Introduction to Finance for Data Scientists

Session 2: Valuation

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Asset valuation

- This morning
  - Assets — stocks, bonds, bank loans, insurance contracts, etc.
  - What they are used for — financing, risk sharing, etc.
  - How they are created and traded — IPOs, exchanges, etc.
- Now
  - What determines the value of assets, i.e., the price at which assets trade
  - Ex.: Lemonade sold 20% of its shares for $319M. How investors came up with this valuation?

Asset valuation

- **Valuation principle:** Value of asset = Value of benefits generated by the asset
  - Benefits usually are in cash: dividends, interests, rents, royalties, etc.
  - Can also be non-monetary, e.g., environmental
  - Called "cash flow" (CF) for brevity
- Two steps
  1. Forecast CF → data science
  2. Value forecasted CF → finance
- Value future and uncertain CF → discount for time and risk

Road map

Risk-free assets

Risky assets

Diversification

Time discounting

Q1. Do you prefer to earn €1 now or €1 in one year? 🤔

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Q2. You are indifferent between €X now and €1 in one year: X=?

- X is the **present value (PV)** of €1 in one year
- r such that $PV = 1/(1+r)$ is the **discount rate**

PV of risk-free cash flow

- More generally €D_t in t years has present value $\dfrac{D_t}{(1+r)^t}$

$\Rightarrow$ Stream of risk-free cash flow $\{D_t\}_{t\geq 1}$ has present value

$$\boxed{PV = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}}$$

- NB
  - Discount rate r is expressed on a per year basis, hence the power t
  - "Risk-free", "safe", "deterministic" $\neq$ "risky", "stochastic"

Valuing risk-free assets

Ex. 1 Consider two assets with the following risk-free cash flow

	Year 1	Year 2	Year 3
Asset A	10	10	10
Asset B	0	0	31

The discount rate is 5% per year

Q. Which project is more valuable? 2 minutes

PV(asset A) =

PV(asset B) =

⇒ Asset . . . has higher present value

Valuing risk-free assets

Ex. 2 US government bond that promises 100 USD in one year. Discount rate = 4% per year

Q1. What is the price of the bond? 2 minutes

$P =$

Valuing risk-free assets

Ex. 2 US government bond that promises 100 USD in one year. Discount rate = 4% per year

Q1. What is the price of the bond? 2 minutes

$P =$

Q2. You buy the bond and hold it for one year. What is your rate of return? 2 minutes

$$r \equiv \frac{\text{what you get}}{\text{what you paid}} - 1 =$$

Risk-free interest rate

- Return on a risk-free asset = discount rate of a risk-free asset ≡ risk-free interest rate (r_f)

    1. r_f is the same for all risk-free assets with cash flow at the same dates in the same currency

    2. r_f depends on the currency

    3. r_f depends on the horizon of cash flow

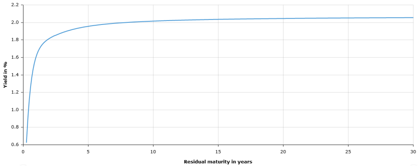
Risk-free interest rate

2. r_f depends on the currency

Australia	4.08%
Austria	2.77%
Belgium	2.79%
Canada	--
Denmark	2.61%
Finland	2.78%
France	2.76%
Germany	2.10%
Greece	4.74%
Ireland	2.74%
Italy	4.71%
Japan	0.25%
Netherlands	2.46%
New Zealand	4.28%
Portugal	3.21%
Spain	3.35%
Sweden	2.23%
Switzerland	1.40%
UK	4.15%
US	3.88%

10-year interest rate on government bonds [source]

3. r_f depends on the horizon of CF



Yield curve (a.k.a. term structure of interest rates) in euro [source]

Remark — Market efficiency

- "Value" and "price" are conceptually different
  - (Present) Value: how much investors should be willing to pay for the asset
  - Price: how much investors actually pay for the asset
- For now, we assume price = value, based on the following argument

  If price < value, investors buy, which pushes price up

  If price > value, investors sell, which pushes price down

  $\Rightarrow$ At the equilibrium of the market, it must be that price = value
- This assumption is called **market efficiency**. We'll return to it in session 3

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Risk aversion

Q1. Do you prefer to earn [€1 for sure] or [either €1.5 or €0.5 with fifty-fifty chance] 🤔

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- **Risk aversion:** Investors are willing to accept a lower expected payoff if risk is lower. Conversely, investors require a higher expected payoff if risk is higher

⇒ Risky cash flows are discounted

Valuing risky assets

- Most assets are risky: stocks, debt with default risk, etc.
- **Valuation principle**: Value of risky asset = PV of cash flow = expected cash flow discounted for time and risk
- An asset with risky payoff $\tilde{D}$ in one year has present value

$$PV = \frac{E[\tilde{D}]}{1+k}$$

where discount rate $k = \underbrace{\text{risk-free rate}}_{\text{time discounting}} + \underbrace{\text{risk premium}}_{\text{risk discounting}}$

- Discount rate = expected return

$$\tilde{r} = \frac{\tilde{D}}{P} - 1 \quad \Rightarrow \quad E(\tilde{r}) = \frac{E(\tilde{D})}{P} - 1 = \frac{E(\tilde{D})}{\frac{E(\tilde{D})}{1+k}} - 1 = k$$

Valuing risky assets **Quiz**

- Consider two stocks of companies specialized in facial recognition

  1. GreatTech has great technology: its expected CF next year is €400 million

  2. LameDuck has so-so technology: its expected CF next year is €100 million

     (NB: Next year's CF is in expectation. Assume same risk for both companies. To simplify, assume no CF after next year.)

Q1. Which stock has the higher price? ⏱ 1 minute

a. GreatTech b. LameDuck c. Both have same price

Q2. Which stock has the higher expected return? ⏱ 1 minute

a. GreatTech b. LameDuck c. Both have same expected return

Valuing risky assets

- An asset with risky cash flow $\{\tilde{D}_t\}_{t \geq 1}$ has present value

$$\boxed{PV = \sum_{t=1}^{\infty} \frac{E[\tilde{D}_t]}{(1+k)^t}}$$

where:

$$\boxed{\text{discount rate } (k) = \text{risk-free rate } (r_f) + \text{risk premium}}$$

NB: The PV formula has several names: *Dividend Discount Model (DDM)* in asset pricing, *Discounted Cash Flow (DCF) method* in corporate finance.

Valuing risky assets

- Expected return = discount rate

$$\boxed{E_t(\tilde{r}_{t+1}) = k}$$

where $\tilde{r}_{t+1} = \frac{\tilde{D}_{t+1}+\tilde{P}_{t+1}}{P_t} - 1$ is the (uncertain) return between t and $t+1$, and $\tilde{P}_{t+1}$ is the (uncertain) price at $t+1$ after the dividend payment

- Riskier assets have a higher risk premium

  i.e., a higher discount rate

  i.e., a higher expected return

Proof that expected return = discount rate

Expected return from t to $t+1$: $E_t(\tilde{r}_{t+1}) = \dfrac{E_t(\tilde{D}_{t+1}) + E_t(\tilde{P}_{t+1})}{P_t} - 1$

Using the PV formula: $P_t = \displaystyle\sum_{\tau=1}^{\infty} \frac{E_t(D_{t+\tau})}{(1+k)^{\tau}}$

Expected price at $t+1$: $E_t(\tilde{P}_{t+1}) = E_t\left[\displaystyle\sum_{\tau=1}^{\infty} \frac{E_{t+1}(D_{t+1+\tau})}{(1+k)^{\tau}}\right]$

Using Bayes rule: $E_t(\tilde{P}_{t+1}) = \displaystyle\sum_{\tau=1}^{\infty} \frac{E_t(D_{t+1+\tau})}{(1+k)^{\tau}}$

Finally: $E_t(\tilde{r}_{t+1}) = \dfrac{(1+k)P_t}{P_t} - 1 = k$

Historical perspective

- Historical returns of main asset classes, 1960–2017[1]

	Moments of return distribution (in % per year)	
	S.D.	Mean
Stocks	17.3	11.3
Real estate	19.3	12.3
Corporate bonds	8.4	7.8
Commodities[2]	24.9	8.7

⇒ Riskier assets have higher expected returns

NB: Commodities are puzzling: high S.D. but not high mean return. Any idea?

[1] Doeswijk, Lam and Swinkels, "Historical Returns of the Market Portfolio," *Review of Asset Pricing Studies* 2019 [pdf]

[2] Commodities include mainly metals (such as gold), energy products (such as oil) and agricultural products

Example — Valuing the stock market

- What should be the valuation of the US stock market?
  - That is, the aggregate value of all companies listed in the US (P_t)
  - Calculate it as a multiple of current earnings (E_t)
- Valuation principle: value = PV of all future dividends
  - US companies pay on average half of earnings as dividends: $D_t = 0.5E_t$
  - Suppose aggregate earnings has constant expected growth rate:

    $$E_t[E_{t+\tau}] = (1+g)^\tau E_t \text{ for } \tau \geq 1 \quad \Rightarrow \quad P_t = \frac{(1+g)0.5E_t}{k-g}$$

  - Discount rate: k = risk-free rate + risk premium

    Risk-free rate = interest rate on US government bonds $\simeq$ 3.5%

    Risk premium: calibrate using historical stock market risk premium $\simeq$ 6%, but lower today, say 4%

  - Earnings growth: calibrate using historical (nominal) GDP growth $\approx$ 6%

$\Rightarrow$ Value of US stocks $\simeq \frac{1.06 \times 0.5}{0.075 - 0.06} \simeq$ 35 times current earnings

Example — Valuing the stock market

- Compare to data: price/earnings ratio of US stock market

[source]

$\Rightarrow$ PV formula + ballpark calibration $\Rightarrow$ correct order of magnitude

Q. How do you interpret the spike around 2000? the drop in 2008? the increasing trend since 2010?

Using the PV formula

Input needed to calculate the PV of a given asset

- Expected future CF: data analysis
- Discount CF at k = risk-free rate + risk premium
- Risk-free rate is observable
- Risk premium = ?

  Which dimension of risk determines the risk premium?

  Variance? No, covariance, because of diversification

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Diversification

Diversification

a Nobel winning idea

Harry M. Markowitz (1990)

Diversification

- A fund invests €10 million spread equally over n companies
- Each €1 invested in company i returns €$(1 + r_i)$ with the same $E(r_i)$ and $V(r_i) \equiv \sigma^2$ for all i
- Portfolio return: $\frac{\sum_{i=1}^{n}(1+r_i)\times 10\,\text{million}/n}{10\,\text{million}} - 1 = \frac{1}{n}\sum_{i=1}^{n} r_i$

Q1. Is it safer to invest in $n = 1$ or $n = 10$ companies?

a. $n = 1$ b. $n = 10$ c. It depends

Diversification

- Consider two investment portfolios

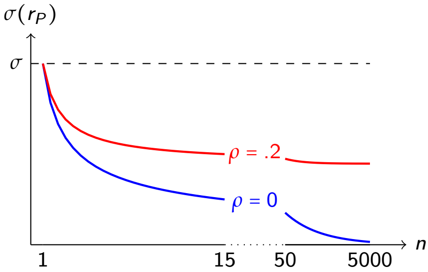
A. $n = 1000$ companies in the same industry: $Cov(r_i, r_j) = 0.2\ \sigma^2$ for $i \neq j$

B. $n = 1000$ companies in unrelated industries: $Cov(r_i, r_j) = 0$ for $i \neq j$

Q. Which portfolio is safer?

A. B. It depends

Diversification



1. Portfolio risk $\downarrow$ ***n***

   Proof: $\sigma(r_P) = \left(\frac{1}{n} + \frac{n-1}{n}\rho\right)^{1/2}\sigma \downarrow n$

2. When $n \to \infty$, portfolio risk does not depend on the total variance of individual asset returns, but on their **covariance**

   Proof: $\lim_{n\to\infty} \sigma(r_P) = \rho^{1/2}\sigma$

Diversification — Summary

- Risk independent between assets can be eliminated by diversification

  $=$ **idiosyncratic (diversifiable) risk**

- Risk correlated between assets cannot be eliminated by diversification

  $=$ **systematic (non-diversifiable) risk**

NB: A portfolio with large n is said to be "well diversified", which does not mean risk-free: idiosyncratic risk is eliminated but systematic risk is not

Diversification — Implication for risk premium

Q. Do you require a higher expected return on portfolio with $\rho = 0$ or portfolio with $\rho = 0.2$?

Diversification — Implication for risk premium

- Idiosyncratic risk does <u>not</u> command a risk premium

- Systematic risk <u>commands</u> a risk premium

- How to measure an asset's systematic risk and the associated risk premium?

  – In previous example with symmetric assets: systematic risk = covariance with other assets

  – More generally: systematic risk = covariance with portfolio of all assets $\Rightarrow$ the Capital Asset Pricing Model (next class)

One last brain-teaser

Do expect high or low expected return on **gold**?

Until tomorrow

- Practice problems on Slack
- See you tomorrow morning ☺