Microeconomics 1

Rational expectations, informational efficiency

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Questions

- Do asset prices reflect assets' fundamental value?

- Important if asset prices are used as signals about the value of new investments

  E.g., is the stock price of Tesla an accurate signal about the value of investing in electric cars?

- Prices depend on agents' expectations

- How do people form expectations?

Road map

Many periods

Rational expectations, market efficiency

Beliefs

Dynamic model

- $t = 0, 1, 2, \ldots, T$
  - At each date t, state of nature ω_t drawn from set Ω_t
  - $\pi(\omega_{t+\tau} \,|\, \omega_t)$: proba of $\omega_{t+\tau}$ occurring at date $t+\tau$ conditional on state ω_t occurring at date t
- Agents $i = 1, \ldots, N$
  - Expected utility $EU_i = E_0 \left[\sum_{t=0}^{T} \beta^t u(c_{it}(\omega_t)) \right]$
  - Endowment of good at each date t: $y_{it}(\omega_t)$ at each date
  - Endowment of assets at date 0: $\overline{n}_{ij}$

Dynamic model

- Assets $j = 1, \ldots, J$
  - Dividend $d_{jt}(\omega_t)$ at each date t
  - Supply $\overline{n}_j = \sum_i \overline{n}_{ij}$
  - Price after dividend payment in state ω_t at date t: $p_{jt}(\omega_t)$
- Complete markets: Arrow security paying off 1 in state ω_t exists for all $\omega_t \in \Omega_t$ for all t

Equilibrium

- Works the same as the two-period economy
  - Equations that hold between dates 0 and 1 in the two-period economy now hold between any two successive dates t and $t+1$ (proofs are omitted)
- To simplify notation: omit ω_t but keep in mind all variables at $t \geq 1$ are random
- IMRS between t and $t+1$ is equalized across agents

$\Rightarrow$ All agents discount future payoffs using the same stochastic discount factor $M_{t+1} = \frac{\beta u'(c_{it+1})}{u'(c_{it})}$ for all i

Asset price

- Asset price

$$\boxed{p_{jt} = E_t\left[M_{t+1}(d_{jt+1} + p_{jt+1})\right]}$$

where $E_t[.]$ denotes expectation conditional on information at date t

- Price = discounted value of next period's payoff
  - As in the two-period model except that next period's payoff now depends on next period's (endogenous) price

Asset price

- Iterating

$$p_{jt} = E_t\left[M_{t+1}d_{jt+1}\right] + E_t\left[M_{t+1}E_{t+1}[M_{t+2}d_{jt+2}]\right] + \ldots$$

and applying the law of iterated expectations

$$\boxed{p_{jt} = \sum_{\tau=t+1}^{T} E_t\left[M_{t+1}\ldots M_\tau d_{j\tau}\right]}$$

- Price = discounted value of future dividends ("fundamental value")

What if $T = +\infty$?

- NB: What if time horizon is infinite ($T = +\infty$)?

- Assume $\sum_{\tau=1}^{+\infty} E_0[M_1 \dots M_\tau d_{j\tau}] < \infty$

Q. Does $p_{jt} = E_t[M_{t+1}(d_{jt+1} + p_{jt+1})]$ still imply

$$p_{jt} = \sum_{\tau=t+1}^{+\infty} E_t[M_{t+1} \dots M_\tau d_{j\tau}] \quad ?$$

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$p_{jt} = \sum_{\tau=t+1}^{+\infty} E_t[M_{t+1} \dots M_\tau d_{j\tau}]$?

- No: limit term $\lim_{t \to \infty} E_t[M_{t+1} \dots M_\tau p_{j\tau}]$ may be $\neq 0$

Rational bubble in infinite horizon

- Any $p_{jt} = p^*_{jt} + b_{jt}$ with

  fundamental value $p^*_{jt} = \sum_{\tau=t+1}^{\infty} E_t[M_{t+1} \dots M_\tau d_{j\tau}]$

  "bubble" component $b_{jt} \geq 0$ such that $b_{jt} = E_t[M_{t+1} b_{jt+1}]$

  is an equilibrium price

- $b_{jt} = 0$ is an equilibrium

- $b_{jt} > 0$ is a "rational bubble." Examples:
  - Deterministic bubble: $b_{jt+1} = R_f b_{jt}$
  - Stochastic bubble: $b_{jt+1} = \begin{cases} G_t b_{jt} & \text{if no crash} \\ 0 & \text{if crash} \end{cases}$

    where $G_t = \frac{1}{E_t[M_{t+1}|\text{no crash}]P[\text{no crash}]}$

Rational bubble: empirical (lack of) evidence

- Giglio, Maggioti and Stroebel, 2016, "No-Bubble Condition: Model-Free Tests in Housing Markets," *Econometrica* [pdf]
- Idea
  - Compare the price of two assets with same dividend stream d
  - Asset A is infinitely-lived $\rightarrow p_A = p^*(d) + b_A$
  - Asset B is very long- but finitely-lived $\rightarrow p_B \approx p^*(d)$
- Empirical setting
  - Leaseholds for houses with 700 to 1,000 years maturity in the UK and Singapore
  - Compare to price of similar houses with infinite-maturity ownership
  - Result: same price $\rightarrow$ no rational bubble
- NB: Rational bubbles ruled out but not the only kind of bubbles

Expected return

- Asset return between t and $t+1$: $R_{jt+1} = \frac{d_{jt+1}+p_{jt+1}}{p_{jt}}$

- Expected return

$$\boxed{E_t\left[M_{t+1}R_{jt+1}\right] = 1}$$

- Risk-free rate between t and $t+1$ (known at t)

$$\boxed{R_{ft+1} = \frac{1}{E_t\left[M_{t+1}\right]}}$$

- Risk premium

$$\boxed{E_t\left[R_{jt+1}\right] - R_{ft+1} = -R_{ft+1}Cov_t\left(M_{t+1}, R_{jt+1}\right)}$$

- NB: Holds whether horizon is finite or infinite

Road map

Many periods

Rational expectations, market efficiency

Beliefs

Time-series predictability

- Suppose you find an (imperfect) predictor of stock market return

  e.g., a variable X such that $Corr(R_{t+1}, X_t) = 0.3$

Q. Does this contradict the complete market model?

Time-series predictability

- Suppose you find an (imperfect) predictor of stock market return

  e.g., a variable X such that $Corr(R_{t+1}, X_t) = 0.3$

Q. Does this contradict the complete market model? not necessarily

Time-series predictability

- Equity risk premium between t and $t+1$ conditional on date t info

  $$E_t[R_{t+1} - R_{ft+1}] = -R_{ft+1} Cov_t(M_{t+1}, R_{t+1})$$

  may be time-varying

  - Because the conditional distribution of next period consumption and return may be time-varying

    e.g., the future may be very uncertain in March 2020

- Empirical evidence

  - Can we find variables X such that $E[R_{t+1} - R_{ft+1} \,|\, X_t]$ depends on X_t, that is, such that X_t predict $R_{t+1} - R_{ft+1}$?

Time-series predictability

- X_t: aggregate dividend yield = agg dividend D_t/agg market cap P_t

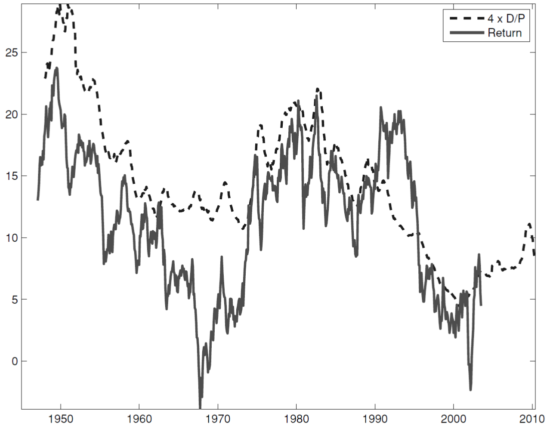

Figure 1. Dividend yield and following 7-year return. The dividend yield is multiplied by four. Both series use the CRSP value-weighted market index.

Updated graph https://johanhombert.github.io/blog/20201121-predictable-stock-market

Time-series predictability

Return-Forecasting Regressions

The regression equation is $R^e_{t\to t+k} = a + b \times D_t/P_t + \varepsilon_{t+k}$. The dependent variable $R^e_{t\to t+k}$ is the CRSP value-weighted return less the 3-month Treasury bill return. Data are annual, 1947–2009. The 5-year regression t-statistic uses the Hansen–Hodrick (1980) correction. $\sigma[E_t(R^e)]$ represents the standard deviation of the fitted value, $\sigma(\hat{b} \times D_t/P_t)$.

Horizon k	b	$t(b)$	R^2	$\sigma[E_t(R^e)]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

$\Rightarrow$ It is possible to predict (imperfectly) the stock market return

- Seminal paper: Shiller (1981) [pdf]. Update: Cochrane (2011) [pdf]
- This result has two possible interpretations

Time-series predictability

- Interpretation 1: The equity risk premium required by agents is time-varying

- This can happen because

  - risk varies over time

  - or risk aversion varies over time

- When investors require a high equity premium, expected returns are high and prices are low, implying that high D/P predicts high return as observed in the data

Agents' expectations

- $E_t[.]$ and $Cov_t(.)$ in the pricing formula $E_t[R_{t+1} - R_{ft+1}] = -R_{ft+1} Cov_t(M_{t+1}, R_{t+1})$ are based on agents' expectations
  - b/c the pricing formula derives from agents' optimizing behavior, which is determined by agents' expectations

$\Rightarrow$ Interpretation 1 and all our analysis so far rely on the assumption that agents' expectations correspond to the true distributions

Rational expectations

- **Definition.** Agents have **rational expectations** if their expectations correspond to the true distribution of random variables conditional on their information set.

- This is the implicit assumption so far

- NB: How may people have rational expectations (RE)?

  - Learn from past data. This requires distributions to be stationary

  - When variables to be predicted depend on other agents' behavior, RE require to predict other agents' expectations, which depends on other agents' expectation, and so on

Market efficiency

- **Definition.** A financial market is **(informationally) efficient** if asset prices at time t are equal to fundamental asset values, i.e., equal to discounted values of expected asset payoff conditional on all information available at time t

  that is, if $p_{jt} = \sum_{\tau=t+1}^{T} E_t \left[M_{t+1} \dots M_\tau d_{j\tau} \right]$ and $E_t[.]$ is calculated using rational expectations

- NB
  - When there is *asymmetric information*, that is, different agents have different information sets, the notion of "available information" becomes ambiguous (available to whom?)
  - See next lecture. For now, consider symmetric information

Market efficiency

- Complete markets + rational expectations is a sufficient condition for market efficiency
  - Proof: our calculation of the equilibrium in the previous slides
- . . . but not a necessary condition: market efficiency may obtain if agents do not have rational expectations but
  - individual mistakes cancel each other such that the average investor has rational expectations

    OR

  - the average investor does not have rational expectations but some investors do, trade on their superior information, pushing prices towards fundamental values

Road map

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Beliefs

Biased expectations

- Suppose agents' date t expectations regarding the distribution of $D_{t+1} + P_{t+1}$ is the true distribution shifted by θ_t
  - $\theta_t > 0$: agents are too optimistic
  - $\theta_t < 0$: agents are too pessimistic
  - $\theta_t = 0$: rational expectations
- Equilibrium price:

$P_t = E_t^b[M_{t+1}(D_{t+1} + P_{t+1})]$ where $E_t^b[.]$ is agents' expectations

$P_t = E_t[M_{t+1}(D_{t+1} + P_{t+1} + \theta_t)]$ where $E[.]$ is based on the true distrib

$\Rightarrow$ When agents are optimistic ($\theta_t > 0$), the price is too high

Biased expectations

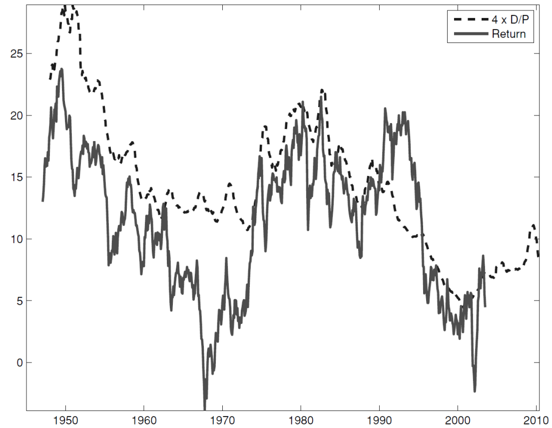
- Expected return according to the true distribution:

$$E_t[R_{t+1} - R_{ft+1}] = -R_{ft+1} Cov_t(M_{t+1}, R_{t+1}) - \theta_t / P_t$$

$\Rightarrow$ When agents are optimistic ($\theta_t > 0$), expected returns are too low

- $\theta_t > 0 \Leftrightarrow P_t$ is high $\Leftrightarrow E_t[R_{t+1} - R_{ft+1}]$ is low

Time-series predictability



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- Interpretation 2: optimism $\Leftrightarrow$ D_t / P_t is low $\Leftrightarrow$ $E_t[R_{t+1}]$ is low

Joint hypothesis problem

- Which interpretation is correct?
- Joint hypothesis problem
  - If we find that the empirical moments do not satisfy $E_t[R_{t+1} - R_{ft+1}] = -R_{ft+1} Cov_t(M_{t+1}, R_{t+1})$, we cannot determine whether
  - ... we estimated Cov using a wrong $u(.)$ and thus a wrong M
  - ... or agents have biased expectations

Survey data

- A solution: ask people!
  - When D/P is high, ask investors if they expect
  - ... high market return (=Interpretation 1: investors have rational expectations and predictability reflects compensation for risk)
  - ... or not (=Interpretation 2: investors have biased expectations)

Survey data

- Greenwood and Shleifer, 2014, "Expectations of Returns and Expected Returns," *Review of Financial Studies* [pdf]
- Investors' stated return expectation$_t$ = (P/D)$_t$ + ... + ϵ_t

Table 3
Determinants of investor expectations

R_{t-12}	3.691
	[7.841]
Log(P/D)	0.909
	[3.220]
Earnings gr.	−0.191
	[−1.291]
Unemployment	−0.065
	[−0.773]
Risk-free rate	−5.094
	[−0.785]
Constant	−3.337
	[−1.930]
N	294
R^2	0.453

Survey data

- Stock $\text{return}_{t \to t+3Y} = \text{Return expectations}_t + \epsilon_t$

Table 6
Forecasting future returns

Panel B: Forecasting thirty-six-month returns

Index*	−5.713
	[−2.678]
Constant	0.825
	[3.825]
N	261
R^2	0.094

$\Rightarrow$ Inconsistent with rational expectations

Some research topics

- Macro finance: financial intermediaries, interest rates, asset prices
- Asset pricing: market (in)efficiency how? when? where? why?
- Corporate finance: impact of market (in)efficiency on firms
- Behavioral finance: biases, belief formation

Bonus slides: tests of rational expectations

- Coibion and Gorodnichenko, 2015, "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review* [pdf]

- Denote

  x_{t+1}: variable being forecast

  $F_t x_{t+1}$: forecast at t

  $E_t x_{t+1}$: RE forecast at t

- Assume

$$F_t x_{t+1} = (1-\lambda) E_t x_{t+1} + \lambda F_{t-1} x_{t+1}$$

  $\lambda = 0$: rational expectations

  $\lambda > 0$: under-reaction

  $\lambda < 0$: over-reaction

$$\Rightarrow \; E_t x_{t+1} - F_t x_{t+1} = \frac{\lambda}{1-\lambda} \left[F_t x_{t+1} - F_{t-1} x_{t+1} \right]$$

- Forecast revision predicts forecast error. Can be estimated by OLS

Bonus slides: tests of rational expectations

- Inflation expectations by professional forecasters

  $F_t \pi_{t+3}$: quarter t forecast of quarter $t+3$ inflation

  π_{t+3}: realized $t+3$ inflation Consensus

- Time-series regression

$$\pi_{t+3} - F_t \pi_{t+3} = a + b \left(F_t \pi_{t+3} - F_{t-1} \pi_{t+3} \right) + \epsilon_t$$

- Result: $b \simeq 1$

$\Rightarrow$ under-reaction relative to rational expectations