

Microeconomics 1

Intertemporal choices and markets

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Road map

Time preference

Debt markets

Endowment economy

Production economy

Evidence

Time preference

Q. Do you prefer to eat a delicious chocolate today or in one year?

Time preference

Q. Do you prefer to eat a delicious chocolate today or in one year?

► People usually prefer immediate utility over delayed utility

- Uncertainty of human life
- Excitement of immediate consumption
- Discomfort of deferring available gratification
- Underestimate future needs
- etc.

Discounted utility

- ▶ Simple representation of time preference: **discounted utility**

$$U(c_0, \dots, c_T) = u(c_0) + \beta u(c_1) + \dots + \beta^T u(c_T)$$

- ▶ $\beta \leq 1$ is the **time discount factor**

- ▶ Implicit assumptions

- Time-separable and stationary
- Exponential discounting

Discounted utility

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Q. A more patient individual has higher or lower beta?

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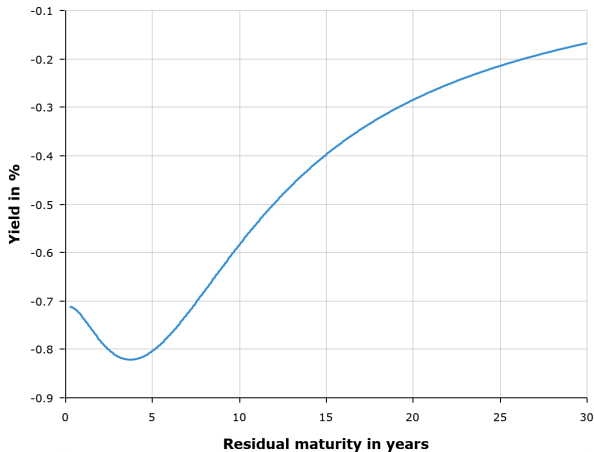
Evidence

Debt markets

- ▶ Financial instrument to move resources across time: debt
- ▶ Total world credit: \$200 tn = 2.5x GDP
 - to firms: \$80 tn
 - to governments: \$70 tn
 - to households: \$50 tn
 - (excl. credit to financial sector to avoid double counting)
- [source: BIS]
- ▶ Who are the lenders? → Other households and firms (mostly through financial institutions)
- ▶ Examples of debt instruments: loans, bonds, bank deposits, etc.

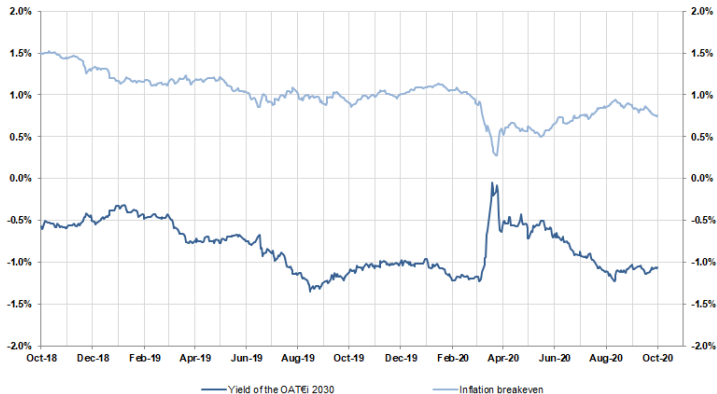
Debt markets

- Term structure of nominal interest rates, Euro Area [source: ECB]



Debt markets

- Real interest rate, France [source: AFT]



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Model

- ▶ Two periods $t = 0, 1$
- ▶ Agents $i = 1, \dots, N$ have preferences $U_i = u(c_{i0}) + \beta u(c_{i1})$ with $u(\cdot)$ increasing concave, time discount factor $\beta < 1$
- ▶ Agents receive endowment of non-storable consumption good: y_{it} for agent i at date t
- ▶ NB: riskless endowment economy
 - Deterministic y_{it} (no risk)
 - Exogenous y_{it} (no production choice)
 - We'll relax both assumptions later

Autarky

- ▶ Suppose no financial instruments
- ▶ Good non storable $\Rightarrow c_{i0} = y_{i0}$ and $c_{i1} = y_{i1}$ for all agents i

Q: This allocation is Pareto optimal: true or false?

Autarky

- ▶ Suppose no financial instruments
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Q: This allocation is Pareto optimal: true or false?

- ▶ False: Pareto improvement
 - Agents with high y_{i1}/y_{i0} consume more at date 0 and less at date 1
 - Agents with low y_{i1}/y_{i0} consume less at date 0 and more at date 1
- ▶ Need a debt instrument

Debt market

- ▶ Debt instrument: risk-free bond
 - Lend 1 unit of consumption at date 0, get back $1 + r$ units at date 1
 - Borrow 1 unit of consump at date 0, pay back $1 + r$ units at date 1
 - r : interest rate determined in equilibrium

Q: The market with a risk-free bond is complete: true or false?

Debt market

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 - r : interest rate determined in equilibrium

Q: The market with a risk-free bond is complete: true or false?

- ▶ True: no risk \rightarrow only possible transaction = move resources between date 0 and date 1 \rightarrow achieved by risk-free bond

Equilibrium definition

- ▶ Equilibrium: all agents maximize and markets clear
- ▶ Agent i chooses (c_{i0}, c_{i1}, b_i) to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

date 0 budget constraint: $c_{i0} + b_i \leq y_{i0}$

date 1 budget constraint: $c_{i1} \leq y_{i1} + (1 + r)b_i$

- ▶ Market clearing

for bond: $\sum_{i=1}^N b_i = 0$

for consumption good at date $t = 1, 2$: $\sum_{i=1}^N c_{it} = \sum_{i=1}^N y_{it}$

Agents' choices

- Consolidate sequential budget constraints by eliminating b_i

$$\Rightarrow \text{inter-temporal budget constraint: } c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r}$$

- Interest rate: relative price of date 1 consump. / date 0 consump.
- LHS: date 0 price of intertemporal consumption
- RHS: date 0 price of intertemporal income

- First order conditions

- w.r.t. c_{i0} : $u'(c_{i0}) - \lambda_i = 0$

where λ_i = multiplier of inter-temporal BC

- w.r.t. c_{i1} : $\beta u'(c_{i1}) - \frac{\lambda_i}{1+r} = 0$

Agents' choices

► FOC $\Rightarrow u'(c_{i0}) = (1 + r)\beta u'(c_{i1})$

► Intuition

- LHS: MU of 1 unit of consumption at date 0
- RHS: MU of saving 1 unit at date 0, lending it, and consuming the proceeds at date 1

► “Euler equation”

Agents' choices

- ▶ Euler equation holds for all agents

⇒ The **intertemporal marginal rate of substitution** $\frac{\beta u'(c_{i1})}{u'(c_{i0})}$ is equalized across all agents and is equal to $\frac{1}{1+r}$

- ▶ Analogy with static case
 - With several goods: MRS between goods is equalized across agents and equal to relative good prices (studied with Tristan)
 - With several states of nature: MRS between states is equalized across agents and equal to relative state prices (studied with Bruno)

QUIZ

Q. For a given intertemporal income $y_{i0} + \frac{y_{i1}}{1+r}$, an individual's consumption at date 0...

QUIZ

Q. For a given intertemporal income $y_{i0} + \frac{y_{i1}}{1+r}$, an individual's consumption at date 0... does not depend on her date 0's income

► “Permanent income hypothesis”

Agents' choices

- ▶ Solve with CRRA utility: $u(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$
 - $\gamma \geq 0$: relative risk aversion
 - IMRS $\frac{\beta u'(c_{i1})}{u'(c_{i0})} = \beta \left(\frac{c_{i1}}{c_{i0}} \right)^{-\gamma}$ is a decreasing fcn of consump growth
- ▶ FOC \Rightarrow consumption growth is equalized across agents

$$\frac{c_{i1}}{c_{i0}} = [(1+r)\beta]^{1/\gamma} \text{ for all } i$$

Market clearing

- ▶ Equilibrium interest rate is determined by market clearing

- ▶ Aggregate FOC across agents: $\sum_{i=1}^N c_{i1} = [(1+r)\beta]^{1/\gamma} \sum_{i=1}^N c_{i0}$

- ▶ Market clearing for good at each date $t=1,2$: $\sum_{i=1}^N c_{it} = \sum_{i=1}^N y_{it} \equiv C_t$

- ▶ NB: mkt clearing for good implies mkt clearing for debt (Say's Law).
Proof: aggregate inter-temporal budget constraints across agents

⇒ Equilibrium interest rate: $1+r = \beta^{-1} \left(\frac{C_1}{C_0} \right)^\gamma$

QUIZ

Q1. The equilibrium interest rate is higher when...

Q2. The equilibrium interest rate is higher when...

Comparative static

- ▶ Higher agg consump growth $\frac{C_1}{C_0} \Rightarrow$ equilibrium interest rate is higher
 - Intuition: agents want to borrow to consume more at date 0 $\Rightarrow r \uparrow$
- ▶ Agents are more impatient (lower β) \Rightarrow eqm interest rate is higher
 - Intuition: agents want to borrow to consume more at date 0 $\Rightarrow r \uparrow$

Debt market

► Who lends and who borrows?

- FOC $\Rightarrow \frac{c_{i1}}{c_{i0}} = \frac{C_1}{C_0}$ (indiv consump gwth = agg consump gwth)

- Inter-temporal BC: $c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r}$

$$\Rightarrow c_{i0} = \frac{y_{i0} + \frac{y_{i1}}{1+r}}{C_0 + \frac{C_1}{1+r}} C_0$$

- Position in bond: $b_i = y_{i0} - c_{i0} = \left(\frac{C_1}{C_0} - \frac{y_{i1}}{y_{i0}} \right) \frac{y_{i0}}{1+r + \frac{C_1}{C_0}}$

► Agents with income growth below agg income growth: lend

Agents with income growth above agg income growth: borrow

Welfare

Q. The equilibrium allocation is Pareto optimal: true or false?

Welfare

Q. The equilibrium allocation is Pareto optimal: true or false?

- True
- Intuition: lending/borrowing gains from trade are exhausted in equilibrium
- First Welfare Theorem applies (complete markets, no externality)

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Production

- ▶ Same model as before + each agent has access to a production technology
 - Invest k units of good at date 0 \rightarrow generate $f(k)$ units of goods at date 1
 - $f(\cdot)$ is the production function: increasing and concave (decreasing return to scale)
 - Equivalence with cost function representation: to produce y units, need $c(y)$ units $\rightarrow c(\cdot) = f^{-1}(\cdot)$
- ▶ Some agents have better technologies than others: $f_i(\cdot) = z_i f(\cdot)$

Equilibrium definition

- ▶ Equilibrium: all agents maximize and markets clear
- ▶ Agent i chooses $(c_{i0}, c_{i1}, k_i, b_i)$ to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

date 0 budget constraint: $c_{i0} + k_i + b_i \leq y_{i0}$

date 1 budget constraint: $c_{i1} \leq y_{i1} + z_i f(k_i) + (1 + r)b_i$

- ▶ Market clearing for bond: $\sum_i b_i = 0$

Market clearing for good at date 0: $\sum_i c_{i0} + \sum_i k_i = \sum_i y_{i0}$

Market clearing for good at date 1: $\sum_i c_{i1} = \sum_i y_{i1} + \sum_i z_i f(k_i)$

Building the intuition

Q1. Agents with high date 0 endowment y_{i0} tend to...

Q2. Agents with high productivity z_i tend to...

Building the intuition

Q1. Agents with high date 0 endowment y_{i0} tend to...

lend but not to invest more

Q2. Agents with high productivity z_i tend to...

borrow to invest more

Agents' choices

- Consolidate budget constraint to eliminate b_i

$$\max_{c_{i0}, c_{i1}, k_i} u(c_{i0}) + \beta u(c_{i1})$$

$$\text{s.t. } c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r} + \left(-k_i + \frac{z_i f(k_i)}{1+r} \right)$$

- Investment decision is independent from consumption decision
- FOC w.r.t. k_i : $z_i f'(k_i) = 1 + r$
 - LHS: marginal productivity of capital
 - RHS: marginal cost of capital

Agents' choices

► $k_i = f'^{-1} \left(\frac{1+r}{z_i} \right)$

- k_i is increasing in productivity z_i (by concavity of $f(\cdot)$)
- k_i does not depend on endowment (y_{i0}, y_{i1})

► FOC w.r.t. consumption: $u'(c_{i0}) = (1+r)\beta u'(c_{i1})$

Equilibrium

- Suppose Cobb-Douglas production function: $f(k) = k^\alpha$, $\alpha \in (0, 1)$

$$\Rightarrow k_i = \left(\frac{\alpha z_i}{1+r} \right)^{\frac{1}{1-\alpha}} \quad \Rightarrow z_i f(k_i) = \left(\frac{\alpha}{1+r} \right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}}$$

- Market clearing for good

$$\text{at date 0: } \sum_i c_{i0} = \sum_i y_{i0} - \sum_i \left(\frac{\alpha z_i}{1+r} \right)^{\frac{1}{1-\alpha}} \quad (\text{C0})$$

$$\text{at date 1: } \sum_i c_{i1} = \sum_i y_{i1} + \sum_i \left(\frac{\alpha}{1+r} \right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}} \quad (\text{C1})$$

- Suppose CRRA utility and aggregate FOC w.r.t. consump across i

$$1+r = \beta^{-1} \left(\frac{\sum_i c_{i1}}{\sum_i c_{i0}} \right)^{1/\gamma} \quad (\text{FOC})$$

- Substitute agg consump in (FOC) using (C0) and (C1) \rightarrow pins down r

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Consumption

► Implications of complete markets

- IMRS of consumption is equalized across agents
- Permanent income hypothesis: current consumption depends on intertemporal income not current income

► Empirically

- People borrow when young, save when middle aged, dis-save when retired: consistent with complete markets
- However, does consumption depend to current income?

Test: consumption response to cash windfalls

Consumption response to cash windfalls

- Souleles, 1999, “The Response of Household Consumption to Income Tax Refunds,” *American Economic Review* [\[paper\]](#)

TABLE 5—THE MARGINAL PROPENSITY TO CONSUME

	Food	Strictly nondurables	Total consumption	Durables	Nonvehicular durables
Refund* $\pi_{(2 \text{ weeks})}$	0.027 (0.017)	0.045** (0.021)	0.344** (0.116)	0.294** (0.122)	0.128** (0.061)
Number of observations	7,622	7,622	7,525	7,525	7,525

Investment

► Implications of complete markets

- Marginal product of investment $z_i f'(k_i)$ is equalized across agents

⇒ More productive agents employ more capital

- Investment does not depend on current income

⇒ Productive agents borrow to invest

► Empirically

- Do firms invest more when they become more productive?

Test: investment response to patent grants

- Does investment depend on wealth?

Test: investment response to cash windfalls

Investment response to patent grants

- Kogan, Papanikolaou, Seru and Stoffman, 2017, “Technological Innovation, Resource Allocation and Growth,” *Quarterly Journal of Economics* [\[paper\]](#)

- Patent grant in year $t = 0$
- Productivity growth $\log(z_{it}) - \log(z_{i0})$ in years $t = 1, \dots, 5$

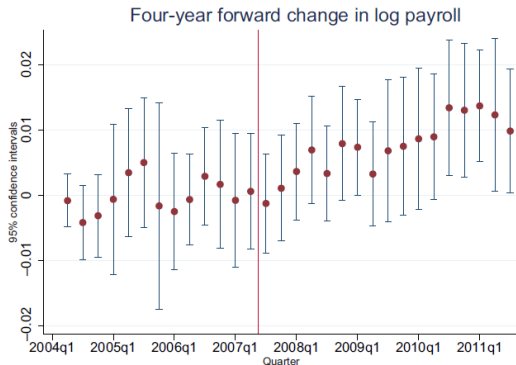
1	2	3	4	5
0.013	0.017	0.019	0.023	0.024
[2.34]	[2.29]	[2.78]	[3.50]	[4.31]

- Capital growth $\log(k_{it}) - \log(k_{i0})$ in years $t = 1, \dots, 5$

1	2	3	4	5
0.010	0.020	0.028	0.033	0.038
[8.24]	[6.89]	[6.07]	[4.66]	[4.33]

Investment response to cash windfalls

- ▶ Barrot and Nanda, 2020, “The Employment Effects of Faster Payment: Evidence from the Federal Quickpay Reform,” *Journal of Finance* [\[paper\]](#)
 - Faster payment to small business gov’t contractors: $y_{i0} \uparrow$, $y_{i1} \downarrow$
 - Firm size growth after the reform



Some research topics

- ▶ Macro finance: debt markets and consumption/investment
- ▶ Household finance: households' consumption and financial decisions
- ▶ Corporate innovation and corporate investment
- ▶ Corporate finance: financing frictions and real effects