CV HW01

Exercise 1.

a) $ax + by + c = 0$

$\Leftrightarrow$

$x^T l = 0$

where $l = (a \quad b \quad c)^T$

$ax + by + c = 0 \Leftrightarrow$

$\Leftrightarrow \overbrace{(x \quad y)}^{x^T} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$

Passing x^T to homogeneous coordinates

$\Leftrightarrow \begin{bmatrix} x & y & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$

where $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = (a \quad b \quad c)^T = L$ $\longrightarrow$

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \cdot L = 0 \quad (=)$$

$$(=) \quad x^T \cdot L = 0$$

~~b) line through two points x and x' is~~
~~$l = x \times x'$~~

b) Intersection of two lines l and l' is the point $x = l \times l'$

Given $l = (a \; b \; c)^T$ and $l' = (a', b', c')^T$,

We want to prove $x = l \times l'$

From the scalar triple product

$(a \times b)^T \cdot c = 0$ if two vectors are parallel

by substitution

$$l \cdot (\underbrace{l \times l'}_{x}) = l' \cdot (\underbrace{l \times l'}_{x}) = 0$$ because $l \parallel l$ and $l' \parallel l'$

$l \cdot x = l' \cdot x = 0$ then x belongs in both lines, so it is the intersection of both.

c) Show that the line through two points x and x' is $l = x \times x'$

The cross product in 3D-space produces a vector that is orthogonal to both vectors in the product. As we are working with homogeneous coordinates a point will have 3 dimensions like a vector $(x, y, 1)$.

So $\overrightarrow{\text{orthogonal}} = x \times x'$ and by the cross product properties $\overrightarrow{\text{orthogonal}} \cdot x = 0$ and $\overrightarrow{\text{orthogonal}} \cdot x' = 0$ as they are orthogonal to each other, And that satisfies the line equation $l \cdot x = 0$ and $l \cdot x' = 0$. We conclude $\overrightarrow{\text{orthogonal}} = l \Rightarrow l = x \times x'$

2. a) b) degrees of freedom

Translation Matrix

$$\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

(2 degrees of freedom)

t_x, t_y

Euclidean transformation Matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

(3 degrees of freedom)

θ, t_x, t_y

Similarity Matrix

$$\begin{pmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

(4 degrees of freedom)

s, θ, t_x, t_y

Affine Matrix

$$\begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 0 \end{pmatrix}$$

(6 degrees of freedom)

a_{xy}, t_x, t_y

Projective Matrix

$$\begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

c) 8 degrees of freedom as it is generally normalized by fixing one element, making the transformation invariant to scalar multiplication.

3. a) Line transformation matrix

line equation

$$l^T x = 0$$

Transformation of points

$$x' = H x$$
$$l' = H l$$

Considering that the transformed points/lines still satisfy $l'^T x' = 0$ let's replace ~~x'~~

$$l'^T H x = 0 \quad (\Leftrightarrow)$$

$$(\Leftrightarrow) \; l'^T H x = l^T x$$

$$(\Leftrightarrow) \; \frac{l'^T H x}{x} = \frac{l^T x}{x} \; (\Leftrightarrow) \; l'^T H = l^T$$

$(\Leftrightarrow)$ $\frac{l'^T H}{H} = \frac{l^T}{H}$

$(\Leftrightarrow)$ $l'^T = (H)^{-1} l^T$

1. d) For all $\alpha \in \mathbb{R}$ the point $y = \alpha x + (1-\alpha)x'$ lies on the line through points x and x'

For any point y the equation must be true

$l^T y = 0$

$l^T_y = l^T (\alpha x + (1-\alpha)x') =$

$= \alpha l^T x + (1-\alpha) l^T x'$

Since the line that goes trough points x and x' is given by $l^T = x \times x'$, we know $l^T x = 0$ and $l^T x' = 0$, by substitution

$= \alpha 0 + (1-\alpha) = 0$ $(\Leftrightarrow)$

$(\Leftrightarrow)$ $l^T y = 0$ is always true so $y \in l^T$