Simulating the Ising model using the Metropolis algorithm

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1 Abstract

2 Introduction

For the purpose of approximating the derivatives of u, three methods are applied: explicit forward Euler, implicit Backward Euler, and the implicit Crank-Nicolson with a time-centered scheme.

3 Methods

$$\nabla^2 u(x,t) = \frac{\partial u(x,t)}{\partial t} \tag{1}$$

$$u_x x = u_t \tag{2}$$

Before approximating the derivatives, the problem discretized. The spacial domain , $x \in [0, L]$, $L = 1$, is discretized over $n+1$ grid points, such that $x_i = i\Delta x$, $i = 0, 1, ..., n, n+1$, where $\Delta x = \frac{1}{n+1}$. Time, $t = [0, T]$, is is expressed as $t_j = j\Delta t$, $j \geq 0$, where $\Delta t = \frac{1}{T}$. The dizcrete approximation of the function u is then defined as $u(x_i, t_j) = u_{i,j}$, with boundary conditions (B.C) $u_{0,j} = 0$ and $u_{n+1,j} = 1$.

The solution to the problem is assumed to be u, and in order to dimdum, a change of variables from u to v is applied, where v is discretized similarly to x: $v(x,t) = v_{i,j}$. $u_{i,j} = v_{i,j} + x \implies v_{i,j} = u_{i,j} - x$. This changes the B.C to

$$v_{0,j} = v_{1,j} = 0$$

Approximating derivatives: Explicit forward Euler:

$$u_t \approx \frac{u(x_i, t_j + \Delta t) - u(x_i, t_j)}{\Delta t} \tag{3}$$

$$u_x x \approx \frac{u(x_i + \Delta x, t_j) - 2u(x, t) + u(x - \delta x, t)}{2\delta x} \tag{4}$$

$$\tag{5}$$

Which, expressed through discretized variables gives

$$u_t \approx \frac{u(x + \Delta x) - 2u(x, t) + u(x - \delta x, t)}{2\delta x} \tag{6}$$

4 Algorithms