General relativity: Bonustest II

May 10, 2017

Give your answers on this sheet. Unless stated in the question, you don't have to provide any reasoning or justifications for your answers. You can answer in english or in swedish.

Max: 14 p. At least 7 p gives 1 point to the exam. At least 11 p gives 2 points to the exam.

1. $A^{\alpha\beta}{}_{\gamma}$ is a rank 3 tensor. Insert indices below to obtain the correct transformation law to new coordinates α', β', γ'. (1 p)

$$A^{\alpha'\beta'}{}_{\gamma'} = \frac{\partial x}{\partial x}\frac{\partial x}{\partial x}\frac{\partial x}{\partial x} A^{\alpha\beta}{}_{\gamma}$$

2. The covariant derivative of a vector ***a***, expressed both with index upstairs and index downstairs, is

$$\nabla_\alpha a^\beta = \partial_\alpha a^\beta + \Gamma^\beta{}_{\alpha\gamma} a^\gamma$$

$$\nabla_\alpha a_\beta = \partial_\alpha a_\beta - \Gamma^\gamma{}_{\alpha\beta} a_\gamma$$

What is the corresponding expression for the covariant derivative of the rank 3 tensor $A^{\alpha\beta}{}_{\gamma}$! (2 p)

$$\nabla_\alpha \; A^{\beta\gamma}{}_{\delta} =$$

3. What kind of object is each of the following quantities? Mark the correct alternative and fill in the appropriate rank! (All correct – 4 p; 6 correct – 3 p; 5 or 4 correct – 2 p. 3 or 2 correct – 1 p.)

(a) number density

☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

(b) energy density

☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

(c) 4-velocity

☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

(d) partial derivative of a vector field: $\partial_\alpha a^\beta$

☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

(e) partial derivative of a scalar field: $\partial_\alpha \Phi$

☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

(f) covariant derivative of a vector field: $\nabla_\alpha a^\beta$

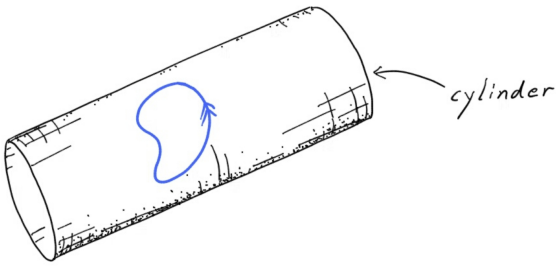
☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

(g) Christoffel symbol $\Gamma^\alpha{}_{\beta\gamma}$

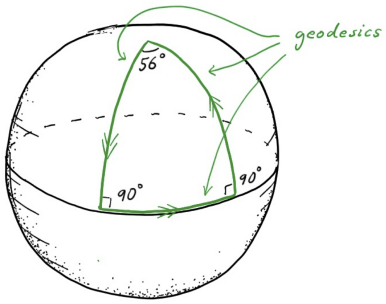
☐ tensor rank __ ☐ comp. of tensor rank __ ☐ neither

4. What are the results of parallel transporting a vector around the indicated loops on the following surfaces? Answer with the rotation angle (if any) as well as with the sense of rotation (clock-wise or anti-clockwise). (3 p)

(a)



(b)



5. The following two expressions ((a) and (b)) involving the Riemann tensor show its geometrical significance in two different ways. Explain the geometrical meaning of each of the expressions in a few words. Also, explain the meaning of each object involved. (A simple drawing in each case is recommended.) (4 p)

(a) $\nabla_u \nabla_u \chi^\alpha = -R^\alpha{}_{\beta\gamma\delta} u^\beta \chi^\gamma u^\delta$

(b) $\delta A^\mu = -R^\mu{}_{\nu\alpha\beta} \delta S^{\alpha\beta} A^\nu$