Introduction to Finance for Data Scientists

Session 2: Valuation

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Asset Valuation

- This morning
  - Assets (stocks, bonds, bank loans, etc.)
  - What they are used for (financing, risk sharing, etc.)
  - How they are created and traded (IPOs, exchanges, etc.)
- Now
  - What determines the value of assets, i.e., the price at which they trade
  - Ex.: Lemonade did an IPO at €29 per share. How did investors come up with this valuation?

Asset Valuation

- Value of asset = value of benefits generated by the asset
- "Benefits"
  - Usually in cash: dividends, interest, rents, royalties, etc.
  - Sometimes non-monetary: control rights, environmental impact
  - Called **cash flow (CF)** for brevity
- Two steps of valuation

1. Forecast CF $\Longrightarrow$ Data
2. Value forecasted CF $\Longrightarrow$ Finance
   - CF are in the future and uncertain $\rightarrow$ valuation must be adjusted accordingly

Road Map

Risk-Free Assets

Risky Assets

Diversification

Time Discounting

Q1. Do you prefer receiving €1 now or €1 in one year? 🤔

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Q2. How much do you value today €1 received in one year?

- This is called the **present value (PV)** of €1 in one year

- r such that $PV = 1/(1+r)$ is the **discount rate**

PV of Risk-Free Cash Flow

- More generally
  - D euros in t years has present value $\dfrac{D}{(1+r)^t}$
  - A stream of risk-free cash flows $\{D_t\}_{t\geq 1}$ has present value

$$\boxed{PV = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}}$$

- Remarks
  - Discount rate r is expressed per year, hence the power t
  - "Risk-free," "safe," "deterministic" $\neq$ "risky," "stochastic"

Valuing Risk-Free Assets

Ex. 1 Consider two assets with the following risk-free cash flows

	Year 1	Year 2	Year 3
Asset A	10	10	10
Asset B	0	0	31

The discount rate is 5% per year

Q1. Which project is more valuable? ⏱ 2 minutes

[spreadsheet]

Valuing Risk-Free Assets

Ex. 2 A US government bond promises 100 USD in one year. The discount rate is 4% per year

Q1. What is the price of the bond? ⏱ 2 minutes

Valuing Risk-Free Assets

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Q1. What is the price of the bond? ⏱ 2 minutes

Q2. You buy the bond and hold it for one year. What is your rate of return? ⏱ 2 minutes

$$\textbf{Return} \equiv \frac{\text{what you get} - \text{what you paid}}{\text{what you paid}} = \frac{\text{what you get}}{\text{what you paid}} - 1$$

Risk-Free Assets

- Typical risk-free assets: debt with no risk of borrower default (otherwise, not risk-free)
  - Government bonds issued by fiscally sound countries (US, Germany)
  - Corporate bonds issued by AAA-rated companies (Microsoft)

Risk-Free Interest Rate

- The **risk-free interest rate** (often called the risk-free rate) is the return on a risk-free asset, and also the discount rate for a risk-free asset

- Properties of the risk-free rate

  1. Same for all risk-free assets with cash flows at the same dates in the same currency
  2. Depends on the currency
  3. Depends on the cash flow horizon

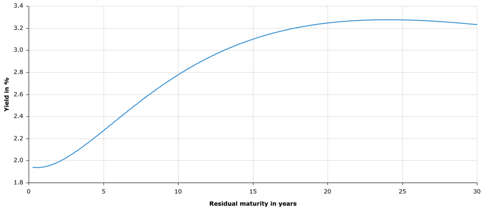
Risk-Free Interest Rate

2. Depends on the currency

Country	Latest yield
Australia	4.32%
Austria	3.05%
Belgium	3.25%
Canada	3.19%
Denmark	2.51%
Finland	3.09%
France	3.53%
Germany	2.71%
Greece	3.43%
Ireland	2.98%
Italy	3.55%
Japan	1.64%
Netherlands	2.87%
New Zealand	4.23%
Portugal	3.11%
Spain	3.25%
Sweden	2.61%
Switzerland	0.28%
UK	4.70%
US	4.10%

Source: Financial Times

3. Depends on the cash flow horizon



Yield curve (a.k.a. term structure of interest rates) in euro. Source: ECB

Remark: Market Efficiency

- "Value" and "price" are conceptually different
  - (Present) Value: how much investors should be willing to pay for an asset
  - Price: how much investors actually pay for an asset
- For now, we assume price = value, based on the following argument

  If price < value, investors buy, pushing price up

  If price > value, investors sell, pushing price down

  $\Rightarrow$ At market equilibrium, price = value
- This property is called **market efficiency**. We will return to it next week

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Risk Aversion

Q1. Do you prefer receiving [€1 for sure] or [€0.5 or €1.5 with a fifty-fifty chance]? 

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- **Risk aversion:** Investors value a risky cash flow less than a safe cash flow, holding expected cash flow fixed

$\Rightarrow$ Risky cash flows are discounted

Valuing Risky Assets

- Most assets are risky: stocks, debt with default risk, etc.

- An asset with risky cash flow $\tilde{D}$ in one year has present value

$$PV = \frac{E[\tilde{D}]}{1+k}$$

where discount rate $k = \underbrace{\text{risk-free rate}}_{\text{time discounting}} + \underbrace{\text{risk premium}}_{\text{risk discounting}}$

Valuing Risky Assets

- More generally, an asset with risky cash flow $\{\tilde{D}_t\}_{t\geq 1}$ has present value

$$\boxed{PV = \sum_{t=1}^{\infty} \frac{E[\tilde{D}_t]}{(1+k)^t}}$$

where:

$$\boxed{\text{discount rate } (k) = \text{risk-free rate} + \text{risk premium}}$$

- Remark: the PV formula has several names

  Dividend Discount Model (DDM) in asset pricing

  Discounted Cash Flow (DCF) method in corporate finance

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Q. Many companies do not pay dividends (e.g., Tesla). The PV formula imply their value is zero: true or false?

Valuing Risky Assets Quiz

Consider the stocks of two companies

- GreatTech has great technology: it will generate significant cash flow
- LameDuck has mediocre technology: its expected cash flow is one-tenth of GreatTech's cash flow (at each future date, with the same discount rate)

Q1. Which stock has the higher price? 1 minute

a. GreatTech b. LameDuck c. Both have the same price

Q2. Which stock has the higher expected return? 1 minute

a. GreatTech b. LameDuck c. Both have the same expected return

Expected Return of Risky Assets

$$\boxed{\text{Expected return} = \text{Discount rate}}$$

- Easy to see with one-period assets

  Realized return: $\tilde{r} = \frac{\tilde{D}}{P} - 1$

  Expected return: $E(\tilde{r}) = \frac{E(\tilde{D})}{P} - 1 = \frac{E(\tilde{D})}{\frac{E(\tilde{D})}{1+k}} - 1 = k$

- The logic goes through with multiple-period assets

  Realized return: $\tilde{r}_{t+1} = \frac{\tilde{D}_{t+1} + (\tilde{P}_{t+1} - P_t)}{P_t}$

  We still have $E_t(\tilde{r}_{t+1}) = k$ (proof on next slide)

Proof that expected return = discount rate

Expected return from t to $t+1$: $E_t(\tilde{r}_{t+1}) = \dfrac{E_t(\tilde{D}_{t+1}) + E_t(\tilde{P}_{t+1})}{P_t} - 1$

Using the PV formula: $P_t = \sum_{\tau=1}^{\infty} \dfrac{E_t(D_{t+\tau})}{(1+k)^{\tau}}$

Expected price at $t+1$: $E_t(\tilde{P}_{t+1}) = E_t\left[\sum_{\tau=1}^{\infty} \dfrac{E_{t+1}(D_{t+1+\tau})}{(1+k)^{\tau}}\right]$

Using Bayes rule: $E_t(\tilde{P}_{t+1}) = \sum_{\tau=1}^{\infty} \dfrac{E_t(D_{t+1+\tau})}{(1+k)^{\tau}}$

Finally: $E_t(\tilde{r}_{t+1}) = \dfrac{(1+k)P_t}{P_t} - 1 = k$

Expected Return of Risky Assets

Discount rate = Risk premium + Risk-free rate = Expected return

$\Rightarrow$ Riskier assets have a higher risk premium
higher discount rate
higher expected return
lower price given expected future cash flows

Historical Perspective

- Historical returns of main asset classes, 1960–2017[1]

	Moments of return distribution (in % per year)	
	Std. dev.	Mean
Stocks	17.3	11.3
Real estate	19.3	12.3
Corporate bonds	8.4	7.8
Commodities[2]	24.9	8.7

⇒ Riskier assets have higher expected returns

Commodities are puzzling: high std. dev. but not a high mean return. Any idea?

[1] Doeswijk, Lam and Swinkels, "Historical Returns of the Market Portfolio," *Review of Asset Pricing Studies*, 2019 [pdf]

[2] Commodities include mainly metals (e.g., gold), energy products (e.g., oil), and agricultural products

Mini-Case: 2022 Stock Market Performance

UPDATED FRI, DEC 30 2022 • 5:23 PM EST

Stocks fall to end Wall Street's worst year since 2008, S&P 500 finishes 2022 down nearly 20%

🤔 What might explain it?

Mini-Case: 2022 Stock Market Performance

- How much did rising interest rates explain the 2022 stock market crash?
- Apply the PV formula to US stocks assuming a constant expected growth rate g of dividends: $E[D_{t+\tau}] = (1+g)^{\tau} D_t$ for $\tau \geq 1$

$$\Longrightarrow P_t = \sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+k)^{\tau}} = \frac{(1+g)D_t}{k-g}$$

- Discount rate: k = risk-free rate + risk premium
- The US 10-year rate rose from 1.5% at end-2021 to 3.5% at end-2022
- Assume risk premium $\simeq 5\%$, dividend growth rate $g \simeq 4\%$

$\Rightarrow$ Implies that stock prices should change by $\dfrac{1/(1.5\% + 5\% - 4\%)}{1/(3.5\% + 5\% - 4\%)} \simeq 0.56$, i.e., fall by 44%

🤔 Directionally correct, but overstates the market decline. Why?

Taking Stock

1. Price = expected cash flow discounted for time and risk

2. Discount rate = expected return = risk-free rate + risk premium

- Remark: 1. and 2. hold under the assumption of market efficiency ("the price is right"). Next week, we'll relax this assumption

- Next question: What determines the risk premium?

  Variance? No – covariance, because of diversification

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Risky Assets

Diversification

Diversification

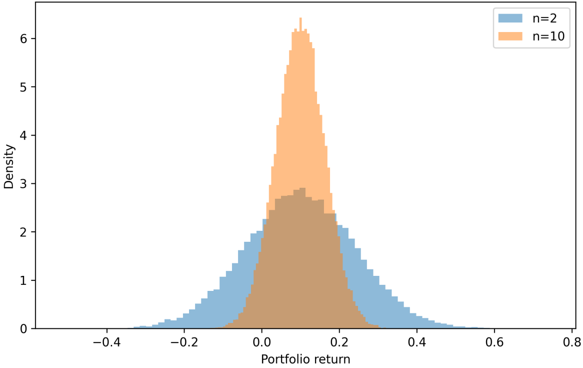
- A fund invests €100 million equally across stocks $i = 1, 2, \ldots, n$
- Each stock i has return r_i with $E(r_i) = \mu$, $V(r_i) = \sigma^2$, and $Corr(r_i, r_j) = \rho$ for all $i \neq j$
- Portfolio return: $r_P = \frac{1}{n} \sum_{i=1}^{n} r_i$

Q1. Is it safer to invest in $n = 2$ or $n = 10$ stocks?

a. $n = 2$ b. $n = 10$ c. It depends

Diversification

- A portfolio with a larger number of assets is less risky — better diversified (provided $\rho < 1$)
- Distribution of portfolio return for $n = 2$ vs. $n = 10$
  ($\mu = 10\%$, $\sigma = 20\%$, $\rho = 0$)



Diversification

- Consider two portfolios

  A: 10 stocks with pairwise correlation $\rho = 0.5$

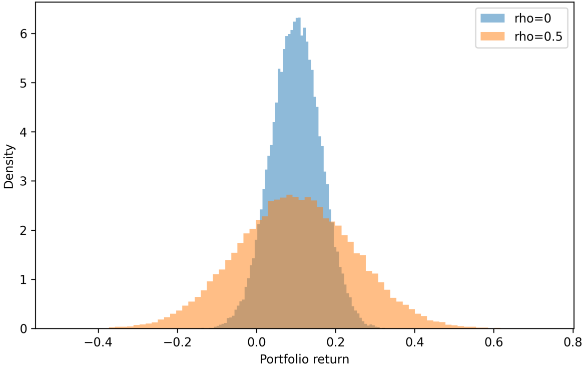
  B: 10 uncorrelated stocks ($\rho = 0$)

Q2. Which portfolio is safer?

A
B
It depends

Diversification

- A portfolio with less correlated assets is less risky — better diversified
- Distribution of portfolio return for $\rho = 0$ vs. $\rho = 0.5$ $(n = 10,\ \mu = 10\%,\ \sigma = 20\%)$

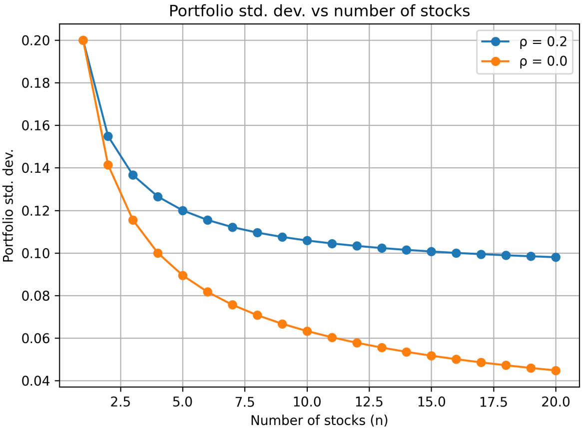


Diversification

Q3. Does portfolio risk go to zero when $n \to \infty$?

a. Yes b. No c. It depends

Diversification



- As $n \to \infty$, portfolio risk does not depend on the total variance of individual assets — but on their **covariance**
- Proof: $V(r_P) = \left(\frac{1}{n} + \frac{n-1}{n}\rho\right)\sigma^2 \xrightarrow[n\to\infty]{} \rho\sigma^2$

Diversification: Summary

- Risk independent across assets can be eliminated by diversification

  = **idiosyncratic (diversifiable) risk**

- Risk correlated across assets cannot be eliminated by diversification

  = **systematic (non-diversifiable) risk**

- Remark: A portfolio with large n is said to be "well diversified," which does not mean risk-free — idiosyncratic risk is eliminated but systematic risk remains

Diversification: Implication for Risk Premium

Q. You would be equally willing to hold a portfolio with $\rho = 0$ and a portfolio with $\rho = 0.2$ if and only if:

a. Both portfolios have the same expected return

b. The portfolio with $\rho = 0$ has a higher expected return

c. The portfolio with $\rho = 0.2$ has a higher expected return

Diversification: Implication for Risk Premium

- Idiosyncratic risk does <u>not</u> command a risk premium

- Systematic risk <u>does</u> command a risk premium

- Systematic risk arises from <u>covariance</u> between assets

- Next class: Quantify the risk premium as a function of systematic risk, i.e., as a function of covariance

Brain-Teaser

Do you expect a high or low expected return on gold?

Until Next Class

- Practice problems on Slack

- Have a great WE and see you next week