$$f\left(x_1,\ldots,x_n\right)=\prod_{i=1}^{n} f\left(x_i\right) \implies f\left(x_{n+1},\ldots,x_{n+m}|x_1,\ldots,x_n\right)=\prod_{i=n+1}^{m} f\left(x_i\right)$$

$$\theta \sim f(\theta)$$

$$Y_1, \ldots, Y_n$$

θ

$$f(y_1, \ldots, y_n)$$

$$= \int f\left(y_1, \ldots, y_n | \theta\right) dF\left(\theta\right)$$

$$= \int \prod_{i=1}^{n} f\left(y_i|\theta\right) dF\left(\theta\right)$$

$$= f\left(y_{\pi_1}, \ldots, y_{\pi_n}\right)$$

$0 - 1$

$$p\left(y_1, \ldots, y_n\right)$$

$$= \int \prod_{i=1}^{n} p\left(y_i|\theta\right) dF\left(\theta\right)$$

$$F(\theta) = \lim_{n \to \infty} P(s_n \le \theta)$$

$$p\left(y_1+\ldots+y_n=s_n\right)=\binom{n}{s_n}p\left(y_1,\ldots,y_n\right)=\binom{n}{s_n}p\left(y_{\pi_1},\ldots,y_{\pi_n}\right)$$

$N > n$

$$p\left(y_1+\ldots+y_n=s_n\right)=\sum p\left(y_1+\ldots+y_n=s_n|y_1+\ldots+y_N=s_N\right)p\left(y_1+\ldots+y_N=s_N\right)$$

b_0

w_0

B_n :

n

$$B_n \sim Bin\left(n, \frac{b_0}{b_0 + w_0}\right)$$

B_0

$$= b_0$$

$$\implies P\left(B_0 = b_0\right)$$

$$= 1$$

$$P\left(B_1 = b_0 + 1\right)$$

$$= \frac{b_0}{b_0 + w_0}$$

$n = 2$

$$P\left(B_2 = b_0\right)$$

$$= \frac{w_0}{w_0 + b_0} \cdot \frac{w_0 + 1}{w_0 + b_0 + 1}$$

$$P\left(B_2 = b_0 + 1\right)$$

$$= \frac{w_0}{w_0 + b_0} \frac{b_0}{w_0 + b_0 + 1} + \frac{b_0}{w_0 + b_0} \frac{w_0}{w_0 + b_0 + 1}$$

$$P\left(B_2 = b_0 + 2\right)$$

$$= \frac{b_0}{w_0 + b_0} \cdot \frac{b_0 + 1}{w_0 + b_0 + 1}$$

$X_1, \ldots, X_n$

Y_n

$$Y_n = \frac{B_n}{B_n + W_n} = \frac{B_n}{b_0 + w_0 + n}$$

$$B_{n+1} = \begin{cases} B_n & \text{con probabilidad } 1 - Y_n \\ B_n + 1 & \text{con probabilidad } Y_n \end{cases}$$

$$E\left(Y_{n+1}|X_1,\ldots,X_n\right)$$

$$= E\left(\frac{B_{n+1}}{b_0 + w_0 + n}|X_1, \ldots, X_n\right)$$

$$= \frac{1}{b_0 + w_0 + n} E\left(B_{n+1} | X_1, \ldots, X_n\right)$$

$$= \frac{1}{b_0 + w_0 + n} \left(B_n \left(1 - Y_n \right) + \left(B_n + 1 \right) Y_n \right)$$

$$= \frac{1}{b_0 + w_0 + n} \left(B_n - B_n Y_n + B_n Y_n + Y_n \right)$$

$$= \frac{1}{b_0 + w_0 + n} \left(B_n + Y_n \right)$$

$$= Y_n$$

$0, 1$

$$p\left(y=k\right)=\int_{0}^{1}p\left(y=k|\theta\right)d\theta$$

$k = 1, \ldots, n$

$$p\left(y=k\right)$$

$$= \int_0^1 p\left(y = k, \theta\right) d\theta$$

$$= \int_0^1 p\left(y=k|\theta\right) \underbrace{p\left(\theta\right)}_{1} d\theta$$

$$= \int_0^1 p\left(y = k|\theta\right) d\theta$$

$$= \int_0^1 \binom{n}{k} \theta^k \, (1-\theta)^{n-k} \, d\theta$$

$$= \binom{n}{k} \int_0^1 \theta^k \left(1-\theta\right)^{n-k} d\theta$$

$$= \binom{n}{k} \frac{\Gamma\left(k+1\right)\Gamma\left(n-k+1\right)}{\Gamma\left(n+2\right)}$$

$$= \binom{n}{k} \frac{k!\,(n-k)!}{(n+1)!}$$

$$= \frac{1}{n+1}$$

$$\int_0^1 \frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)} \theta^{\alpha-1} \left(1-\theta\right)^{\beta-1} d\theta$$

$$\implies \int_0^1 \theta^{\alpha-1} (1-\theta)^{\beta-1} \, d\theta$$

$$= \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\implies \int_0^1 \theta^k (1-\theta)^{n-k} \, d\theta$$

$$= \frac{\Gamma(k+1)\,\Gamma(n-k+1)}{\Gamma(k+1+n-k+1)} = \frac{\Gamma(k+1)\,\Gamma(n-k+1)}{\Gamma(n+2)}$$

$\theta \sim$

$$(\alpha, \beta)$$

y

$$p\left(\theta|y\right)$$

$$\propto p\left(y|\theta\right)p\left(\theta\right)$$

$$= \binom{n}{y} \theta^{y} (1-\theta)^{n-y} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$$\propto \theta^{\alpha+y-1}\,(1-\theta)^{\beta+n-y-1}$$

$$\Longrightarrow \theta | y$$

~

$$(\alpha + y, \beta + n - y)$$

$$E\left(\theta|y\right)$$

$$= \frac{\alpha + y}{\alpha + y + \beta + n - y}$$

$$= \frac{\alpha + y}{\alpha + \beta + n}$$

$$= \frac{\alpha}{\alpha + \beta + n} + \frac{y}{\alpha + \beta + n}$$

$$= \frac{\alpha+\beta}{\alpha+\beta}\frac{\alpha}{\alpha+\beta+n} + \frac{n}{n}\frac{y}{\alpha+\beta+n}$$

$$= \frac{\alpha+\beta}{\alpha+\beta+n}\underbrace{\frac{\alpha}{\alpha+\beta}}_{E(\theta)}+\frac{n}{\alpha+\beta+n}\frac{y}{n}$$

$$(1, 1)$$

$(\theta|y)$

$$= \frac{(\alpha + y)(\beta + n - y)}{(\alpha + y + \beta + n - y)^2 (\alpha + y + \beta + n - y)}$$

$$= \frac{(\alpha + y)(\beta + n - y)}{(\alpha + \beta + n)^2 (\alpha + \beta + n + 1)}$$

$$= \frac{(1+y)(1+n-y)}{(2+n)^2(2+n+1)} = \frac{(1+y)(1+n-y)}{(2+n)^2(n+3)} \leq \frac{1}{12} =$$

(θ)

$$\frac{(\alpha+y)(\beta+n-y)}{(\alpha+\beta+n)^2(\alpha+\beta+n+1)} \leq \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$y \sim$

$$(n, \theta)$$

$$p(\theta) \propto \theta^{-1}(1-\theta)^{-1}$$

$$\phi = \log\left(\frac{\theta}{1-\theta}\right) = h(\theta)$$

$$p\left(\theta\right)$$

$$= \underbrace{p\left(\phi\right)}_{\propto 1} \left| \frac{d\phi}{d\theta} \right|$$

$$\propto \left| \frac{d}{d\theta} \log\left(\frac{\theta}{1-\theta}\right) \right|$$

$$= \frac{1}{\theta} + \frac{1}{1-\theta}$$

$$= \theta^{-1} (1-\theta)^{-1}$$

$$y = 0$$

$$n = 0$$

$$= \binom{n}{y} \theta^{y} \left(1-\theta\right)^{n-y} \theta^{-1} \left(1-\theta\right)^{-1}$$

$$\overline{y} = 150$$

20

180

$y|\theta$

$$\sim N\left(\theta, \sigma^2\right)$$

θ

$$\sim N\left(\mu_0, \tau_0^2\right)$$

$$= \prod_{i=1}^{n} \left[\left(2\pi\sigma^2\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2} \left(y_i - \theta\right)^2 \right) \right] \left(2\pi\tau_0^2\right)^{-1/2} \exp\left(-\frac{1}{2\tau_0^2} \left(\theta - \mu_0\right)^2 \right)$$

$$\propto \exp\left(\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i-\theta)^2\right)\exp\left(\frac{1}{2\tau_0^2}(\theta-\mu_0)^2\right)$$

$$= \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - \theta\right)^2 - \frac{1}{2\tau_0^2}\left(\theta - \mu_0\right)^2\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i^2 - 2y_i\theta + \theta^2\right) - \frac{1}{2\tau_0^2}\left(\theta^2 - 2\theta\mu_0 + \mu^2\right)\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(\theta^2 - 2y_i\theta\right) - \frac{1}{2\tau_0^2}\left(\theta^2 - 2\theta\mu_0\right)\right)$$

$$\theta|y \sim N\left(\frac{\frac{1}{\sigma^2}\mu_0 + \frac{n}{\tau_0^2}\overline{y}}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}}, \frac{1}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}}\right)$$

$$\theta|y \sim N\left(\frac{\frac{1}{20^2}180 + \frac{n}{40^2}150}{\frac{1}{20^2} + \frac{n}{40^2}}, \frac{1}{\frac{1}{20^2} + \frac{n}{40^2}}\right)$$

$$\tilde{y}|y \sim N\left(\widetilde{\mu}, \widetilde{\sigma}^2\right)$$

$$\widetilde{\mu}$$

$$= E(\tilde{y}|y)$$

$$= E\left(E\left(\tilde{y}|\theta, y\right)|y\right)$$

$$= E\left(E\left(\tilde{y}|\theta\right)|y\right)$$

$$= E(\theta|y)$$

$$= \frac{\frac{1}{\sigma^2}\mu_0 + \frac{n}{\tau_0^2}\overline{y}}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}}$$

$$\widetilde{\sigma}^2$$

$$= var\left(\tilde{y}|y\right)$$

$$= var\left(E\left(\tilde{y}|\theta\right)|y\right) + E\left(var\left(\tilde{y}|\theta\right)|y\right)$$

$$= var\left(\theta|y\right) + E\left(\sigma^2|y\right)$$

$$= \frac{1}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}} + \sigma^2$$

$$\tilde{y}|y \sim N\left(\frac{\frac{1}{20^2}180 + \frac{n}{40^2}150}{\frac{1}{20^2} + \frac{n}{40^2}}, \frac{1}{\frac{1}{\sigma^2} + \frac{n}{\tau_0^2}} + \sigma^2\right)$$

$y_1 \ldots, y_5$

1

$$p\left(y_i|\theta\right) \propto \frac{1}{1+\left(y_i-\theta\right)^2}$$

$\theta \sim$

$(0, 1)$

$$l(\theta|y) = \prod_{i=1}^{n} \frac{1}{1 + (y_i - \theta)^2}$$

x_i

$$p\left(\mu, \sigma^2\right) \propto \left(\sigma^2\right)^{-3/2}$$

$$p\left(\mu, \sigma^2 | \mathbf{x}\right)$$

$$\propto p\left(\mu, \sigma^2\right) p\left(\mathbf{x} | \mu, \sigma^2\right)$$

$$= \left(\sigma^2\right)^{-3/2} \sigma^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum \left(x_i - \mu\right)^2\right)$$

$$= \left(\sigma^2\right)^{-3/2} \sigma^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum \left(x_i - \overline{x}\right)^2 + n\left(\overline{x} - \mu\right)^2\right)$$

$$= \left(\sigma^2\right)^{-3/2} \sigma^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\left(\sum\left(x_i - \overline{x}\right)^2 + n\left(\overline{x} - \mu\right)^2\right)\right)$$

$$= \left(\sigma^2\right)^{-3/2-n/2} \exp\left(-\frac{1}{2\sigma^2}\left(\left(n-1\right)s^2 + n\left(\overline{x}-\mu\right)^2\right)\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\left(n\left(\overline{x}-\mu\right)^2\right)\right)$$

$$\implies \mu|\mathbf{x}, \sigma^2$$

$$\sim N\left(\overline{x}, \sigma^2/n\right)$$

$$\propto \left(\sigma^2\right)^{-3/2-n/2} \exp\left(-\frac{1}{2\sigma^2}\left(\left(n-1\right)s^2 + n\left(\overline{x}-\mu\right)^2\right)\right)$$

$$= \left(\sigma^2\right)^{-3/2-n/2} \exp\left(-\frac{1}{2\sigma^2}\left(n-1\right)s^2\right) \underbrace{\exp\left(-\frac{1}{2\sigma^2} n\left(\overline{x}-\mu\right)^2\right)\left(\frac{\sigma}{n}\right)^{-1/2}}_{\text{kernel de una normal}} \left(\frac{\sigma}{n}\right)^{1/2}$$

$$\implies p\left(\sigma^2|\mathbf{x}\right)$$

$$\propto \left(\sigma^2\right)^{-(n/2+1)} \exp\left(-\frac{1}{2\sigma^2}\left(n-1\right)s^2\right)$$

$\sigma^2|\mathbf{x}$

$$\sim \chi^2 - \mathbf{inv}\left(n, s^2\right)$$

$\boldsymbol{\mu}$

$$p\left(\mu|\mathbf{x}\right)$$

$$= \int_0^\infty p\left(\mu, \sigma^2 | \mathbf{x}\right) d\sigma^2$$

$$\propto \int_0^\infty \left(\sigma^2\right)^{-3/2} \sigma^{-n/2} \exp\left(-\frac{1}{2\sigma^2} A\right) d\sigma^2$$

A

$$= \sum \left(x_i - \overline{x}\right)^2 + n\left(\overline{x} - \mu\right)^2$$

$$z = A/\left(2\sigma^2\right)$$

$$\sigma^2 = A/2z$$

$$d\sigma^2 = -\frac{A}{2z^2}dz$$

$$\propto \int_0^\infty \left(\frac{A}{2z}\right)^{-3/2-n/2} \exp\left(-z\right) \left(-\frac{A}{2z^2}\right) dz$$

$$\propto \left(\frac{A}{2z}\right)^{-1/2-n/2}$$

$$s = 1 \ldots, S$$

$$\mu^{(s)} \sim p\left(\mu|\mathbf{y}\right)$$

$$\sigma^{2(s)} \sim p\left(\sigma^2|\mathbf{y}\right)$$

$$y^s \sim p\left(x|\mu^{(s)}, \sigma^{2(s)}\right)$$

$$p\left(\mu, \sigma^2\right) = p\left(\mu|\sigma^2\right) p\left(\sigma^2\right)$$

$$= p\left(\mu|\sigma^2\right) p\left(\sigma^2\right) p\left(\mathbf{x}|\mu, \sigma^2\right)$$

$=$

β, σ^2

$$p\left(\beta, \sigma^2\right) \propto \sigma^{-2}$$

$$y|\beta, \sigma^2, X$$

$$\sim N\left(X\beta, \sigma^2 I\right)$$

$$p\left(y|\beta,\sigma^2,X\right)$$

$$\propto \det\left(\sigma^2 I\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\beta\right)^t \left(y - X\beta\right)\right)$$

$$= \left(\sigma^2\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\beta\right)^t\left(y - X\beta\right)\right)$$

$$\implies p\left(\beta, \sigma^2 | y, X\right)$$

$$\propto \left(\sigma^2\right)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\beta\right)^t \left(y - X\beta\right)\right)\sigma^{-2}$$

$$= \left(\sigma^2\right)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\beta\right)^t \left(y - X\beta\right)\right)$$

β

σ^2

$$\hat{\beta}, \sigma^2 \left(X^t X\right)^{-1}$$

$$p\left(\beta, \sigma^2 | y\right)$$

$$= p\left(\sigma^2|\beta, y\right) p\left(\beta|y\right)$$

$$\Longrightarrow$$

$$p\left(\beta|y\right)=\frac{p\left(\beta,\sigma^{2}|y\right)}{p\left(\sigma^{2}|\beta,y\right)}=\frac{p\left(y|\beta,\sigma^{2}\right)p\left(\beta,\sigma^{2}\right)}{p\left(\sigma^{2}|\beta,y\right)}$$

$$= p\left(\beta|\sigma^2, y\right) p\left(\sigma^2|y\right)$$

$$p\left(\sigma^2|y\right)=\frac{p\left(y|\beta,\sigma^2\right)p\left(\beta,\sigma^2\right)}{p\left(\beta|\sigma^2,y\right)}$$

$$p\left(\beta|\sigma^2, y\right)$$

$$= \frac{p\left(\beta, \sigma^2 |, y\right)}{p\left(\sigma^2 | y\right)}$$

$$\propto p\left(\beta, \sigma^2 |, y\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\left(y - X\beta\right)^t\left(y - X\beta\right)\right)$$

$$= \exp\left(-\frac{1}{2\sigma^2}\left(y - X\hat{\beta}\right)^t\left(y - X\hat{\beta}\right) - \frac{1}{2\sigma^2}\left(\beta - \hat{\beta}\right)^t X^t X\left(\beta - \hat{\beta}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\left(\beta-\hat{\beta}\right)^t X^t X\left(\beta-\hat{\beta}\right)\right)$$

$$\implies \beta|\sigma^2, y$$

$$\sim N\left(\hat{\beta}, \sigma^2\left(X^t X\right)^{-1}\right)$$

$$p\left(\sigma^2|y\right)$$

$$= \frac{p\left(\beta, \sigma^2 |, y\right)}{p\left(\beta | \sigma^2, y\right)}$$

$$\propto \frac{\left(\sigma^2\right)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\beta\right)^t \left(y - X\beta\right)\right)}{\det\left(\sigma^2\left(X^t X\right)^{-1}\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}\left(\beta - \hat{\beta}\right)^t X^t X \left(\beta - \hat{\beta}\right)\right)}$$

$$= \frac{\left(\sigma^2\right)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\hat{\beta}\right)^t \left(y - X\hat{\beta}\right) - \frac{1}{2\sigma^2}\left(\beta - \hat{\beta}\right)^t X^t X \left(\beta - \hat{\beta}\right)\right)}{\det\left(\sigma^2 \left(X^t X\right)^{-1}\right)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}\left(\beta - \hat{\beta}\right)^t X^t X \left(\beta - \hat{\beta}\right)\right)}$$

$$= \frac{\left(\sigma^2\right)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\hat{\beta}\right)^t \left(y - X\hat{\beta}\right)\right)}{\det\left(\sigma^2 \left(X^t X\right)^{-1}\right)^{-1/2} \exp\left(0\right)}$$

$$= \frac{\left(\sigma^2\right)^{-n/2-2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\hat{\beta}\right)^t \left(y - X\hat{\beta}\right)\right)}{\left(\sigma^{2k}\right)^{-1/2} \det\left(\left(X^t X\right)^{-1}\right)^{-1/2} \exp\left(0\right)}$$

$$\propto \left(\sigma^2\right)^{-n/2-2+k/2} \exp\left(-\frac{1}{2\sigma^2}\left(y - X\hat{\beta}\right)^t \left(y - X\hat{\beta}\right)\right)$$

$$\implies \sigma^2 | y$$

$$\sim Inv - \chi^2 \left(n - k, s^2\right)$$

$$\widetilde{y}$$

a

b

$$\pi\left(x\right)=\Phi\left(\beta_0+\beta_1 x|\mu,\sigma^2\right)$$

$$Y_i | x_i$$

$$(\pi (x_i))$$

$$\implies \pi(x)$$

$$= \Phi\left(\beta_0 + \beta_1 x | \mu, \sigma^2\right)$$

$$\pi\,(x)$$

$$= \Phi_{0,1}\left(\frac{\beta_0 + \beta_1 x - \mu}{\sigma}\right)$$

$$= \Phi_{0,1}\left(\beta_0^* + \beta_1^* x\right)$$

$$1 - \alpha$$