Introduction to Finance for Data Scientists

Session 3: Asset Management

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Refresher From Last Class

- Asset price = expected cash flow discounted for time and risk

- Discount rate = Expected return = Risk-free rate + Risk premium

- Risk premium depends on systematic (non-diversifiable) risk

- Systematic risk depends on covariance with other assets

Today's Menu

How to evaluate an investment strategy's performance: does it deliver returns above and beyond compensation for risk?

- We will study two metrics of an asset's or portfolio's risk-adjusted performance

1. **Sharpe ratio**: evaluates **portfolio** performance
2. **Alpha**: evaluates performance of **individual** assets or portfolios within a broader portfolio

Appetizer

- Market data: [download]

- US stock returns, monthly, 1980-2019

Stock Market Data

1. Select the stock of Apple for the month December 2019

2. How much was Apple's stock price (`prc`) at the end of Dec 2019?

3. How many shares had Apple issued (`shrout`) by Dec 2019?

4. How much was Apple's market capitalization?

Road Map

Sharpe Ratio

CAPM

(Proof of CAPM)

Sharpe Ratio

- The **Sharpe ratio** of portfolio P is the average excess return relative to its standard deviation

$$\boxed{SR_P = \frac{E(r_P - r_f)}{\sigma(r_P - r_f)}}$$

(Excess return = portfolio return – risk-free rate)

- Intuitively, the Sharpe ratio is higher when expected return is higher and risk is lower

- The Sharpe ratio is widely used to evaluate fund managers' performance

- Another intuition: the Sharpe ratio is like the signal-to-noise ratio of returns

Sharpe Ratio in Data

- Use stock return data for 2010-2019

`ticker`	stock identifier
`ret`	stock return
`rf`	risk-free rate

- Construct a portfolio P of 5 equally-weighted stocks

- Portfolio return: $r_{P,t} = \sum_{i=1}^{5} \frac{1}{5} r_{i,t}$

- Compute its annualized Sharpe ratio

  "Annualized" means converting monthly returns to % per year:
  → multiply expected return by 12
  → multiply s.d. of return by $\sqrt{12}$ (assume i.i.d.)
  → overall, annualized SR = monthly SR $\times\sqrt{12}$

Sharpe Ratio

How does the portfolio Sharpe ratio compare to the individual stocks' Sharpe ratios?

🤔

Sharpe Ratio

How does the portfolio Sharpe ratio compare to the individual stocks' Sharpe ratios?

🤔

- Portfolio SR > average SR of individual stocks, due to diversification

Sharpe Ratio

- The Sharpe ratio is used to evaluate the risk-return tradeoff of the overall portfolio
  - A risk-averse investor cares about the std. dev. of its portfolio
- When evaluating an individual asset (or a portfolio part of a broader portfolio), the std. dev. is not the appropriate risk measure
  - The contribution of the individual asset to the overall portfolio risk is determined by the asset's covariance with the portfolio

$\Rightarrow$ This is the logic of the CAPM

Road Map

Sharpe Ratio

CAPM

(Proof of CAPM)

CAPM

- According to the Capital Asset Pricing Model (CAPM), an asset's risk premium should depend on its **covariance with the market portfolio**
- Market portfolio = all traded assets
- Intuition
  - A well-diversified investor holds the market portfolio
  - The contribution of asset i to the portfolio risk (variance) depends on the covariance of asset i with the portfolio:

    low covariance ⇒ diversification, reduces risk
    high covariance ⇒ increases risk
  - Hence, higher covariance commands larger risk premium

CAPM

- CAPM formula: the expected return on asset i should be

$$\boxed{E(r_i) = r_f + \beta_i \times (E(r_m) - r_f)}$$

  - r_f: risk-free rate
  - r_m: return on the market portfolio
  - $\beta_i = Cov(r_i, r_m)/V(r_m)$: asset i's **market beta** (systematic, non-diversifiable risk), measuring its comovement with the market portfolio
  - $E(r_m) - r_f$: the **market risk premium**
  - $\beta_i \times (E(r_m) - r_f)$: asset i's **risk premium**

CAPM Assumptions

- The CAPM formula can be derived under the assumptions that investors (i) have a mean-variance objective and (ii) can perfectly diversify

- see bonus slides for a heuristic proof

- For now, assume CAPM assumptions hold. We will discuss and relax them later

Market Beta of Main Asset Classes

- The CAPM logic and formula apply equally to individual assets and to portfolios within a broader portfolio

- Historical returns of main asset classes, 1960–2017

	Moments of return distribution (in % per year)		
	Std. dev.	Mean	Beta
Stocks	17.3	11.3	1.5
Real estate	19.3	12.3	1.3
Corporate bonds	8.4	7.8	0.5
Commodities	24.9	8.7	0.1

- Commodities have a low mean return despite high std. dev. because they have low beta

Alpha

- CAPM: $E(r_i - r_f) = \beta_i \times E(r_m - r_f)$

- If an asset's expected return > CAPM prediction ⇒ attractive risk-adjusted return

- Such an asset has a positive **CAPM alpha**:
  $\alpha_i \equiv E(r_i - r_f) - \beta_i \times E(r_m - r_f)$

- Conversely, an asset with $\alpha_i < 0$ has a poor risk-adjusted return

- A fairly priced asset has $\alpha_i = 0$

- CAPM with mispricing: $\boxed{E(r_i - r_f) = \alpha_i + \beta_i \times E(r_m - r_f)}$

CAPM in Data

- How to estimate α_i and β_i?
- The CAPM formula is the expected value of the OLS regression equation of $r_i - r_f$ on $r_m - r_f$:

$$r_i - r_f = \alpha_i + \beta_i(r_m - r_f) + \epsilon$$

  which can be estimated via a **time-series regression**
- α_i: regression intercept
- β_i: regression coefficient of $r_m - r_f$ on $r_i - r_f$
- ϵ_i: error term uncorrelated with $r_m - r_f$
- Remarks
  - OLS estimator is $\beta_i^{OLS} = \frac{Cov(r_i - r_f, r_m - r_f)}{V(r_m - r_f)}$, which is indeed equal to β_i in the CAPM formula
  - Adding r_f within Cov or Var is irrelevant because r_f is deterministic

Example

- Use same monthly stock return data as before

`ret`	stock return
`rf`	risk-free rate
`mkt`	market return

- Estimate the CAPM for Ford's stock

Q. What are Ford's α and β? How do you interpret them?

Example

- $\hat{\beta} = 1.3$ (s.e. = .1)

  $\beta > 1$ means Ford is quite procyclical: when market goes up 1%, Ford stock goes up 1.3% on average

  $\hat{\alpha} = 0.0009$ (s.e. = .005)

  - alpha is not statistically different from zero: Ford delivered average stock return in line with its systematic risk

Quiz

Q1. Ford has a higher expected return than the market portfolio: true or false?

Q2. Ford has a more favorable risk-return tradeoff than the market portfolio: true or false?

Q3. Which industries have $\beta < 1$?

Q4. How do you interpret ϵ in the CAPM equation?

CAPM with Portfolios

- Consider this portfolio

Stock	Ticker	Amount invested	Portfolio weight (w_i)
Ford	F	\$100	0.25
General Electric	GE	\$100	0.25
American Electric Power	AEP	\$200	0.50

- Portfolio return = weighted average of individual returns

$$r_{pt} = \sum_i w_i r_{it}$$

CAPM with Portfolios

- Portfolio beta = weighted average of individual betas

$$\beta_p = \sum_i w_i \beta_i \qquad \text{because covariance is a linear operator}$$

	Portfolio weight	Market beta
F	0.25	1.31
GE	0.25	1.13
AEP	0.50	.33
Portfolio		.78

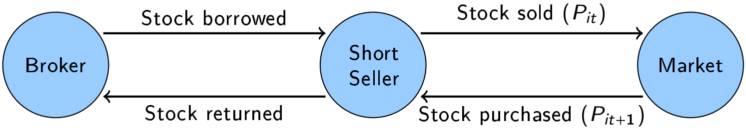
- Portfolio alpha = weighted average of individual alphas

Q. Can portfolio weights be negative? 🤔

Short Selling

Q. Can portfolio weights be negative?

- Yes: short selling



Profit on a **short position** (i.e., short selling): $P_{it} - P_{it+1}$

$=$ minus profit on a **long position** (i.e., buying): $P_{it+1} - P_{it}$

Long-Short Portfolio

- The risk-free asset can also be included in the portfolio such that the portfolio has the same amount in long stock positions and in short positions

	Amount	Weight
Risk-free	long \$100	1
F	long \$100	1
GE	short \$100	–1

Portfolio return = $r_f + r_F - r_{GE}$

Portfolio beta = $\beta_F - \beta_{GE}$

Portfolio alpha = $\alpha_F - \alpha_{GE}$

Long-Short Portfolio

- Hedge funds commonly construct long-short portfolios, buying assets with positive α and short-selling assets with negative α

- If successful, such long-short portfolios have:
  - Positive alpha
  - Low beta ($\beta = 0$ if long and short legs of the portfolio have same β)

    $\rightarrow$ It is why they are called *hedge* funds
  - High Sharpe ratio

To Be Continued

- More on quantitative asset management on Friday: guest speaker + class on market efficiency

see you tomorrow!

Bonus Slides

Road Map

Sharpe Ratio

CAPM

(Proof of CAPM)

Bonus Slides: Heuristic Derivation of the CAPM

- **CAPM:** Under the following assumptions

  (i) investors have mean-variance preferences

  (ii) and investors can perfectly diversify,

  then the risk premium on asset i's is

$$E(r_i) - r_f = \beta_i \times (E(r_m) - r_f)$$

  where $\beta_i = \dfrac{Cov(r_i, r_m)}{V(r_m)}$ and

  - r_m: return on the market portfolio

    $r_m = \sum_i w_i^m r_i \quad$ where $w_i^m = \frac{\text{Asset } i\text{'s market capitalization}}{\text{Sum of all assets' market capitalizations}}$

  - r_f: risk-free rate = return on risk-free government bonds

Bonus Slides: Heuristic Derivation of the CAPM

- Assumption (i): investors have mean-variance preferences, i.e., they assign subjective value $E(r) - \frac{1}{2}\gamma V(r)$ to risky return r

  $\gamma \geq 0$ is the coefficient of risk aversion: investors with higher γ are more risk averse, investors with $\gamma = 0$ are risk neutral

- Determine the optimal portfolio for a mean-variance investor
  - Denote x_i the % of the portfolio invested in each risky asset $i = 1, \ldots n$, the rest $\left(1 - \sum_i x_i\right)$ % is invested in the risk-free asset
  - Portfolio return: $r_P = \sum_i x_i r_i + \left(1 - \sum_i x_i\right) r_f$
  - Optimal portfolio solves $\max_{x_1,\ldots,x_n} E(r_P) - \frac{\gamma}{2} V(r_P)$

Bonus Slides: Heuristic Derivation of the CAPM

$$E(r_P) = r_f + \sum_i x_i (E(r_i) - r_f)$$

$$V(r_P) = \sum_{i,j} x_i x_j Cov(r_i, r_j)$$

– First order condition w.r.t. x_i:

$$\frac{dE(r_P)}{dx_i} = E(r_i) - r_f$$

$$\frac{dV(r_P)}{dx_i} = 2\sum_j x_j Cov(r_i, r_j) = 2Cov(r_i, r_P)$$

$$\Rightarrow \quad \underbrace{E(r_i) - r_f}_{\substack{\text{Asset i's marginal} \\ \text{contribution to} \\ \text{portfolio} \\ \text{expected return}}} - \frac{\gamma}{2} \underbrace{2Cov(r_i, r_P)}_{\substack{\text{Asset i's marginal} \\ \text{contribution to} \\ \text{portfolio risk}}} = 0 \qquad (1)$$

Bonus Slides: Heuristic Derivation of the CAPM

- Assumption (ii): investors can perfectly diversify (i.e., can choose $x_i \neq 0$ for all i, and they hold the market portfolio (i.e., x_i proportional to market portfolio weight w_i^m)

  (NB: The second part of assumption (ii) can be derived as an implication of the first part.)

$\Rightarrow$ $r_P = r_f + s\, r_m$ where $s = \sum_i x_i$

(1) $\Rightarrow E(r_i) - r_f = \gamma\, s\, Cov(r_i, r_m)$

This also holds for the market portfolio (proof: sum over i with weights w_i) $\Rightarrow E(r_M) - r_f = \gamma\, s\, V(r_M)$

Therefore $E(r_i) = r_f + \beta_i \times (E(r_m) - r_f)$ where $\beta_i = \frac{Cov(r_i, r_m)}{V(r_m)}$