FYS3120 Classical Mechanics and Electrodynamics

Problem set 6

March 4, 2019

Problem 1 This problem is an exercise in using space-time diagrams (Minkowski diagrams) when studying how elementary relativistic effects are represented in different inertial reference frames.

A railway carriage is moving in a straight line with constant velocity v relative to the earth. The earth is considered as an inertial reference frame S, and in this reference frame the moving carriage has the length L. The situation is shown in Fig. 4, where A and B indicate points on the rear wall and front wall of the carriage, respectively. C is a point in the middle of the carriage.

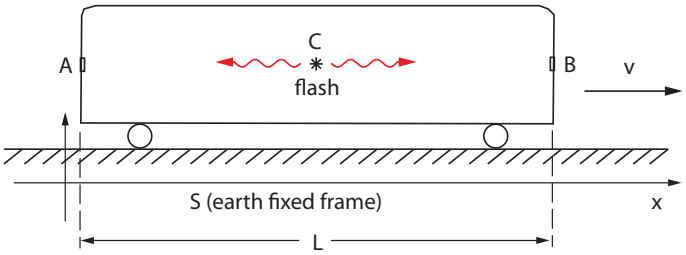

Figure 1: Diagram of the railway carriage.

a) In Fig. 2 we have drawn the world line (space-time trajectory) for the midpoint C in a two-dimensional Minkowski diagram of reference frame S. Draw the world lines for the points A and B in the same diagram and show that the angle α between these lines and the time axis is given by $\tan\alpha = v/c$. (Choose the origin of the coordinate system in S so that A has coordinate $x = 0$ at time $t = 0$.)

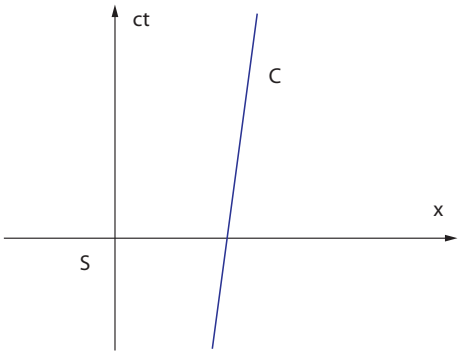

Figure 2: Minkowski diagram for reference frame S.

- $ct'\sin\alpha = vt$, $ct'\cos\alpha = ct$, thus $\tan\alpha = v/c$

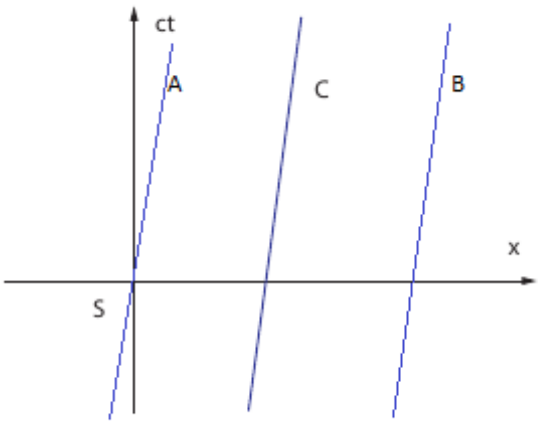

Figure 3: 1.a) Minkowski diagram for reference frame S, including world lines for A and B.

At a given time t_0 a flash tube is discharged at point C. We will call this event (space-time point) E_0. Some of the light will propagate backwards in the compartment and some will propagate forwards. Let E_1 and E_2 be the events where the light signals hit the rear wall and front wall, respectively. Let us assume that the light is reflected from A and B, and that the two reflected light signals meet at a space-time point E_3.

b) Draw the world lines of the light signals as well as the four events E_0, E_1, E_2 and E_3 in the Minkowski diagram of reference frame S.

c) We introduce the co-moving reference frame S' of the carriage. Explain why E_1 and E_2 are simultaneous in this reference frame and why E_0 and E_3 are at the same point in space in S'. Is this consistent with the drawing of point **b)**?

- E_1 and E_2 are simultaneous in RF S', as the distance from C to either A or B is the same, and for an observer at rest in S', the points A and B are not moving, thus the light signals will use the same time from C to either A or B. With the same logic, E_3 will occur at the same point as E_0 as the reflected light signals will arrive at the same time at C, observed from within S'. This is consistent with the drawing made in b), as that drawing shows the events as seen from RF S, not S' - although in S, E_3 does not occur at C.

d) Draw the straight line from from E_1 to E_2 in the Minkowski diagram of S and show that the angle between the x-axis and this line is α.

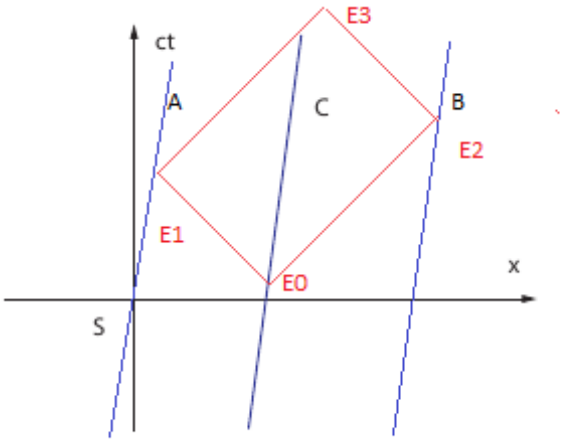

Figure 4: 1.b) Minkowski diagram for reference frame S, with events $E_i, i = 0, 1, 2, 3$

e) Show that if a signal should connect the two space-time points E_1 and E_2 it must have the velocity c^2/v, which is greater than c and thus forbidden.

Problem 2 A thin rigid rod has rest length L_0 (length measured in its rest frame). It moves relative to an inertial reference frame S', so that the midpoint A of the rod has the time dependent coordinates $x'_A = 0$, $y'_A = ut'$, $z'_A = 0$, with u as the velocity of the rod. In this reference frame the rod is at all times parallel to the x' axis.

a) Let B be one of the end points of the rod. What are the time dependent coordinates of this point measured in S'?

- $x'_B = L_0/2, y'_B = ut', z'_B = 0$

b) The inertial frame S' moves with velocity v along the x-axis relative to another inertial frame S. (The axes of the two frames are parallel.) Find the space coordinates (x, y, z) of the points A and B as functions of the time coordinate t in the reference frame S. (Remember that if the time coordinate t in S is fixed, the time coordinates t'_A and t'_B of the points A and B are not the same.)

c) Show that the the rod is not oriented along the x-axis in S, by calculating the ratio $\tan\phi = (y_B - y_A)/(x_B - x_A)$. What is the length of the rod measured in this frame?