

$$\nabla \phi \psi = \partial^i \phi(x^j) \psi(x^j) \hat{e}_i =$$

$$\nabla \phi \psi \rightarrow$$

$$= \phi \partial^i \psi(x^j) \hat{e}_i + \psi \partial^i \phi(x^j) \hat{e}_i =$$

$$\frac{\partial \phi \psi}{\partial x} \hat{i} + \frac{\partial \phi \psi}{\partial y} \hat{j} + \frac{\partial \phi \psi}{\partial z} \hat{k}$$

$$\left( \psi \frac{\partial \phi}{\partial x} + \phi \frac{\partial \psi}{\partial x} \right) \hat{i} + \left[ \frac{\psi \partial \phi}{\partial y} + \frac{\phi \partial \psi}{\partial y} \right] \hat{j} + \left[ \frac{\phi \partial \psi}{\partial z} + \frac{\psi \partial \phi}{\partial z} \right] \hat{k}$$

$$\psi \left[ \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k} \right] + \phi \left[ \frac{\partial \psi}{\partial x} \hat{i} + \frac{\partial \psi}{\partial y} \hat{j} + \frac{\partial \psi}{\partial z} \hat{k} \right]$$

$$\psi \nabla \phi + \phi \nabla \psi = \nabla \phi \psi$$

$$= \phi \Delta \psi + \psi \Delta \phi$$

$$\nabla \cdot (\nabla \times a)$$

$$(\nabla \times a) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_1 & a_2 & a_3 \end{vmatrix} = \left[ \frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z} \right] \hat{i} - \left[ \frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z} \right] \hat{j} + \dots$$

$$\dots \left[ \frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y} \right] \hat{k}$$

$$\nabla \cdot (\nabla \times a)$$

$$\frac{\partial}{\partial x} \left[ \frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z} \right] - \frac{\partial}{\partial y} \left[ \frac{\partial a_3}{\partial x} - \frac{\partial a_1}{\partial z} \right] + \frac{\partial}{\partial z} \left[ \frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y} \right]$$

$$\underbrace{\frac{\partial^2 a_3}{\partial y \partial x}}_{\text{purple}} - \underbrace{\frac{\partial^2 a_2}{\partial z \partial x}}_{\text{green}} - \underbrace{\frac{\partial^2 a_3}{\partial y \partial x}}_{\text{purple}} + \underbrace{\frac{\partial^2 a_1}{\partial y \partial z}}_{\text{red}} + \underbrace{\frac{\partial^2 a_2}{\partial z \partial x}}_{\text{green}} - \underbrace{\frac{\partial^2 a_1}{\partial z \partial y}}_{\text{red}} = 0$$