Sistemas Inteligentes

Introducción ML

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Sistemas Inteligentes

15 de noviembre 2017

Biblio and Resources

- Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani. An Introduction to Statistical Learning, Springer, 2017.

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- Daphne Koller, Nir Friedman, Probabilistic Graphical Models. MIT Press. 2009.
- Andrew Ng ML handouts

Tools

- Octave

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- Python, Notebooks

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Machine Learning

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- Tom Mitchell (1998) Well-posed learning problem: A computer is said to learn from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E.
- Mitchell: Machine Learning is the study of computer algorithms that improve automatically through experience.

ML- Example

BUSINESS INSIDER POLITICS

An artificial intelligence system that correctly predicted the last 3 elections says Trump will win

Pamela Engel

Oct. 28, 2016, 8:24 PM 152,066

Applications

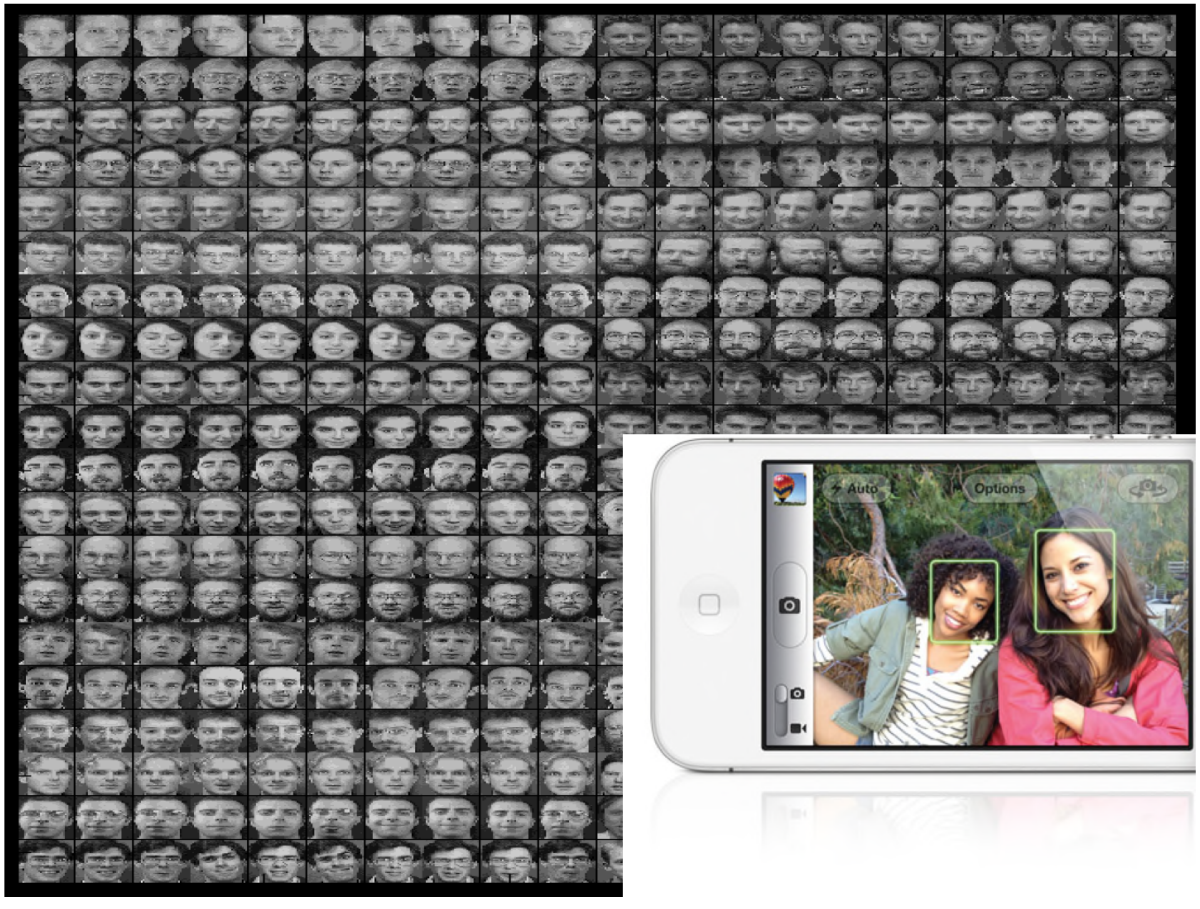
Machine Learning Applications (Nando de Freitas)

Applications

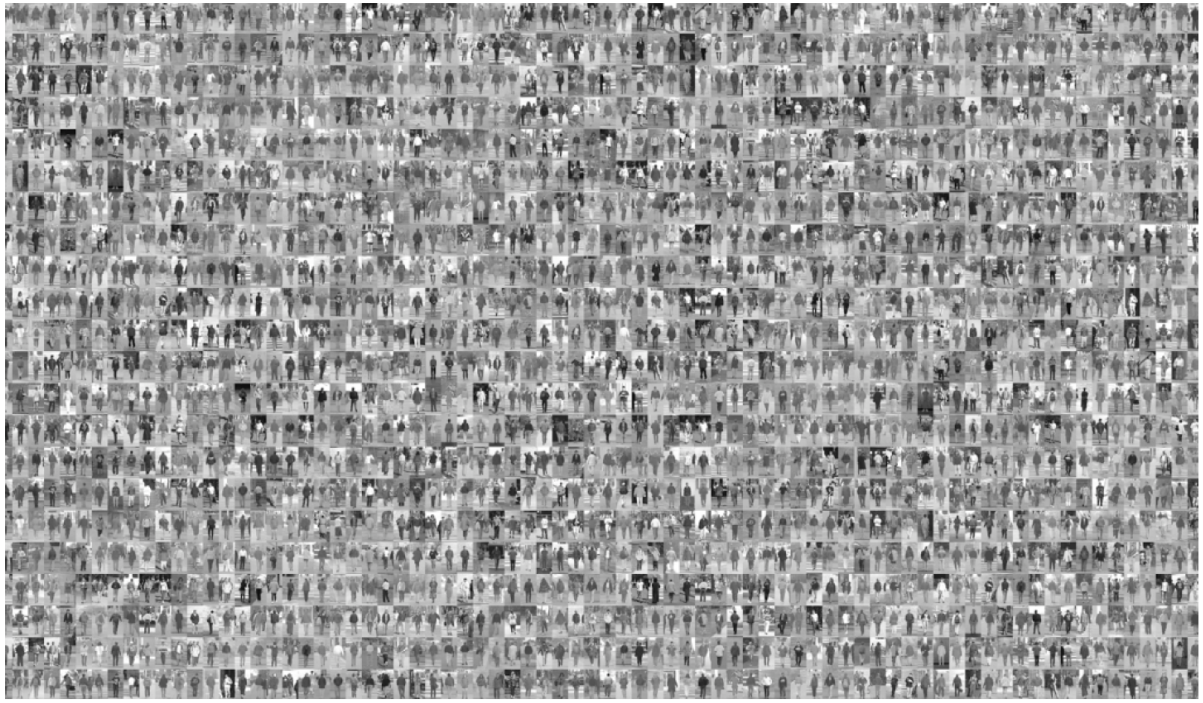
Application: Invariant recognition in natural images

[Thomas Serre 2012]

Applications



Applications

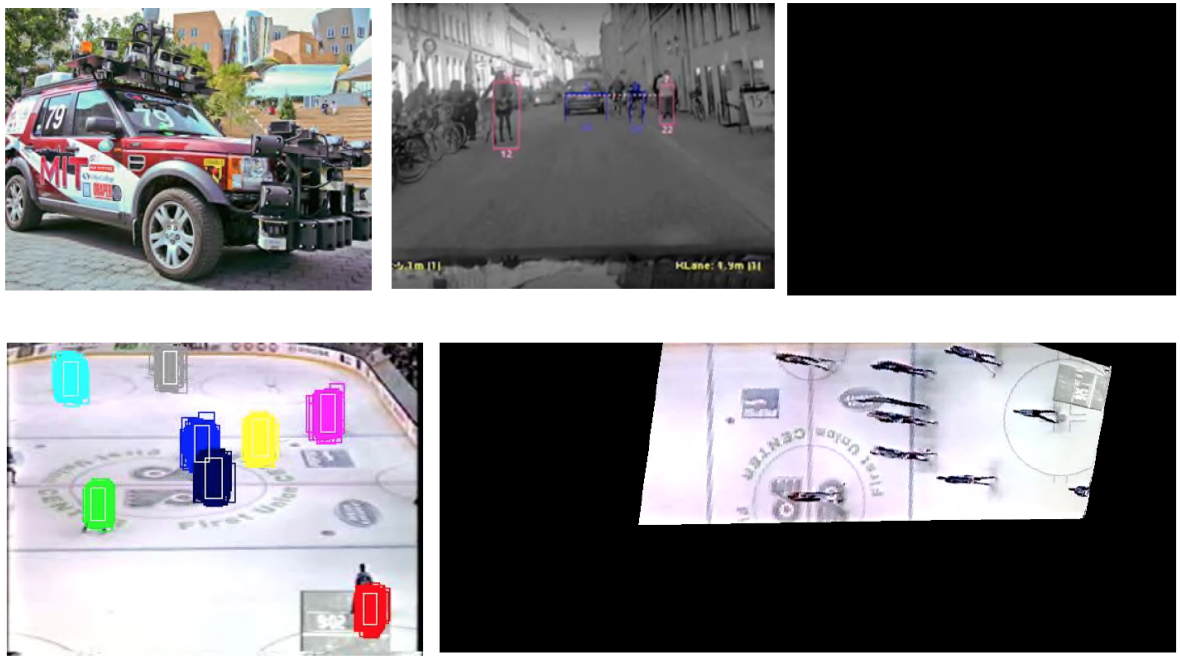


Millions of labeled examples are used to build real-world applications, such as pedestrian detection

[Tomas Serre]

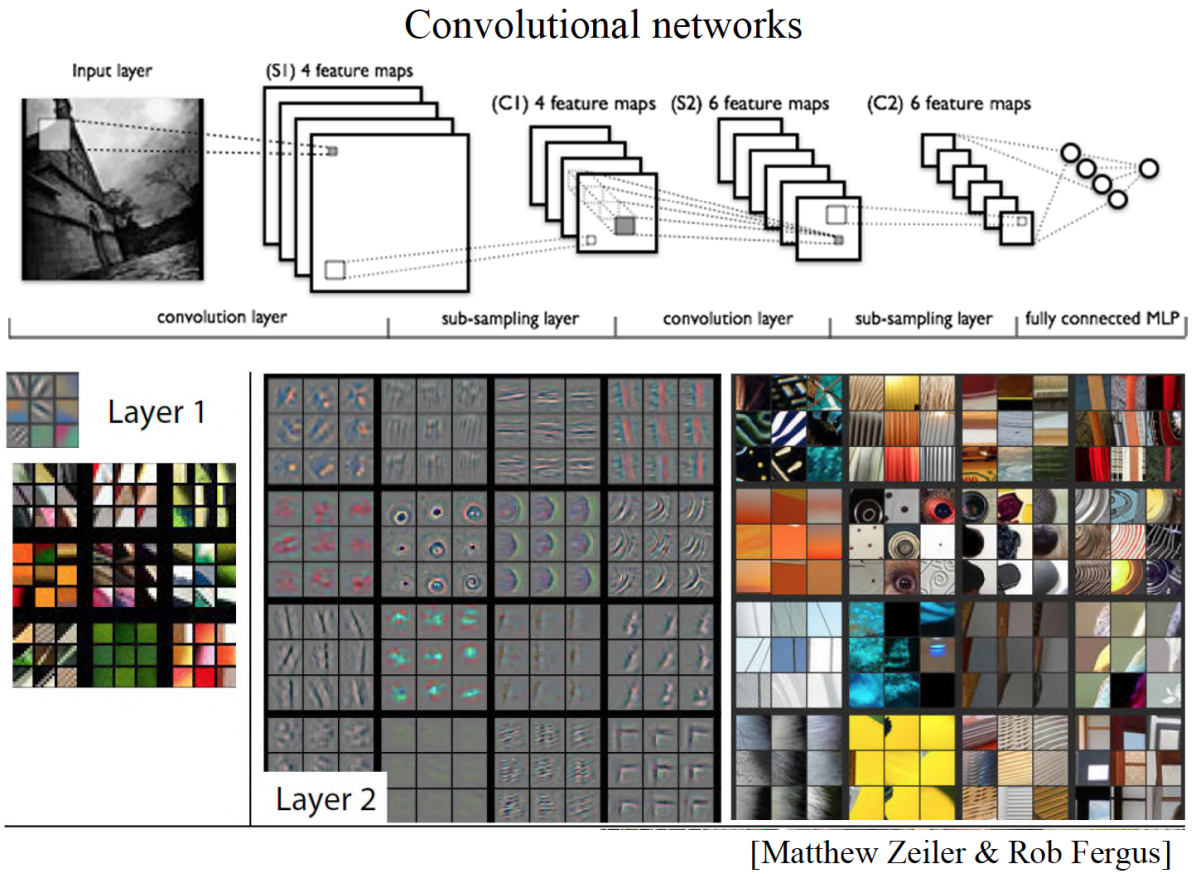
Applications

More applications of the same idea



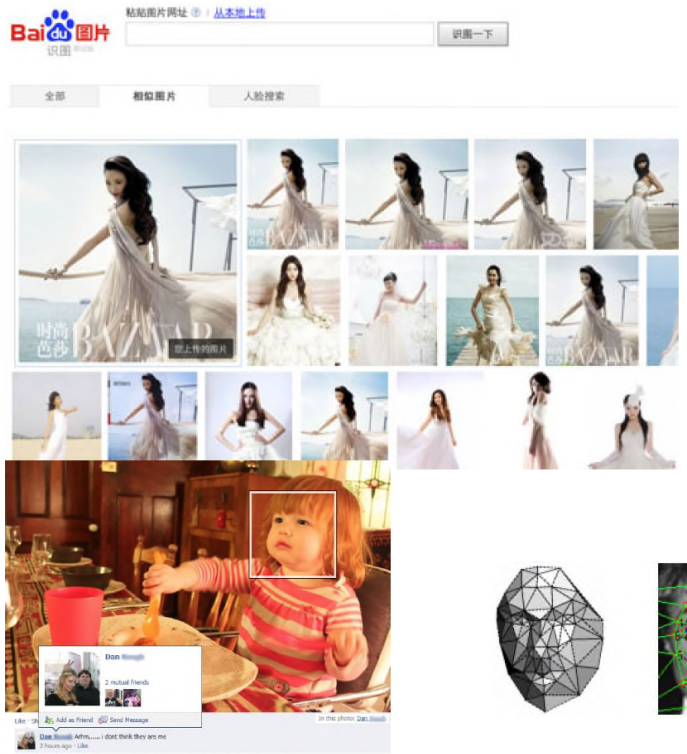
[Okuma, Taleghani, dF, Little, Lowe, 2004]
Best Cognitive Vision Paper- ECCV

Applications



Applications

Machines that learn to recognise what they see and hear are at the heart of Apple, Google, Amazon, Facebook, Netflix, Microsoft, etc.



Applications

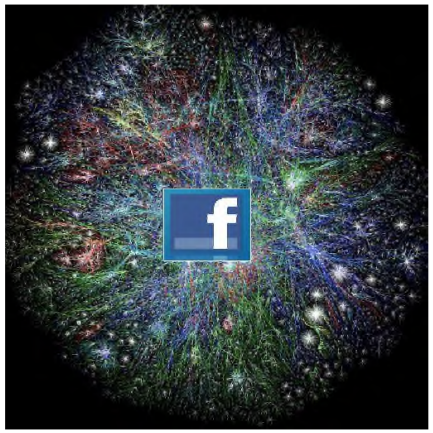
Review sentiment and summarization

Paul Bettany did a great role as the tortured father whose favorite little girl dies tragically of disease.
For that, he deserves all the credit.
However, the movie was mostly about exactly that, keeping the adventures of Darwin as he gathered data for his theories as incomplete stories told to children and skipping completely the disputes regarding his ideas.
Two things bothered me terribly: the soundtrack, with its whiny sound, practically shoving sadness down the throat of the viewer, and the movie trailer, showing some beautiful sceneries, the theological musings of him and his wife and the enthusiasm of his best friends as they prepare for a battle against blind faith, thus misrepresenting the movie completely.
To put it bluntly, if one were to remove the scenes of the movie trailer from the movie, the result would be a non descript family drama about a little child dying and the hardships of her parents as a result.
Clearly, not what I expected from a movie about Darwin, albeit the movie was beautifully interpreted.

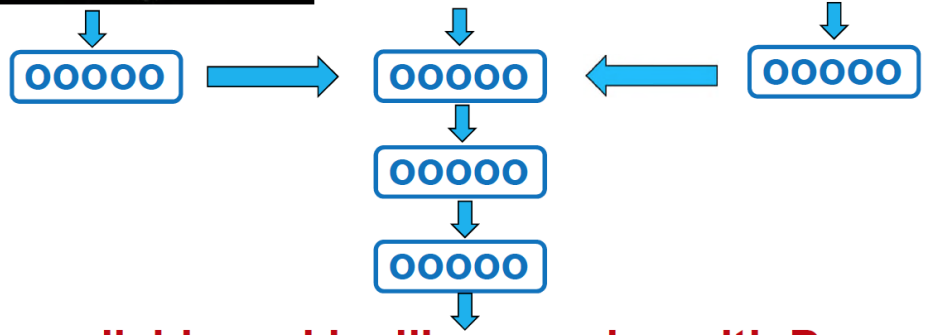
[Kotzias, Denil, Blunsom & NdF, 2014]

Applications

Structured queries and outputs

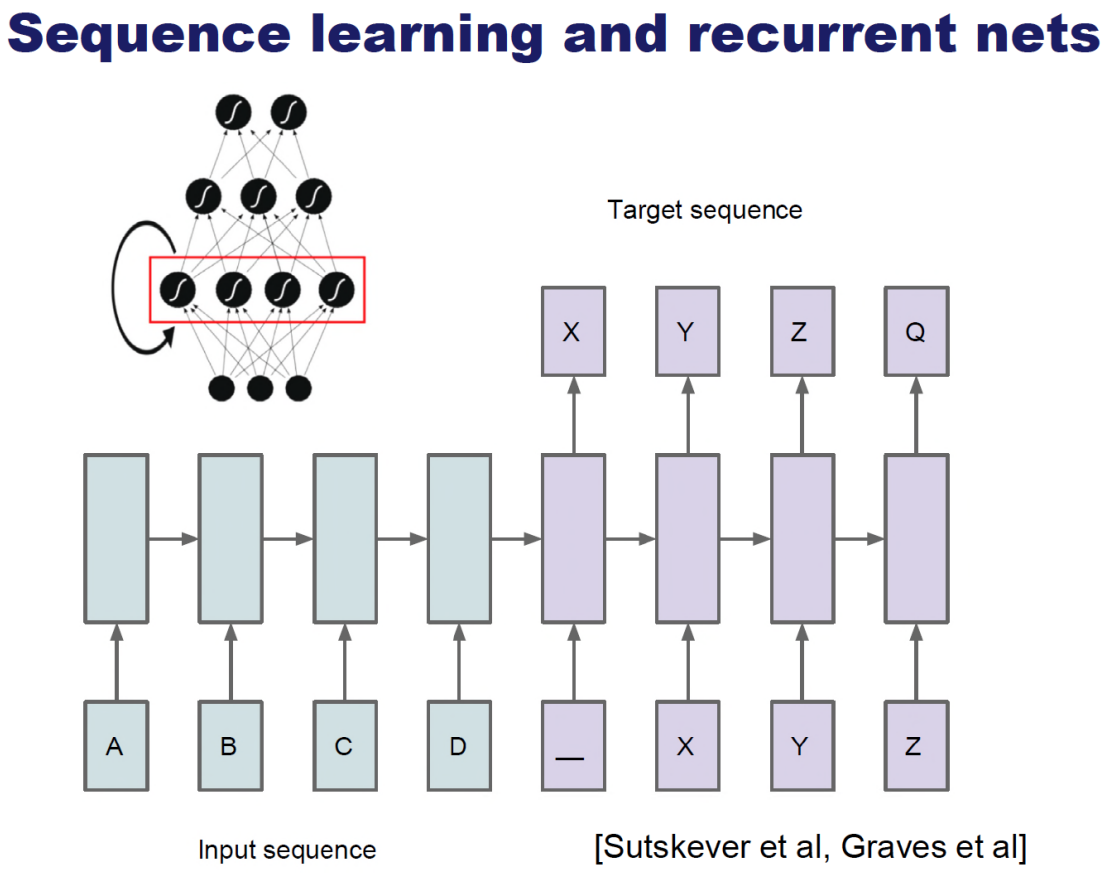


Who's likely to want to watch this movie with me on Friday?



Phil is available and he likes movies with Downey JR

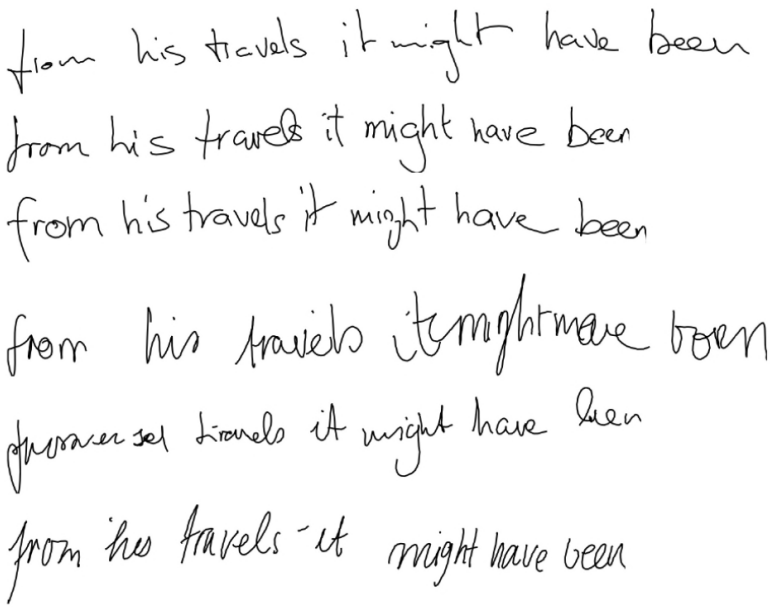
Applications



Applications

Sequence learning and recurrent nets

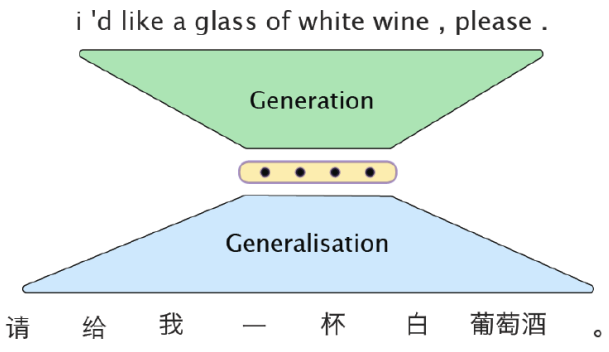
Which is Real?

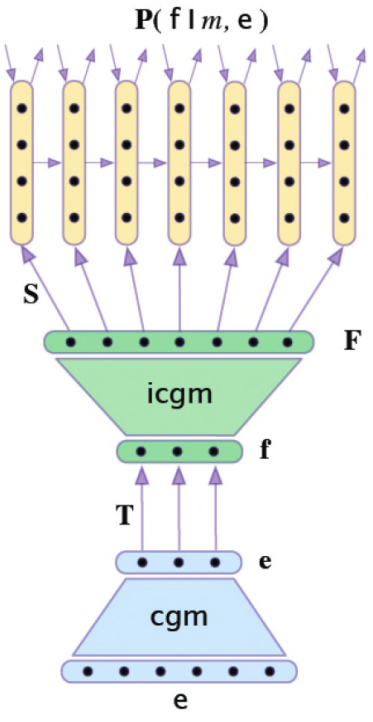


[Alex Graves]

Applications

Siamese nets, machine translation

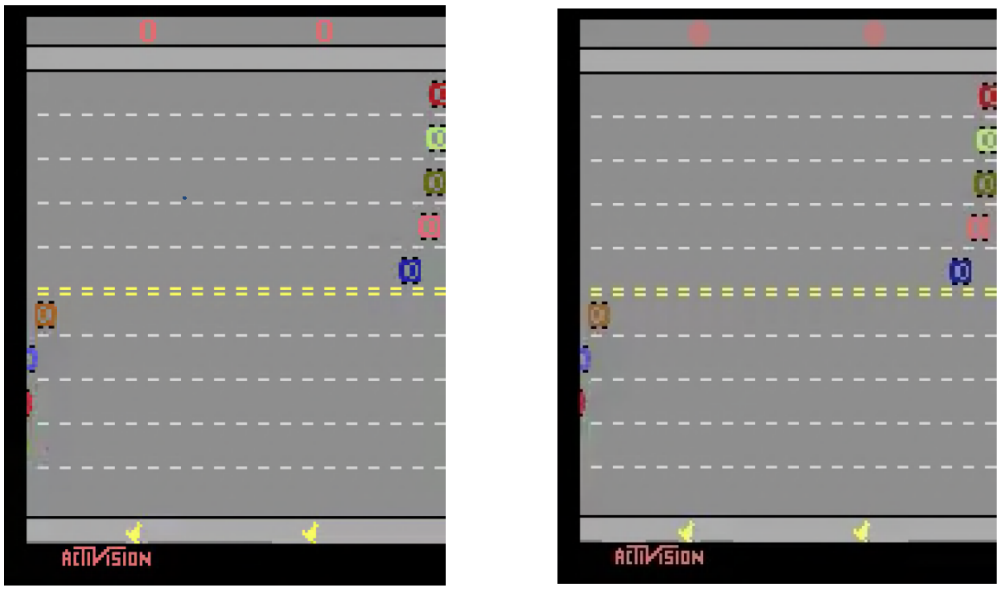




[Le Cun et al, Blunsom et al]

Applications

Imitation learning for Atari



[Dejan Markovikj, Miroslav Bogdanovic, Misha Denil, NdF 2014]

ML Applications

ML deals with the problem of extracting features from data so as to solve many different predictive tasks:

- Forecasting (e.g. Energy demand prediction, finance)

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- Summarizing (e.g. News, social media sentiment)
- Decision making (e.g. AI, robotics)

When to apply ML

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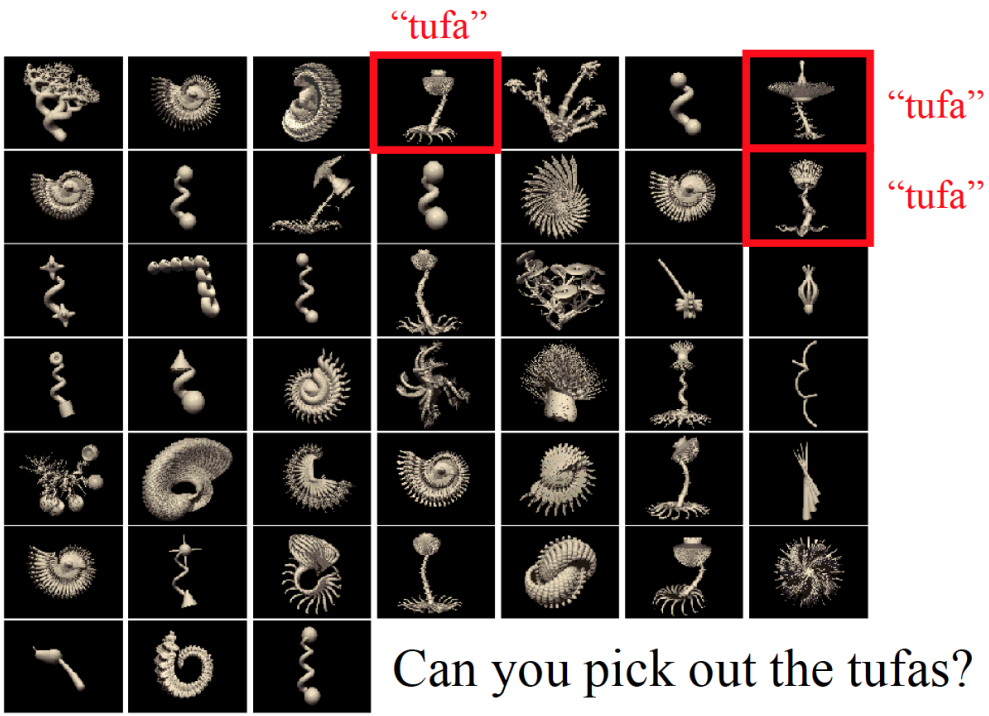
- Solution changes with time (e.g. Tracking, temperature control, preferences)
- Solutions needs to be adapted to particular cases (e.g. Biometrics, personalization)
- The problem size is to vast for our limited reasoning capabilities (e.g. calculating webpage ranks)

ML Challenge

Challenge: One-shot learning



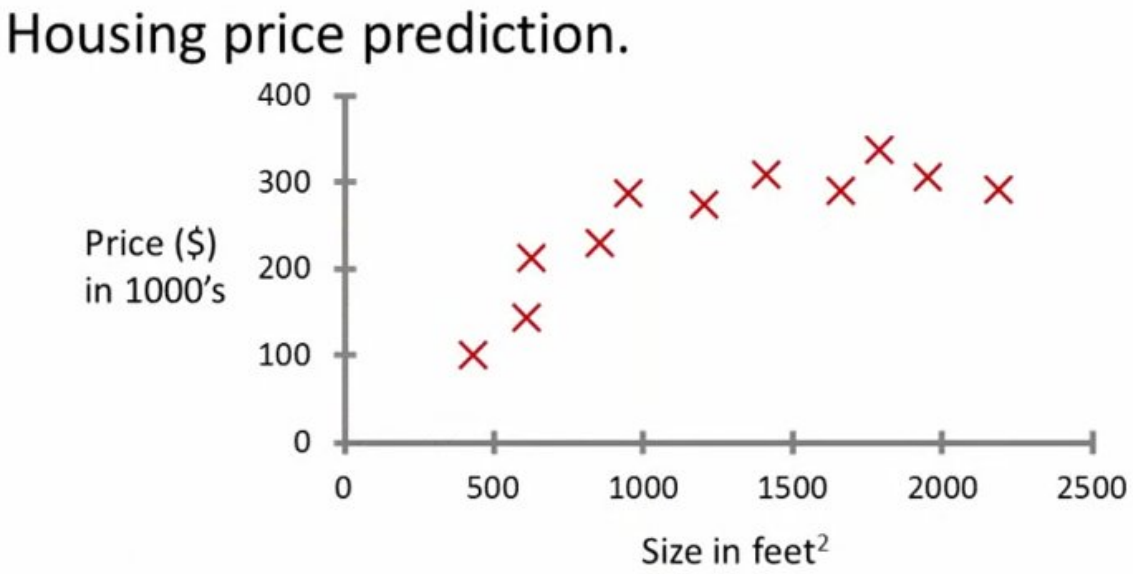
Can you pick out the tufas?

Josh Tenenbaum

Learning

Supervised and Unsupervised Learning

Aprendizaje Supervisado



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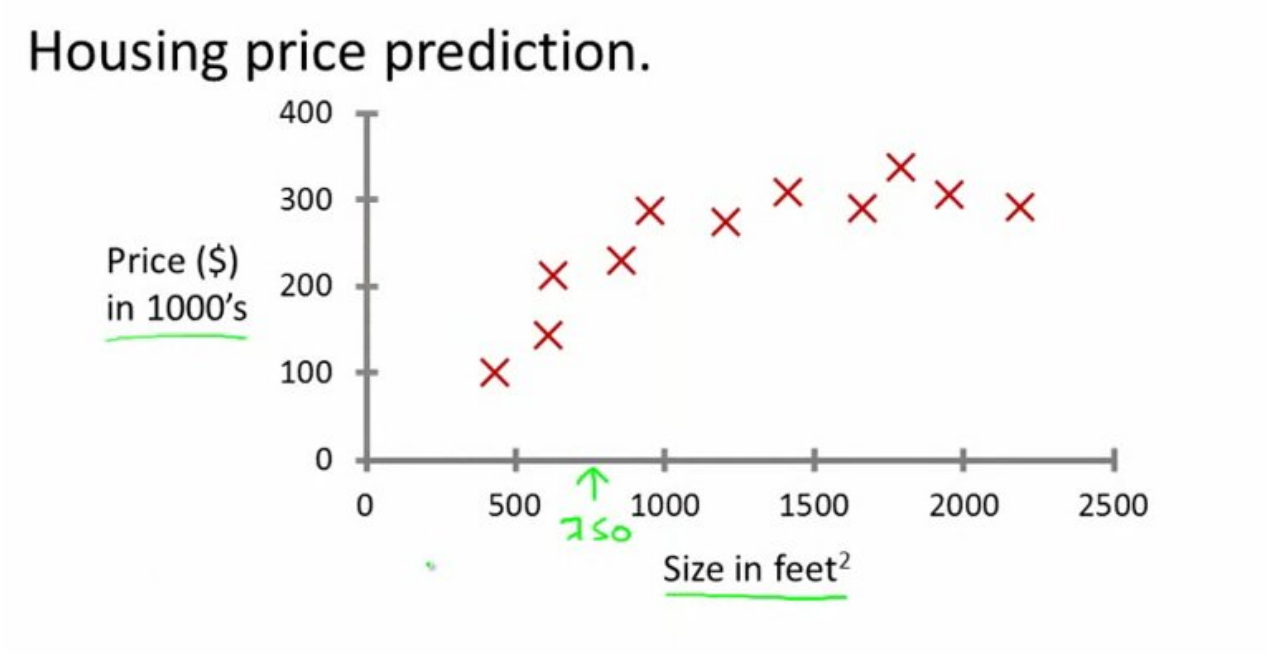

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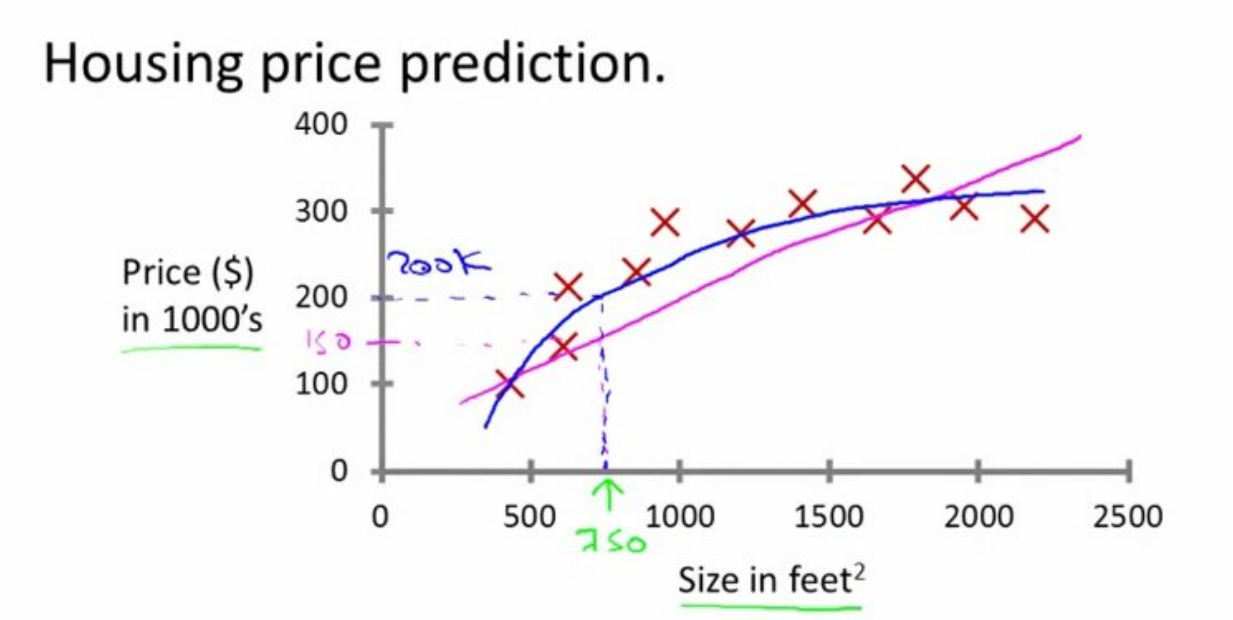

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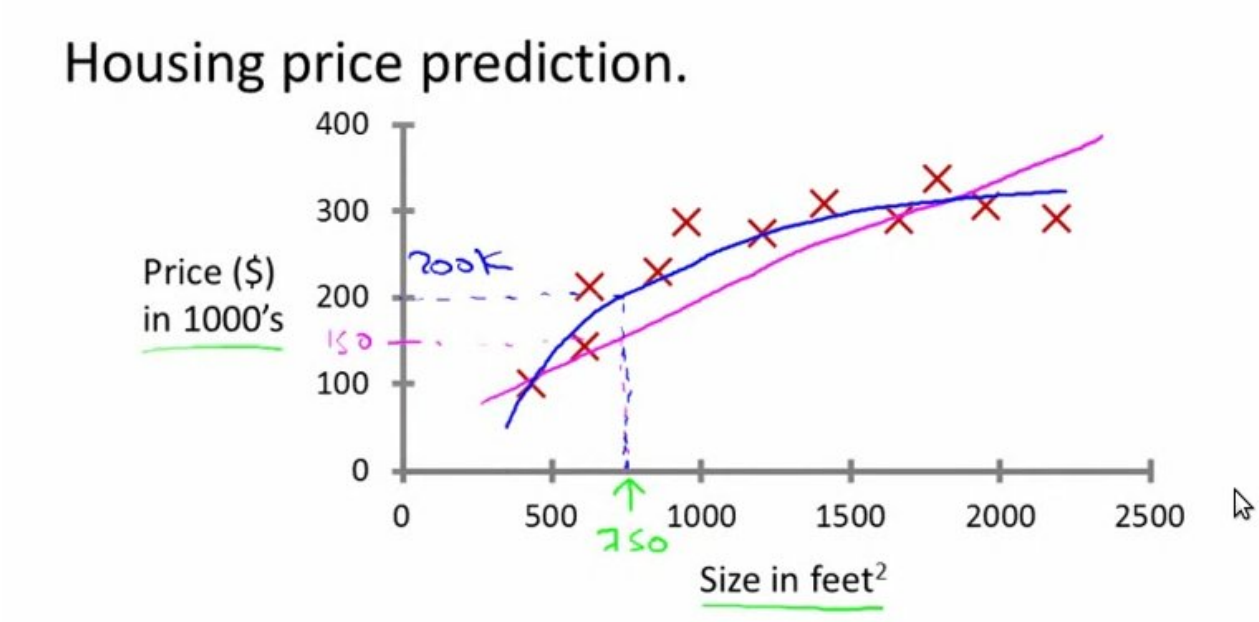
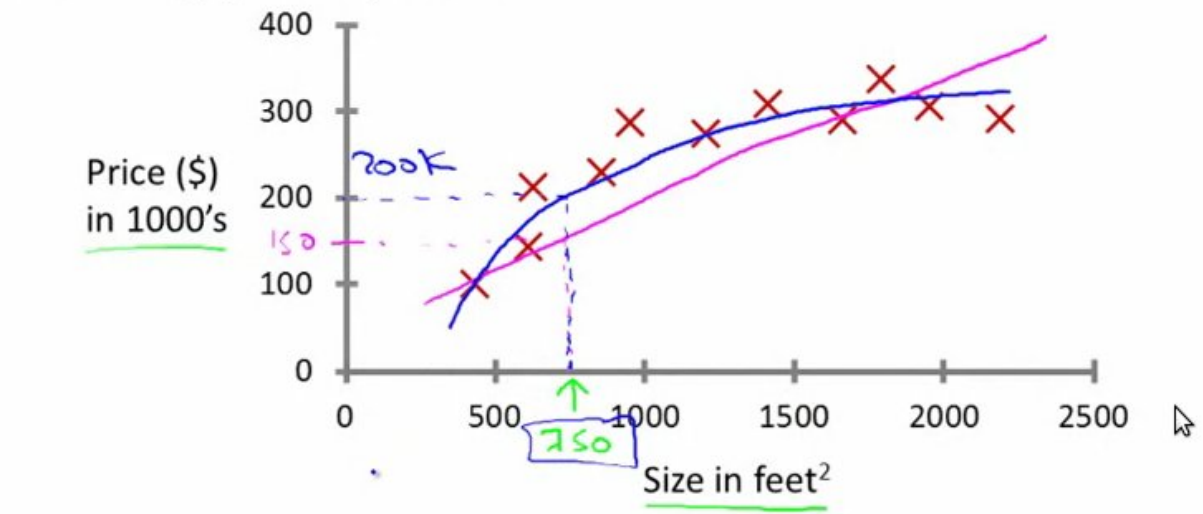


Supervised Learning

"right answers" given

Aprendizaje Supervisado

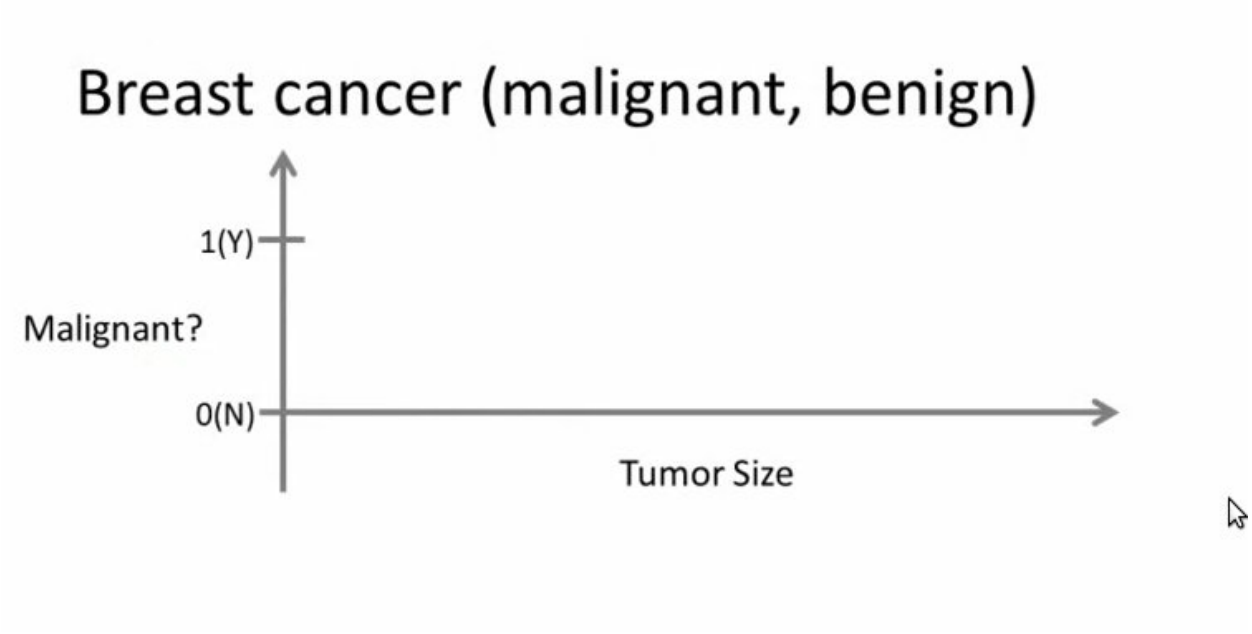


Supervised Learning

"right answers" given

Regression: Predict continuous valued output (price)

Aprendizaje Supervisado



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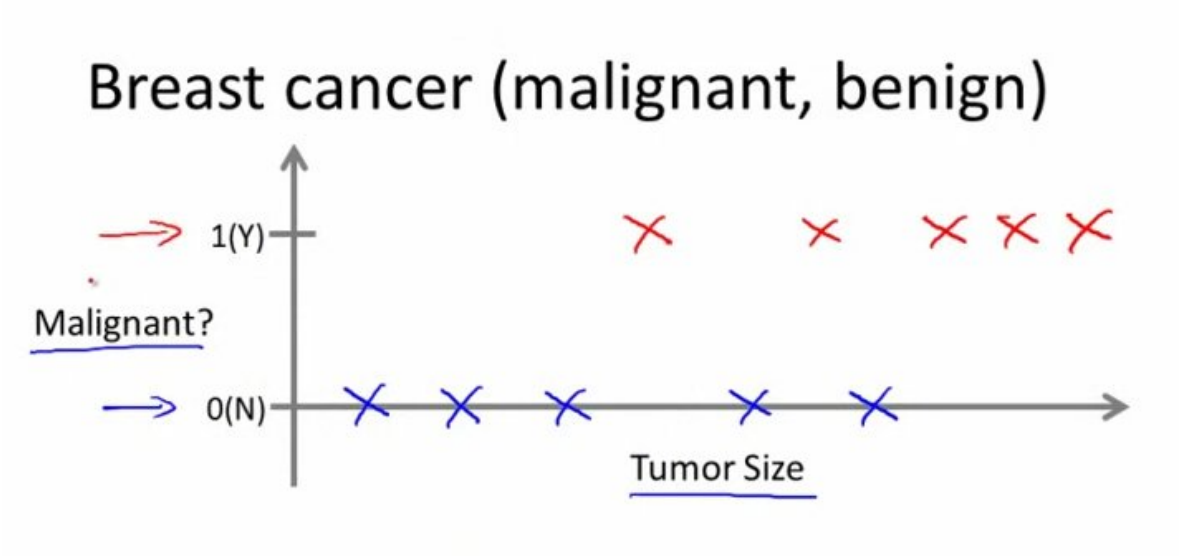

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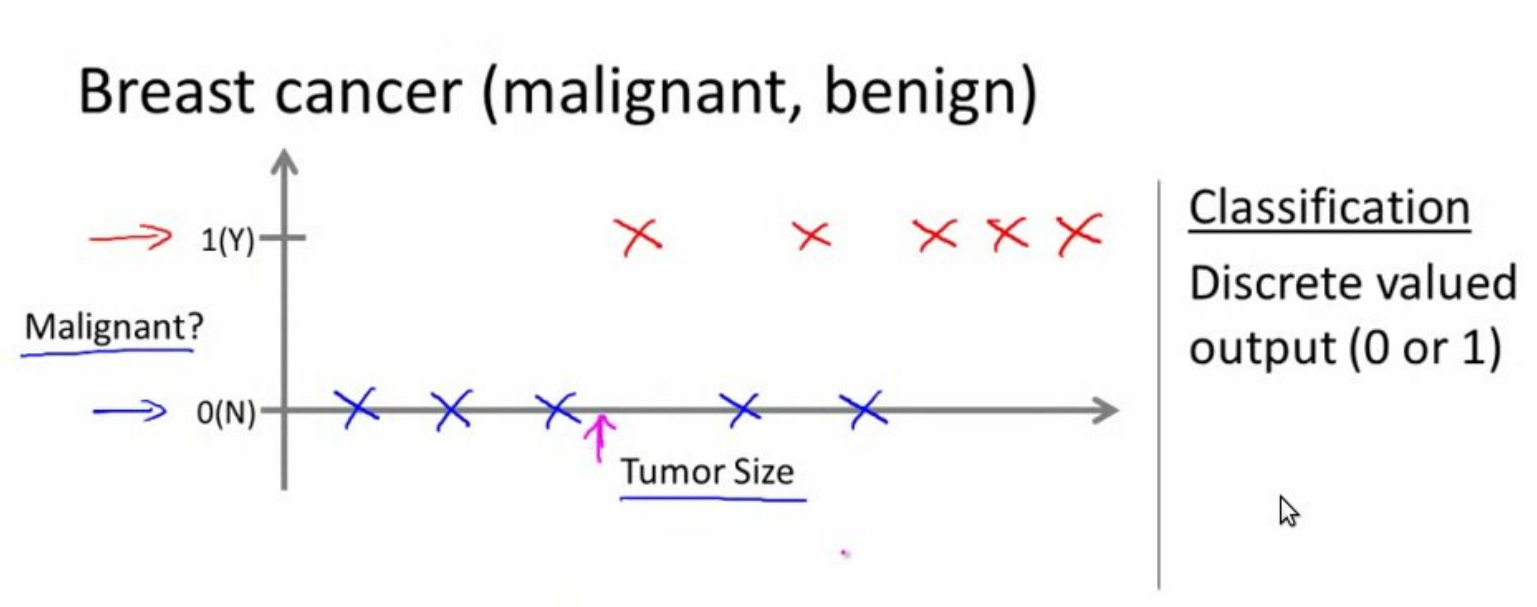

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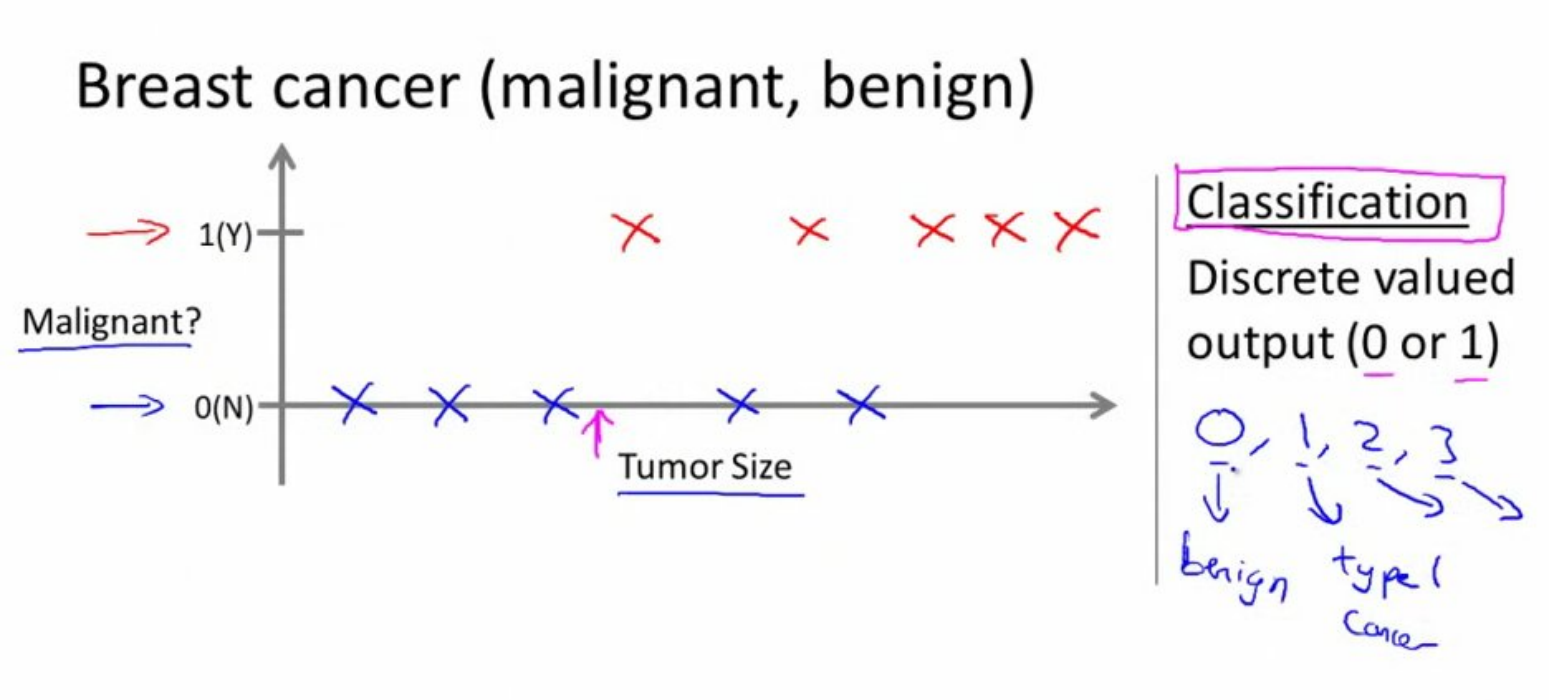

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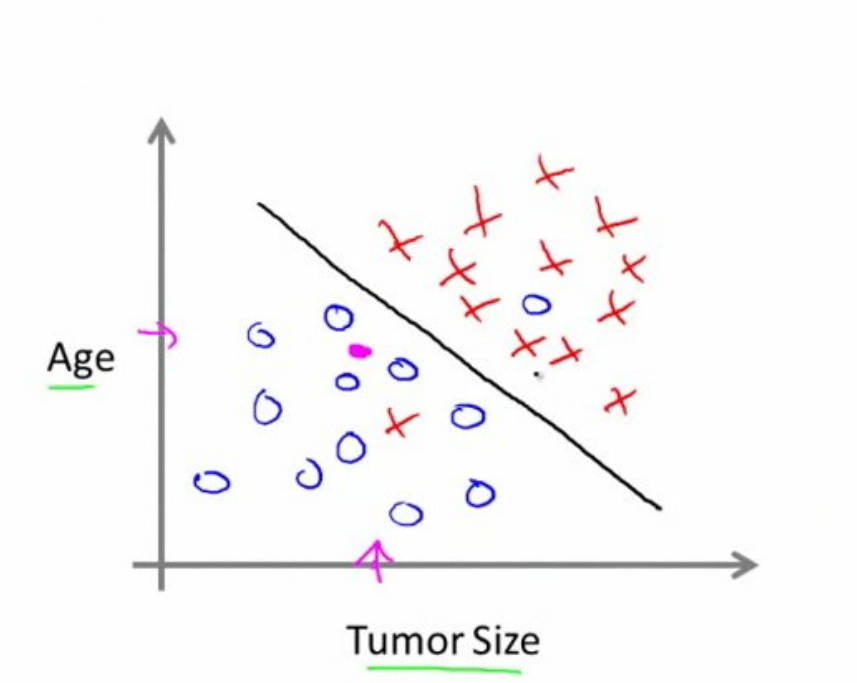
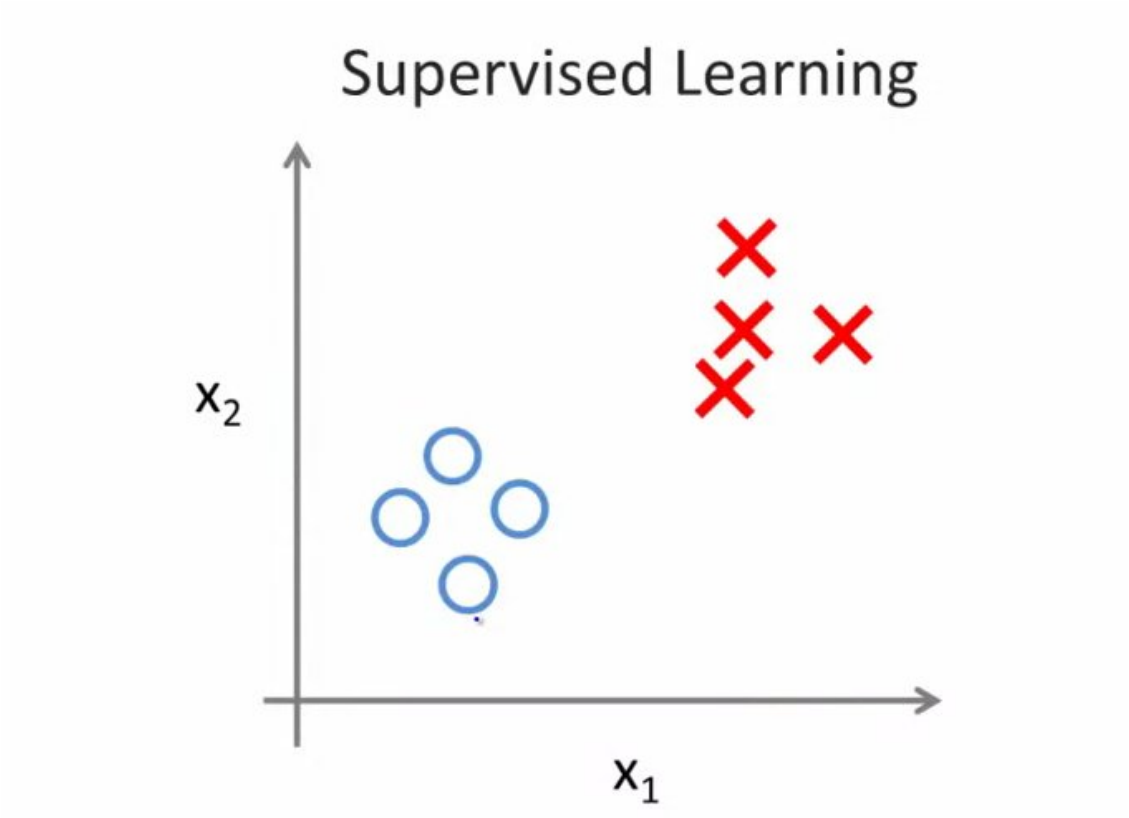


Aprendizaje No Supervisado



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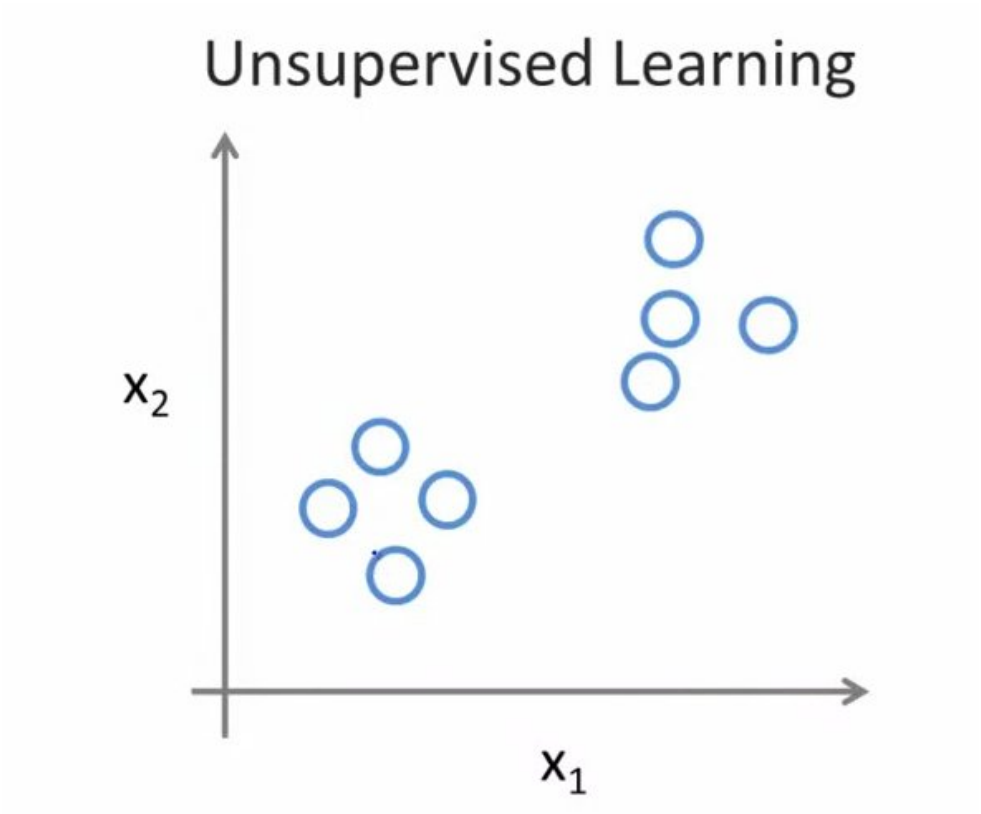

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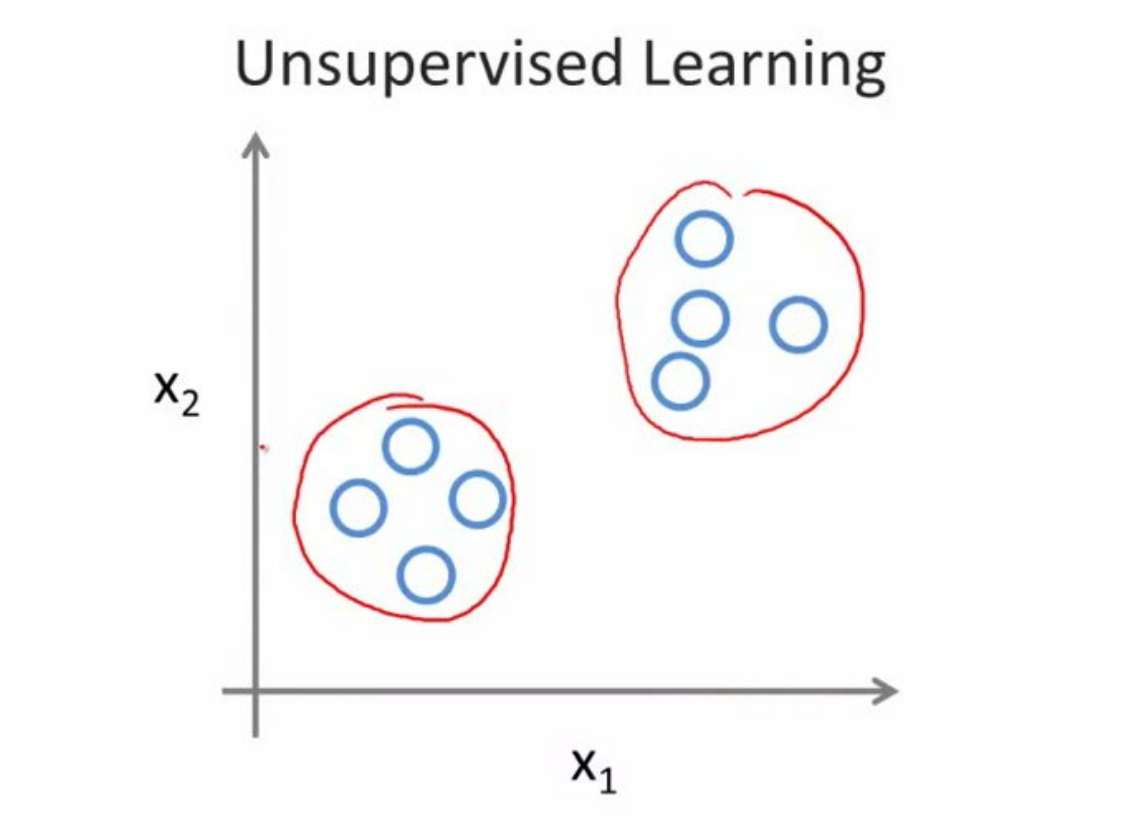

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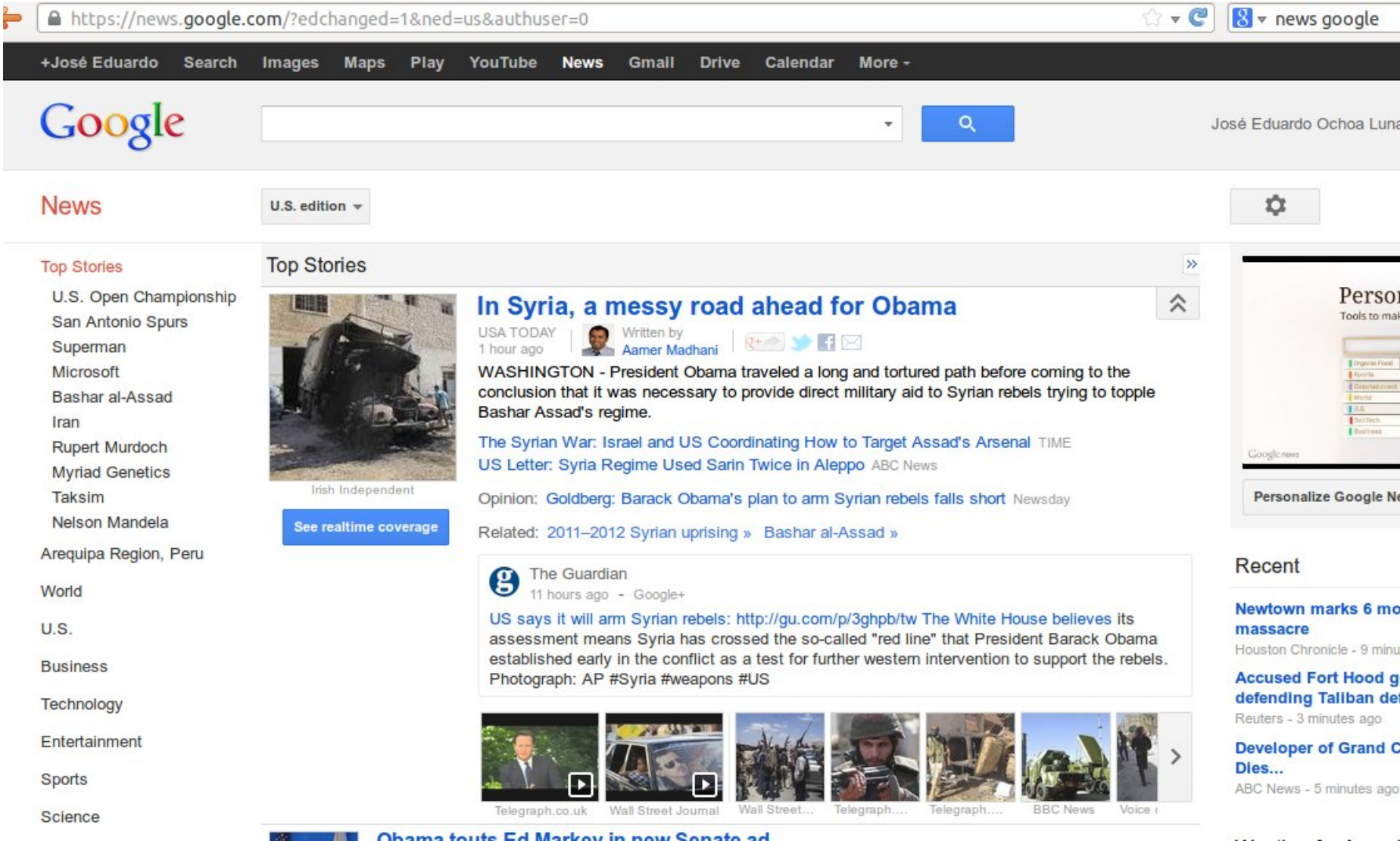
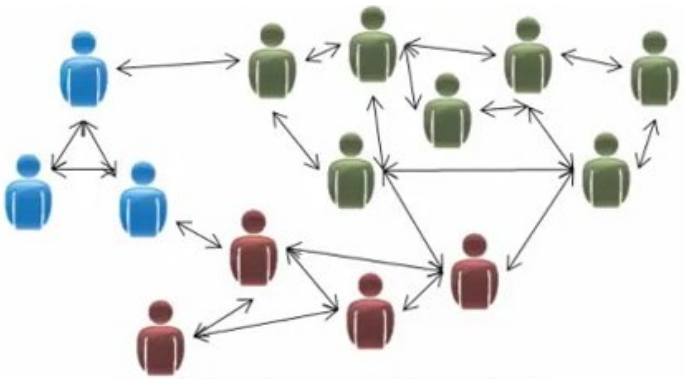


Aprendizaje No Supervisado

Organize computing clusters



Social network analysis

Market segmentation

Astronomical data analysis

Linear Regression

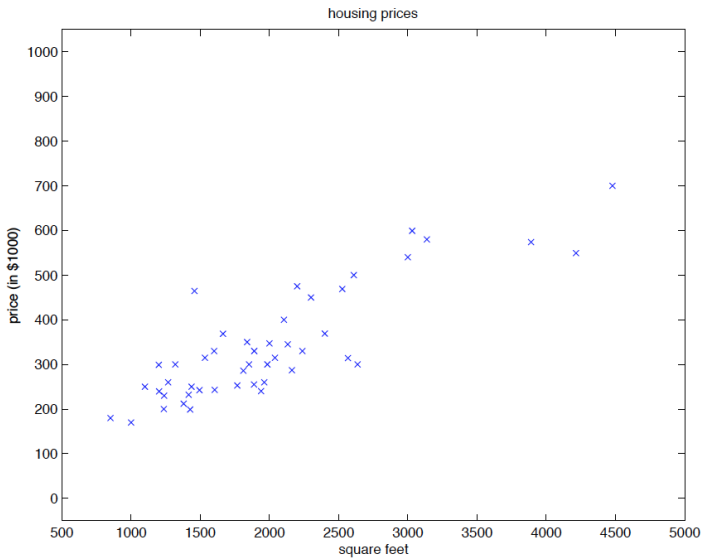
Linear Regression

Example

We have a dataset giving the living areas and prices of 47 houses from Portland, Oregon:

Living area (feet^2)	Price (1000$s)
2104	400
1600	330
2400	369
1416	232
3000	540
⋮	⋮

Data



Given data like this, how can we learn to predict the prices of other houses in Portland, as a function of the size of their living areas?

Notation

- $x^{(i)}$ denotes the input variables (living area), also called input features

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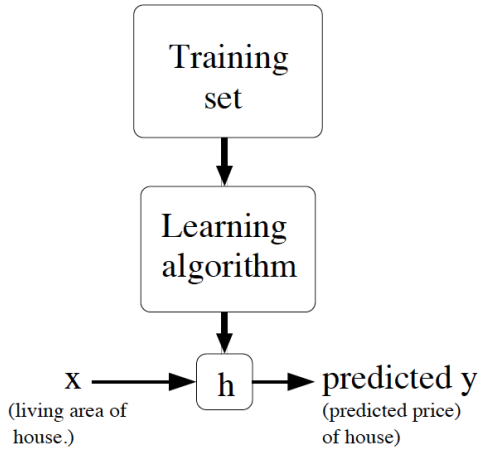
- X denotes the space of input values and Y the space of output values.

The problem

Goal: given a training set, to learn a function $h : X \to Y$ so that $h(x)$ is a good predictor for the corresponding value of y. h is called a hypothesis



Linear Regression

X's are two dimensional vectors. $x_1^{(i)}$ is the living area of the i-th house in the training set. $x_2^{(i)}$ is its number of bedrooms.

Living area (feet^2)	#bedrooms	Price (1000$s)
2104	3	400
1600	3	330
2400	3	369
1416	2	232
3000	4	540
⋮	⋮	⋮

How to represent functions / hypotheses h in a computer?

Linear Regression II

We decide to approximate y as a linear function of x:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

- θ_i's are the parameters (weights), parameterizing the space of linear functions mapping from X to Y.

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- θ and x are vectors, n is the number of input variables

Cost function

- How do we pick the parameters θ?

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- make $h(x)$ close to y, at least for the training examples

Cost function

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- make $h(x)$ close to y, at least for the training examples
- To formalize, we define a function that measures, how to close the $h(x^{(i)})$'s are to the corresponding $y^{(i)}$'s, the cost function:

$$J(\theta) = \frac{1}{2} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Cost function

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- least-squares cost function

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- repeatedly changes θ to make $J(\theta)$ smaller
- until hopefully we converge to a value of θ that minimizes $J(\theta)$

Gradient Descent Algorithm

- consider the gradient descent algorithm, which starts with some initial θ and performs the update:

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- repeatedly takes a step in the direction of steepest decrease of J
- we have to work out what is the partial derivative term

Gradient Descent Algorithm - Partial Derivatives

only one training example (x, y)

$$\begin{aligned}
\frac{\partial}{\partial \theta_j} &= \frac{\partial}{\partial \theta_j} \frac{1}{2}(h_\theta(x) - y)^2 \\
&= 2 \cdot \frac{1}{2}(h_\theta(x) - y) \cdot \frac{\partial}{\partial \theta_j}(h_\theta(x) - y) \\
&= (h_\theta(x) - y) \cdot \frac{\partial}{\partial \theta_j}\left(\sum_{i=0}^{n} \theta_i x_i - y\right) \\
&= (h_\theta(x) - y) x_j
\end{aligned}$$

LMS update rule

For a single training example, this gives the update rule

$$\theta_j := \theta_j + \alpha(y^{(i)} - h_\theta(x^{(i)}))x_j^{(i)}$$

The least mean squares update rule (Widrow-Hoff learning rule)

- the magnitude of the update is proportional to the error term $(y^{(i)} - h_\theta(x^{(i)}))$
- if in a training example our prediction nearly matches the actual value of $y^{(i)}$, then parameter are unchanged
- if the prediction has a large error then a larger change will be made

Batch Gradient Descent

For a training set:
Repeat until convergence $\{$

$$\theta_j := \theta_j + \alpha \sum_{i=1}^{m} (y^{(i)} - h_\theta(x^{(i)}))x_j^{(i)} \quad \text{for every } j$$

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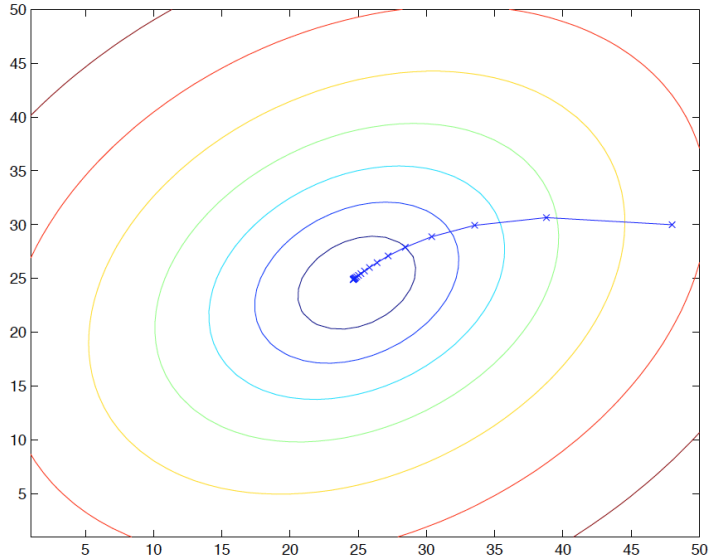
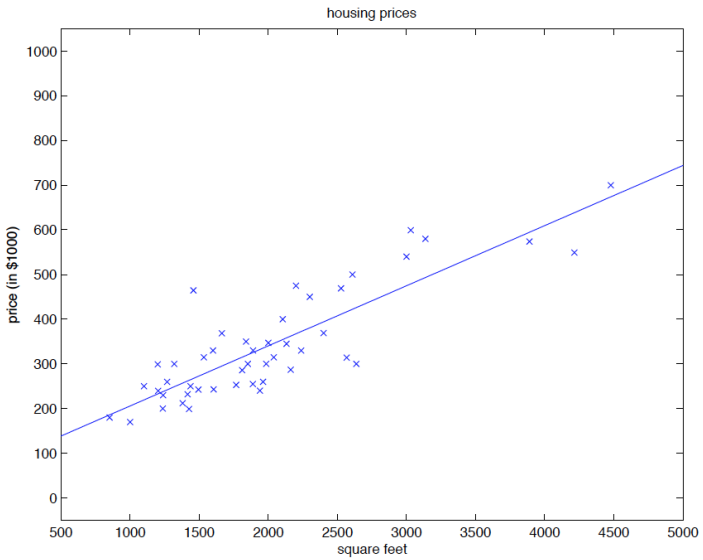
- The entire training set is considered on every step (batch gradient descent)
- This method can be susceptible to local minima in general
- The optimization problem for linear regression has only one global optima (J is a convex function)
- gradient descent always converge (assuming α is not too large)

Gradient Descent convergence



Ellipses are the contours of a quadratic function. The x's mark the successive values of θ that gradient descent went through

Gradient Descent fit



predict housing price as a function of living area: $\theta_0 = 71.27$, $\theta_1 = 0.1345$

Stochastic Gradient Descent

Loop {

for i=1 to m, {

$$\theta_j := \theta_j + \alpha \left(y^{(i)} - h_\theta(x^{(i)}) \right) x_j^{(i)} \qquad \text{(for every } j\text{)}.$$

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- we repeatedly run through the training set, and each time we encounter a training example, we update the parameters
- Batch gradient descent has to scan through the entire training set (costly if m is large)
- Often SGD gets θ close to the minimum much faster than batch GD

Probabilistic Interpretation

Let us asume that the target variables and the inputs are related via the equation

$$y^{(i)} = \theta^T x^{(i)} + \epsilon^{(i)},$$

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$$p(\epsilon^{(i)}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\epsilon^{(i)})^2}{2\sigma^2}\right)$$

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This implies

$$p(y^{(i)}|x^{(i)};\theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y^{(i)} - \theta^T x^{(i)})^2}{2\sigma^2}\right)$$

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- This quantity is typically viewed a function of $\vec{y}$ (and perhaps X), for a fixed value of θ
- When view this as a function of θ, it is called the likelihood function

Likelihood Function

$$L(\theta) = L(\theta; X, \vec{y}) = p(\vec{y}|X; \theta)$$

By the independence assumption on the $\epsilon^{(i)}$'s , this can also be written

$$\begin{aligned} L(\theta) &= \prod_{i=1}^{m} p(y^{(i)}|x^{(i)}; \theta) \\ &= \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y^{(i)} - \theta^T x^{(i)})^2}{2\sigma^2}\right) \end{aligned}$$

We should choose θ to maximize $L(\theta)$ (maximum likelihood)

log Likelihood

Instead of maximizing $L(\theta)$, we can also maximize the log likelihood:

$$\begin{aligned} l(\theta) &= \log L(\theta) \\ &= \log \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y^{(i)}-\theta^T x^{(i)})^2}{2\sigma^2}\right) \\ &= \sum_{i=1}^{m} \log \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y^{(i)}-\theta^T x^{(i)})^2}{2\sigma^2}\right) \\ &= m \log \frac{1}{\sqrt{2\pi\sigma}} - \frac{1}{\sigma^2} \cdot \frac{1}{2} \sum_{i=1}^{m} (y^{(i)} - \theta^T x^{(i)})^2 \end{aligned}$$

hence, maximizing $l(\theta)$ gives the same answer as minimizing

$$\frac{1}{2} \sum_{i=1}^{m} (y^{(i)} - \theta^T x^{(i)})^2$$