Exercise Round 12

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2) $\overrightarrow{O'p'} \cdot (\overrightarrow{O'O} \times \overrightarrow{Op}) = 0 \Leftrightarrow x'^T E x = 0$

$\overrightarrow{Op} = x = (x, y, 1)^T$

$\overrightarrow{O'p'} = x' = (x', y', 1)^T$

$\overrightarrow{O'O} = -t$ → t is the translation matrix between the 2 cameras

The vector $\overrightarrow{Op}$ in 2nd camera frame is given by $Rx$ as transforms points from first frame to the second

$$\overrightarrow{O'p'} \cdot (\overrightarrow{O'O} \times \overrightarrow{Op}) = 0 \quad (\Leftrightarrow)$$

by substitution

$$(\Leftrightarrow) \quad x'^T \underbrace{((-t) \times (Rx))}_{(-t)_\times Rx \Rightarrow [t_\times] R x} = 0$$

by defining $E = [t_\times] R$ and substituting

$$(\Leftrightarrow) \quad x'^T E x = 0 \text{ where } E = [t_\times] R$$

=

3/ a) $d = 1 \quad b = 6 \quad f = 1$

$$Z_p = \frac{B \cdot f}{d} = 6$$

b) $d_{min} = 1_{pixel} \cdot 0{,}01\,mm = 0{,}01\,mm = 0{,}001\,cm$

$$Z_p \le \frac{6 \cdot 1}{0{,}001} \Leftrightarrow 6\,000\,cm = 60\,m$$

$Z_p$ is inverse proportional to $d$ so for points where $d < 1_{pixel}$,

$Z_p > 60m$

c) $Q(3, 0, 3) \xrightarrow[\text{coordinates}]{\text{homogeneous}} (3, 0, 3, 1)^T$

To project Q into the image plane of the left camera, we use:

$$Q' = \underset{[I\ 0]}{P_l}\, Q$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \overset{\text{normalizing}}{=} \begin{bmatrix} 3/3 \\ 0/3 \\ 3/3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

corresponding epipolar line, we need to calculate $E = [t]_\times$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -6 \\ 6 & 6 & 0 \end{bmatrix}$$ and the equation for the line

$$l = E x_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -6 \\ 0 & 6 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -6 \\ 6 \end{bmatrix} \rightarrow -6y + 6 = 0$$