Lecture 18 : Black hole thermodynamics

A "classical" black hole paradox.

Two important black hole theorems:

- "No hair theorem":
  An equilibrium black hole is fully characterized by its Mass M, Angular momentum J and Charge Q.

- The total black hole area can never decrease.

Suggestion: $S_{BH} = A$

How does the area depend upon M, J, Q?

Simplest case $(J = 0,\ Q = 0)$:

$$A = 16\pi M^2$$
$$dA = 32\pi M \, dM$$
$$dM = \frac{1}{32\pi M} dA = \frac{\kappa}{8\pi} dA$$

General case: $dM = \frac{\kappa}{8\pi} dA + \Omega \, dJ + \Phi \, dQ$

Note similarity with first and second law of thermodynamics!

Problem: If a black hole has an entropy it should also have a temperature. Classically impossible!

Hawking radiation!

Bekenstein: $dS + dS_{BH} \geq 0$

- Bekenstein bound
- Holographic principle

Penrose diagram of evaporating black hole.

Information paradox

Black holes are strange objects, also from the perspective of thermodynamics. And we don't need quantum mechanics to realize that something strange is going on. Here is what we might call "the black hole entropy problem".

As any system, a black hole should be associated with an entropy. Otherwise we would be able to lower the entropy of the universe just by disposing high entropy systems into a black hole! But if a black hole has entropy, it should also have a temperature, by the definition of temperature $\frac{1}{T} = \frac{dS}{dE}$ (where the energy should be the mass of the black hole). But if it has a temperature we expect it to be able to emit energy if the surroundings have lower temperature. In particular, we expect it to emit black body radiation, as any hot object. But how could a black hole emit energy — that seems contrary to the very definition of it.

The black hole entropy problem:

$S_{tot} \geq 0 \Rightarrow$ b.h. entropy $S \Rightarrow$ b.h. temp. $T \Rightarrow$

$\Rightarrow$ b.h. black body radiation

(or $T_{BH} = 0 \Rightarrow S_{BH} = \infty$) strange from a QM point of view, where the number of states for finite energy tends to be finite.

In the early 70's this problem was sharpend by two important observations about black holes:

After it has settled down, and to an external obs.

The no-hair "theorem": Any black hole is fully characterized by just three parameters:

- Mass M
- Angular momentum J
- Charge Q

Hawking's area theorem: The total black hole area can never decrease: $dA \geq 0$

Since the black hole should be characterized only by M, J and Q, its area A must depend upon these. In particular we may ask how the area changes as these parameters are changed. It is possible to show that

"surface gravity" (itself a function of M, J and Q)

$$dM = \frac{\kappa}{8\pi} dA + \Omega\, dJ + \Phi\, dQ$$

angular velocity

electric potential

(This is written in a way to make it look similar to the first law of thermodynamic!)

To better understand what this means, let us only consider the simplest case, where $J=0$ and $Q=0$.

$$J=0,\ Q=0: \quad A = 4\pi r_s^2 = 16\pi M^2$$

$$\Rightarrow \quad dA = 32\pi M\, dM$$

$$dM = \frac{\kappa}{8\pi} dA \quad \text{where} \quad \kappa = \frac{1}{4M}$$

which you actually have shown (in Problem) is the surface gravity for Schwarzschild black hole. $(\xi^\alpha \nabla_\alpha \xi_\beta = \kappa\, \xi_\beta)$

Intuitively, κ is the acc. needed to keep an object at the horizon, as seen from infinity.

Now if this result is combined with Hawking's area theorem it becomes very tempting to identify the black hole area A with its entropy:

$$S_{BH} \sim A$$

If we do that, then we also have to make the identification

$$\kappa \sim T$$

But then we have the problem as noted above: Black holes do not radiate! Moreover, it is strange to say that the black hole entropy is the area, since from thermodynamics we know that, even if the total entropy cannot decrease, the entropy of a non-isolated system can decrease (entropy can flow from one system to another). But the area of a single black hole can, according to Hawking's theorem, never decrease.

These problems where solved in the late 70's, when Hawking showed that, when taking quantum field theory into account, black holes actually do radiate as perfect black bodies with a temperature proportional to the surface gravity.

Thereby the classical entropy problem is solved, and the proportionality factors between area – entropy and surface gravity – temperature, are fixed:

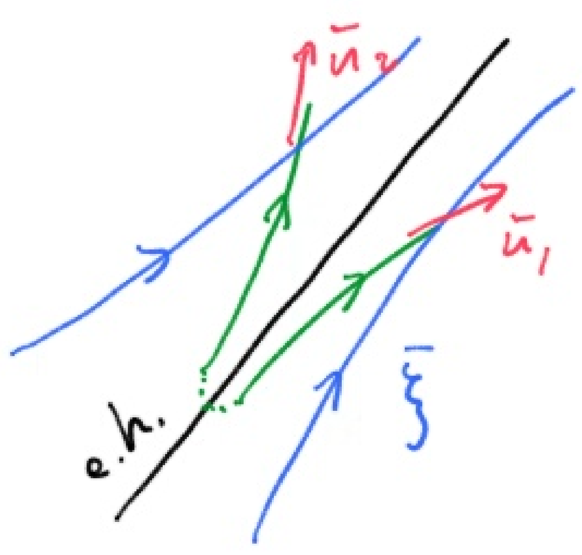
$$S_{BH} = \frac{A}{4\hbar G} \qquad\qquad k\,T_H = \frac{\hbar \kappa}{2\pi}$$

Planck area $\frac{\hbar G}{c^3}$

Hence, the entropy of a black hole is $\frac{1}{4}$ of its area measured in units of Planck area.

Notice the (unusual!) occurence of both $\hbar$ and G, indicating that this is a truly quantum gravitational result!

Intuitive picture: virtual particle pairs



$$0 = \underbrace{-\bar{u}_1 \cdot \bar{\xi}(x_1)}_{\text{energy at infinity}} \; \underbrace{- \; \bar{u}_2 \cdot \bar{\xi}(x_2)}_{\text{can be } < 0 \text{ since } \bar{\xi}(x_2) \text{ spacelike}}$$

When a black hole Hawking radiates it must loose energy, and hence its area decreases. Hence, Hawking's classical area theorem does not hold. But we now can replace it by something that makes more sense thermodynamically:

$$\underbrace{dS}_{\text{usual entropy}} + \underbrace{dS_{BH}}_{\text{black hole entropy} \sim \text{area}} \geqslant 0$$

– generalized second law

So this solves the classical entropy paradox. But it immediately leads to, what is considered as an even more serious problem. Before coming to that, let's discuss an interesting consequence of our new second law, pointed out by Bekenstein.

The Bekenstein bound

First, note a peculiar thing about this black hole entropy. From ordinary thermodynamics, we know that entropy is an extensive variable: The entropy for two systems are just the sum of the entropies for each system. This is usually the same as the statement that entropy scales with the volume.

$\boxed{?}$ Remember the reason for this:

$$S = \ln \Omega$$

total number of states

For two independent systems

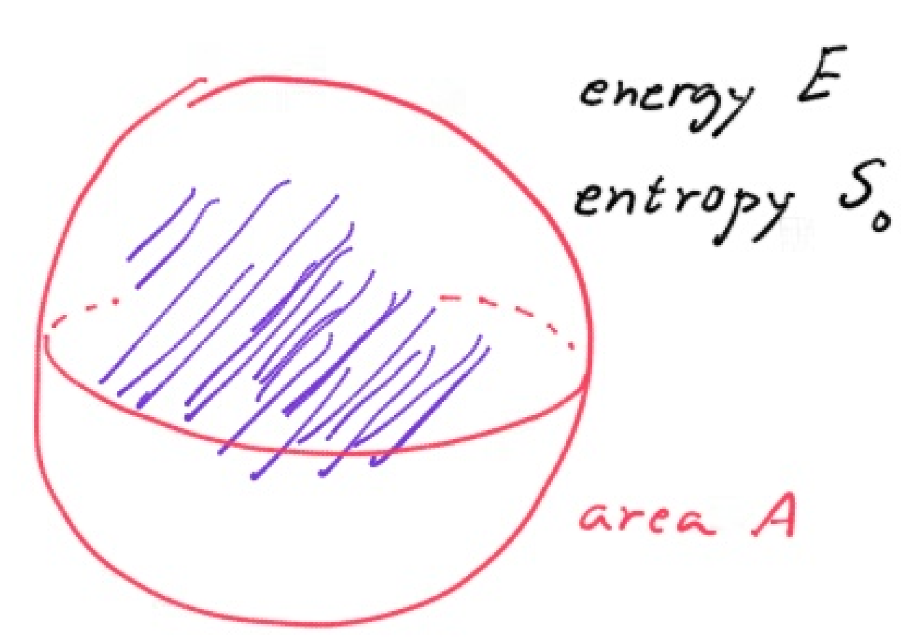
$$\Omega = \Omega_1 \cdot \Omega_2 \quad \Rightarrow \quad S = S_1 + S_2$$

So the fact that entropy scales with volume is closely related to some notion of locality: the number of states in one system is independent of other systems.

Now, what is strange is that this volume-scaling does not hold for the black hole entropy — it scales as the area!

Bekenstein realized that, provided we accept the generalized second law, this area scaling must be much more general. Here is the argument.

Consider an isolated system of energy E and entropy S_0.



Also, consider a sphere that encloses the system, and assume that this sphere has area A.

Let M_A = mass of black hole with area A

Then $E \leq M_A$, otherwise the system would be a black hole with $E = M_A$ and $S_0 = S_{BH}$.

Now, suppose we were to add energy $M_A - E$ to the system (keeping the sphere fixed). This would make the system into a black hole.

Because of the second law we then must have:

$$S_{BH} \geq S_0 + S'$$

entropy of resulting b.h. — entropy of added energy

But this implies that

$$\Longrightarrow \quad S_0 \leq S_{BH} = \frac{A}{4\hbar G}$$

Thus we reach the conclusion

$$\boxed{\text{The maximal entropy contained in a region bounded by area } A \text{ is } \quad S_{max} = \frac{A}{4\hbar G}}$$

— the Bekenstein bound!

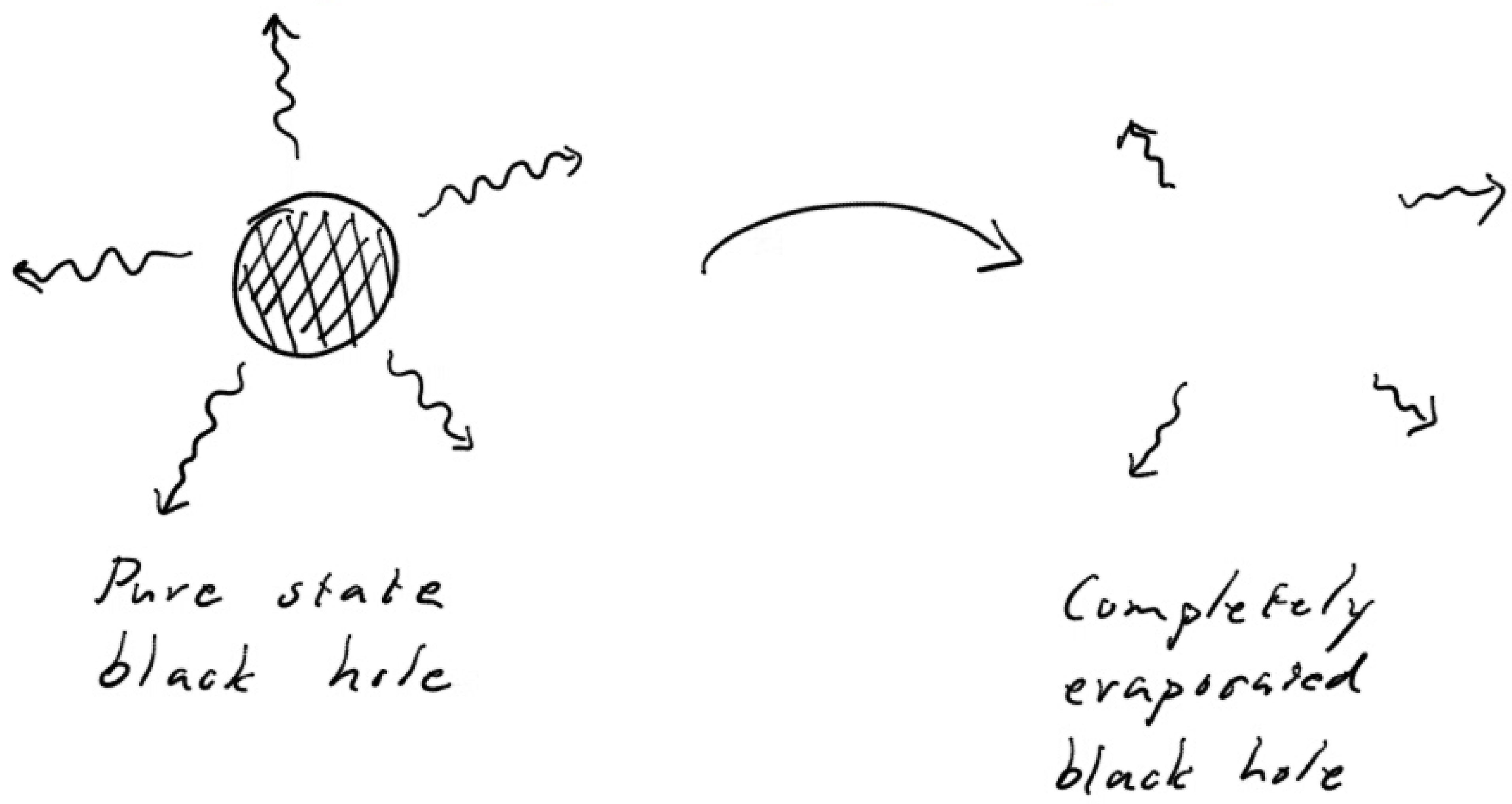
Hence, the <u>maximal</u> entropy also has to scale as area, not as volume!

This is one of the motivations behind the so called "holographic principle": In quantum gravity, a system bounded by area A should be fully described by no more than $\frac{A}{4\hbar G}$ degrees of freedom, that is, of the order of one degree of freedom per Planck area.

The information paradox

A black hole that Hawking-radiates will loose energy and therefore get smaller. Note that smaller M means higher temperature ($K \sim \frac{1}{M}$!) so then it will radiate even more, and loose energy even faster. There seems to be nothing to stop this process — so eventually the black hole will have evaporated away completely!

Now, from a QM point of view this is very strange. Suppose that we form a black hole from a pure state. After a long but finite time it will have evaporated completely into thermal radiation. But since QM evolution is unitary the state should still be pure.

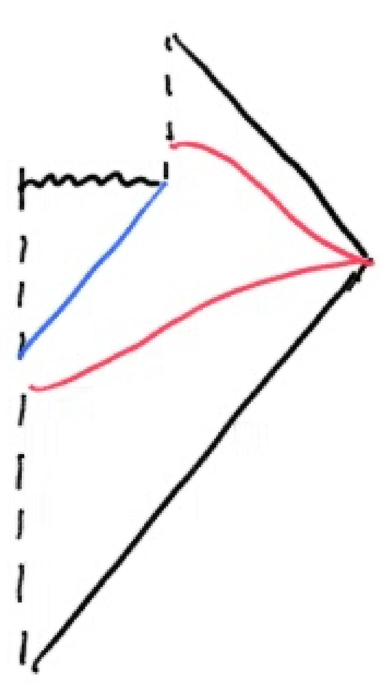


In other words, no information should be lost in the process. All information that ever went into the black hole should somehow still be there, but now in the radiation.

But how did the information get out from the black hole?

It turns out to be difficult to come up with a way for the information to get out. There are, as I see it, two difficulties to be overcome. First, if the black hole is spherically symmetric, then so is its evaporation process and it should be possible to draw a Penrose diagram.

This is the diagram that seems most reasonable:

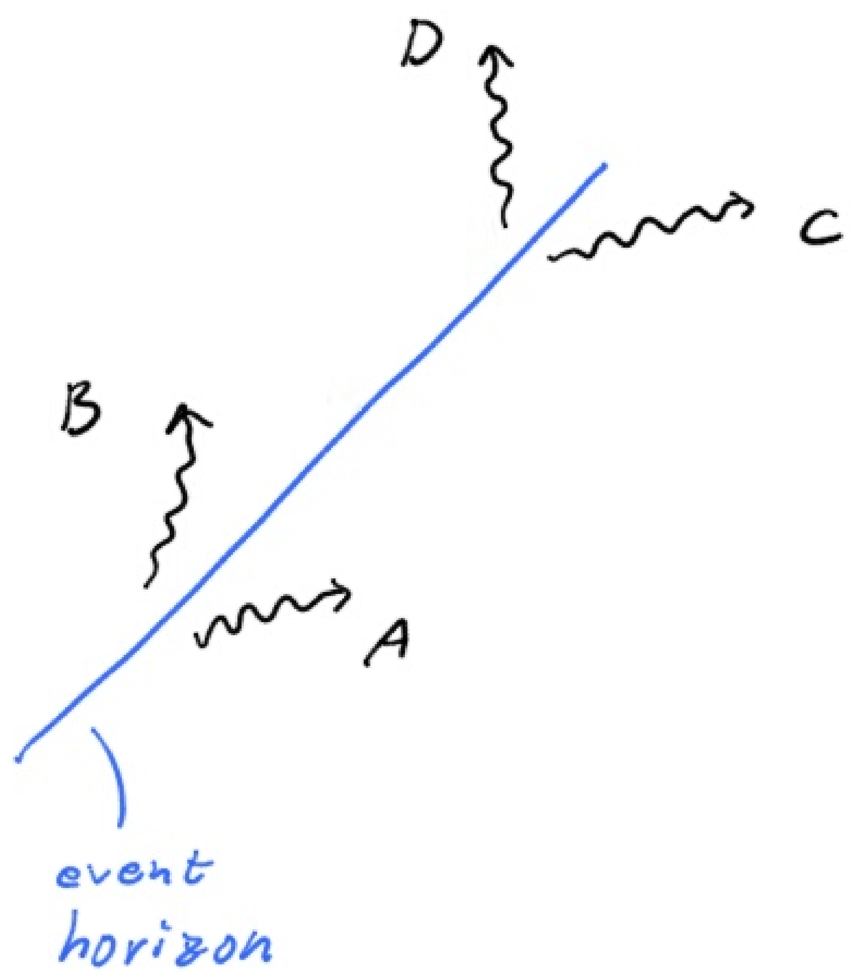


It is difficult to see how the information could get out, so that both spacelike hypersurfaces, before and after the black hole collapse and evaporation, could contain the same information.

But without a full theory of quantum gravity we cannot be sure that this is the right way to draw the diagram.

But there is still a problem to get the information out.

Consider two parts of the Hawking radiation, one early and one late:



To obtain Hawkings result, A has to be maximally entangled with B, and C with D.

But if the full Hawking radiation should have any chance of being a pure state, then the early radiation A must be "purified" by the late radiation C.

But if A is purified by B (which it is since it is maximally entangled with B) it cannot also be purified by C.
Hence, A and B cannot be maximally entangled. But this means that the quantum field is <u>not</u> in its vacuum state near the horizon. It turns out that it, in that case, then must be in a highly exited state near the horizon, creating a so called firewall there!

So we might get the information out, but only at the cost of a firewall. That would be very strange from a classical general relativistic point of view.