*Topics in Asset Pricing*

# Lecture 2: Limits to Arbitrage

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# Explaining mispricing

Need to explain two things:

A. Why does mispricing appear?

- Institutional reasons
- Behavioral reasons

B. Why does mispricing persist? $\rightarrow$ "Limits to arbitrage"

1. Imperfect diversification
   - Fundamental risk
   - Noise trading risk
2. Funding constraints
   - Capital flows
   - Leverage constraints

# Fundamental risk

- Idea: arbitrageurs are imperfectly diversified $\Rightarrow$ limits their willingness to exploit mispricing, even if no systematic risk
- Risky asset in zero net supply
  - $t = 1$: demand shock $D \lesseqgtr 0$, (endogenous) price $P$
  - $t = 2$: payoff $\widetilde{V} \sim \mathcal{N}(V, \sigma^2)$
- Risk-free asset, normalize $r_f = 0$
- Mass $A$ of competitive arbitrageurs
  - Endowment $W_1$
  - Buy $X$ shares of risky asset
  - Consume $W_2 = W_1 + X(\widetilde{V} - P)$
  - CARA utility: $u(W_2) = -e^{-\gamma W_2}$
- Equilibrium

$$\boxed{P = V + \frac{\gamma}{A}\sigma^2 D}$$

# Index effect (Wurgler and Zhuravskaya, 2002)

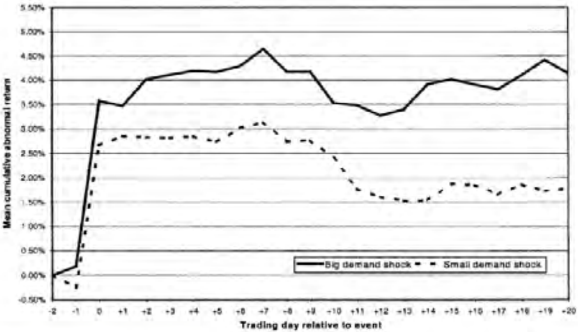



FIG. 1.—Mean cumulative abnormal returns for stocks added to the S&P 500, by size of index fund demand shock. The sample includes 191 stocks that were added to the S&P 500 between September 1976 and September 1989 and were not the subject of contemporaneously reported news. To control for the level of arbitrage risk, we exclude stocks in the extreme two quartiles of the arbitrage-risk distribution (measure $A_1$). We split the remaining 96 stocks into above-median and below-median demand shock groups. Cumulative abnormal returns are calculated by summing returns over the CRSP value-weighted market index. The standard error of the estimates in each series is approximately .40% at day 0, .90% at day +10, and 1.10% at day +20.

# Index effect (Wurgler and Zhuravskaya, 2002)

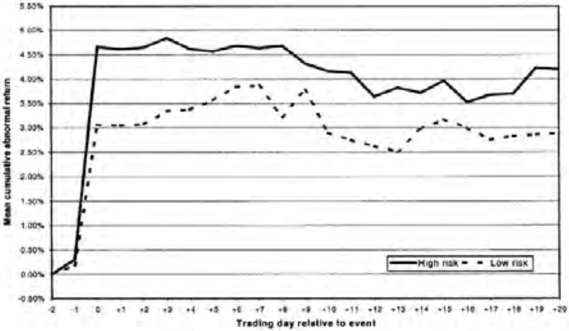



FIG. 2.—Mean cumulative abnormal returns for stocks added to the S&P 500, by level of arbitrage risk. The sample includes 191 stocks that were added to the S&P 500 between September 1976 and September 1989 and were not the subject of contemporaneously reported news. To control for the size of the index fund demand shock, we exclude stocks in the extreme two quartiles of the shock size distribution. We split the remaining 96 stocks into above-median and below-median arbitrage-risk groups, using measure $A_1$. Cumulative abnormal returns are calculated by summing returns over the CRSP value-weighted market index. The standard error of the estimates in each series is approximately .40% at day 0, .80% at day +10, and 1.00% at day +20.

# Noise trading risk (De Long, Shleifer, Summers, Waldmann, 1990)

- OLG model

  at $t$: endowment $w$ invested

  at $t+1$: consume, $u(c_{t+1}) = -e^{-\gamma c_{t+1}}$

- Risk-free asset

  dividend $r$ every period

  perfectly elastic supply at price $= 1$

- "Risky" asset

  dividend $r$ every period

  supply $= 1$, (endogenous) price $p_t$

- $\mu$ noise traders believe $E_t^n[p_{t+1}] = E_t[p_{t+1}] + \theta_t, \quad \theta_t \sim \mathcal{N}(\theta^*, \sigma_\theta^2)$
- $1-\mu$ arbitrageurs hold correct beliefs
- Equilibrium

$$\boxed{p_t = 1 + \frac{\mu(\theta_t - \theta^*)}{1+r} + \frac{\mu\theta^*}{r} - \frac{\gamma\mu^2\sigma_\theta^2}{r(1+r)^2}}$$

# Noise trading risk (DSSW 1990)

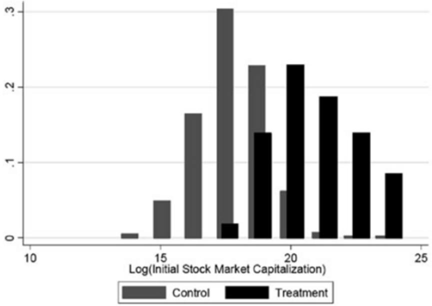
When arbs have short horizons:

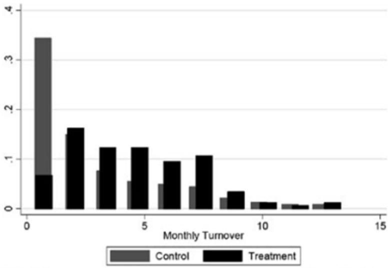
- "Noise traders create their own space"
- Excess volatility
- Return predictability (reversal)
- Noise traders don't necessarily die out

# Noise trading risk: Evidence (Foucault, Sraer, Thesmar, 2011)

**Table II**
**Summary Statistics**

| | Control | | | Treated | | |
|---|---|---|---|---|---|---|
| | Mean | *SD* | Obs. | Mean | *SD* | Obs. |
| Panel A: Full Sample Statistics | | | | | | |
| Market capitalization (bn € ) | 0.2 | 1.9 | 32,301 | 6.5 | 1.5 | 7,596 |
| Turnover (%) | 1.6 | 2.8 | 33,228 | 4.7 | 4.4 | 7,612 |
| Volatility1 (%) | 3.0 | 1.2 | 22,783 | 2.4 | 0.9 | 7,398 |
| Volatility2 (%) | 3.0 | 1.2 | 22,783 | 2.4 | 0.9 | 7,398 |
| Volatility3 (%) | 2.6 | 1.3 | 22,783 | 2.1 | 0.9 | 7,398 |
| Pimpact ($\times 10^6$) | 10.2 | 26.5 | 24,232 | 0.1 | 1.5 | 7,484 |
| Autocov ($\times 10^4$) | −0.5 | 2.8 | 21,947 | −0.2 | 1.9 | 7,377 |
| Bid–ask spread/midquote (%) | 7.0 | 7.8 | 7,793 | 1.8 | 1.0 | 2,731 |



# Noise trading risk: Evidence (Foucault, Sraer, Thesmar, 2011)

Table V
**The Impact of the Reform on Retail Trading Activity**

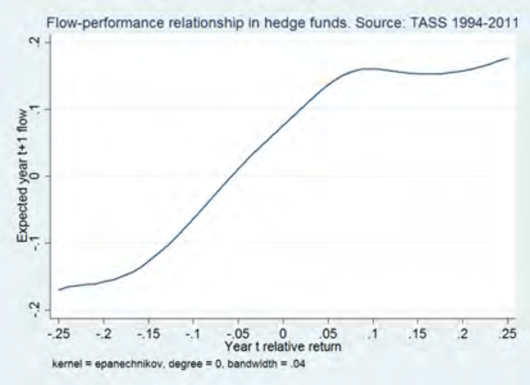
| | DD (1) | Quartile Matching (2) | Percentage Difference Matching (3) | Propensity Score Matching (4) |
|---|---|---|---|---|
| **Panel A: Dependent Variable: #Buys** | | | | |
| Treated×Post ($\beta_2$) | $\boxed{-0.020^{***}}$ | – | – | – |
| | [−4.15] | | | |
| Treated | 0.002 | – | – | – |
| | [0.37] | | | |
| Post ($\delta_1$) | $-0.009^{*}$ | $-0.026^{***}$ | $-0.026^{***}$ | $-0.019^{***}$ |
| | [−1.81] | [−6.35] | [−5.77] | [−4.14] |
| Constant | $0.046^{***}$ | $0.031^{***}$ | $0.030^{***}$ | $0.024^{***}$ |
| | [9.35] | [6.64] | [4.77] | [2.92] |
| Observations | 29,214 | 6,790 | 4,208 | 5,007 |
| $R^2$ | 0.01 | 0.04 | 0.02 | 0.01 |
| **Panel B: Dependent Variable: #Sells** | | | | |
| Treated×Post ($\beta_2$) | $\boxed{-0.022^{***}}$ | – | – | – |
| | [−4.77] | | | |
| Treated | 0.009 | – | – | – |
| | [1.61] | | | |
| Post ($\delta_1$) | −0.005 | $-0.024^{***}$ | $-0.029^{***}$ | $-0.023^{***}$ |
| | [−1.14] | [−6.26] | [−6.48] | [−6.16] |
| Constant | $0.038^{***}$ | $0.032^{***}$ | $0.033^{***}$ | $0.029^{***}$ |
| | [8.19] | [7.13] | [5.97] | [4.47] |
| Observations | 29,214 | 6,790 | 4,208 | 5,007 |
| $R^2$ | 0.01 | 0.05 | 0.04 | 0.02 |

# Noise trading risk: Evidence (Foucault, Sraer, Thesmar, 2011)

| | DD (1) | Quartile Matching (2) | Percentage Difference Matching (3) | Propensity Score Matching (4) |
|---|---|---|---|---|
| **Panel A: Dependent Variable: Volatility2 (Implication 1)** | | | | |
| Treated×Post ($\beta_2$) | $\boxed{-0.297^{***}}$ | – | – | – |
| | $\boxed{[-5.47]}$ | | | |
| Treated | $-0.472^{***}$ | – | – | – |
| | [−8.52] | | | |
| Post ($\delta_1$) | 0.200 | $\boxed{-0.194^{***}}$ | $\boxed{-0.172^{***}}$ | $\boxed{-0.274^{***}}$ |
| | [1.60] | $\boxed{[-2.97]}$ | $\boxed{[-2.71]}$ | $\boxed{[-3.25]}$ |
| Constant | $2.877^{***}$ | $-0.227^{***}$ | $-0.192^{***}$ | $-0.238^{***}$ |
| | [30.80] | [−4.41] | [−3.31] | [−3.52] |
| Observations | 30,181 | 7,398 | 4,552 | 5,652 |
| $R^2$ | 0.06 | 0.01 | 0.01 | 0.01 |
| **Panel B: Dependent Variable: Autocovariance of Returns (Implication 2)** | | | | |
| Treated×Post ($\beta_2$) | $\boxed{0.293^{***}}$ | – | – | – |
| | $\boxed{[3.24]}$ | | | |
| Treated | $0.109^{*}$ | – | – | – |
| | [1.74] | | | |
| Post ($\delta_1$) | $-0.484^{***}$ | $\boxed{0.611^{***}}$ | $\boxed{0.329^{**}}$ | $\boxed{0.437^{**}}$ |
| | [−5.19] | $\boxed{[4.06]}$ | $\boxed{[2.18]}$ | $\boxed{[2.26]}$ |
| Constant | $-0.231^{***}$ | −0.137 | $-0.172^{*}$ | −0.118 |
| | [−2.81] | [−1.27] | [−1.85] | [−1.06] |
| Observations | 29,325 | 7,378 | 4,512 | 5,578 |
| $R^2$ | 0.01 | 0.02 | 0.00 | 0.01 |

# Arbitrageurs horizon

- Endogenous short horizon because of flow-performance relation




- Can even amplify non-fundamental fluctuations (Shleifer and Vishny, 1997)

# Slow moving capital (Mitchell, Pedersen and Pulvino, 2007)

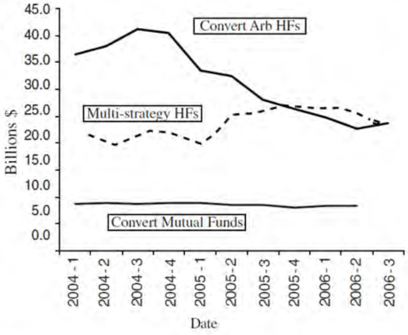



FIGURE 1. ADJUSTED HOLDINGS OF CONVERTIBLE BONDS IN BILLIONS OF DOLLARS

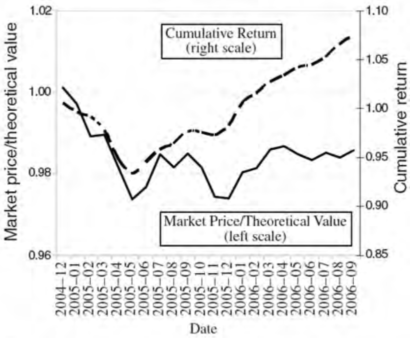
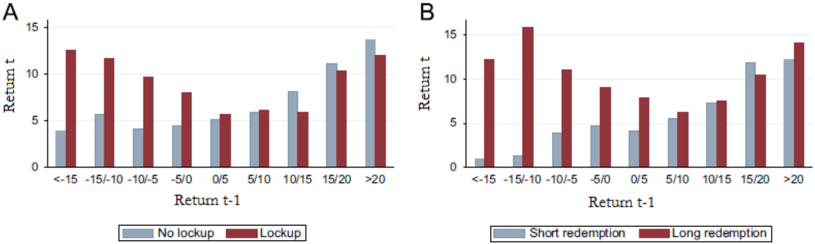



FIGURE 2. PRICE-TO-THEORETICAL-VALUE OF CONVERTIBLE BONDS, AND RETURN OF CONVERTIBLE BOND HEDGE FUNDS (2004/12-2006/09)

- Models: Vayanos and Woolley (2013), Moreira (2019)

# Arb capital structure (Hombert and Thesmar, 2014)

- Can arbs increase the duration of their liabilities?



# Leverage constraints

- Idea: arbitrageurs face funding constraints
- Model: equity issuance impossible, debt issuance constrained
  - Key papers: Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), Gârleanu and Pedersen (2011)
  - Today: simple version of these models
- Asset in zero net supply
  - $t = 1$: trade at price $P_1$, supply shock for $\frac{S+P_1-V}{P_1}$ shares ($0<S<V$)
  - $t = 2$: payoff $V$
- Risk-free asset, normalize $r_f = 0$
- Mass 1 of competitive arbitrageurs
  - Endowment $W_1$; position $X_1$ in asset; max $W_2 = W_1 + X_1(V - P_1)$
  - Leverage constraint: $mP_1|X_1| \leq W_1$ (margin or haircut $m \in [0, 1]$)
- Equilibrium: $\boxed{P_1 = V - S + \frac{W_1}{m}}$ if $W_1 < mS$, else $P_1 = V$
- What if $S < 0$?

# Amplification

- Arbs start at $t = 1$ with $X_0 > 0$ shares of asset:

$$W_1 = W_0 + X_0(P_1 - P_0)$$

- $\Rightarrow$ Amplification mechanism ("fire sales")

# LoOP violation & Contagion

- Extend to several assets $i = 1, ..., I$
  - $t = 1$: selling pressure $\frac{S_i + P_i - V}{P_i}$, price $P_i$
  - $t = 2$: payoff $V$
- Equilibrium

$$\boxed{P_i = \frac{V}{1 + \lambda m_i} \qquad \text{for all } i}$$

where $\lambda > 0$ such that $\sum_i m_i \left( S_i - \frac{\lambda m_i}{1+\lambda m_i} V \right) = W_1$

- Violation of Law of One Price
- Contagion

# Evidence: Gârleanu and Pedersen (2011)

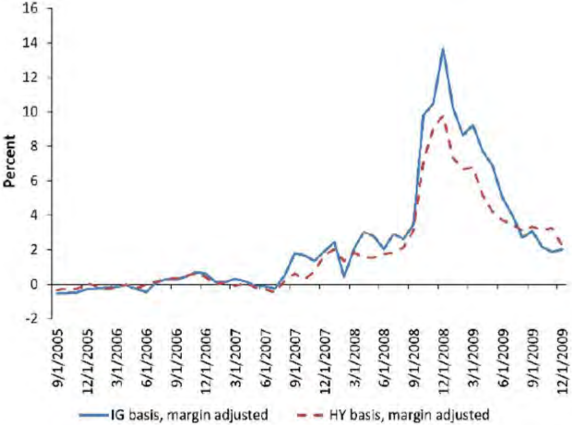



**Figure 6**
**Investment grade (IG) and high yield (HY) CDS–bond bases, adjusted for their margins**
This figure shows the CDS–bond basis, computed as the yield spread for corporate bonds minus the CDS spread (adjusted to account for certain differences between CDS and bonds), averaged across IG and HY bonds, respectively. Our model predicts that the basis should line up in the cross-section according to the margin differences. Since IG corporate bonds have a margin around 25% and IG CDS have margins around 5%, the IG margin differential is 20%. Hence, the adjusted IG basis is basis/0.20. Similarly, we estimate that the HY margin differential is around 50% so the HY adjusted basis is basis/0.50. We adjust the level of each series by subtracting the average during the first two years, 2005–2006, when credit was easy so margin effects played a small role. Consistent with the idea that the expected profit per margin use is constant in the cross-section, we see that the adjusted bases track each other.

# Chen, Chen, He, Liu, Xie (2020)

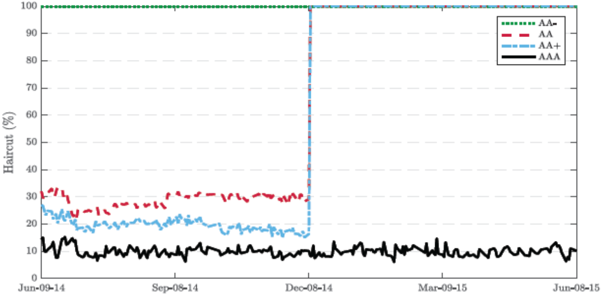



Figure 3: **Average repo haircut on the exchange market.** This figure plots the average daily haircut on the exchange market for dual-listed enterprise bonds in each of the four rating categories. The sample period is from 6/9/2014 to 6/8/2015.

# Chen, Chen, He, Liu, Xie (2020)

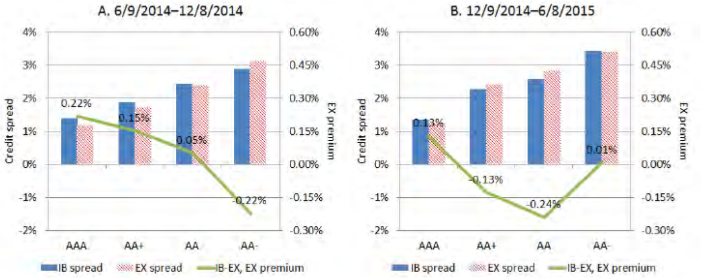



Figure 4: **Exchange premia before and after the event.** This figure plots the average credit spreads for each of the four rating categories on the interbank market and the exchange, along with the average exchange premium. Panels A and B show the results for the 6 months before and after the event date 12/8/2014, respectively.

# What determines margins?

- Gromb and Vayanos (2002): assume risk-free debt
  - $\widetilde{V} \in [V_{\text{min}}, V_{\text{max}}] \Rightarrow m = 1 - \frac{V_{\text{min}}}{P}$
  - Fostel and Geanakoplos (2015) derive risk-free margin loan as optimal contract in binomial model $\widetilde{V} \in \{V_{\text{min}}, V_{\text{max}}\}$
- Brunnermeier and Pedersen (2009): assume Var-at-Risk rule
  - $\alpha$-VaR $\Leftrightarrow$ $m$ such that $\Pr(\frac{\tilde{V}-P}{P} < -m) = \alpha$
  - Adrian and Shin (2014) derive VaR rule as optimal debt contract in presence of risk-shifting

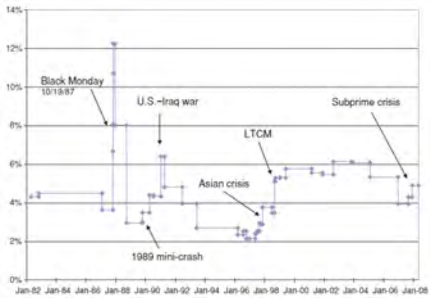
# What determines margins?

- $m$ depends on distribution of $\tilde{V} \rightarrow$ how is it estimated?
- Case 1: constant volatility, distribution of $\tilde{V}$ known
  - e.g., Gromb-Vayanos 2002
  - $\Rightarrow$ stabilizing margins because $P \downarrow \Rightarrow m \downarrow$
- Case 2: time-varying volatility
  - Brunnermeier-Pedersen 2009: ARCH volatility $\sigma_{t+1} = \bar{\sigma} + \theta|\Delta V_t|$ estimated using $|\Delta P_t|$
  - $\Rightarrow$ destabilizing margins because $P \downarrow\downarrow \Rightarrow m \uparrow$ ("liquidity spiral")

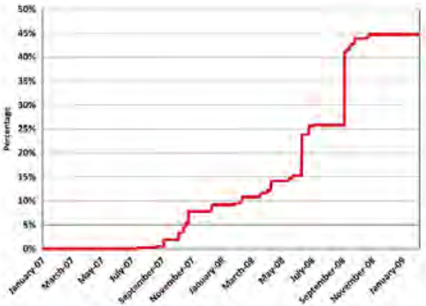
# Margins

Margin on S&P 500 futures

(Brunnermeier and Pedersen, 2009)




Haircut on repo transactions (average across all collateral excluding US treasuries)
(Gorton and Metrick, 2011)



# Financial intermediaries leverage

- Adrian and Shin (2010)

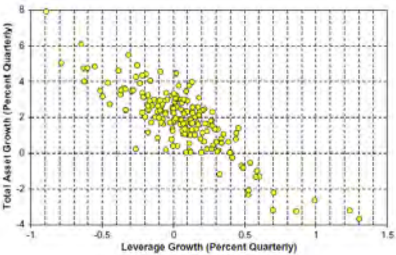



Fig. 1. Total assets and leverage of household.

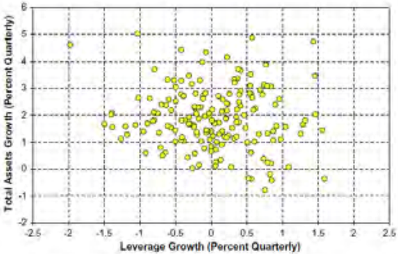



Fig. 2. Total assets and leverage of non-financial, non-farm corporates.

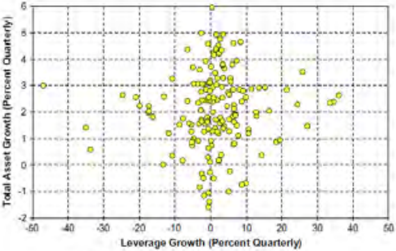



Fig. 3. Total assets and leverage of commercial banks.

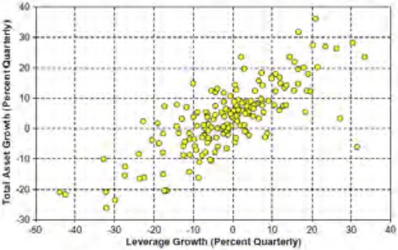



Fig. 4. Total assets and leverage of security brokers and dealers.

# Leverage constraints: Risk premium

- Arbs choose $X_{i0}$ at $t = 0 \rightarrow$ endogenize $P_{i0}$
  - To simplify: no leverage constraint at $t = 0$
- $S_i$, $m_i$ random, realized at $t = 1 \Rightarrow P_{i1}$, $\lambda$, $W_1$ also random
- Date 0 equilibrium

$$P_{i0} = E_0[P_{i1}] + \frac{Cov_0(\lambda, P_{i1})}{E_0[1+\lambda]} < E_0[P_{i1}]$$

$\Rightarrow$ Expected returns

$$\boxed{E[R_{i1}] = -\frac{1}{E_0[1+\lambda]} Cov_0(\lambda, R_{i1})}$$

- Empirical evidence?
  - Challenge: proxy for $\lambda$
  - "Intermediary asset pricing": financial intermediaries balance sheet
  - Adrian, Etula and Muir (2014) vs. He, Kelly and Manela (2017)

# Adrian, Etula and Muir (2014)

- $\lambda_t \sim (\text{broker-dealer leverage})^{-1}$
- Intuition: deleveraging $\Leftrightarrow$ bad times

| Panel A: Prices of Risk | | | | | | |
|---|---|---|---|---|---|---|
| | CAPM | FF | FF, Mom | FF, Mom, PC1 | $\boxed{\text{LevFac}}$ | LevMkt |
| Intercept | 3.39 | 3.16 | 1.06 | 0.66 | 0.12 | −0.19 |
| $t$-FM | 3.55 | 4.09 | 1.51 | 1.14 | 0.06 | −0.21 |
| $t$-Shanken | 3.54 | 4.03 | 1.34 | 1.01 | 0.04 | −0.14 |
| LevFac | | | | | 62.21 | 60.97 |
| $t$-FM | | | | | 4.62 | 5.29 |
| $t$-Shanken | | | | | 3.12 | 3.65 |
| Mkt | 3.06 | 2.30 | 4.54 | 4.89 | | 5.46 |
| $t$-FM | 0.99 | 0.80 | 1.59 | 1.71 | | 1.75 |
| $t$-Shanken | 0.99 | 0.80 | 1.58 | 1.70 | | 1.55 |
| SMB | | 1.76 | 1.57 | 1.63 | | |
| $t$-FM | | 0.93 | 0.83 | 0.87 | | |
| $t$-Shanken | | 0.93 | 0.82 | 0.86 | | |
| HML | | 3.33 | 4.37 | 4.34 | | |
| $t$-FM | | 1.45 | 1.90 | 1.89 | | |
| $t$-Shanken | | 1.45 | 1.86 | 1.85 | | |
| MOM | | | 7.82 | 7.75 | | |
| $t$-FM | | | 2.94 | 2.91 | | |
| $t$-Shanken | | | 2.92 | 2.89 | | |
| PC1 | | | | 14.99 | | |
| $t$-FM | | | | 1.03 | | |
| t-Shanken | | | | 0.93 | | |

# Adrian, Etula and Muir (2014)

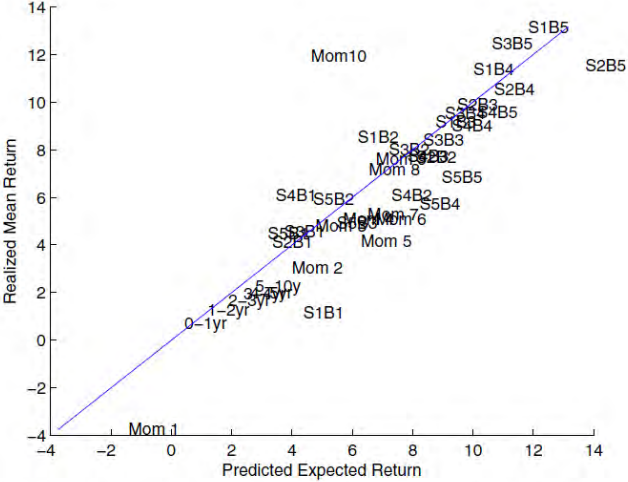



**Figure 1. Realized versus predicted mean returns: leverage factor.** We plot the realized mean excess returns of 35 equity portfolios (25 size- and book-to-market-sorted portfolios and 10 momentum-sorted portfolios) and six Treasury bond portfolios (sorted by maturity) against the mean excess returns predicted by our single-factor financial intermediary leverage model, estimated without an intercept ($E[R^e] = \beta_{lev}\lambda_{lev}$). The sample period is 1968Q1 to 2009Q4. Data are quarterly, but returns are expressed in percent per year.

# He, Kelly and Manela (2017)

- $\lambda_t \sim (\text{broker-dealer capital ratio})^{-1} = \text{leverage}$
- Intuition: low capital ratio $\Leftrightarrow$ bad times
- Opposite to Adrian-Etula-Muir

| | FF25 | US bonds | Sov. bonds | Options | CDS | Commod. | FX | All |
|---|---|---|---|---|---|---|---|---|
| Capital | 6.88 | 7.56 | 7.04 | 22.41 | 11.08 | 7.31 | 19.37 | 9.35 |
| | (2.16) | (2.58) | (1.66) | (2.02) | (3.44) | (1.90) | (3.12) | (2.52) |
| Market | 1.19 | 1.42 | 1.24 | 2.82 | 1.11 | −0.55 | 10.14 | 1.49 |
| | (0.78) | (0.82) | (0.32) | (0.67) | (0.41) | (−0.25) | (2.17) | (0.80) |
| Intercept | 0.48 | 0.41 | 0.34 | −1.11 | −0.39 | 1.15 | −0.94 | −0.00 |
| | (0.36) | (1.44) | (0.33) | (−0.31) | (−2.77) | (0.83) | (−0.83) | (−0.00) |
| $R^2$ | 0.53 | 0.84 | 0.81 | 0.99 | 0.67 | 0.25 | 0.53 | 0.71 |
| MAPE, % | 0.34 | 0.13 | 0.32 | 0.14 | 0.18 | 1.15 | 0.44 | 0.63 |
| MAPE-R, % | 0.40 | 0.26 | 0.45 | 0.68 | 0.39 | 1.40 | 0.62 | 0.63 |
| RRA | 2.71 | 3.09 | 2.52 | 8.90 | 3.61 | 2.88 | 8.26 | 3.69 |
| Assets | 25 | 20 | 6 | 18 | 20 | 23 | 12 | 124 |
| Quarters | 172 | 148 | 65 | 103 | 47 | 105 | 135 | 172 |

# He, Kelly and Manela (2017)

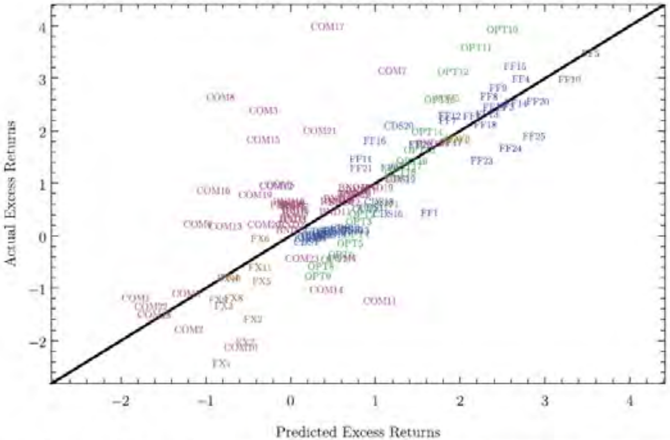



**Fig. 2.** Pricing errors: all portfolios. Actual average percent excess returns on all tested portfolios versus predicted expected returns using their risk exposures (betas) with respect to shocks to the intermediary capital ratio and the excess return on the market. Test portfolios are abbreviated based on their asset class: equities (FF), US bonds (BND), foreign sovereign bonds (SOV), options (OPT), CDS, commodities (COM), and foreign exchange (FX). Distance from the 45-degree line represents pricing errors (alphas). Betas are estimated in a first-stage time-series regression. The quarterly sample is 1970Q1–2012Q4. The intermediary capital ratio is the ratio of total market equity to total market assets (book debt plus market equity) of primary dealer holding companies. Shocks to capital ratio are defined as AR(1) innovations in the capital ratio, scaled by the lagged capital ratio.

# Leverage constraints: Welfare

- Need to model noise traders and specify their utility function
- First Welfare Theorem doesn't apply
- Pecuniary externality: arbs do not internalize that their actions affect the leverage constraints of other arbs through prices
- Gromb and Vayanos (2002), Lorenzoni (2008)

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