Homework 4

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$$\mathbf{Z}_{(3,2)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \ \mathbf{G}_{(2,2)} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}, \ \mathbf{R}_{(3,3)} = \begin{bmatrix} \sigma_\epsilon^2 & 0 & 0 \\ 0 & \sigma_\epsilon^2 & 0 \\ 0 & 0 & \sigma_\epsilon^2 \end{bmatrix}$$

$$\mathbf{ZG}_{(3,2)} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_1^2 + \sigma_{12} & \sigma_{12} + \sigma_2^2 \\ \sigma_1^2 + 2\sigma_{12} & \sigma_{12} + 2\sigma_2^2 \end{bmatrix}, \ \mathbf{Z}^T_{(2,3)} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\mathbf{ZGZ}^T_{(3,3)} = \begin{bmatrix} \sigma_1^2 & \sigma_1^2 + \sigma_{12} & \sigma_1^2 + 2\sigma_{12} \\ \sigma_1^2 + \sigma_{12} & \sigma_1^2 + 2\sigma_{12} + \sigma_2^2 & \sigma_1^2 + 3\sigma_{12} + 2\sigma_2^2 \\ \sigma_1^2 + 2\sigma_{12} & \sigma_1^2 + 3\sigma_{12} + 2\sigma_2^2 & \sigma_1^2 + 4\sigma_{12} + 4\sigma_2^2 \end{bmatrix}$$

$$\mathbf{V}_{(3,3)} = \mathbf{ZGZ}^T + \mathbf{R} = \begin{bmatrix} \sigma_1^2 + \sigma_\epsilon^2 & \sigma_1^2 + \sigma_{12} & \sigma_1^2 + 2\sigma_{12} \\ \sigma_1^2 + \sigma_{12} & \sigma_1^2 + 2\sigma_{12} + \sigma_2^2 + \sigma_\epsilon^2 & \sigma_1^2 + 3\sigma_{12} + 2\sigma_2^2 \\ \sigma_1^2 + 2\sigma_{12} & \sigma_1^2 + 3\sigma_{12} + 2\sigma_2^2 & \sigma_1^2 + 4\sigma_{12} + 4\sigma_2^2 + \sigma_\epsilon^2 \end{bmatrix}$$

$$Cov[Y_{i1}, Y_{i2}] > Cov[Y_{i1}, Y_{i3}]$$
$$\implies \sigma_1^2 + \sigma_{12} > \sigma_1^2 + 2\sigma_{12}$$
$$\implies 0 > \sigma_{12}$$

$Cov[Y_{i1}, Y_{i2}] > Cov[Y_{i1}, Y_{i3}]$ holds true if σ_{12} is negative. That is the covariance parameter σ_{12} from the **G** matrix is negative.

$$Cov[Y_{i1}, Y_{i3}] < Cov[Y_{i2}, Y_{i3}]$$

$$\implies \sigma_1^2 + 2\sigma_{12} < \sigma_1^2 + 3\sigma_{12} + 2\sigma_2^2$$
$$\implies 0 < \sigma_{12} + 2\sigma_2^2$$
$$\implies |\,\sigma_{12}\,| < 2\sigma_2^2$$

$Cov[Y_{i1}, Y_{i3}] < Cov[Y_{i2}, Y_{i3}]$ holds true if $|\sigma_{12}| < 2\sigma_2^2$. That is the absolute value of the covariance parameter σ_{12} from the **G** matrix is is less than twice the variance parameter σ_2^2 from the same **G** matrix.