CV HW 02

Exercise 1. Pinhole camera

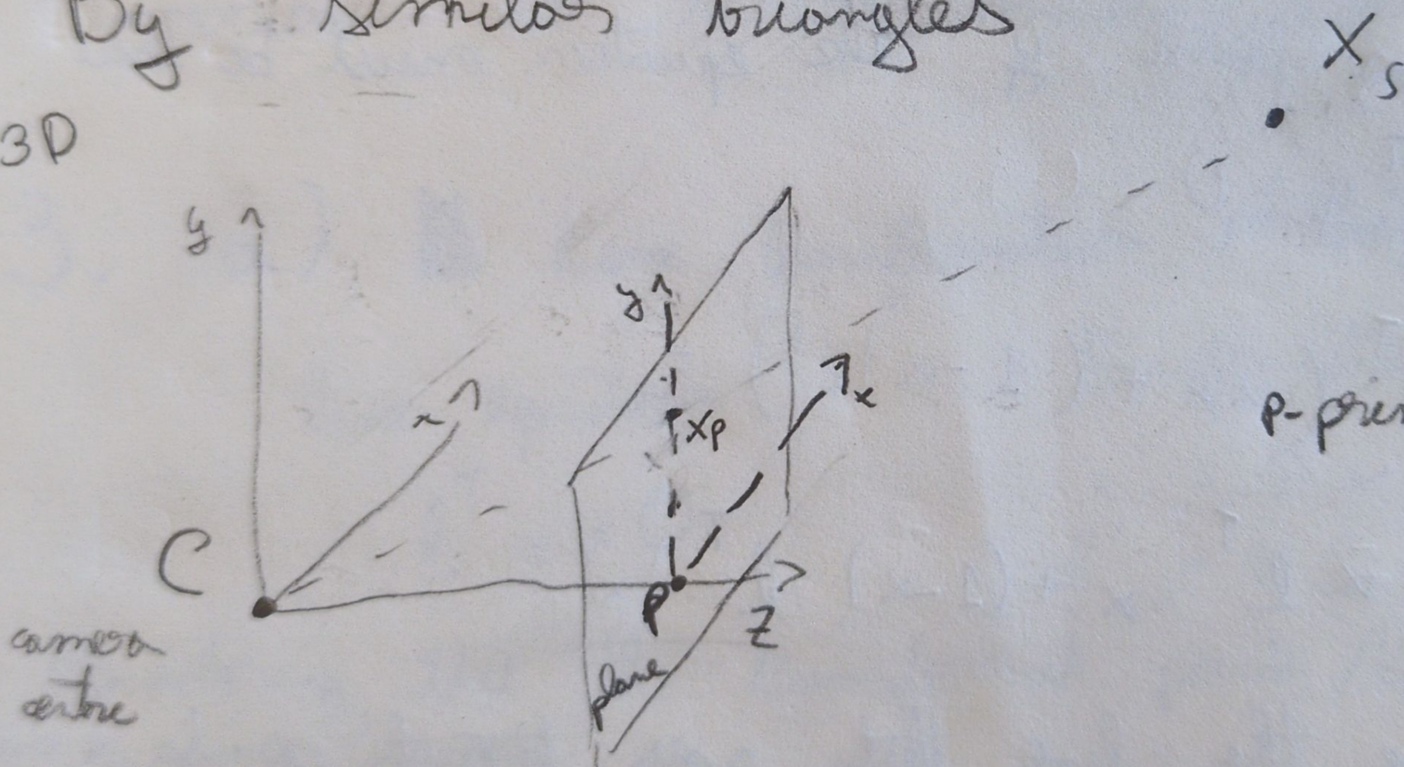
Point in space $\mathbf{x}_s = [x_s, y_s, z_s]^T$

Point in plane $x_p = [x_p, y_p]^T$

f - focal length

By similar triangles

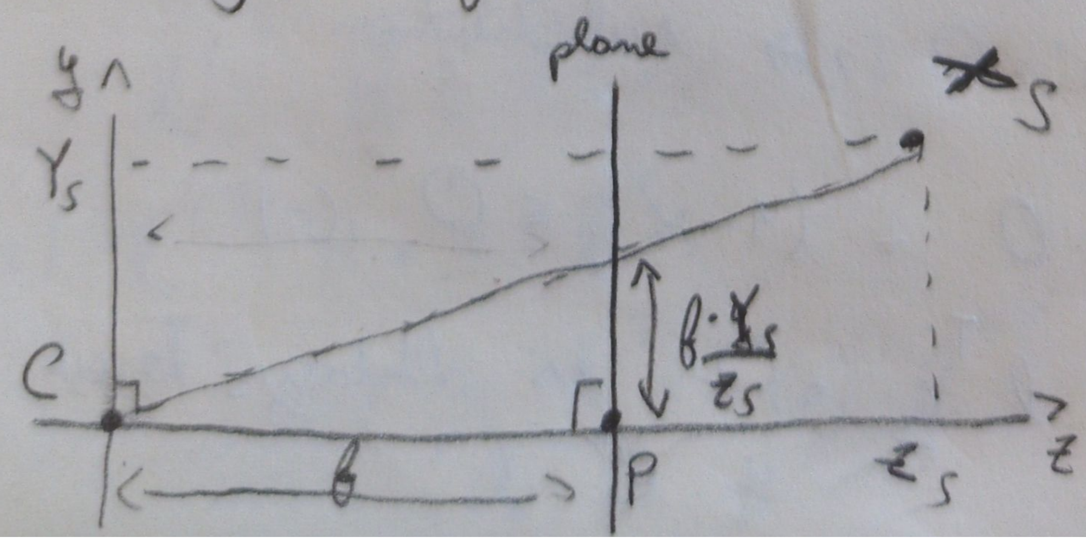
3D



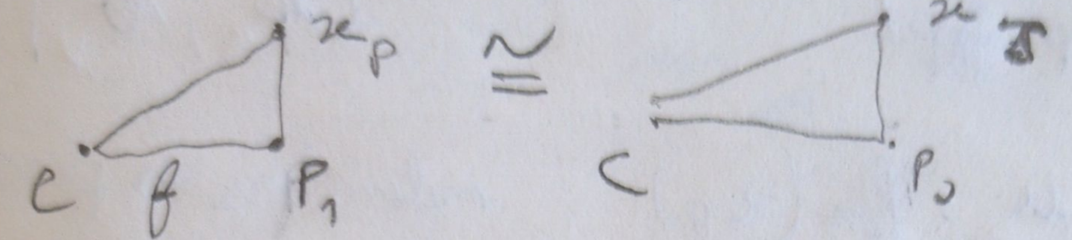
p-principal point

$\overline{CP} = f$ = focal length

2D



Since the projection onto the image is on the same line through the original point and the camera center, the triangles formed by



P_1, P_0 (projections)

are similar

From the proportion of the sides

x-coordinates:

$$\frac{x_p}{f} = \frac{x_s}{z_s} \iff x_p = f\,\frac{x_s}{z_s}$$

y-coordinates

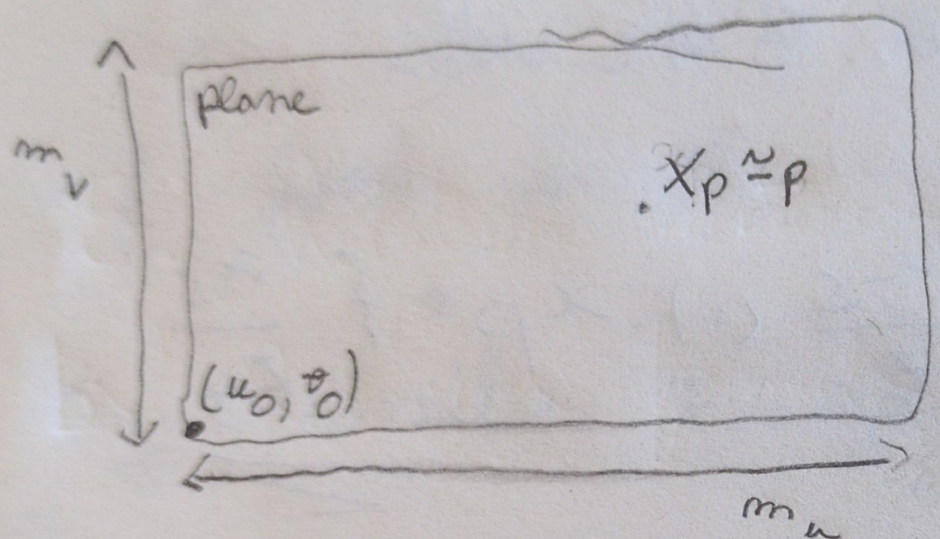
$$\frac{y_p}{f} = \frac{y_s}{z_s} \iff y_p = f\,\frac{y_s}{z_s}$$

Exercise 2. Pixel coordinate frame

a) $X_{p(x_p, y_p)} \xrightarrow{f} P_{(u, v)}$

$u = f_x(x_p)$ milimeters → pixels

$v = f_y(y_p)$



As m_u, m_v is the pixel coordinates per unit distance, to convert $u = x_p \cdot m_u + u_0$

$$(u, v) = \left(x_p \cdot m_u + u_0, \; y_p \cdot m_v + v_0\right)$$

//

b) u is parallel to x so

$$u = x_p \cdot m_u + u_0,$$

And angle θ between u and v
So v is no longer parallel to the y-axis.
It has components across x and y-axis.

$$v = (x_p \cdot \sin\theta + y_p \cos\theta) \cdot m_v + v_0$$

Exercise 3. Intrinsic camera calibration

$$K = \begin{bmatrix} m_u & 0 & u_0 \\ 0 & m_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

$$(u, v)^{p} = (x_p \cdot m_u + u_0 \,,\; y_p \cdot m_v + v_0)$$

4. Camera projection matrix

$$x_{cam} = R\, x_{world} + t$$

P - projection matrix

R - rotation

K - internal camera parameters

t - translation

R, t - external camera parameters

internal matrix

external matrix

$$\begin{pmatrix} f & sf & u_0 \\ 0 & \gamma f & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} R_{3\times 3} & t_{1\times 3} \\ 0 & 1 \end{pmatrix}$$

X_{cam} = Internal parameters · External Parameters · X_{world}

$$= \underbrace{K \cdot R,t}_{P} \cdot X_{world}$$

$$= K\,[R\,|t) \rightarrow P_{3\times4}$$

5. Rotation matrise

b.) Rodrigues formula

$$Rx = \cos\theta\, x + \sin\theta\, u \times X + (1-\cos\theta)(u\cdot x)\,u$$

Rotation matrise + Translation vector t

$\vec{u} \rightarrow$ rotation axis

$\vec{x} \rightarrow$ vector to rotate

$\theta \rightarrow$ angle to rotate

$R_x \rightarrow$ rotated vector

Decomposing $\vec{x}$ into parallel and perpendicular components to the $\vec{u}$ axis

$$\vec{x}_{\parallel} = (\vec{x}\cdot\vec{u})\,\vec{u} \qquad \vec{x}_{\perp} = \vec{x} - \vec{x}_{\parallel}$$

$$= \vec{x} - (\vec{x}\cdot\vec{u})\,\vec{u} = -\vec{u}\times(\vec{u}\times\vec{x})$$

Where we have the triple product formula

$a \times (b \times c) = (a \cdot c) b - (a \cdot b) c$

- The vector $\vec{u} \times \vec{x}_\perp = \vec{u} \times \vec{x}$ is copy of $\vec{x}_\perp$ rotated $90°$ around $\vec{u}$, therefore $\vec{x}_\perp$ and $\vec{u} \times \vec{x}$ the same length

In the rotation, the component $\vec{x}_\parallel$ will not change while the $\vec{x}_\perp$ will rotate its direction in the plane determined by $\vec{x}_\perp$ and $\vec{u} \times \vec{x}$

$$R_{\vec{x}_\perp} = \cos(\theta) \vec{x}_\perp + \sin(\theta) \vec{u} \times \vec{x}$$

Polar coordinates to Cartesian

$$r = r \cos(\theta) e_x + r \sin(\theta) e_y$$

Now

$$R_x = R_{x_\perp} + R_{x_\parallel} = R_{x_\parallel} + \cos(\theta) \vec{x}_\perp + \sin(\theta) \vec{u} \times \vec{x}$$

By substitution ($\vec{x}_\parallel = \vec{x} - \vec{x}_\perp$)

$$R_x = \cos(\theta) \vec{x} + (1 - \cos\theta)(\vec{u} \cdot \vec{x}) \vec{u} + \sin(\theta) \vec{u} \times \vec{x}$$

a) Rodrigues formula rotates $\vec{x}$ by an angle θ around a vector $\vec{u}$ by decomposing it into its parallel and perpendicular components perpendicular to $\vec{u}$, and rotating only the perpendicular component