Microeconomics 1

Session 9: Intertemporal choices and markets

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Road map

Time preference

Debt markets

Endowment economy

Production economy

Evidence

Time preference

Q. Do you prefer to eat a delicious chocolate today or in one year?

Time preference

Q. Do you prefer to eat a delicious chocolate today or in one year?

- People usually prefer immediate utility over delayed utility
  - Uncertainty of human life
  - Excitement of immediate consumption
  - Discomfort of deferring available gratification
  - Underestimate future needs
  - etc.

Discounted utility

- Simple representation of time preference: discounted utility

$$U(c_0, \ldots, c_T) = u(c_0) + \beta u(c_1) + \ldots + \beta^T u(c_T)$$

- $\beta \leq 1$ is the time discount factor
- Implicit assumptions
  - Time-separable and stationary
  - Exponential discounting

Discounted utility

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Q. A more patient individual has higher or lower beta?

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Q. A more patient individual has higher or lower beta? higher

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Debt markets

- Financial instrument to move resources across time: debt
- Total world credit: \$200 tn = 2.5x GDP

  to firms: \$80 tn

  to governments: \$70 tn

  to households: \$50 tn

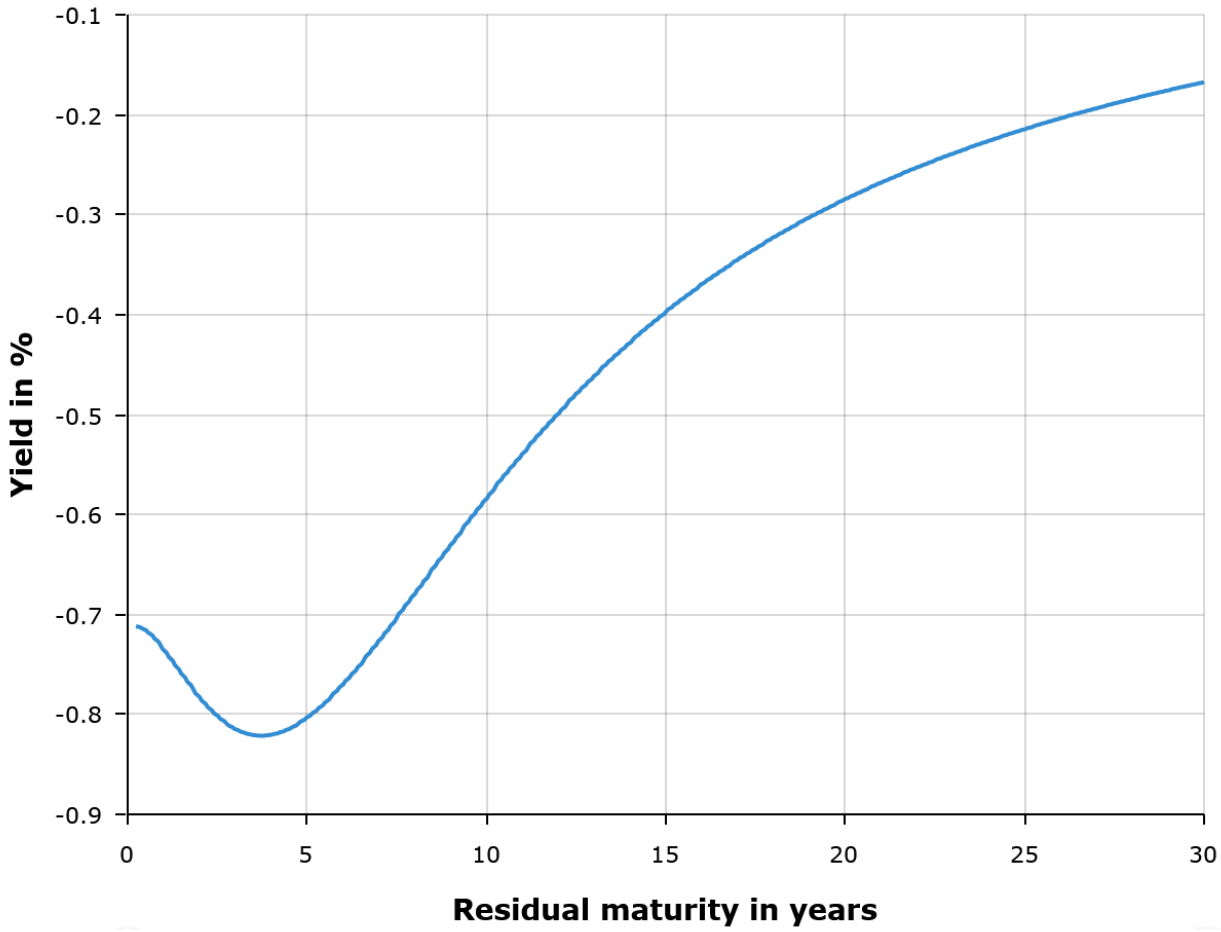
  (excl. credit to financial sector to avoid double counting)

  [source: BIS]

- Who are the lenders? $\rightarrow$ Other households and firms (mostly through financial institutions)
- Examples of debt instruments: loans, bonds, bank deposits, etc.

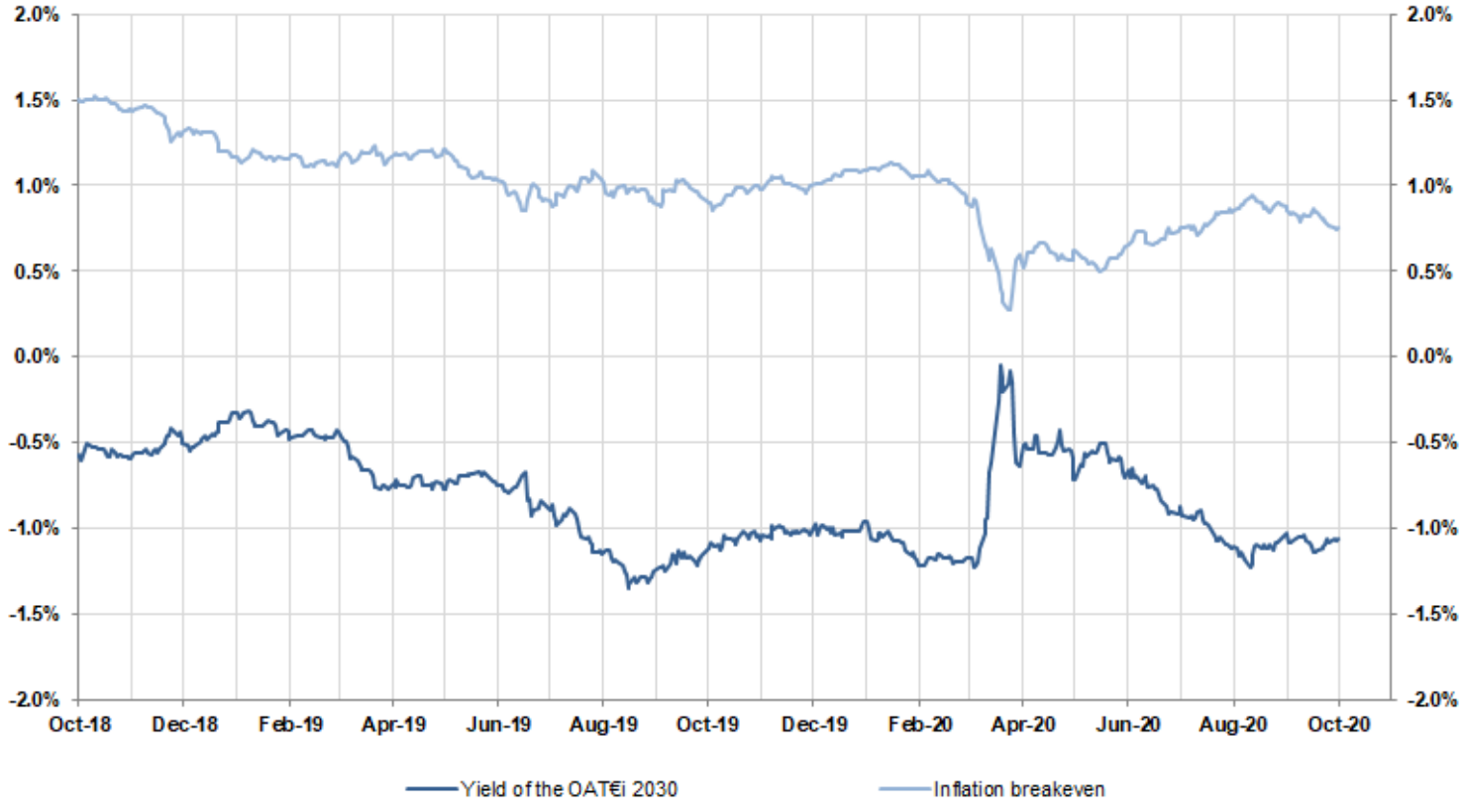
Debt markets

- Term structure of nominal interest rates, Euro Area [source: ECB]



Debt markets

- Real interest rate, France [source: AFT]



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Model

- Two periods $t = 0, 1$
- Agents $i = 1, ..., N$ have preferences $U_i = u(c_{i0}) + \beta u(c_{i1})$ with $u(.)$ increasing concave, time discount factor $\beta < 1$
- Agents receive endowment of non-storable consumption good: y_{it} for agent i at date t
- NB: riskless endowment economy
  - Deterministic y_{it} (no risk)
  - Exogenous y_{it} (no production choice)
  - We'll relax both assumptions later

Autarky

- Suppose no financial instruments
- Good non storable $\Rightarrow$ $c_{i0} = y_{i0}$ and $c_{i1} = y_{i1}$ for all agents i

Q: This allocation is Pareto optimal: true or false?

Autarky

- Suppose no financial instruments
- Good non storable $\Rightarrow$ $c_{i0} = y_{i0}$ and $c_{i1} = y_{i1}$ for all agents i

Q: This allocation is Pareto optimal: true or false?

- False: Pareto improvement
  - Agents with high y_{i1}/y_{i0} consume more at date 0 and less at date 1
  - Agents with low y_{i1}/y_{i0} consume less at date 0 and more at date 1
- Need a debt instrument

Debt market

- Debt instrument: risk-free bond
  - Lend 1 unit of consumption at date 0, get back $1 + r$ units at date 1
  - Borrow 1 unit of consump at date 0, pay back $1 + r$ units at date 1
  - r: interest rate determined in equilibrium

Q: The market with a risk-free bond is complete: true or false?

Debt market

- Debt instrument: risk-free bond
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  - r: interest rate determined in equilibrium

Q: The market with a risk-free bond is complete: true or false?

- True: no risk $\rightarrow$ only possible transaction = move resources between date 0 and date 1 $\rightarrow$ achieved by risk-free bond

Equilibrium definition

- Equilibrium: all agents maximize and markets clear
- Agent i chooses (c_{i0}, c_{i1}, b_i) to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

  date 0 budget constraint: $c_{i0} + b_i \leq y_{i0}$

  date 1 budget constraint: $c_{i1} \leq y_{i1} + (1 + r) b_i$
- Market clearing

  for bond: $\sum_{i=1}^{N} b_i = 0$

  for consumption good at date $t = 1, 2$: $\sum_{i=1}^{N} c_{it} = \sum_{i=1}^{N} y_{it}$

Agents' choices

- Consolidate sequential budget constraints by eliminating b_i

  $\Rightarrow$ inter-temporal budget constraint: $c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r}$

  - Interest rate: relative price of date 1 consump. / date 0 consump.
  - LHS: date 0 price of intertemporal consumption
  - RHS: date 0 price of intertemporal income

- First order conditions

  - w.r.t. c_{i0}: $u'(c_{i0}) - \lambda_i = 0$

    where λ_i = multiplier of inter-temporal BC

  - w.r.t. c_{i1}: $\beta u'(c_{i1}) - \frac{\lambda_i}{1+r} = 0$

Agents' choices

- FOC $\Rightarrow$ $\quad u'(c_{i0}) = (1+r)\beta u'(c_{i1})$

- Intuition

  - LHS: MU of 1 unit of consumption at date 0

  - RHS: MU of saving 1 unit at date 0, lending it, and consuming the proceeds at date 1

- "Euler equation"

Agents' choices

- Euler equation holds for all agents

  $\Rightarrow$ The intertemporal marginal rate of substitution $\frac{\beta u'(c_{i1})}{u'(c_{i0})}$ is equalized across all agents and is equal to $\frac{1}{1+r}$

- Analogy with static case

  - With several goods: MRS between goods is equalized across agents and equal to relative good prices (studied with Tristan)

  - With several states of nature: MRS between states is equalized across agents and equal to relative state prices (studied with Bruno)

QUIZ

Q. For a given intertemporal income $y_{i0} + \frac{y_{i1}}{1+r}$, an individual's consumption at date 0...

QUIZ

Q. For a given intertemporal income $y_{i0} + \frac{y_{i1}}{1+r}$, an individual's consumption at date 0... does not depend on her date 0's income

- "Permanent income hypothesis"

Agents' choices

- Solve with CRRA utility: $u(c) = (c^{1-\gamma} - 1)/(1-\gamma)$
  - $\gamma \geq 0$: relative risk aversion
  - IMRS $\frac{\beta u'(c_{i1})}{u'(c_{i0})} = \beta \left(\frac{c_{i1}}{c_{i0}}\right)^{-\gamma}$ is a decreasing fcn of consump growth
- FOC $\Rightarrow$ consumption growth is equalized across agents

$$\frac{c_{i1}}{c_{i0}} = [(1+r)\beta]^{1/\gamma} \text{ for all } i$$

Market clearing

- Equilibrium interest rate is determined by market clearing
- Aggregate FOC across agents: $\sum_{i=1}^{N} c_{i1} = [(1+r)\beta]^{1/\gamma} \sum_{i=1}^{N} c_{i0}$
- Market clearing for good at each date t=1,2: $\sum_{i=1}^{N} c_{it} = \sum_{i=1}^{N} y_{it} \equiv C_t$
- NB: mkt clearing for good implies mkt clearing for debt (Say's Law).
  Proof: aggregate inter-temporal budget constraints across agents

$\Rightarrow$ Equilibrium interest rate: $1 + r = \beta^{-1} \left(\frac{C_1}{C_0} \right)^{\gamma}$

QUIZ

Q1. The equilibrium interest rate is higher when...

Q2. The equilibrium interest rate is higher when...

Comparative static

- Higher agg consump growth $\frac{C_1}{C_0} \Rightarrow$ equilibrium interest rate is higher
  - Intuition: agents want to borrow to consume more at date 0 $\Rightarrow r \uparrow$
- Agents are more impatient (lower β) $\Rightarrow$ eqm interest rate is higher
  - Intuition: agents want to borrow to consume more at date 0 $\Rightarrow r \uparrow$

Debt market

- Who lends and who borrows?
  - FOC $\Rightarrow \frac{c_{i1}}{c_{i0}} = \frac{C_1}{C_0}$ (indiv consump gwth = agg consump gwth)
  - Inter-temporal BC: $c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r}$

    $\Rightarrow c_{i0} = \frac{y_{i0} + \frac{y_{i1}}{1+r}}{C_0 + \frac{C_1}{1+r}} C_0$
  - Position in bond: $b_i = y_{i0} - c_{i0} = \left(\frac{C_1}{C_0} - \frac{y_{i1}}{y_{i0}} \right) \frac{y_{i0}}{1+r+\frac{C_1}{C_0}}$
- Agents with income growth below agg income growth: lend

  Agents with income growth above agg income growth: borrow

Welfare

Q. The equilibrium allocation is Pareto optimal: true or false?

Welfare

Q. The equilibrium allocation is Pareto optimal: true or false?

- True
- Intuition: lending/borrowing gains from trade are exhausted in equilibrium
- First Welfare Theorem applies (complete markets, no externality)

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Production

- Same model as before + each agent has access to a production technology
  - Invest k units of good at date 0 $\rightarrow$ generate $f(k)$ units of goods at date 1
  - $f(.)$ is the production function: increasing and concave (decreasing return to scale)
  - Equivalence with cost function representation: to produce y units, need $c(y)$ units $\rightarrow$ $c(.) = f^{-1}(.)$
- Some agents have better technologies than others: $f_i(.) = z_i f(.)$

Equilibrium definition

- Equilibrium: all agents maximize and markets clear
- Agent i chooses $(c_{i0}, c_{i1}, k_i, b_i)$ to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

  date 0 budget constraint: $c_{i0} + k_i + b_i \leq y_{i0}$

  date 1 budget constraint: $c_{i1} \leq y_{i1} + z_i f(k_i) + (1 + r) b_i$
- Market clearing for bond: $\sum_i b_i = 0$

  Market clearing for good at date 0: $\sum_i c_{i0} + \sum_i k_i = \sum_i y_{i0}$

  Market clearing for good at date 1: $\sum_i c_{i1} = \sum_i y_{i1} + \sum_i z_i f(k_i)$

Building the intuition

Q1. Agents with high date 0 endowment y_{i0} tend to...

Q2. Agents with high productivity z_i tend to...

Building the intuition

Q1. Agents with high date 0 endowment y_{i0} tend to...

lend but not to invest more

Q2. Agents with high productivity z_i tend to...

borrow to invest more

Agents' choices

- Consolidate budget constraint to eliminate b_i

$$\max_{c_{i0}, c_{i1}, k_i} u(c_{i0}) + \beta u(c_{i1})$$

$$\text{s.t. } c_{i0} + \frac{c_{i1}}{1+r} \leq y_{i0} + \frac{y_{i1}}{1+r} + \left(-k_i + \frac{z_i f(k_i)}{1+r} \right)$$

- Investment decision is independent from consumption decision
- FOC w.r.t. k_i: $z_i f'(k_i) = 1 + r$
  - LHS: marginal productivity of capital
  - RHS: marginal cost of capital

Agents' choices

- $k_i = f'^{-1}\left(\frac{1+r}{z_i}\right)$
  - k_i is increasing in productivity z_i (by concavity of $f(.)$)
  - k_i does not depend on endowment (y_{i0}, y_{i1})
- FOC w.r.t. consumption: $u'(c_{i0}) = (1+r)\beta u'(c_{i1})$

Equilibrium

- Suppose Cobb-Douglas production function: $f(k) = k^{\alpha}$, $\alpha \in (0,1)$

$$\Rightarrow k_i = \left(\frac{\alpha z_i}{1+r}\right)^{\frac{1}{1-\alpha}} \qquad \Rightarrow z_i f(k_i) = \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}}$$

- Market clearing for good

$$\text{at date 0: } \sum_i c_{i0} = \sum_i y_{i0} - \sum_i \left(\frac{\alpha z_i}{1+r}\right)^{\frac{1}{1-\alpha}} \tag{C0}$$

$$\text{at date 1: } \sum_i c_{i1} = \sum_i y_{i1} + \sum_i \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}} \tag{C1}$$

- Suppose CRRA utility and aggregate FOC w.r.t. consump across i

$$1 + r = \beta^{-1} \left(\frac{\sum_i c_{i1}}{\sum_i c_{i0}}\right)^{1/\gamma} \tag{FOC}$$

- Substitute agg consump in (FOC) using (C0) and (C1) $\rightarrow$ pins down r

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Consumption

- Implications of complete markets
  - IMRS of consumption is equalized across agents
  - Permanent income hypothesis: current consumption depends on intertemporal income not current income
- Empirically
  - People borrow when young, save when middle aged, dis-save when retired: consistent with complete markets
  - However, does consumption depend to current income?

    Test: consumption response to cash windfalls

Consumption response to cash windfalls

- Souleles, 1999, "The Response of Household Consumption to Income Tax Refunds," *American Economic Review* [paper]

TABLE 5—THE MARGINAL PROPENSITY TO CONSUME

	Food	Strictly nondurables	Total consumption	Durables	Nonvehicular durables
Refund* $\pi_{(2\ \text{weeks})}$	0.027	0.045**	0.344**	0.294**	0.128**
	(0.017)	(0.021)	(0.116)	(0.122)	(0.061)
Number of observations	7,622	7,622	7,525	7,525	7,525

Investment

- Implications of complete markets
  - Marginal product of investment $z_i f'(k_i)$ is equalized across agents

    $\Rightarrow$ More productive agents employ more capital
  - Investment does not depend on current income

    $\Rightarrow$ Productive agents borrow to invest
- Empirically
  - Do firms invest more when they become more productive?

    Test: investment response to patent grants
  - Does investment depend on wealth?

    Test: investment response to cash windfalls

Investment response to patent grants

- Kogan, Papanikolaou, Seru and Stoffman, 2017, "Technological Innovation, Resource Allocation and Growth," *Quarterly Journal of Economics* [paper]

  - Patent grant in year $t = 0$
  - Productivity growth $\log(z_{it}) - \log(z_{i0})$ in years $t = 1, \ldots, 5$

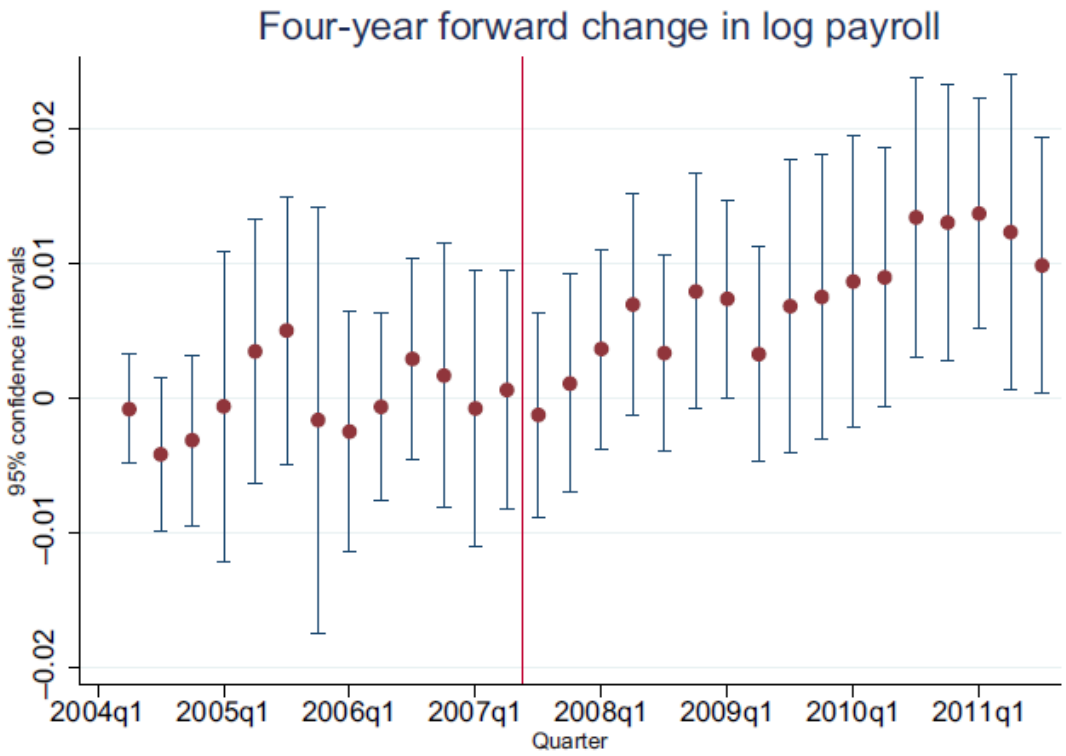
1	2	3	4	5
0.013	0.017	0.019	0.023	0.024
[2.34]	[2.29]	[2.78]	[3.50]	[4.31]

  - Capital growth $\log(k_{it}) - \log(k_{i0})$ in years $t = 1, \ldots, 5$

1	2	3	4	5
0.010	0.020	0.028	0.033	0.038
[8.24]	[6.89]	[6.07]	[4.66]	[4.33]

Investment response to cash windfalls

- Barrot and Nanda, 2020, "The Employment Effects of Faster Payment: Evidence from the Federal Quickpay Reform," *Journal of Finance* [paper]
  - Faster payment to small business gov't contractors: $y_{i0} \uparrow$, $y_{i1} \downarrow$
  - Firm size growth after the reform



Some research topics

- Macro finance: debt markets and consumption/investment
- Household finance: households' consumption and financial decisions
- Corporate innovation and corporate investment
- Corporate finance: financing frictions and real effects