

Question 2

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Part A

$$Y_{ij} = \beta_0 + \beta_1 treatment_{ij} + \beta_2 time_{ij} + \beta_3(treatment_{ij} * time_{ij}) + b_i + \epsilon_{ij}$$

- * $b_i \sim N(0, \sigma_b^2)$
- * $\epsilon_{ij} \sim N(0, \sigma^2)$
- * $Corr(\epsilon_{ij}, \epsilon_{ik}) = 0$
- * $Corr(b_i, \epsilon_{ij}) = 0$

Part B

In the model above, the fixed effects are the treatment, time, and the interaction term of treatment and time, whose coefficients are represented by β_i . The random effects for the model above are the subject ID's, represented by b_i .

Part C

```
##               Estimate Std. Error      df    t value      Pr(>|t|)
## (Intercept)    80.11214904  0.83882705   37.50971  95.504966  2.147974e-46
## factor(trt)2     1.21644032  1.11353140   37.52302   1.092417  2.816144e-01
## time            0.12145189  0.02695975  199.06278   4.504933  1.130501e-05
## factor(trt)2:time 0.03398007  0.03667787  199.15549   0.926446  3.553356e-01

##               variance
## Intercept     10.678
## Residual       1.212
```

Part D

The estimated variance of the random intercepts is 10.678. The differences between subjects is accounting for 89.81% of variability in strength, relative to the variability within subjects.

Part E

The fixed effect for the second treatment group is not significant ($p > 0.05$). However, the fixed effect for the covariate of interest time, is significant ($p < 0.001$). The average increase in strength for a one day increase is 0.12145. The interaction between linear time measured in days, and treatment group is not significant ($p > 0.05$).

Part F

The random intercept term, when treatment group is 1, was statistically significant ($p < 0.001$). The average population-level strength at baseline (time = 0 days) for treatment group 1 was 80.11. This is interpretable, because measurements for baseline were taken at 0 days.

Part G

While it appeared from our plots that there may be a difference between the two treatment groups in strength, nothing in our analysis has suggest a statistically significant difference between the two treatment groups.

Appendix

```
knitr::opts_chunk$set(echo = TRUE, include = TRUE, warning = FALSE)

library(haven)
library(lmerTest)

data = read_dta('exercise_therapy.dta')

# Part C
model0 = lmer(y ~ (1 | id) + factor(trt) + time + factor(trt) * time , data = data)
output = summary(model0)
fixed = coef(output)

rand = data.frame(
  'variance' = c(10.678, 1.212)
)

rownames(rand) = c('Intercept', 'Residual')

fixed
rand

# Part D
sigma_b = 10.678
sigma = 1.212
icc = sigma_b / (sigma + sigma_b)
```