

# COM3105 Advanced Algorithms

## Homework 2

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### Question

(5 points)

Given a simple graph  $G = (V, E)$  (no loops or multiple edges), consider the following deterministic algorithm:

- start with an arbitrary partition.
- If moving a vertex from one part to the other increases the number of crossing edges, we will move it.
- We do this until there is no such vertex left.

Why does this algorithm terminate?

What can you say about the number of edges crossing when the algorithm terminates?

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### Answer

#### Termination of the Algorithm:

- Let  $C$  be the set of edges crossing the partition.
- Let state  $i$  be the state of the partition after  $i$  moves have been made.
- For each state  $i$ , we iteratively check a vertex  $v_j$  (where  $0 \leq j < |V|$ ) to see if moving it increases  $|C|$ .
- If moving a vertex  $v_j$  increases  $|C|$  we make this move and move to state  $i + 1$  and restart the same process.
- Checking each state  $i$  takes  $\leq |V|$  steps to check, where  $V$  is finite.
- As  $V$  is finite, for a simple graph,  $E$  is also finite, so the maximum  $|C|$  is  $|E|$ .
- Therefore, in the worst case:
  - at state 0,
    - ◊  $|C| = 0$  (no edges in partition)
    - ◊ Adding  $v_{|V|-1}$ , increases  $|C|$  by 1.
  - at state 1,
    - ◊  $|C| = 1$
    - ◊ Adding  $v_{|V|-1}$ , increases  $|C|$  by 1.
  - at state  $i$ ,
    - ◊  $|C| = i$
    - ◊ Adding  $v_{|V|-1}$ , increases  $|C|$  by 1.

- at state  $|E|$ ,
  - ◊  $|C| = |E|$
  - ◊ Adding  $v_{|V|-1}$ , cannot increase  $|C|$  by 1, as  $|C|$  is saturated.
  - ◊ Algorithm stops as no more edges to add.
- So  $|E|$  edges being added to  $C$  and taking  $|V|$  checks at each state.
- Hence the runtime is  $\mathcal{O}(|E| * |V|)$ .
- Both  $E$  and  $V$  are finite, so the algorithm terminates

### **Number of Crossing Edges at Termination:**

As the algorithm doesn't make any clever choices about which vertex to move, it is possible that the algorithm terminates in a local maximum.

However, we can prove that the algorithm terminates with a minimum number of crossing edges.

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