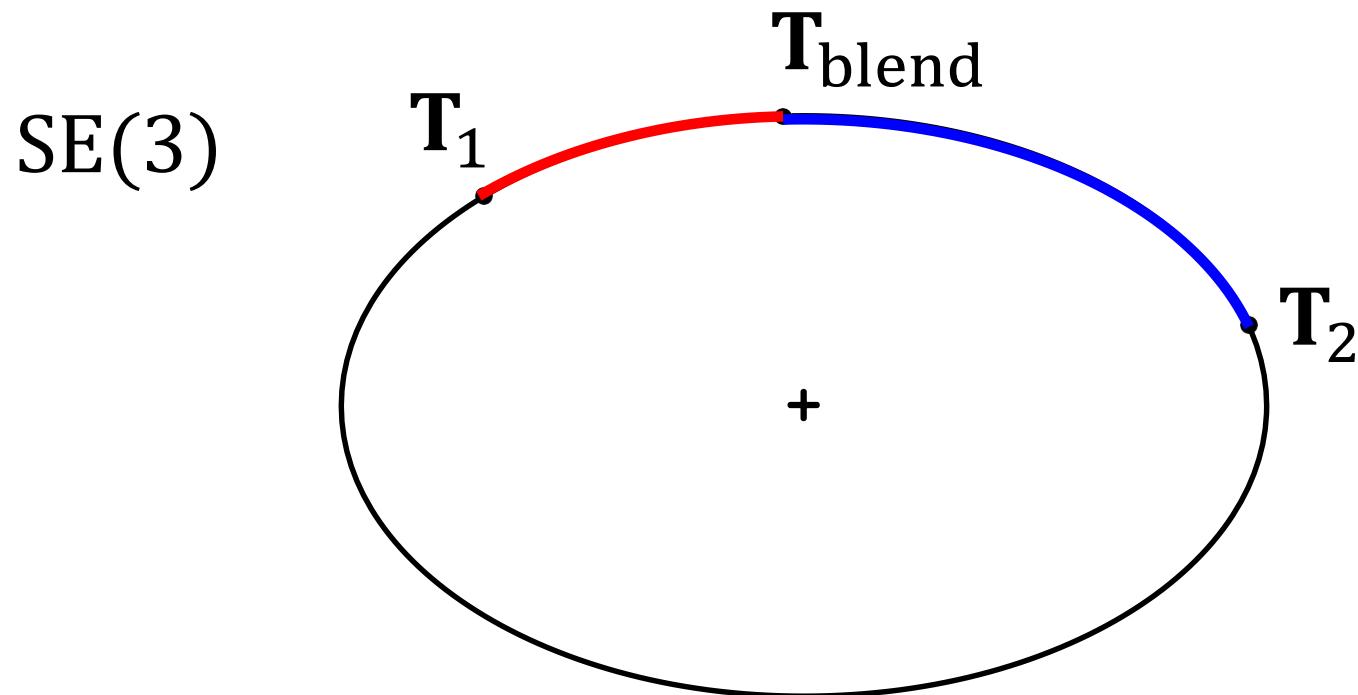
SE(3)



Intrinsic blending

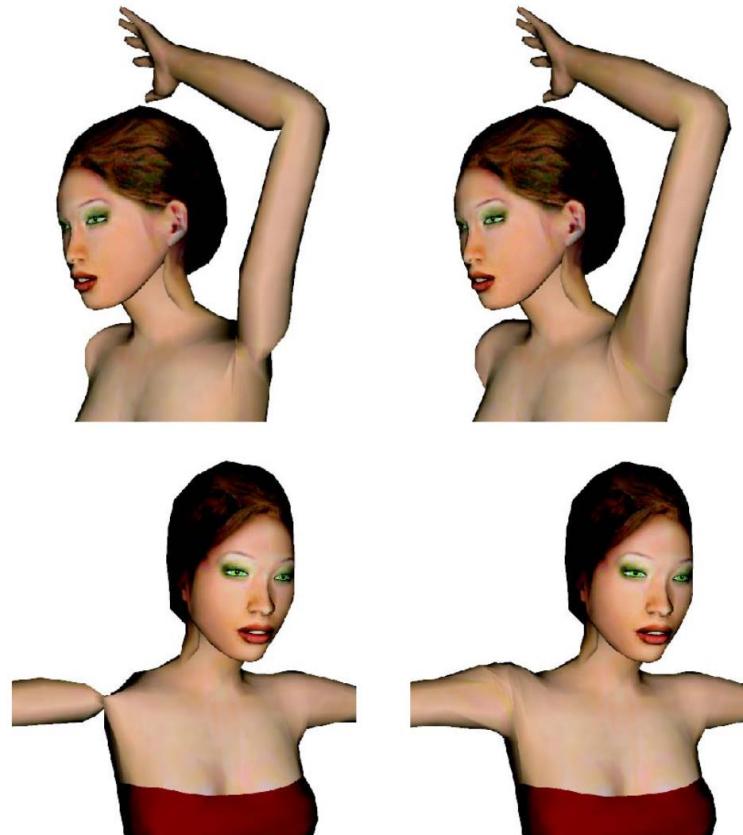


Intrinsic blending



Dual-Quaternion Skinning (DQS)

- Approximation of intrinsic averages in $SE(3)$



Ladislav Kavan, Steven Collins, Jiri Zara, Carol O'Sullivan. *Geometric Skinning with Approximate Dual Quaternion Blending*, ACM Transaction on Graphics, 27(4), 2008.

Dual-Quaternion

- Dual number

$$x = a + b\varepsilon$$

where $\varepsilon^2 = 0$

- Conjugate

$$\bar{x} = \overline{a + b\varepsilon} = a - b\varepsilon$$

- Multiplication

$$(a + b\varepsilon)(c + d\varepsilon) = ac + (ad + bc)\varepsilon$$

Dual-Quaternion

- Dual quaternion

$$\hat{q} = q_0 + \varepsilon q_\varepsilon$$

where $\varepsilon^2 = 0$

A good note of dual-quaternion:

<https://faculty.sites.iastate.edu/jia/files/inline-files/dual-quaternion.pdf>

Dual-Quaternion

- Scalar Multiplication

$$s\hat{\mathbf{q}} = s\mathbf{q}_r + s\mathbf{q}_\varepsilon\varepsilon$$

- Addition

$$\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2 = \mathbf{q}_{r1} + \mathbf{q}_{r2} + \varepsilon(\mathbf{q}_{\varepsilon 1} + \mathbf{q}_{\varepsilon 2})$$

- Multiplication

$$\hat{\mathbf{q}}_1 \hat{\mathbf{q}}_2 = \mathbf{q}_{r1}\mathbf{q}_{r2} + \varepsilon(\mathbf{q}_{r1}\mathbf{q}_{\varepsilon 2} + \mathbf{q}_{r2}\mathbf{q}_{\varepsilon 1})$$

Dual-Quaternion

- Dual quaternion

$$\hat{\mathbf{q}} = \mathbf{q}_0 + \varepsilon \mathbf{q}_\varepsilon$$

- Conjugation

$$\text{I: } \hat{\mathbf{q}}^* = \mathbf{q}_0^* + \varepsilon \mathbf{q}_\varepsilon^*$$

$$\text{II: } \hat{\mathbf{q}}^\circ = \mathbf{q}_0 - \varepsilon \mathbf{q}_\varepsilon$$

$$\begin{aligned}\text{III: } \hat{\mathbf{q}}^\star &= \mathbf{q}_0^* - \varepsilon \mathbf{q}_\varepsilon^* \\ &= (\hat{\mathbf{q}}^*)^\circ = (\hat{\mathbf{q}}^\circ)^*\end{aligned}$$

$$(\hat{\mathbf{q}}_1 \hat{\mathbf{q}}_2)^\times = \hat{\mathbf{q}}_2^\times \hat{\mathbf{q}}_1^\times$$

- Norm

$$\|\hat{\mathbf{q}}\| = \sqrt{\hat{\mathbf{q}}^* \hat{\mathbf{q}}} = \|\mathbf{q}_0\| + \frac{\varepsilon (\mathbf{q}_0 \cdot \mathbf{q}_\varepsilon)}{\|\mathbf{q}_0\|}$$

Dual-Quaternion

- Norm

$$\|\hat{q}\| = \sqrt{\hat{q}^* \hat{q}} = \|q_0\| + \frac{\varepsilon (q_0 \cdot q_\varepsilon)}{\|q_0\|}$$

- Unit dual quaternion: $\|\hat{q}\| = 1$

which requires:

- $\|q_0\| = 1$
- $q_0 \cdot q_\varepsilon = 0$

Dual-Quaternion

- Like quaternion, any rigid transformation $T \in SE(3)$ can be converted into a dual-quaternion

$$T\mathbf{x} = R\mathbf{x} + \mathbf{t}$$

$$T = [R \mid t] \rightarrow \hat{\mathbf{q}} = \mathbf{q}_0 + \varepsilon \mathbf{q}_\varepsilon$$

$$\mathbf{q}_0 = \mathbf{r} \quad \text{quaternion of } R$$

$$\mathbf{q}_\varepsilon = \frac{1}{2}\mathbf{t}\mathbf{r} \quad \text{pure quaternion } \mathbf{t} = (0, \mathbf{t})$$

Dual-Quaternion

- Transform a vector using unit dual quaternion

$$\hat{v}' = \hat{q}\hat{v}\hat{q}^*$$

Where

$$\begin{aligned}\hat{v} &= 1 + \varepsilon(0, v) \\ &= (1, 0, 0, 0) + \varepsilon(0, v_x, v_y, v_z)\end{aligned}$$

$$\begin{aligned}\text{III: } \hat{q}^* &= q_0^* - \varepsilon q_\varepsilon^* \\ &= (\hat{q}^*)^\circ = (\hat{q}^\circ)^*\end{aligned}$$

Dual-Quaternion

- Like quaternion, any rigid transformation $T \in SE(3)$ can be converted into a dual-quaternion

$$T = [R \mid t] \rightarrow \hat{\mathbf{q}} = \mathbf{q}_0 + \varepsilon \mathbf{q}_\varepsilon$$

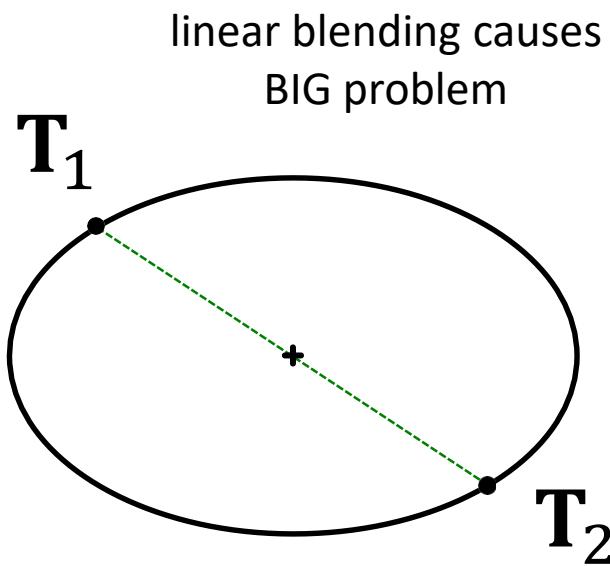
$$\mathbf{q}_0 = \mathbf{r}$$

$$\mathbf{q}_\varepsilon = \frac{1}{2} \mathbf{t} \mathbf{r}$$

$\hat{\mathbf{q}}$ and $-\hat{\mathbf{q}}$ represent the same transformation

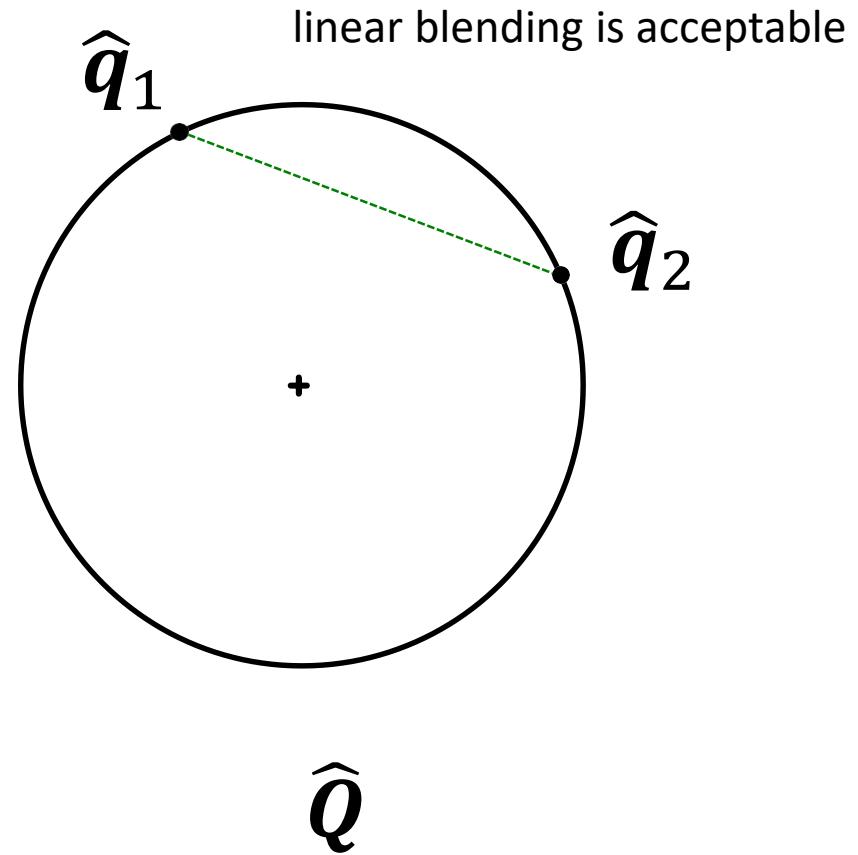
\widehat{Q} is a **double cover** of $SE(3)$

Double Cover Visualized



linear blending causes
BIG problem

$\text{SE}(3)$



linear blending is acceptable

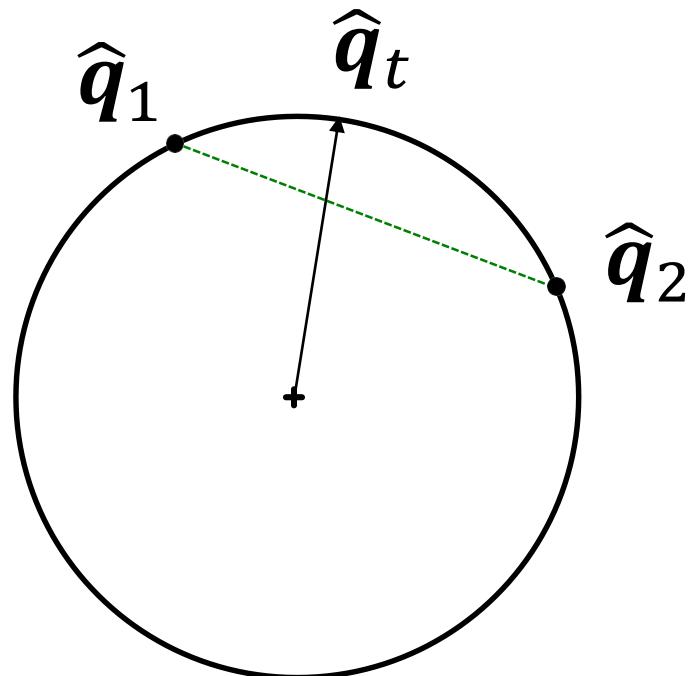
\hat{Q}

Interpolating Dual-Quaternion

$$\hat{q}_t = (1 - t)\hat{q}_0 + t\hat{q}_1$$



$$\hat{q}_t = \frac{(1 - t)\hat{q}_0 + t\hat{q}_1}{\|(1 - t)\hat{q}_0 + t\hat{q}_1\|}$$

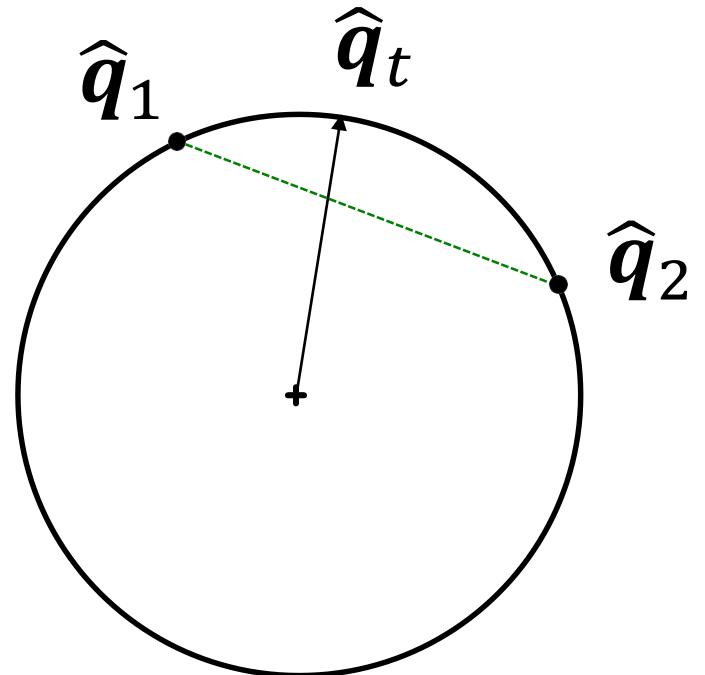


Dual-Quaternion Linear Blending

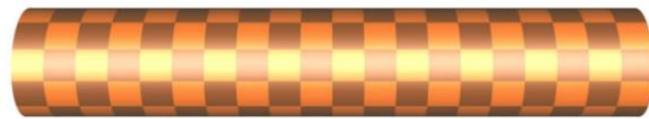
$$\hat{\mathbf{q}}_t = (1 - t)\hat{\mathbf{q}}_0 + t\hat{\mathbf{q}}_1$$



$$\hat{\mathbf{q}}_t = \frac{(1 - t)\hat{\mathbf{q}}_0 + t\hat{\mathbf{q}}_1}{\|(1 - t)\hat{\mathbf{q}}_0 + t\hat{\mathbf{q}}_1\|}$$



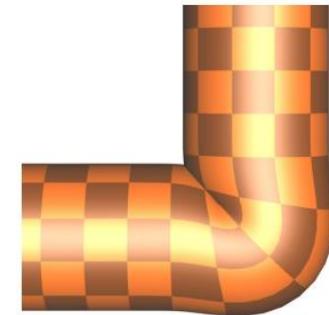
Dual-Quaternion Skinning (DQS)



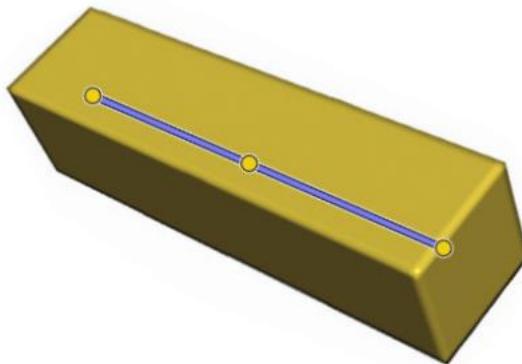
Rest pose



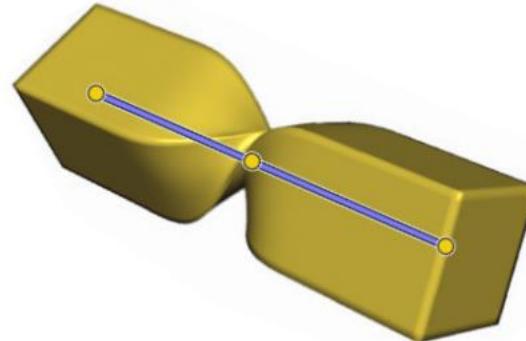
Dual quaternions: twist



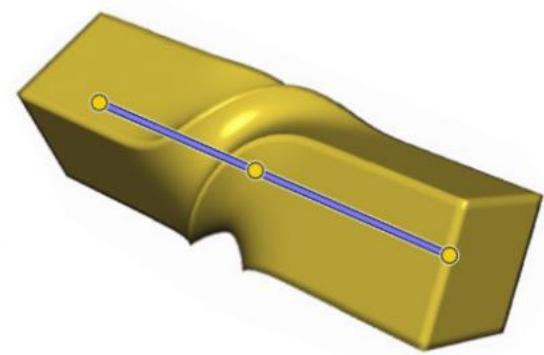
Dual quaternions: bend



Rest pose



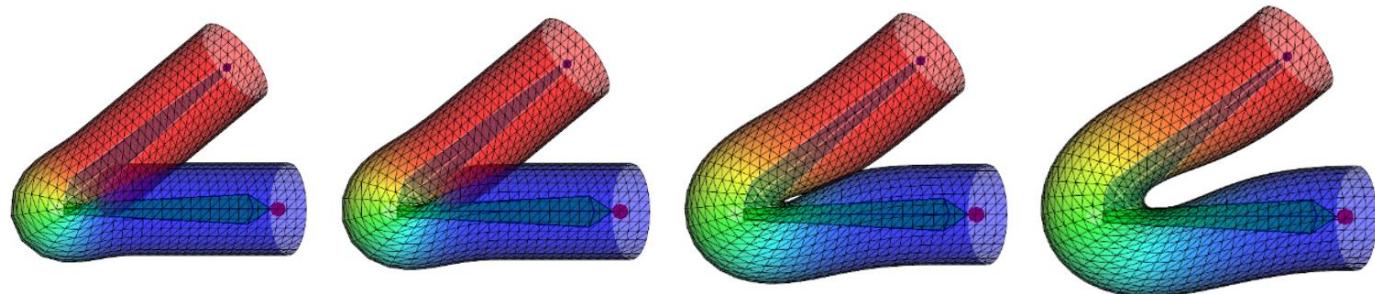
Linear blend skinning



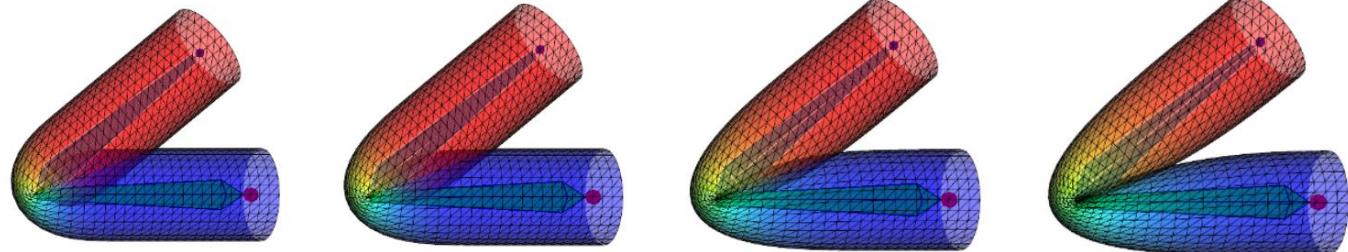
Dual quaternion skinning

Budging Artifact of DQS

Dual Quats



Linear
Blend



Weight
diffusion

+

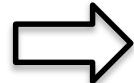
++

+++

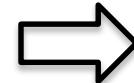
++++

Non-rigid transformations

1. Scaling



2. Rotations



Two-phase skinning

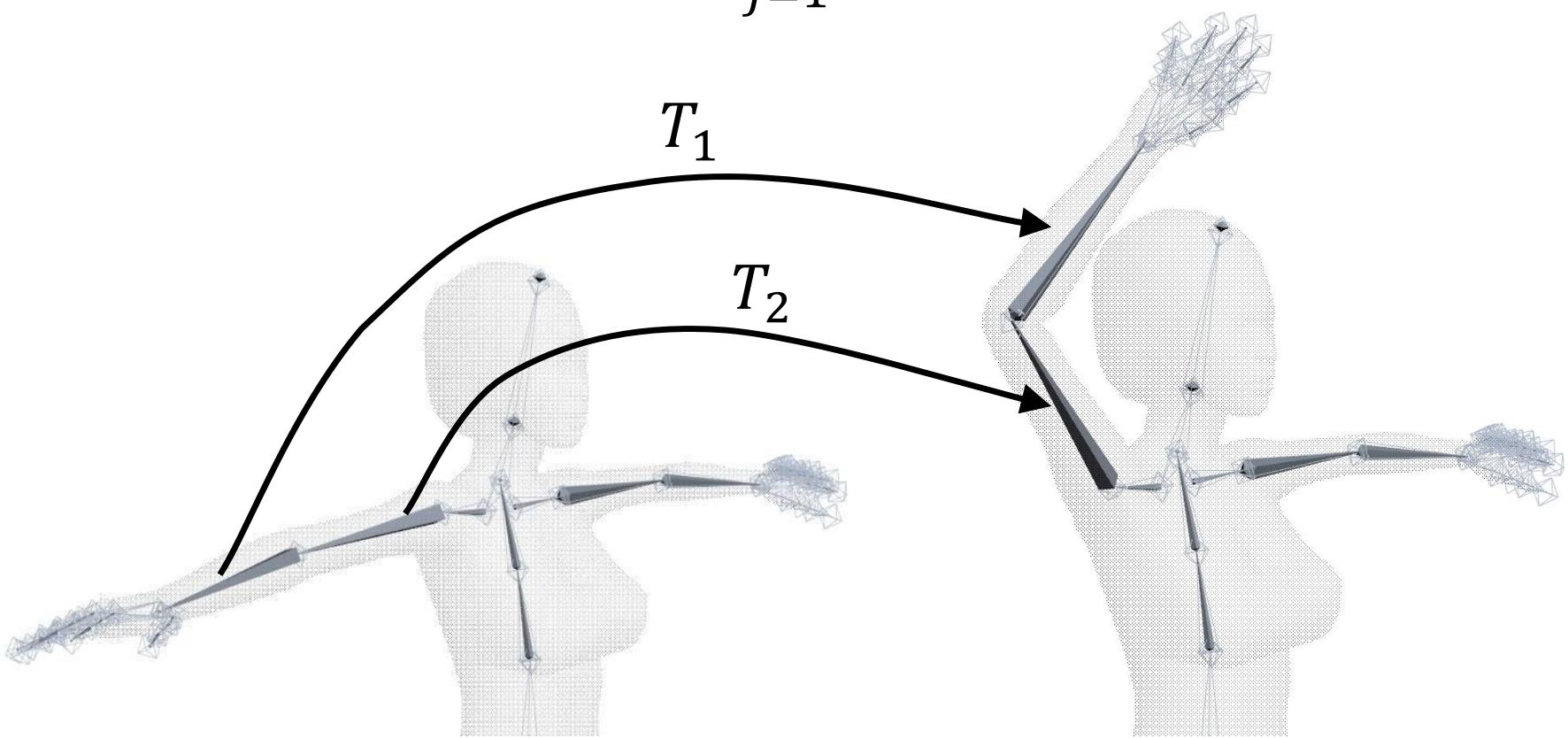
Enhanced for production use by Disney

[Lee et al. 2013]



Linear Blend Skinning (LBS)

$$x'_i = \sum_{j=1}^m \omega_{ij} T_j x_i$$



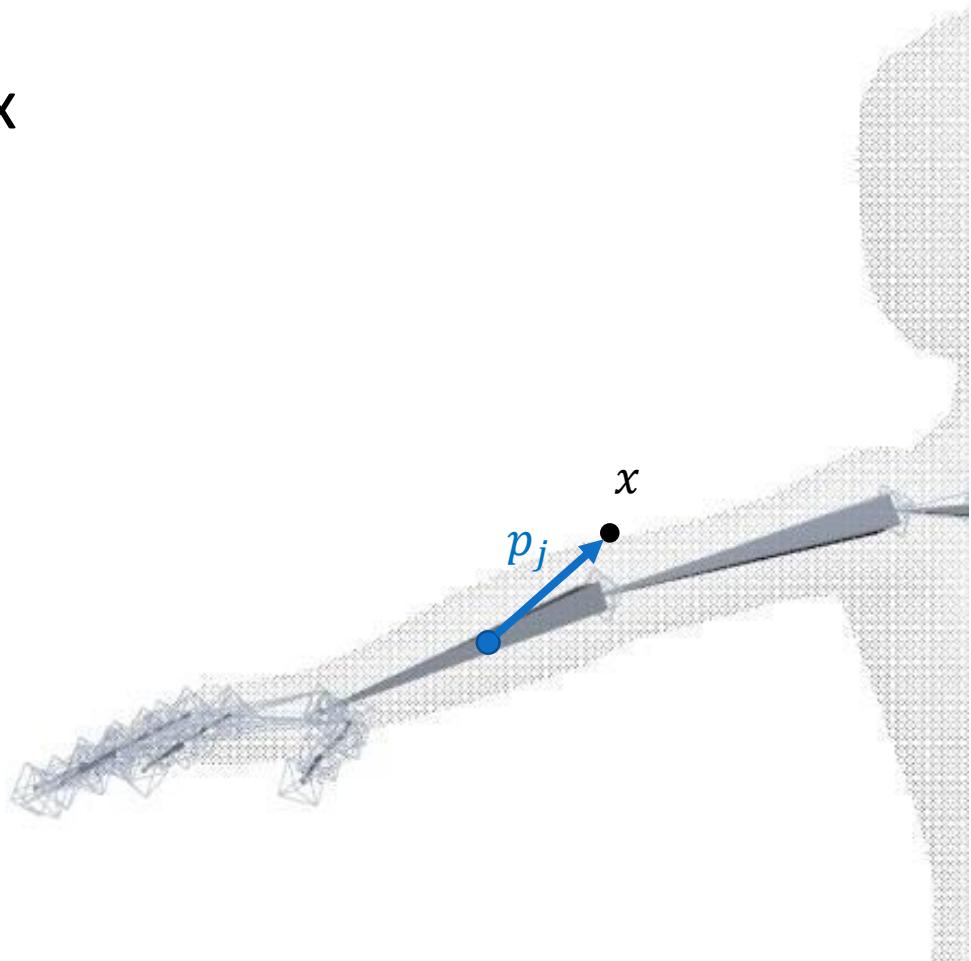
Linear Blend Skinning (LBS)

- Consider a single vertex

$$x' = \sum_{j=1}^m \omega_j T_j x$$

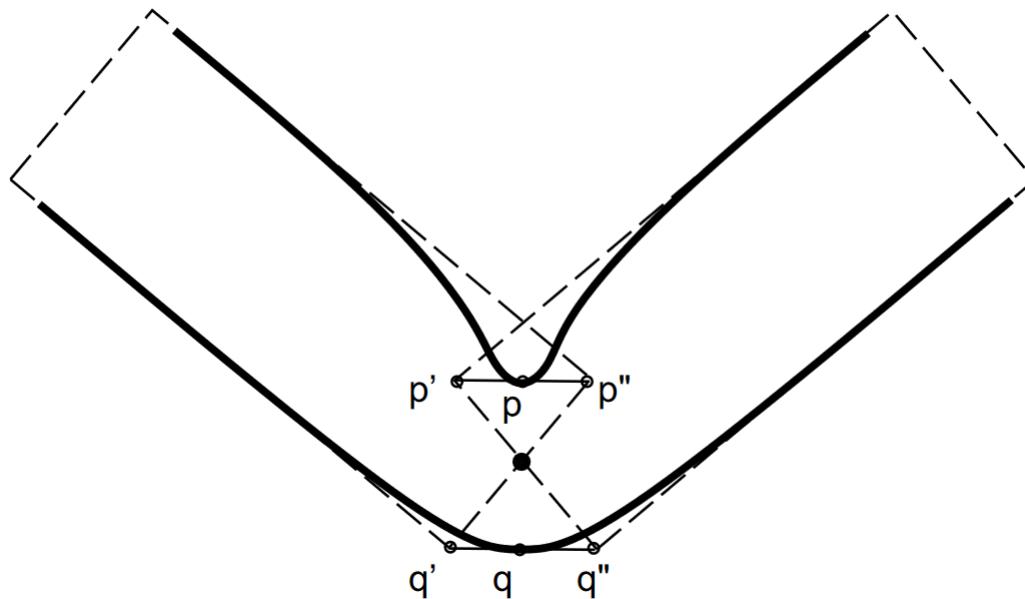
$$= \sum_{j=1}^m \omega_j T_j^\delta \hat{\mathbf{T}}_j^{-1} \mathbf{x}$$

$$= \sum_{j=1}^m \omega_j T_j^\delta \mathbf{p}_j$$



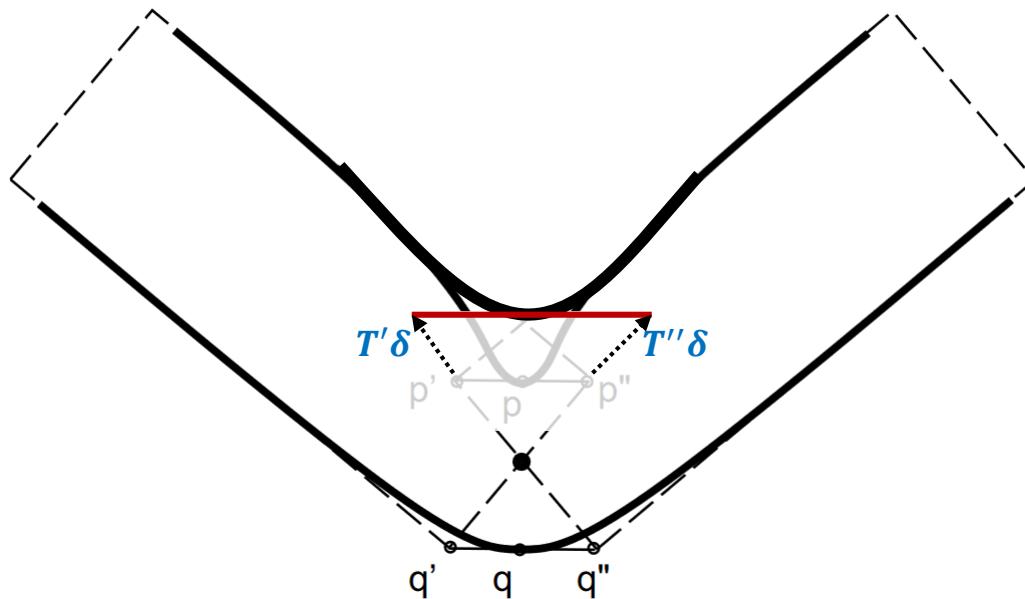
How to Correct LBS?

$$\boldsymbol{x}' = \sum_{j=1}^m \omega_j T_j \boldsymbol{x}$$



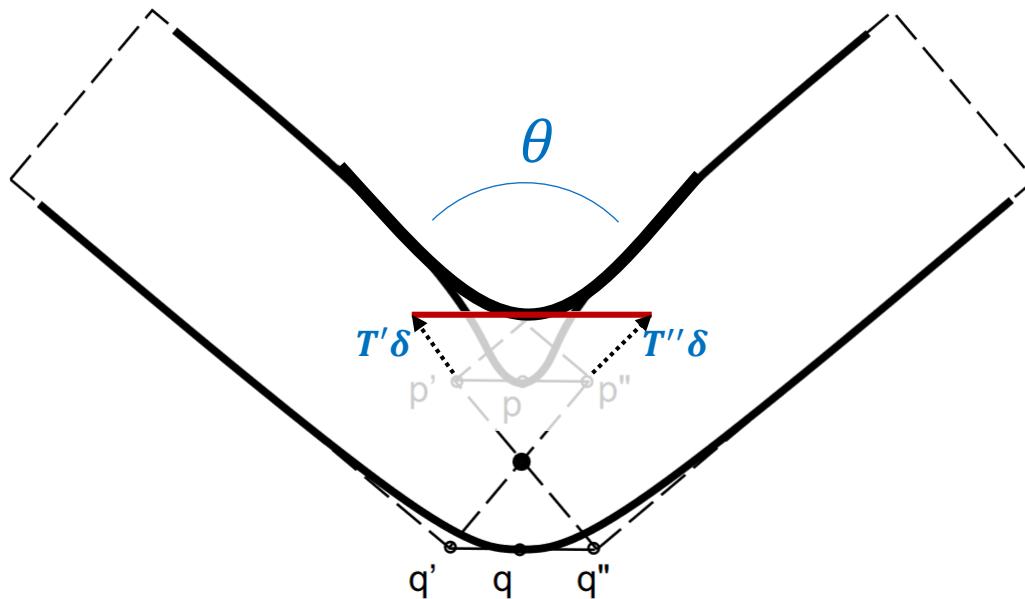
How to Correct LBS?

$$x' = \sum_{j=1}^m \omega_j T_j(\mathbf{x} + \boldsymbol{\delta}(x))$$



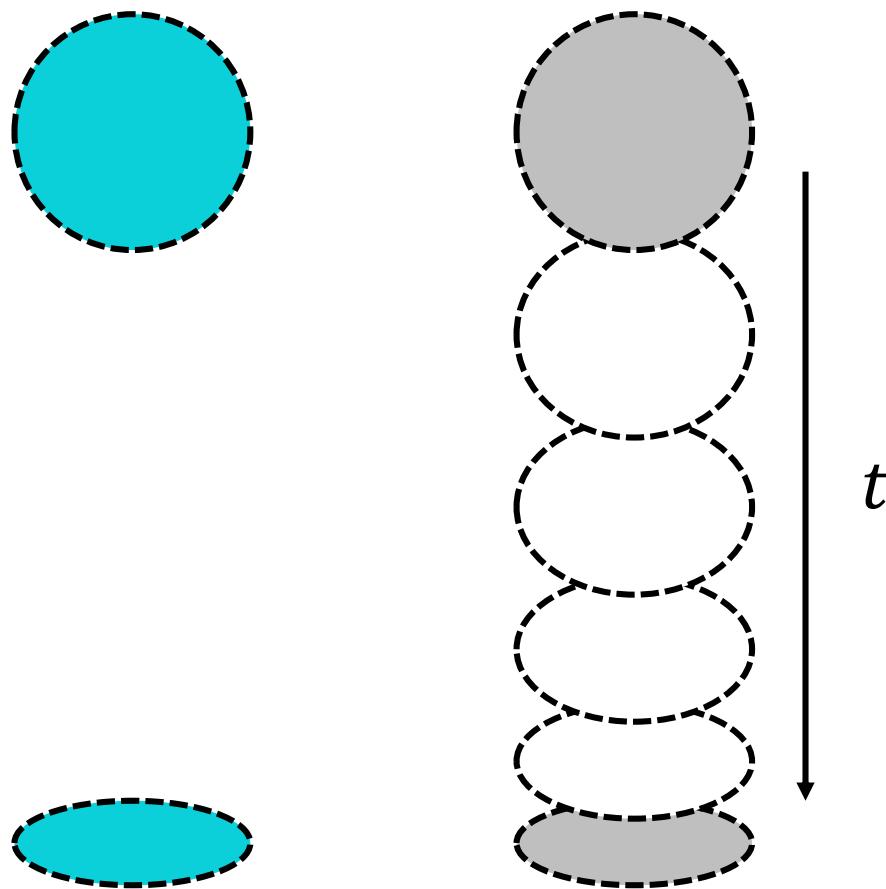
How to Correct LBS?

$$x' = \sum_{j=1}^m \omega_j T_j(x + \delta(x, \theta))$$

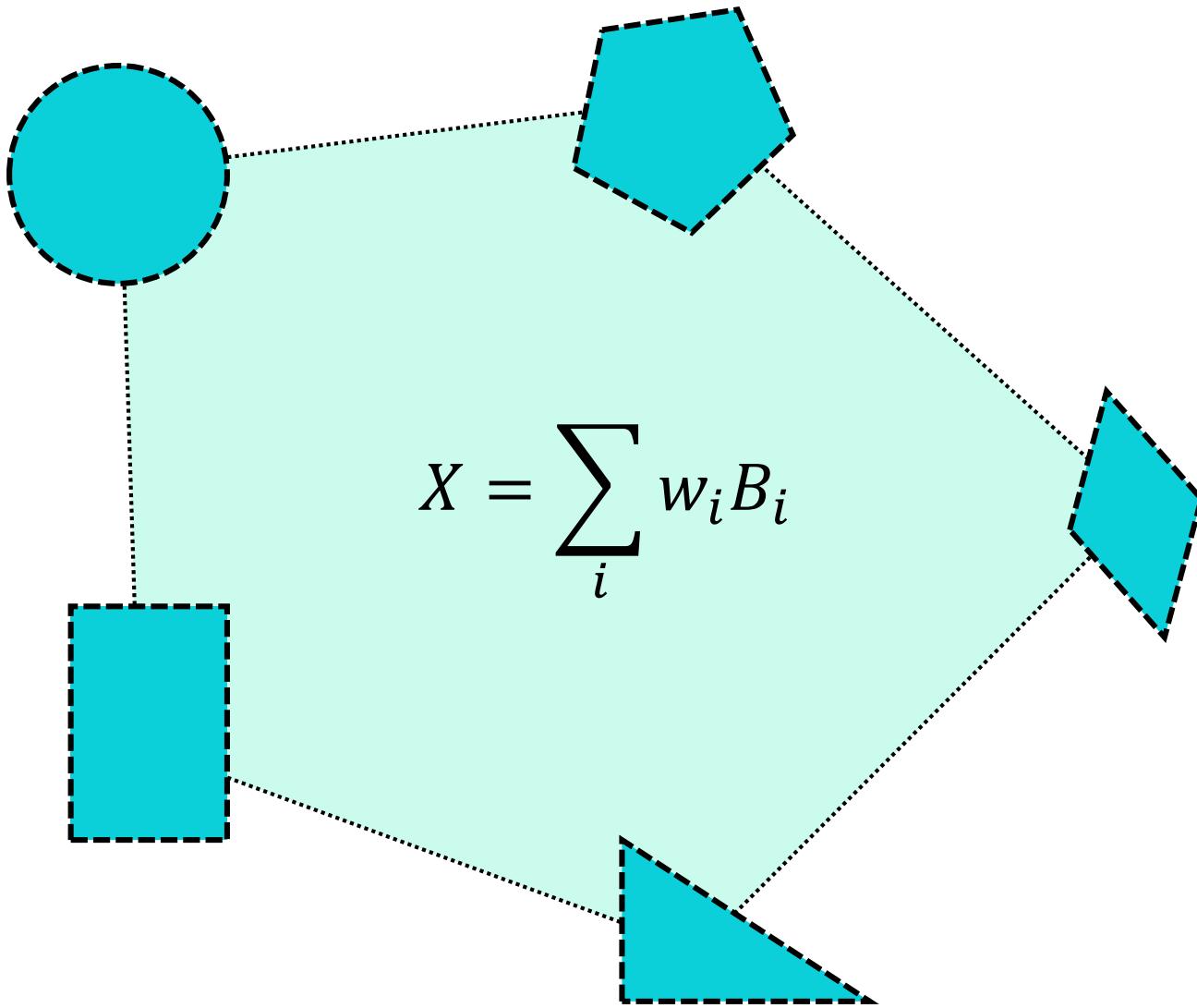


这样才值之后
不会下凹，从而
改善了原本的
skinny 牛模型
为了不改变取 $\delta(x - \theta)$

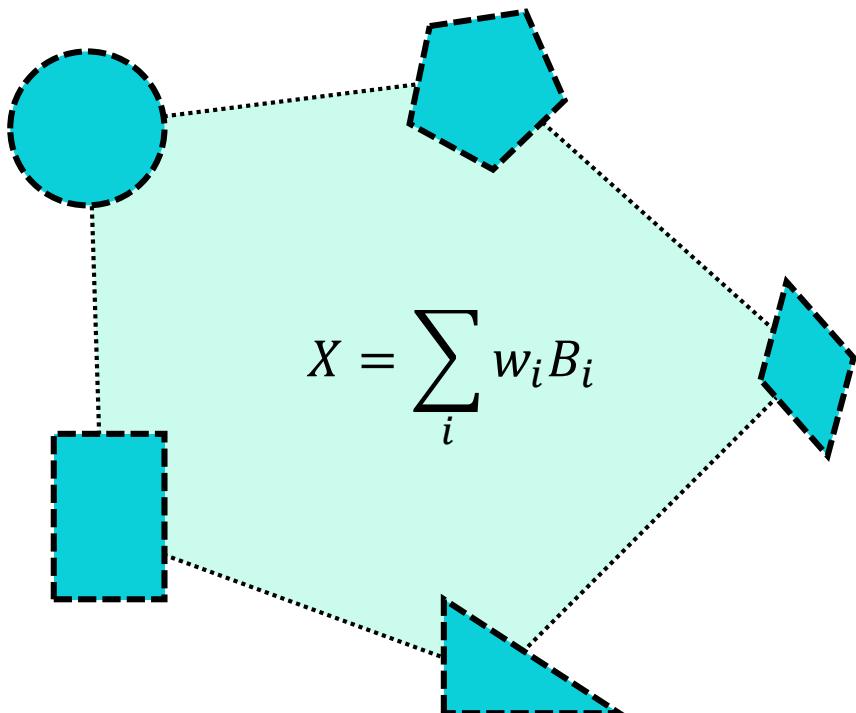
Example-based Shape Deformation



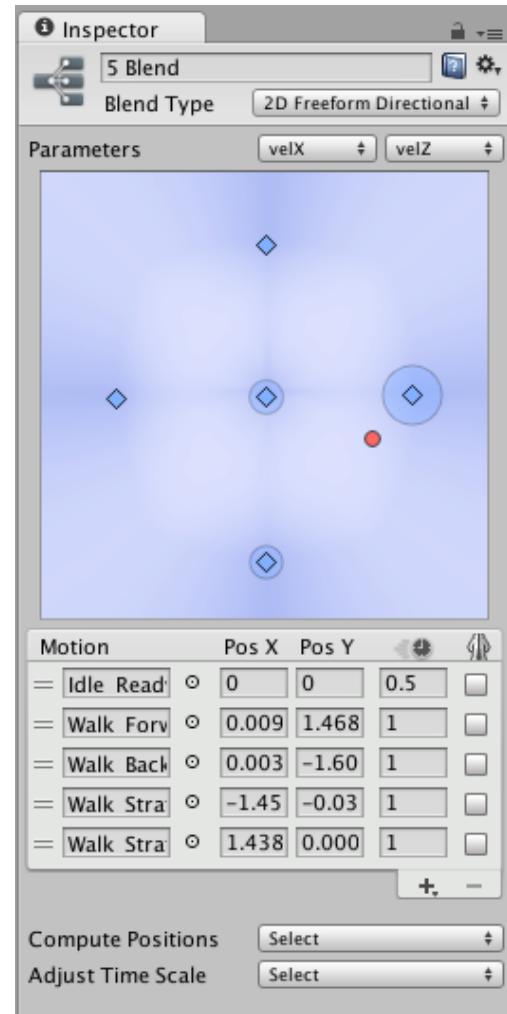
Blendshapes / Blend Space



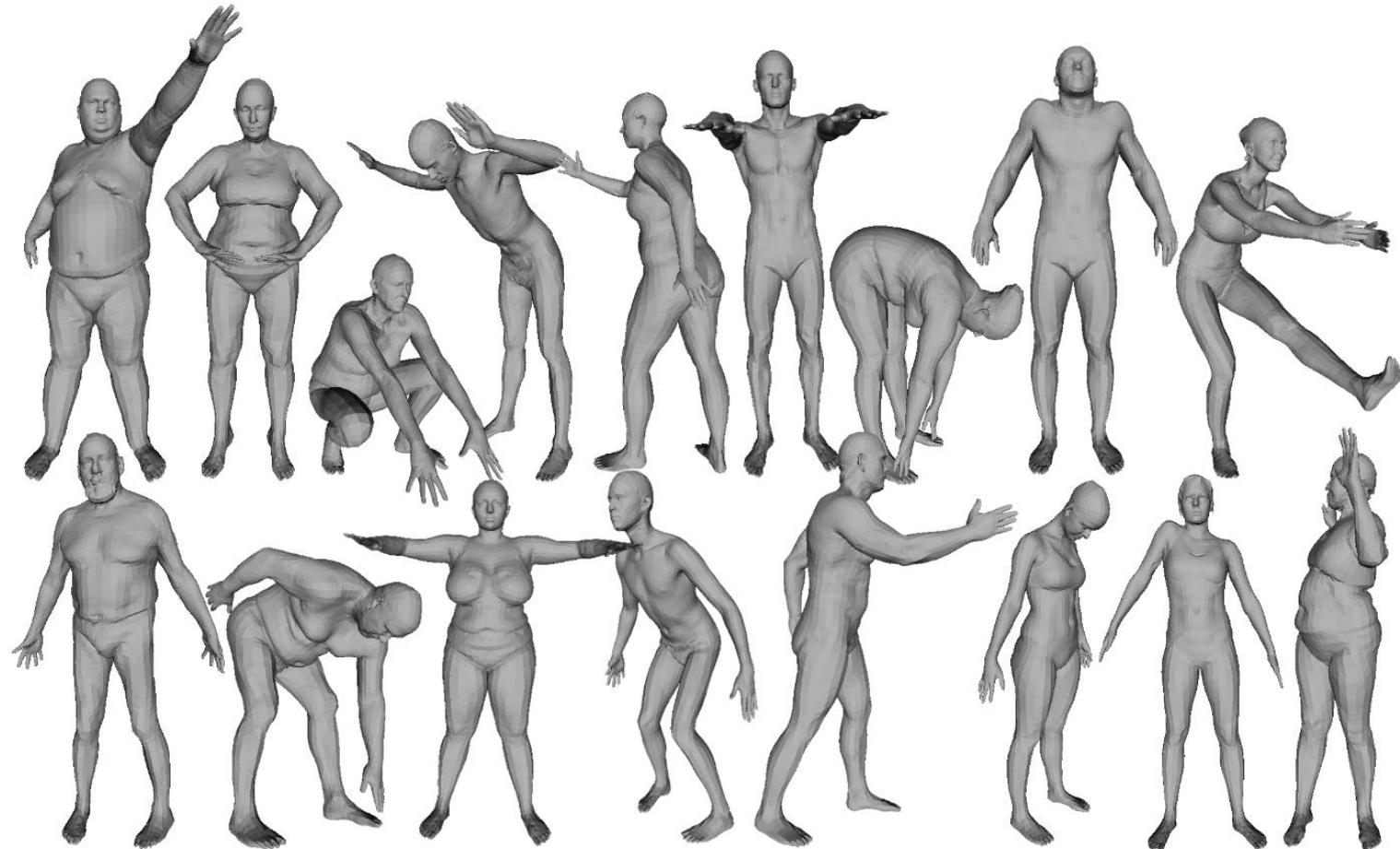
Blendshapes / Blend Space



由一组基构成的linear system
同时也是对animation有用，可以把多个姿势生成一个新姿势



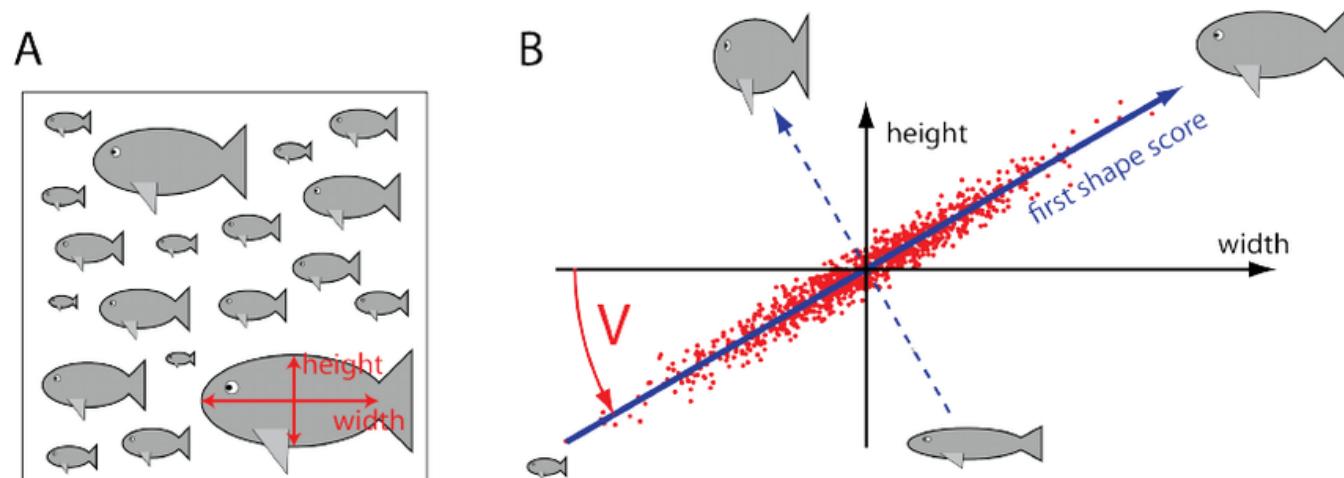
How to deal with massive examples?



[SMPL: A Skinned Multi-Person Linear Model]

Principal Component Analysis (PCA)

- A technique for
 - finding out the correlations among dimensions
 - dimensionality reduction



Werner, Steffen & Rink, Jochen & Friedrich, Benjamin. (2014). Shape Mode Analysis Exposes Movement Patterns in Biology: Flagella and Flatworms as Case Studies. PloS one. 9. 10.1371/journal.pone.0113083.

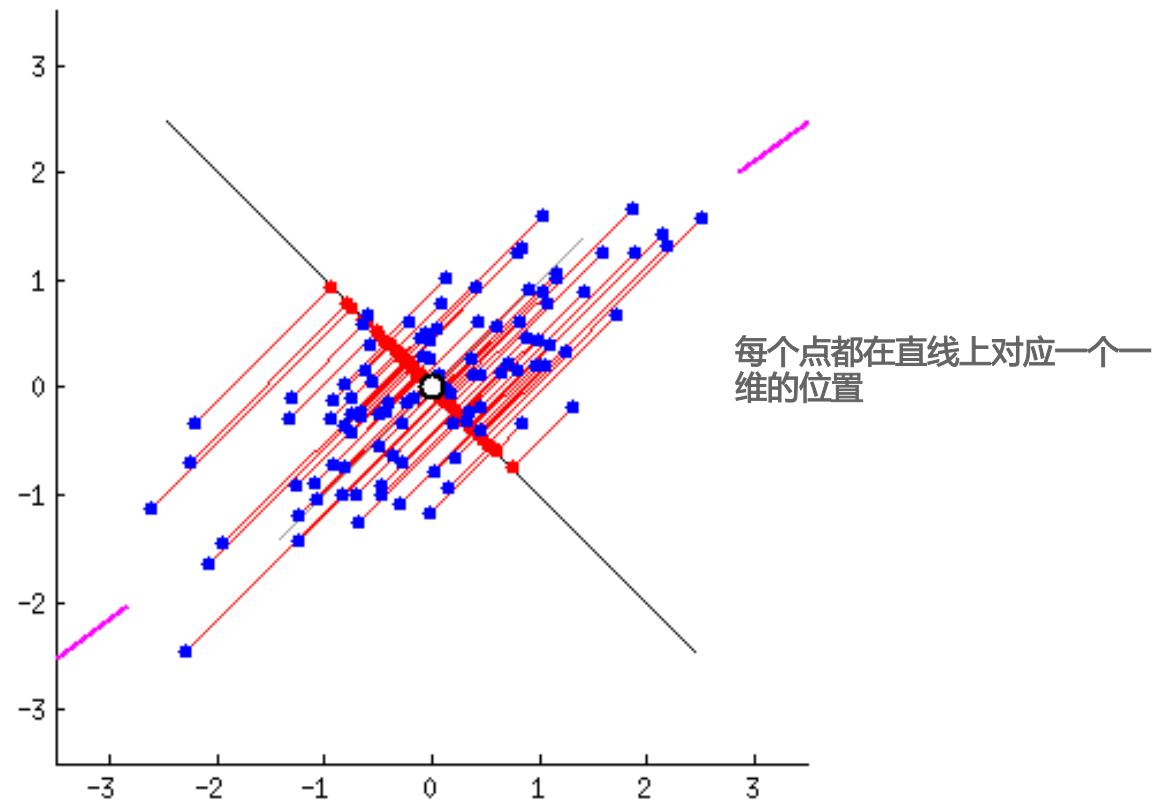
PCA in a Nutshell

- Given a dataset $\{\mathbf{x}_i\}$, $\mathbf{x}_i \in \mathbb{R}^N$, then PCA gives

$$\mathbf{x}_i = \bar{\mathbf{x}} + \sum_{k=1}^n w_{i,k} \mathbf{u}_k$$

- \mathbf{u}_k is the k -th **principal component**
 - A direction in \mathbb{R}^N along which the projection of $\{\mathbf{x}_i\}$ has the k -th maximal **variance**
- $w_{i,k} = (\mathbf{x}_i - \bar{\mathbf{x}}) \cdot \mathbf{u}_k$ is the **score** of \mathbf{x}_i on \mathbf{u}_k

PCA in a Nutshell



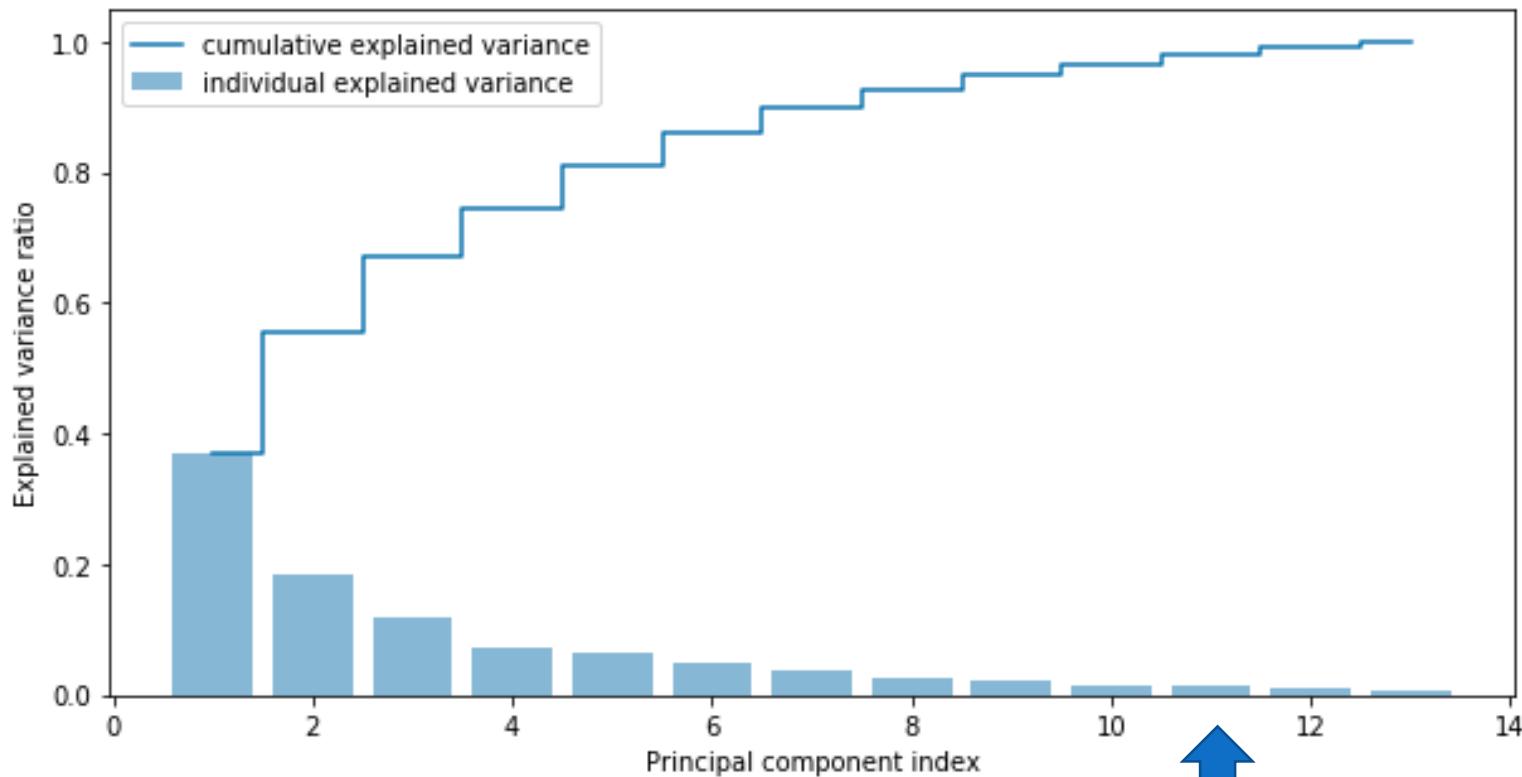
PCA in a Nutshell

- Given a dataset $\{x_i\}, x_i \in \mathbb{R}^N$, the PCA can be computed by apply eigen decomposition on **covariance matrix**

$$X^T X = U \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix} U^T$$

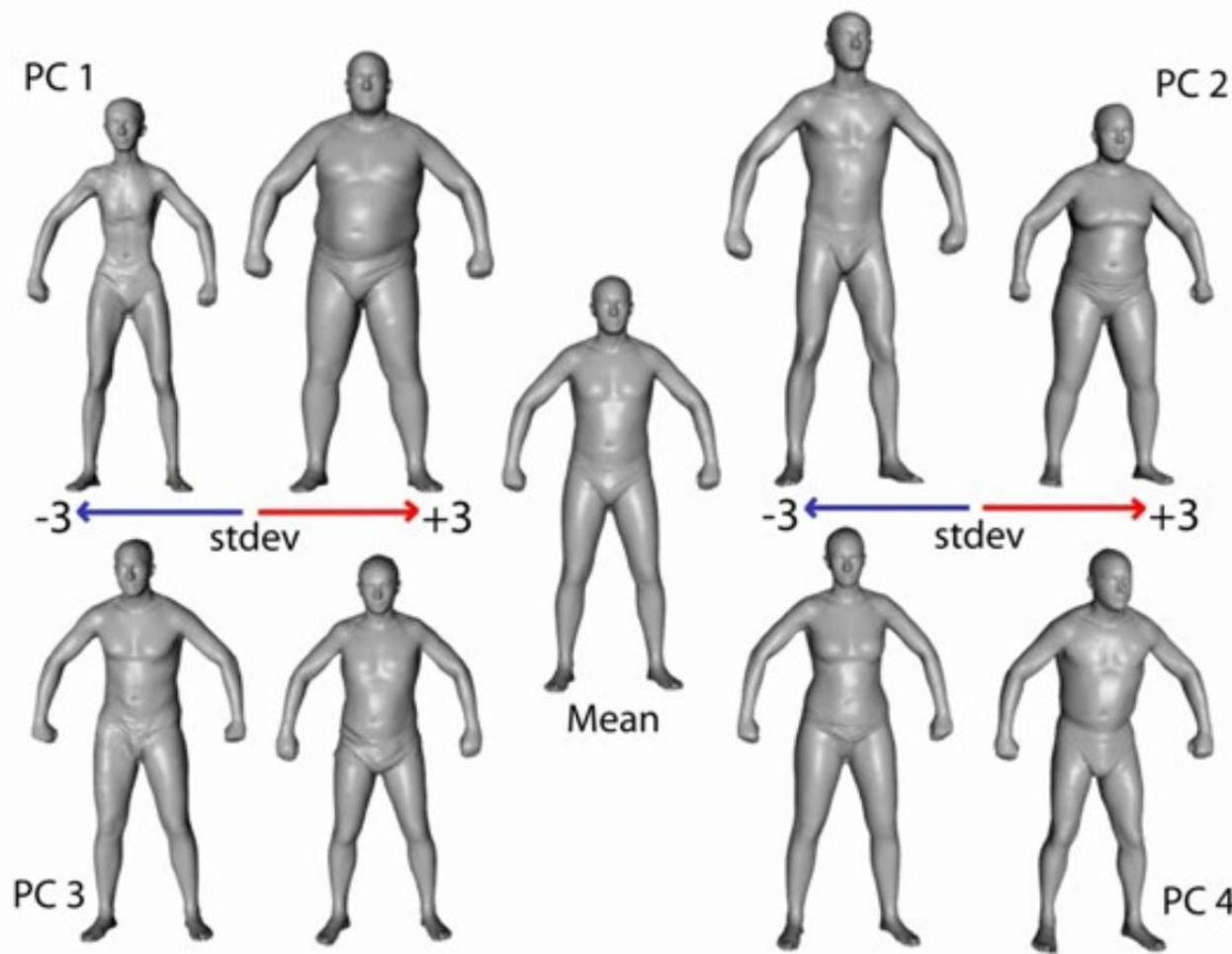
- $X = [x_0 - \bar{x}, x_1 - \bar{x}, \dots, x_N - \bar{x}]^T$
- $\lambda_i \geq \lambda_j$ when $i < j$, corresponds to the **Explained Variance**
- $U = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N]$

PCA in a Nutshell



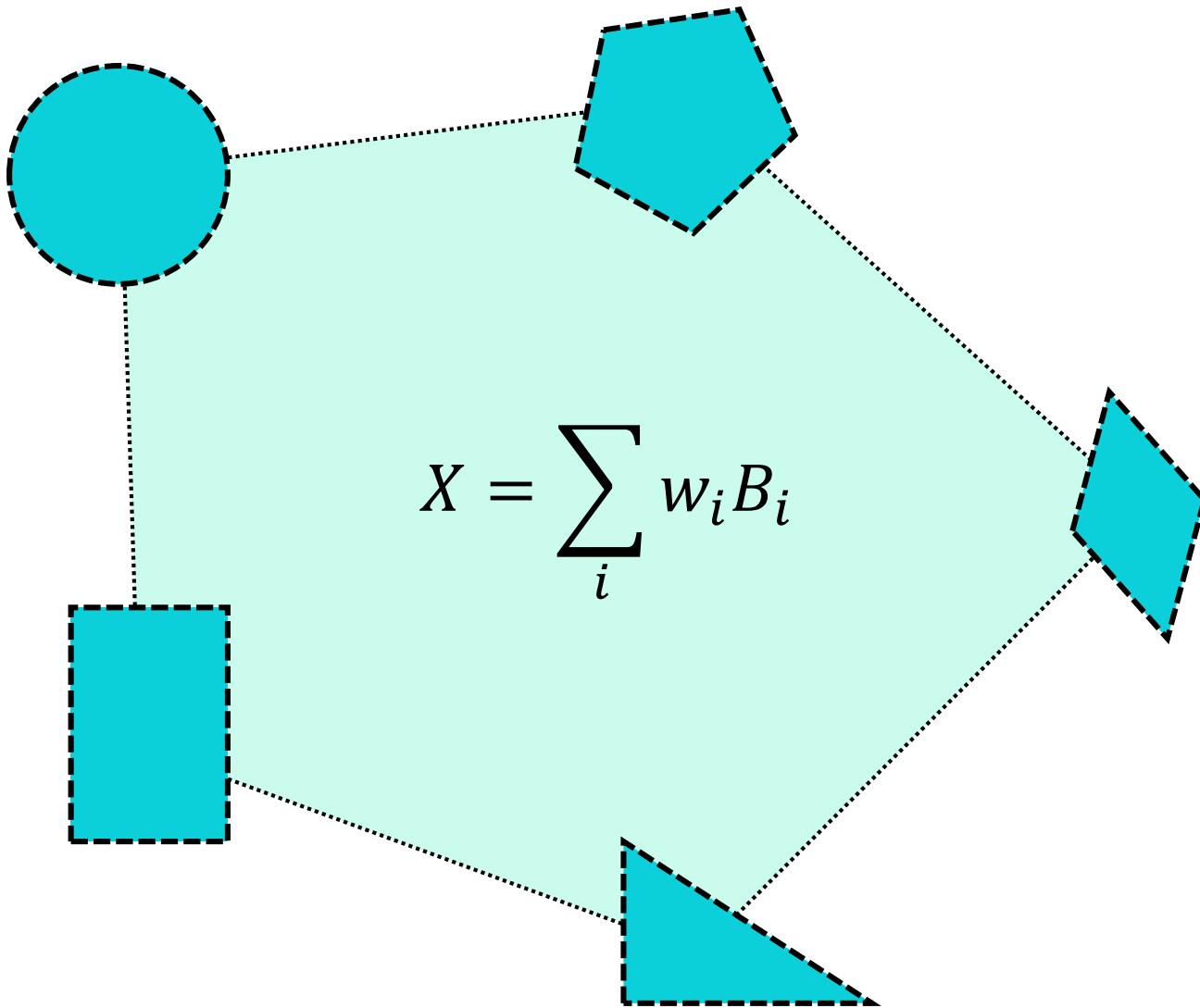
Choose n principal components that explains enough (e.g. 95%) of the variance

PCA over Body Shapes

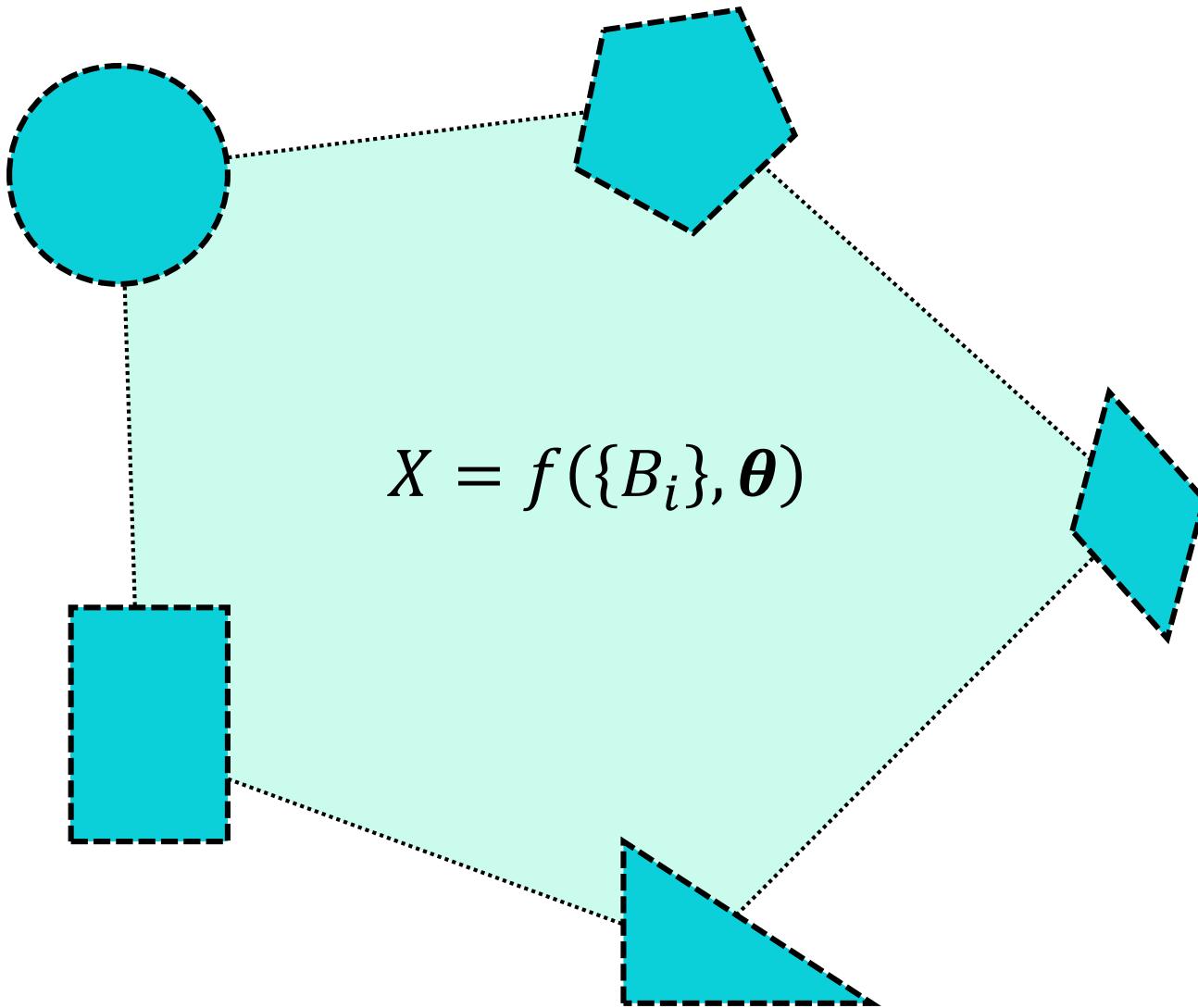


Dragomir Anguelov, Praveen Srinivasan, Daphne Koller, Sebastian Thrun, Jim Rodgers, and James Davis. 2005. *SCAPE: shape completion and animation of people*. ACM Trans. Graph. 24, 3 (July 2005), 408–416.

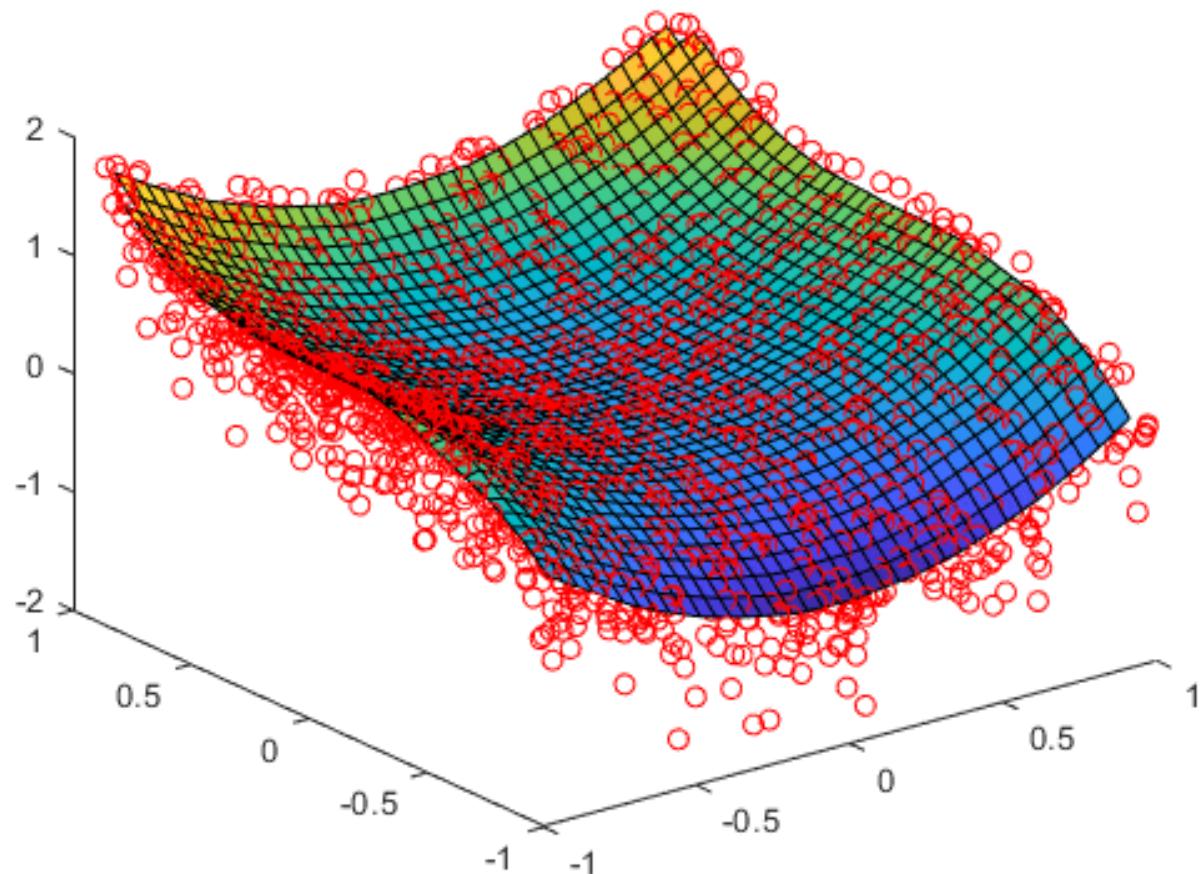
Blendshapes / Blend Space



Pose Space Deformation

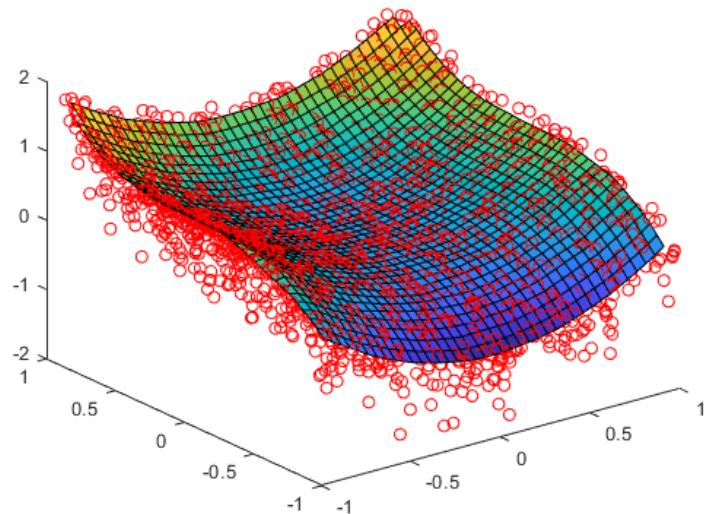


Scattered Data Interpolation

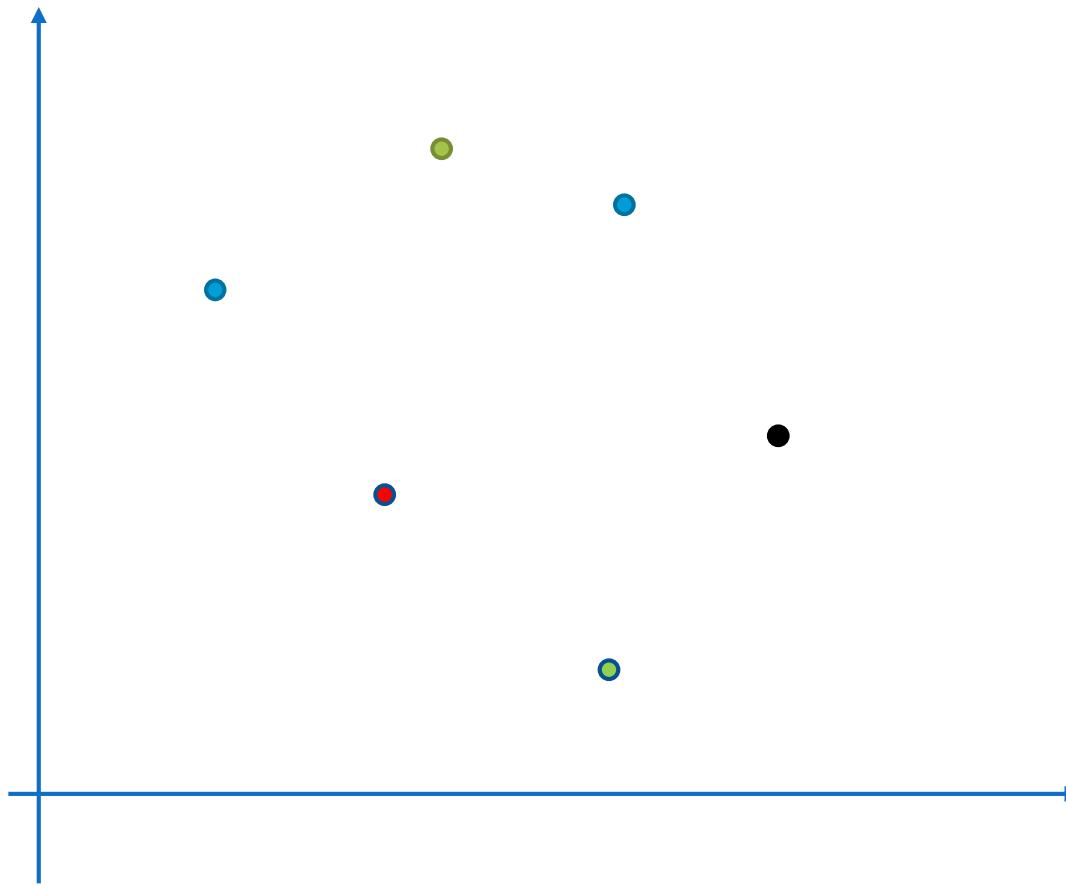


Scattered Data Interpolation

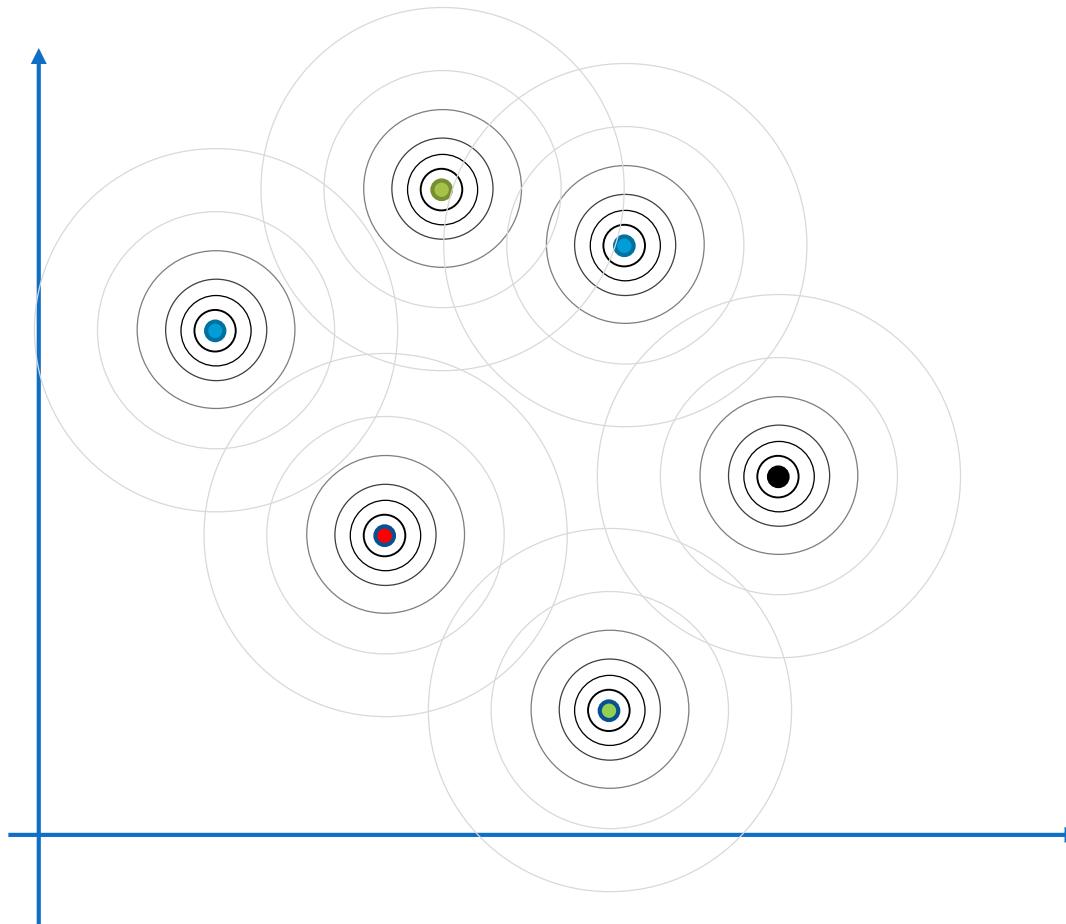
- Linear
- Least squares
- Splines
- Inverse distance weighting
- Gaussian process
- Radial Basis Function
-



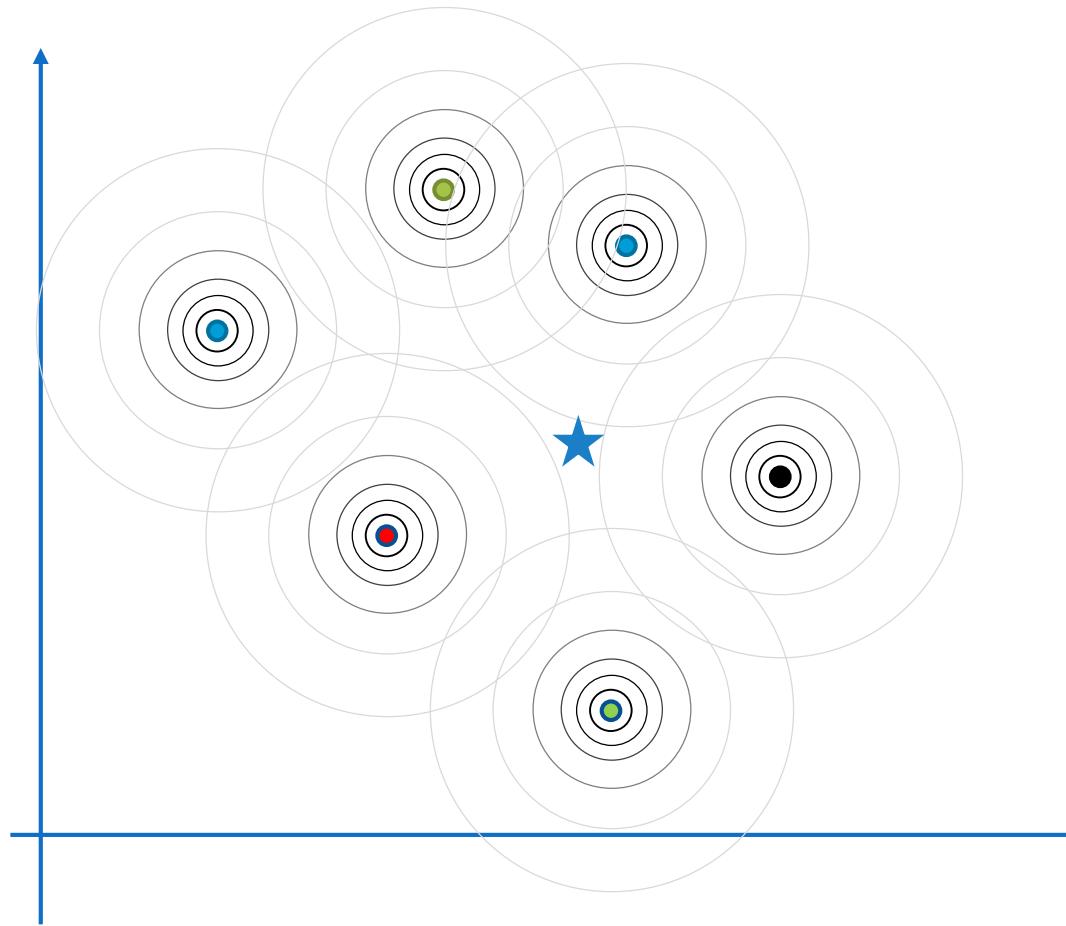
Radial Basis Function (RBF) Interpolation



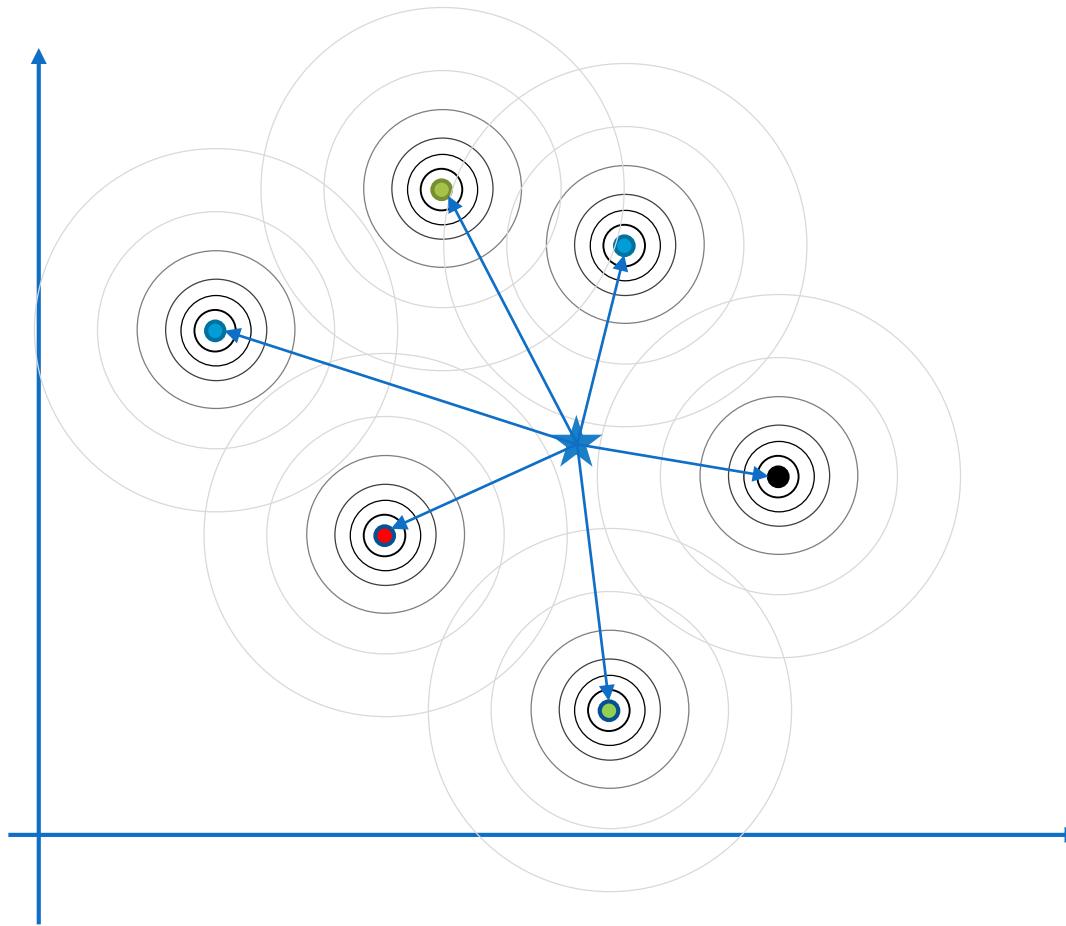
Radial Basis Function (RBF) Interpolation



Radial Basis Function (RBF) Interpolation

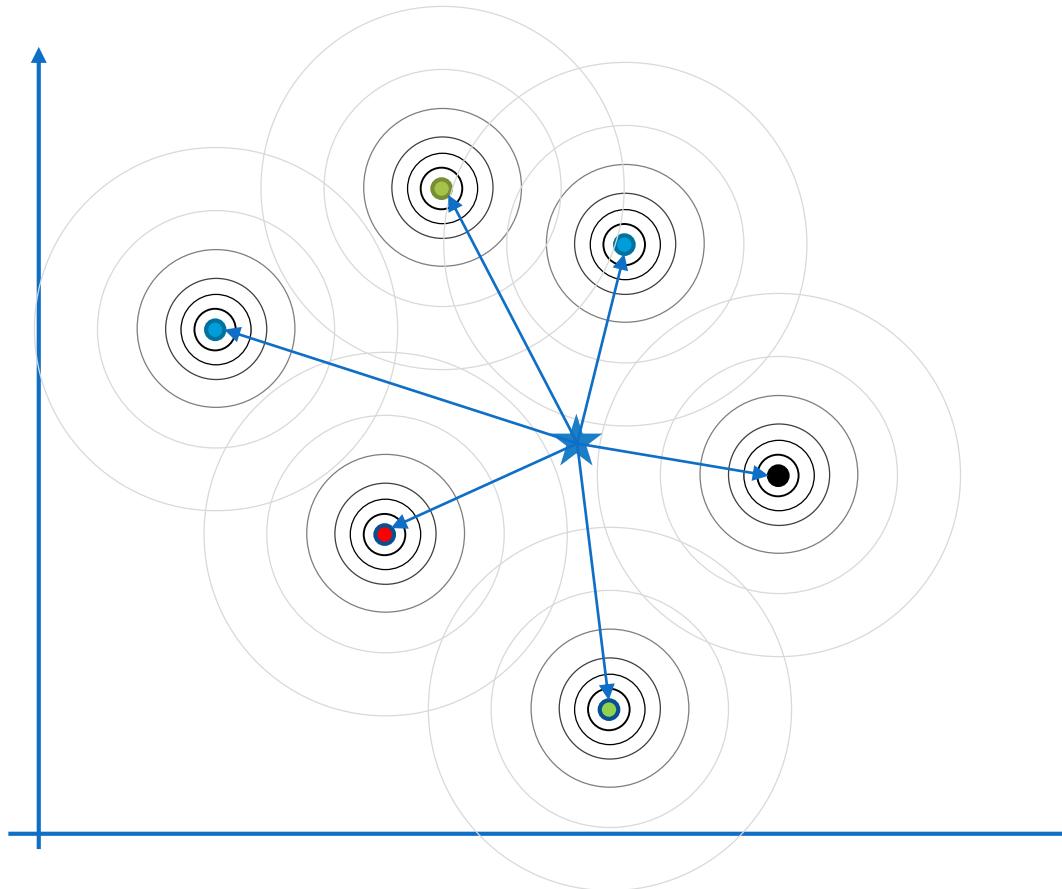


Radial Basis Function (RBF) Interpolation



Radial Basis Function (RBF) Interpolation

$$x = \sum_{i=1}^K w_i \varphi(\|x - x_i\|)$$



Radial Basis Function (RBF) Interpolation

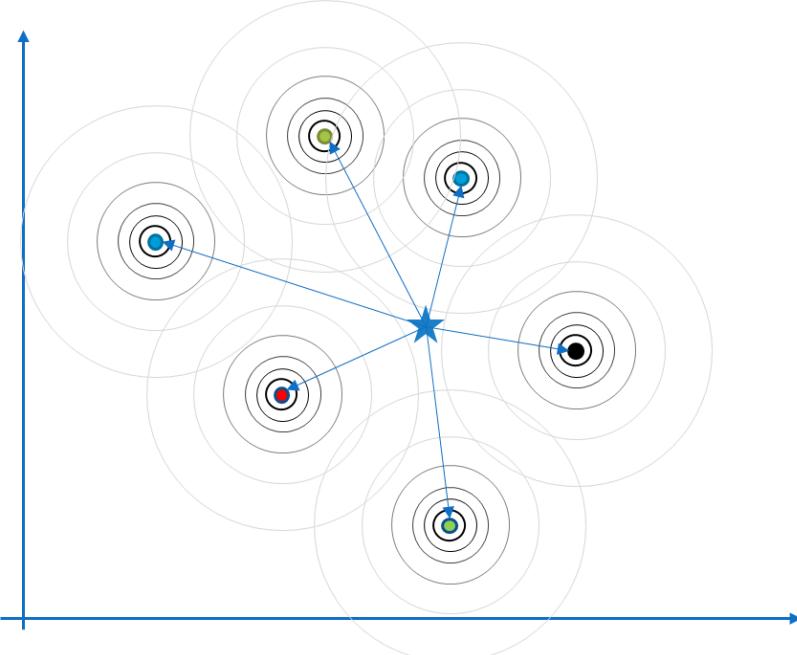
$$x = \sum_{i=1}^K w_i \varphi(\|x - x_i\|)$$

How to compute w_i ?

Radial Basis Function (RBF) Interpolation

$$x = \sum_{i=1}^K w_i \varphi(\|x - x_i\|)$$

How to compute w_i ?



$$\begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,K} \\ R_{2,1} & R_{2,2} & \ddots & \vdots \\ \vdots & & \ddots & \vdots \\ R_{K,1} & \cdots & \cdots & R_{K,K} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_K \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix}$$

$$R_{i,j} = \varphi(\|x_i - x_j\|)$$

Radial Basis Function (RBF)

$$x = \sum_{i=1}^K w_i \varphi(\|x - x_i\|)$$

- Gaussian: $\varphi(r) = e^{-(r/c)^2}$
- Inverse multiquadric: $\varphi(r) = \frac{1}{\sqrt{r^2 + c^2}}$
- Thin plate spline: $\varphi(r) = r^2 \log r$
- Polyharmonic splines: $\varphi(r) = \begin{cases} r^k, & k = 2n + 1 \\ r^k \log r, & k = 2n \end{cases}$

Pose Space Deformation

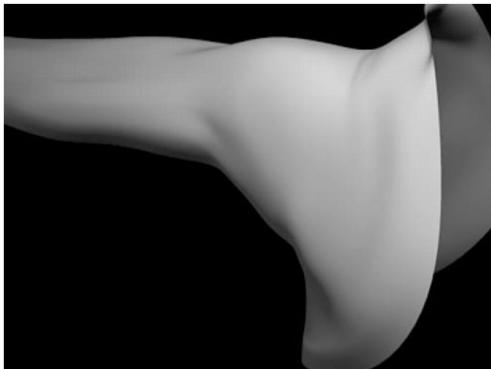
$$\mathbf{x}' = \sum_{j=1}^m \omega_j T_j(\mathbf{x} + \boldsymbol{\delta}(\mathbf{x}, \theta))$$

- $\mathbf{x}' = SKIN(PSD(\mathbf{x}))$
- PSD is implemented as RBF interpolation
PSD: local

J. P. Lewis, Matt Cordner, and Nickson Fong. 2000. *Pose space deformation: a unified approach to shape interpolation and skeleton-driven deformation*. In *Proceedings of the 27th annual conference on Computer graphics and interactive techniques (SIGGRAPH '00)*, ACM Press/Addison-Wesley Publishing Co., USA, 165–172.

Pose Space Deformation

PSD



LBS



J. P. Lewis, Matt Cordner, and Nickson Fong. 2000. *Pose space deformation: a unified approach to shape interpolation and skeleton-driven deformation*. In *Proceedings of the 27th annual conference on Computer graphics and interactive techniques (SIGGRAPH '00)*, ACM Press/Addison-Wesley Publishing Co., USA, 165–172.

Issues

- Per-shape or per-vertex interpolation
 - Should we interpolate a shape as a whole?
- Local or global interpolation?
 - Should a vertex be affected by all joints?
- Interpolation algorithm?
 - Is RBF the only choice?

Example-based Skinning (EBS) vs. Skeleton Subspace Deformation (SSD)

*SSD (LBS, DQS, etc.)

- **Good:** Easy to control
- **Good:** Good quality
- **Good:** Pose-dependent details (e.g. bulging muscle and extruding veins)
- **Bad:** Creating examples can be cumbersome
- **Bad:** Extra storage for examples
- **Bad:** Interpolation needs careful tuning

Other Example-based Skinning

- Automatically compute weights using examples
- Example-based simulation
 - We will touch this part later...

SMPL Model

- A widely adopted human model in ML/CV
- Learned on real scan data
- Combines SSD and EBS techniques

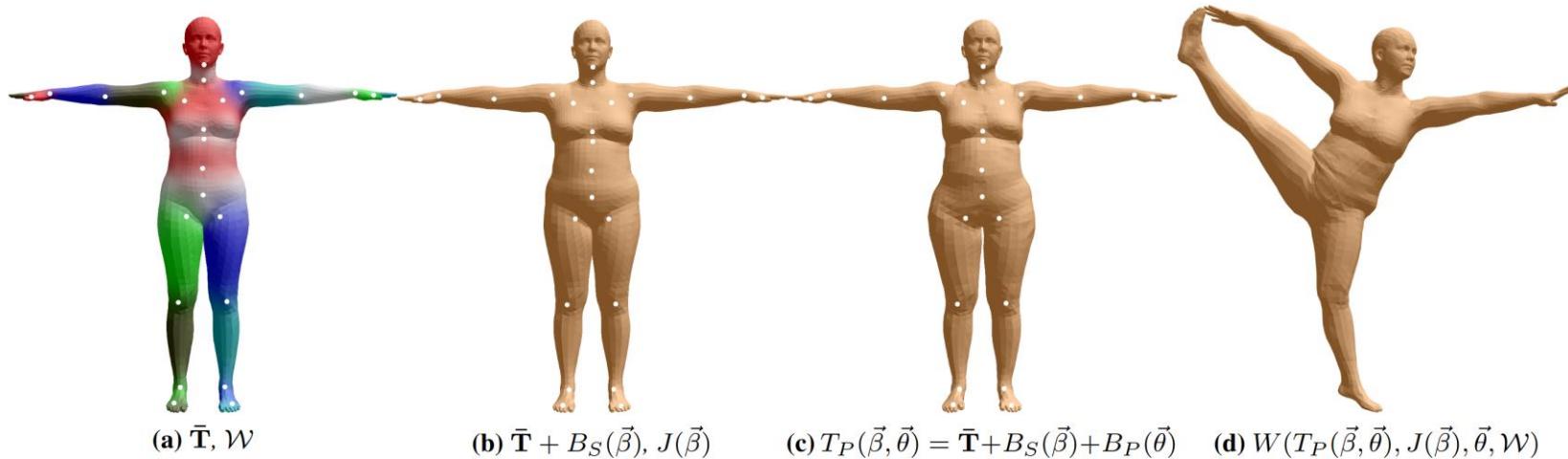


Figure 3: SMPL model. (a) Template mesh with blend weights indicated by color and joints shown in white. (b) With identity-driven blendshape contribution only; vertex and joint locations are linear in shape vector $\vec{\beta}$. (c) With the addition of pose blend shapes in preparation for the split pose; note the expansion of the hips. (d) Deformed vertices reposed by dual quaternion skinning for the split pose.

[SMPL: A Skinned Multi-Person Linear Model]

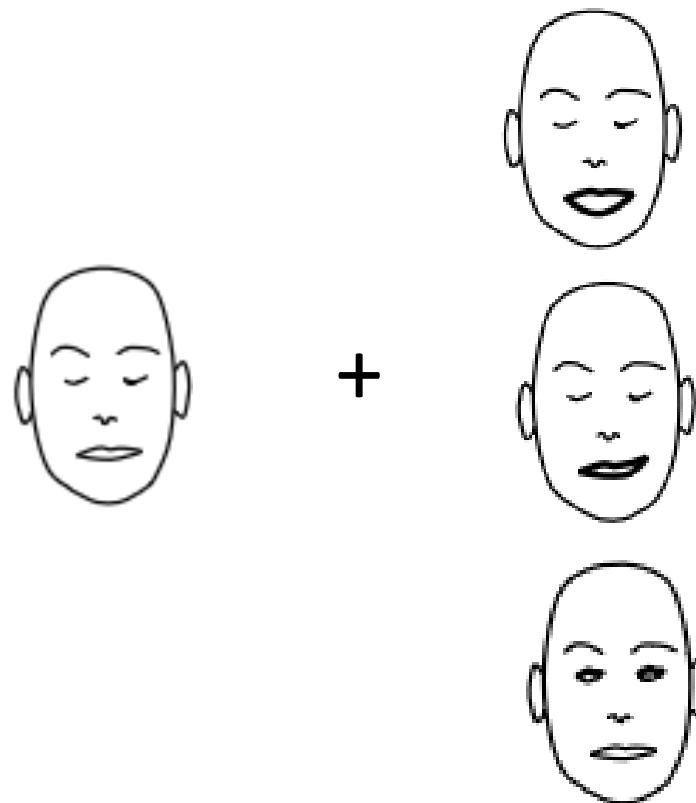
Facial Animation



[UnrealEngine]

Facial Animation

Facial Animation = Identity + Expression



Facial Animation

Facial Animation = Identity + Expression

$$X = X_0 + \sum_i \beta_i B_i^{\text{ID}} + \sum_j \theta_j B_j^{\text{Exp}}$$

X_0 是一个平均脸，identity是不做表情的基础脸部形状，如大脸之类的，实际上是base；
expression是一个基于example的参数，实际上混合了PCA和EXAMPLE

Facial Animation

Facial Animation = Identity + Expression

$$X = X_0 + \sum_i \beta_i B_i^{\text{ID}} + \sum_j \theta_j B_j^{\text{Exp}}$$

平均脸 捏脸 调表情

Facial Blendshapes

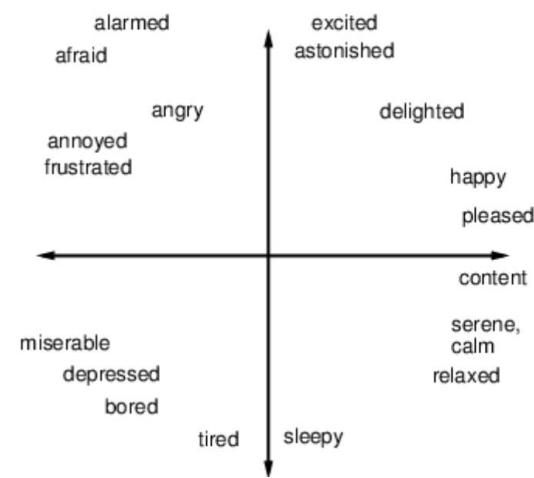
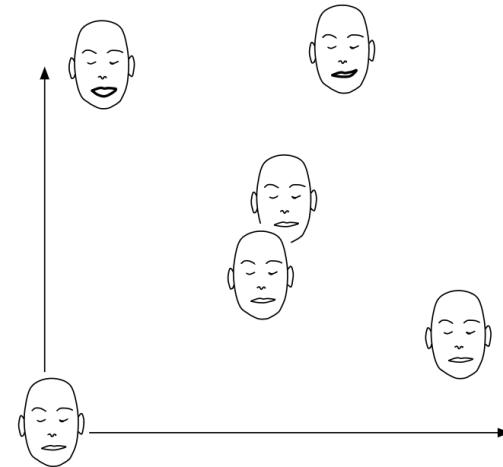
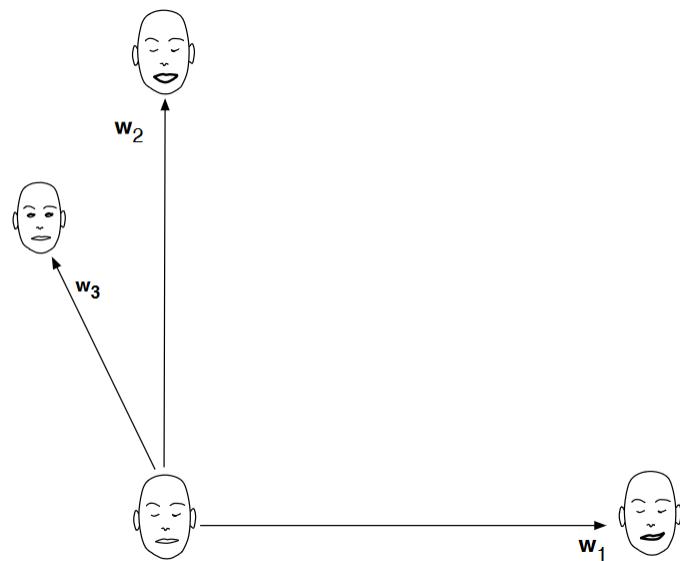


<http://www.santhoshkoneru.com/facial-blendshapes>

A Typical Set of Blendshapes (ARKit)

- eyeBlinkLeft
- eyeLookDownLeft
- eyeLookInLeft
- eyeLookOutLeft
- eyeLookUpLeft
- eyeSquintLeft
- eyeWideLeft
- eyeBlinkRight
- eyeLookDownRight
- eyeLookInRight
- eyeLookOutRight
- eyeLookUpRight
- eyeSquintRight
- eyeWideRight
- jawForward
- jawLeft
- jawRight
- jawOpen
- mouthClose
- mouthFunnel
- mouthPucker
- mouthRight
- mouthLeft
- mouthSmileLeft
- mouthSmileRight
- mouthFrownRight
- mouthFrownLeft
- mouthDimpleLeft
- mouthDimpleRight
- mouthStretchLeft
- mouthStretchRight
- mouthRollLower
- mouthRollUpper
- mouthShrugLower
- mouthShrugUpper
- mouthPressLeft
- mouthPressRight
- mouthLowerDownLeft
- mouthLowerDownRight
- mouthUpperUpLeft
- mouthUpperUpRight
- browDownLeft
- browDownRight
- browInnerUp
- browOuterUpLeft
- browOuterUpRight
- cheekPuff
- cheekSquintLeft
- cheekSquintRight
- noseSneerLeft
- noseSneerRight
- tongueOut
- gemfield

Blendshapes vs. Example-based Skinning



Blendshapes vs. Example-based Skinning

$$X = X_0 + \sum_i \beta_i B_i^{\text{ID}} + \sum_j \theta_j (B_j^{\text{Exp}} + \delta(\{\beta_i, \theta_j\}))$$

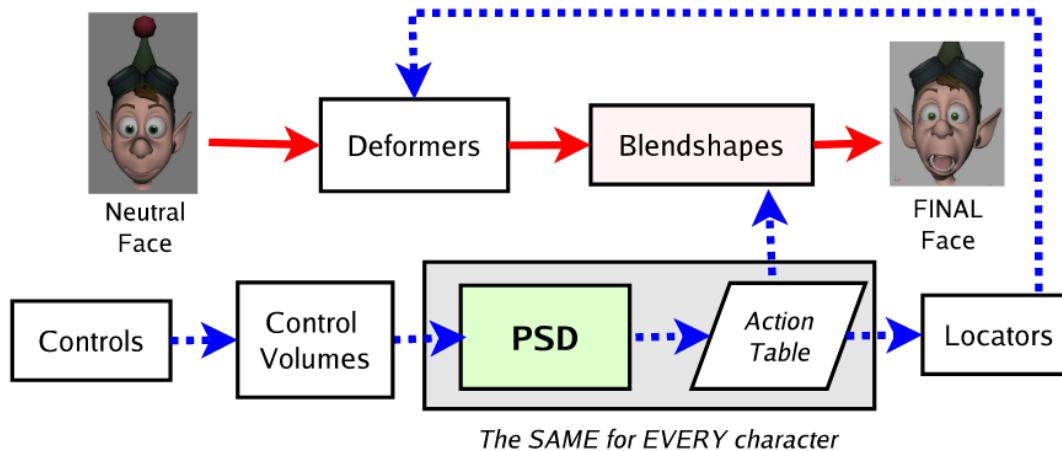
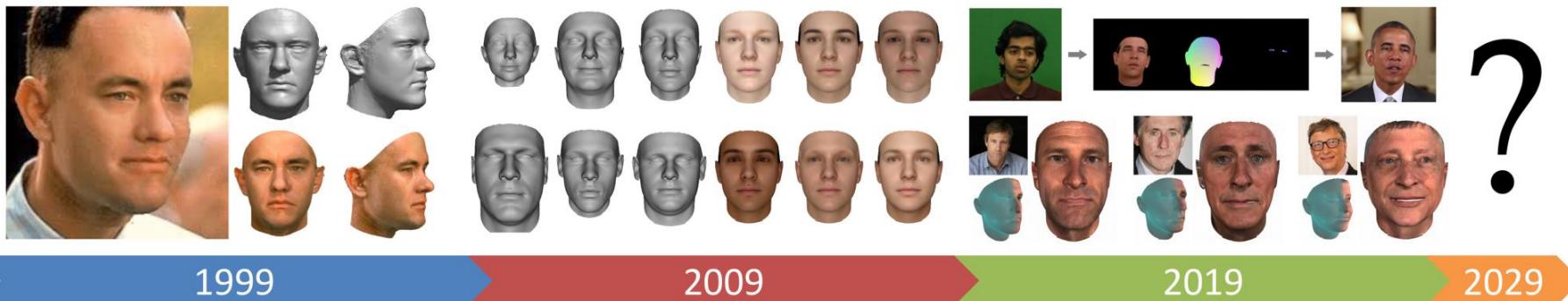
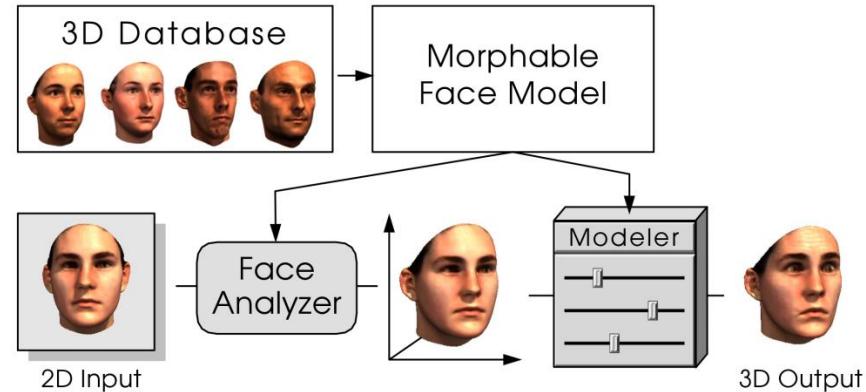


Figure 2: *Architecture of Hybrid Approach*

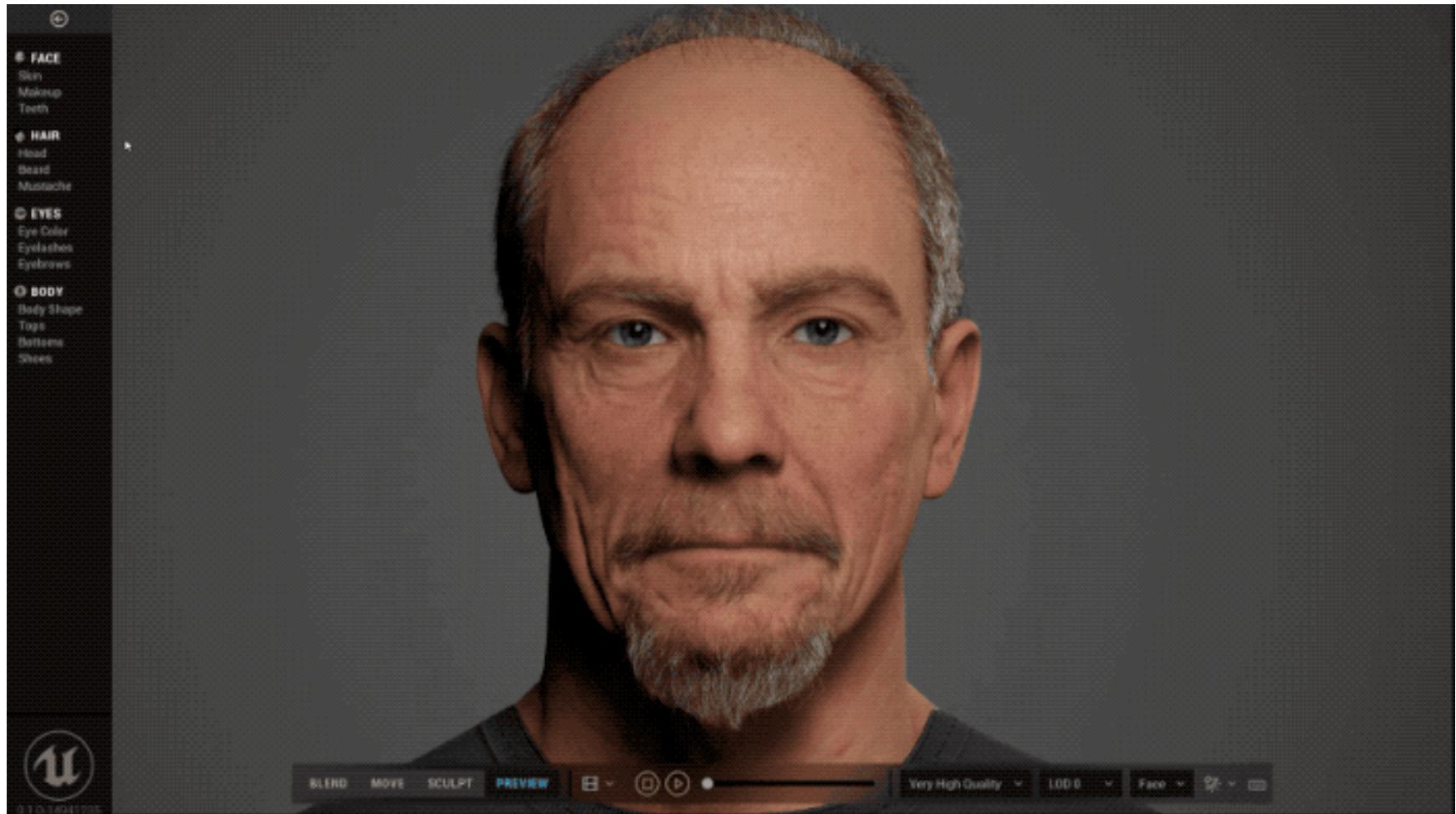
David Komorowski, Vinod Melapudi, Darren Mortillaro, and Gene S. Lee. 2010. **A hybrid approach to facial rigging**. In *ACM SIGGRAPH ASIA 2010 Sketches (SA '10)*, Association for Computing Machinery, New York, NY, USA, 1–2.

Morphable Face Models

$$X = X_0 + \sum_i \beta_i B_i^{\text{ID}} + \sum_j \theta_j B_j^{\text{Exp}}$$

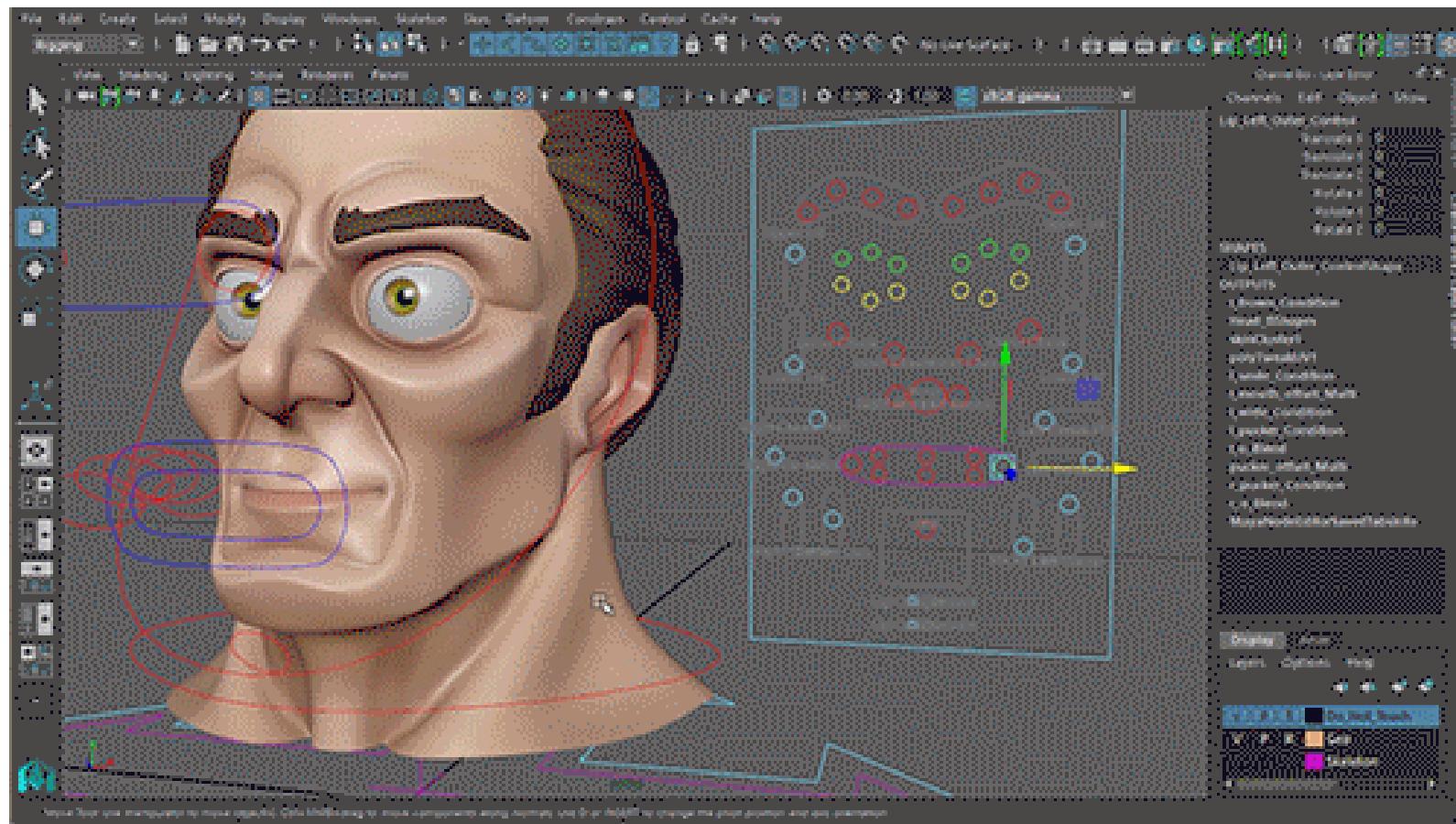


Morphable Face Models



Meta Human - UnrealEngine

How to Animate a Face?

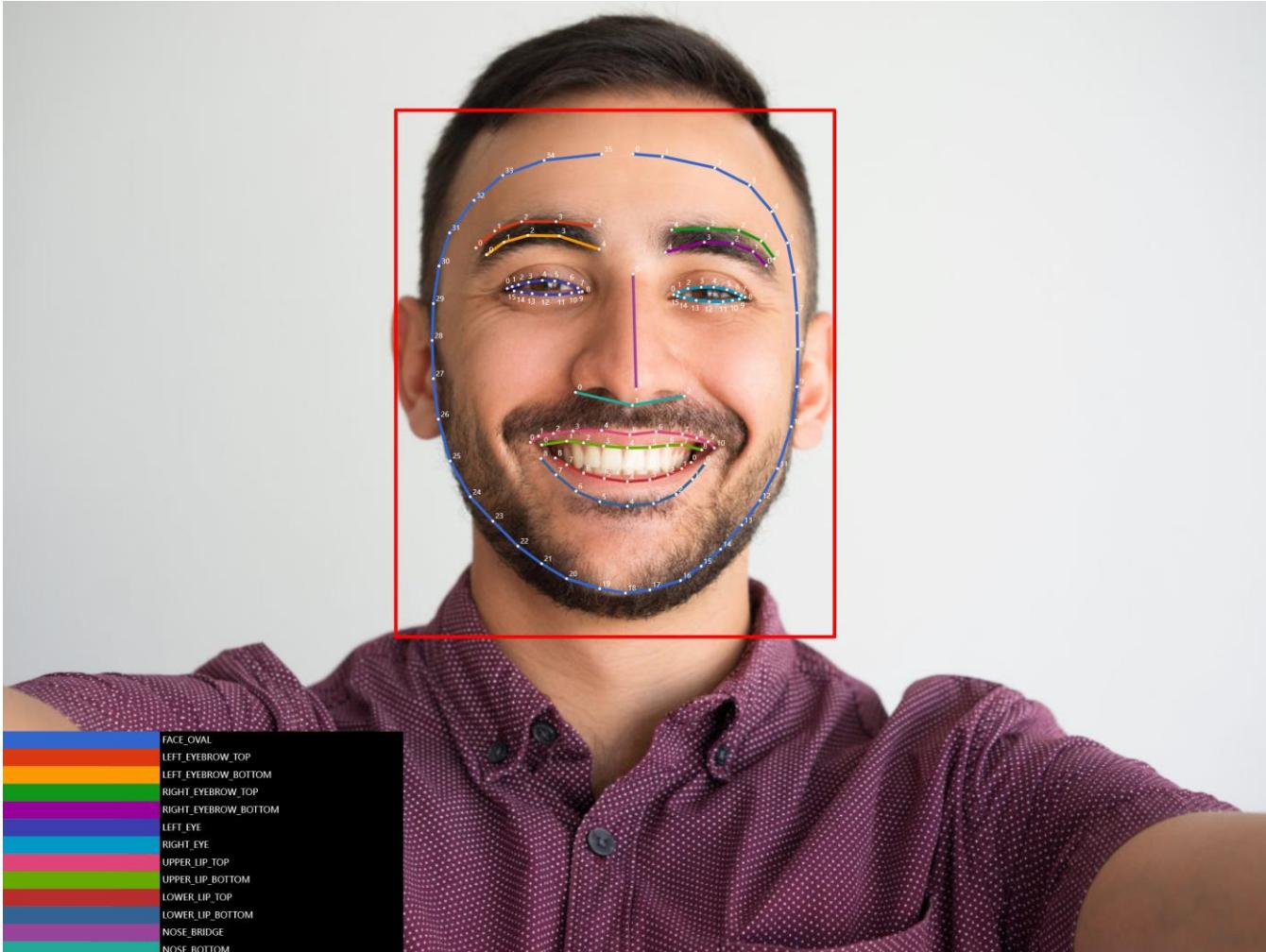


Face Tracking



<https://developers.google.com/ml-kit/vision/face-detection>

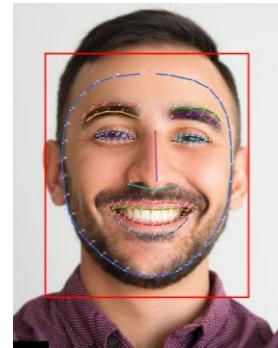
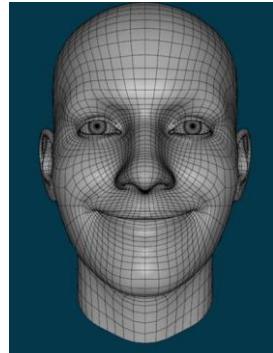
Face Tracking



<https://developers.google.com/ml-kit/vision/face-detection>

Face Tracking

$$\min_{\beta_i, \theta_j} \left\| X_0 + \sum_i \beta_i B_i^{\text{ID}} + \sum_j \theta_j B_j^{\text{Exp}} - Y \right\| + E(\beta_i, \theta_j)$$



Speech-driven



gifs.com

Uncovered Topics

- Skinning weight reduction and compression
- Skeleton extraction and auto-rigging

本节内容

- 几何形状的编辑
 - 自由形变 (Free-form Deformation)
 - 基于优化的形变
- 蒙皮形变 (Skinning Deformation)
 - 线性蒙皮
 - Linear-Blend Skinning (LBS)
 - Multi-linear Skinning
 - 非线性蒙皮
 - Dual-quaternion Skinning (DQS)
 - 基于样例的蒙皮
 - Blendshapes
 - 人脸动画

有没有问题？

Any Questions