Question 2

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Part A

$$Y_{ij} = \beta_0 + \beta_1 treatment_{ij} + \beta_2 time_{ij} + \beta_3 (treatment_{ij} * time_{ij}) + b_i + \epsilon_{ij}$$

* $b_i \sim N(0, \sigma_b^2)$
* $\epsilon_{ij} \sim N(0, \sigma^2)$
* $Corr(\epsilon_{ij}, \epsilon_{ik}) = 0$
* $Corr(b_i, \epsilon_{ij}) = 0$

Part B

In the model above, the fixed effects are the treatment, time, and the interaction term of treatment and time, whose coefficients are represented by β_i. The random effects for the model above are the subject ID's, represented by b_i.

Part C

```
model0 = lmer(y ~ (1  | id) + factor(trt) + time + factor(trt) * time , data = data)
output = summary(model0)
fixed = coef(output)

rand = data.frame(
  'variance' = c(10.678, 1.212)
)

rownames(rand) = c('Intercept', 'Residual')

fixed
```

```
##                      Estimate Std. Error        df   t value     Pr(>|t|)
## (Intercept)       80.11214904 0.83882705  37.50971 95.504966 2.147974e-46
## factor(trt)2       1.21644032 1.11353140  37.52302  1.092417 2.816144e-01
## time               0.12145189 0.02695975 199.06278  4.504933 1.130501e-05
## factor(trt)2:time  0.03398007 0.03667787 199.15549  0.926446 3.553356e-01
```

```
rand
```

```
##           variance
## Intercept   10.678
## Residual     1.212
```

Part D

```
sigma_b = 10.678
sigma = 1.212
icc = sigma_b / (sigma + sigma_b)
```

The estimated variance of the random intercepts i s 10.678. The differences between subjects is accounting for 89.81% of variablility in strength, relative to the variability within subjects.

Part E

The fixed effect for the second treatment group is not significant ($p > 0.05$). However, the fixed effect for the covariate of interest time, is significant ($p < 0.001$). The average increase in strength for a one day increase is 0.12145. The interaction between linear time measured in days, and treatment group is not significant ($p > 0.05$).

Part F

The random intercept term, when treatment group is 1, was statistically significant ($p < 0.001$). The average population-level strength at baseline (time = 0 days) for treatment group 1 was 80.11. This is interpretable, because measurements for baseline were taken at 0 days.

Part G

While it appeared from our plots that there may be a difference between the two treatment groups in strength, nothing in our anaylsis has suggest a statistically significant difference between the two treatment groups.