Problem set 1 - FYS3120

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0.1 Problem 2

Consider first a an Atwood machine with two parts; m_1 and m_2, moving vertically on a massless pulley, with a rope of fixed length, l, connecting the masses. Each mass has one degree of freedom (vertical movement along the y-axis), while the system set up imposes one constraint equation: $y_1 + y_2 = l$. Thus, a two body Atwood machine has $d = 1$ degree of freedom.

A compound Atwood machine of three masses, such as the one in fig.2 in the assignment paper, may be thought of as two separate two body Atwood machines, similar to the one described above. One Atwood machine with two parts; the part with mass m_1, and the part composed of the pulley to which m_2 and m_3 are connected, pulley 2, with mass $m_2 + m_3$. The other Atwood machine is composed of the parts with m_2 and m_3. Each of the two Atwood machines has one degree of freedom, for a total of $d = 2$ degrees of freedom.

$d = 2 \implies q = 2$ general coordinates: the vertical distances Y_1 and Y_2, where Y_1 is y coordinate of pulley 2, and Y_2 is the distance from pulley 2 to the mass m_2. So;

$$\begin{aligned} y_1 &= Y_1 - l_1 \\ y_3 &= Y_2 - l_2 \\ y_2 &= Y_2 \end{aligned}$$

Thus

$$T = \frac{1}{2}m_1(\dot{Y_1})^2 + \frac{1}{2}m_2(\dot{Y_2})^2 + \frac{1}{2}m_3(\dot{Y_2})^2 = \frac{1}{2}m_1(\dot{Y_1})^2 + \frac{1}{2}(m_2 + m_3)(\dot{Y_2})^2$$
$$V = m_1 g(Y_1 - l_1) + m_2 g(Y_2 - l_2) + m_3 g Y_2$$

Such that $L = T - V = \frac{1}{2}m_1(\dot{Y_1})^2 + \frac{1}{2}m_2(\dot{Y_2})^2 + \frac{1}{2}m_3(\dot{Y_2})^2 = \frac{1}{2}m_1(\dot{Y_1})^2 + \frac{1}{2}(m_2 + m_3)(\dot{Y_2})^2 - m_1 g(Y_1 - l_1) - m_2 g(Y_2 - l_2) - m_3 g Y_2$