物理仿真

北京大学 前沿计算研究中心

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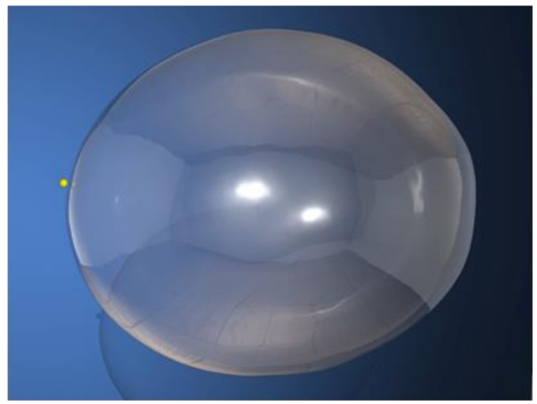
Ruan, Liangwang, et al. "Solid-fluid interaction with surface-tension-dominant contact." *ACM Transactions on Graphics (TOG)* 40.4 (2021): 1-12.

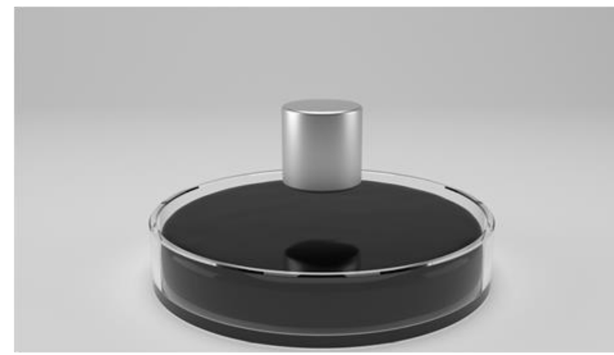
Zhu, Bo, et al. "A new grid structure for domain extension." *ACM Transactions on Graphics (TOG)* 32.4 (2013): 1-12.

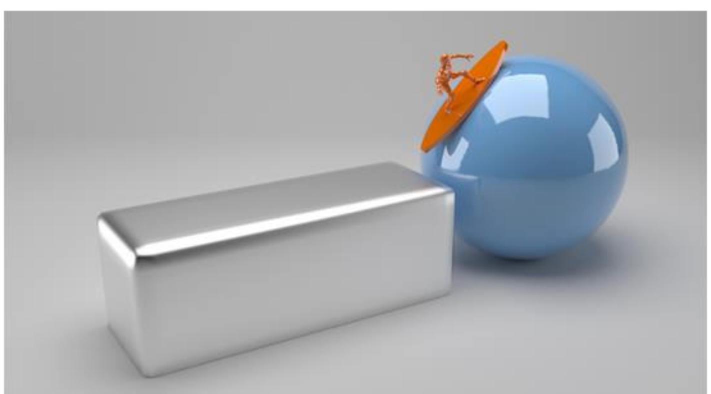
Wang, Huamin. "GPU-based simulation of cloth wrinkles at submillimeter levels." ACM Transactions on Graphics (TOG) 40.4 (2021): 1-14.

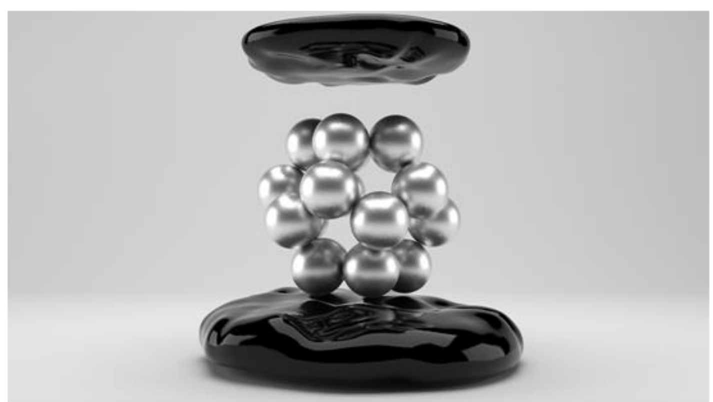
Stomakhin, Alexey, et al. "A material point method for snow simulation." *ACM Transactions on Graphics (TOG)* 32.4 (2013): 1-10.



Zhu, Bo, et al. "Codimensional surface tension flow on simplicial complexes." *ACM Transactions on Graphics (TOG)* 33.4 (2014): 1-11.







Sun, Yuchen, et al. "A material point method for nonlinearly magnetized materials." *ACM Transactions on Graphics (TOG)* 40.6 (2021): 1-13.

物理仿真

- 质点系统
  - 弹簧质点
- 软体仿真
  - 对象：
    - 一维：绳索
    - 二维：薄壳物体、衣服
    - 三维：体软体
  - 现象：
    - 弹性形变与非弹性形变
    - 撕裂、破碎、爆炸
- 刚体仿真
- 流体仿真
  - 理想流体：水、空气
  - 粘性流体、非牛顿流体
  - 拟流体以及相关现象
    - 沙、雪、烟
- 其他
  - 声音
  - 锈蚀、老化、燃烧
  - 电磁
- 多物理场耦合

Kinematic or Dynamic?

Mass

$$F = ma$$

Position
Velocity
Acceleration
……

Force
Torque
……

Kinematics

Dynamics

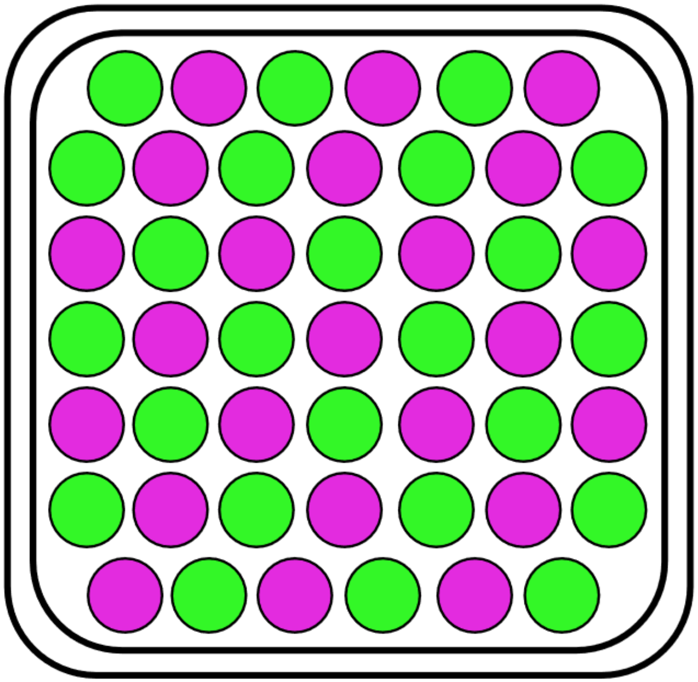
不考虑质量的就是
Kinematics

Homogeneous or Heterogeneous

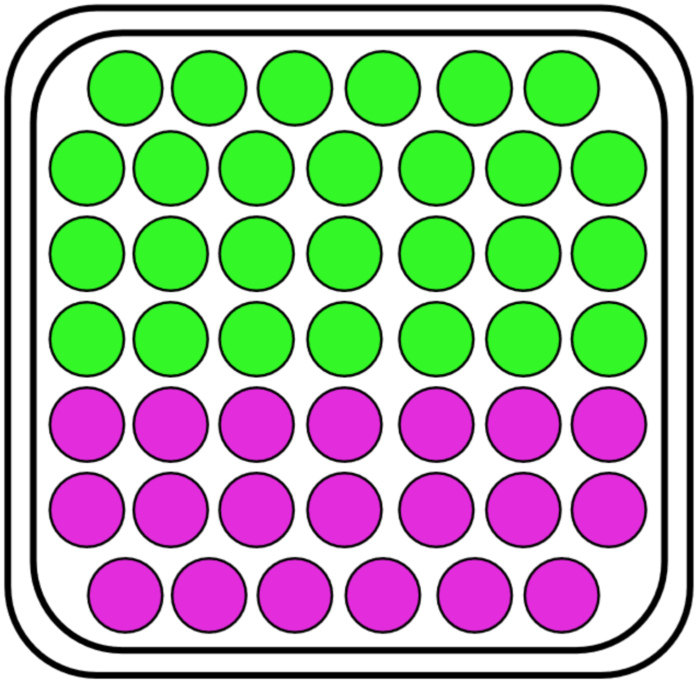
Homogeneous

Heterogeneous

Homogeneous or Heterogeneous



Homogeneous 同质



Heterogeneous 不同类

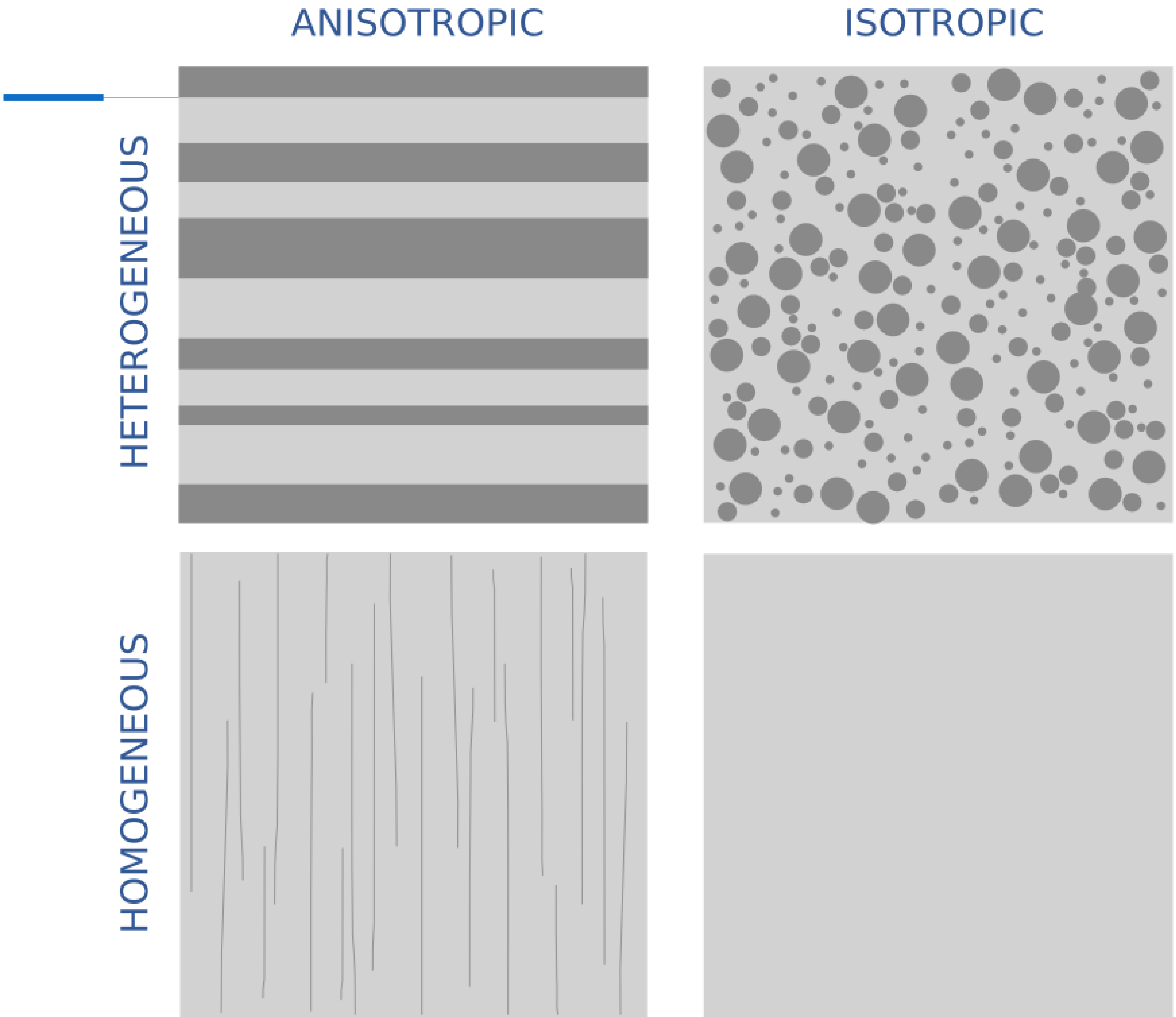
Isotropic or Anisotropic?

Isotropic

Anisotropic

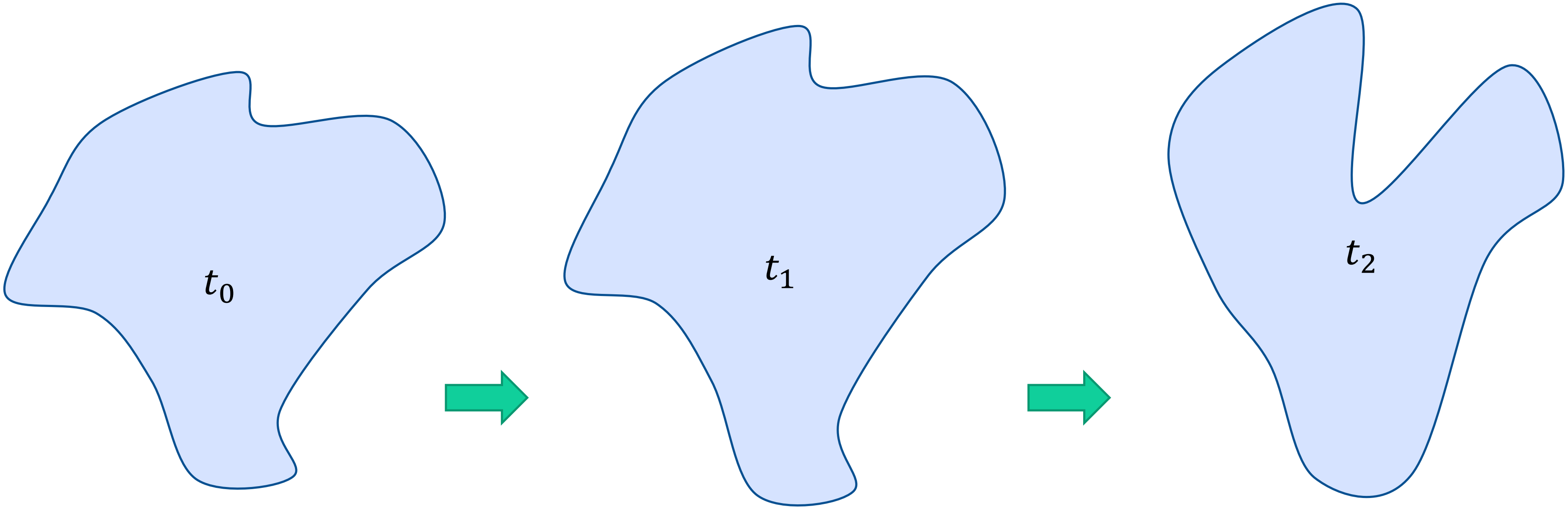
Isotropic or Anisotropic?

晶体各向异性，打成粉之后各向同性


ANISOTROPIC
ISOTROPIC
HETEROGENEOUS
HOMOGENEOUS

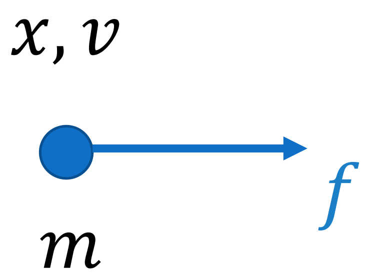
What is Simulation

$$X = X(t)$$



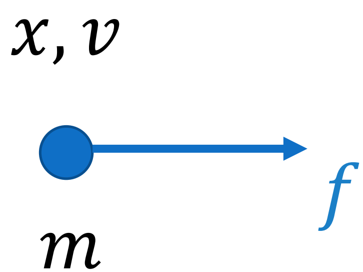
Dynamics of a Particle

$$x = x(t)$$



Dynamics of a Particle

$$x = x(t)$$



$$f = ma$$

$$a = \dot{v}$$

$$v = \dot{x}$$

Dynamics of a Particle

$$x = x(t)$$

$$f = ma$$

$$a = \dot{v}$$

$$v = \dot{x}$$

$$\Rightarrow$$

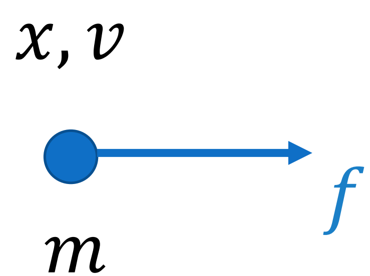
$$a = f/m$$

$$v = v_0 + \int_{t_0}^{t} a dt$$

$$\mathrm{x} = \mathrm{x}_0 + \int_{t_0}^{t} v dt$$

Dynamics of a Particle

$$x = x(t)$$



$$f = ma$$

$$a = \dot{v}$$

$$v = \dot{x}$$

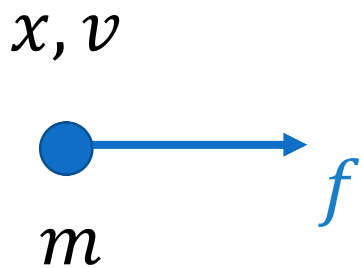
$$a = f/m$$

$$v = v_0 + at$$

$$\mathrm{x} = x_0 + v_0 t + \frac{1}{2} at$$

Dynamics of a Particle

$$x = x(t)$$



$$f = ma$$

$$a = \dot{v}$$

$$v = \dot{x}$$

$$a = f(x, v, t)/m$$

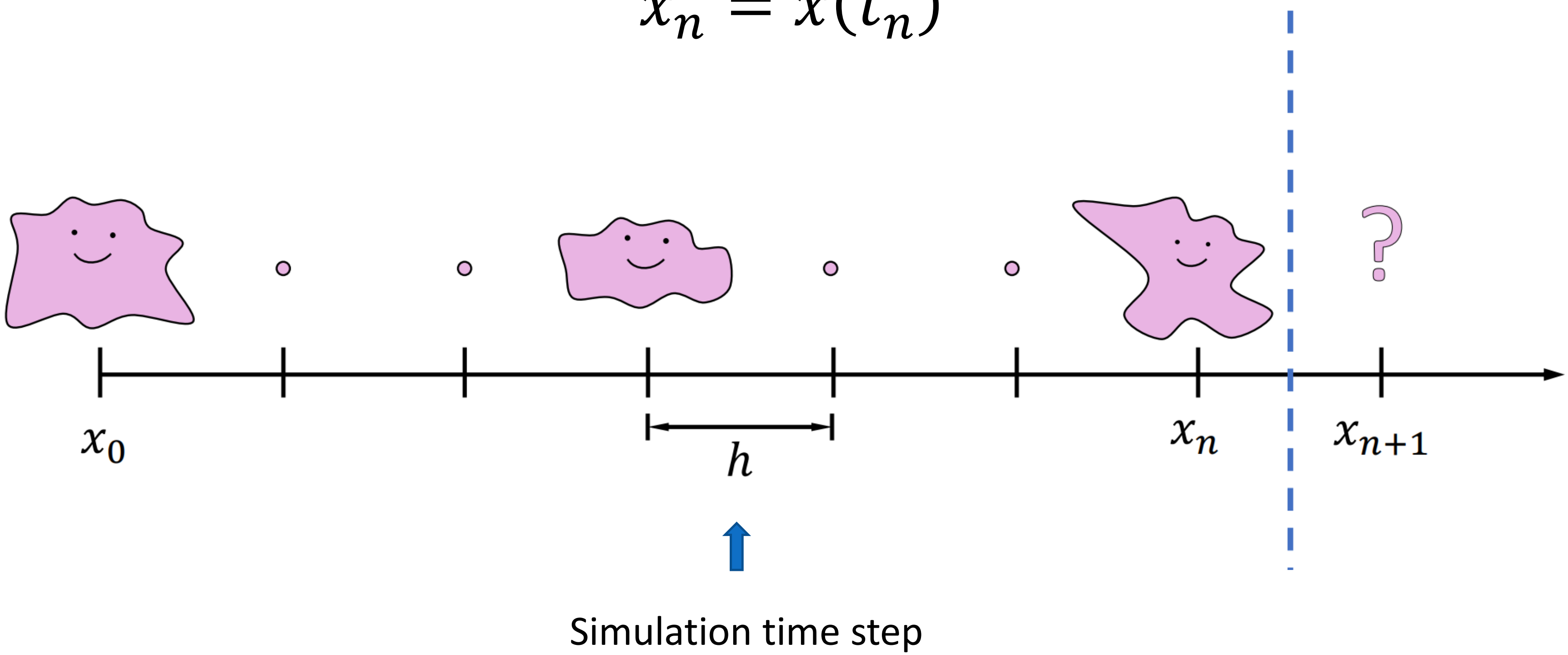
$$v = v_0 + \int_{t_0}^{t} a dt$$

$$\mathrm{x} = \mathrm{x}_0 + \int_{t_0}^{t} v dt$$

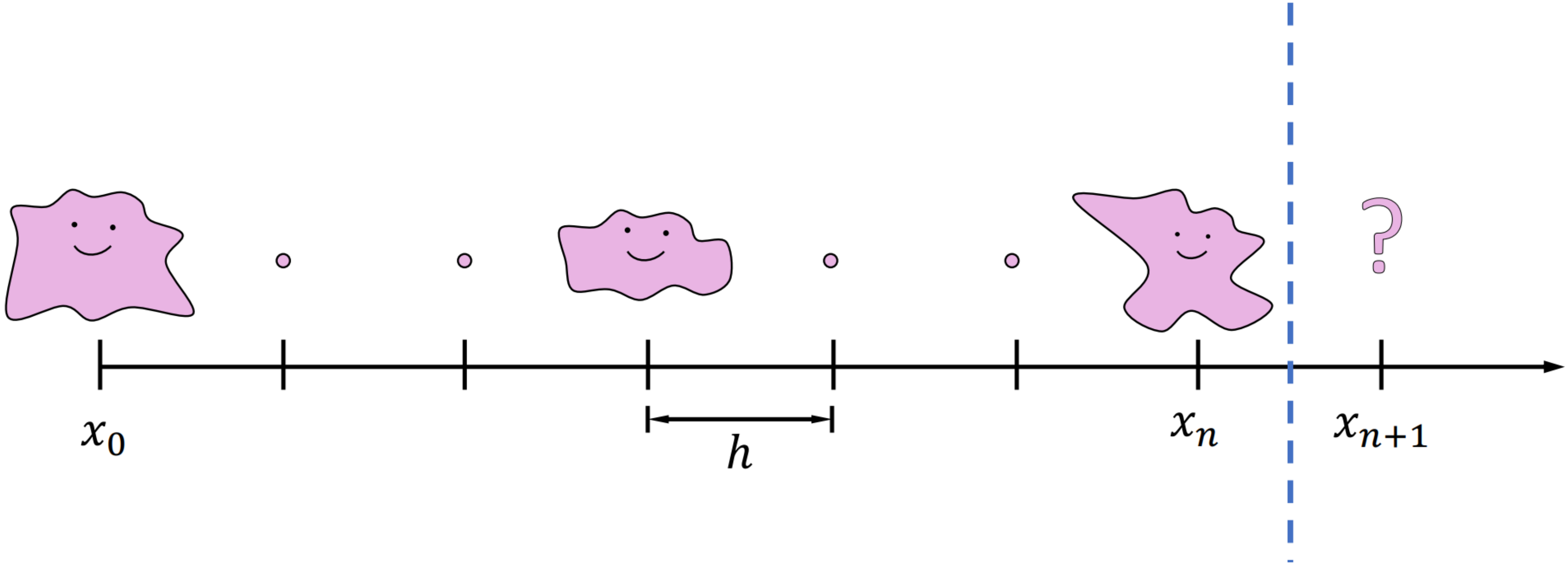
Temporal Discretization

$$x = x(t)$$

$$x_n = x(t_n)$$



Temporal Discretization



$$a = f(x, v, t)/m$$

$$v = v_0 + \int_{t_0}^{t} a dt$$

$$x = x_0 + \int_{t_0}^{t} v dt$$

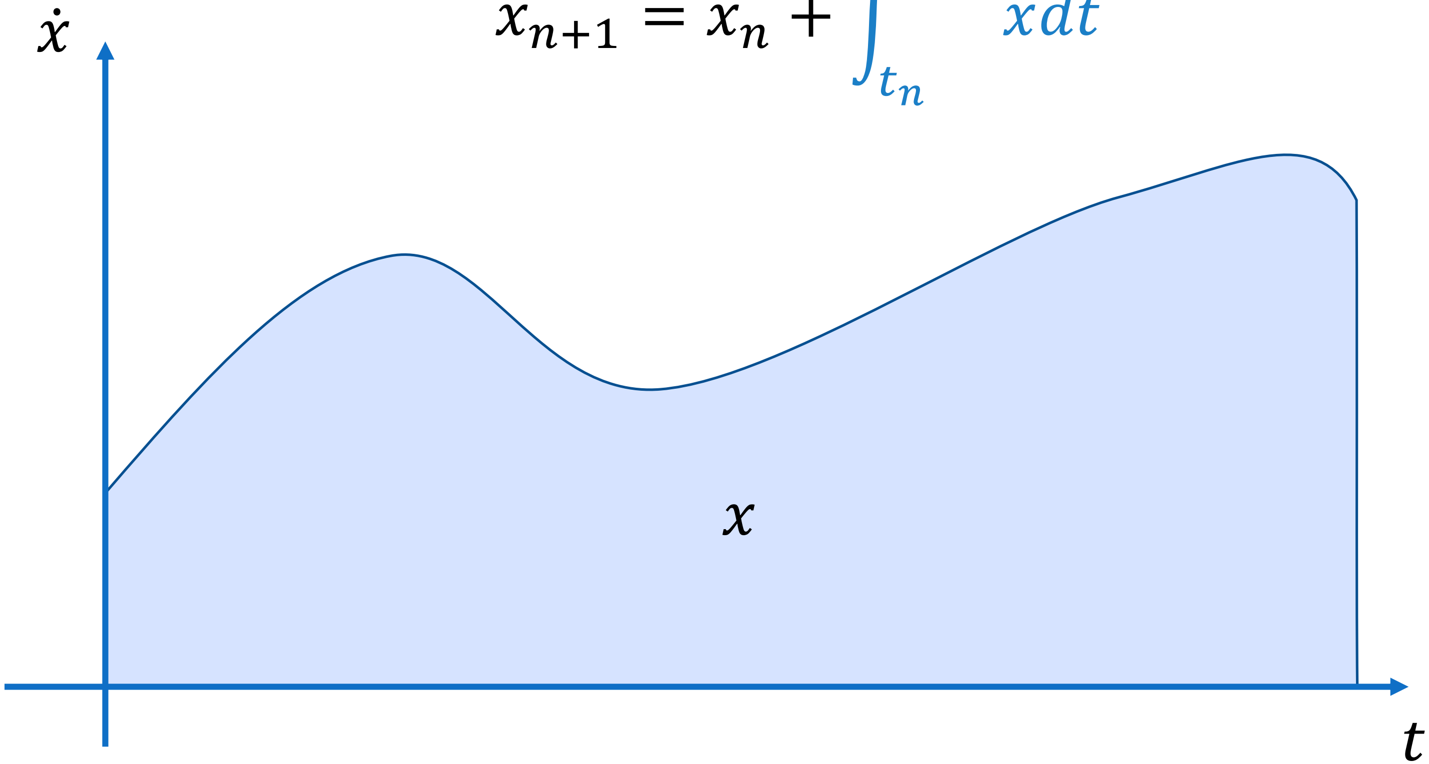
$$a = f(x, v, t)/m$$

$$v_{n+1} = v_n + \int_{t_n}^{t_{n+1}} a dt$$

$$x_{n+1} = x_n + \int_{t_n}^{t_{n+1}} \mathrm{v} dt$$

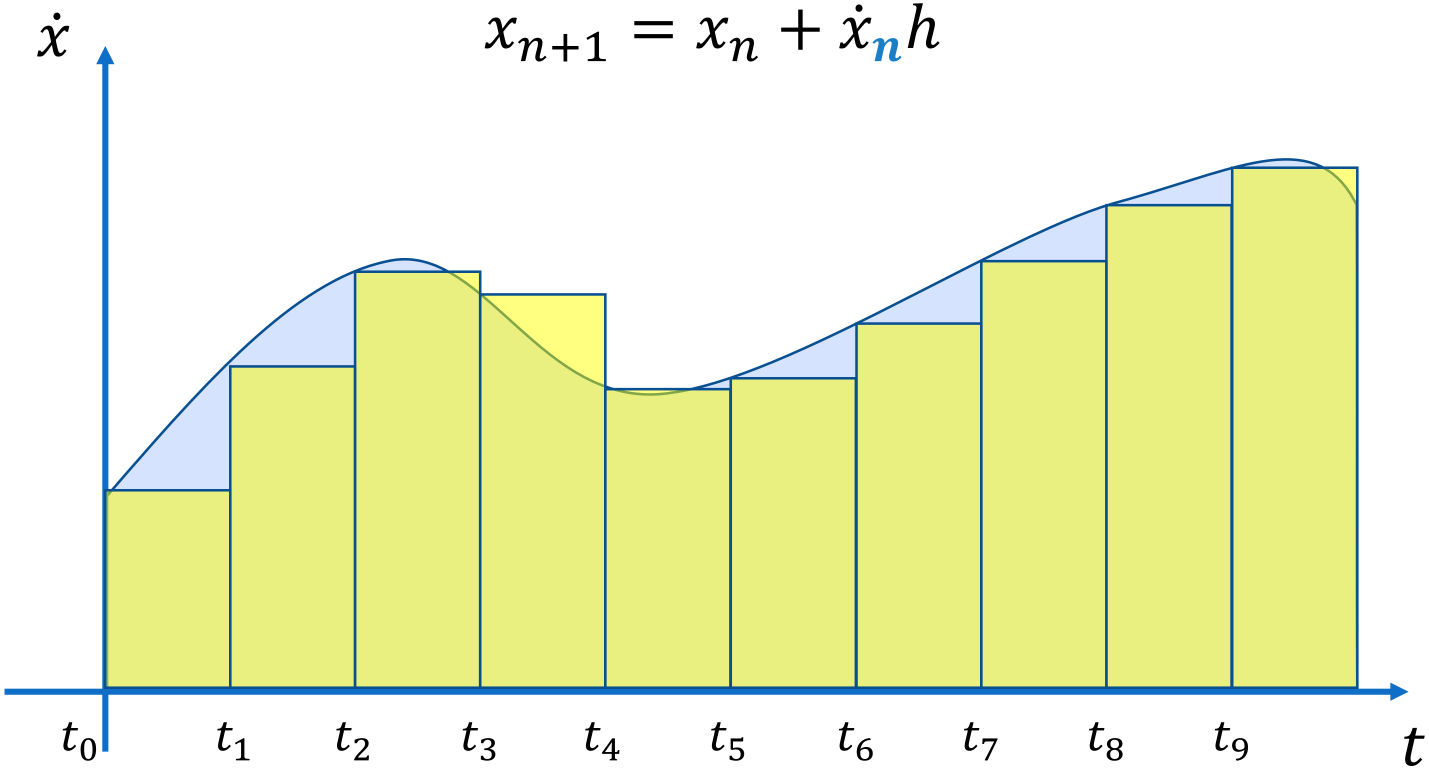
Numerical Integration

$$x_{n+1} = x_n + \int_{t_n}^{t_{n+1}} \dot{x}\,dt$$

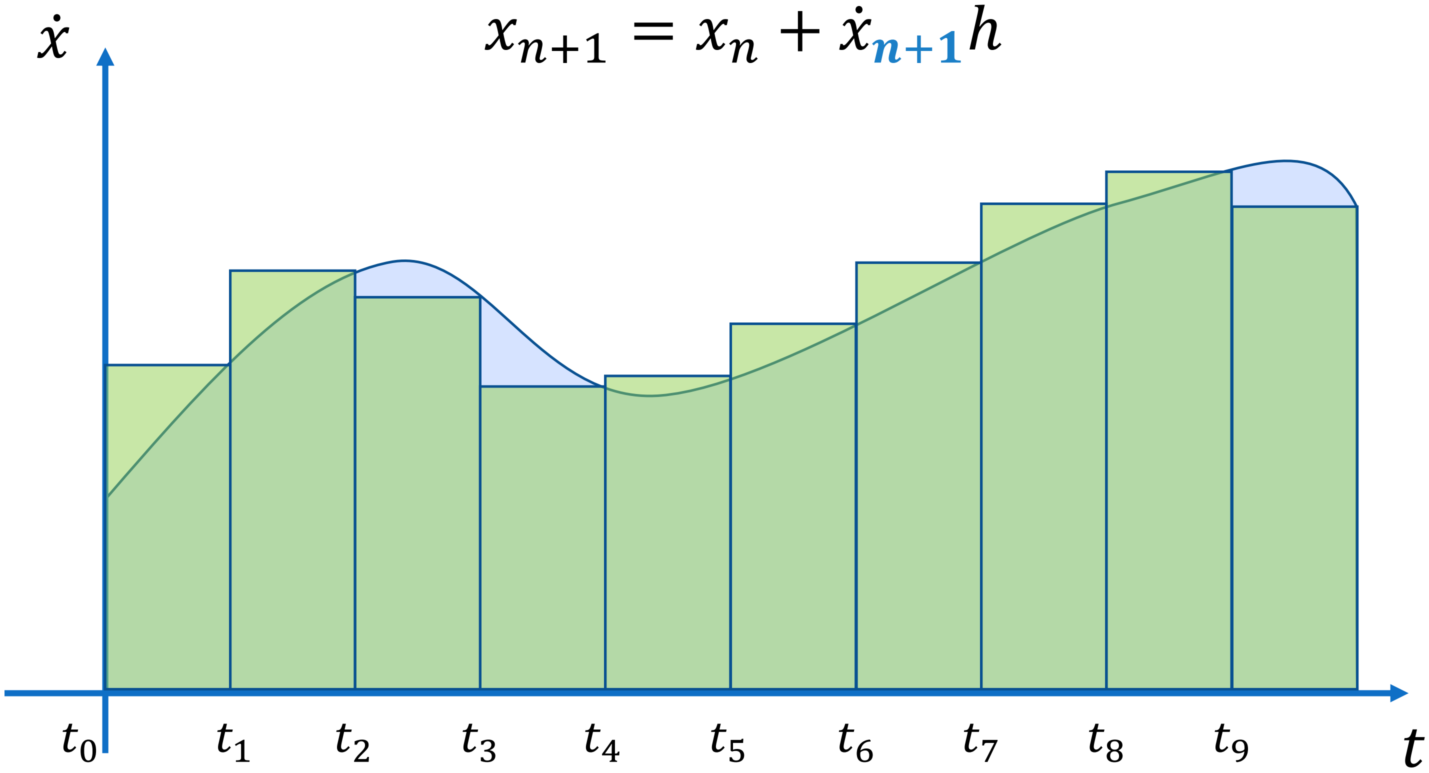


Numerical Integration

$$x_{n+1} = x_n + \dot{x}_n h$$



Numerical Integration



Numerical Integration

- Explicit/Forward Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n}} h$$

- Implicit/Backward Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n+1}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n+1}} h$$

Numerical Integration

- Explicit/Forward Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n}} h$$

- Implicit/Backward Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n+1}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n+1}} h$$

Requires information from the future

Numerical Integration

- Explicit/Forward Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n}} h$$

- Implicit/Backward Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n+1}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n+1}} h$$

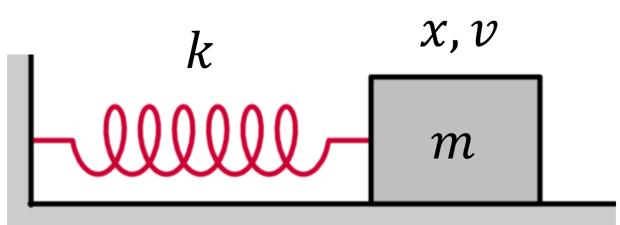
- Symplectic / Semi-implicit Euler Integration

$$v_{n+1} = v_n + a_{\boldsymbol{n}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n+1}} h$$

Mass on a Spring

$$\begin{bmatrix} v_n \\ x_n \end{bmatrix} = A^n \begin{bmatrix} v_0 \\ x_0 \end{bmatrix}$$

$\det|A^n| = (\det A)^n$



$$f = -kx$$

$|A| = 1 + kh$

Explicit Euler Integration

$$v_{n+1} = v_n - \frac{kx_n}{m}h$$
$$x_{n+1} = x_n + v_n h$$

$|A| = 1$

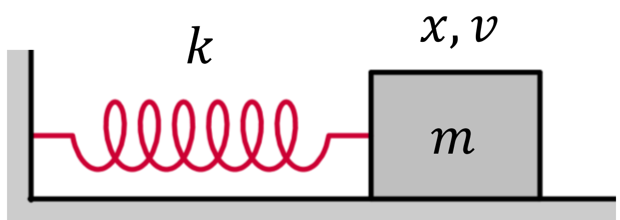
Semi-implicit Euler Integration

$$v_{n+1} = v_n - \frac{kx_n}{m}h$$
$$x_{n+1} = x_n + v_{n+1} h$$

Implicit Euler Integration

$$v_{n+1} = v_n - \frac{kx_{n+1}}{m}h$$
$$x_{n+1} = x_n + v_{n+1} h$$

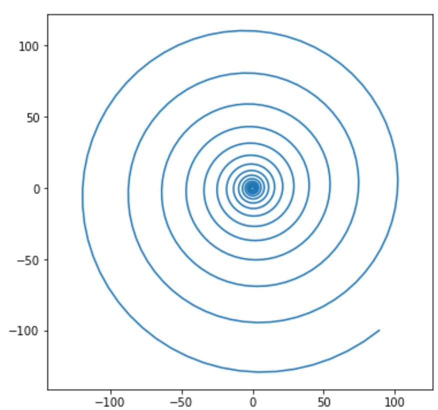
Mass on a Spring



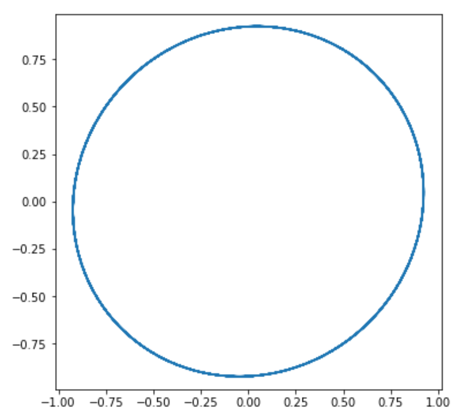
$$f = -kx$$

隐式欧拉能量不断减少，显式欧拉能量不断增加（步长越小越稳定）

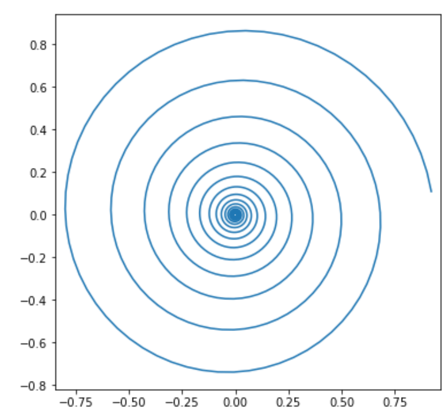
Explicit Euler Integration



Semi-implicit Euler Integration



Implicit Euler Integration



Numerical Integration

- Explicit/Forward Euler
  Symplectic/Semi-implicit Euler
  - Fast, no need to solve equations
  - Can be unstable under large time step

- Implicit/Backward Euler
  - Rock stable (unconditionally)
  - Slow, need to solve a large problem

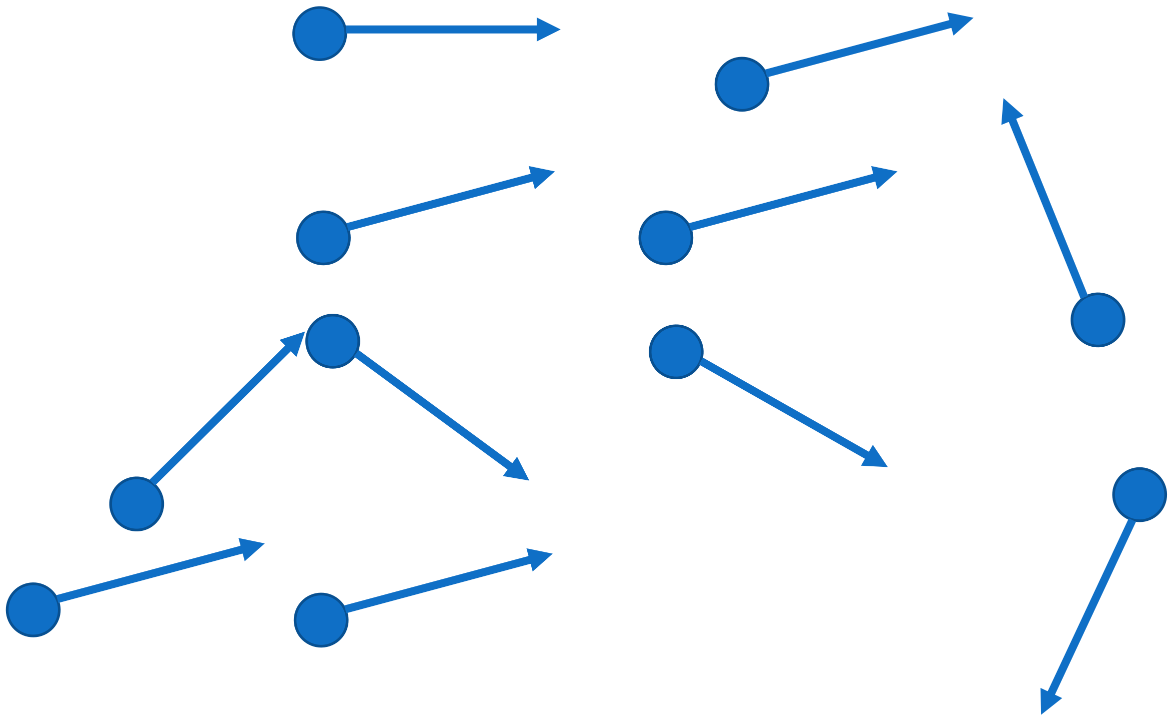
More Advanced Integration

- Runge–Kutta methods
- Variational integration

Particle Systems System

Particle Systems

- A set of (identical) simulated particles $\{x_i\}$



Particle Systems

- Simulation Loop
  - Clear forces
    - Prevent force accumulation
  - Calculate forces
    - Sum all forces into accumulators
  - Update
    - Loop over particles, update x_i and v_i using the corresponding integrator

x

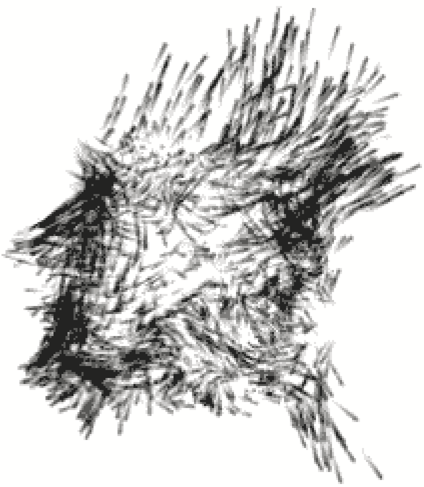
v

f

m

Particle Systems

- Forces
  - Constant
    - Gravity
  - Position/time dependent
    - Force field
  - Velocity-dependent
    - Damping, dragging
  - Others
    - Contacts, bouncing
    - Spring



Realtime?

- A few related concepts
  - Wall clock / real world time T
  - Simulation clock t
    - Advance $\boldsymbol{h}$ seconds every simulation step
  - $t \geq T$ → realtime simualtion
- Synchronization between the two worlds
  - Sleep when necessary

Example: Particle System in Unity

Everything to know about the PARTICLE SYSTEM
https://www.youtube.com/watch?v=FEA1wTMJAR0

Mass-Spring System

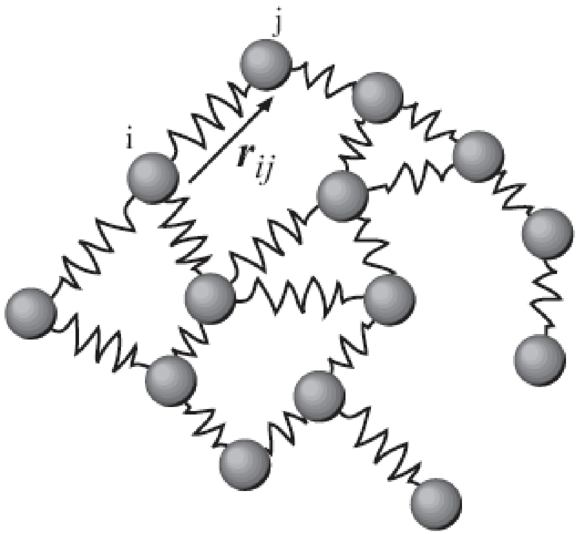
Mass Spring System

$$\boxed{\mathbf{f}_i = -\mathbf{f}_j}$$

$$f_{ij} = -k\left(\|x_i - x_j\| - l_0\right)\frac{x_i - x_j}{\|x_i - x_j\|}$$

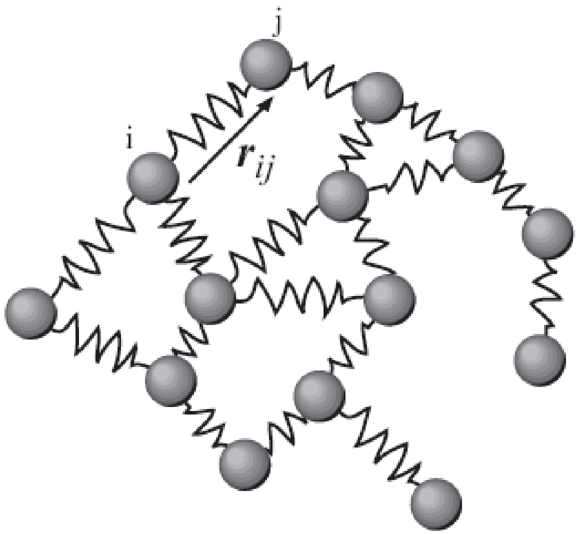
$$f_{ji} = -k\left(\|x_j - x_i\| - l_0\right)\frac{x_j - x_i}{\|x_j - x_i\|}$$

Mass Spring System



$$f_i = \sum_{j \in N(i)} f_{ij}$$

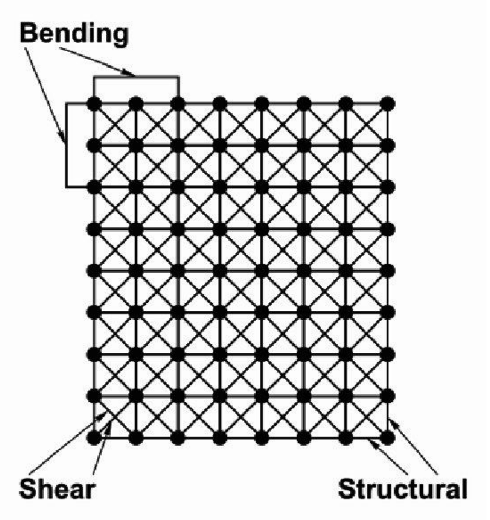
Damping?

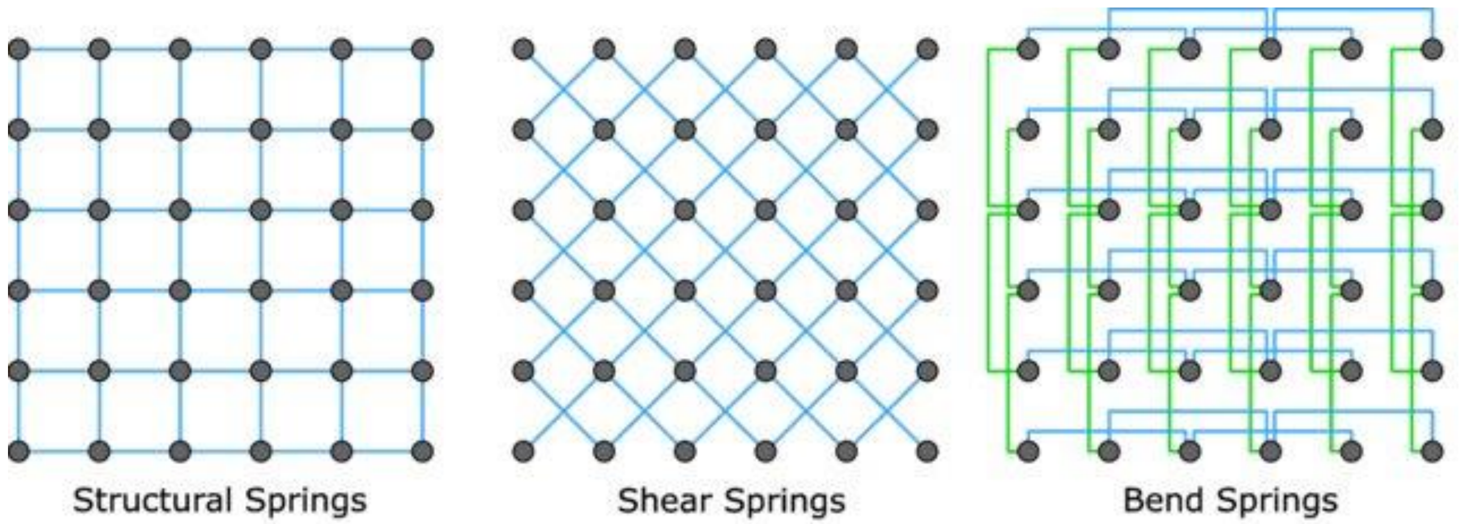


边的遍历

$$f_i = \sum_{j \in N(i)} f_{ij} - k_d (v_i - v_j)$$

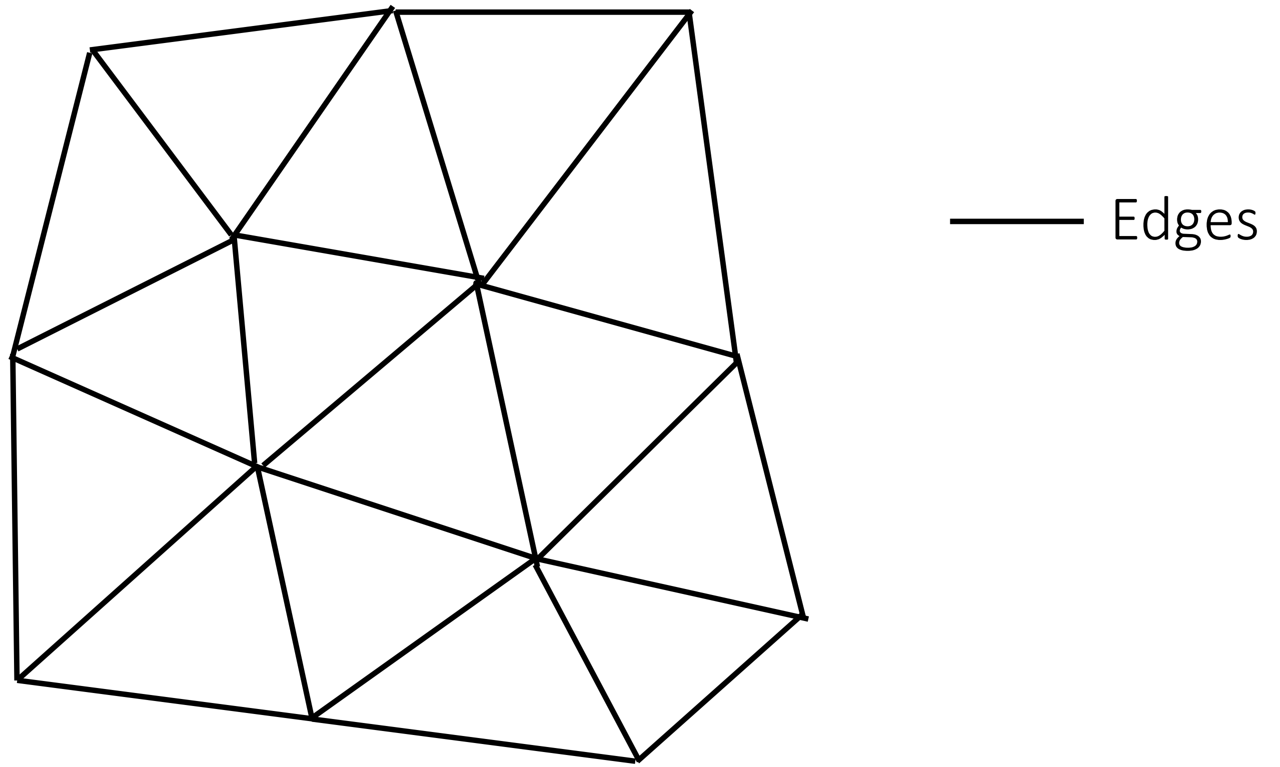
Structured Network





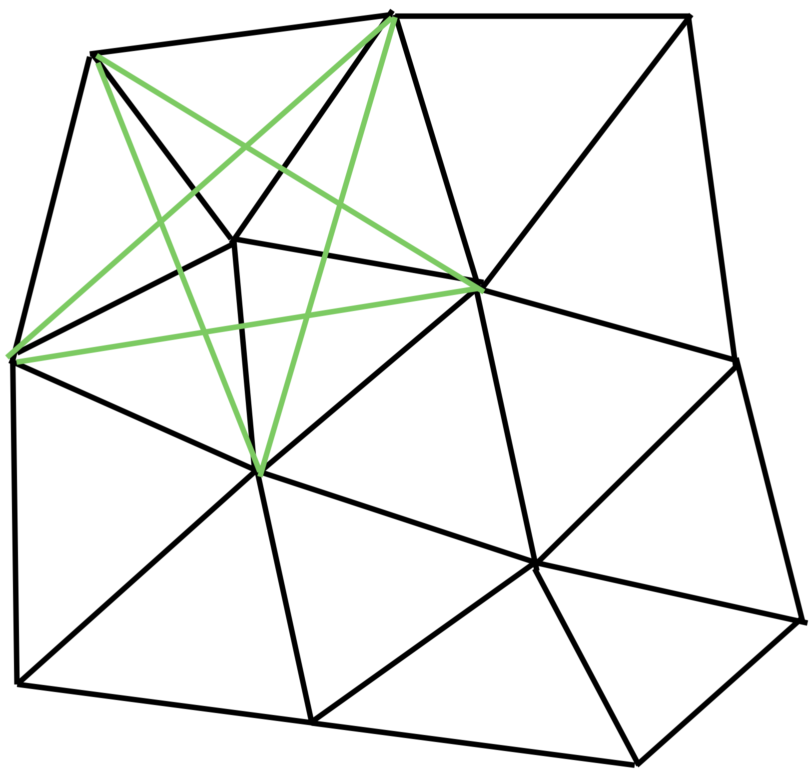
Structured Network

- For a triangle mesh



Structured Network

- For a triangle mesh



Edges

Bending 二阶连接
(every neighboring
triangle pair) 稳定性

Mass Spring System

- Simulation Loop
  - Clear forces
    - Prevent force accumulation
  - Calculate forces
    - Sum all forces into accumulators
  - Update
    - Loop over particles, update x_i and v_i using the corresponding integrator

$f_i \mathrel{+}= f_{ij}, \quad f_j \mathrel{-}= f_{ij}$

$$\boxed{\begin{array}{c} x \\ v \\ f \\ M \end{array}}$$

Updating

- (Semi-) Explicit Euler Integration
  - Need small time step

Explicit Euler Integration

$$v_{n+1} = v_n + hM^{-1}f_n h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n}} h$$

Semi-implicit Euler Integration

$$v_{n+1} = v_n + hM^{-1}f_n$$
$$x_{n+1} = x_n + v_{\boldsymbol{n+1}} h$$

Implicit Integration

Implicit Euler Integration

$$v_{n+1} = v_n + M^{-1} f_{\boldsymbol{n+1}} h$$
$$x_{n+1} = x_n + v_{\boldsymbol{n+1}} h$$

$$\Downarrow$$

$$x_{n+1} = x_n + h v_n + h^2 M^{-1} f(x_{n+1})$$

Implicit Integration as Optimization

$$x_{n+1} = x_n + hv_n + h^2 M^{-1} f(x_{n+1})$$

$$\Updownarrow$$

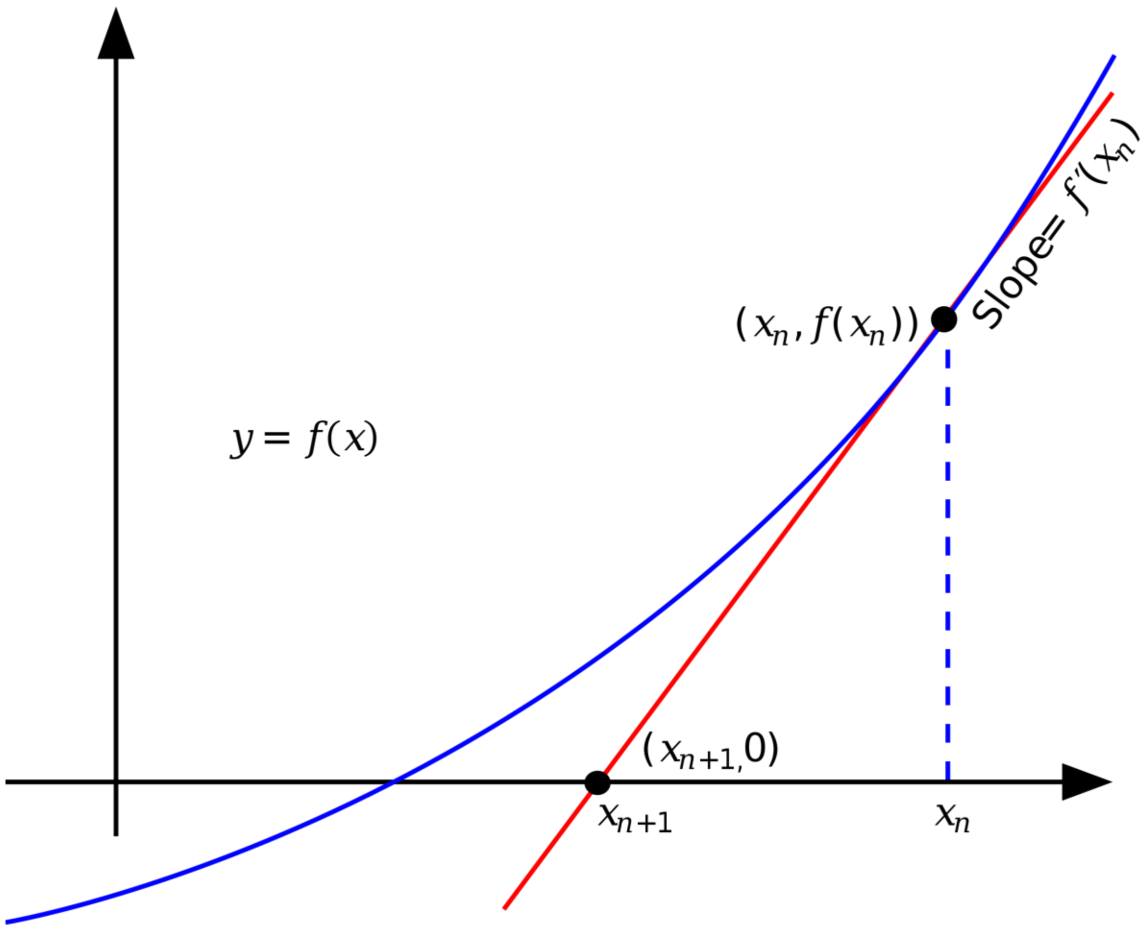
$$x_{n+1} = \text{argmax}_x \, F(x)$$

$$= \text{argmax}_x \frac{1}{2h^2} \|x - x_n - hv_n\|_M^2 + E(x)$$

$$\boxed{\|\mathbf{x}\|_{\mathbf{M}}^2 = \mathbf{x}^{\mathrm{T}} \mathbf{M} \mathbf{x}}$$

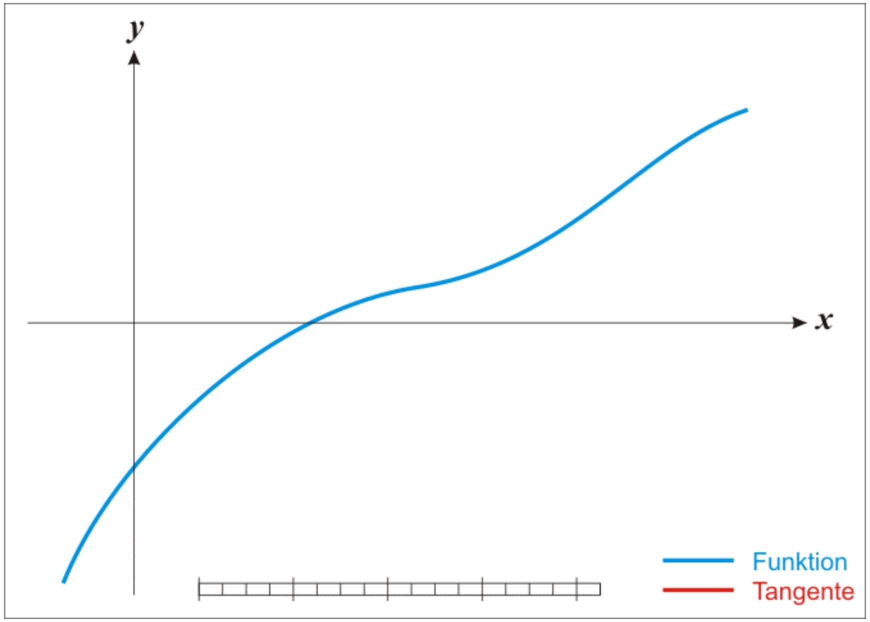
Elastic Energy

Newton-Raphson Method



Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



Simulation by Newton's Method

Specifically to simulation, we have: $\boxed{F(\mathbf{x}) = \frac{1}{2\Delta t^2}\left\|\mathbf{x} - \mathbf{x}^{[0]} - \Delta t\mathbf{v}^{[0]}\right\|_{\mathbf{M}}^2 + E(\mathbf{x})}$

$$\boxed{\nabla F\left(\mathbf{x}^{(k)}\right) = \frac{1}{\Delta t^2}\mathbf{M}\left(\mathbf{x}^{(k)} - \mathbf{x}^{[0]} - \Delta t\mathbf{v}^{[0]}\right) - \mathbf{f}\left(\mathbf{x}^{(k)}\right)}$$

$$\boxed{\frac{\partial^2 F\left(\mathbf{x}^{(k)}\right)}{\partial \mathbf{x}^2} = \frac{1}{\Delta t^2}\mathbf{M} + \mathbf{H}\left(\mathbf{x}^{(k)}\right)}$$

Initialize $\mathbf{x}^{(0)}$, often as $\mathbf{x}^{[0]}$ or $\mathbf{x}^{[0]} + \Delta t\mathbf{v}^{[0]}$

For $k = 0 \dots K$

Solve $\left(\frac{1}{\Delta t^2}\mathbf{M} + \mathbf{H}\left(\mathbf{x}^{(k)}\right)\right)\Delta\mathbf{x} = -\frac{1}{\Delta t^2}\mathbf{M}\left(\mathbf{x}^{(k)} - \mathbf{x}^{[0]} - \Delta t\mathbf{v}^{[0]}\right) + \mathbf{f}\left(\mathbf{x}^{(k)}\right)$

$\mathbf{x}^{(k+1)} \leftarrow \mathbf{x}^{(k)} + \Delta\mathbf{x}$

If $\|\Delta\mathbf{x}\|$ is small then break

$\mathbf{x}^{[1]} \leftarrow \mathbf{x}^{(k+1)}$

$\mathbf{v}^{[1]} \leftarrow \left(\mathbf{x}^{[1]} - \mathbf{x}^{[0]}\right) / \Delta t$