Simulating the Ising model using the Metropolis algorithm

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1 Abstract

2 Introduction

3 Methods

$$\nabla^2 u(x,t) = \frac{\partial u(x,t)}{\partial t} \tag{1}$$

$$u_x x = u_t \tag{2}$$

$L = 1$ Before approximating the derivatives, the PDE is discretized. The spacial domain, $x \in [0,1]$, is discretized over n grid points, such that $x_i = i\Delta x$, where $\Delta x = \frac{1}{n+1}$

of the function $u(x,t)$, using ρ, where ρ is a dimensionless variable, so that $u = u(\rho)$ and $\rho \in [0,1] \implies \rho = \frac{x}{L}$. Application of this dimensionless variable, together with grouping of other variables yields the following:
$\frac{d^2u(\rho)}{d\rho^2} = -\frac{FL^2}{R}u(\rho) = -\lambda u(\rho)$.

In order to numerically solve the problem, the range of ρ is discretized over N gridpoints, with a step length $h = \frac{\rho_{max}-\rho_{min}}{N}$, so that $\rho_i = ih$. The dizcrete approximation of the function u is defined as $u(x_i) = u_i$, with boundary conditions (B.C) $u_0 = u_{n+1} = 0$. The second derivative of u is approximated by using (**??**).

4 Algorithms