*Microeconomics 1*

# Intertemporal choices and markets

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# Contact


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# Road map

Time preference

Debt markets

Exchange economy

Production economy

Evidence

# Time preference

**Q.** Do you prefer to eat a delicious chocolate today or in one year?

# Time preference

**Q.** Do you prefer to eat a delicious chocolate today or in one year?

- People usually prefer immediate utility over delayed utility
  - Uncertainty of human life
  - Excitement of immediate consumption
  - Discomfort of deferring available gratification
  - Underestimate future needs
  - etc.

# Discounted utility

- Simple representation of time preference:

  $$U(c_0, \ldots, c_T) = u(c_0) + \beta u(c_1) + \ldots + \beta^T u(c_T)$$

- $u(.)$ is the instantaneous or per-period utility

  $U(.)$ is the intertemporal or discounted utility

- $\beta \leq 1$ is the time discount factor

- Implicit assumptions

  - Exponential discounting
  - $U(.)$ is additively time-separable
  - $u(.)$ is stationary

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**Q.** A more patient individual has higher or lower beta? higher

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# Debt markets

- Financial instrument to move resources across time: debt

- Total world credit: \$200 tn = 2.5x GDP

  to firms: \$80 tn

  to governments: \$70 tn

  to households: \$50 tn

  (excl. credit to financial sector to avoid double counting)

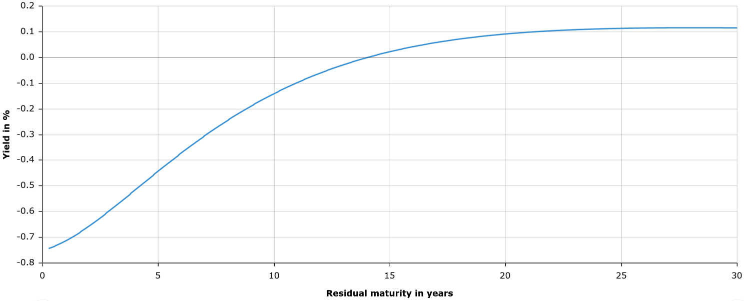
  [source: BIS]

- Who are the lenders? $\rightarrow$ Other households and firms (mostly through financial institutions)

- Examples of debt instruments: loans, bonds, bank deposits, etc.

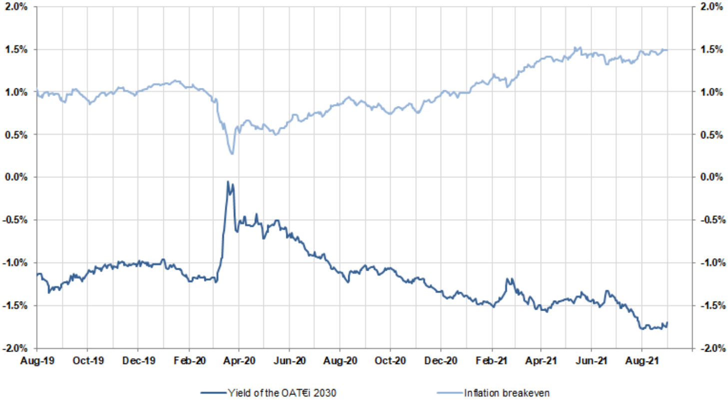
# Debt markets

- Term structure of nominal interest rates, Euro Area [source: ECB]



# Debt markets

- Real interest rate, France [source: AFT]



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# Model

- Two periods $t = 0, 1$
- Agents $i = 1, ..., N$ have preferences $U_i = u(c_{i0}) + \beta u(c_{i1})$ with $u(.)$ increasing concave, time discount factor $\beta < 1$
- Agents receive endowment of non-storable consumption good: $y_{it}$ for agent $i$ at date $t$
- NB: riskless exchange economy
  - Deterministic $y_{it}$ (no risk)
  - Exogenous $y_{it}$ (no production choice)
  - We'll relax both assumptions later

# Autarky

- Suppose no financial instruments
- Good non storable $\Rightarrow c_{i0} = y_{i0}$ and $c_{i1} = y_{i1}$ for all agents $i$

**Q:** This allocation is Pareto optimal: true or false?

# Autarky

- Suppose no financial instruments
- Good non storable $\Rightarrow c_{i0} = y_{i0}$ and $c_{i1} = y_{i1}$ for all agents $i$

**Q:** This allocation is Pareto optimal: true or false?

- False: Pareto improvement
  - Increase date 0 consumption and decrease date 1 consumption of agents with high $y_{i1}/y_{i0}$
  - Decrease date 0 consumption and increase date 1 consumption of agents with low $y_{i1}/y_{i0}$
- Need a debt instrument

# Debt market

- Debt instrument: risk-free bond
  - Lend 1 unit of consumption at date 0, get back $1 + r$ units at date 1
  - Borrow 1 unit of consump at date 0, pay back $1 + r$ units at date 1
  - $r$: interest rate determined in equilibrium

**Q:** The market with a risk-free bond is complete: true or false?

# Debt market

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  - $r$: interest rate determined in equilibrium

**Q:** The market with a risk-free bond is complete: true or false?

- True: no risk $\rightarrow$ only possible transaction $=$ move resources between date 0 and date 1 $\rightarrow$ achieved by risk-free bond

# Equilibrium definition

- Equilibrium: all agents maximize and markets clear

- Agent $i$ chooses $(c_{i0}, c_{i1}, b_i)$ to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

  date 0 budget constraint: $c_{i0} + b_i \leq y_{i0}$

  date 1 budget constraint: $c_{i1} \leq y_{i1} + (1 + r)b_i$

- Market clearing

  for bond: $\sum_{i=1}^{N} b_i = 0$

  for consumption good at date $t = 1, 2$: $\sum_{i=1}^{N} c_{it} = \sum_{i=1}^{N} y_{it}$

# Agents' choices

- Consolidate sequential budget constraints by eliminating $b_i$

  $\Rightarrow$ intertemporal budget constraint: $c_{i0} + \dfrac{c_{i1}}{1+r} \leq y_{i0} + \dfrac{y_{i1}}{1+r}$

  - Interest rate: relative price of date 1 consump. / date 0 consump.
  - LHS: date 0 price of intertemporal consumption
  - RHS: date 0 price of intertemporal income

- First order conditions

  - w.r.t. $c_{i0}$: $\quad u'(c_{i0}) - \lambda_i = 0$

    where $\lambda_i =$ multiplier of intertemporal BC

  - w.r.t. $c_{i1}$: $\quad \beta u'(c_{i1}) - \frac{\lambda_i}{1+r} = 0$

# Agents' choices

- FOC $\Rightarrow \quad u'(c_{i0}) = (1+r)\beta u'(c_{i1})$

- Intuition

  - LHS: MU of 1 unit of consumption at date 0

  - RHS: MU of saving 1 unit at date 0, lending it, and consuming the proceeds at date 1

- "Euler equation"

# Agents' choices

- Euler equation holds for all agents

  $\Rightarrow$ The intertemporal marginal rate of substitution $\frac{\beta u'(c_{i1})}{u'(c_{i0})}$ is equalized across all agents and is equal to $\frac{1}{1+r}$

- Analogy with static case
  - With several goods: MRS between goods is equalized across agents and equal to relative good prices (studied with Tristan)
  - With several states of nature: MRS between states is equalized across agents and equal to relative state prices (studied with Bruno)

# QUIZ

**Q.** For a given intertemporal income $y_{i0} + \frac{y_{i1}}{1+r}$, an individual's consumption at date 0...

# QUIZ

**Q.** For a given intertemporal income $y_{i0} + \frac{y_{i1}}{1+r}$, an individual's consumption at date 0... does not depend on her date 0's income

- "Permanent income hypothesis"

# Agents' choices

- Solve with CRRA utility: $u(c) = (c^{1-\gamma} - 1)/(1-\gamma)$
    - $\gamma \geq 0$: relative risk aversion
    - IMRS $\frac{\beta u'(c_{i1})}{u'(c_{i0})} = \beta \left(\frac{c_{i1}}{c_{i0}}\right)^{-\gamma}$ is a decreasing fcn of consump growth
- FOC $\Rightarrow$ consumption growth is equalized across agents

$$\frac{c_{i1}}{c_{i0}} = [(1+r)\beta]^{1/\gamma} \text{ for all } i$$

# Market clearing

- Equilibrium interest rate is determined by market clearing

- Aggregate FOC across agents: $\sum_{i=1}^{N} c_{i1} = [(1+r)\beta]^{1/\gamma} \sum_{i=1}^{N} c_{i0}$

- Market clearing for good at each date $t$=1,2: $\sum_{i=1}^{N} c_{it} = \sum_{i=1}^{N} y_{it} \equiv C_t$

- NB: mkt clearing for good implies mkt clearing for debt (Say's Law).
  Proof: aggregate intertemporal budget constraints across agents

$\Rightarrow$ Equilibrium interest rate: $1 + r = \beta^{-1} \left( \frac{C_1}{C_0} \right)^{\gamma}$

# QUIZ

**Q1.** The equilibrium interest rate is higher when...

**Q2.** The equilibrium interest rate is higher when...

# Comparative static

- Higher agg consump growth $\frac{C_1}{C_0} \Rightarrow$ equilibrium interest rate is higher
  - Intuition: agents want to borrow to consume more at date 0 $\Rightarrow r \uparrow$
- Agents are more impatient (lower $\beta$) $\Rightarrow$ eqm interest rate is higher
  - Intuition: agents want to borrow to consume more at date 0 $\Rightarrow r \uparrow$

# Debt market

**Q.** Who lends and who borrows in equilibrium?

# Debt market

**Q.** Who lends and who borrows in equilibrium?

Combine FOC: $\frac{c_{i1}}{c_{i0}} = \frac{C_1}{C_0}$ (indiv consump gwth = agg consump gwth)

with intertemporal BC: $c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r}$

$\Rightarrow$ $c_{i0} = \frac{y_{i0}+\frac{y_{i1}}{1+r}}{C_0+\frac{C_1}{1+r}} C_0$

$\Rightarrow$ Position in bond: $b_i = y_{i0} - c_{i0} = \left(\frac{C_1}{C_0} - \frac{y_{i1}}{y_{i0}}\right) \frac{y_{i0}}{1+r+\frac{C_1}{C_0}}$

- Agents with income growth $<$ agg income growth lend ($b_i > 0$)

  Agents with income growth $>$ agg income growth borrow ($b_i < 0$)

# Welfare

**Q.** The equilibrium allocation is Pareto optimal: true or false?

# Welfare

**Q.** The equilibrium allocation is Pareto optimal: true or false?

- True
- Intuition: lending/borrowing gains from trade are exhausted in equilibrium
- First Welfare Theorem applies (complete markets, no externality)

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# Production

- Same model as before + each agent has access to a production technology
  - Invest $k$ units of good at date 0 $\rightarrow$ generate $f(k)$ units of goods at date 1
  - $f(.)$ is the production function: increasing and concave (decreasing return to scale)
  - Equivalence with cost function representation: to produce $y$ units, need $c(y)$ units $\rightarrow$ $c(.) = f^{-1}(.)$
- Some agents have better technologies than others: $f_i(.) = z_i f(.)$

# Equilibrium definition

- Equilibrium: all agents maximize and markets clear

- Agent $i$ chooses $(c_{i0}, c_{i1}, k_i, b_i)$ to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

  date 0 budget constraint: $c_{i0} + k_i + b_i \leq y_{i0}$

  date 1 budget constraint: $c_{i1} \leq y_{i1} + z_i f(k_i) + (1 + r) b_i$

- Market clearing for bond: $\sum_i b_i = 0$

  Market clearing for good at date 0: $\sum_i c_{i0} + \sum_i k_i = \sum_i y_{i0}$

  Market clearing for good at date 1: $\sum_i c_{i1} = \sum_i y_{i1} + \sum_i z_i f(k_i)$

# Building the intuition

**Q1.** Agents with high date 0 endowment $y_{i0}$ tend to...

**Q2.** Agents with high productivity $z_i$ tend to...

# Building the intuition

**Q1.** Agents with high date 0 endowment $y_{i0}$ tend to...

lend, not invest more

**Q2.** Agents with high productivity $z_i$ tend to...

borrow in order to invest more

# Agents' choices

- Consolidate budget constraint to eliminate $b_i$

$$\max_{c_{i0}, c_{i1}, k_i} u(c_{i0}) + \beta u(c_{i1})$$

$$\text{s.t. } c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r} + \left(-k_i + \frac{z_i f(k_i)}{1+r}\right)$$

- Investment decision is independent from consumption decision
- FOC w.r.t. $k_i$: $\quad z_i f'(k_i) = 1 + r$
  - LHS: marginal productivity of capital
  - RHS: marginal cost of capital

# Agents' choices

- $k_i = f'^{-1}\left(\frac{1+r}{z_i}\right)$
  - $k_i$ is increasing in productivity $z_i$ (by concavity of $f(.)$)
  - $k_i$ does not depend on endowment $(y_{i0}, y_{i1})$
- FOC w.r.t. consumption: $u'(c_{i0}) = (1+r)\beta u'(c_{i1})$

# Equilibrium

- Suppose Cobb-Douglas production function: $f(k) = k^{\alpha}$, $\alpha \in (0, 1)$

$$\Rightarrow k_i = \left(\frac{\alpha z_i}{1+r}\right)^{\frac{1}{1-\alpha}} \qquad \Rightarrow z_i f(k_i) = \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}}$$

- Market clearing for good

$$\text{at date 0: } \sum_i c_{i0} = \sum_i y_{i0} - \sum_i \left(\frac{\alpha z_i}{1+r}\right)^{\frac{1}{1-\alpha}} \tag{C0}$$

$$\text{at date 1: } \sum_i c_{i1} = \sum_i y_{i1} + \sum_i \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}} \tag{C1}$$

- Suppose CRRA utility and aggregate FOC w.r.t. consump across $i$

$$1 + r = \beta^{-1} \left(\frac{\sum_i c_{i1}}{\sum_i c_{i0}}\right)^{\gamma} \tag{FOC}$$

- Substitute agg consump in (FOC) using (C0) and (C1) $\rightarrow$ pins down $r$

# Road map

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# Consumption

- Implications of complete markets

    1. IMRS of consumption is equalized across agents

    2. Permanent income hypothesis: current consumption depends on intertemporal income not current income

- Empirically

    1. People borrow when young, save when middle aged, dis-save when retired: consistent with complete markets

    2. Does consumption depend to current income?

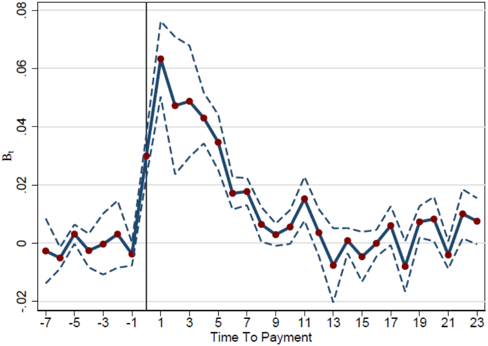
        Test: consumption response to cash windfall

# Consumption response to stimulus payment

- Baker, Farrokhnia, Meyer, Pagel and Yannelis, 2020, "Income, Liquidity, and the Consumption Response to the 2020 Economic Stimulus Payments" [pdf]

**Figure 4: Spending Around Stimulus Payments - Regression Estimates**

Notes: This figure shows estimates of $\beta_k$ from $c_{it} = \alpha_i + \alpha_t + \sum_{k=-7}^{23} \beta_k \mathbb{1}[t = k]_{it} + \varepsilon_{it}$. The sample includes all users in our sample period (both those who do and do not receive stimulus payments). The solid line shows point estimates of $\beta_k$, while the dashed lines show 95% confidence interval. Date and individual times day of week fixed effects are included. Standard errors are clustered at the user level. Time to payment is equal to zero on the day of receiving the stimulus check. Source: SaverLife.




$\Rightarrow$ Inconsistent with complete markets

# Investment

- Implications of complete markets

  1. Marginal product of investment $z_i f'(k_i)$ is equalized across agents

     $\Rightarrow$ More productive agents employ more capital

  2. Investment does not depend on current income

     $\Rightarrow$ Productive agents borrow to invest

- Empirically

  1. Do firms invest more when they become more productive?

     Test: investment response to patent grants

  2. Does investment depend on wealth?

     Test: investment response to cash windfalls

# Investment response to patent grants

- Kogan, Papanikolaou, Seru and Stoffman, 2017, "Technological Innovation, Resource Allocation and Growth," *Quarterly Journal of Economics* [pdf]
  - Patent grant in year $t = 0$
  - Productivity growth $\log(z_{it}) - \log(z_{i0})$ in years $t = 1, \ldots, 5$

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 0.013 | 0.017 | 0.019 | 0.023 | 0.024 |
| [2.34] | [2.29] | [2.78] | [3.50] | [4.31] |

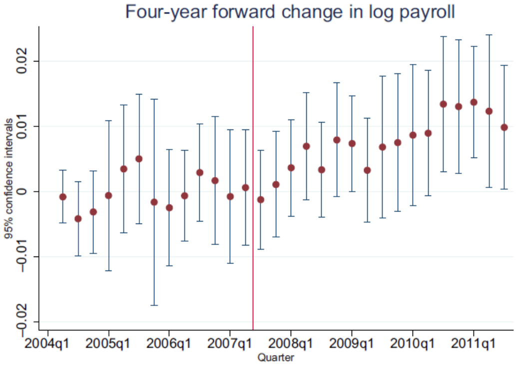
  - Capital growth $\log(k_{it}) - \log(k_{i0})$ in years $t = 1, \ldots, 5$

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 0.010 | 0.020 | 0.028 | 0.033 | 0.038 |
| [8.24] | [6.89] | [6.07] | [4.66] | [4.33] |

$\Rightarrow$ Consistent with complete markets

# Investment response to cash windfalls

- Barrot and Nanda, 2020, "The Employment Effects of Faster Payment: Evidence from the Federal Quickpay Reform," *Journal of Finance* [pdf]
  - Faster payment to small business gov't contractors: $y_{i0} \uparrow$ but no change in intertemporal income
  - Firm size growth after the reform




$\Rightarrow$ Inconsistent with complete markets

# Some research topics

- Macro finance: debt markets and consumption/investment
- Household finance: households' consumption and financial decisions
- Corporate innovation and corporate investment
- Corporate finance: financing frictions and real effects