Microeconomics 1

Intertemporal choices under uncertainty

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Outline of the lecture

- Extend the 2-period production economy to the case with risk

  $\approx$ framework in previous class (2-period, no risk) + framework studied with Bruno (1-period, risk)

- Application: climate policy

Road map

Two-period economy with risk

The Ramsey rule

Two periods economy with risk

- With Bruno: one-period economy with risk

- Last class: two-period economy with no risk

- Now: two-period economy with risk

Model

- Two periods $t = 0, 1$
- State of nature at $t = 1$: ω drawn from set Ω, proba $\pi(\omega)$
- Agents $i = 1, ..., N$
  - Maximize expected discounted utility $EU_i = u(c_{i0}) + \beta E[u(c_{i1}(\omega))]$
    $\beta < 1$, $u' > 0$, $u'' < 0$
  - Endowment of good: y_{i0} at date 0; $y_{i1}(\omega)$ at date 1 in state ω
  - Endowment of asset j at date 0: $\overline{n}_{ij}$

Model

- Assets $j = 1, \ldots, J$
  - Stocks, bonds, houses, etc.
  - Produce $d_j(\omega)$ units of good at date 1 in state ω
  - Supply $\overline{n}_j = \sum_i \overline{n}_{ij}$
- Aggregate endowment of consumption good
  - Date 0: $\sum_i y_{i0}$
  - Date 1 state ω: $\sum_i y_{i1}(\omega) + \sum_j \overline{n}_j d_j(\omega)$

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Model

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- Only if the set of assets is sufficiently rich to deliver the payoff of all Arrow securities paying off in the next period
- Formally: if and only if the matrix of asset payoff $(d_j(\omega))_{(\omega,j)\in\Omega\times\{1,\dots,J\}}$ has rank $Card(\Omega)$

- From now on, assume markets are complete
  - Assume the set of assets has sufficiently rich payoff structure

    OR

  - Assume the Arrow security paying 1 in state ω exists for all $\omega \in \Omega$

Agent's problem

- Agent i maximizes $u(c_{i0}) + \sum_{\omega \in \Omega} \beta \pi(\omega) u(c_{i1}(\omega))$ by choice of
  - Consumption profile: c_{i0}, $c_{i1}(\omega)$
  - Holdings of each asset j: n_{ij}
  - Holdings of each state ω-Arrow security: $a_i(\omega)$
- Subject to budget constraints
  - Date 0: $c_{i0} + \sum_{j=1}^{J} n_{ij} p_j + \sum_{\omega \in \Omega} a_i(\omega)\phi(\omega) \leq y_{i0} + \sum_{j=1}^{J} \overline{n}_{ij} p_j$
  - Date 1 state ω: $c_{i1}(\omega) \leq y_{i1}(\omega) + \sum_{j=1}^{J} n_{ij} d_j(\omega) + a_i(\omega)$

  where $p_j =$ date 0 price of asset j

  $\phi(\omega) =$ date 0 price of state ω-Arrow security

First order conditions

- Substitute $c_{i1}(\omega)$ in expected utility using date 1 budget constraint
- First order conditions
  - w.r.t. c_{i0}: $\quad u'(c_{i0}) - \lambda_i = 0$

    where $\lambda_i =$ multiplier of date 0 budget constraint
  - FOC w.r.t. $a_i(\omega)$: $\quad \beta\pi(\omega)u'(c_{i1}(\omega)) - \lambda_i\phi(\omega) = 0$

State prices

$\Rightarrow$ $\phi(\omega)u'(c_{i0}) = \pi(\omega)\beta u'(c_{i1}(\omega))$

- Intuition: move 1 unit of consumption from date 0 to date 1 state ω
- LHS: MU loss at date 0
- RHS: expected MU gain at date 1

$\Rightarrow$ State price: $\phi(\omega) = \dfrac{\pi(\omega)\beta u'(c_{i1}(\omega))}{u'(c_{i0})}$

- LHS: relative price of consumption in date 1 state ω / price of consumption at date 0
- RHS: intertemporal marginal rate of substitution $IMRS_i(\omega)$ between date 1 state ω consumption and date 0 consumption

Marginal rates of substitution

- FOC holds for all agents $\Rightarrow$ $IMRS_i(\omega)$ equalized across agents
  - As in the riskless economy (cf. last class)
- MRS between states $\frac{\pi(\omega_1)u'(c_{i1}(\omega_1))}{\pi(\omega_2)u'(c_{i1}(\omega_2))}$ are equalized across agents and equal to relative state prices $\frac{\phi(\omega_1)}{\phi(\omega_2)}$
  - As in the static risky economy (cf. classes with Bruno)

Asset prices

- FOC w.r.t. n_{ij}: $\sum_\omega \pi(\omega)\beta u'(c_{i1}(\omega))d_j(\omega) - \lambda_i p_j = 0$

  and using $\lambda_i = u'(c_{i0})$:

  $$p_j u'(c_{i0}) = \sum_\omega \pi(\omega)\beta u'(c_{i1}(\omega))d_j(\omega)$$

- Intuition
  - Reduce consumption at date 0 to buy one unit of asset j and consume its payoff at date 1
  - LHS: MU loss at date 0
  - RHS: expected MU gain at date 1

Asset prices

$$\Rightarrow \quad p_j = E\left[\frac{\beta u'(c_{i1}(\omega))}{u'(c_{i0})} d_j(\omega)\right]$$

where $\frac{\beta u'(c_{i1}(\omega))}{u'(c_{i0})} = \frac{IMRS_i(\omega)}{\pi(\omega)} \equiv M(\omega)$ is equalized across agents

- $M(\omega)$ is the stochastic discount factor that prices all assets:

$$\boxed{p_j = E\left[M(\omega) d_j(\omega)\right]}$$

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Q2. Holding fixed $E[d_j(\omega)]$, an asset has a higher price if it has... higher payoff in recessions and lower payoff in expansions

- Consumption in recession is more valuable because MU is higher

Expected returns

- Can be rewritten in terms of asset return $R_j(\omega) = d_j(\omega)/p_j$

$$\boxed{E\left[M(\omega)R_j(\omega)\right] = 1}$$

- Applies to assets $1, \ldots, J$ and to any other security in zero net supply

Building the intuition

Q3. Asset j has higher expected return when $Cov(M(\omega), R_j(\omega))$ is positive or negative?

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Negative covariance: low return when M is high i.e. in bad time when MU is high $\Rightarrow$ unattractive risk profile $\Rightarrow$ require higher average return to compensate

Risk-free rate

- One-period risk-free bond
  - Pays off 1 in all states at date 1
  - Price at date 0: $p_f = E\left[M(\omega)\right]$
  - Return: $\dfrac{1}{p_f} \equiv R_f$

$\Rightarrow$ Risk-free rate $\boxed{R_f = \dfrac{1}{E\left[M(\omega)\right]}}$

Systematic risk & Risk premium

- Asset j's risk premium ($\equiv$ expected return in excess of risk-free rate)

$$\boxed{E\left[R_j(\omega)\right] - R_f = -R_f \, Cov\left(M(\omega), R_j(\omega)\right)}$$

- The risk premium does not depend on the variance of the asset's payoff but on the <u>covariance</u> with aggregate risk
  - Idiosyncratic risk (uncorrelated with aggregate risk) can be diversified away $\Rightarrow$ does not command a risk premium
  - Systematic risk (correlated with aggregate risk) cannot be diversified away $\Rightarrow$ commands a risk premium

- Intuition: Tradeoff between risk and expected return
  - Assets with low return in bad time (i.e. when consumption is low and MU is high) have an unattractive risk profile

    $\Rightarrow$ They must have a lower price i.e. a higher average return so that agents are willing to hold these assets

Quiz

- Suppose markets are complete and that
  - return on gold is very volatile but has zero correlation with the business cycle and consumption
  - return on the stock market is less volatile but positively correlated with the business cycle and consumption

Q. Rank $E[R_{gold}]$, $E[R_{stock}]$ and R_f

Quiz

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$$E[R_{stock}] > E[R_{gold}] = R_f$$

Present value and discount rate

- $p_j = E[M(\omega)d_j(\omega)]$ is also called the present value of risky payoff $d_j(\omega)$

- If you studied finance in a business school, you learned that

$$p_j = \frac{E[d_j(\omega)]}{1 + \text{discount rate}}$$

- The two formulas are equivalent and asset j's discount rate is $E[R_j(\omega)] = R_f - R_f Cov\left(M(\omega), R_j(\omega)\right)$

Equilibrium

- Equilibrium: all agents maximize and markets clear

- Market clearing for

  date 0 good: $\sum_{i=1}^{I} c_{i0} = \sum_{i=1}^{I} y_{i0}$

  date 1 state ω good: $\sum_{i=1}^{I} c_{i1}(\omega) = \sum_{i=1}^{I} y_{i1}(\omega) + \sum_j \overline{n}_j d_j(\omega)$

  asset j: $\sum_{i=1}^{I} n_{ij} = \overline{n}_j$

  state ω-Arrow security: $\sum_{i=1}^{I} a_i(\omega) = 0$

Road map

Two-period economy with risk

The Ramsey rule

Production choices

- So far: exchange economy, i.e., investment and production unmodeled, $\overline{n}_j$ and $d_j(\omega)$ exogenous
- Now: production economy
- Suppose agent i has investment opportunity:
  - $t = 0$: investment cost ε (small relative to the economy)
  - $t = 1$: output $d(\omega)\varepsilon$ in each state ω

Q. At which condition should this investment be carried out?

1. The condition depends on the vector $d(\omega)$: true or false?
2. The condition depends on the identity of agent i: true or false?

- Necessary and sufficient condition:

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- Necessary and sufficient condition: $E[M(\omega)d(\omega)] > 1$

Example: Climate policy

- Suppose each ton of carbon emission today generates certain costs of adaptation to climate change of 500 in 100 years

- Should we implement a green policy that reduces carbon emission today at a cost of 50 per ton?

- Yes if and only if $50 < \frac{500}{(1+r_{100})^{100}}$ where r_{100} is the annualized 100-year discount rate of a risk-free investment

- r_{100} is not observed
  - 100-year bonds don't (barely) exist
  - Even if they did, their price would not reflect social preferences because generations living in 100 years are not born yet (see macro courses: eqm in OLG models is in general not socially optimal)

- Let's calibrate r_{100}

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Q2. This argument is stronger when marginal utility decreases fast: true or false? true: in this case, (dis)utility of future costs is very small

CRRA-lognormal case

Closed-form formula in the case:

- CRRA utility
  - $u(c) = c^{1-\gamma}/(1-\gamma)$, relative risk aversion $\gamma > 0$
- and lognormal random variables
  - Log aggregate consumption: $c_t = \log(C_t)$ where $C_t = \sum_i c_{it}$
  - Log return: $r_j = \log(R_j)$
  - Assume (c_1, r_j) is jointly normally distributed
  - NB: We now have a continuum of states

CRRA-lognormal case

- Stochastic discount factor: $M = \beta \left(\frac{C_1}{C_0} \right)^{-\gamma}$

- Take log: $m \equiv \log(M) = \log(\beta) - \gamma \Delta c$

  where $\Delta c \equiv \log(C_1 / C_0)$ is aggregate consumption growth

- Take log of $E[MR_j] = 1$:

  $$\log(\beta) + E(r_j) - \gamma E(\Delta c) + \frac{1}{2} Var(r_j - \gamma \Delta c) = 0$$

  where we have used that if X is normal, $E\left(e^X\right) = e^{E(X) + \frac{1}{2} Var(X)}$

CRRA-lognormal case

- Risk-free rate

$$r_f = -\log(\beta) + \gamma E(\Delta c) - \frac{1}{2}\gamma^2 Var(\Delta c)$$

- Checking intuitions
  - Richer future generations (higher $E(\Delta c)$) $\Rightarrow$ higher discount rate
  - Even more so if MU decreases fast (γ high)
- NB
  - The formula for r_f in the special case with no aggregate risk ($Var(\Delta c) = 0$) is called the Ramsey rule
  - Agg risk lowers r_f due to precautionary saving (see problem set)
- Let's calibrate β, γ, Δc and $Var(\Delta c)$ at a 100-year horizon

$\beta = ?$

- When c_0 and c_1 are for the same individual
  - β reflects psychological traits
  - β can be elicited from individual choices
  - First Welfare Theorem: social planner should use agents' β
  - NB: ...unless individuals behave impatiently due to lack of self-control or other behavioral mistakes. In this case, should the social planner use a higher β? Paternalism vs. liberal ethics
- When c_0 and c_1 are for different generations
  - β reflects the weight on future vs. current generation in social welfare
  - $\beta = 1$ as the only morally justifiable choice?

100-year risk-free discount rate

- An aggressive calibration (Nordhaus 2008)

  γ: RRA $= 2$

  $E(\Delta c)$: Avg growth rate of agg consump $= 2\%$ per year

  $\sigma(\Delta c)$: S.D. of agg consump $= 2\%$ per year

  $\Rightarrow$ $r_f = 3.9\%$ per year

  $\frac{500}{1.039^{100}} = 11$

- A conservative calibration (Stern 2007)

  $\gamma = 1$; $E(\Delta c) = 1.3\%$; $\sigma(\Delta c) = 2\%$

  $\Rightarrow$ $r_f = 1.3\%$ per year

  $\frac{500}{1.013^{100}} = 137$

Uncertain cost of climate change

- Suppose cost of climate change is uncertain and still 500 in expectation

$\Rightarrow$ Climate policy is a risky investment

- Expected return on risky asset

$$E(r_j) = r_f - \frac{1}{2} Var(r_j) + \gamma Cov(r_j, \Delta c)$$

Take exponential:

$$E(R_j) = R_f \, e^{\gamma Cov(r_j, \Delta c)}$$

Uncertain cost of climate change

- Suppose cost of climate change can vary by factor x2: S.D. of log cost $= \ln(2)$

- …and is higher if economic growth is higher (with corr. $= 1$)

  Q. This increases the discount rate on the green policy i.e. makes it less valuable: true or false?

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  $\gamma Cov(r_j, \Delta c) = 1.4\%$ using $\gamma = 1$ and $\sigma(\Delta c) = 2\%$

  $\Rightarrow \; R_j \simeq \; R_f + 1.4\%$

- . . . and is higher if economic growth is lower (with corr. -1)

  $\Rightarrow \; R_j \simeq \; R_f - 1.4\%$