COMPUTER VISION #W05

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Exercise 01

1) $ax + by - d = 0$

where $a^2 + b^2 = 1$

$P(x_i, y_i)$

$\overline{Pl} = |ax_i + by_i - d|$

The distance from a point to a line in perpendicular to the line is given by

$$\frac{|Ax_i + By_i + c|}{\sqrt{\underbrace{a^2 + b^2}_{1}}} = \frac{|ax_i + By_i + c|}{\sqrt{1} = 1}$$

$= |ax_i + by_i + c|$ ✓

$c = -d$

2) n points

$$E = \sum_{i=1}^{n} (a x_i + b y_i - d)^2$$

To find the minimal E

$$\frac{\partial E}{\partial d} = \frac{\partial}{\partial d} E$$

$$= \sum_{i=1}^{n} 2(a x_i + b y_i - d)$$

$$= 2 \sum_{i=1}^{n} (a x_i + b y_i - d)$$

Solving to 0 (zero)

$$2 \sum_{i=1}^{n} (a x_i + b y_i - d) = 0 \quad (=)$$

$$(=) \sum_{i=1}^{n} (a x_i + b y_i - d) = 0$$

$$(=) \sum_{i=1}^{n} a x_i + \sum_{i=1}^{n} b y_i - n d = 0$$

$$(=) n d = \sum_{i=1}^{n} a x_i + \sum_{i=1}^{n} b y_i$$

$$(=) d = \frac{\sum_{i=1}^{n} a x_i + \sum_{i=1}^{n} b y_i}{n}$$

3) $\varepsilon = \sum_{i=1}^{m} (a x_i + b y_i - d)^2$

$$= \sum_{i=1}^{m} \left(a x_i + b y_i - \frac{\sum_{i=1}^{m} a x_i + \sum_{i=1}^{m} b y_i}{m} \right)^2$$

$$= \sum_{i=1}^{m} \left(a\left(x_i - \underbrace{\frac{1}{m}\left(\sum_{j=1}^{m} x_j\right)}_{\substack{\text{mean} \\ \bar{x}}}\right) + b\left(y_i - \underbrace{\frac{1}{m}\left(\sum_{j=1}^{m} y_j\right)}_{\substack{\text{mean} \\ \bar{y}}}\right)\right)^2$$

$$= \sum_{i=1}^{m} \left(a(x_i - \bar{x}) + b(y_i - \bar{y}) \right)^2$$

$$\boxed{(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2}$$

$$= \sum_{i=1}^{m} \left(a^2 (x_i - \bar{x})^2 + 2ab (x_i - \bar{x})(y_i - \bar{y}) + b^2 (y_i - \bar{y}) \right)$$

$$= \left\| \underbrace{\begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \cdots & \cdots \\ x_m - \bar{x} & y_m - \bar{y} \end{bmatrix}}_{U} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2$$

$$= (a \quad b)\, U^T \, U \begin{pmatrix} a \\ b \end{pmatrix}$$

//

4) Minimize $|| U (a\ b^T) ||$ where $a^2+b^2=1$

$(a\ b)^T$ = eigenvector $(U^T U)$

$$d = \frac{1}{n}\left(a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i\right)$$

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