# Question 3

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## 0.1 $\bar{\xi} = \frac{1}{n}\sum_{i=1}^{n} I(Y_i = 0)$

$$Var[\bar{\xi}] = Var\left[\frac{1}{n}\sum_{i=1}^{n} I(Y_i = 0)\right]$$

$$Var[\bar{\xi}] = \frac{1}{n^2}\, Var\left[\sum_{i=1}^{n} I(Y_i = 0)\right]$$

$I(Y_i) \sim bernoulli(p)$ and a sum of $bernoulli(p)$ are $binomial(n,p)$, hence:

$$Var[\bar{\xi}] = \frac{1}{n^2}\, np(1-p)$$

$$Var[\bar{\xi}] = \frac{p(1-p)}{n}$$

Now, $p = e^{-y}$:

$$Var[\bar{\xi}] = \frac{e^{-y}(1-e^{-y})}{n}$$

## 0.2 $\bar{\xi} = e^{-\lambda}$

$$Var[\bar{\xi}] = Var[g(\hat{\lambda})] - [g'(\hat{\lambda})]^2(Var[\hat{\lambda}])$$

$$Var[\bar{\xi}] = Var[g(\hat{\lambda})] - [g'(\hat{\lambda})]^2(Var[\hat{\lambda}])$$

Now using; $g(\hat{\lambda}) = e^{-\lambda}$; $g'(\hat{\lambda}) = -e^{-\lambda}$; $[g(\hat{\lambda})]^2 = e^{-2\lambda}$:

$$Var[\bar{\xi}] = [g'(\hat{\lambda})]^2(Var[\hat{\lambda}])$$

$$Var[\bar{\xi}] = e^{-2\lambda}\left(\frac{\lambda}{n}\right)$$

$$Var[\bar{\xi}] = \frac{\lambda e^{-2\lambda}}{n}$$