Incentive Constrained Dynamic Asset Pricing

Johan Hombert (HEC Paris)

HEC PARIS

Summer School “Incentives in Finance”
30 August 2023

Promises

- Financial instruments = promises of future payments

- How to enforce promises? Backed by assets (cash flow and/or re-sell value)

- However, in many contexts, assets are imperfectly pledgeable i.e. creditors cannot extract full asset value

- Impact of imperfect pledgeability on risk sharing and asset pricing?

Preview of Literature

1. AP with (exogenous) financing frictions

   1a. AP with leverage constraints (Gromb Vayanos 02, Brunnermeier Pedersen 09, Gârleanu Pedersen 11)

   - Risk-free debt only
   - Leverage constraint $\Rightarrow$ distorted asset prices, shocks amplification

   1b. Intermediary AP (He Krishnamurthy 13), macro-fin. (Brunnermeier Sannikov14)

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   - Missing risk-sharing market $\Rightarrow$ distorted asset prices, shocks amplification

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      - State-contingent liabilities
      - Imperfectly pledgeable income ⇒ imperfect risk sharing

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Today's lecture: AP with endogenously incomplete markets (Biais Hombert Weill 21)

- State-contingent liabilities + imperfectly pledgeable collateral
- $\Rightarrow$ Imperfect risk sharing (as in 2.) and distort asset prices (related to but different/new relative to 1a. and 1b.)

Two-Period Model

- $t = 0, 1$
- State $s = 1, \ldots, S$ realized at $t = 1$
- Securities
  - Trees (collateral) indexed by j

    trade at $t = 0$: price p_j

    dividend at $t = 1$: $d_j(s) \geq 0$
  - Complete set of Arrow securities indexed by s

    trade at $t = 0$: price $q(s)$

    payoff 1 at $t = 1$ in state s
- Agents indexed by i

  endowed with $\bar{n}_{ij} > 0$ trees and non-tradable income e_{i0} and $e_{i1}(s)$

  trade securities at $t = 0$

  consume at $t = 0, 1$, expected utility $u_i(c_{i0}) + \beta \sum_s \pi(s) u_i(c_{i1}(s))$

Agents' Problem

Agents maximize expected utility by choice of

- consumption: c_{i0}, $c_{i1}(s)$ for each state s
- Arrow positions: $a_i(s) \lesseqgtr 0$ for each state s
- tree positions: $n_{ij} \geq 0$ for each tree j

  NB: Allowing short tree positions does not change the equilibrium because they can be replicated using short Arrow positions.

subject to

- budget constraints
- incentive-compatibility constraint

Budget Constraints

At $t = 0$

$$\underbrace{c_{i0}}_{\text{consump.}} + \underbrace{\sum_j p_j \, n_{ij}}_{\text{trees portfolio}} + \underbrace{\sum_s q(s) \, a_i(s)}_{\text{Arrow portfolio}} \leq \underbrace{\sum_j p_j \, \bar{n}_{ij} + e_{i0}}_{\text{endowment}}$$

At $t = 1$ for each s

$$\underbrace{c_{i1}(s)}_{\text{consump.}} = \underbrace{\sum d_j(s) \, n_{ij}}_{\text{trees payoff}} + \underbrace{a_i(s)}_{\text{Arrow payoff}} + \underbrace{e_{i1}(s)}_{\text{endowment}}$$

Default Costs

- Short Arrow positions = liabilities

- Suppose that, if an agent defaults, creditor seizes debtor's assets but recovers only fraction $(1 - \theta)$ of asset value

- Default costs = fraction θ of asset value

  mortgages (Campbell-Giglio-Pathak 11)

  non-financial firms (Andrade-Kaplan 98)

  financial firms (Fleming and Sarkar 14, Credit Suisse AT1s)

Limited Commitment

- Suppose debtor has can threaten to default and make a take-it-or-leave-it offer to creditor
  - At $t = 1$, debtor can renegotiate liab down to fraction $1 - \theta$ of asset value
  - At $t = 0$, debtor cannot credibly pledge more than $1 - \theta$ of asset value
- Suppose non-tradable income $e_{i1}(s)$ is fully non-pledgeable

$\Rightarrow$ **IC constraint:**

$$\underbrace{-a_i(s)}_{\text{liabilities}} \leq \underbrace{(1-\theta) \sum_j d_j(s)\, n_{ij}}_{\text{pledgeable income}}$$

Equilibrium

- Definition: allocation and prices such that agents maximize given prices and markets clear
- Existence because IC only imposes additional linear constraints
- Constrained Pareto optimality because prices don't show up in IC $\Rightarrow$ no pecuniary externality

Imperfect Risk Sharing

Alvarez & Jermann 2000, Chien & Lustig 2010, Rampini & Viswanathan 2010

- FOC / consumption

$$\boxed{\frac{\pi(s)\, u_i'(c_{i1}(s)) + \mu_i(s)}{u_i'(c_{i0})} = q(s)}$$

- MRS not equalized across agents $\Rightarrow$ **imperfect risk sharing**

Imperfect Risk Sharing

Example 1: Aggregate Risk

- 2 agents with CRRA utility $\gamma_1 < \gamma_2$ 1 is risk-tolerant, 2 is risk-averse
- 2 states s_1 and s_2
- 1 tree $d(s_1) < d(s_2)$ s_1 bad state, s_2 good state
- $e = 0$
- Frictionless economy ($\theta = 0$): risk-tolerant agent insures risk-averse agent against bad state

  risk-tolerant sells Arrow against bad state: $a_1(s_1) < 0$

  which risk-averse purchases by selling Arrow against good state: $a_2(s_2) < 0$

  MRS is equalized across agents

- With IC problems: if $\theta > \overline{\theta}$

  IC of risk-tolerant binds in bad state

  IC of risk-averse binds in good state

  risk-averse consumes too little and risk-tolerant too much in bad state

Imperfect Risk Sharing

Example 2: Idiosyncratic Risk

- 2 agents with CRRA utility $\gamma_1 = \gamma_2$
- 2 states s_1 and s_2
- 1 tree $d(s_1) = d(s_2)$ — no aggregate risk
- $e_{11}(s_1) = e_{21}(s_2) > e_{11}(s_2) = e_{21}(s_1)$ — state s_i is good state for agent i
- Frictionless equilibrium ($\theta = 0$): idiosyncratic risk is perfectly shared

  agent 1 sells Arrow against state s_1: $a_1(s_1) < 0$

  agent 2 sells Arrow against state s_2: $a_2(s_2) < 0$

  MRS is equalized across agents

- With IC problems: if $\theta > \overline{\theta}$

  IC of agent 1 binds in state s_1

  IC of agent 2 binds in state s_2

  agent i consumes too much and other agent too little in state s_i

Arrow Securities Pricing

- For each state s, there is at least one unconstrained agent

  Liabilities (short Arrow positions) create IC problem. By market clearing, for each state s, there must be at least one agent who is long state s-Arrow security

$\Rightarrow$ Arrow securities are priced by unconstrained agents' MRS

$$q(s) = \max_i \frac{\pi(s) u_i'(c_{i1}(s))}{u_i'(c_{i0})}$$

Endogenous Segmentation

Biais, Hombert & Weill (2021)

- FOC / tree holdings

$$p_j \;\geq \sum_s q(s)\, d_j(s) - \sum_s \frac{\mu_i(s)}{\lambda_i}\, \theta\, d_j(s) \quad \text{with} = \text{if agent } i \text{ holds tree } j$$

tree price $\geq$ agent i's valuation for tree j

- Agent i's valuation for tree j

  1st term: PV of cash flow at state prices (common value)

  2nd term: shadow IC cost for agent i (*endogenously varies across agents*)

$\Rightarrow$ **Segmentation:** each tree is held by agents who value it most

Because they have the lowest shadow IC cost of holding it

Endogenous Segmentation

Example 1: Aggregate Risk

- Same as previous example 1 but now with many trees
- State s_1 is bad state: $\sum_j d_j(s_1) < \sum_j d_j(s_2)$
- Trees differ by exposure to aggregate risk: higher $\frac{d_j(s_2)}{d_j(s_1)} \Leftrightarrow$ riskier
- Equilibrium
  - Risk-tolerant (HF) insures risk-averse (PF) against bad state
  - IC of HF binds in bad state
  - If PF holds risky trees $\Rightarrow$ needs more insurance against bad state $\Rightarrow$ requires larger HF liabilities in bad state $\Rightarrow$ tightens IC
  - To minimize HF liabilities in bad state, PF holds safe trees and HF holds risky trees
- By contrast, in frictionless economy, tree holdings are indeterminate because they can be replicated by Arrow securities

Deviation from Law of One Price

- Price = max of private valuation across agents

$$\boxed{p_j \;=\; \max_i \; \sum_s q(s)\, d_j(s) - \sum_s \frac{\mu_i(s)}{\lambda_i}\, \theta\, d_j(s)}$$

$\Rightarrow$ **Basis** relative to Arrow replicating portfolio

- Underlying < Replicating derivative: $p_j \leq \sum_s q(s) d_j(s)$
- Not an arbitrage: buy tree/sell Arrow would violate IC

Deviation from Law of One Price

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- **More generally:** Tree priced below any replicating portfolio of long positions in trees and/or Arrow securities
  - Proof: Private valuations linear in dividends + convexity of max
  - Inequality is strict if there is no agent with long positions in all the securities in the replicating portfolio
  - Intuition: Tree is bundle of risks that cannot be traded separately, whereas replicating portfolio is same bundle of risks that can be traded separately $\Rightarrow$ Basis reflects value of stripping

Example

- Convertible bond = straight bond + call of stock
- Price of convertible < price of straight bond + price of call
  - In model and in data (Mitchell-Pulvino 12)
- "Convertible arbitrage" strategy
  - Buy convertible
  - Issue replicating portfolio and sell specific risks to different clienteles
  - Constrained by ability to issue liabilities (in model and in practice)

Literature on LoOP Deviation

- Exogenous leverage constraints (Gromb Vayanos 02, Brunnermeier Pedersen 09, Gârleanu Pedersen 11)
  - Two assets with same CF, different pledgeability parameter
  - Less pledgeable $\Rightarrow$ lower price
- Endogenous incomplete markets (Biais Hombert Weill 21)
  - Underlying vs. replicating portfolio with same CF, same pledgeability
  - Underlying $\Rightarrow$ lower price
  - Basis always in same direction, does not rely on exogenous difference in pledgeability

Concave Beta Pricing

- Price = max of private valuation across agents

$$p_j \;=\; \max_i \;\sum_s q(s)\, d_j(s) - \sum_s \frac{\mu_i(s)}{\lambda_i}\,\theta\, d_j(s) \;\equiv\; \max_i \;E[M_i(s) d_j(s)]$$

  where $M_i(s) = \frac{1}{\pi(s)}\left[q(s) - \frac{\mu_i(s)}{\lambda_i}\,\theta\right]$ is agent i's SDF

- Project tree returns $R_j(s) = d_j(s)/p_j$ onto factors

$$R_j(s) = E[R_j(s)] + \sum_{k=1}^{K} \beta_{j,k} F_k(s) + \epsilon_j(s)$$

  and suppose $\epsilon_j(s)$ is unpriced risk [formally, factors span agents' SDFs: $Cov(F_k(s), M_i(s)) = 0$]

- **Concave Beta Pricing:** expected return $E[R_j(s)]$ is concave in factor exposure $(\beta_{jk})_{k=1}^{K}$
  - Proof: concavity of max
  - Same intuition as before: intermediate-beta tree can be replicated by, and is therefore cheaper, than portfolio of low-beta trees + high-beta trees

Literature on SML Slope

- Assume stock market risk is the only priced risk
- Exogenous leverage constraints (Frazzini Pedersen 14)
  - SML is flatter than in frictionless economy
- Endogenous incomplete markets (Biais Hombert Weill 21)
  - SML is concave
  - Consistent with evidence (Frazzini Pedersen 14, Hong Sraer 16)

Dynamics

- $t = 0, 1, 2, \ldots, T$
- Transition matrix $\pi(s_t, s_{t+1})$
- If agent defaults, creditor recovers $(1-\theta)$ of asset value $=$ dividend $+$ price

  $\Rightarrow$ IC constraint

$$-a_i(s_{t+1}) \leq (1-\theta) \sum_j \left[d_j(s_{t+1}) + p_j(s_{t+1})\right] n_{ij}(s_t)$$

liabilities pledgeable income

- New effects
  - Pecuniary externality: price in the constraint $\Rightarrow$ equilibrium is not constrained Pareto efficient
  - Shocks affect next period's wealth distribution and tightness of IC
- Avenues for research
  - Calibration
  - Amplification of shocks
  - Financial intermediation (heterogeneous θ_i)

Thank You and Enjoy the Workshop! 🎉