
Counterfactual Fairness

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Abstract

1 Machine learning can impact people with legal or ethical consequences when
2 it is used to automate decisions in areas such as insurance, lending, hiring, and
3 predictive policing. In many of these scenarios, previous decisions have been made
4 that are unfairly biased against certain subpopulations, for example those of a
5 particular race, gender, or sexual orientation. Since this past data may be biased,
6 machine learning predictors must account for this to avoid perpetuating or creating
7 discriminatory practices. In this paper, we develop a framework for modeling
8 fairness using tools from causal inference. Our definition of *counterfactual fairness*
9 captures the intuition that a decision is fair towards an individual if it the same in
10 (a) the actual world and (b) a counterfactual world where the individual belonged
11 to a different demographic group. We demonstrate our framework on a real-world
12 problem of fair prediction of success in law school.

13 1 Contribution

14 Machine learning has spread to fields as diverse as credit scoring [18], crime prediction [5], and loan
15 assessment [22]. Decisions in these areas may have ethical or legal implications, so it is necessary for
16 the modeler to think beyond the objective of maximizing prediction accuracy and consider the societal
17 impact of their work. For many of these applications, it is crucial to ask if the predictions of a model
18 are *fair*. Training data can contain unfairness for reasons having to do with historical prejudices or
19 other factors outside an individual's control. In 2016, the Obama administration released a report¹
20 which urged data scientists to analyze "how technologies can deliberately or inadvertently perpetuate,
21 exacerbate, or mask discrimination."

22 There has been much recent interest in designing algorithms that make fair predictions [4, 6, 9, 10, 12,
23 14–17, 19, 21, 30–33]. In large part, the literature has focused on formalizing fairness into quantitative
24 definitions and using them to solve a discrimination problem in a certain dataset. Unfortunately, for
25 a practitioner, law-maker, judge, or anyone else who is interested in implementing algorithms that
26 control for discrimination, it can be difficult to decide *which* definition of fairness to choose for the
27 task at hand. Indeed, we demonstrate that depending on the relationship between a protected attribute
28 and the data, certain definitions of fairness can actually *increase discrimination*.

29 In this paper, we introduce the first explicitly causal approach to address fairness. Specifically, we
30 leverage the causal framework of Pearl [26] to model the relationship between protected attributes
31 and data. We describe how techniques from causal inference can be effective tools for designing fair
32 algorithms and argue, as in DeDeo [8], that it is essential to properly address causality in fairness.
33 In perhaps the most closely related work, Johnson et al. [13] make similar arguments but from a
34 non-causal perspective.

¹<https://obamawhitehouse.archives.gov/blog/2016/05/04/big-risks-big-opportunities-intersection-big-data-and-civil-rights>

In Section 2, we provide a summary of basic concepts in fairness and causal modeling. In Section 3, we provide the formal definition of *counterfactual fairness*, which enforces that a distribution over possible predictions for an individual should remain unchanged in a world where an individual’s protected attributes had been different in a causal sense. In Section 4, we describe an algorithm to implement this definition, while distinguishing it from existing approaches. In Section 5, we illustrate the algorithm with a case of fair assessment of law school success.

2 Background

This section provides a basic account of two separate areas of research in machine learning, which are formally unified in this paper. We suggest Berk et al. [1] and Pearl et al. [25] as references for further reading.

Throughout this paper, we will use the following notation. Let A denote the set of *protected attributes* of an individual, variables that must not be discriminated against in a formal sense defined differently by each notion of fairness discussed. The decision of whether an attribute is protected or not is taken as a primitive in any given problem, regardless of the definition of fairness adopted. Moreover, let X denote the other observable attributes of any particular individual, U the set of relevant latent attributes which are not observed, and let Y denote the outcome to be predicted, which itself might be contaminated with historical biases. Finally, \hat{Y} is the *predictor*, a random variable that depends on A , X and U , and which is produced by a machine learning algorithm as a prediction of Y .

2.1 Fairness

The goal of fairness in machine learning is to design automated algorithms that make fair predictions across various demographic groups. This unfairness can arise in several ways. Consider for instance historically biased distributions, where individuals with different protected attributes A may have different attributes X such as their current level of wealth, which is then used for credit scoring. Disparities may be due to discriminative measures in hiring practices and perpetuated by reduced financial support for the education of minorities due to bad credit ratings.

There has been a wealth of recent work on fair algorithms. These include fairness through unawareness [10], individual fairness [9, 14, 21, 32], demographic parity/disparate impact [30], and equality of opportunity [12, 31]. For simplicity we often assume A is encoded as a binary attribute, but this can be generalized.

Definition 1 (Fairness Through Unawareness (FTU)). *An algorithm is fair so long as any protected attributes A are not explicitly used in the decision-making process.*

Any mapping $\hat{Y} : X \rightarrow Y$ that excludes A satisfies this. Initially proposed as a baseline, the approach has found favor recently with more general approaches such as Grgic-Hlaca et al. [10]. Despite its compelling simplicity, FTU has a clear shortcoming as elements of X can contain discriminatory information analogous to A that may not be obvious at first. The need for expert knowledge in assessing the relationship between A and X was highlighted in the work on individual fairness:

Definition 2 (Individual Fairness (IF)). *An algorithm is fair if it gives similar predictions to similar individuals. Formally, if individuals i and j are similar apart from their protected attributes A_i, A_j then $\hat{Y}(X^{(i)}, A^{(i)}) \approx \hat{Y}(X^{(j)}, A^{(j)})$.*

As described in [9], the notion of similarity must be carefully chosen, requiring an understanding of the domain at hand beyond black-box statistical modeling. This can also be contrasted against population level criteria such as

Definition 3 (Demographic Parity (DP)). *A predictor \hat{Y} satisfies demographic parity if $P(\hat{Y}|A = 0) = P(\hat{Y}|A = 1)$.*

Definition 4 (Equality of Opportunity (EO)). *A predictor \hat{Y} satisfies equality of opportunity if $P(\hat{Y} = 1|A = 0, Y = 1) = P(\hat{Y} = 1|A = 1, Y = 1)$.*

These criteria can be incompatible in general, as discussed in [1, 7, 19]. Following the motivation of IF and [13], we propose that knowledge about relationships between all attributes should be taken

into consideration, even if strong assumptions are necessary. Moreover, it is not immediately clear for any of these approaches in which ways historical biases can be tackled. We approach such issues from an explicit causal modeling perspective.

2.2 Causal Models and Counterfactuals

We follow Pearl [24], and define a causal model as a triple (U, V, F) of sets such that

- U is a set of latent **background** variables, which are factors not caused by any variable in the set V of **observable** variables;
- F is a set of functions $\{f_1, \dots, f_n\}$, one for each $V_i \in V$, such that $V_i = f_i(pa_i, U_{pa_i})$, $pa_i \subseteq V \setminus \{V_i\}$ and $U_{pa_i} \subseteq U$. Such equations are also known as **structural equations** [2].

The notation “ pa_i ” refers to the “parents” of V_i and is motivated by the assumption that the model factorizes as a directed graph, here assumed to be a directed acyclic graph (DAG). The model is causal in that, given a distribution $P(U)$ over the background variables U , we can derive the distribution of a subset $Z \subseteq V$ following an **intervention** on $V \setminus Z$. An intervention on variable V_i is the substitution of equation $V_i = f_i(pa_i, U_{pa_i})$ with the equation $V_i = v$ for some v . This captures the idea of an agent, external to the system, modifying it by forcefully assigning value v to V_i , for example as in a randomized experiment.

The specification of F is a strong assumption but allows for the calculation of **counterfactual** quantities. In brief, consider the following counterfactual statement, “the value of Y if Z had taken value z ”, for two observable variables Z and Y . By assumption, the state of any observable variable is fully determined by the background variables and structural equations. The counterfactual is modeled as the solution for Y for a given $U = u$ where the equations for Z are replaced with $Z = z$. We denote it by $Y_{Z \leftarrow z}(u)$ [24], and sometimes as Y_z if the context of the notation is clear.

Counterfactual inference, as specified by a causal model (U, V, F) given evidence W , is the computation of probabilities $P(Y_{Z \leftarrow z}(U) \mid W = w)$, where W, Z and Y are subsets of V . Inference proceeds in three steps, as explained in more detail in Chapter 4 of Pearl et al. [25]: 1. **Abduction**: for a given prior on U , compute the posterior distribution of U given the evidence $W = w$; 2. **Action**: substitute the equations for Z with the interventional values z , resulting in the modified set of equations F_z ; 3. **Prediction**: compute the implied distribution on the remaining elements of V using F_z and the posterior $P(U \mid W = w)$.

3 Counterfactual Fairness

Given a predictive problem with fairness considerations, where A, X and Y represent the protected attributes, remaining attributes, and output of interest respectively, let us assume that we are given a causal model (U, V, F) , where $V \equiv A \cup X$. We postulate the following criterion for predictors of Y .

Definition 5 (Counterfactual fairness). *Predictor \hat{Y} is counterfactually fair if under any context $X = x$ and $A = a$,*

$$P(\hat{Y}_{A \leftarrow a}(U) = y \mid X = x, A = a) = P(\hat{Y}_{A \leftarrow a'}(U) = y \mid X = x, A = a), \quad (1)$$

for all y and for any value a' attainable by A .

This is closely related to **actual causes** [11], or token causality in the sense that, to be fair, A should not be a cause of \hat{Y} in any individual instance. In other words, changing A while holding things which are not causally dependent on A constant will not change the distribution of \hat{Y} . We also emphasize that counterfactual fairness is an individual-level definition. This is substantially different from comparing different individuals that happen to share the same “treatment” $A = a$ and coincide on the values of X , as discussed in Section 4.3.1 of [25] and the Supplementary Material². Differences between X_a and $X_{a'}$ must be caused by variations on A only. Notice also that this definition is agnostic with respect to how good a predictor \hat{Y} is, which we discuss in Section 4.

Relation to individual fairness. IF is agnostic with respect to its notion of similarity metric, which is both a strength (generality) and a weakness (no unified way of defining similarity). Counterfactuals

²We strongly suggest that reviewers look at it.

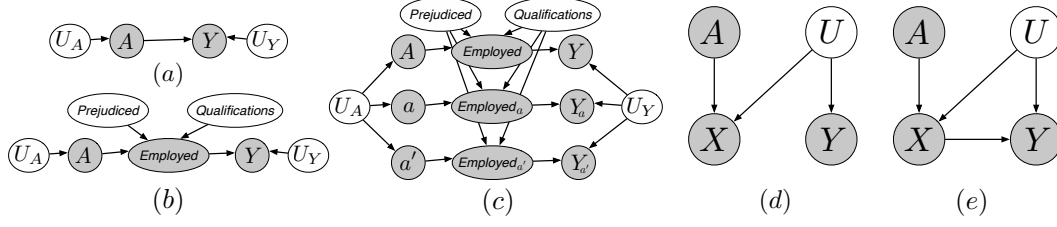


Figure 1: (a) The graph corresponding to a causal model with A being the protected attribute and Y some outcome of interest, with background variables assumed to be independent. (b) Expanding the model to include an intermediate variable indicating whether the individual is employed with two (latent) background variables **Prejudiced** (if the person offering the job is prejudiced) and **Qualifications** (a measure of the individual’s qualifications). (c) A twin network representation of this system [24] under two different counterfactual levels for A . This is created by copying nodes descending from A , which inherit unaffected parents from the factual world. (d),(e) Two causal models for different real-world fair prediction scenarios. See Section 3 for discussion.

and similarities are related, as in the classical notion of distances between “worlds” corresponding to different counterfactuals [20]. Defining \hat{Y} as a deterministic function of $W \subset A \cup X \cup U$, as in several of our examples to follow, then IF can be defined by treating equally two individuals with the same W in a way that is also counterfactually fair.

Relation to Pearl et al. [25]. In Example 4.4.4 of [25], the authors condition instead on X , A , and the observed realization of \hat{Y} , and calculate the probability of the counterfactual realization $\hat{Y}_{A \leftarrow a'}$ differing from the factual. This example conflates the predictor \hat{Y} with the outcome Y , of which we remain agnostic in our definition but which is used in the construction of \hat{Y} as in Section 4. Our framing makes the connection to machine learning more explicit.

3.1 Implications

One simple but important implication of the definition of counterfactual fairness is the following:

Lemma 1. *Let \mathcal{G} be the causal graph of the given model (U, V, F) . Then \hat{Y} will be counterfactually fair if it is a function of the non-descendants of A .*

Proof. Let W be any non-descendant of A in \mathcal{G} . Then $W_{A \leftarrow a}(U)$ and $W_{A \leftarrow a'}(U)$ have the same distribution by the three inferential steps in Section 2.2. Hence, the distribution of any function \hat{Y} of the non-descendants of A is invariant with respect to the counterfactual values of A . \square

This does not exclude using a descendant W of A as a possible input to \hat{Y} . However, this will only be possible in the case where the overall dependence of \hat{Y} on A disappears, which will not happen in general. Hence, Lemma 1 provides the most straightforward way to achieve counterfactual fairness.

Ancestral closure of protected attributes. Suppose that a parent of a member of A is not in A . Counterfactual fairness allows for the use of it in the definition of \hat{Y} . If this seems counterintuitive, then we argue that the fault should be at the postulated set of protected attributes rather than with the definition of counterfactual fairness, and that typically we should expect set A to be closed under ancestral relationships given by the causal graph. For instance, if *Race* is a protected attribute, and *Mother’s race* is a parent of *Race*, then it should also be in A .

Dealing with historical biases. The explicit difference between \hat{Y} and Y allows us to tackle historical biases. For instance, let Y be an indicator of whether a client defaults on a loan, while \hat{Y} is the actual decision of giving the loan. Consider the DAG $A \rightarrow Y$, shown in Figure 1(a) with the explicit inclusion of set U of independent background variables. Y is the objectively ideal measure for decision making, the binary indicator of the event that the individual defaults on a loan. If A is postulated to be a protected attribute, then the predictor $\hat{Y} = Y = f_Y(A, U)$ is not counterfactually fair, with the arrow $A \rightarrow Y$ being (for instance) the result of a world that punishes individuals in a way that is out of their control. Figure 1(b) shows a finer-grained model, where the path is mediated

by a measure of whether the person is employed, which is itself caused by two background factors: one representing whether the person hiring is prejudiced, and the other the employee’s qualifications. In this world, A is a cause of defaulting, even if mediated by other variables³. The counterfactual fairness principle however forbids us from using Y : using the twin network of Pearl [24], we see in Figure 1(c) that Y_a and $Y_{a'}$ need not be identically distributed given the background variables.

In contrast, any function of variables not descendants of A can be used a basis for fair decision making. This means that any variable \hat{Y} defined by $\hat{Y} = g(U)$ will be counterfactually fair for any function $g(\cdot)$. Hence, given a causal model, the functional defined by the function $g(\cdot)$ minimizing some predictive error for Y will satisfy the criterion, as proposed in Section 4.1. We are essentially learning a projection of Y into the space of fair decisions, removing historical biases as a by-product.

3.2 Further Examples

To give further intuition for counterfactual fairness, we will consider two real-world fair prediction scenarios: **insurance pricing** and **crime prediction**. Each of these correspond to one of the two causal graphs in Figure 1(d),(e). The Supplementary Material provides a more mathematical discussion of these examples with more detailed insights.

Scenario 1: The Red Car. A car insurance company wishes to price insurance for car owners by predicting their accident rate Y . They assume there is an unobserved factor corresponding to aggressive driving U , that (a) causes drivers to be more likely have an accident, and (b) causes individuals to prefer red cars (the observed variable X). Moreover, individuals belonging to a certain race A are more likely to drive red cars. However, these individuals are no more likely to be aggressive or to get in accidents than any one else. We show this in Figure 1(d). Thus, using the red car feature X to predict accident rate Y would seem to be an unfair prediction because it may charge individuals of a certain race more than others, even though no race is more likely to have an accident. Counterfactual fairness agrees with this notion, as X is a descendent of A but U is not. Interestingly, we can show (Supplementary Material) that in a linear model, regression Y on A and X is equivalent to regressing on U , so off-the-shelf regression here is counterfactually fair. Regressing Y on X alone obeys the FTU criterion but is not counterfactually fair, so *omitting A (FTU) may introduce unfairness into an otherwise fair world.*

Scenario 2: High Crime Regions. A city government wants to estimate crime rates by neighborhood to allocate policing resources. Its analyst constructed training data by merging (1) a registry of residents containing their neighborhood X and race A , with (2) police records of arrests, giving each resident a binary label with $Y = 1$ indicating a criminal arrest record. Due to historically segregated housing, the location X depends on A . Locations X with more police resources have larger numbers of arrests Y . And finally, U represents the totality of socioeconomic factors and policing practices that both influence where an individual may live and how likely they are to be arrested and charged. This can all be seen in Figure 1(e).

In this example, higher observed arrest rates in some neighborhoods are due to greater policing there, not because people of different races are any more or less likely to break the law. The label $Y = 0$ does not mean someone has never committed a crime, but rather that they have not been caught. *If individuals in the training data have not already had equal opportunity, algorithms enforcing EO will not remedy such unfairness.* In contrast, a counterfactually fair approach would model differential enforcement rates using U and base predictions on this information rather than on X directly.

In general, we need a multistage procedure in which we first derive latent variables U , and then based on them we minimize some loss with respect to Y . This is the core of the algorithm discussed next.

4 Implementing Counterfactual Fairness

As discussed in the previous Section, we need to relate \hat{Y} to Y if the predictor is to be useful, and we restrict \hat{Y} to be a (parameterized) function of the non-descendants of A in the causal graph following

³For example, if the function determining employment $f_E(A, P, Q) \equiv I_{(Q>0, P=0 \text{ or } A \neq a)}$ then an individual with sufficient qualifications and prejudiced potential employer may have a different counterfactual employment value for $A = a$ compared to $A = a'$, and a different chance of default.

209 Lemma 1. We next introduce an algorithm, then discuss assumptions that can be used to express
210 counterfactuals.

211 4.1 Algorithm

212 Let $\hat{Y} \equiv g_\theta(U, X_{\not\sim A})$ be a predictor parameterized by θ , such as a logistic regression or a neural
213 network, and where $X_{\not\sim A} \subseteq X$ are non-descendants of A . Given a loss function $l(\cdot, \cdot)$ such as
214 squared loss or log-likelihood, and training data $\mathcal{D} \equiv \{(A^{(i)}, X^{(i)}, Y^{(i)})\}$ for $i = 1, 2, \dots, n$, we
215 define $L(\theta) \equiv \sum_{i=1}^n \mathbb{E}[l(y^{(i)}, g_\theta(U^{(i)}, x_{\not\sim A}^{(i)})) \mid x^{(i)}, a^{(i)}]/n$ as the empirical loss to be minimized
216 with respect to θ . Each expectation is with respect to random variable $U^{(i)} \sim P_{\mathcal{M}}(U \mid x^{(i)}, a^{(i)})$
217 where $P_{\mathcal{M}}(U \mid x, a)$ is the conditional distribution of the background variables as given by a causal
218 model \mathcal{M} that is available by assumption. If this expectation cannot be calculated analytically,
219 Markov chain Monte Carlo (MCMC) can be used to approximate it as in the following algorithm.

220 1: **procedure** FAIRLEARNING(\mathcal{D}, \mathcal{M}) ▷ Learned parameters $\hat{\theta}$
221 2: For each data point $i \in \mathcal{D}$, sample m MCMC samples $U_1^{(i)}, \dots, U_m^{(i)} \sim P_{\mathcal{M}}(U \mid x^{(i)}, a^{(i)})$.
222 3: Let \mathcal{D}' be the augmented dataset where each point $(a^{(i)}, x^{(i)}, y^{(i)})$ in \mathcal{D} is replaced with the
223 corresponding m points $\{(a^{(i)}, x^{(i)}, y^{(i)}, u_j^{(i)})\}$.
224 4: $\hat{\theta} \leftarrow \operatorname{argmin}_{\theta} \sum_{i' \in \mathcal{D}'} l(y^{(i')}, g_\theta(U^{(i')}, x_{\not\sim A}^{(i')}))$.
225 5: **end procedure**

226 At prediction time, we report $\tilde{Y} \equiv \mathbb{E}[\hat{Y}(U^*, x_{\not\sim A}^*) \mid x^*, a^*]$ for a new data point (a^*, x^*) .

227 **Deconvolution perspective.** The algorithm can be understood as a deconvolution approach that,
228 given observables $A \cup X$, extracts its latent sources and pipelines them into a predictive model. We
229 advocate that *counterfactual assumptions must underlie all approaches that claim to extract the*
230 *sources of variation of the data as “fair” latent components*. As an example, Louizos et al. [21] start
231 from the DAG $A \rightarrow X \leftarrow U$ to extract $P(U \mid X, A)$. As U and A are not independent given X in this
232 representation, a type of penalization is enforced to create a posterior $P_{\text{fair}}(U \mid A, X)$ that is close
233 to the model posterior $P(U \mid A, X)$ while satisfying $P_{\text{fair}}(U \mid A = a, X) \approx P_{\text{fair}}(U \mid A = a', X)$.
234 But *this is neither necessary nor sufficient for counterfactual fairness*. The model for X given A
235 and U must be justified by a causal mechanism, and that being the case, $P(U \mid A, X)$ requires no
236 postprocessing. As a matter of fact, model \mathcal{M} can be learned by penalizing empirical dependence
237 measures between U and pa_i for a given V_i (e.g. Mooij et al. [23]), but this concerns \mathcal{M} and not \tilde{Y} ,
238 and is motivated by explicit assumptions about structural equations, as described next.

239 4.2 Designing the Input Causal Model

240 Model \mathcal{M} must be provided to algorithm FAIRLEARNING. Causal models always require strong
241 assumptions, but counterfactuals in particular are typically presented in terms of structural equations
242 which are in general unfalsifiable. We point out that we do not need to specify a fully deterministic
243 model, and structural equations can be relaxed as conditional distributions. In particular, the concept
244 of counterfactual fairness holds under three levels of assumptions of increasing strength:

245 **Level 1.** Build \hat{Y} using only the observable non-descendants of A . This only requires partial
246 causal ordering and no further causal assumptions, but in many problems there will be few, if any,
247 observables which are not descendants of protected demographic factors.

248 **Level 2.** Postulate background latent variables that act as non-deterministic causes of observable
249 variables, based on explicit domain knowledge and learning algorithms⁴. Information about X is
250 passed to \tilde{Y} via $P(U \mid x, a)$.

251 **Level 3.** Postulate a fully deterministic model with latent variables. For instance, the distribution
252 $P(V_i \mid pa_i)$ can be treated as an additive error model, $V_i = f_i(pa_i) + e_i$ [27]. The error term e_i then
253 becomes an input to \hat{Y} as calculated from the observed variables. This maximizes the information
254 extracted by the fair predictor \hat{Y} .

⁴In some domains, it is actually common to build a model entirely around latent constructs with few or no
observable parents nor connections among observed variables [2].

5 Illustration: Law School Success

We illustrate our approach on a practical problem that requires fairness, the *prediction of success in law school*. A second problem, *separating actual and perceived criminality in police stops*, is described in the Supplementary Material. Following closely the usual framework for assessing causal models in the machine learning literature, the goal of this experiment is to quantify how our algorithm behaves with finite sample sizes while assuming ground truth compatible with a synthetic model.

Problem definition: Law school success

The Law School Admission Council conducted a survey across 163 law schools in the United States [29]. It contains information on 21,790 law students such as their entrance exam scores (LSAT), their grade-point average (GPA) collected prior to law school, and their first year average grade (FYA).

Given this data, a school may wish to predict if an applicant will have a high FYA. The school would also like to make sure these predictions are not biased by an individual’s race and sex. However, the LSAT, GPA, and FYA scores, may be biased due to social factors. We compare our framework with two unfair baselines: 1. **Full**: the standard technique of using all features, including sensitive features such as race and sex to make predictions; 2. **Unaware**: fairness through unawareness, where we do not use race and sex as features. For comparison, we generate predictors \hat{Y} for all models using logistic regression.

Fair prediction. As described in Section 4.2, there are three ways in which we can model a counterfactually fair predictor of FYA. Level 1 uses any features which are not descendants of race and sex for prediction. Level 2 models latent ‘fair’ variables which are parents of observed variables. These variables are independent of both race and sex. Level 3 models the data using an additive error model, and uses the independent error terms to make predictions. These models make increasingly strong assumptions corresponding to increased predictive power. We split the dataset 80/20 into a train/test set, preserving label balance, to evaluate the models.

As we believe LSAT, GPA, and FYA are all biased by race and sex, we cannot use any observed features to construct a counterfactually fair predictor as described in Level 1.

In Level 2, we postulate that a latent variable: a student’s **knowledge** (K), affects GPA, LSAT, and FYA scores. The causal graph corresponding to this model is shown in Figure 2, (**Level 2**). This is a short-hand for the distributions:

$$\begin{aligned} \text{GPA} &\sim \mathcal{N}(b_G + w_G^K K + w_G^R R + w_G^S S, \sigma_G), & \text{FYA} &\sim \mathcal{N}(w_F^K K + w_F^R R + w_F^S S, 1), \\ \text{LSAT} &\sim \text{Poisson}(\exp(b_L + w_L^K K + w_L^R R + w_L^S S)), & K &\sim \mathcal{N}(0, 1) \end{aligned}$$

We perform inference on this model using an observed training set to estimate the posterior distribution of K . We use the probabilistic programming language Stan [28] to learn K . We call the predictor constructed using K , **Fair** K .

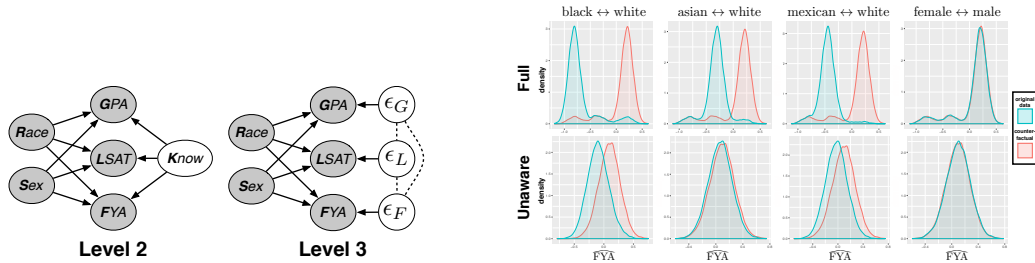


Figure 2: **Left:** A causal model for the problem of predicting law school success fairly. **Right:** Density plots of predicted FYA_a and $\text{FYA}_{a'}$.

In Level 3, we model GPA, LSAT, and FYA as continuous variables with additive error terms independent of race and sex (that may in turn be correlated with one-another). This model is shown

Table 1: Prediction results using logistic regression. Note that we must sacrifice a small amount of accuracy to ensuring counterfactually fair prediction (Fair K , Fair Add), versus the models that use unfair features: GPA, LSAT, race, sex (Full, Unaware).

	Full	Unaware	Fair K	Fair Add
RMSE	0.873	0.894	0.929	0.918

in Figure 2, (**Level 3**), and is expressed by:

$$\begin{aligned} \text{GPA} &= b_G + w_G^R R + w_G^S S + \epsilon_G, \quad \epsilon_G \sim p(\epsilon_G) \\ \text{LSAT} &= b_L + w_L^R R + w_L^S S + \epsilon_L, \quad \epsilon_L \sim p(\epsilon_L) \\ \text{FYA} &= b_F + w_F^R R + w_F^S S + \epsilon_F, \quad \epsilon_F \sim p(\epsilon_F) \end{aligned}$$

We estimate the error terms ϵ_G, ϵ_L by first fitting two models that each use race and sex to individually predict GPA and LSAT. We then compute the residuals of each model (e.g., $\epsilon_G = \text{GPA} - \hat{Y}_{\text{GPA}}(R, S)$). We use these residual estimates of ϵ_G, ϵ_L to predict FYA. We call this *Fair Add*.

Accuracy. We compare the RMSE achieved by logistic regression for each of the models on the test set in Table 1. The **Full** model achieves the lowest RMSE as it uses race and sex to more accurately reconstruct FYA. Note that in this case, this model is not fair even if the data was generated by one of the models shown in Figure 2 as it corresponds to Scenario 3. The (also unfair) **Unaware** model still uses the unfair variables GPA and LSAT, but because it does not use race and sex it cannot match the RMSE of the **Full** model. As our models satisfy counterfactual fairness, they trade off some accuracy. Our first model **Fair K** uses weaker assumptions and thus the RMSE is highest. Using the Level 3 assumptions, as in **Fair Add** we produce a counterfactually fair model that trades slightly stronger assumptions for lower RMSE.

Counterfactual fairness. We would like to empirically test whether the baseline methods are counterfactually fair. To do so we will assume the true model of the world is given by Figure 2, (**Level 2**). We can fit the parameters of this model using the observed data and evaluate counterfactual fairness by sampling from it. Specifically, we will generate samples from the model given either the observed race and sex, or *counterfactual* race and sex variables. We will fit models to both the original and counterfactual sampled data and plot how the distribution of predicted FYA changes for both baseline models. Figure 2 shows this, where each row corresponds to a baseline predictor and each column corresponds to the counterfactual change. In each plot, the blue distribution is density of predicted FYA for the original data and the red distribution is this density for the counterfactual data. If a model is counterfactually fair we would expect these distributions to lie exactly on top of each other. Instead, we note that the **Full** model exhibits counterfactual unfairness for all counterfactuals except sex. We see a similar trend for the **Unaware** model, although it is closer to being counterfactually fair. To see why these models seem to be fair w.r.t. to sex we can look at weights of the DAG which generates the counterfactual data. Specifically the DAG weights from (male,female) to GPA are (0.93,1.06) and from (male,female) to LSAT are (1.1,1.1). Thus, these models are fair w.r.t. to sex simply because of a very weak causal link between sex and GPA/LSAT.

6 Conclusion

We have presented a new model of fairness we refer to as *counterfactual fairness*. It allows us to propose algorithms that, rather than simply ignoring protected attributes, are able to take into account the different social biases that may arise towards individuals based on ethically sensitive attributes and compensate for these biases effectively. We experimentally contrasted our approach with previous fairness approaches and show that our explicit causal models capture these social biases and make clear the implicit trade-off between prediction accuracy and fairness in an unfair world. We propose that fairness should be regulated by explicitly modeling the causal structure of the world. Criteria based purely on probabilistic independence cannot satisfy this and are unable to address *how* unfairness is occurring in the task at hand. By providing such causal tools for addressing fairness questions we hope we can provide practitioners with customized techniques for solving a wide array of fairness modeling problems.

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