Lecture 14

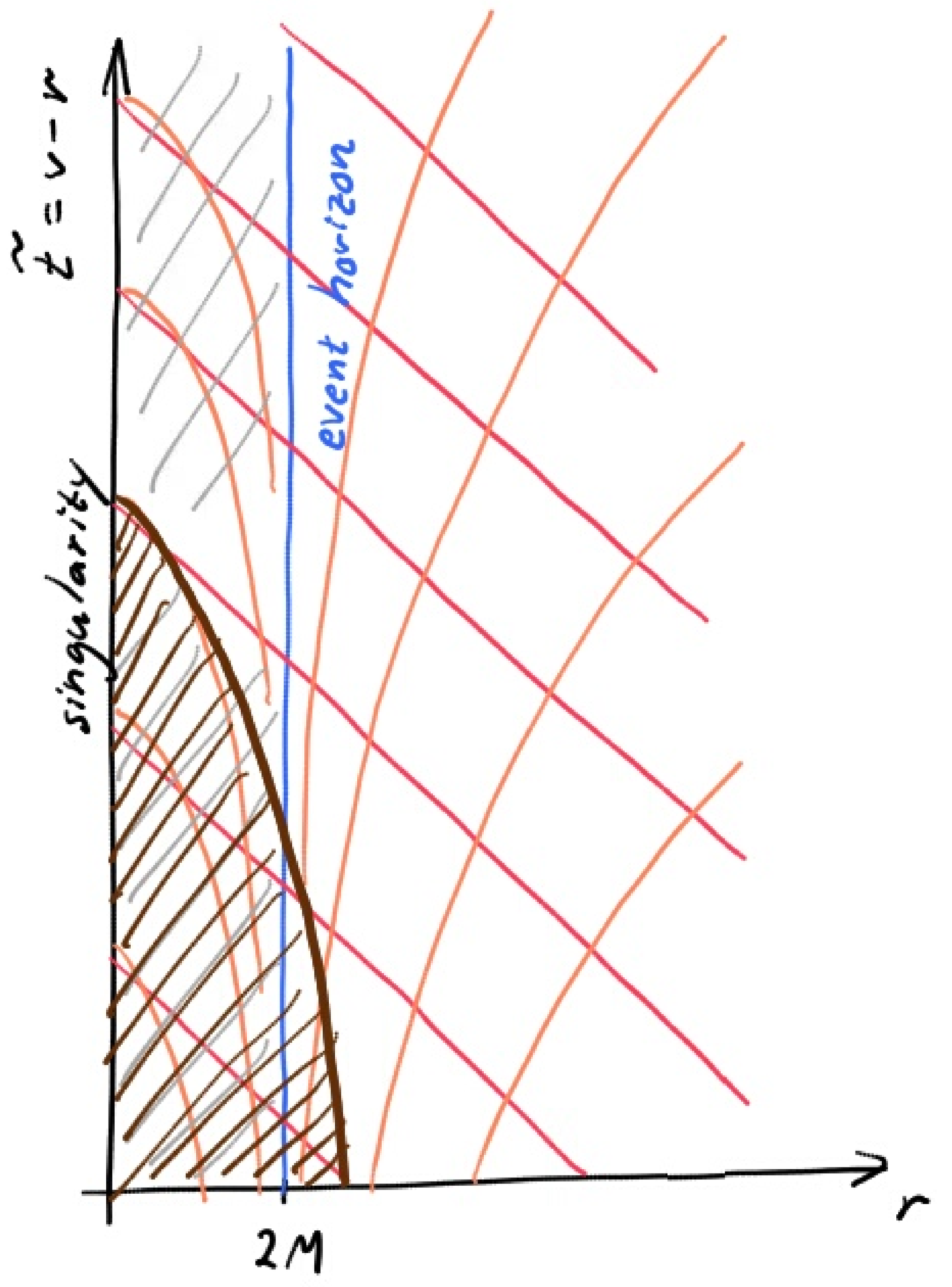
Collapse to a black hole

- horizon area
  - $16\pi M^2$ — stays constant after collapse!
  - More matter falls in:
    1) the horizon area increases
    2) the horizon depends on the future

Kruskal coordinates

- relevant regions (Schwarzschild / Eddington-Finkelstein)
- without collapsing matter:

  the Kruskal extension



Last time we drew an Eddington-Finkelstein diagram of a (spherically symmetric) collapsing star.

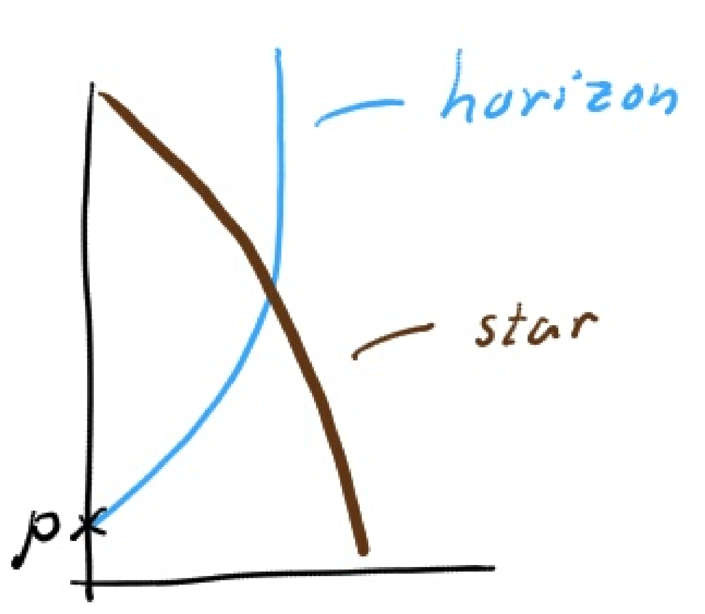
Let us now look more closely on the event horizon in this diagram.

After the collapse the event horizon is at the constant value $r = 2M$:

Event horizon: $r_s = 2M$ — Schwarzschild radius

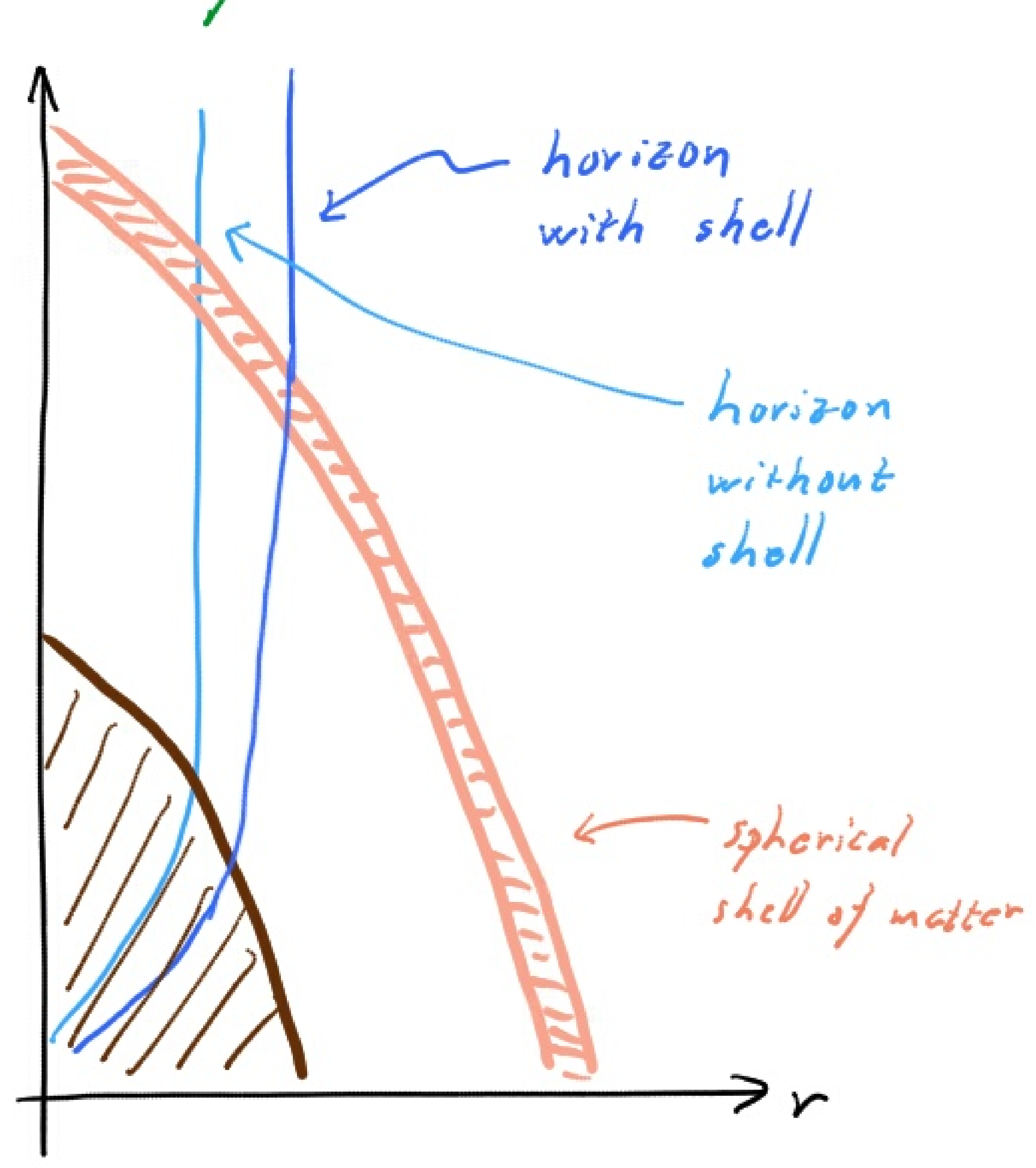
$$\text{area } A = 4\pi r_s^2 = 16\pi M^2$$

Before the collapse there was no event horizon, so during the collapse the event horizon must grow. In fact, if we trace the light rays comprising the horizon backwards in time, they must converge at some point p in the center of the star shortly before the collapse.



This shows how the horizon "grows up", and, in this case of perfect spherical symmetry, is the forward light cone of the point p.

Now, what if more matter falls into the black hole later? To keep the discussion simple let us preserve the spherical symmetry by imagining an infalling spherical shell of matter which falls in some time after the black hole has formed.



Note that the shell changes the position of the horizon also before the shell has fallen in.

Thus, the exact position of the event horizon depends on what happens in the future!

The Kruskal extension

To see what this is consider first a new set of coordinates for the Schwarzschild geometry called

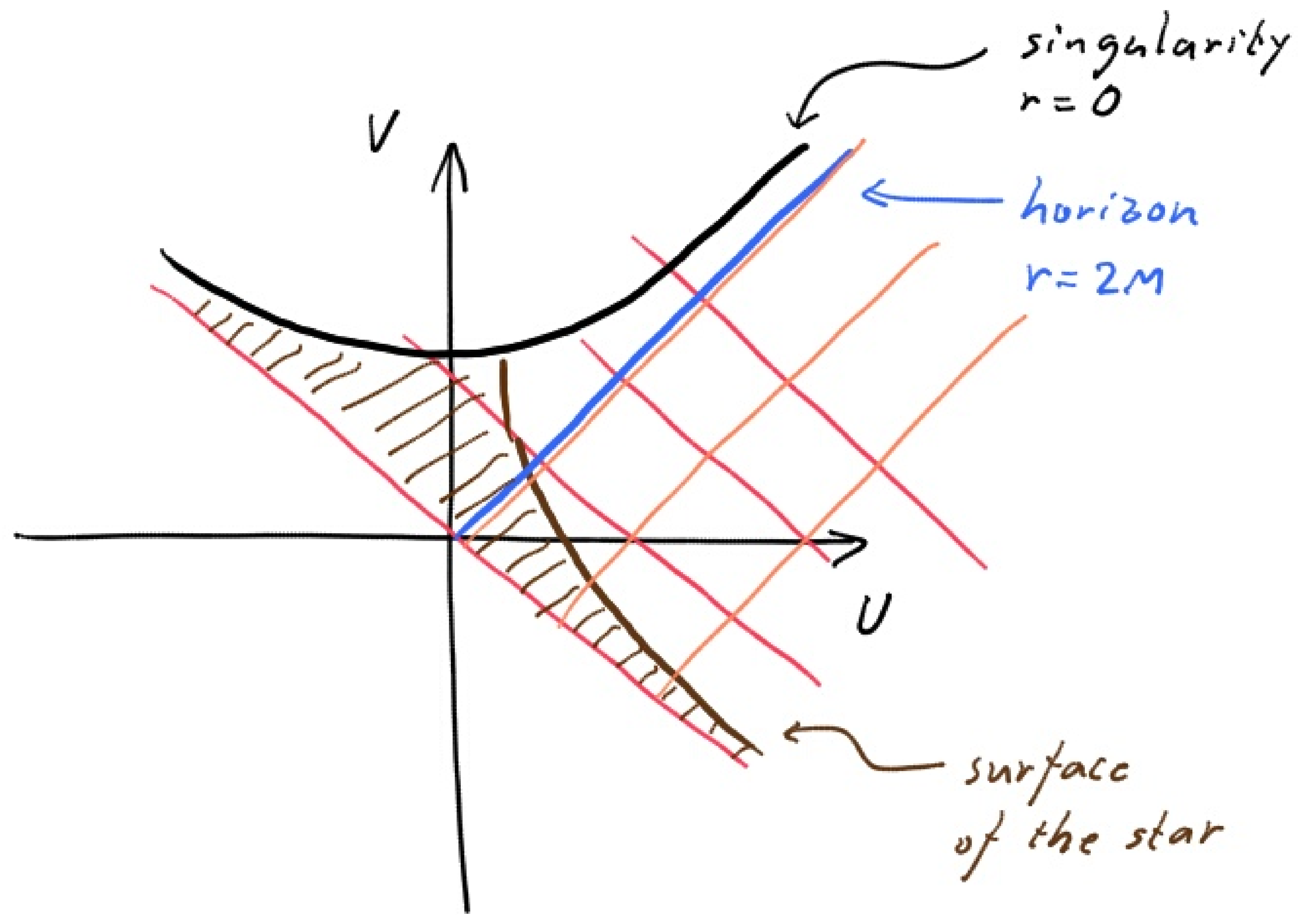
Kruskal coordinates : $U = U(r, t)$
$V = V(r, t)$

(See Hartle for the exact transformation, and the resulting line element.)

What is the point with these coordinates?

Remember that the E.-F. coord. are such that all ingoing lightrays are "45-degree lines". Nothing can stop us from requiring that also the outgoing light should be drawn as 45-degree lines – this is what the Kruskal coordinates does!

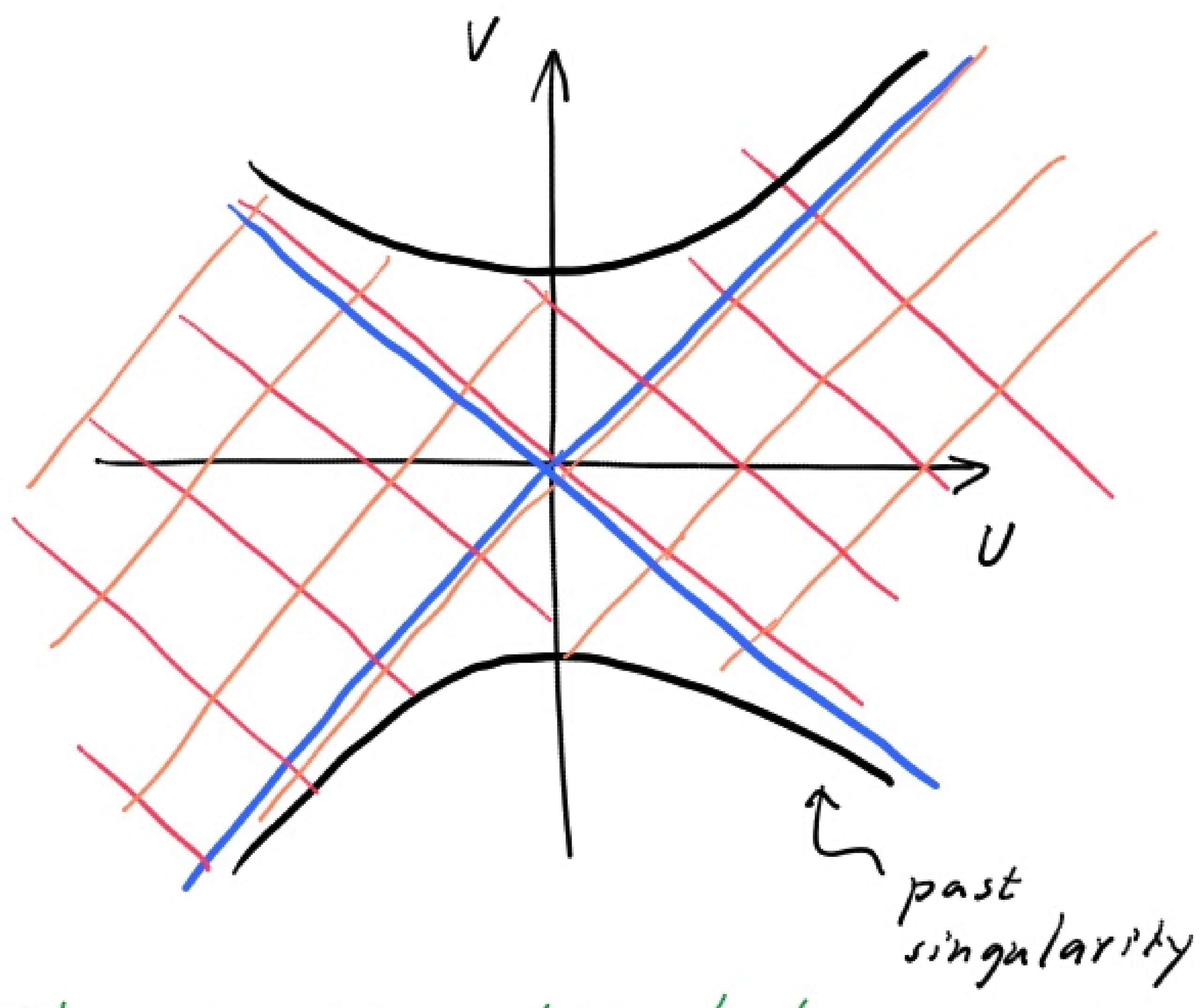
This is the result:



In this set of coord. it is obvious that the singularity is a spacelike line.

Note that this diagram (as the corresponding E.-F.-diagram) has no meaning below the surface of the star.

However, Einstein's field equation does not require that there should be any star in the middle: We could consider the Schwarzsch. solution as part of a global vacuum solution, without any matter at all! But if we take away the matter we find that we must extend the spacetime to the $V < -U$ part of the diagram. Otherwise, timelike geodesics followed backwards in time would just end in finite proper time without hitting any singularity.



This is the Kruskal extension of the Schwarzschild solution!

Now, remember that each point in this diagram actually represents a sphere, since we have suppressed the angular coordinates. The line element of that sphere is $r^2 d\Omega^2$.

Therefore the origin $(U=0, V=0)$ itself is a sphere with an area corresponding to the Schwarzschild radius.

It turns out that the hypersurface $V=0$ (a 3 dim. space) has the geometry of a wormhole. It connects the two exterior regions with a throat. The smallest area of that throat is at the origin and therefore has an area $16\pi M^2$.

This is the Einstein-Rosen bridge!

Note that it is not a traversible wormhole!

Therefore, this does <u>not</u> show that black holes are gates to other universes! First, the Kruskal extension is <u>not</u> the correct geometry of a realistic black hole. Second, even if it somewhere existed a true Kruskal black hole, it would not be possible to traverse its wormhole to the other side!