FYS3120 Classical Mechanics and Electrodynamics

Problem set 2

January 30, 2019

Problem 1

1.a)

$\frac{\partial L}{\partial t}$ is the expression for the explicit time dependence of the Lagrangian with respect to time - where all other variables than time is fixed. $\frac{dL}{dt}$ is, in contrast, the total (explicit and implicit) dependence of the Lagrangian on time, which includes the dependence of all variables in the Lagrangian with respect to time - for example how a spatial coordinate (in the Lagrangian) varies with respect to time.

1.b)

Problem 2

2.a)

$q = \{q_i\}_{i=1}^{i=2}, q_1 = \theta, q_2 = \phi$, where θ is the angle between the uppermost rod (rod 1) and the vertical plane, and ϕ is the angle between the other rod (rod 2) and the horizontal plane.

Taking the moment of inertia of rod 1 about it's fixed end, $I_1 = ml^2/3$, the speed of the center of mass of rod 2, v_2, and the moment of inertia about the center of mass of rod 2, $I_{2,CM}$: $K = \frac{1}{2}I_1\dot{\theta}^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I_{2,CM}\dot{\phi}^2 = \frac{1}{6}ml^2\dot{\theta}^2 + \frac{3}{6}ml^2\dot{\theta}^2 + \frac{1}{24}ml^2\dot{\phi}^2 = \frac{2}{3}ml^2\dot{\theta}^2 + \frac{1}{24}ml^2\dot{\phi}^2$.

Taking the distance from the fixed surface to the center of mass of each rod, $y = -l\cos\theta$: $V = mgy_1 + mgy_2 = -mg\frac{l}{2}cos\theta - mglcos\theta = -\frac{3}{2}mglcos\theta$. Thus: $L = K - V = \frac{2}{3}ml^2\dot{\theta}^2 + \frac{1}{24}ml^2\dot{\phi}^2 + \frac{3}{2}mglcos\theta$

2.b)

Lagrange's equations for the system:

$$\frac{dL}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} = 0 \implies$$
$$\frac{dL}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} = \frac{4}{3}ml^2\ddot{\theta} + \frac{3}{2}mglsin\theta = 0$$
$$\frac{dL}{dt}(\frac{\partial L}{\partial \dot{\phi}}) - \frac{\partial L}{\partial \phi} = \frac{1}{12}ml^2\ddot{\phi} = 0$$

For small oscillations about equilibrium position ($\theta = 0$), $\sin\theta \approx \theta$, so:

$$\frac{4}{3}ml^2\ddot{\theta} + \frac{3}{2}mglsin\theta \approx \frac{4}{3}ml^2\ddot{\theta} + \frac{3}{2}mgl\theta = 0 \implies$$
$$\ddot{\theta}(\theta) = -\frac{9}{8}\frac{g}{l}\theta \implies \dot{\theta} = -\frac{9}{8}\frac{g}{l}\theta t + c, c = \theta_0 = 0 \to \dot{\theta} = -\frac{9}{8}\frac{g}{l}\theta t$$

Problem 3

3.a)