

$$f(x_1, \dots, x_n) = \prod_{i=1}^n f(x_i) \implies f(x_{n+1}, \dots, x_{n+m} | x_1, \dots, x_n) = \prod_{i=n+1}^m f(x_i)$$







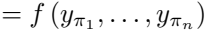


1999

$$= \int f(y_1, \dots, y_n | \theta) dF(\theta)$$

$$= \int \prod_{i=1}^n f(y_i | \theta) dF(\theta)$$







2021-2022

$$= \int \prod_{i=1}^n p(y_i | \theta) dF(\theta)$$



$$P(y_1 + \dots + y_n = s_n) = \binom{n}{s_n} P(y_1, \dots, y_n) = \binom{n}{s_n} P(y_{\pi_1}, \dots, y_{\pi_n})$$



$$p(y_1 + \dots + y_n = s_n) = \sum p(y_1 + \dots + y_n = s_n | y_1 + \dots + y_N = s_N) p(y_1 + \dots + y_N = s_N)$$











$$B_n \sim \text{Bin}\left(n, \frac{b_0}{b_0 + w_0}\right)$$





THE WORLD OF







$b_0$

$=$

---

$b_0$

$+$

$w_0$



123456

$$= \frac{w_0}{w_0 + b_0} - \frac{w_0 + 1}{w_0 + b_0 + 1}$$

A pixelated, black and white graphic of the text "P.E.S.O. + 1". The characters are rendered in a bold, blocky font with a dithered or pixelated texture. The "P" and "E" are large and prominent, followed by a smaller "S", then "O", a plus sign, and finally "1". The entire graphic is set against a plain white background.

$$= \frac{w_0}{w_0 + b_0 + 1} + \frac{b_0}{w_0 + b_0 + 1}$$



A pixelated, black and white graphic of the text "P.O. + 2". The characters are rendered in a bold, blocky font with a dithered or pixelated texture. The "P" and "O" are large and prominent, followed by a plus sign and the number "2". The entire graphic is set against a plain white background.

$$= \frac{b_0}{w_0 + b_0} \cdot \frac{b_0 + 1}{w_0 + b_0 + 1}$$





$$Y_n = \frac{B_n}{B_n + W_n} = \frac{B_n}{b_0 + w_0 + n}$$

$$B_{n+1} = \begin{cases} B_n & \text{con probabilidad } 1 - Y_n \\ B_n + 1 & \text{con probabilidad } Y_n \end{cases}$$

Explain the following:

1. The following are the main components of the business system:

1.1. The business system is a complex of interrelated elements.

$$= E\left(\frac{B_{n+1}}{b_0 + w_0 + n} \middle| x_1, \dots, x_n\right)$$



$$= \frac{1}{b_0 + w_0 + n} E(B_{n+1} | X_1, \dots, X_n)$$

$$= \frac{1}{b_0 + w_0 + n} (B_n (1 - y_n) + (B_n + 1) y_n)$$

$$= \frac{1}{b_0 + w_0 + n} (B_n - B_n Y_n + B_n Y_n + Y_n)$$

$$= \frac{1}{b_0 + w_0 + x(B_n + Y_n)}$$

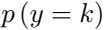




$$p(y = \kappa) = \int_0^1 p(y = \kappa | \theta) d\theta$$







$$= \int_0^1 p(y = k, \theta) d\theta$$

$$= \int_0^1 p(y = k | \theta) \underbrace{p(\theta)}_1 d\theta$$

$$= \int_0^1 p(y = k | \theta) d\theta$$

$$= \int_0^1 \binom{n}{k} \theta^k (1-\theta)^{n-k} d\theta$$

$$= \binom{n}{k} \int_0^1 \theta^k (1-\theta)^{n-k} d\theta$$

$$= \binom{n}{k} \frac{\Gamma(k+1)\Gamma(n-k+1)}{\Gamma(n+2)}$$

$$= \binom{n}{k} \frac{k!(n-k)!}{(n+1)!}$$



1

=

\_\_\_\_\_

$n$

+

1

$$\int_0^1 \frac{\Gamma(a+\beta)}{\Gamma(a)\Gamma(\beta)} \theta^{a-1} (1-\theta)^{\beta-1} d\theta$$

$$\Rightarrow \int_0^1 \theta^{\alpha-1} (1-\theta)^{\beta-1} d\theta$$

$=$

$$\frac{\Gamma(a)\Gamma(\beta)}{\Gamma(a+\beta)}$$

$$\Rightarrow \int_0^1 \theta^k (1-\theta)^{n-k} d\theta$$

$$\begin{aligned}
 &= \frac{\Gamma(k+1)\Gamma(n-k+1)}{\Gamma(k+1+n-k+1)} = \frac{\Gamma(k+1)\Gamma(n-k+1)}{\Gamma(n+2)}
 \end{aligned}$$









2019

QWERTY QWERTY

$$= \binom{n}{y} \theta^y (1-\theta)^{n-y} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$$x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(-1)(-1)}}{2(-1)}$$











$$a + y$$

$$a + y + \beta + x - y$$

$a$

+

$y$

=

—

$a$

+

$\beta$

+

$n$

$$= \frac{a}{a + \beta + n} + \frac{y}{a + \beta + n}$$

$$\begin{aligned}
 &= \frac{a + \beta}{a + \beta} \frac{a}{a + \beta + n} + \frac{n}{n} \frac{y}{a + \beta + n}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\alpha + \beta}{\alpha + \beta + n} \underbrace{\frac{\alpha}{\alpha + \beta}}_{E(\theta)} + \frac{n}{\alpha + \beta + n} \frac{y}{n}
 \end{aligned}$$







$$= \frac{(a+y)(\beta+n-y)}{(a+y+\beta+n-y)^2(a+y+\beta+n-y)}$$

$$= \frac{(a+y)(\beta+n-y)}{(a+\beta+n)^2(a+\beta+n+1)}$$

$$\begin{aligned}
 &= \frac{(1+y)(1+n-y)}{(2+n)^2(2+n+1)} = \frac{(1+y)(1+n-y)}{(2+n)^2(n+3)} < \frac{1}{12} =
 \end{aligned}$$



$$\frac{(a+y)(\beta+n-y)}{(a+\beta+n)^2(a+\beta+n+1)} \leq \frac{a\beta}{(a+\beta)^2(a+\beta+1)}$$





000001100011



$$\phi = \log\left(\frac{\theta}{1-\theta}\right) = \eta(\theta)$$



$$= \underbrace{p(\phi)}_{\propto 1} \left| \frac{d\phi}{d\theta} \right|$$

$$\propto \left| \frac{d}{d\theta} \log \left( \frac{\theta}{1-\theta} \right) \right|$$

=

1  
—  
0

+

1  
—  
1 — 0

11

11

11

11

11







$$= \binom{n}{y} \theta^y (1-\theta)^{n-y} \theta^{-1} (1-\theta)^{-1}$$

















WORLD



WAVELENGTHS

$$= \prod_{i=1}^n \left[ (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{1}{2\sigma^2} (y_i - \theta)^2\right) \right] (2\pi\tau_0^2)^{-1/2} \exp\left(-\frac{1}{2\tau_0^2} (\theta - \mu_0)^2\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^n (y_i - \theta)^2\right) \exp\left(-\frac{1}{2\tau_0^2}(\theta - \mu_0)^2\right)$$

$$= \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^n (y_i - \theta)^2 - \frac{1}{2\tau_0^2} (\theta - \mu_0)^2 \right)$$

$$\propto \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^n (y_i^2 - 2y_i\theta + \theta^2) - \frac{1}{2\tau_0^2} (\theta^2 - 2\theta\mu_0 + \mu^2) \right)$$

$$\propto \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^n (\theta^2 - 2y_i\theta) - \frac{1}{2\tau_0^2} (\theta^2 - 2\theta\mu_0) \right)$$



$$\theta|y \sim N\left(\frac{\frac{1}{\sigma^2}\mu_0 + \frac{n}{\tau_0^2}\bar{y}}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}}, \frac{1}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}}\right)$$

$$\theta|y \sim \mathcal{N}\left(\frac{\frac{1}{20^2}180 + \frac{n}{40^2}150}{\frac{1}{20^2} + \frac{n}{40^2}}, \frac{1}{\frac{1}{20^2} + \frac{n}{40^2}}\right)$$

WAVELENGTH





1992-1993





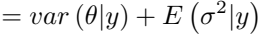


$$= \frac{\frac{1}{\sigma^2} \mu_0 + \frac{n}{\tau_0^2} \bar{y}}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}}$$





$\frac{1}{2} \pi$



$$= \frac{1}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}} + \sigma^2$$

$$\tilde{y}|y \sim \mathcal{N}\left(\frac{\frac{1}{20^2}180 + \frac{n}{40^2}150}{\frac{1}{20^2} + \frac{n}{40^2}}, \frac{1}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}} + \sigma^2\right)$$







$$p(v_i) \propto \frac{1}{1 + (v_i - \theta)^2}$$





$$\ell(\theta|y) = \prod_{i=1}^n \frac{1}{1 + (y_i - \theta)^2}$$



Вопросы по  
математике

POWERS | EX



expanding the  
range of  
products  
available  
to  
the  
public  
and  
the  
range of  
products  
available  
to  
the  
public

$$= (\sigma^2)^{-3/2} \sigma^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum (x_i - \mu)^2\right)$$

$$= (\sigma^2)^{-3/2} \sigma^{-n/2} \exp \left( -\frac{1}{2\sigma^2} \sum (x_i - \bar{x})^2 + n(\bar{x} - \mu)^2 \right)$$

$$= (\sigma^2)^{-3/2} \sigma^{-n/2} \exp \left( -\frac{1}{2\sigma^2} \left( \sum (x_i - \bar{x})^2 + n(\bar{x} - \mu)^2 \right) \right)$$

$$= (\sigma^2)^{-3/2-n/2} \exp\left(-\frac{1}{2\sigma^2}\left((n-1)s^2 + n(\bar{x} - \mu)^2\right)\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\left(\pi(\bar{x}-\mu)^2\right)\right)$$



W E R O S E



$$\propto (\sigma^2)^{-3/2-n/2} \exp\left(-\frac{1}{2\sigma^2}\left((n-1)s^2 + n(\bar{x} - \mu)^2\right)\right)$$

$$= (\sigma^2)^{-3/2-n/2} \exp\left(-\frac{1}{2\sigma^2}(n-1)s^2\right) \underbrace{\exp\left(-\frac{1}{2\sigma^2}n(\bar{x}-\mu)^2\right) \left(\frac{\sigma}{n}\right)^{-1/2} \left(\frac{\sigma}{n}\right)^{1/2}}_{\text{kernel de una normal}}$$

kernel de una normal

POPEX

$$\propto (\sigma^2)^{-(n/2+1)} \exp\left(-\frac{1}{2\sigma^2}(n-1)s^2\right)$$



2020-2021







$$= \int_0^{\infty} p(\mu, \sigma^2 | x) d\sigma^2$$

$$\propto \int_0^{\infty} (\sigma^2)^{-3/2} \sigma^{-n/2} \exp\left(-\frac{1}{2\sigma^2} A\right) d\sigma^2$$



$$= \int_0^1 dx \left( x^2 + \pi \left( x - \frac{1}{x} \right)^2 \right)$$

2020

A pixelated, black and white graphic of the text "APR 22". The letters are rendered in a bold, blocky font with a dithered or pixelated texture. The "A" is particularly prominent, with a thick vertical stroke. The "P" has a curved top. The "R" is also blocky. The "2" is composed of several horizontal and vertical segments. The "22" is written in a similar style. The overall image has a low-resolution, digital-art feel.

002

11

1

A 22

$$\propto \int_0^{\infty} \left( \frac{A}{2z} \right)^{-3/2 - n/2} \exp(-z) \left( -\frac{A}{2z^2} \right) dz$$



$$\alpha\left(\frac{A}{2z}\right)^{-1/2-\pi/2}$$



1992-93

2020-2021

www.2020

$\psi(x) = \psi(x)$

$\frac{d}{dx} \left( x^2 \right) = 2x$







2020-2021

WORLD'S

WORLD OF

POPO2X

$$\propto \det(\sigma^2 I)^{-1/2} \exp\left(-\frac{1}{2\sigma^2} (y - X\beta)^t (y - X\beta)\right)$$

$$= (\sigma^2)^{-1/2} \exp\left(-\frac{1}{2\sigma^2} (y - X\beta)^t (y - X\beta)\right)$$

Handwritten text in a cursive script, likely a signature or name, rendered in grayscale. The text is highly stylized and difficult to decipher, but appears to be a single line of writing.

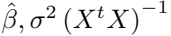


$$\propto (\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} (y - X\beta)^t (y - X\beta)\right) \sigma^{-2}$$

$$= (\sigma^2)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2} (y - X\beta)^t (y - X\beta)\right)$$







POE 2019

Handwritten text in a cursive script, likely a signature or name, rendered in grayscale. The text is split into two lines by a vertical separator. The first line contains the characters "P" followed by a stylized "D" and a "2". The second line contains a stylized "B", a comma, a stylized "V", another stylized "D", and a "2".





$$p(\beta|y) = \frac{p(\beta, \sigma^2|y)}{p(\sigma^2|\beta, y)} = \frac{p(y|\beta, \sigma^2)p(\beta, \sigma^2)}{p(\sigma^2|\beta, y)}$$

==> 2020, 2020, 2020

$$p(\sigma^2|y) = \frac{p(y|\beta, \sigma^2) p(\beta, \sigma^2)}{p(\beta|\sigma^2, y)}$$

PO2020

$$= \frac{p(\beta, \sigma^2 | y)}{p(\sigma^2 | y)}$$

expanding our  
operations in  
the United States  
and Europe.

$$\propto \exp\left(-\frac{1}{2\sigma^2}(y-X\beta)^t(y-X\beta)\right)$$

$$= \exp \left( -\frac{1}{2\sigma^2} (y - X\hat{\beta})^t (y - X\hat{\beta}) - \frac{1}{2\sigma^2} (\beta - \hat{\beta})^t X^t X (\beta - \hat{\beta}) \right)$$



$$\propto \exp\left(-\frac{1}{2\sigma^2}(\beta-\hat{\beta})^t X^t X (\beta-\hat{\beta})\right)$$



$$\sim N(\theta, \sigma^2 (X^t X)^{-1})$$

2021

$$= \frac{p(\beta, \sigma^2 | y)}{p(\beta | \sigma^2, y)}$$

$$\propto \frac{(\sigma^2)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2} (y - X\beta)^t (y - X\beta)\right)}{\det\left(\sigma^2 (X^t X)^{-1}\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2} (\beta - \hat{\beta})^t X^t X (\beta - \hat{\beta})\right)}$$

$$\begin{aligned}
 &= \frac{(\sigma^2)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2} \left(y - X\hat{\beta}\right)^t \left(y - X\hat{\beta}\right) - \frac{1}{2\sigma^2} \left(\beta - \hat{\beta}\right)^t X^t X \left(\beta - \hat{\beta}\right)\right)}{\det\left(\sigma^2 (X^t X)^{-1}\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2} \left(\beta - \hat{\beta}\right)^t X^t X \left(\beta - \hat{\beta}\right)\right)}
 \end{aligned}$$

$$= \frac{(\sigma^2)^{-n/2-2} \exp \left( -\frac{1}{2\sigma^2} \left( y - X\hat{\beta} \right)^t \left( y - X\hat{\beta} \right) \right)}{\det \left( \sigma^2 (X^t X)^{-1} \right)^{-1/2} \exp (0)}$$



$$\begin{aligned}
 &= \frac{(\sigma^2)^{-n/2-2} \exp \left( -\frac{1}{2\sigma^2} \left( y - X\hat{\beta} \right)^t \left( y - X\hat{\beta} \right) \right)}{(\sigma^{2k})^{-1/2} \det \left( (X^t X)^{-1} \right)^{-1/2} \exp (0)}
 \end{aligned}$$

$$\propto (\sigma^2)^{-n/2-2+k/2} \exp\left(-\frac{1}{2\sigma^2}(y-X\beta)^t(y-X\beta)\right)$$



*1920-1921*







$$\pi(x) = \Phi(x) + \psi(x)$$









$\Phi_{\text{PO}} + \Phi_{\text{P1x}}$  |  $\Phi_{\text{P2x}}$



$$= \Phi_{0,1} \left( \frac{\beta_0 + \beta_1 x - \mu}{\sigma} \right)$$

