Module 4: Probability models

Slide Show Subtitle

1. Stochastic and deterministic models

2. Rental cars

The probability of car rented at place A, being in place A after n rentals can be described by a recursion. For each step step n, the probability for the car ending in A is the probability for the car starting in A and ending in A, plus the probability for the car starting in B and ending in A.

```
Pa[0] := 1
Pa[n_] := Pa[n - 1] * 0.6 + (1 - Pa[n - 1]) * 0.3
```

The probability can be converted to an equation of the following form:

```
PaEqn[n]:=1-Sum[0.4*0.3^m,{m,0,n-1}]
```

Should one wish to calculate the probability for a car starting at place B ending in A, one could use the same recursion as above, but replace the initial probability with zero. This since the probability of a car starting in B being in place A after zero rentals is zero, except for quantum cars which may be at both places at any moment.

2b. Long run proportions

2c. Expected proportion of cars, given starting values

Apart from knowing the expected number of cars after a given number of iterations, one might want to extend the number in a number of ways to make it more usable, for example by taking the following into account:

- Support different rental periods or rental intensities at different stations, the current model assumes that car rentals happens in synchronized steps at both stations.
- Support more than two stations, in a real-world setting we would have to handle multiple stations, for each station pair (A,B) we would need a probability for a car picked up at A to be returned at B.
- Answer questions such as: *After three months, what is the probability that there will be less than five cars at any station?*

3. Analyzing a Markov chain

4. Risk of infection

Given the problem description there are essentially four different scenarious, each with their own probability distribution.

Diagnosed	Actual state	Equation	Probability
Healthy	Healthy	99.7 % * 97 %	96.709 %
Healthy	Sick	0.3 % * 1 %	0.003 %
Sick	Healthy	99.7 % * 3 %	2.991 %
Sick	Sick	0.3 % * 99 %	0.297 %

To get the probability for someone being diagnosed as sick to actually be sick, we simply divide the probability of correctly being diagnosed sick with the total probability of being diagnosed sick.

$P(\text{Sick} \mid \text{Diagnosed sick}) = \frac{0.297\,\%}{0.297\,\% + 2.991\,\%} \approx 9.8\,\%$

5. Bayesian networks

6. Will it rain tomorrow?

To judge the risk of rain on a specific day, we choose to broaden and generalize the problem given.

```
p[n_] := RandomInteger[1];
Table[p[n], {n, 365}]
2 + 2
```

7. Reflection on previous module