Topics in Asset Pricing

Assignment 5: Under- and over-reaction — To be submitted in pdf format to hombert@hec.fr before the beginning of the class on March 30th.

There are three dates $t = 0, 1, 2$. A risky asset is traded at $t = 0$ and $t = 1$ and pays off $V = 1 + \epsilon$ at $t = 2$. Investors are competitive, risk neutral and have a zero discount rate. At $t = 1$, investors receive a public signal s about ϵ. The unconditional distribution of ϵ has mean zero and the distribution conditional on the signal has mean $E[\epsilon|s] = s$. Investors do not have rational expectations: denote investors' forecasts by $F[.]$. Investors have unbiased forecasts at $t = 0$: $F[\epsilon] = F[s] = 0$. But they update in a non-Bayesian way at $t = 1$:

$$F[\epsilon|s] = (1 - \lambda)s, \tag{1}$$

where $\lambda < 1$ may be positive or negative.

Question 1 Which values of λ may be interpreted as under-reaction? over-reaction? rational expectations?

Question 2 What is the equilibrium price at $t = 1$?

Question 3 What is the equilibrium price at $t = 0$?

Question 4 Calculate the (rational) expectation of the asset return between $t = 1$ and $t = 2$ as a function of λ and s.

Question 5 Calculate the (rational) expectation of the asset return between $t = 1$ and $t = 2$ as a function of λ and R_1.

Question 6 When does the model predicts return momentum? return reversal? Explain the intuition in a few words.

Question 7 Alice has data on N stocks whose returns are described by this simple model. The realization of s is i.i.d. across stocks. Alice runs the following linear regression in the cross-section of stock returns:

$$R_{2,i} = a + b\,R_{1,i} + u_i \qquad i = 1, \ldots, N.$$

What is the expected value of the OLS estimate of b? [You will denote $\kappa \equiv Cov(s, \frac{s}{1+(1-\lambda)s})/Var(s) \approx 1$.]