Besselfunctions of the first kind

Johannes Kruse

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Bessel functions, first defined by the mathematician Daniel Bernoulli and then generalized by Friedrich Bessel, are canonical solutions y(x) of Bessel's differential equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \frac{\mathrm{d}y}{\mathrm{d}x} + (x^2 + \alpha^2) y = 0 \qquad (1)$$

for an arbitrary complex number α, the order of the Bessel function. Although α and $-\alpha$ produce the same differential equation, it is conventional to define different Bessel functions for these two values in such a way that the Bessel functions are mostly smooth functions of α.

The most important cases are when α is an integer or half-integer. Bessel functions for integer α are also known as cylinder functions or the cylindrical harmonics because they appear in the solution to Laplace's equation in cylindrical coordinates. Spherical Bessel functions with half-integer α are obtained when the Helmholtz equation is solved in spherical coordinates.

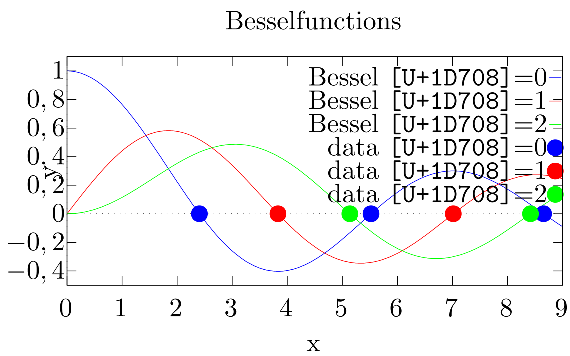

Figure 1: Illustration of the exponential function.