Advanced Finance

Practice Problems

[#2] Trading and Market Microstructure

Problem 1: Limit Order Book and Transaction Costs

A stock trades in a centralized exchange with a limit order book. Suppose the limit order book at a given point in time is as follows.

# shares	Bid price	Ask price	# shares
50	29.50	30.50	100
150	29.00	31.00	100
100	28.50	31.50	100
150	28.00	32.00	200

The bid side of the limit order book reads as follows. There are limit orders standing in the book offering to buy up to 50 shares at a price of 29.50 per share. There are additional limit orders offering to buy up to 150 shares at a price of 29.00 per share, and so on. A trader submitting a market order to sell 100 shares would have his order executed at a price of 29.50 for the first 50 shares and at 29.00 for the next 50 shares. The ask side displays the limit orders to sell against which market orders to buy are executed.

a. What is the price per share you pay if you buy 50 shares with a market order?

b. What is the average price per share you pay if you buy 150 shares with a market order?

c. What is the average price per share you pay if you buy 300 shares with a market order?

The transaction cost of an order is evaluated as the additional cost per share relative the mid-quote (mid-point of bid and ask) before the order is executed.

d. How much is the transaction cost per share in each of the three cases considered in parts a, b and c? Do you agree with the statement on slide 36 of the lecture slides that transaction costs grow more than proportionally with the number of shares traded because the price drifts against the trader?

Problem 2: Glosten-Milgrom Model

A stock trades in a centralized market with a limit order book. There are three types of traders in this market.

- Market makers submit limit orders.
- Informed traders have advanced data analytics that provide them with information about the value of the stock. They trade using market orders.
- Noise traders have no information about the stock. They trade using market orders.

a. What are the two broad types of reasons why noise traders may trade even if they have no information? Give a specific example for each case.

When market makers analyze historical order flow data, they estimate that:

- a fraction z of market orders come from informed traders ("informed orders") and a fraction $1 - z$ of market orders come from noise traders ("uninformed orders"), both for buy orders and for sell orders;
- when there is an informed buy order (resp. sell order) and the market maker's prior estimate of the stock value is V, the price subsequently increases to V plus 30 cents (resp. V minus 30 cents) on average;
- when there is an uninformed buy or sell order and the market maker's prior estimate of the stock value is V, the price subsequently remains V on average.

Initially, everyone agrees that the fundamental value of one share of the stock is €50.

b. If the bid price is €49.90 and the ask price is €50.10, market makers' average profit/loss when they trade against an uninformed order is:

   1. profit of 20 cents
   2. profit of 10 cents
   3. zero
   4. loss of 10 cents
   5. loss of 20 cents

c. If the bid price is €49.90 and the ask price is €50.10, market makers' average profit/loss when they trade against an informed order is:

   1. profit of 20 cents
   2. profit of 10 cents

3. zero
4. loss of 10 cents
5. loss of 20 cents

There is some competition between market makers such that their average profit across all informed and uninformed orders is 5 cents per trade, both for buy orders and for sell orders.

d. Calculate the ask price and the bid price as a function of the fraction of informed trades z. [To check your result, you should find that the bid-ask spread is 70 cents when there are no noise traders in the market.]

e. Explain in words why the bid-ask spread is higher when there are more informed traders in the market.

f. Using your previous answers, assess whether noise traders benefit or suffer from the presence of informed traders.

g. Calculate the average profit per trade for an informed trader as a function of z. Show that this average profit is the same for buy orders and for sell orders.

h. Using your previous answers, assess whether informed traders benefit or suffer from the presence of other informed traders.

Problem 3: Payment For Order Flow

Consider a limit order market as in Problem 2 (you should solve Problem 2 before attacking this problem). The fraction of informed traders is such that the bid price is €49.80 and the ask price is €50.20.

RobinDesBois is an online trading app that offers retail investors to trade stocks for free. The app's users submit their orders on the app, and RobinDesBois sends the orders to a market maker. The market maker executes the orders at the prevailing bid and ask prices.

When the market maker analyzes historical RobinDesBois order flow data, it estimates that users of the app are noise traders.

a. Why is the market maker ready to pay RobinDesBois to execute RobinDesBois users' orders?

b. What is the maximum amount per order that the market maker is ready to pay to execute RobinDesBois users' orders?

The market maker pays 12 cents per order to RobinDesBois to execute the users' orders.

c. Calculate the profit per order for RobinDesBois and for the market maker.

d. Is trading really free for RobinDesBois users?