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Speculative Betas

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ABSTRACT

The risk and return trade-off, the cornerstone of modern asset pricing theory, is often of the wrong sign. Our explanation is that high-beta assets are prone to speculative overpricing. When investors disagree about the stock market's prospects, high-beta assets are more sensitive to this aggregate disagreement, experience greater divergence of opinion about their payoffs, and are overpriced due to short-sales constraints. When aggregate disagreement is low, the Security Market Line is upward-sloping due to risk-sharing. When it is high, expected returns can actually decrease with beta. We confirm our theory using a measure of disagreement about stock market earnings.

THERE IS COMPELLING EVIDENCE that high-risk assets often deliver lower expected returns than low-risk assets. This is contrary to the risk-return trade-off at the heart of neoclassical asset pricing theory. The high-risk, low-return puzzle literature, which dates back to Black (1972) and Black, Jensen, and Scholes (1972), shows that low-risk stocks, as measured by a stock's comovement with the stock market or Sharpe's (1964) capital asset pricing model (CAPM) beta, have significantly outperformed high-risk stocks over the last 30 years.[1] Baker, Bradley, and Wurgler (2011) further show that since January 1968 the cumulative performance of stocks has actually been declining with beta. For instance,

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[1] A nonexhaustive list of studies includes Blitz and Vliet (2007), Cohen, Polk, and Vuolteenaho (2005), and Frazzini and Pedersen (2014).

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a dollar invested in a value-weighted portfolio of the lowest quintile of beta stocks would have yielded \$96.21 (\$15.35 in real terms) at the end of December 2010, while a dollar invested in the highest quintile of beta stocks would have yielded around \$26.39 (\$4.21 in real terms). Relatedly, Baker, Bradley, and Wurgler (2011) and Frazzini and Pedersen (2014) both point out that a strategy of shorting high-beta stocks and buying low-beta stocks generates excess profits as large as famous excess stock return predictability patterns such as the value growth or price momentum effects.[2]

In early work, Black (1972) originally tries to reconcile a flat Security Market Line by relaxing one of the central CAPM assumptions of borrowing at the risk-free rate. He shows that, in the presence of borrowing constraints, risk-tolerant investors desiring more portfolio volatility will demand high-beta stocks since they cannot simply lever up the tangency portfolio. However, borrowing constraints can only deliver a flatter Security Market Line relative to the CAPM, not a downward-sloping one; investors would not bid up high-beta stock prices to the point of having lower returns than low-beta stocks. Indeed, it is difficult to get a downward-sloping line even if one introduces noise traders as in Delong et al. (1990) or liquidity shocks as in Campbell, Grossman, and Wang (1993), since noise traders or fundamental risk in these models lead to higher expected returns.[3]

In contrast to Black (1972), we provide a theory for the high-risk and low-return puzzle even when investors can borrow at the risk-free rate. We show that relaxing the other CAPM assumptions of homogeneous expectations and costless short-selling can deliver rich patterns in the Security Market Line, including an inverted-U shape or even a downward-sloping line. Our model starts from a CAPM framework, in which firms' cash flows follow a one-factor model and there are a finite number of securities so that markets are incomplete. We allow investors to disagree about the market or common factor of firms' cash flows and prohibit *some* investors from short-selling. Investors only disagree about the mean of the common factor of cash flows. There are two groups of investors, buyers such as retail mutual funds (MFs) that cannot short and arbitrageurs such as hedge funds (HFs) that can short.

Substantial evidence supports both of these assumptions. First, there is time-varying disagreement among professional forecasters' and households' expectations about many macroeconomic state variables such as market earnings, industrial production (IP) growth, and inflation (Cukierman and Wachtel

[2] The value-growth effect (Fama and French (1992), Lakonishok, Shleifer, and Vishny (1994)), buying stocks with low price-to-fundamental ratios and shorting those with high ratios, generates a reward-to-risk or Sharpe (1964) ratio that is two-thirds of a zero-beta-adjusted strategy of buying low-beta stocks and shorting high-beta stocks. The corresponding figure for the momentum effect (Jegadeesh and Titman (1993)), buying the past year's winning stocks and shorting the past year's losing stocks, is roughly three-fourths of the long low-beta, short high-beta strategy.

[3] Indeed, most behavioral models would not deliver such a pattern. In Barberis and Huang (2001), mental accounting by investors still leads to a positive relationship between risk and return. The exception is the model of overconfident investors and the cross section of stock returns in Daniel, Hirshleifer, and Subrahmanyam (2001), which might yield a negative relationship as well but not the new patterns for beta that we document below.

(1979), Kandel and Pearson (1995), Mankiw, Reis, and Wolfers (2004), Lamont (2002)). Such aggregate disagreement might emanate from many sources including heterogeneous priors or cognitive biases like overconfidence.[4] Second, short-sales constraints bind for some investors for institutional reasons rather than due to the physical cost of shorting.[5] For instance, many investors in the stock market such as retail MFs, which in 2010 had 20 trillion dollars of assets under management, are prohibited by charter from shorting either directly (Almazan et al. (2004)) or indirectly through the use of derivatives (Koski and Pontiff (1999)). Indeed, only a smaller subset of investors, such as HFs with 1.8 trillion dollars in assets under management in 2010, can and do short.

Our main result is that high-beta assets are overpriced compared to low-beta assets when disagreement about the common factor of firms' cash flows is high. If investors disagree about the mean of the common factor, then their forecasts for the payoffs of high-beta stocks will naturally diverge more than their forecasts for low-beta ones. In other words, beta amplifies disagreement about the macroeconomy. Because of short-sales constraints, high-beta stocks, which are more sensitive to aggregate disagreement than low-beta stocks, are only held in equilibrium by optimists, as pessimists are sidelined. This greater divergence of opinion creates overpricing of high-beta stocks compared to low-beta stocks (Miller (1977) and Chen, Hong, and Stein (2002)).[6] Arbitrageurs attempt to correct this mispricing but their limited risk-bearing capacity results in limited shorting, leading to equilibrium overpricing.[7]

That more disagreement on high-beta stocks leads to overpricing of these stocks is far from obvious in an equilibrium model like ours. Optimistic investors can achieve large exposure to the common factor by buying high-beta stocks or levering up low-beta ones. If high-beta stocks are overpriced, optimistic investors should favor the levering up of low-beta assets, which could potentially undo the initial mispricing. The key reason why this does not occur in our model is that, when markets are incomplete (which is implicit in all theories of limits of arbitrage, as in Delong et al. (1990) or Shleifer and Vishny (1997), and most modern asset pricing models (Merton (1987)), idiosyncratic risk matters for investors' portfolios. In our context, while levering up low-beta stocks increases the exposure to the common factor, it also magnifies the

[4] See Hong and Stein (2007) for a discussion of the various rationales. A large literature starting with Odean (1999) and Barber and Odean (2001) argues that retail investors engage in excessive trading due to overconfidence.

[5] See Lamont (2004) for a discussion of the many rationales for the bias against shorting in financial markets, including historical events such as the Great Depression in which short-sellers were blamed for the crash of 1929.

[6] A general disagreement structure about both means and covariances of asset returns with short-sales restrictions in a CAPM setting is developed in Jarrow (1980). He shows that short-sales restrictions in one asset might increase the prices of others. It turns out that a focus on a simpler one-factor disagreement structure about common cash flows yields closed-form solutions and a host of testable implications for the cross section of asset prices that would otherwise not be possible.

[7] High-beta stocks might also be more difficult to arbitrage because of incentives for benchmarking and other agency issues (Brennan (1993), Baker, Bradley, and Wurgler (2011)).

idiosyncratic risk that investors have to bear. This role of idiosyncratic volatility as a limit of arbitrage is motivated by a number of empirical papers that show that idiosyncratic risk is the biggest impediment to arbitrage (Pontiff (1996), Wurgler and Zhuravskaya (2002)). It leads, in equilibrium, to a situation where levering up low-beta stocks ends up being less efficient than buying high-beta stocks when speculating on the common factor of firms' cash flows. In other words, higher beta assets are naturally more speculative.

Our model yields the following key testable implications. When macrodisagreement is low, all investors are long and short-sales constraints do not bind. The traditional risk-sharing motive leads high-beta assets to attract a lower price or higher expected return. For high enough aggregate disagreement, the relationship between risk and return takes on an inverted-U shape. For assets with a beta below a certain cutoff, expected returns are increasing in beta as there is little disagreement about these stocks' cash flows and therefore short-selling constraints do not bind in equilibrium. But for assets with a beta above an equilibrium cutoff, disagreement about dividends becomes sufficiently large that the pessimist investors are sidelined. This speculative overpricing effect can dominate the risk-sharing effect and the expected returns of high-beta assets can actually be lower than those of low-beta ones. As disagreement increases, the cutoff level of beta below which all investors are long falls and the fraction of assets experiencing binding short-sales constraints increases.[8]

We test these predictions using a monthly time series of disagreement about market earnings. Disagreement about a stock's cash flow is measured by the standard deviation of analysts' forecasts of the long-term growth of earnings per share (EPS), as in Diether, Malloy, and Scherbina (2002). The aggregate disagreement measure is a beta-weighted average of the stock-level disagreement measure for all stocks in our sample, similar in spirit to Yu (2011). The weighting by beta in our proxy for aggregate disagreement is suggested by our theory. Stocks with very low beta have by definition almost no sensitivity to aggregate disagreement, and hence their disagreement should mostly reflect idiosyncratic disagreement. Aggregate disagreement can be high during both down-markets, like the recessions of 1981 to 1982 and 2007 to 2008, and up-markets, like the dot-com boom of the late 1990s (Figure 1). Panel C of Figure 6 shows the 12-month excess returns on 20 β-sorted portfolios (see Section II.B. for details on the construction of these portfolios). In months with low aggregate disagreement (defined as the bottom quartile of the aggregate disagreement distribution and represented by triangles), we see that returns are, in fact, increasing with beta. However, in months with high aggregate disagreement, however (defined as the top quartile of the disagreement distribution and represented by diamonds), the risk-return relationship has an inverted-U

[8] When aggregate disagreement is so large that pessimists are sidelined on all assets, the relationship between risk and return is downward-sloping as the entire market becomes overpriced. We assume that all assets in our model have a strictly positive loading on the aggregate factor. Thus, it is always possible that pessimists want to be short an asset, provided that aggregate disagreement is large enough.

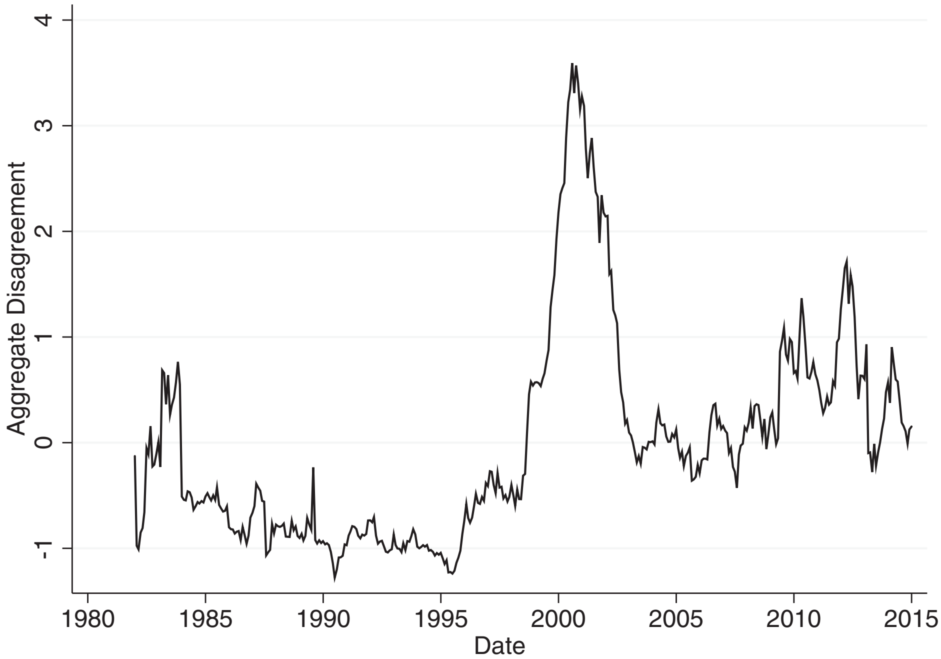


Figure 1. Time series of aggregate disagreement. Sample period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). Each month, we calculate for each stock the standard deviation of analyst forecasts of long-run growth in EPS, our measure of stock-level disagreement. We also estimate for each stock $i\,\hat{\beta}_{i,t-1}$, the stock market beta of stock i at the end of the previous month. These betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. Aggregate disagreement is the monthly $\hat{\beta}_{i,t-1}$-weighted average stock-level disagreement.

shape. In these months, the two top and bottom portfolios have average excess returns net of the risk-free rate of around 0%, while the portfolios around the median portfolio have average excess returns of around 8%. This inverted-U shape relationship is formally estimated in the context of a standard Fama and Macbeth (1993) analysis where the excess return/beta relationship is shown to be strictly more concave when aggregate disagreement is large.[9]

Our baseline analysis assumes that stocks' cash flow process is homoskedastic. When we allow for heteroskedasticity, our main asset pricing equation is slightly modified. Intuitively, a large idiosyncratic variance makes optimist investors reluctant to demand too much of a stock. Thus, a stock with a large cash flow beta—and therefore whose expected cash flow is high from the optimists' point of view—may nonetheless have little demand from the optimists if the stock has high idiosyncratic variance. In equilibrium, this low demand from optimists will drive down the price and make pessimists long this asset.

[9] The "square" portfolio, which corresponds to the monthly coefficient estimate of a regression of portfolio returns on the portfolio's β^2, is a portfolio that goes long the top three and bottom six β-sorted portfolios and short the remaining portfolios. It thus captures intuitively the inverted-U shape of the Security Market Line in our theoretical analysis.

As a result, such a stock may not experience the same speculative overpricing as a stock with a similar cash flow beta but lower idiosyncratic variance. In other words, stocks experience overpricing only when the ratio of their cash flow beta to idiosyncratic variance is high enough. Below a certain cutoff in this ratio, stocks are priced as in the CAPM and the partial Security Market Line (the relationship between expected returns and β for stocks below this cutoff) is upward-sloping and independent of aggregate disagreement. Above the cutoff, the partial Security Market Line has a slope that strictly decreases with aggregate disagreement. We confirm this additional prediction in the data.

Our findings are consistent with Diether, Malloy, and Scherbina (2002) and Yu (2011), who find that dispersion of earnings forecasts predicts low returns in the cross section and for the market return in the time series, respectively, as predicted in models with disagreement and short-sales constraints. Our particular focus is on the theory and the empirics of the Security Market Line as a function of aggregate disagreement. Importantly, we show that the patterns observed in the data are not simply a function of high-beta stocks performing badly during down-markets nor are they a function of high-disagreement stocks underperforming.

Finally, in an overlapping generations (OLGs) extension of our static model, we show that these predictions also hold in a dynamic setting where disagreement follows a two-state Markov chain. Investors anticipate that high-beta assets are more likely to experience binding short-sales constraints in the future, and hence have a potentially higher resale price than low-beta ones relative to fundamentals (Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006)). Since disagreement is persistent, this pushes up the price of high-beta assets in both the low and high disagreement states. At the same time, since the price of high-beta assets varies more with aggregate disagreement, these stocks carry an extra risk premium. We use this dynamic model to show that a basic simulation of the model can yield economically significant flattenings of the Security Market Line using reasonable levels of disagreement and risk-aversion among investors.

Our paper proceeds as follows. We present the model in Section I. We describe the data used in our empirical analysis in Section II. We present the empirical analysis in Section III. We conclude in Section IV. All proofs are in the Internet Appendix.[10]

I. Model

A. Static Setting

We consider an economy populated with a continuum of investors of mass one. There are two periods, $t = 0, 1$. There are N risky assets and the risk-free

[10] The Internet Appendix is available in the online version of this article on the *Journal of Finance* website.

rate is exogenously set at r. Risky asset i delivers a dividend $\tilde{d}_i$ at date 1, which is given by

$$\forall i \in \{1, \ldots, N\}, \tilde{d}_i = d + b_i \tilde{z} + \tilde{\epsilon}_i.$$

The common factor in stock i's dividend is $\tilde{z}$, with $\mathbb{E}[\tilde{z}] = 0$ and $\mathrm{Var}[\tilde{z}] = \sigma_z^2$. The idiosyncratic component in stock i's dividend is $\tilde{\epsilon}_i$, with $\mathbb{E}[\tilde{\epsilon}_i] = 0$ and $\mathrm{Var}[\tilde{\epsilon}_i] = \sigma_\epsilon^2$. By definition, for all $i \in [1, N]$, $\mathrm{Cov}(\tilde{z}, \tilde{\epsilon}_i) = 0$. The cash flow beta of asset i is b_i and assumed to be strictly positive. Each asset i is in supply $s_i = \frac{1}{N}$ and we assume w.l.o.g. that[11]

$$0 < b_1 < b_2 < \cdots < b_N.$$

Assets in the economy are indexed by their cash flow betas, which are increasing in i. The share-weighted average b in the economy is set to 1 ($\sum_{i=1}^N \frac{b_i}{N} = 1$).

Investors are divided into two groups. A fraction α of them hold heterogeneous beliefs and cannot short. We call these buyers MFs, which, in practice, are prohibited from shorting by charter. These investors are divided in two groups of mass $\frac{1}{2}$, A and B, who disagree about the mean value of the aggregate shock $\tilde{z}$. Group A believes that $\mathbb{E}^A[\tilde{z}] = \lambda$, while group B believes that $\mathbb{E}^B[\tilde{z}] = -\lambda$. We assume without loss of generality (w.l.o.g.) that $\lambda > 0$ so that investors in group A are the optimists and investors in group B the pessimists.

A fraction $1 - \alpha$ of investors hold homogeneous and correct beliefs and are not subject to the short-sales constraint. We index these investors by a (for "arbitrageurs"). For concreteness, one might interpret these buyers as HFs, which can generally short at little cost. That these investors have homogeneous beliefs is simply assumed for expositional convenience. Heterogeneous priors for unconstrained investors wash out in the aggregate and thus have no impact on equilibrium asset prices in our model.

Investors maximize their date-1 wealth and have mean-variance preferences:

$$U(\tilde{W}^k) = \mathbb{E}^k[\tilde{W}^k] - \frac{1}{2\gamma}\mathrm{Var}(\tilde{W}^k),$$

where $k \in \{a, A, B\}$ and γ is investors' risk tolerance. Investors in group A or B maximize under the constraint that their holding of stocks has to be greater than zero.

B. Equilibrium

The following theorem characterizes the equilibrium:

THEOREM 1: *Let* $\theta = \frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}$ *and define* $(u_i)_{i \in [0, N+1]}$ *such that* $u_{N+1} = 0$,

[11] This normalization of supply to $1/N$ is without loss of generality. If asset i is in supply s_i, then what matters is the ranking of assets along the $\frac{b_i}{s_i}$ dimension. The rest of the analysis is then unchanged.

$u_i = \frac{1}{\gamma N b_i}(\sigma_\epsilon^2 + \sigma_z^2(\sum_{j<i} b_j^2)) + \frac{\sigma_z^2}{\gamma}(\sum_{j\geq i} \frac{b_j}{N})$ *for* $i \in [1, N]$*, and* $u_0 = \infty$*. u is a strictly decreasing sequence. Let* $\bar{i} = \min\{k \in [0, N+1] | \lambda > u_k\}$*. There exists a unique equilibrium, in which asset prices are given by*

$$P_i(1+r) = \begin{cases} d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) & \textit{for } i < \bar{i} \\ d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) + \underbrace{\frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_\epsilon^2}{N}\right)}_{\pi^i = \textit{speculative premium}} & \textit{for } i \geq \bar{i} \end{cases}, \tag{1}$$

where $\omega(\lambda) = \frac{\lambda\gamma - \frac{\sigma_z^2}{N}(\sum_{i\geq\bar{i}} b_i)}{\sigma_z^2(1+\sigma_z^2(\sum_{i<\bar{i}} \frac{b_i^2}{\sigma_\epsilon^2}))}$.

PROOF: See Internet Appendix Section I.A. ■

The main intuition underlying the equilibrium is that there is more disagreement among investors about the expected dividends of high-b_i assets relative to low-b_i assets. Above a certain level of b_i ($b_i \geq b_{\bar{i}}$), investors sufficiently disagree that the pessimists, that is, investors in group B, would optimally short these stocks. However, this is impossible because of the short-sales constraint. These high-b stocks thus experience a speculative premium since their price reflects disproportionately the belief of the optimists, that is, investors in group A. As aggregate disagreement grows, the cash flow beta of the marginal asset—the asset above which group B investors are sidelined—decreases and a larger fraction of assets experiences overpricing.[12]

We can derive a number of comparative static results regarding this equilibrium. The first relates to overpricing. When short-sales constraints are binding, that is, for assets $i \geq \bar{i}$, the difference between the equilibrium price and the price that would prevail in the absence of short-sales constraints (i.e., when MFs can short without restriction or when $\alpha = 0$) is given by

$$\pi^i = \frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_\epsilon^2}{N}\right). \tag{2}$$

This term, which we call the speculative premium, captures the degree of overpricing due to the short-sales constraints. The following corollary explores how this speculative premium varies with aggregate disagreement, cash flow betas, and the fraction of agents that face short-sales constraints.

[12] The condition defining the marginal asset $\bar{i}, \bar{i} = \min\{k \in [0, N+1] | \lambda > u_k\}$, corresponds to an N-asset generalization of the condition defining the equilibrium with short-sales constraints in the one-asset model of Chen, Hong, and Stein (2002). The intuition for this condition is that disagreement has to be larger than the risk premium for bearing the risk of an asset for short-sales constraints to bind, as otherwise, even pessimists would like to be long the risky asset. The sequence (u_i), which plays a key role in this condition, corresponds to the pessimists' equilibrium holding in asset i. Naturally, the u_is depend on the risk tolerance γ, the supply of risky assets $1/N$, and the covariance of asset i with other assets.

COROLLARY 1: *Assets with high cash flow betas, that is, $i \geq \bar{i}$, are overpriced (relative to the benchmark with no short-sales constraints or when $\alpha = 0$) and the amount of overpricing, defined as the difference between the price and the benchmark price in the absence of short-sales constraints, is increasing with disagreement λ, with cash flow betas b_i, and with the fraction of investors that are short-sales constrained α. Furthermore, an increase in aggregate disagreement λ leads to a larger increase in mispricing for assets with larger cash flow betas.*

PROOF: See Internet Appendix Section I.B. ■

The second comparative static we consider relates to the holdings observed in equilibrium. Remember that HFs (i.e., investors in group a) are not restricted in their ability to short. Intuitively, HFs short assets with large mispricing, that is, high-b assets. As aggregate disagreement increases, mispricing increases, so that HFs end up shorting more. Since an increase in aggregate disagreement leads to a larger relative increase in mispricing for higher-b stocks, the corresponding increase in shorting is also larger for high-b stocks. In other words, there is a weakly increasing relationship between shorting by HFs and b. Provided that λ is large enough, this relationship becomes strictly steeper as aggregate disagreement increases. We summarize these comparative statics in the following corollary.

COROLLARY 2: *Group A investors are long all assets. Group B investors are long assets $i < \bar{i} - 1$ and have zero holdings of assets $i \geq \bar{i}$. There exists $\hat{\lambda} > 0$ such that, provided $\lambda > \hat{\lambda}$, there exists $\tilde{i} \in [\bar{i}, N]$ such that (1) group A investors short high cash flow beta assets, that is, assets $i \geq \tilde{i}$, (2) the dollar amount of stocks being shorted in equilibrium increases with aggregate disagreement λ, and (3) the sensitivity of shorting to aggregate disagreement is higher for high cash flow beta assets.*

PROOF: See Internet Appendix Section I.C. ■

C. Beta and Expected Return

We now restate the equilibrium in terms of expected excess returns and relate them to the familiar market β from the CAPM. We denote by $\tilde{r}_i^e$ the excess return per share for asset i and by $\tilde{R}_m^e$ the excess return per share for the market portfolio. The market portfolio is defined as the portfolio of all assets in the market. The value of the market portfolio is $P_m = \sum_{j=1}^{N} s_j P_j = \sum_{j=1}^{N} \frac{P_j}{N}$ since we have normalized the supply of stocks to $\frac{1}{N}$. Then, by definition,

$$\tilde{R}_i^e = d + b_i \tilde{z} + \tilde{\epsilon}_i - (1+r)P_i \text{ and } \tilde{R}_m^e = \sum_{i=1}^{N} s_i \tilde{R}_i^e = \sum_{i=1}^{N} \frac{1}{N} \tilde{R}_i^e = d + \tilde{z} + \sum_{i=1}^{N} \frac{\tilde{\epsilon}_i}{N} - (1+r)P_m.$$

Define $\beta_i = \frac{\mathrm{Cov}(\tilde{R}^e_i, \tilde{R}^e_m)}{\mathrm{Var}(\tilde{R}^e_m)} = \frac{b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}$ to be the stock market beta of stock i. By definition, the expected excess return per share of stock i is simply given by

$$\mathbb{E}[\tilde{R}^e_i] = d - (1+r)P_i.$$

Using Theorem 1, we can express this expected excess return per share on stock i as follows:[13]

$$\mathbb{E}[\tilde{R}^e_i] = \begin{cases} \beta_i \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} & \textit{for } i < \bar{i} \\ \beta_i \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1 - \theta\omega(\lambda)\right) + \theta\frac{\sigma_\epsilon^2}{\gamma N}\left(1 + \omega(\lambda)\right) & \textit{for } i \geq \bar{i} \end{cases}. \quad (3)$$

This representation follows directly from Theorem 1: we simply express the price of asset i as a function of the market beta of asset i, β_i. For $\alpha = 0$ (and hence $\theta = 0$), investors have homogeneous beliefs, and thus λ does not affect the expected returns of the assets. In fact, when $\alpha = 0$, there are no binding short-sales constraints, so $\bar{i} = N + 1$ and we can simply rewrite for all $i \in [1, N]$: $\mathbb{E}[\tilde{R}^e_i] = \beta_i \mathbb{E}[\tilde{R}^e_m]$, that is, the standard CAPM formula. However, when a fraction $\alpha > 0$ of investors are short-sales constrained and aggregate disagreement is large enough, $\bar{i} \leq N$ and the expected return per share for assets $i \geq \bar{i}$ depend on aggregate disagreement λ.

More precisely, the Security Market Line is then piecewise linear. For low-beta assets ($\beta_i < \beta_{\bar{i}}$), expected excess returns are solely driven by risk-sharing: higher-β assets are more exposed to market risk and thus command a higher expected return. When β crosses a certain threshold ($\beta \geq \beta_{\bar{i}}$), however, expected excess returns are also driven by speculation, in the sense that pessimists are sidelined from these high-beta stocks: over this part of the Security Market Line, higher-beta assets, which are more exposed to aggregate disagreement, command a larger speculative premium, and thus receive smaller expected returns than what they would absent disagreement. Note that, provided λ is large enough, the Security Market Line can even be downward-sloping over the interval $[\beta_{\bar{i}}, \beta_N]$, that is, for speculative assets.

We illustrate the role of aggregate disagreement on the shape of the Security Market Line in Figure 2, where the Security Market Line is plotted for three possible levels of λ: $\lambda^0 < \lambda^1 < \lambda 2$. The Security Market Line is simply the 45-degree line when $\lambda = \lambda^0 = 0$ as seen in Panel A of Figure 2). We assume that λ^1 is large enough that a strictly positive fraction of assets experience binding short-sales constraints and hence speculative mispricing (assets above $\bar{i}$): expected returns are increasing with beta but at a lower pace for assets above the endogenous marginal asset $\bar{i}$ (Figure 2, Panel B). When $\lambda = \lambda^2 > \lambda^1$ (Figure 2, Panel C), the slope of the Security Market Line for assets $i \geq \bar{i}$ is negative, that is, the Security Market Line has an inverted-U shape.

In our empirical analysis below, we approach the relationship between expected excess returns and β by looking at the concavity of the Security Market

[13] The derivation of this formula can be found in Internet Appendix Section I.D.

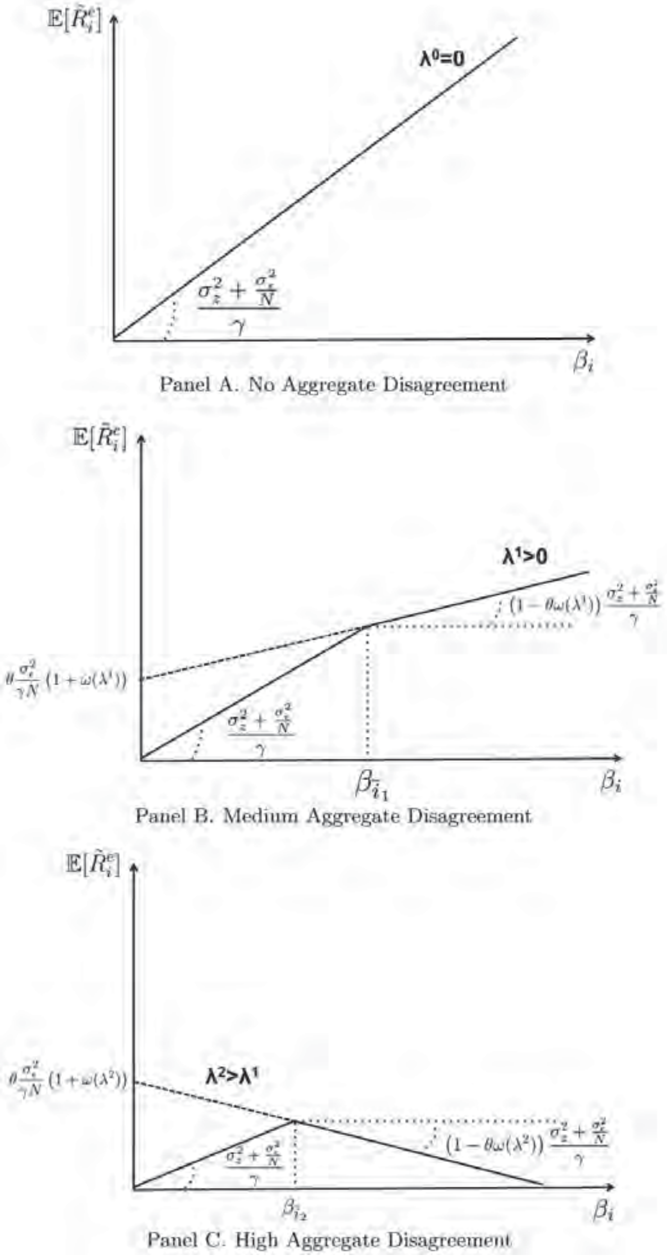


Figure 2. Security Market Line for different levels of aggregate disagreement.

Line and how this concavity is related to our empirical proxy for aggregate disagreement. More precisely, for each month, we estimate a cross-sectional regression of realized excess returns of 20 β-sorted portfolios on the portfolio β and the portfolio β^2. The time series of the coefficient estimate on β^2 represents a time series of returns that essentially goes long the low- and high-beta portfolios and short the portfolios around the median-beta portfolio. In effect, the Security Market Line described in equation (3) predicts that this square portfolio should experience lower returns when aggregate disagreement is high, or in other words, that the Security Market Line should be more concave when aggregate disagreement is high. This is our first main empirical prediction.

PREDICTION 1: *In low-disagreement months, the Security Market Line is upward-sloping. In high-disagreement months, the Security Market Line has a kink. Its slope is strictly positive for low-β assets but strictly lower (and potentially negative) for high-β assets. The Security Market Line should be more concave following months with high aggregate disagreement; equivalently, a portfolio long low- and high-beta assets and short medium-beta assets should experience lower performance following months of high aggregate disagreement.*

A consequence of the previous analysis is that the slope of the Security Market Line should also decrease following a month of high aggregate disagreement. However, this is a weaker prediction of the model since it does not exploit the specificity of our model, namely, the kink in the Security Market Line, which, as we will see in Section III, is an important feature of the data.

COROLLARY 3: *Let $\hat{\mu}$ be the coefficient estimate of a cross-sectional regression of realized returns $(\tilde{R}^e_i)_{i\in[1,N]}$ on $(\beta_i)_{i\in[1,N]}$ and a constant. The coefficient $\hat{\mu}$ decreases with aggregate disagreement λ and with the fraction of short-sales-constrained agents in the economy α. Furthermore, the negative effect of aggregate disagreement λ on $\hat{\mu}$ is larger when there are fewer arbitrageurs in the economy (i.e., when α increases).*

PROOF: See Internet Appendix Section I.E. ■

D. Discussion of Assumptions

Our theory relies on two fundamental ingredients, disagreement and short-sales constraints. Both are important. In the absence of disagreement, all investors share the same portfolio and, since there is a strictly positive supply of assets, this portfolio is long only. Thus, the short-sales constraint is irrelevant—it never binds—and the standard CAPM results apply. In the absence of short-sales constraints, the disagreement between group A and group B investors washes out in the market-clearing condition and prices simply reflect the average belief, which we have assumed to be correct.

The model also relies on important simplifying assumptions. First is that, in our framework, investors disagree only on the expectation of the aggregate factor, $\tilde{z}$. A more general setting would allow investors to also disagree on the idiosyncratic component of stock dividends $\tilde{\epsilon}_i$. If the idiosyncratic disagreement on a stock is not systematically related to the stock's cash flow beta, then our analysis is left unchanged since whatever mispricing is created by idiosyncratic disagreement does not affect the shape of the Security Market Line in a systematic fashion. If idiosyncratic disagreement is positively correlated with stocks' cash flow beta, then the impact of aggregate disagreement on the Security Market Line becomes even stronger. This is because there are now two sources of overpricing linked systematically with b_i, one coming from aggregate disagreement, and the other from this additional idiosyncratic disagreement.

As we show in Section II.C below, the overall disagreement on the earnings growth of high-beta stocks is much larger than the disagreement on low-beta stocks, *even* in months with low aggregate disagreement. This finding suggests that idiosyncratic disagreement is, in fact, larger for high-beta stocks. This conforms to standard intuition on the characteristics of high- and low-beta stocks.

The second key assumption in the model is that investors disagree only on the first moment of the aggregate factor $\tilde{z}$ and not on the second moment σ_z^2. From a theoretical viewpoint, this is not so different. In the same way that β scales disagreement regarding $\bar{z}$, β scales disagreement about σ_z^2. In other words, if we label the group that underestimates σ_z^2 as optimists and the group that overestimates σ_z^2 as pessimists, then optimists are more optimistic about the utility derived from holding a high-β asset than a low-β asset, while pessimists are more pessimistic about the utility derived from holding a high-β asset than a low-β asset. Again, high-β assets are more sensitive to disagreement about the variance of the aggregate factor σ_z^2 than are low-β assets. As in our model, this would naturally lead to high β stocks being overpriced when the disagreement about σ_z^2 is large. However, while empirical proxies for disagreement about the mean of the aggregate factor can be constructed, it is not clear how one would proxy for disagreement about its variance.

The third key assumption imposed by the model is that the dividend process is homoskedastic. In the next section, we relax this assumption and allow the dividend process of different assets to have heterogeneous levels of idiosyncratic volatility.

E. Heteroskedastic Idiosyncratic Variance

Our results in Theorem 1 in the static case are derived under the assumption that idiosyncratic shocks to the dividend process are homoskedastic, that is, $\forall i \in [1, N], \sigma_i^2 = \sigma_\epsilon^2$. This assumption is easily relaxed. When dividends are assumed to be heteroskedastic, assets need to be ranked in ascending order of $\frac{b_i}{\sigma_i^2}$, which is equivalent to ranking them in ascending order of $\frac{\beta_i}{\sigma_i^2}$. In Internet Appendix Section I.A, we show that the unique equilibrium then features a marginal asset $\bar{i}$, such that

$$\mathbb{E}[\tilde{R}_i^e] = \begin{cases} \beta_i \frac{\sigma_z^2 + \sum_{j=1}^N \frac{\sigma_j^2}{N^2}}{\gamma} & \textit{for } \frac{\beta_i}{\sigma_i^2} < \frac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2} \\ \beta_i \frac{\sigma_z^2 + \sum_{j=1}^N \frac{\sigma_j^2}{N^2}}{\gamma} \left(1 - \theta\omega(\lambda)\right) + \theta \frac{\sigma_i^2}{\gamma N} \left(1 + \omega(\lambda)\right) & \textit{for } \frac{\beta_i}{\sigma_i^2} \geq \frac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2} \end{cases}. \quad (4)$$

Intuitively, consider a stock with a high cash flow beta. Relative to pessimists, optimistic investors believe that this stock is likely to have a high dividend. If the stock has low idiosyncratic variance (σ_i^2), this will lead to high demand from optimists for this stock. In equilibrium, this will sideline pessimists from the stock and lead to speculative overpricing. However, if the stock has high idiosyncratic variance, optimists will be reluctant to demand large quantities of the stock despite their optimistic valuation. As a result, pessimists may be required to be long the stock in equilibrium, in which case the stock will be fairly priced. Thus, the equilibrium features a cutoff in the ratio of cash flow beta to idiosyncratic variance.

In particular, the pricing formula in equation (4) says that, for stocks i with β_i/σ_i^2 below the cutoff $\beta_{\bar{i}}/\sigma_{\bar{i}}^2$ (i.e., *nonspeculative stocks*), the slope of the partial Security Market Line (the relationship between expected returns and β for assets below the cutoff) does not depend on aggregate disagreement. For stocks i with a ratio β_i/σ_i^2 above this cutoff (i.e., *speculative stocks*), the partial Security Market Line is still linear in β but its slope is strictly decreasing with aggregate disagreement. The asset pricing equation (4) also predicts that, for these mispriced assets, idiosyncratic variance is priced and the price of idiosyncratic risk increases with aggregate disagreement. This is related to the previous intuition: all else equal, an asset with high idiosyncratic variance will receive smaller demand by optimists, which in equilibrium will drive down its price and drive up its expected return. This leads to our second main empirical prediction.

PREDICTION 2: *Define speculative assets as assets with a high ratio of β_i/σ_i^2. Then, the slope of the relationship between expected returns and β for these assets decreases strictly with aggregate disagreement. Conversely, for nonspeculative assets—assets with a low ratio of β_i/σ_i^2—the relationship between expected returns and β is independent of aggregate disagreement.*

F. Infinite Number of Assets

We analyze the case in which markets become complete and N goes to infinity. To simplify the discussion, we assume that, for any N, the number of assets, assets in the cross section always have cash flow betas that are bounded in $[\underline{b}, \bar{b}]$. We adapt our previous notation to denote by b_i^N the cash flow beta of asset i when the cross section has N assets, with $i \leq N$. Our assumption is that for all $N \in \mathbb{N}$ and $i \leq N$, $0 < \underline{b} < b_i^N < \bar{b} < \infty$. Under this assumption, we show that in the limiting case, where $N \to \infty$, asset returns always admit a linear CAPM representation. In particular, the slope of the Security Market Line is independent of λ as long as $\lambda \leq \frac{\sigma_z^2}{\gamma}$ and is strictly decreasing with λ when $\lambda > \frac{\sigma_z^2}{\gamma}$.

Since $u_i^N = \frac{1}{\gamma N b_i^N}(\sigma_\epsilon^2 + \sigma_z^2(\sum_{j<i}(b_j^N)^2)) + \frac{\sigma_z^2}{\gamma}(\sum_{j \geq i}\frac{b_j^N}{N})$ and the b_i are bounded, it is direct to see that when $N \to \infty, u_1^N \to \frac{\sigma_z^2}{\gamma}$ and $u_N^N \to l\frac{\sigma_z^2}{\gamma}$, where $l = \lim_{N\to\infty}\sum_{j<N}\frac{(b_j^N)2}{N b_N^N}$ and $l < 1$ since for all $j < N, b_j^N < b_N^N$.

Our first result is that if λ is small enough (i.e., $\lambda\gamma < l\sigma_z^2 = \gamma \lim u_N^N$), then, at the limit $N \to \infty$, no asset will experience binding short-sales constraints, so that asset returns will be independent of λ and the standard CAPM formula will apply: $\mathbb{E}[\tilde{R}_i^e] = \beta_i \mathbb{E}[\tilde{R}_m^e]$, with $\mathbb{E}[\tilde{R}_m^e]$ independent of λ.

Our second result is that, provided λ is large enough (i.e., $\lambda\gamma > \sigma_z^2 = \gamma \lim u_1^N$), then at the limit all assets will experience binding short-sales constraints. In this case, expected returns at the limit are given by

$$\mathbb{E}[\tilde{R}_i^e] = \beta_i \left((1+\theta)\frac{\sigma_z^2}{\gamma} - \lambda\theta \right) = \beta_i \mathbb{E}[\tilde{R}_m^e(\lambda)].$$

The Security Market Line is linear as in the previous case, but its slope is now strictly decreasing with aggregate disagreement λ. In particular, if $\lambda\gamma > \frac{1+\theta}{\theta}\sigma_z^2$, then the Security Market Line is strictly decreasing.

The final case occurs when $\sigma_z^2 > \lambda\gamma > l\sigma_z^2$. For any finite i, we know that $\lim u_i^N = \frac{\sigma_z^2}{\gamma}$. Thus, at the limit, the marginal asset has to be such that $\lim \bar{i}^N = \infty$. But, we know that $\omega(\lambda) \to 0$, so the speculative premium at the limit is also zero. As a result, at the limit, asset returns will be independent of λ and the standard CAPM formula will again apply: $\mathbb{E}[\tilde{R}_i^e] = \beta_i \mathbb{E}[\tilde{R}_m^e]$, with $\mathbb{E}[\tilde{R}_m^e]$ independent of λ.

G. Dynamics

G.1. Setup

We now consider a dynamic extension of the previous model, where we also allow for heteroskedasticity in dividend shocks. This extension is done in the context of an OLG framework. Time is infinite, $t = 0, 1, \ldots \infty$. Each period t, a new generation of investors of mass one is born and invests in the stock market to consume the proceeds at date $t+1$. Thus, at date t, the new generation is buying assets from the current old generation (born at date $t-1$), which has to sell its entire portfolio in order to consume. Each generation consists of two groups of investors: arbitrageurs or HFs, in proportion $1-\alpha$, and MFs in proportion α. Investors have mean-variance preferences with risk tolerance parameter γ. There are N assets, whose dividend process is given by

$$\tilde{d}_t^i = d + b^i \tilde{z}_t + \tilde{\epsilon}_t^i,$$

where $\mathbb{E}[\tilde{z}] = 0, \mathrm{Var}[\tilde{z}] = \sigma_z^2, \mathbb{E}[\tilde{\epsilon}^i] = 0, \mathrm{Var}[\tilde{\epsilon}^i] = \sigma_i^2$, and $\frac{1}{N}\sum_{i=1}^N b_i = 1$. We normalize the assets to be ranked in ascending order of $\frac{b_i}{\sigma_i^2}$:

$$0 < \frac{b_1}{\sigma_1^2} < \frac{b_2}{\sigma_2^2} < \cdots < \frac{b_N}{\sigma_N^2}.$$

The timeline of the model appears in Figure 3. MFs born at date t hold heterogeneous beliefs about the expected value of $\tilde{z}_{t+1}$. Specifically, there are two groups of MFs: investors in group A—the optimists—whose expectations about $\tilde{z}_{t+1}$ are such that $\mathbb{E}_t^A[\tilde{z}_{t+1}] = \tilde{\lambda}_t$, and investors in group B—the

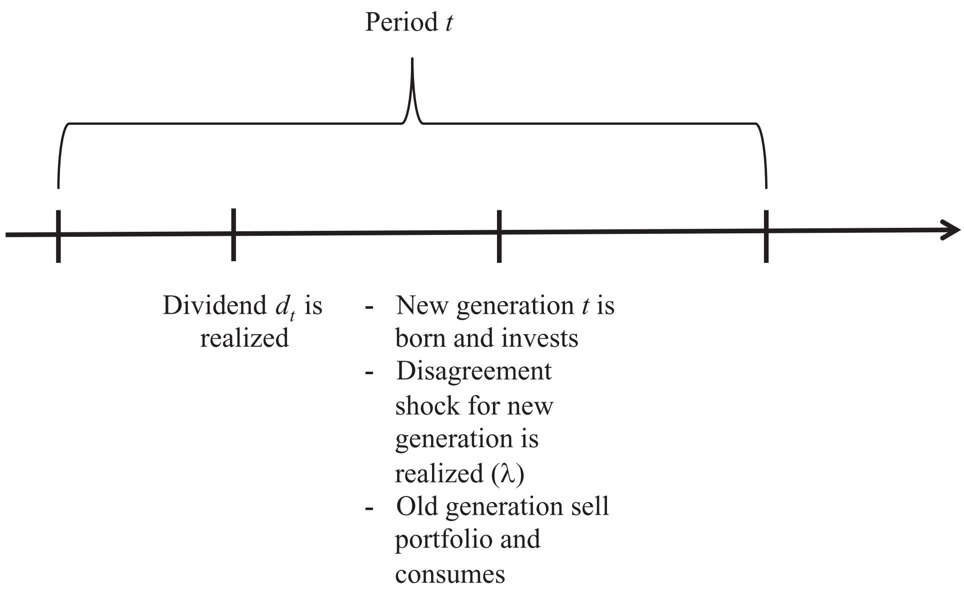


Figure 3. Timeline of the dynamic model of Section I.G.

pessimists—whose expectations about $\tilde{z}_{t+1}$ are such that $\mathbb{E}^B[\tilde{z}_{t+1}] = -\tilde{\lambda}_t$.[14] Finally, we assume that $\tilde{\lambda}_t \in \{0, \lambda > 0\}$ is a two-state Markov process with persistence $\rho \in]1/2, 1[$.

Call $P_t^i(\tilde{\lambda})$ the price of asset i at date t when realized aggregate disagreement is $\tilde{\lambda}_t \in \{0, \lambda\}$, and define $\Delta P_t^i = P_t^i(\lambda) - P_t^i(0)$. Let $\mu_i^k(\tilde{\lambda}_t)$ be the number of shares of asset i purchased by investors in group k when realized aggregate disagreement is $\tilde{\lambda}_t \in \{0, \lambda\}$, and let λ_t^k be the realized belief at date t for investors in group $k \in \{\text{a, A, B}\}$.[15] We first compute the date $t+1$ wealth of investors in group $k \in \{\text{a, A, B}\}$, born at date t and with portfolio holdings $(\mu_i^k(\tilde{\lambda}_t))_{i \in [1,N]}$:

$$\tilde{W}_{t+1}^k = \left(\sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) b_i \right) \tilde{z}_{t+1} + \sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) \tilde{\epsilon}_{t+1}^i + \sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) \left(d + P_{t+1}^i(\tilde{\lambda}_{t+1}) - (1+r) P_t^i(\lambda_t) \right).$$

Thus, for investors in group k, their own expected wealth at date $t+1$ and its associated variance are given by

$$\begin{cases} \mathbb{E}^k[\tilde{W}^k] = \left(\sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) b_i \right) \lambda_t^k + \sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) \\ \qquad \left(d + \mathbb{E}[P_{t+1}^i(\tilde{\lambda}_{t+1}) | \tilde{\lambda}_t] - (1+r) P_t^i(\lambda_t) \right) \\ \mathrm{Var}[\tilde{W}^k] = \left(\sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) b_i \right)^2 \sigma_z^2 + \sum_{i \le N} \left(\mu_i^k(\tilde{\lambda}_t) \right)^2 \sigma_i^2 \\ \qquad + \rho(1-\rho) \left(\sum_{i \le N} \mu_i^k(\tilde{\lambda}_t) \left(\Delta P_{t+1}^i \right) \right)^2. \end{cases}$$

[14] Most of the assumptions made in this model are discussed in Section I.A in the context of our static model.

[15] $\lambda_t^a = 0$, $\lambda_t^A = \lambda$, and $\lambda_t^B = -\lambda$ when $\tilde{\lambda}_t = \lambda$ or $\lambda_t^a = \lambda_t^A = \lambda_t^B = 0$ when $\tilde{\lambda}_t = 0$ and $\lambda_t^a = 0$.

Relative to the static model, there are two notable changes. First, investors value the resale price of their holding at date 1 (the $\mathbb{E}[P^i_{t+1}(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t]$ term in expected wealth). Second, investors now bear the corresponding risk that the resale prices move with aggregate disagreement $\tilde{\lambda}_t$ (this is, in our binomial setting, the $\rho(1-\rho)(\sum_{i\leq N}\mu^k_i(\tilde{\lambda}_t)(\Delta P^i_{t+1}))^2$ term in wealth variance).

G.2. Equilibrium

The following theorem characterizes the equilibrium of this economy.

THEOREM 2: *Define* $(v_i)_{i\in[0,N+1]}$ *such that* $v_{N+1}=0$, $v_i=\frac{\sigma_z^2}{N}(\sum_{k\geq i}b_k)+\frac{1}{N}\frac{\sigma_i^2}{b_i}(1+\sigma_z^2\sum_{k<i}\frac{b_k^2}{\sigma_k^2})$ *for* $i\in[1,N]$, *and* $v_0=\infty$. *The sequence* v *is strictly decreasing. Let* $\bar{\imath}=\min\{k\in[0,N+1]|\lambda>v_k\}$. *There exists a unique equilibrium. In this equilibrium, short-sales constraints bind only for the group of pessimist investors (i.e., group B) in the high disagreement states* ($\tilde{\lambda}_t=\lambda>0$) *for assets* $i\geq\bar{\imath}$. *The speculative premium on these assets is given by*

$$\pi^j=\frac{\theta}{\gamma}\left(b_j\frac{\lambda\gamma-\frac{\sigma_z^2}{N}\sum_{k\geq\bar{\imath}}b_k}{1+\sigma_z^2\left(\sum_{i<\bar{\imath}}\frac{b_i^2}{\sigma_i^2}\right)}-\frac{\sigma_j^2}{N}\right).$$

Finally, define

$$\Gamma^\star=\frac{-(1+r)+(2\rho-1)+\sqrt{\left((1+r)-(2\rho-1)\right)^2+\frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{\imath}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}>0.$$

In equilibrium, asset returns are given by

$$\begin{cases}\mathbb{E}[R^j(\lambda)]=\mathbb{E}[R^j(0)]=\dfrac{1}{\gamma}\left(b_j\sigma_z^2+\dfrac{\sigma_i^2}{N}\right) & \textit{for } j<\bar{\imath}\\ \mathbb{E}[R^j(0)]=\dfrac{1}{\gamma}\left(b_j\sigma_z^2+\dfrac{\sigma_i^2}{N}+\rho(1-\rho)\dfrac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right) & \textit{for } j\geq\bar{\imath}\\ \mathbb{E}[R^j(\lambda)]=\dfrac{1}{\gamma}\left(b_j\sigma_z^2+\dfrac{\sigma_i^2}{N}+\rho(1-\rho)\dfrac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right) & \\ \qquad-\dfrac{1+r-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j & \textit{for } j\geq\bar{\imath}.\end{cases}$$

PROOF: See Internet Appendix Section I.F. ■

Our characterization of how disagreement affects the Security Market Line in our static model still carries over to this dynamic model with heteroskedasticity. Low b/σ^2 assets (i.e., $j<\bar{\imath}$) are never shorted since there is not enough disagreement among investors to make the pessimist investors willing to go short, even in the high-disagreement states. Thus, the price of these assets is the same in both states of nature and similar to the standard CAPM case. In the high-aggregate-disagreement state ($\tilde{\lambda}=\lambda>0$), pessimist investors, that is, investors in group B, want to short high-b assets to the extent that these assets are not too risky (i.e., assets j such that $\frac{b_j}{\sigma_j^2} \geq \frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}$), but are prevented from doing so by the short-sale constraint.[16] This leads to overpricing of these assets relative to the benchmark without disagreement.

A consequence of the previous analysis is that the price of assets $j \geq \bar{i}$ depends on the realization of aggregate disagreement. There is an extra source of risk embedded in these assets: their resale price is more exposed to aggregate disagreement. These assets are thus riskier and command an extra risk premium relative to lower-b assets. This extra risk premium takes the form $\frac{1}{\gamma}\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j$. Relative to a benchmark without disagreement (and where expected returns are always equal to $\frac{1}{\gamma}(b_j\sigma_z^2 + \frac{\sigma_i^2}{N})$), high-$b$ assets have higher expected returns in low-disagreement states (because of the extra risk premium). In high-disagreement states, holding σ^2 constant, the expected returns of high-b assets are strictly lower than in low-disagreement states, since the large disagreement about next-period dividends leads to overpricing. Thus, in high-disagreement states, the slope of the relationship between expected returns and cash flow betas holding σ^2 constant is smaller for assets with a high ratio of cash flow beta to idiosyncratic variance (i.e., assets $j \geq \bar{i}$) than assets $j < \bar{i}$. Whether the expected returns of high-b assets are lower or higher than in the benchmark without disagreement depends on the relative size of the extra risk premium and the speculative premium. In the data, however, aggregate disagreement is persistent, that is, ρ is close to one. A first-order Taylor expansion of $\Gamma^\star$ around $\rho = 1$ gives $\Gamma^\star \approx \sum_{j\geq\bar{i}} \frac{\pi_j}{N}$, so that, in the vicinity of $\rho = 1$, $\mathbb{E}[R^j(\lambda)] < \frac{1}{\gamma}(b_j\sigma_z^2 + \frac{\sigma_j^2}{N})$. Intuitively, when aggregate disagreement is persistent, the resale price risk is very limited, since there is only a small probability that the price of high-b assets will change next period. Thus, the speculative premium term dominates and expected returns of high-b assets are lower than under the no-disagreement benchmark. We summarize these findings in the following proposition.

COROLLARY 4:

(i) *In low-disagreement states ($\tilde{\lambda} = 0$), conditional on σ_i^2, expected returns $\mathbb{E}R_j^e$ are strictly increasing with cash flow beta b_j. Because of resale price risk, the slope of the return/cash flow beta relationship is higher for assets $j \geq \bar{i}$ than for assets $j < \bar{i}$.*

(ii) *In high-disagreement states ($\tilde{\lambda} = \lambda > 0$), conditional on σ_i^2, expected returns $\mathbb{E}R_j^e$ are strictly increasing with cash flow beta b_j for assets $j < \bar{i}$. For assets $j \geq \bar{i}$, the slope of the return/cash flow beta relationship can be either higher or lower than for assets $j < \bar{i}$. There exists $\rho^\star < 1$ such that, for $\rho \geq \rho^\star$, this slope is strictly lower for $b \geq b_{\bar{i}}$ than for $b < b_{\bar{i}}$.*

[16] In the low-disagreement state, $\tilde{\lambda} = 0$ so that there is no disagreement among investors and hence there cannot be any binding short-sales constraint.

(iii) *Conditional on σ_i^2, expected returns $\mathbb{E}R_j^e$ can decrease strictly with cash flow beta b_j for assets $j \geq \bar{\imath}$ in high-disagreement states, provided ρ is close to one and λ is large enough.*

(iv) *Conditional on σ_i^2, the slope of the returns/cash flow beta relationship for assets $j \geq \bar{\imath}$ is strictly lower in high-disagreement states ($\tilde{\lambda} = \lambda > 0$) than in low-disagreement states ($\tilde{\lambda} = 0$).*

PROOF: See Internet Appendix Section I.G. ■

H. Calibration

In this section, we present a simple calibration of the dynamic model presented in the previous section. The objective of this calibration is to see what magnitude of aggregate disagreement is required to obtain a significant distortion in the Security Market Line. We use the following parameters. First, ρ is set to 0.95. This estimate is obtained by dividing our time series into high and low aggregate disagreement months (i.e., above and below the median of aggregate disagreement) and computing the probability of transitioning from high to low disagreement ($\mathbb{P} = 0.05$) and from low to high disagreement ($\mathbb{P} = 0.05$). We set α to 0.66 (i.e., $\theta = 0.5$), which corresponds to the fraction of the stock market held by MFs and retail investors, for which the cost of shorting is presumably nontrivial.

The most difficult parameter for us to set is N. We show in Section I.F that, when N is large, the Security Market Line can be upward- or downward-sloping but not inverted-U shaped. However, we argue that a large N is not a good calibration for our model. In the presence of such fixed costs, investors will trade a much smaller number of assets than the overall number of assets in the market. Of course, introducing fixed costs of trading in our model complicates the analysis substantially. We defer a full treatment of this more complex model to further research. In particular, with fixed trading costs, the choice of which asset to trade becomes endogenous. We believe, however, that the main elements of our analysis would remain unchanged and we highlight here how this endogenous asset selection affects our analysis.

When investors face fixed trading costs, in equilibrium, there is a segmentation of the market. All else equal, optimists would tend to buy the segment of high-cash-flow beta assets, as opposed to our current model with no trading costs where they trade all assets. While pessimists would only trade on the segment of low-beta assets, as in our model, one notable difference with our current setup is that the pessimists would now be the only investors holding these low-beta assets. As a consequence, the low-beta assets would be underpriced. This effect would reinforce our results as the underpriced low-beta securities would make the Security Market Line "kinkier."

With fixed trading costs, arbitrageurs also need to decide which assets to trade. First, in equilibrium, they need to hold the segment of intermediate-cash-flow beta securities. To the extent that mispricing on high- and low-beta assets is not large—that is, aggregate disagreement λ is not too large—the risk premium they receive for holding these intermediate-beta assets will be greater than the arbitrage premium they would receive from shorting the high-beta securities. As disagreement λ increases, a fraction of arbitrageurs will start shorting the high-beta assets. In this case, arbitrageurs engage in shorting in an amount such that the utility they derive from shorting the high-beta stocks is equal to the utility of holding the intermediate-beta stocks. Thus, as λ increases, the amount of arbitrage capital devoted to shorting the high-beta stocks increases. At the same time, however, optimistic MFs increase their leverage to bet on high-beta stocks. This increase in optimistic MFs' demand may well dominate the effect of increased shorting by arbitrageurs.[17]

Beyond trading costs, there exist additional reasons why MF managers invest among a restricted set of stocks. Most MF managers are benchmarked to indices, such as the Morningstar Large Cap Growth Index or the Russell 1000 Growth Index. These indices typically have only a few hundred stocks as constituent members. Hence, because of their index or tracking mandates, most mutual fund managers form their portfolios based on a universe of only a few hundred stocks. Retail investors also trade within a restricted universe of stocks, as it is well known that these investors typically consider buying stocks that they are familiar with, such as stocks headquartered near where they live or stocks with a high advertising presence (Huberman (2001) or Barber and Odean (2008)). To the extent that the betas of the securities these investors consider are evenly distributed, our model can be directly applied using the average number of stocks held by each investor as the N in our model.

Consistent with N being small for MF investors, Griffin and Xu (2009) show that from 1980 to 2004, which overlaps with our sample period, the average number of stocks held by MFs is between 43 and 119. Consistent with N being even smaller for retail investors, Kumar and Lee (2006) document, using a data set from a large U.S. retail broker in the 1990s, that the average retail investor holds a four-stock portfolio. Fewer than 5% of retail investors hold more than 10 stocks.

As noted in Barber and Odean (2000), in 1996, approximately 47% of equity investments in the United States were held directly by households and 14% by MFs, although these shares change quite a bit over time. As such, $N = 50$ seems in the relevant range for the typical number of stocks held by long only investors. The calibration we perform below is not very sensitive to small changes in N around $N = 50$.[18] As expected, however, when N becomes very large, we get the result we derived theoretically in Section I.F when solving the complete market case: the Security Market Line can be upward- or downward-sloping but not inverted-U shaped as is the case when N is smaller and in the calibration we perform below.

We set the values of b_i such that $b_i = \frac{2i}{N+1}$. Thus, cash flow betas are bounded between zero and two and have an average value of one. We implement our

[17] In a simple three-asset version of this model with trading costs, we can show that mispricing is, in fact, increasing with λ.

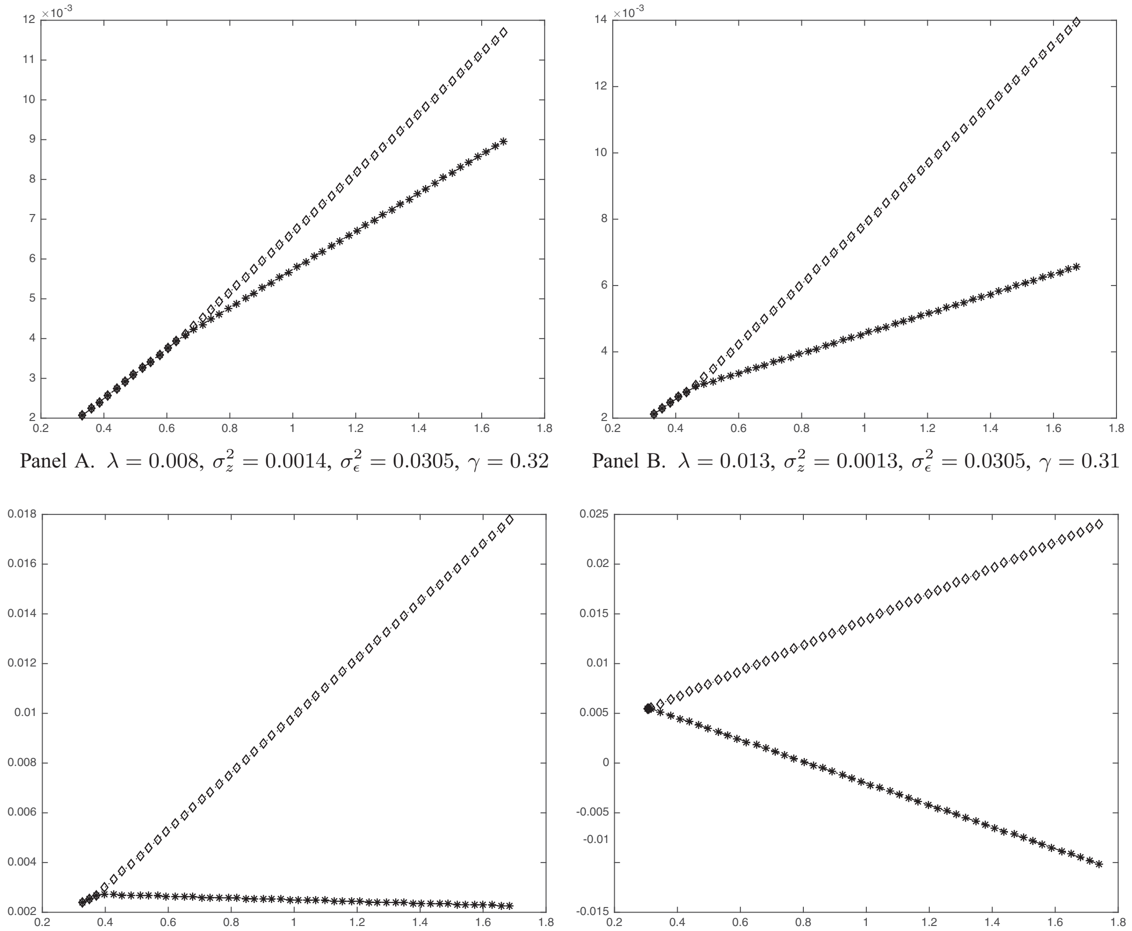
[18] We perform calibrations using $N = 25$ and $N = 75$ and find similar qualitative results.

calibration as followings. We set a value for λ. We then find the values for σ_z^2, σ_ϵ^2, and γ such that the model matches the following empirical moments, computed over the 1981 to 2011 period: (i) the average volatility of the monthly market return (0.2% monthly), (ii) the average idiosyncratic variance of monthly stock returns (3.5% monthly), and (iii) the average expected excess return of the market (0.63% monthly).[19] Finally, we borrow from Campbell, Grossman, and Wang (1993) and calibrate a CARA model using dollar returns by setting the dividend to have an asset price equal to one. We report four calibrations in Figure 4:

1. $\lambda = 0.008$, which implies $\sigma_\epsilon^2 = 0.0305$, $\sigma_z^2 = 0.0014$, and $\gamma = 0.32$. In equilibrium, 38 of the 50 assets are shorted. This level of disagreement corresponds to 20% of σ_z. Figure 4(a) plots the Security Market Line for these parameter values. Figure 4(a) shows that, for this level of disagreement, the distortion in the Security Market Line is limited. Even in the high-aggregate-disagreement state, the Security Market Line is upward-sloping with a slope close to its slope in the low-aggregate-disagreement state.
2. $\lambda = 0.013$, which implies $\sigma_\epsilon^2 = 0.0305$, $\sigma_z^2 = 0.0013$, and $\gamma = 0.31$. In equilibrium, 45 of the 50 assets are shorted. This level of disagreement corresponds to 35% of σ_z. Figure 4(b) shows that, for this level of disagreement, the distortion in the Security Market Line becomes noticeable. In the high-aggregate-disagreement state, the Security Market Line is still upward-sloping for all β, but with a much smaller slope for assets with $b_i \geq b_{\bar{i}}$. The Security Market Line is kink-shaped in the high-aggregate-disagreement state.
3. $\lambda = 0.022$, which implies $\sigma_\epsilon^2 = 0.0305$, $\sigma_z^2 = 0.0011$, and $\gamma = 0.27$. In equilibrium, 47 of the 50 assets are shorted. This level of disagreement corresponds to 65% of σ_z. Figure 4(c) shows that, for this level of disagreement, in the high-aggregate-disagreement state, the Security Market Line has an inverted-U shape.
4. $\lambda = 0.05$, which implies $\sigma_\epsilon^2 = 0.0305$, $\sigma_z^2 = 0.0005$, and $\gamma = 0.16$. In equilibrium, 48 of the 50 assets are shorted. This level of disagreement corresponds to 187% of σ_z. Figure 4(d) shows that, for this level of disagreement, in the high-aggregate-disagreement state, the Security Market Line is downward-sloping. Moreover, we also see in Figure 4(d) that assets with beta greater than 0.9 have a negative expected excess return in the high-aggregate-disagreement state.

Overall, these calibrations support the idea that, for reasonable levels of disagreement, the Security Market Line in the high-aggregate-disagreement

[19] The volatility and average excess return on the market are computed directly from the monthly market return series obtained from Ken French's website. To compute the average idiosyncratic variance of stock returns, we first estimate a CAPM equation for each stock in our sample using monthly excess returns; we then compute the variance of the residuals from this equation for each stock. Finally, we define the average idiosyncratic variance as the average of these variances across all stocks in our sample.



Panel A. $\lambda = 0.008$, $\sigma_z^2 = 0.0014$, $\sigma_\epsilon^2 = 0.0305$, $\gamma = 0.32$

Panel B. $\lambda = 0.013$, $\sigma_z^2 = 0.0013$, $\sigma_\epsilon^2 = 0.0305$, $\gamma = 0.31$

Panel C. $\lambda = 0.022$, $\sigma_z^2 = 0.0011$, $\sigma_\epsilon^2 = 0.0305$, $\gamma = 0.27$

Panel D. $\lambda = 0.05$, $\sigma_z^2 = 0.00012$, $\sigma_\epsilon^2 = 0.304$, $\gamma = 0.11$

Figure 4. Calibration of the dynamic model. This figure plots the Security Market Line in the high-aggregate-disagreement state (diamonds) and in the low-aggregate-disagreement state (stars) obtained from the simulation of the dynamic model. Across simulations, we use the following parameters: $\theta = 0.5$, $N = 50$, and $\rho = 0.95$. Each of the four panels sets a value of λ (0.008 in Panel A, 0.013 in Panel B, 0.022 in Panel C, and 0.05 in Panel D), and then finds the values of σ_z^2, σ_ϵ^2, and γ that match the empirical average idiosyncratic variance of monthly stock returns, the empirical variance of the monthly market return, and the empirical average return on the market portfolio over the sample period.

state will be significantly distorted relative to the low-aggregate-disagreement state.

II. Data and Variables

A. Data Source

The data in this paper are collected from two main sources. U.S. stock return data come from the CRSP tape, and analyst forecasts come from the I/B/E/S analyst forecast database. The I/B/E/S data start in December 1981.

We begin with all available common stocks on CRSP between December 1981 and December 2014. Then, for each month, we exclude penny stocks with a share price below \$5 and microcaps, defined as stocks in the bottom two deciles of the monthly market capitalization distribution using NYSE break points. The betas are computed with respect to the value-weighted market returns provided on Ken French's website. Excess returns are in excess of the U.S. Treasury bill rate, which we also download from Ken French's website. We use analyst forecasts of the EPS long-term growth rate (LTG) as our main proxy for investors' beliefs regarding the future prospects of individual stocks. These data come from the I/B/E/S database. As explained in detail in Yu (2011), the long-term forecast has several advantages. First, it features prominently in valuation models. Second, it is less affected by a firm's earnings guidance than short-term forecasts. Because the long-term forecast is an expected growth rate, it is directly comparable across firms or over time.

B. β-Sorted Portfolios

We follow the literature in constructing beta portfolios as follows. Each month, we use the past 12 months of daily returns to estimate the market beta of each stock in that cross section. This is done by regressing a stock's excess return on the contemporaneous excess market return as well as five lags of the market return to account for the illiquidity of small stocks (Dimson (1979)). Our measure of β is then the sum of the six OLS coefficients.

We next sort stocks each month into 20 β portfolios based on these preranking betas, using only stocks in the NYSE to define the β thresholds. We compute the daily returns on these portfolios, both equal- and value-weighted. Postranking βs are then estimated using a similar market model—regressing each portfolio's daily return on the excess market return, as well as five lags of the market return, and adding up these six OLS coefficients. The postranking βs are computed using the full sample period (Fama and French (1992)). Table I presents descriptive statistics for the 20 β-sorted portfolios. The 20 β-sorted portfolios exhibit a significant spread in β, with the postranking full-sample β of the bottom portfolio equal to 0.43 and that of the top portfolio equal to 1.78.

C. Measuring Aggregate Disagreement

Our measure of aggregate disagreement is similar in spirit to Yu (2011). We first measure stock-level disagreement as the dispersion in analyst forecasts of the EPS LTG. We then aggregate this stock-level disagreement measure, weighting each stock by its preranking β.[20] Intuitively, our model suggests that there are two components to the overall disagreement on a stock-dividend process: a component coming from the disagreement about the aggregate factor $\tilde{z}$—the λ in our model—and a component coming from disagreement about the

[20] The preranking βs are constructed as detailed in Section II.B.

Table I

Summary Statistics for 20 β-Sorted Porftolios

Sample period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Preformation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to 1 of 20 value-weighted portfolios based on NYSE break points. The table reports the full sample β of each of these 20 portfolios, computed using a similar risk model. *Median Vol.* is the median preranking volatility of stocks in the portfolio. $R_t^{(1)}$ is the return of the portfolio from t to $t+1$, and $R_t^{(12)}$ from t to $t+11$. *Stock Disp.* is the average stock-level disagreement for stocks in each portfolio, where stock-level disagreement is defined as the standard deviation of analysts' forecasts of long-run EPS growth. *% Mkt. Cap.* is the average ratio of market capitalization of stocks in the portfolio divided by the total market capitalization of stocks in the sample. *N* stocks is the number of stocks on average in each portfolio.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
β	0.43	0.49	0.55	0.60	0.67	0.69	0.77	0.79	0.83	0.89	0.95	0.98	1.02	1.11	1.13	1.21	1.26	1.39	1.50	1.78
Median Vol.	1.56	1.14	1.10	1.11	1.13	1.14	1.15	1.17	1.22	1.23	1.28	1.33	1.34	1.43	1.50	1.58	1.70	1.88	2.26	3.16
$R_{i,t}^{(1)}$	0.12	0.39	0.63	0.61	0.61	0.73	0.87	0.61	0.79	0.60	0.67	0.64	0.53	0.57	0.73	0.54	0.58	0.44	−0.17	−0.72
$R_{i,t}^{(12)}$	3.21	5.58	7.98	7.74	8.19	7.94	8.10	8.15	8.92	6.88	8.28	8.51	7.46	7.82	8.08	7.86	8.23	6.62	−0.55	−11.69
Stock Disp.	2.97	2.76	2.81	2.78	2.89	3.02	3.24	3.35	3.44	3.38	3.54	3.56	3.60	3.72	3.94	3.93	4.29	4.71	5.03	6.91
% Mkt. Cap.	2.77	3.58	4.15	4.73	4.86	5.00	5.32	5.44	5.49	5.84	5.60	5.58	5.62	5.63	5.36	5.45	5.31	5.09	5.21	7.48
N stocks	171	143	145	147	149	154	157	160	161	169	164	166	167	167	167	168	175	186	210	365

idiosyncratic factor $\tilde{\epsilon}_i$. We are interested in constructing an empirical proxy for the first component only. To that end, disagreement about low-β stocks should only play a minor role since disagreement about a low-β stock has to come mostly from idiosyncratic disagreement—in the limit, disagreement about a $\beta = 0$ stock can only come from idiosyncratic disagreement. Thus, we weight each stock's disagreement by the stock's preranking β.[21]

To assess the robustness of our analysis, we use two alternative proxies for aggregate disagreement. The first of these alternative measures is the analyst forecast dispersion of Standard & Poor's (S&P) 500 index annual EPS. The problem with this top-down measure is that far fewer analysts forecast this quantity, making it less attractive compared to our bottom-up measure. While our preferred measure of aggregate disagreement is constructed using thousands of individual stock forecasts, on average, only 20 or so analysts in the sample cover the S&P 500 EPS. Our second alternative proxy is an index of the dispersion of macroforecasts from the Survey of Professional Forecasters (SPF). More precisely, we use the first principal component of the cross-sectional standard deviation of forecasts on GDP, IP, corporate profit, and unemployment from Li and Li (2014).

To simplify the reading of the tables in the paper, all of our time-series measures of aggregate disagreement are standardized to have zero mean and a variance of one. Table II presents summary statistics on the time-series variables used in the paper. Figure 1 plots the time series of our baseline disagreement measure. It peaks during the 1981 to 1982 recession, the dot-com bubble of the late 1990s, and the recent recession of 2008. When fundamentals are more uncertain, there is more scope for disagreement among investors. In other words, the aggregate disagreement series is not the same as the business cycle, as we see high disagreement in both growth and recessionary periods.

In Figure 5, we highlight the role played by aggregate disagreement on the relationship between stock-level disagreement and β. This figure is constructed as follows. We divide our time series into high-aggregate-disagreement months (diamonds) and low-aggregate-disagreement months (triangles), where high- (low-) aggregate disagreement months are defined as those in the top (bottom) quartiles of the in-sample distribution of aggregate disagreement. Then, for each of our 20 β-sorted portfolios, we plot the value-weighted average of the stock-level dispersion in analyst earnings forecasts against the postranking full sample β of the value-weighted portfolio. Stock-level disagreement increases with β. Moreover, this relation is steeper in months with high aggregate disagreement relative to months with low aggregate disagreement. Thus, consistent with the premise of our model, we find that β does scale up aggregate disagreement.

[21] In Table I.IAIII, we show that our main results are robust to weighting using compressed betas and weighting using the product of beta and size. These measures also use preranking betas.

Table II
Summary Statistics for Time-Series Variables

To construct Agg. Disp, we start from the CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). Each month, for each stock, we calculate the standard deviation of analysts' forecasts of long-run EPS growth, which is our measure of stock-level disagreement. We also estimate for each stock $i\,\hat{\beta}_{i,t-1}$, the stock market beta of stock i at the end of the previous month. These betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. *Agg. Disp*. is the monthly $\hat{\beta}_{i,t-1}$-weighted average of stock-level disagreement. *Agg. Disp. (compressed)* uses $0.5\hat{\beta}_i + 0.5$ as the weight for stock i instead of $\hat{\beta}_i$. *Agg. Disp* (*β Value Weight)* uses $\hat{\beta}_i$ (Market Value)$_i$ as the weight for stock i instead of $\hat{\beta}_i$. *Top-down Disp*. is the monthly standard deviation of analysts' forecasts of annual S&P 500 earnings, scaled by the average forecast of S&P 500 earnings. *SPF Disp*. is the first principal component of the standard deviation of forecasts for GDP, IP, corporate profit, and the unemployment rate in the SPF and is taken from Li and Li (2014). These measures of aggregate disagreement are standardized to have an in-sample mean of zero and a standard deviation of one. D/P is the aggregate dividend-to-price ratio from Robert Shiller's website. $R^{(12)}_{m,t}$, $SMB^{(12)}_t$, $HML^{(12)}_t$, $UMD^{(12)}_t$ are the 12-month monthly returns on the market, SMB, HML, and UMD portfolios from Ken French's website and are expressed in %. *TED* is the TED spread and *Inflation* is the yearly inflation rate. The sample period is from 12/1981 to 12/2014, and summary statistics are displayed for months in which both Agg. Disp and $R^{(12)}_{m,t}$ are nonmissing.

	Mean	Std. Dev.	p10	p25	Median	p75	p90	Obs.
Agg. Disp.	−0.00	1.00	−0.96	−0.79	−0.21	0.42	1.28	385
Agg. Disp. (compressed)	0.00	1.00	−1.04	−0.79	−0.16	0.57	1.46	385
Agg. Disp. (β · Value weight)	0.00	1.00	−1.00	−0.84	−0.33	0.67	1.47	385
Top-down Disp.	0.00	1.00	−0.43	−0.37	−0.27	−0.13	0.64	353
SPF Disp.	0.00	1.00	−1.00	−0.78	−0.14	0.44	1.06	361
$\mathrm{R}^{(12)}_{m,t}$	8.82	16.95	−15.47	0.41	10.53	19.73	27.93	385
$\mathrm{SMB}^{(12)}_t$	1.23	9.91	−9.46	−5.32	0.18	6.93	14.11	385
$\mathrm{HML}^{(12)}_t$	3.96	13.32	−10.45	−4.52	3.11	10.77	17.50	385
$\mathrm{UMD}^{(12)}_t$	7.15	16.56	−9.29	−0.91	7.58	16.92	25.53	385
D/P	2.58	1.09	1.41	1.76	2.17	3.24	4.21	385
Inflation	0.03	0.01	0.01	0.02	0.03	0.03	0.04	385
TED spread	0.72	0.57	0.20	0.32	0.56	0.91	1.34	385

III. Empirical Analysis

A. Aggregate Disagreement and the Concavity of the Security Market Line

A.1. Main Analysis

Our empirical analysis examines how the Security Market Line is affected by aggregate disagreement. To this end, in Figure 6, we first present the empirical relationship between β and excess returns. For each of the 20 β portfolios in our sample, we compute the average excess forward return for high- (diamonds) and low- (triangles) disagreement months (defined as top versus bottom quartile of aggregate disagreement). Given the persistence in aggregate disagreement, we run this analysis using several horizons: 3 months (Panel A), 6 months (Panel B), 12 months (Panel C), and 18 months (Panel D). The portfolio returns $r^{(k)}_{P,t}$ for $k = 3, 6, 12,$ and 18 are value-weighted.

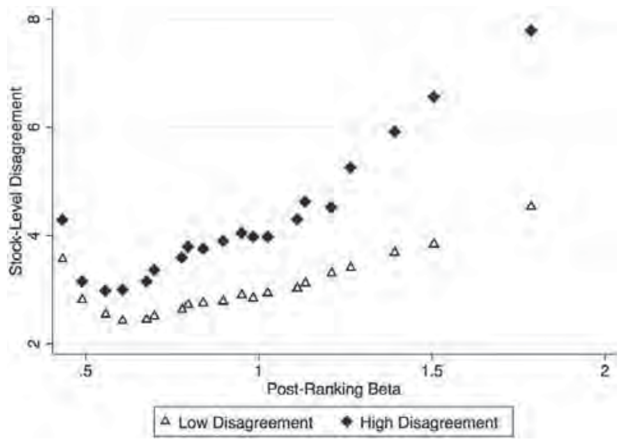


Figure 5. Stock-level disagreement and β. Sample period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price $<$ \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Preformation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to 1 of 20 portfolios based on NYSE break points. The graph plots the value-weighted average stock-level disagreement of stocks in the 20 β-sorted portfolios for months in the bottom quartile of aggregate disagreement (triangles) and months in the top quartile of aggregate disagreement (diamonds). Stock-level disagreement is the standard deviation of analysts' forecasts of long-run EPS growth. Aggregate disagreement is the monthly β-weighted average of this stock-level disagreement measure.

While the relationship between excess forward returns and β is quite noisy at the 3- and 6-month horizons, two striking facts emerge at the 12- and 18-month horizons. First, the average excess returns-to-β relationship is mostly upward-sloping for months with low aggregate disagreement, except for the top β portfolio that exhibits a somewhat lower average return. This result is generally consistent with our theory, which holds that low aggregate disagreement means low or even no mispricing and hence a strictly upward-sloping Security Market Line. Second, in months of high aggregate disagreement, the excess returns-to-β relationship appears to exhibit the inverted-U shape predicted by the theory.

To formally test our Prediction 1, we run the following two-stage Fama and MacBeth (1973) regressions in Table III. Each month, we first estimate the following cross-sectional regression over our 20 β-sorted portfolios:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \text{ where } P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample postranking beta of the P^{th} beta-sorted portfolio.[22] We retrieve from this analysis a time series of coefficient estimates for κ_t, π_t, and ϕ_t.

[22] We also check hermite polynomials in this specification but the quadratic functional fits the best.

Table III

Disagreement and Concavity of the Security Market Line Sample Period: 12/1981 to 12/2014

Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Preformation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to 1 of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE break points. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \cdot \beta_P + \phi_t \cdot (\beta_P)^2 + \epsilon_{P,t}, \quad P = 1, \ldots, 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample postranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = c_1 + \psi_1 \cdot Agg.\ Disp._{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = c_2 + \psi_2 \cdot Agg.\ Disp._{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = c_3 + \psi_3 \cdot Agg.\ Disp._{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns (1) and (5) control for $Agg.\ Disp._{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analyst forecasts of long-run EPS. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past 12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *,**, and *** indicate statistically different from zero at the 10%, 5%, and 1% level of significance, respectively.

	ϕ_t				π_t				κ_t			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	−6.37**	−5.54*	−9.97***	−10.00***	7.39*	10.84**	16.14***	15.43***	−4.48**	−3.52*	−3.75	−3.23
	(−2.06)	(−1.90)	(−3.05)	(−3.08)	(1.68)	(2.09)	(2.65)	(2.59)	(−2.31)	(−1.69)	(−1.49)	(−1.32)

(*Continued*)

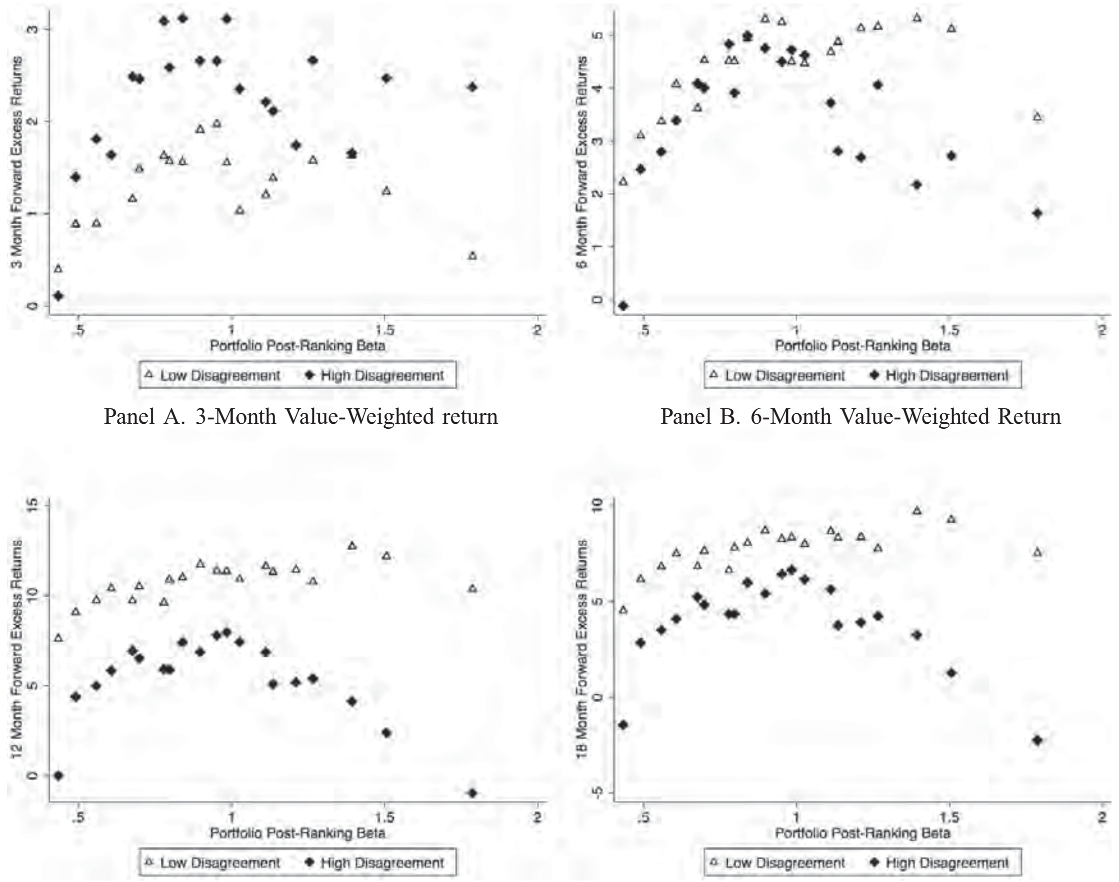
Table III—*Continued*

Dep. Var:	ϕ_t (1)	ϕ_t (2)	ϕ_t (3)	ϕ_t (4)	π_t (5)	π_t (6)	π_t (7)	π_t (8)	κ_t (9)	κ_t (10)	κ_t (11)	κ_t (12)
Panel A: Value-Weighted Portfolios												
$R_{m,t}^{(12)}$		−0.15	−0.22*	−0.22*		0.92***	1.05***	1.07***		0.26**	0.21	0.20
		(−1.18)	(−1.87)	(−1.84)		(3.57)	(4.09)	(4.00)		(2.00)	(1.60)	(1.46)
$HML_t^{(12)}$		−0.54**	−0.40**	−0.40**		0.53	0.37	0.41		0.24	0.24	0.21
		(−2.44)	(−2.10)	(−2.11)		(1.31)	(0.97)	(1.08)		(1.31)	(1.29)	(1.13)
$SMB_t^{(12)}$		0.28	0.48**	0.48**		−0.11	−0.38	−0.39		−0.17	−0.13	−0.12
		(1.04)	(2.00)	(1.99)		(−0.21)	(−0.73)	(−0.75)		(−0.66)	(−0.46)	(−0.46)
$UMD_t^{(12)}$		−0.00	0.06	0.06		0.00	−0.09	−0.07		−0.01	0.02	0.00
		(−0.03)	(0.56)	(0.57)		(0.02)	(−0.39)	(−0.32)		(−0.06)	(0.13)	(0.03)
D/P_{t-1}			−2.26	−2.35			0.86	−1.04			1.70	3.08
			(−0.97)	(−0.85)			(0.17)	(−0.19)			(0.60)	(1.01)
Inflation_{t-1}			−5.91***	−5.96***			9.26**	8.19*			−2.44	−1.67
			(−3.61)	(−3.67)			(2.24)	(1.81)			(−1.08)	(−0.63)
Ted Spread_{t-1}				0.15				3.30				−2.39
				(0.08)				(0.90)				(−1.17)
Constant	−6.25**	−3.13	−3.67	−3.69	14.07***	4.02	4.41	4.02	2.38	−0.58	−0.36	−0.07
	(−2.55)	(−1.20)	(−1.52)	(−1.50)	(2.99)	(0.75)	(0.84)	(0.72)	(1.04)	(−0.22)	(−0.13)	(−0.02)

(*Continued*)

Table III—*Continued*

	ϕ_t				π_t				κ_t			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	−6.80**	−4.85***	−6.48***	−6.32***	9.81**	10.20**	10.17**	9.04**	−3.43*	−3.12	−0.67	0.25
	(−2.55)	(−2.62)	(−3.13)	(−3.12)	(2.25)	(2.52)	(2.34)	(2.20)	(−1.95)	(−1.48)	(−0.32)	(0.13)
$R_{m,t}^{(12)}$		−0.22**	−0.30***	−0.30***		1.09***	1.24***	1.26***		0.16	0.11	0.10
		(−2.54)	(−3.53)	(−3.41)		(6.35)	(6.51)	(6.14)		(1.62)	(1.12)	(0.88)
$HML_t^{(12)}$		−0.69***	−0.65***	−0.66***		0.87***	0.89***	0.96***		0.22	0.13	0.08
		(−4.41)	(−5.01)	(−5.32)		(2.71)	(3.08)	(3.29)		(1.39)	(0.88)	(0.51)
$SMB_t^{(12)}$		0.01	0.12	0.12		0.69**	0.58*	0.58*		−0.11	−0.16	−0.15
		(0.05)	(0.86)	(0.86)		(2.51)	(1.85)	(1.81)		(−0.69)	(−0.88)	(−0.87)
$UMD_t^{(12)}$		−0.08	−0.03	−0.03		0.06	−0.01	0.02		0.03	0.04	0.01
		(−1.12)	(−0.55)	(−0.62)		(0.39)	(−0.07)	(0.14)		(0.28)	(0.34)	(0.13)
D/P$_{t-1}$			1.31	1.74			−6.12	−9.14*			4.80**	7.26***
			(0.61)	(0.67)			(−1.36)	(−1.76)			(2.37)	(3.28)
Inflation$_{t-1}$			−4.68***	−4.44***			7.13**	5.44			−0.90	0.48
			(−3.60)	(−3.20)			(2.07)	(1.41)			(−0.48)	(0.22)
Ted Spread$_{t-1}$				−0.75				5.22				−4.25**
				(−0.43)				(1.36)				(−2.23)
Constant	−9.73***	−4.51**	−4.41**	−4.32**	21.57***	7.30	6.43	5.81	−0.92	−3.27	−2.47	−1.97
	(−4.75)	(−2.31)	(−2.34)	(−2.20)	(4.97)	(1.59)	(1.38)	(1.18)	(−0.46)	(−1.39)	(−1.00)	(−0.76)
N	385	385	385	385	385	385	385	385	385	385	385	385



Panel A. 3-Month Value-Weighted return

Panel B. 6-Month Value-Weighted Return

Panel C. 12-Month Value-Weighted Return

Panel D. 18-Month Value-Weighted Return

Figure 6. Excess returns, β, and aggregate disagreement. Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Preformation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to 1 of 20 value-weighted portfolios based on NYSE break points. The graph plots the average excess return over the next 3 months (Panel A), 6 months (Panel B), 12 months (Panel C), and 18 months (Panel D) of the 20 β-sorted portfolios for months in the bottom quartile of aggregate disagreement (triangles) and months in the top quartile of aggregate disagreement (diamonds). Aggregate disagreement is the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analyst forecasts of long-run EPS growth.

Note that our analysis focuses here on 12-month returns. Since our aggregate disagreement variable is persistent, our results tend to be stronger over longer horizons (Summers (1986), Campbell and Shiller (1988)). For robustness, in Table I.AII, we present the results from a similar analysis using different horizons.

The time series of coefficient estimates ϕ_t is the dependent variable of interest in our analysis. Given the postranking β of our β-sorted portfolios, this ϕ_t series corresponds to the excess returns on a portfolio that goes long the two bottom β portfolios ($P = 1$ to 2) as well as the two top β portfolios ($P = 19$ and 20) and short the remaining portfolios. As we explain in Section I.C, this portfolio's returns capture, each month, the concavity of the Security Market Line. Prediction 1 states that, when aggregate disagreement is higher, this portfolio should have significantly lower returns.

To examine the evidence in support of Prediction 1, in a second stage, we regress the ϕ_t time series on $Agg.Disp._{t-1}$ only (Column (1)), where $Agg.Disp.$ stands for the monthly β-weighted average stock-level disagreement introduced in Section II.C and is measured in month $t-1$. Noxy-Marx (2014) shows that the returns on defensive equity strategies load significantly on standard risk factors. Although our portfolio of interest is not a slope portfolio but rather the square portfolio, in Column (2), we nonetheless follow Noxy-Marx (2014) and control for the 12-month returns of the Fama and French (1992) factors and the Jegadeesh and Titman (1993) momentum factor measured in month t.[23] Column (3) adds the dividend-to-price ratio D/P_{t-1} and the year-over-year inflation rate measured in month $t-1$, $Inflation_{t-1}$, from Cohen, Polk, and Vuolteenaho (2005). Column (4) adds the $TedSpread$ measured in month $t-1$ from Frazzini and Pedersen (2014). Columns (5) to (8) and (9) to (12) rerun the above analysis after replacing the dependent variables with, respectively, the estimated κ_t and π_t. In these estimations, standard errors are Newey and West (1987) adjusted and allow for 11 lags of serial correlation.

Panel A of Table III presents the results from the second-stage regressions using value-weighted portfolios. A higher $Agg.Disp._{t-1}$ is associated with a smaller ϕ_t, that is, a more concave Security Market Line, or equivalently, lower average returns of the square portfolio. The t-statistics are between -1.9 and -4 depending on the specification. Importantly, the estimate is significant by itself even without any controls, although the inclusion of the D/P ratio and the year-over-year past inflation rate increases the effect of aggregate disagreement on the concavity of the Security Market Line.

Interestingly, we see in Table III that a higher return on the high-minus-low portfolio (HML) from t to $t+11$ is correlated with a more negative ϕ_t—a more concave Security Market Line. We believe that this result is consistent with a simple extension of our model. Note first that, even in the absence of disagreement, our model generates a value-growth effect through risk—high-risk stocks have lower prices and higher expected returns (Berk (1995)). To abstract from this effect, one can simply define the fundamental value of a stock as its expected dividend minus its risk premium—$F = d - \frac{1}{\gamma}(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N})$. This is the fundamental price an investor would expect to pay for the stock based purely on risk-based valuation. As in Daniel, Hirshleifer, and Subrahmanyam (2001), we then define the price-to-fundamental ratio as $P-F$ and the return on the high-minus-low HML portfolio as the return of the long-short portfolio that goes long the stock with the lowest price-to-fundamental ratio and short

[23] Noxy-Marx (2014) also shows that a significant part of the returns on defensive equity strategies is driven by exposure to a profitability factor. In unreported regressions, available from the authors upon request, we show that the inclusion of this additional factor does not affect our results.

the stock with the highest price-to-fundamental ratio. In our static model, as long as $b_N\omega(\lambda) > \frac{\sigma_\epsilon^2}{N}$, this ratio corresponds to the return on a portfolio short asset N and long any asset $k \in [1, \bar{i} - 1]$. The return on this portfolio is given by $(b_k - b_N)\frac{\sigma_z^2}{\gamma} + \frac{\theta}{\gamma}(b_N\omega(\lambda)\sigma_z^2 - \frac{\sigma_\epsilon^2}{N})$. In particular, the return on this HML portfolio is strictly increasing with λ. Thus, a larger return on the HML portfolio will be associated with a smaller slope of the Security Market Line. Empirically, to the extent that our proxy for aggregate disagreement λ is measured with noise, we should thus expect the return to HML to have a significant and negative correlation with the concavity of the Security Market Line. This is precisely what we observe in columns (2) to (4) of Table III. This result, although not the main point of the paper, is novel in that it connects the failure of the CAPM to HML through time variation in aggregate disagreement.

In contrast, we see that a larger contemporaneous return on SMB corresponds to a more convex Security Market Line. Inflation comes in with a negative sign—the higher is inflation, the more concave or flatter the Security Market Line. The $TedSpread$ is not significantly related to the concavity of the Security Market Line. Panel B of Table III presents the results from a similar analysis using equal-weighted β-sorted portfolios. The results in Panel B are quantitatively similar to those in Panel A, with a higher level of statistical significance. Overall, consistent with Prediction 1, we find that a higher level of aggregate disagreement is associated with a more concave Security Market Line in the following months.

A.2. Robustness Checks

In the Internet Appendix, we present results of a battery of robustness checks for the above results.

In Table II.AI, we show analogous results to Table III but the preranking βs are now estimated by regressing monthly stock returns over the past three years on the contemporaneous market returns. The results are quantitatively very similar to those in Table III.

In Table II.AII, we use different horizons for the portfolio returns used in the first-stage regression—namely 1, 3, 6, and 18 months. While the effect of disagreement on the concavity of the Security Market Line is insignificant when using a 1- or 3-month horizon (but of the right sign), it is significant when using a 6- or 18-month horizon. Note that, once D/P_{t-1} and $Inflation_{t-1}$ are included in the regression, aggregate disagreement becomes significantly negatively correlated with the concavity of the Security Market Line at all horizons. The fact that the short-horizon results are weaker is to be expected given the literature on long-horizon predictability associated with persistent predictor variables and the fact that *Agg.Disp.* is persistent.

In Table II.AIII, we use the alternative measures of aggregate disagreement introduced in Section II.C. In Panel A, aggregate disagreement is constructed using preranking *compressed* β (i.e., $\beta = 0.5\hat{\beta} + 0.5$) to weight the stock-level disagreement measure. In Panel B, we use $\beta \times$ value-weights to define aggregate disagreement. In Panel C, disagreement is the "top-down" measure of market disagreement used in Yu (2011), which is calculated as the standard deviation of analyst forecasts of annual S&P 500 earnings, scaled by the average forecast on S&P 500 earnings. In Panel D, disagreement is the first principal component of the monthly cross-sectional standard deviation of forecasts on GDP, IP, corporate profit, and the unemployment rate in the SPF and is taken from Li and Li (2014). All these series are standardized to have mean zero and variance one. In all specifications, especially those that include the additional covariates, we get results that are quantitatively similar to our baseline results presented in Table III, although all of the estimated coefficients are less significant than in our baseline specification.[24]

A potential concern with our analysis is that the results are simply a recast of the results in Diether, Malloy, and Scherbina (2002): high-beta stocks experience more idiosyncratic disagreement, especially in high-aggregate-disagreement months, so that the effect of aggregate disagreement on the Security Market Line works entirely through idiosyncratic disagreement. In Table II.AIV, we show that this is not the case. Specifically, we replicate the analysis of Table I, but in the first-stage regression, we now control for the logarithm of the average disagreement on the stocks in each of the 20 β-sorted portfolios. Again, our results are virtually unchanged by inclusion of this additional control in the first-stage regression.

Additional empirical concerns with the analysis in Table III are that high-beta stocks have higher idiosyncratic volatility and idiosyncratic stocks have lower returns (Ang et al. (2006)), perhaps especially when aggregate disagreement is high. In Section III.B, we test the asset pricing equation from our model when dividends are allowed to be heteroskedastic. However, we can also amend our methodology to include, in the first-stage regression, a control for the median idiosyncratic volatility of stocks in each of the 20 β-sorted portfolios.[25] The results are presented in Table II.AV. The point estimates are quantitatively similar to those obtained in Table III, although the statistical significance is slightly lower (t-statistics ranging from 1.6 to 2.4 in the value-weighted specification and from 2 to 2.6 in the equal-weighted specification). Our main finding is thus robust to controlling directly, in the first-stage regressions, for the idiosyncratic volatility of the stocks included in the 20 β-sorted portfolios.

A.3. Disagreement and the Slope of the Security Market Line

Corollary 3 shows that the slope of the Security Market Line should decrease with aggregate disagreement. Although we argue in Section I.C that this is a

[24] In addition, Li (2014) tests our model using dispersion of macroforecasts for each of these macrovariables separately. But rather than using 20 β portfolios, he forms optimal tracking portfolios for each of these macrovariables and calculates each stock's macrobeta with respect to these macrotracking portfolios and finds that, when aggregate disagreement is high, higher macrobeta stocks underperform lower macrobeta stocks.

[25] We use the logarithm of idiosyncratic volatility as a control variable in the first-stage regression to account for the skewness in this variable.

weaker test of our model—since it fails to account for the kinks in the Security Market Line predicted in the model—in Table II.AVI, we nonetheless present a test for this prediction. This test is again a two-stage procedure. In the first stage, each month we regress the excess return on the 20 β-sorted portfolios on their postranking full sample β:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \cdot \beta_P + \epsilon_{P,t}, \qquad P = 1, ..., 20.$$

Here, π_t, the slope of the Security Market Line in month t, is the variable of interest. More specifically, π_t represents the 12-month excess return of a "slope" portfolio in month t—a portfolio that goes long the portfolios with above-average β and short the portfolio with below-average β. Column (1) of Table II.AVI shows that, by itself, aggregate disagreement in month $t-1$ predicts a significantly flatter Security Market Line in the following month. In Column (2), we see that introducing the contemporaneous four-factor return in the regression absorbs most of the effect of disagreement on the slope of the Security Market Line. However, we also see in this column, as well as in Columns (3) and (4), that a higher high-minus-low 12-month return in month t is associated with a significantly flatter Security Market Line at t.[26] As we explain above, this result is a natural prediction of our model, since aggregate disagreement leads to the mispricing of high-beta securities, which then mean-revert. Interestingly, the inclusion of D/P_{t-1} and $Inflation_{t-1}$ in Columns (3) and (4) makes the effect of aggregate disagreement on the slope of the Security Market Line significant again. Consistent with Cohen, Polk, and Vuolteenaho (2005), a higher level of $Inflation_{t-1}$ leads to a flatter Security Market Line. As in Frazzini and Pedersen (2014), $TedSpread$ does not significantly explain the average excess return of the slope portfolio.

B. Heteroskedastic Idiosyncratic Variance

We now turn our attention to our main test for Prediction 2, which holds that the slope of the Security Market Line is more sensitive to aggregate disagreement for stocks with a high β_i/σ_i^2 ratio relative to stocks with a low β_i/σ_i^2 ratio.

To test this prediction, we need to ascribe a value for the threshold $\frac{\beta_i}{\sigma_i^2}$ defining speculative and nonspeculative stocks. Our strategy is to use as a baseline specification a threshold corresponding to the median $\frac{\beta_i}{\sigma_i^2}$ ratio and then to assess the robustness of the results to this particular choice. More precisely, our test for Prediction 2 is based on a three-stage approach. In the first stage, we rank stocks each month based on their preranking ratio of β to σ^2 and define as speculative (nonspeculative) stocks all stocks with a ratio above (below) the NYSE median ratio: $\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE median$\frac{\hat{\beta}}{\hat{\sigma}^2}$ ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq$ NYSE median$\frac{\hat{\beta}}{\hat{\sigma}^2}$). This creates two groups of stocks for each month t: speculative and nonspeculative.

[26] This result is consistent with that in Noxy-Marx (2014). The novelty here is that our model proposes an explanation for this correlation.

Then, within each of these two groups, we rerank stocks in ascending order based on their estimated beta at the end of the previous month and assign them to 1 of 20 beta-sorted portfolios again using NYSE break points. We compute the full-sample beta of these 40 value-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. The resulting full-sample beta is given by $\beta_{P,s}$, where $P = 1, \ldots, 20$ and s $\in$ {speculative, nonspeculative}. Table IV presents descriptive statistics for the resulting 40 portfolios. We see that (1) the constructed portfolios generate significant spreads in the postranking full-sample βs and ex post, and (2) the β/σ^2 ratio of the β-sorted portfolios created from speculative stocks is much higher than that of the β-sorted portfolios created from nonspeculative stocks: the average β/σ^2 ratio for speculative stocks is 0.61, while it is only 0.25 for nonspeculative stocks.[27]

Next, for each of these two groups of portfolios ($s \in$ {speculative, nonspeculative}), each month we estimate the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios, and t is a month:

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the value-weighted 12-month excess return of portfolio (P, s). In contrast to $\beta_{P,s}$, $\sigma^2_{P,s,t-1}$ has a large skew, so we use the logarithm of $\sigma_{P,s,t-1}$ in the cross-sectional regressions to limit the effect of outliers on the regression estimates. We retrieve a time series of monthly estimated coefficients: $\iota_{s,t}$, $\chi_{s,t}$, and $\varrho_{s,t}$. Finally, as we do for Table III, in a third stage, we regress each of these series on $Agg.Disp._{t-1}$, the contemporaneous four-factor alphas ($R_{m,t}$, HML_t, SMB_t, and UMD_t), and a set of additional forecasting variables measured in month $t-1$, namely, D/P_{t-1}, $Inflation_{t-1}$, and $TedSpread_{t-1}$. Standard errors are Newey and West (1987) adjusted and allow for 11 lags of serial correlation.

Figure 7 summarizes our findings graphically. In this figure, we compute for the 20 β-sorted portfolios constructed from speculative stocks (Panel B) and the 20 β-sorted portfolios constructed from nonspeculative stocks, the average excess 12-month return for high- (diamonds) and low- (triangles) disagreement months (defined as top versus bottom quartile of aggregate disagreement). For nonspeculative stocks, we see that the Security Market Line is not related in a clear way with aggregate disagreement. For speculative stocks, however, Figure 7 suggests that, when aggregate disagreement is high, the Security Market Line exhibits an inverted-U shape, while there is no such kink in months with low aggregate disagreement. This first pass at the data is consistent with Prediction 2, which holds that aggregate disagreement makes the Security Market Line flatter only for speculative stocks.

Table V reports results from the actual regression analysis. Panel A of this table reports the estimation results of the third-stage regression when using

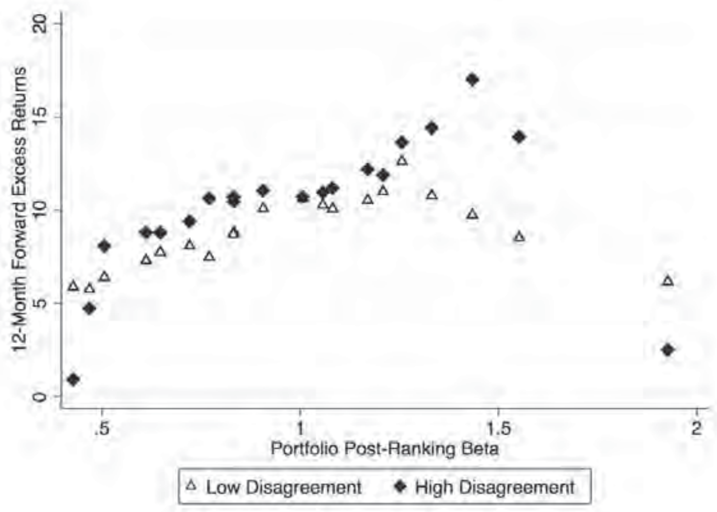
[27] This is true for all but the top β portfolio created from speculative stocks, which has a β/σ^2 ratio of 0.25 only. Excluding this portfolio does not change our analysis qualitatively.

Table IV

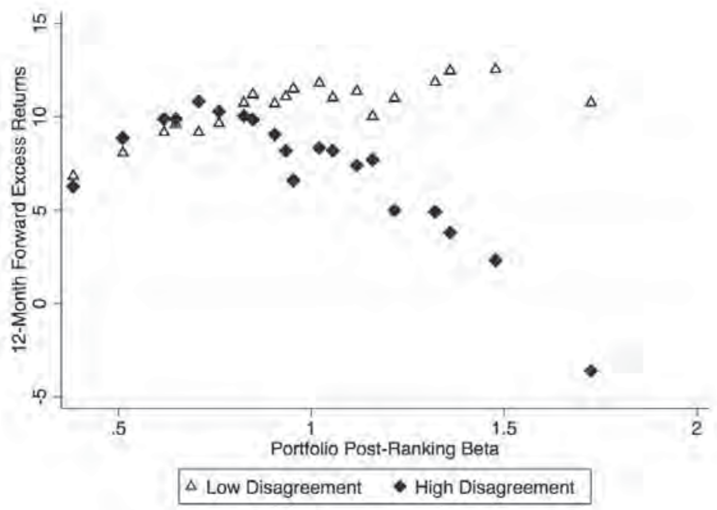
Summary Statistics for 20 β-Sorted Porftolios: Speculative versus Nonspeculative Stocks

Sample period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price <\$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the estimated ratio of beta to idiosyncratic variance ($\frac{\beta}{\sigma^2}$) at the end of the previous month. Preformation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to two groups: speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to 1 of 20 value-weighted beta-sorted portfolios based on NYSE break points. The table reports statistics for these portfolios: Panel A for nonspeculative stocks and Panel B for speculative stocks. β is the postranking full-sample β, and is computed using a similar risk model as that used to compute the preranking β. *Median Vol*. is the median preranking volatility of stocks in the portfolio. $\frac{\beta}{\sigma^2}$ is the ratio of β to the square of *Median Vol*. $R_t^{(1)}$ is the return of the portfolio from t to $t+1$, and $R_t^{(12)}$ from t to $t+11$. *Stock Disp*. is the average stock-level disagreement for stocks in each portfolio, where stock-level disagreement is defined as the standard deviation of analysts' forecasts for long-run EPS growth. *% Mkt. Cap*. is the average ratio of market capitalization of stocks in the portfolio divided by the total market capitalization of stocks in the sample. *N* stocks is the number of stocks on average in each portfolio.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Panel A: Nonspeculative Stocks																				
β	0.42	0.46	0.50	0.61	0.64	0.72	0.77	0.83	0.83	0.90	1.00	1.05	1.08	1.17	1.20	1.25	1.33	1.43	1.55	1.92
Median Vol.	1.85	1.35	1.29	1.40	1.38	1.46	1.54	1.62	1.71	1.80	1.86	1.93	2.01	2.15	2.26	2.33	2.43	2.70	2.92	4.09
$\frac{\beta}{\sigma^2}$	0.12	0.25	0.30	0.31	0.33	0.33	0.32	0.31	0.28	0.27	0.29	0.28	0.26	0.25	0.23	0.22	0.22	0.19	0.18	0.11
$R_{i,t}^{(1)}$	0.05	0.21	0.34	0.35	0.57	0.46	0.79	0.59	0.66	0.82	0.82	0.61	0.61	0.93	0.43	0.90	0.75	0.51	0.76	–0.02
$R_{i,t}^{(12)}$	1.89	3.01	3.93	5.70	5.88	7.57	6.77	9.75	9.07	9.03	8.83	8.34	7.90	10.35	10.15	12.73	10.80	11.21	11.58	–2.78
Stock Disp.	3.12	2.85	3.06	3.56	3.34	3.36	3.54	3.80	3.87	4.16	4.74	4.73	4.83	4.95	5.17	5.66	5.96	6.06	6.24	7.99
% Mkt. Cap.	4.31	3.95	4.76	5.15	5.56	5.62	5.90	5.65	5.47	5.57	5.19	5.00	5.06	4.74	4.81	4.61	4.58	4.55	5.14	8.70
N stocks	96	73	71	75	81	84	87	89	93	96	97	98	100	101	108	111	118	126	151	339
Panel B: Speculative Stocks																				
β	0.38	0.51	0.61	0.64	0.70	0.76	0.82	0.84	0.90	0.93	0.95	1.02	1.05	1.11	1.15	1.21	1.32	1.36	1.47	1.72
Median Vol.	0.78	0.86	0.93	0.98	1.01	1.02	1.05	1.09	1.13	1.17	1.17	1.24	1.27	1.34	1.39	1.47	1.56	1.75	1.94	2.60
$\frac{\beta}{\sigma^2}$	0.62	0.68	0.70	0.66	0.68	0.71	0.73	0.70	0.69	0.68	0.68	0.65	0.65	0.62	0.59	0.56	0.54	0.44	0.39	0.25
$R_{i,t}^{(1)}$	0.62	0.81	0.54	0.78	0.76	0.66	0.47	0.61	0.63	0.55	0.83	0.48	0.46	0.73	0.43	0.65	0.49	–0.24	0.00	–0.91
$R_{i,t}^{(12)}$	8.51	9.47	8.71	8.24	7.38	7.10	6.49	7.43	7.96	7.58	7.41	7.90	7.31	7.06	7.55	6.54	4.57	–1.52	–2.38	–11.62
Stock Disp.	1.95	2.17	2.33	2.66	2.76	2.90	2.98	3.07	3.16	3.24	3.23	3.17	3.31	3.49	3.50	3.75	4.21	4.55	5.03	6.57
% Mkt. Cap.	4.16	4.48	4.98	5.05	5.19	5.37	5.88	5.84	5.73	5.50	5.35	5.24	5.30	5.17	4.98	4.68	5.01	5.15	4.81	6.64
N stocks	57	58	58	60	63	64	65	67	68	67	68	69	67	66	65	66	65	66	68	90



Panel A. 12-Month Value-Weighted Return for Nonspeculative Stocks



Panel B. 12-Month Value-Weighted Return for Speculative Stocks

Figure 7. Excess returns, β, and aggregate disagreement: Speculative vs. nonspeculative stocks. Sample period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the estimated ratio of beta to idiosyncratic variance ($\frac{\beta}{\sigma^2}$) at the end of the previous month. Preformation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to two groups: speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are then ranked in ascending order of their estimated beta at the end of the previous month and are assigned to 1 of 20 value-weighted beta-sorted portfolios based on NYSE break points. The graph plots the average excess return over the next 12 months for the 20 β-sorted portfolios for months in the bottom quartile of aggregate disagreement (triangles) and months in the top quartile of aggregate disagreement (diamonds). Aggregate disagreement is the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analyst forecasts of long-run EPS growth. Panel A plots these excess returns for nonspeculative stocks, Panel B for the speculative stocks.

Table V

Disagreement and Slope of the Security Market Line: Speculative versus Nonspeculative Stocks

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Preformation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to two groups: speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to 1 of 20 beta-sorted portfolios using NYSE break points. We compute the full sample beta of these 2 × 20 value-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full-sample beta, where $P = 1, \ldots, 20$ and s ∈ {speculative, nonspeculative}. We estimate each month the following cross-sectional regression, where P is one of the 20 β-sorted portfolios, s ∈ {speculative, nonspeculative}, and t denotes the month:

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \cdot \beta_{P,s} + \varrho_{s,t} \cdot \ln(\sigma_{P,s,t-1}) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the value-weighted 12-month excess return of portfolio (P, s). We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = c_{1,s} + \psi_{1,s} \cdot Agg.\ Disp._{t-1} + \delta^{m}_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_{t} + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_{t} + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_{t} + \sum_{x \in X} \delta^{x}_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = c_{2,s} + \psi_{2,s} \cdot Agg.\ Disp._{t-1} + \delta^{m}_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_{t} + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_{t} + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_{t} + \sum_{x \in X} \delta^{x}_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = c_{3,s} + \psi_{3,s} \cdot Agg.\ Disp._{t-1} + \delta^{m}_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_{t} + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_{t} + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_{t} + \sum_{x \in X} \delta^{x}_{3,s} x_{t-1} + \nu_{t,s}. \end{cases}$$

Columns (1) and (5) control for $Agg.\ Disp._{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_{t}$), SMB ($SMB^{(12)}_{t}$), and UMD ($UMD^{(12)}_{t}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past 12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *,**, and *** indicate statistically different from zero at 10%, 5%, and 1% level of significance, respectively.

	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative Stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE Median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	−12.77**	−6.96**	−10.69***	−11.29***	2.77	2.54	1.62	0.89	5.45*	5.89**	10.34***	11.16***
	(−2.49)	(−2.13)	(−2.71)	(−2.86)	(1.05)	(1.03)	(0.57)	(0.31)	(1.69)	(2.11)	(3.20)	(3.41)

(*Continued*)

Table V—*Continued*

	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative Stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE Median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
$R_{m,t}^{(12)}$		0.67***	0.63***	0.64***		−0.06	−0.07	−0.06		0.36***	0.41***	0.40***
		(4.83)	(4.26)	(4.28)		(−0.45)	(−0.48)	(−0.38)		(2.86)	(3.43)	(3.34)
$HML_t^{(12)}$		−0.72***	−0.60***	−0.57**		0.17	0.19	0.24		0.70***	0.56***	0.51***
		(−2.93)	(−2.67)	(−2.49)		(1.01)	(1.16)	(1.43)		(3.33)	(3.12)	(2.92)
$SMB_t^{(12)}$		0.61**	0.76***	0.76***		−0.35	−0.31	−0.32		−0.50**	−0.69***	−0.68***
		(2.44)	(2.86)	(2.88)		(−1.36)	(−1.31)	(−1.36)		(−2.18)	(−3.06)	(−3.17)
$UMD_t^{(12)}$		0.04	0.08	0.10		−0.20*	−0.19**	−0.17*		0.03	−0.02	−0.04
		(0.27)	(0.59)	(0.75)		(−1.94)	(−2.04)	(−1.67)		(0.24)	(−0.15)	(−0.38)
D/P_{t-1}			−2.69	−4.31			−0.57	−2.51			3.45	5.63
			(−0.65)	(−0.94)			(−0.20)	(−0.80)			(1.02)	(1.45)
$Inflation_{t-1}$			−4.04*	−4.95**			−1.11	−2.19			4.57***	5.80***
			(−1.80)	(−2.58)			(−0.49)	(−1.16)			(2.97)	(3.95)
Ted $Spread_{t-1}$				2.80				3.35				−3.78*
				(0.95)				(1.06)				(−1.90)
Constant	−1.30	−5.46***	−6.03***	−6.36***	0.03	1.73	1.60	1.20	9.95***	4.38**	5.10***	5.54***
	(−0.35)	(−2.63)	(−2.65)	(−2.85)	(0.01)	(0.87)	(0.81)	(0.63)	(3.49)	(2.40)	(2.62)	(2.92)

(*Continued*)

Table V—*Continued*

	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel B: Nonspeculative Stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq$ NYSE Median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	0.07	3.35	−3.44	−3.83	−4.69	−3.96*	−1.88	−2.20	2.69	2.83	7.27***	7.66***
	(0.04)	(1.21)	(−1.37)	(−1.58)	(−1.33)	(−1.85)	(−0.71)	(−0.84)	(0.88)	(1.18)	(3.22)	(3.42)
$R_{m,t}^{(12)}$		0.30*	0.34***	0.34***		0.30**	0.23*	0.23*		0.50***	0.53***	0.52***
		(1.79)	(3.11)	(3.02)		(2.52)	(1.85)	(1.88)		(3.61)	(4.81)	(4.47)
$HML_t^{(12)}$		−0.43	−0.21	−0.19		−0.41***	−0.49***	−0.47***		1.01***	0.87***	0.85***
		(−1.61)	(−0.91)	(−0.80)		(−2.95)	(−4.16)	(−3.77)		(4.46)	(4.67)	(4.85)
$SMB_t^{(12)}$		0.25	0.46**	0.46**		0.97***	0.95***	0.95***		−0.64***	−0.81***	−0.81***
		(0.82)	(2.34)	(2.33)		(4.95)	(5.68)	(5.77)		(−2.64)	(−4.50)	(−4.53)
$UMD_t^{(12)}$		0.16	0.18	0.19		−0.15*	−0.13*	−0.12		−0.01	−0.05	−0.06
		(1.02)	(1.45)	(1.55)		(−1.78)	(−1.66)	(−1.52)		(−0.06)	(−0.42)	(−0.51)
D/P$_{t-1}$			−9.47***	−10.50***			5.39	4.54			4.20*	5.22*
			(−3.33)	(−4.00)			(1.54)	(1.46)			(1.73)	(1.74)
Inflation$_{t-1}$			−2.03	−2.60			−2.31	−2.78			3.66*	4.23*
			(−0.60)	(−0.66)			(−0.87)	(−0.93)			(1.74)	(1.76)
Ted Spread$_{t-1}$				1.78				1.47				−1.77
				(0.55)				(0.66)				(−0.75)
Constant	0.87	−1.47	−3.15	−3.36	0.12	−1.08	−0.21	−0.39	8.70***	1.11	1.93	2.14
	(0.29)	(−0.54)	(−1.28)	(−1.31)	(0.04)	(−0.50)	(−0.10)	(−0.18)	(2.75)	(0.47)	(0.87)	(0.92)
N	385	385	385	385	385	385	385	385	385	385	385	385

portfolios constructed from speculative stocks. Panel B presents results from portfolios constructed from nonspeculative stocks. The first four columns provide the estimation results for our main coefficient of interest $\chi_{s,t}$, which measures the slope of the Security Market Line conditional on the idiosyncratic variance of portfolios. Consistent with Prediction 2, an increase in aggregate disagreement in month $t-1$ is associated with a significantly flatter Security Market Line—holding portfolio variance constant—only in Panel A, that is, only for stocks with a β/σ^2 ratio above the NYSE median ratio. In Panel B, where the portfolios are formed from stocks with a β/σ^2 ratio below the NYSE median ratio, aggregate disagreement is not significantly related to the slope of the SML. Across our four specifications, which add additional controls, including the contemporaneous four-factor returns, the results are similar: higher aggregate disagreement in month $t-1$ leads to a significantly flatter slope of the Security Market Line (with t-statistics ranging from 2.1 to 2.9) when β-sorted portfolios are formed using speculative stocks, while there is no significant relationship between aggregate disagreement and the slope of the Security Market Line for these portfolios constructed using nonspeculative stocks. These results are consistent with Prediction 2.

Columns (5) to (8) of Table V present the regression estimates when $\varrho_{s,t}$ is the dependent variable, where $\varrho_{s,t}$ represents the effect of (log) idiosyncratic variance on the returns of these β-sorted portfolios. Our model predicts that for speculative stocks, stocks with high idiosyncratic variance should have higher expected returns, especially when aggregate disagreement is high. In Panel A, columns (5) to (8), we see that both the constant and the coefficients on $Agg.Disp._{t-1}$ are positive, consistent with our model, but they are not statistically significant. In some of the specifications below (most notably, the specification using equal-weighted portfolios), we find that these coefficients are not only positive but also statistically significant. However, Table V shows that our model does not fully capture how idiosyncratic variance is priced in the cross section of stock returns.

In Panel B, we find that (1) idiosyncratic variance has no significant effect on the returns of β-sorted portfolios constructed from nonspeculative stocks (the constant is insignificant and small in magnitude), and for these nonspeculative portfolios, and (2) an increase in disagreement is associated with a lower effect of idiosyncratic variance on the returns of these β-sorted portfolios (this effect is insignificant in all but column (6) where the t-statistic is 1.8). This negative sign is inconsistent with our model since, in the model, nonspeculative stocks should have returns that are independent of aggregate disagreement. However, in differential terms, these results could be reconciled with the model to the extent that they show that the effect of aggregate disagreement on the price of idiosyncratic variance is significantly larger for speculative stocks than for nonspeculative stocks.

We confirm the robustness of this analysis by performing a battery of additional tests. In Table VI, we use equal-weighted portfolios instead of value-weighted portfolios. We obtain even more supporting evidence in that the coefficients of interest are both economically larger and statistically more significant.

Table VI

Disagreement and Slope of the Security Market Line: Speculative versus Nonspeculative Stocks; Equal-Weighted Portfolios

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Preformation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to two groups: speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to 1 of 20 beta-sorted portfolios using NYSE break points. We compute the full-sample beta of these 2 × 20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s ∈ {speculative, nonspeculative}. We estimate each month the following cross-sectional regression, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, nonspeculative} and t denotes the month:

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \cdot \ln(\sigma_{P,s,t-1}) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the equal-weighted 12-months excess return of portfolio (P, s). We then estimate second-stage regressions in the time-series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = c_{1,s} + \psi_{1,s} \cdot Agg.\ Disp._{t-1} + \delta^m_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = c_{2,s} + \psi_{2,s} \cdot Agg.\ Disp._{t-1} + \delta^m_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = c_{3,s} + \psi_{3,s} \cdot Agg.\ Disp._{t-1} + \delta^m_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{3,s} x_{t-1} + \nu_{t,s}. \end{cases}$$

Columns (1) and (5) control for $Agg.\ Disp._{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *,**, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance, respectively.

	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative Stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE Median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	−19.02***	−11.37***	−15.08***	−15.74***	8.53***	6.24**	6.74	6.70*	9.16***	7.00***	11.39***	12.24***
	(−3.08)	(−2.93)	(−3.23)	(−3.42)	(2.72)	(1.99)	(1.62)	(1.73)	(2.68)	(2.60)	(3.67)	(3.83)

(*Continued*)

Table VI—*Continued*

	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative Stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE Median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
$R_{m,t}^{(12)}$		0.55***	0.49***	0.50***		0.23	0.24	0.24		0.45***	0.52***	0.50***
		(3.08)	(2.89)	(2.93)		(1.57)	(1.49)	(1.45)		(2.96)	(3.95)	(3.90)
$HML_t^{(12)}$		−1.18***	−1.07***	−1.03***		0.74***	0.73***	0.73***		1.08***	0.95***	0.90***
		(−4.84)	(−4.60)	(−4.36)		(3.98)	(3.69)	(3.86)		(5.93)	(6.14)	(5.79)
$SMB_t^{(12)}$		0.45*	0.62**	0.61**		0.02	−0.01	−0.01		−0.09	−0.28	−0.28
		(1.79)	(2.50)	(2.43)		(0.07)	(−0.05)	(−0.05)		(−0.44)	(−1.61)	(−1.58)
$UMD_t^{(12)}$		−0.13	−0.08	−0.06		−0.04	−0.05	−0.05		0.06	0.00	−0.02
		(−0.67)	(−0.44)	(−0.38)		(−0.23)	(−0.30)	(−0.29)		(0.41)	(0.01)	(−0.20)
D/P_{t-1}			−1.81	−3.57			−0.03	−0.15			2.47	4.73
			(−0.66)	(−1.08)			(−0.01)	(−0.05)			(0.91)	(1.43)
$Inflation_{t-1}$			−5.04**	−6.03**			1.01	0.94			5.59***	6.86***
			(−2.03)	(−2.27)			(0.38)	(0.36)			(3.28)	(3.87)
Ted $Spread_{t-1}$				3.05				0.20				−3.90**
				(1.08)				(0.07)				(−2.04)
Constant	−8.00*	−7.79**	−8.23**	−8.60***	7.02**	2.34	2.36	2.34	13.79***	5.24**	5.81**	6.28***
	(−1.95)	(−2.49)	(−2.46)	(−2.61)	(2.36)	(0.69)	(0.67)	(0.65)	(4.26)	(2.28)	(2.44)	(2.71)

(*Continued*)

Table VI—*Continued*

	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel B: Nonspeculative Stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq$ NYSE Median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	0.61	3.62	−1.50	−1.77	−5.26	−3.54	−0.74	−1.84	4.68*	2.42	5.53**	6.61***
	(0.27)	(0.98)	(−0.38)	(−0.47)	(−1.41)	(−1.26)	(−0.20)	(−0.55)	(1.82)	(1.17)	(2.49)	(3.14)
$R_{m,t}^{(12)}$		−0.01	0.05	0.06		0.83***	0.67***	0.69***		0.31*	0.46***	0.44***
		(−0.04)	(0.30)	(0.31)		(4.24)	(3.67)	(4.08)		(1.68)	(4.24)	(3.97)
$HML_t^{(12)}$		−0.78**	−0.61**	−0.59*		0.10	−0.01	0.06		1.04***	0.96***	0.90***
		(−2.35)	(−1.97)	(−1.96)		(0.50)	(−0.05)	(0.29)		(4.51)	(4.52)	(4.53)
$SMB_t^{(12)}$		0.22	0.35	0.35		0.95***	0.95***	0.95***		−0.26	−0.46**	−0.45**
		(0.53)	(1.05)	(1.05)		(2.60)	(3.31)	(3.54)		(−1.12)	(−2.05)	(−2.13)
$UMD_t^{(12)}$		−0.08	−0.07	−0.06		0.01	0.06	0.09		0.15	0.06	0.04
		(−0.42)	(−0.41)	(−0.37)		(0.10)	(0.54)	(0.72)		(0.87)	(0.51)	(0.33)
D/P$_{t-1}$			−8.17**	−8.88**			9.17*	6.24			−2.06	0.82
			(−1.98)	(−2.21)			(1.86)	(1.46)			(−0.74)	(0.27)
Inflation$_{t-1}$			−0.32	−0.72			−5.35*	−7.00**			8.43**	10.05***
			(−0.10)	(−0.20)			(−1.75)	(−2.46)			(2.58)	(2.94)
Ted Spread$_{t-1}$				1.23				5.09				−4.98**
				(0.29)				(1.39)				(−2.02)
Constant	−4.46	−1.01	−2.42	−2.56	5.94	−3.03	−1.59	−2.20	9.34***	1.77	1.63	2.22
	(−1.27)	(−0.33)	(−0.82)	(−0.87)	(1.56)	(−1.14)	(−0.60)	(−0.90)	(2.86)	(0.79)	(0.77)	(1.07)
N	385	385	385	385	385	385	385	385	385	385	385	385

As mentioned above, we even find some support with these equal-weighted portfolios for the prediction relating aggregate disagreement to the price of idiosyncratic variance.

In Table II.AVII, the preranking βs are estimated by regressing monthly stock returns over the past three years on contemporaneous market returns. The results are essentially similar. In Table II.AVIII, we rerun the analysis of Table V using portfolio returns over a horizon of 1, 3, 6, and 18 months. The results are not statistically significant using the one-month horizon. However, for the three-month horizon and above, the results are consistent with the baseline 12-month horizon result shown in Table V: overall, the prediction that aggregate disagreement leads to a flatter Security Market Line for speculative stocks is strongly supported in the data, while the prediction relating aggregate disagreement to the price of idiosyncratic risk finds only mixed support.[28]

Our analysis so far has used an arbitrary cutoff to define speculative stocks, namely, the median NYSE β/σ^2 ratio. In Figure 8, we rerun the analysis we performed in Table V but we use different cutoffs to define speculative versus nonspeculative stocks. For each of these cutoffs, we plot the coefficient estimate of the regression of aggregate disagreement on the slope of the SML $\chi_{s,t}$, obtained from the specification in column (2), which includes only the realized four-factor returns as controls. We select this specification as it typically yields the smallest point estimates. The left (right) panel displays the results obtained for portfolios formed from speculative (nonspeculative) stocks. The cutoffs we use range from the 30$^{\text{th}}$ percentile to the 70$^{\text{th}}$ percentile of the NYSE distribution of the β_i/σ_i^2 ratio. Across all of these specifications, we consistently find that aggregate disagreement leads to a flatter Security Market line only for speculative stocks. The effect of disagreement on the slope of the Security Market Line for speculative stocks becomes larger (in absolute value) as the threshold to define speculative stocks becomes more conservative.

IV. Conclusion

We show that incorporating the speculative motive for trade into asset pricing models yields strikingly different results from the risk-sharing or liquidity motives. High-beta assets are more speculative because they are more sensitive to disagreement about common cash flows. Hence, they experience greater divergence of opinion and, in the presence of short-sales constraints for some investors, end up being overpriced relative to low-beta assets. When aggregate disagreement is low, the risk-return relationship is upward-sloping. As aggregate disagreement rises, the slope of the Security Market Line is piecewise constant, higher in the low-beta range and potentially negative for the high-beta range. Empirical tests using measures of disagreement based on security analyst forecasts are consistent with these predictions. We believe that our simple and tractable model provides a plausible explanation for part of the

[28] This is true except at the three-month horizon, where the predictions relating aggregate disagreement to the price of β and the price of idiosyncratic variance are verified.

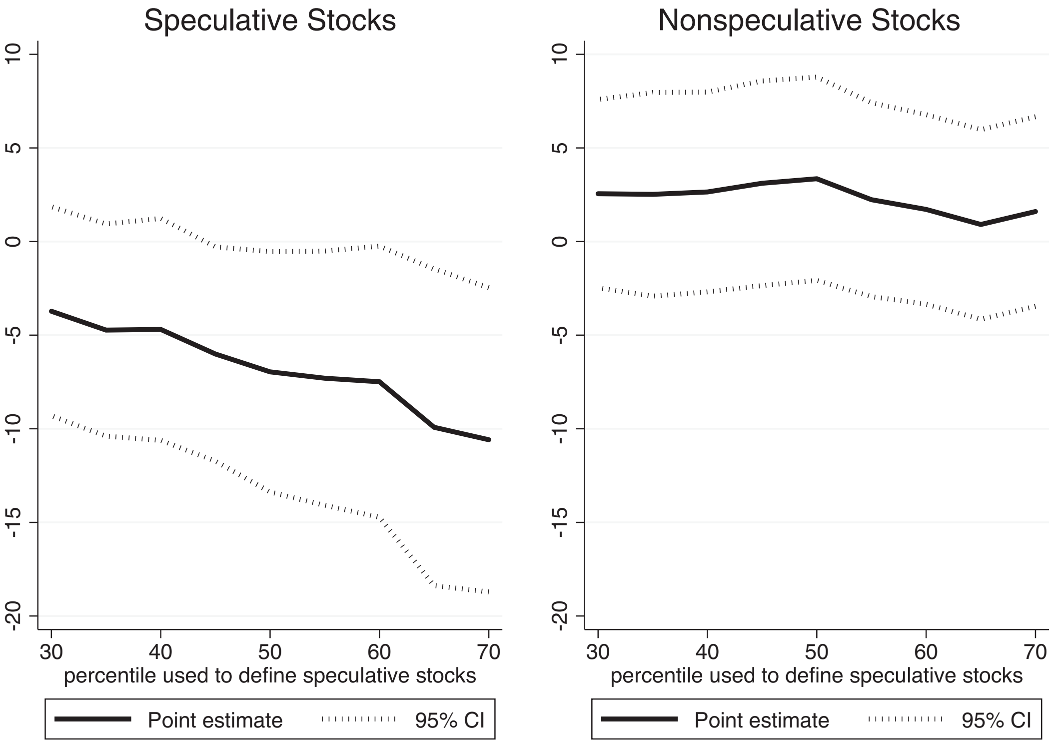


Figure 8. Disagreement and slope of the Security Market Line: Speculative versus nonspeculative stocks. Sample period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE break points). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the estimated ratio of beta to idiosyncratic variance ($\frac{\beta}{\sigma^2}$) at the end of the previous month. Preformation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and five lags justified the market returns. The ranked stocks are assigned to two groups: speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and assigned to 1 of 20 value-weighted beta-sorted portfolios using NYSE break points. We compute the full-sample beta of these 40 portfolios (20 beta-sorted portfolios for speculative stocks and 20 for nonspeculative stocks) using the same market model.The ranked stocks are assigned to two groups: speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE q^{th} percentile of $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) market model. $\beta_{P,s}$ is the resulting full-sample beta, where $P = 1, \ldots, 20$ and $s \in$ {speculative, nonspeculative}. We then estimate the following cross-sectional regression, where P is 1 of the 20 β-sorted portfolios, $s \in$ {speculative, nonspeculative} and t denotes the month where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated in month $t-1$ and $r^{(12)}_{P,s,t}$ is the 12-month excess return of portfolio (P, s). We estimate the following time-series regression with Newey-West (1987) adjusted standard errors allowing for 11 lags: $Agg.Disp._{t-1}$ is the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analyst forecasts of long-run EPS growth, $R^{(12)}_{m,t}$ ($HML^{(12)}_t$, $SMB^{(12)}_t$, and $UMD^{(12)}_t$) is the 12-month excess return from t to $t+11$ on the market (HML, SMB, and UMD). The thresholds used to define speculative stocks are q= 30, 35, 40, 45, 50, 55, 60, 65, and 70$^{\text{th}}$ percentile of the distribution of $\frac{\beta}{\sigma^2}$. The figure plots the estimated $\xi_{spec.}$ (left panel) and $\xi_{nonspec.}$ (right panel) for each of these thresholds and their 95% confidence interval.

high-risk, low-return puzzle. The broader thrust of our analysis is to point out that one can construct a behavioral macrofinance model in which aggregate sentiment can influence the cross section of asset prices in nontrivial ways.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix

Internet Appendix to "Speculative Betas"*

This internet appendix has two sections. Section I contains all the proofs for the paper. Section II contains additional tables.

I. Proofs of the Model

A. Proof of Theorem 1

Proof. We solve the model here allowing for heteroskedastic dividends σ_i^2. Theorem 1 can then be proved as the special case $\sigma_\epsilon^2 = \sigma_i^2$. We assume that assets are ranked in ascending order of β/σ^2.

We first posit an equilibrium structure. We then check ex-post that it is indeed an equilibrium and that it is a unique equilibrium. Let $\bar{i} \in [2, N]$ and let μ_i^m be the share holdings of asset k by group m, where $m \in \{a, A, B\}$. Consider an equilibrium where group B investors are long assets $i < \bar{i}$ and hold no position (i.e., $\mu_i^B = 0$) for assets $i \geq \bar{i}$, and group A investors are long all assets $i \in [1, N]$. Since group A investors are long all assets, their holdings satisfy the following first-order condition:

$$\forall i \in [1, N]: \quad d + \lambda b_i - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} b_k \mu_k^A\right) b_i \sigma_z^2 + \mu_i^A \sigma_i^2\right).$$

Since group B investors are long only assets $i < \bar{i}$, their holdings for these assets must also satisfy the following first-order condition:

$$\forall i \in [1, \bar{i}-1], \quad d - \lambda b_i - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) b_i \sigma_z^2 + \mu_i^B \sigma_i^2\right).$$

For assets $i \geq \bar{i}$, group B investors have zero holdings and so $\mu_i^B = 0$. For these assets, it must be the case that the group B investors' marginal utility of holding the asset, taken at the equilibrium holdings, is strictly negative (otherwise, group B investors would have an incentive to increase their holdings). This is equivalent to

$$\forall i \geq \bar{i}, \quad d - \lambda b_i - P_i(1+r) - \frac{1}{\gamma}\left(\left(\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) b_i \sigma_z^2\right) < 0.$$

Finally, since arbitrageurs are not short-sales constrained, their holdings always satisfy the following first-order condition:

$$\forall i \in [1, N], \quad d - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} b_k \mu_k^a\right) b_i \sigma_z^2 + \mu_i^a \sigma_i^2\right).$$

The market clearing condition for asset i is simply $\alpha\frac{\mu_i^A + \mu_i^B}{2} + (1-\alpha)\mu_i^a = \frac{1}{N}$. We sum the first-order conditions of investors a, A, and B for assets $i < \bar{i}$, and of investors a and A for assets $i \geq \bar{i}$, weighting the sum by the size of each investor group ($\frac{\alpha}{2}$ for groups A and B, and $1-\alpha$ for group a). This results in the following system of equations:

$$\begin{cases} d - P_i(1+r) = \dfrac{1}{\gamma}\left(b_i \sigma_z^2 + \dfrac{\sigma_i^2}{N}\right) & \text{for } i < \bar{i} \\ \left(1 - \dfrac{\alpha}{2}\right)(d - P_i(1+r)) + \dfrac{\alpha}{2}\lambda b_i = \dfrac{1}{\gamma}\left(b_i \sigma_z^2 + \dfrac{\sigma_i^2}{N} - \dfrac{\alpha}{2}\sigma_z^2 b_i \displaystyle\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) & \text{for } i \geq \bar{i}. \end{cases} \tag{1}$$

Let $S^B = \sum_{k=1}^{\bar{i}-1} b_k \mu_k^B$, where S^B represents the exposure of group B investors to the aggregate factor $\tilde{z}$. We look for an expression for S^B. We start by using the first-order condition of group B investors on assets $k < \bar{i}$ and plug in the equilibrium price of assets $k < \bar{i}$

*Harrison Hong and David A. Sraer, 2016, Internet Appendix to "Speculative Betas", Journal of Finance vol no., pages, http://www.afa.jof.org/supplements.asp. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

found in the first equation of system (1):

$$\forall k < \bar{i}, \qquad -\lambda\gamma b_k + b_k\sigma_z^2 + \frac{\sigma_k^2}{N} = S^B b_k \sigma_z^2 + \mu_k^B \sigma_k^2.$$

We can now multiply the previous equation by b_k, divide it by σ_k^2 for all $k < \bar{i}$, and then sum up the resulting equations for $k < \bar{i}$. This results in

$$S^B = -\lambda\gamma \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right) - S\sigma_z^2 \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right) + \sigma_z^2 \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right) + \sum_{k<\bar{i}} \frac{b_k}{N}. \tag{2}$$

From the previous expression, we can now derive S^B as

$$S^B = 1 - \frac{\left(\sum_{k\geq\bar{i}} \frac{b_k}{N}\right) + \lambda\gamma \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)}{1 + \sigma_z^2 \left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)}.$$

Now that we have a closed-form expression for S^B, we simply plug it into the second equation of system 1. Define $\theta = \frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}$. The price of assets $i \geq \bar{i}$ is then given by

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right) + \underbrace{\frac{\theta}{\gamma}\left(b_i\sigma_z^2 \underbrace{\frac{\lambda\gamma - \sigma_z^2 \sum_{k\geq\bar{i}} \frac{b_k}{N}}{\sigma_z^2\left(1 + \sigma_z^2\left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_i^2}\right)\right)}}_{=\omega(\lambda)} - \frac{\sigma_i^2}{N}\right)}_{\pi^i = \text{speculative premium}}. \tag{3}$$

The first equation of system 1 provides us with a simple expression for the price of assets $i < \bar{i}$:

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right). \tag{4}$$

To derive the conditions under which the proposed equilibrium is indeed an equilibrium (i.e., $\bar{i}$ is indeed the marginal asset), we need to derive the equilibrium holdings of group B investors:

$$\mu_i^{B,\star} = \begin{cases} \dfrac{1}{N} + \dfrac{b_i}{\sigma_i^2}\left(\dfrac{\sigma_z^2\left(\sum_{i\geq\bar{i}} \frac{b_i}{N}\right) - \lambda\gamma}{1 + \sigma_z^2\left(\sum_{i<\bar{i}} \frac{b_i^2}{\sigma_i^2}\right)}\right) & \text{for } i < \bar{i} \\ 0 \quad \text{for } i \geq \bar{i}. \end{cases}$$

We are now ready to derive the conditions under which the proposed equilibrium is indeed an equilibrium. The marginal asset is asset $\bar{i}$ if and only if $\frac{\partial U^B}{\partial \mu_{\bar{i}}^B}(\mu^{B,\star}) < 0$ and $\mu_{\bar{i}-1}^B \geq 0$, where $\mu^{B,\star}$ is group B investors' holdings derived above. The condition that the marginal utility of investing in asset $\bar{i}$ for pessimist agents is equivalent to $\pi_{\bar{i}} > 0$, so that $\bar{i}$ is the marginal asset if and only if

$$\frac{\sigma_z^2}{\gamma N}\sum_{k\geq\bar{i}} b_k + \frac{1}{\gamma N \frac{b_{\bar{i}-1}}{\sigma_{\bar{i}-1}^2}}\left(1 + \sigma_z^2 \sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right) \geq \lambda > \frac{\sigma_z^2}{\gamma N}\sum_{k\geq\bar{i}} b_k + \frac{1}{\gamma N \frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}}\left(1 + \sigma_z^2 \sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right).$$

Let $u_k = \frac{1}{\gamma N \frac{b_k}{\sigma_k}}\left(1 + \sigma_z^2\left(\sum_{i<k} \frac{b_i^2}{\sigma_i^2}\right)\right) + \frac{\sigma_z^2}{\gamma}\left(\sum_{i\geq k} \frac{b_i}{N}\right)$. Clearly, u_k is a strictly decreasing sequence as

$$
\begin{aligned}
u_{i-1} - u_i &= \frac{1}{\gamma N \frac{b_{i-1}}{\sigma_{i-1}^2}} \left(1 + \sigma_z^2 \left(\sum_{j<i-1} \frac{b_j^2}{\sigma_j^2}\right)\right) + \frac{\sigma_z^2}{\gamma} \left(\sum_{j \geq i-1} \frac{b_j}{N}\right) - \frac{1}{\gamma N \frac{b_i}{\sigma_i^2}} \left(1 + \sigma_z^2 \left(\sum_{j<i} \frac{b_j^2}{\sigma_j^2}\right)\right) - \frac{\sigma_z^2}{\gamma} \left(\sum_{j \geq i} \frac{b_j}{N}\right) \\
&= \frac{1}{\gamma N} \left(1 + \sigma_z^2 \left(\sum_{j<i-1} \frac{b_j^2}{\sigma_j^2}\right)\right) \left(\frac{\sigma_{i-1}^2}{b_{i-1}} - \frac{\sigma_i^2}{b_i}\right) - \frac{1}{\gamma N \frac{b_i}{\sigma_i^2}} \sigma_z^2 \frac{b_{i-1}^2}{\sigma_{i-1}^2} + \frac{\sigma_z^2}{\gamma N} b_{i-1} \\
&= \frac{1}{\gamma N} \left(1 + \sigma_z^2 \left(\sum_{j<i-1} \frac{b_j^2}{\sigma_j^2}\right)\right) \left(\frac{\sigma_{i-1}^2}{b_{i-1}} - \frac{\sigma_i^2}{b_i}\right) + \frac{\sigma_z^2}{\gamma N} \frac{b_{i-1}^2}{\sigma_{i-1}^2} \left(\frac{\sigma_{i-1}^2}{b_{i-1}} - \frac{\sigma_i^2}{b_i}\right) \\
&= \frac{1}{\gamma N} \left(1 + \sigma_z^2 \left(\sum_{j<i} \frac{b_j^2}{\sigma_j^2}\right)\right) \left(\frac{\sigma_{i-1}^2}{b_{i-1}} - \frac{\sigma_i^2}{b_i}\right) > 0.
\end{aligned}
$$

Define $u_0 = +\infty$ and $u_{N+1} = 0$. Then the sequence $(u_i)_{i \in [0,N+1]}$ spans $\mathbb{R}^+$ and the marginal asset is simply defined as $\bar{\imath} = \min\{k|\lambda > u_k\}$. We know that $\bar{\imath} > 0$ since $u_0 = +\infty$. If $\bar{\imath} = N+1$, then group B investors are long all assets and all the previous formulas apply except that there is no asset such that $i \geq \bar{\imath}$. If $\bar{\imath} \in [1, n]$, then the equilibrium has the proposed structure, that is, investors B are long only assets $i < \bar{\imath}$.

We have so far assumed that $\bar{\imath} > 1$. The equilibrium is easily derived when $\bar{\imath} = 1$, that is, when all assets are overpriced. In this case, $S^B = 0$ and we have

$$
d - (1+r)P_i = \frac{1}{\gamma}(1+\theta)\left(b_i \sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) - \theta \lambda b_i.
$$

This corresponds to the formula derived in Theorem 1 where we define $\sum_{i<1} b_i^2 = 0$. Moreover, $\bar{\imath} = 1$ is an equilibrium if and only if $\mu_1^{B,\star} < 0$, which is equivalent to $\lambda < u_N$, as stated in the theorem.

We now show that the equilibrium is unique. Let $J = j|\mu_j^B > 0 -$ J be the set of assets that pessimists are long. It is straightforward to show that prices are given by in this case

$$
P_i(1+r) = \begin{cases} d - \dfrac{1}{\gamma}\left(b_i \sigma_z^2 + \dfrac{\sigma_\epsilon^2}{N}\right) & \text{for } i \in J \\ d - \dfrac{1}{\gamma}\left(b_i \sigma_z^2 + \dfrac{\sigma_i^2}{N}\right) + \underbrace{\dfrac{\theta}{\gamma}\left(b_i \left(\dfrac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i \notin J} b_i\right)}{1 + \sigma_z^2 \left(\sum_{i \in J} \frac{b_i^2}{\sigma_i^2}\right)}\right) - \dfrac{\sigma_i^2}{N}\right)}_{\pi^i = \text{speculative premium}} & \text{for } i \notin J \end{cases} \tag{5}
$$

The holdings of the pessimists can then be written as

$$
\mu_i^{B,\star} = \begin{cases} \dfrac{1}{N} + \dfrac{b_i}{\sigma_i^2}\left(\dfrac{\sigma_z^2\left(\sum_{i \notin J} \frac{b_i}{N}\right) - \lambda\gamma}{1 + \sigma_z^2\left(\sum_{i \in J} \frac{b_i^2}{\sigma_i^2}\right)}\right) & \text{for } i \in J \\ 0 \quad \text{for } i \notin J \end{cases}
$$

This implies that for $j \in J$, we need to have $\frac{b_j}{\sigma_j^2}\left(\frac{\lambda\gamma - \sigma_z^2\left(\sum_{i \notin J} \frac{b_i}{N}\right)}{1 + \sigma_z^2\left(\sum_{i \in J} \frac{b_i^2}{\sigma_i^2}\right)}\right) < \frac{1}{N}$, and for $i \notin J$ that $b_i \left(\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i \notin J} b_i\right)}{1 + \sigma_z^2\left(\sum_{i \in J} \frac{b_i^2}{\sigma_i^2}\right)}\right) > \frac{\sigma_i^2}{N}$.

Thus, for all $j \in J$ and for all $i \notin J$,

$$
\frac{b_i}{\sigma_i^2} > \frac{1}{N}\left(\frac{1 + \sigma_z^2\left(\sum_{i \in J} \frac{b_i^2}{\sigma_i^2}\right)}{\lambda\gamma - \sigma_z^2\left(\sum_{i \notin J} \frac{b_i}{N}\right)}\right) > \frac{b_j}{\sigma_j^2}.
$$

The equilibrium structure is necessarily in the form of a cutoff and hence our equilibrium is unique.

■

B. Proof of Corollary 1

Proof. Corollary 1 characterizes overpricing. Overpricing for assets $i \geq \bar{\imath}$ is defined as the difference between the equilibrium price and the price that would prevail in the absence of heterogeneous beliefs and short-sales constraints ($\alpha = 0$). Overpricing is simply equal to the speculative premium:

$$
\forall i \geq \bar{\imath}, \quad \text{Overpricing}^i = \pi^i = \frac{\theta}{\gamma}\left(b_i \sigma_z^2 \omega(\lambda) - \frac{\sigma_i^2}{N}\right).
$$

By definition of the equilibrium, $\lambda > u_{\bar{i}}$, which is equivalent to $\frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}\sigma_z^2\omega(\lambda) > \frac{1}{N}$. Since assets are ranked in ascending order of $\frac{b_i}{\sigma_i^2}$, this implies that for $i \geq \bar{i}$, $\pi^i > 0$ and assets $i \geq \bar{i}$ are in fact overpriced. That mispricing is increasing with the fraction of short-sales-constrained investors α follows as θ is a strictly increasing function of α. That mispricing increases with b_i and decreases with σ_i^2 also follows from the definition of mispricing:[1]

$$\forall j > i \geq \bar{i}, \quad \text{Overpricing}^j - \text{Overpricing}^i = \frac{\theta}{\gamma}\sigma_z^2\omega(\lambda)(b_j - b_i).$$

■

C. Proof of Corollary 2

Proof. Corollary 2 characterizes the amount of shorting in the equilibrium. We first need to derive the equilibrium holdings of arbitrageurs. Group a holdings need to satisfy the following first-order condition:

$$\forall i \in [1, N], \quad d - P_i(1+r) = \frac{1}{\gamma}\left(b_i\sigma_z^2\left(\sum_{k=1}^{N}\mu_k^a b_k\right) + \mu_i^a\sigma_i^2\right)$$

Define $S^a = \sum_{k=1}^{N}\mu_k^a b_k$. Using the equilibrium pricing equation in equation 3 and equation 4, this first-order condition can be rewritten as

$$\forall k \in [1, N], \quad b_k\sigma_z^2 + \frac{\sigma_k^2}{N} - \gamma\pi^k 1_{k\geq\bar{i}} = b_k\sigma_z^2 S^a + \mu_k^a\sigma_k^2.$$

We multiply each of these equations by b_k, divide them by σ_k^2, and sum up the resulting equations for all $i \in [1, N]$ to obtain

$$S^a = 1 - \frac{\gamma\sum_{k\geq\bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1 + \sigma_z^2\left(\sum_{k=1}^{N}\frac{b_k^2}{\sigma_k^2}\right)}$$

We can now plug this expression for S^a in group a investors' first-order conditions derived above. This yields the following expression for group a investors' holdings of assets $i \in [1, N]$:

$$\forall i \in [1, N], \quad \mu_i^a\sigma_i^2 = \frac{\sigma_i^2}{N} - \gamma\pi^i 1_{\{i\geq\bar{i}\}} + b_i\sigma_z^2\frac{\gamma\sum_{k\geq\bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1 + \sigma_z^2\left(\sum_{k=1}^{N}\frac{b_k^2}{\sigma_k^2}\right)}.$$

First note that if $i < \bar{i}$, $\mu_i^a > 0$, so that arbitrageurs are long assets $i < \bar{i}$. Now consider the case $i \geq \bar{i}$. Notice from the expression of the speculative premium that

$$\forall k, i \geq \bar{i}, \quad \pi_k + \frac{\theta\sigma_k^2}{\gamma N} = \frac{b_k}{b_i}\left(\pi^i + \frac{\theta\sigma_i^2}{\gamma N}\right).$$

Thus, multiplying the previous expression by b_k, dividing by σ_k^2, and summing over all $k \geq \bar{i}$, we have

$$\sum_{k\geq\bar{i}} b_k\frac{\pi_k}{\sigma_k^2} + \frac{\theta}{\gamma N}\left(\sum_{k\geq\bar{i}} b_k\right) = \left(\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)\left(\frac{\pi^i + \frac{\theta\sigma_i^2}{\gamma N}}{b_i}\right).$$

Thus, for $i \geq \bar{i}$,

$$
\begin{aligned}
\gamma\pi^i - b_i\sigma_z^2 \frac{\gamma \sum_{k\geq\bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} &= \gamma\pi^i - \frac{b_i\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\left(\left(\sum_{k\geq\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)\left(\frac{\gamma\pi^i + \frac{\theta}{N}\sigma_i^2}{b_i}\right) - \frac{\theta}{N}\left(\sum_{k\geq\bar{i}} b_k\right)\right) \\
&= \gamma\pi^i \frac{1+\sigma_z^2\left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} - \frac{\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\frac{\theta}{N}\left(\sum_{k\geq\bar{i}}\sigma_i^2\frac{b_k^2}{\sigma_k^2} - b_i\sum_{k\geq\bar{i}} b_k\right) \\
&= \theta\left[b_i\left(\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i\geq\bar{i}} b_i\right)}{1+\sigma_z^2\left(\sum_{i=1}^N \frac{b_i^2}{\sigma_i^2}\right)}\right) - \frac{\sigma_i^2}{N}\frac{1+\sigma_z^2\left(\sum_{k<\bar{i}} \frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\right. \\
&\qquad \left. - \frac{1}{N}\frac{\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\left(\sigma_i^2\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2} - b_i\sum_{k\geq\bar{i}} b_k\right)\right] \\
&= \theta\left[b_i\frac{\lambda\gamma}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} - \frac{\sigma_i^2}{N}\right].
\end{aligned}
$$

We can now derive the actual holdings of arbitrageurs on assets $i \geq \bar{i}$:

$$
\forall i \geq \bar{i}, \quad \mu_i^a = \frac{1+\theta}{N} - \theta\frac{b_i}{\sigma_i^2}\frac{\lambda\gamma}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}.
$$

Notice that arbitrageurs' holdings are decreasing with i since $\frac{b_i}{\sigma_i^2}$ increases strictly with i. There is at least one asset shorted by group a investors provided that $\mu_N^a < 0$, which is equivalent to $\lambda > \hat{\lambda} = \frac{1+\theta}{\theta}\frac{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}{N}\frac{\sigma_N^2}{\gamma b_N}$. If this holds, there exists a unique $\tilde{i} \in [1, N]$ such that $\mu_i^a < 0 \Leftrightarrow i \geq \tilde{i}$. We already know that $\tilde{i} \geq \bar{i}$ since for $i < \bar{i}$, group a investors holdings are strictly positive. It follows from the expression for group a investors' holdings that provided $i \geq \tilde{i}$, we have

$$
\frac{\partial|\mu_i^a|}{\partial\lambda} > 0, \quad \frac{\partial|\mu_i^a|}{\partial\frac{b_i}{\sigma_i^2}} > 0, \quad \text{and} \quad \frac{\partial^2|\mu_i^a|}{\partial\lambda\partial b_i} > 0.
$$

There is more shorting of assets with a larger ratio of cash-flow beta to idiosyncratic variance, there is more shorting the larger is aggregate disagreement, and the effect of aggregate disagreement on shorting is larger for assets with a high ratio of cash-flow beta to idiosyncratic variance. ∎

D. Proof of Formula (3) for Expected Excess Returns

Proof. From Theorem 1, we know that

$$
P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_I^2}{N}\right) + \mathbf{1}_{i\geq\bar{i}}\frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_I^2}{N}\right).
$$

Denote by $\tilde{R}_i^e$ the percentage excess return per share on asset i. Define $\tilde{R}_i^e = d + b_i\tilde{z} + \tilde{\epsilon}_i - (1+r)P_i$ and $\mathbb{E}[\tilde{r}_i^e] = d - (1+r)P_i$. Further, define the market portfolio as the portfolio of all assets in the market. Since all assets have a supply of 1/N, the excess return per share on the market portfolio is $\tilde{R}_m^e = \sum_{j=1}^N \frac{\tilde{R}_j^e}{N}$. Let $P_m = \sum_{j=1}^N \frac{P_j}{N}$ be the price of the market portfolio. Stock i's beta is defined as $\beta_i = \frac{\mathrm{Cov}\left(\tilde{R}_i^e, \tilde{R}_m^e\right)}{Var\left(\tilde{R}_m^e\right)}$ and can be written as

$$
\beta_i = \frac{b_i\sigma_z^2 + \frac{\sigma_i^2}{N}}{\sigma_z^2 + \sum_{k=1}^N \frac{\sigma_k^2}{N^2}}, \quad \text{so that} \quad b_i\sigma_z^2 = \beta_i\left(\sigma_z^2 + \sum_{k=1}^N \frac{\sigma_k^2}{N^2}\right) - \frac{\sigma_i^2}{N}.
$$

We can thus substitute b_i by β_i in the price formula and derive an expression for expected excess returns per share as a function of β_i:

$$
\mathbb{E}[\tilde{R}_i^e] = \beta_i\frac{\sigma_z^2 + \sum_{k=1}^N \frac{\sigma_k^2}{N^2}}{\gamma}\left(1 - \mathbf{1}_{i\geq\bar{i}}\theta\omega(\lambda)\right) + \theta\frac{\sigma_i^2}{\gamma N}\mathbf{1}_{i\geq\bar{i}}(1+\omega(\lambda)).
$$

$\blacksquare$

E. Proof of Corollary 3

Proof. We make this proof in the context of homoskedastic dividends: $\sigma_i^2 = \sigma_\epsilon^2$. We can write actual excess returns as

$$\tilde{R}_i^e = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} + \tilde{\eta}^i & \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} (1 - \theta\omega(\lambda)) + \dfrac{\sigma_\epsilon^2}{\gamma N} \theta (1 + \omega(\lambda)) + \tilde{\eta}^i & \text{for } i \geq \bar{i} \end{cases},$$

where $\tilde{\eta_i} = b_i \tilde{z} + \tilde{\epsilon_i}$.

Using the fact that, by definition, $\sum_{i=1}^N b_i = \sum_{i=1}^N \beta_i = N$, a cross-sectional regression of realized excess returns per share $\left(\tilde{R}_i^e\right)_{i \in [1,N]}$ on $(\beta_i)_{i \in [1,N]}$ and a constant would deliver the following coefficient estimate:

$$\begin{aligned} \hat{\mu} &= \frac{\sum_{i=1}^N \beta_i \tilde{R}_i - \sum_{i=1}^N \tilde{R}_i}{\sum_{i=1}^N \beta_i^2 - N} \\ &= \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} \left(1 + \frac{\gamma}{\sigma_z^2} \tilde{z} - \left(\frac{\sum_{i \geq \bar{i}} \beta_i^2 - \sum_{i \geq \bar{i}} \beta_i}{\sum_{i=1}^N \beta_i^2 - N} \right) \theta\omega(\lambda) \right) + \frac{\sum_{i \geq \bar{i}} (\beta_i - 1)}{\sum_{i=1}^N \beta_i^2 - N} \frac{\sigma_\epsilon^2}{\gamma N} \theta \left(1 + \omega(\lambda)\right). \end{aligned}$$

Let $\frac{u_{\bar{i}-1}}{\gamma} > \lambda_1 > \lambda_2 > \frac{u_{\bar{i}}}{\gamma}$, and let $\bar{i}_1$ ($\bar{i}_2$) the threshold associated with disagreement λ_1 (λ_2). We have that $\bar{i}_1 = \bar{i_2} = \bar{i}$. Thus,

$$\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2) = -\frac{1}{\gamma} \frac{\theta\left(\omega(\lambda_1) - \omega(\lambda_2)\right)}{\sum_{i=1}^N \beta_i^2 - N} \left(\sigma_z^2 \left(\sum_{i \geq \bar{i}}^N \beta_i^2 - \sum_{i \geq \bar{i}}^N \beta_i \right) + \frac{\sigma_\epsilon^2}{N} \sum_{i \geq \bar{i}}^N (\beta_i - 1)^2 \right).$$

We show that $\sum_{i \geq \bar{i}} \beta_i^2 \geq \sum_{i \geq \bar{i}} \beta_i$. Since average β is one, we can write β_i as $\beta_i = 1 + y_i$ with y_i such that $\sum y_i = 0$. Using this decomposition, we have that

$$\sum_{i=1}^N \beta_i^2 = N + 2 \underbrace{\sum_{i=1}^N y_i}_{=0} + \sum_{i=1}^N y^2 > N = \sum_{i=1}^N \beta_i.$$

Thus, the relationship is true for $\bar{i} = 1$. Now assume it is true for $\bar{i} = k > 1$. We have $\sum_{i \geq k+1} \beta_i^2 - \sum_{i \geq k+1} \beta_i = \sum_{i \geq k} \beta_i^2 - \sum_{i \geq k} \beta_i + \beta_k - \beta_k^2$. Either $\beta_k > 1$, in which case it is evident that $\sum_{i \geq k+1} \beta_i^2 - \sum_{i \geq k+1} \beta_i > 0$ as $\beta_k > 1$ implies that $\beta_i > 1$ for $i \geq k$, or $\beta_k \leq 1$, in which case $\beta_k - \beta_k^2 > 0$ and using the recurrence assumption, $\sum_{i \geq k+1} \beta_i^2 - \sum_{i \geq k+1} \beta_i > 0$. This proves that $\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2) < 0$.

We show now that for all $i \in [1, N]$, $\hat{\mu}(\lambda)$ is continuous in u_i where u_i is the sequence defined in Theorem 1 and calculated as $b_i \sigma_z^2 \omega(u_i) = \frac{\sigma_\epsilon^2}{N}$. When $\lambda = u_i^+$, we have $\bar{i} = i$. When $\lambda = u_i^-$, we have $\bar{i} = i + 1$. Notice that $\omega(\lambda)$ is continuous in u_i and

$$\omega(u_i^-) = \omega(u_i^+) = \omega(u_i) = \frac{\sigma_\epsilon^2}{\sigma_z^2} \frac{1}{N\gamma} \frac{1}{b_i}.$$

Thus,

$$\begin{aligned} \hat{\mu}\left(u_i^+\right) - \hat{\mu}\left(u_i^-\right) &= -\frac{\theta}{\gamma} \frac{\beta_i - 1}{\sum_{k=1}^N \beta_k^2 - N} \left(-\beta_k \omega(u_k) \left(\sigma_z^2 + \frac{\sigma_\epsilon^2}{N} \right) + \frac{\sigma_\epsilon^2}{N} \left(1 + \omega(u_k)\right) \right) \\ &= -\frac{\theta}{\gamma} \frac{\beta_i - 1}{\sum_{k=1}^N \beta_k^2 - N} \left(-b_i \sigma_z^2 \omega(u_i) + \frac{\sigma_\epsilon^2}{N} \right) = 0 \quad \text{by definition of } u_i. \end{aligned}$$

Therefore, $\hat{\mu}$ is continuous and strictly decreasing for λ in $]u_{i+1}, u_i[$ and is continuous at $\lambda = u_i$, so that it is overall strictly decreasing in aggregate disagreement λ. Since the derivative of the slope of the Security Market Line w.r.t. λ is linear in θ, it is trivial to show that $\frac{\partial^2 \hat{\mu}}{\partial \lambda \partial \theta} < 0$, that is, the negative effect of λ on the slope of the Security Market Line is stronger when there is a larger fraction of short-sales-constrained agents, that is, when θ is larger.

We can show that the slope of the Security Market Line, $\hat{\mu}$, is strictly decreasing with θ, the fraction of short-sales-constrained investors in the model. Since the marginal asset $\bar{i}$ is independent of θ and since we have already shown that $\sum_{i \geq \bar{i}} \beta_i^2 - \sum_{i \geq \bar{i}} \beta_i$, we have that

$$\frac{\partial \hat{\mu}}{\partial \theta} = -\frac{\omega(\lambda)}{\gamma \left(\sum_{i=1}^N \beta_i^2 - N \right)} \left(\frac{\sigma_\epsilon^2}{N} \sum_{i \geq \bar{i}} (\beta_i - 1)^2 + \sigma_z^2 \left(\sum_{i \geq \bar{i}} \beta_i^2 - \sum_{i \geq \bar{i}} \beta_i \right) \right) < 0.$$

$\blacksquare$

F. Proof of Theorem 2

Proof. We first consider the case in which $\tilde{\lambda}_t = 0$. There is no disagreement among investors, so all investors are long all assets $i \in [1, N]$. There is thus a unique first-order-condition for all investor types – for all $j \in [1, N]$ and $k = a$, A or B:

$$d - (1+r)P_t^j(0) + \mathbb{E}_t[P_{t+1}^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = 0] = \frac{1}{\gamma}\left(b_j\sigma_z^2 \sum_{i\leq N}\mu_i^k(0)b_i + \mu_j^k(0)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^k(0)\left(\Delta P_{t+1}^i\right)\right)\right).$$

Summing this equation across investor types, using the market-clearing condition, and dropping the time subscript leads to[2]

$$d - (1+r)P^j(0) + \mathbb{E}_t[P^j(\tilde{\lambda}_{+1})|\tilde{\lambda}_t = 0] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\left(\sum_{i\leq N}\frac{(\Delta P^i)}{N}\right)\right). \tag{6}$$

Consider now the case in which $\lambda_t = \lambda$. Importantly, investors disagree on the expected value of the aggregate factor $\tilde{z}_{t+1}$, but they agree on the expected value of asset i's resale price $\mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})]$. This is because investors agree to disagree, so they recognize the existence of the next generation of investors with heterogeneous beliefs – and in particular, with beliefs different from theirs. However, they nevertheless evaluate the $t+1$ expected dividend stream differently. We proceed as in the static model. We assume there is a marginal asset $\bar{i}$ such that there are no binding short-sales constraints for assets $j < \bar{i}$ and strictly binding short-sales constraints for assets $j \geq \bar{i}$. We check ex post the conditions under which this is indeed an equilibrium. Under the proposed equilibrium structure, the first-order condition of the three groups of investors born at date t for assets $j < \bar{i}$ is easily written since, in the proposed equilibrium structure, these assets do not experience binding short-sales constraints:

$$\begin{gathered} d + b_j\lambda_t^k - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = \lambda] = \\ \tfrac{1}{\gamma}\left(b_j\sigma_z^2\left(\textstyle\sum_{i\leq N}\mu_i^k(\lambda)b_i\right) + \mu_j^k(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P^j\left(\textstyle\sum_{i\leq N}\mu_i^k(\lambda)\left(\Delta P^i\right)\right)\right). \end{gathered}$$

Summing across investor types (using the weight of each investor group) and using the market-clearing condition leads to

$$\forall j < \bar{i}, \quad d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{+1})|\tilde{\lambda}_t = \lambda] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\sum_{i\leq N}\frac{\Delta P^i}{N}\right). \tag{7}$$

Subtracting equation (6) – prices in the low-disagreement state – from equation (7) leads to

$$\forall j < \bar{i}, \quad -(1+r)\Delta P^j + \rho\Delta P^j - (1-\rho)\Delta P^j = 0 \Leftrightarrow P^j(\lambda) = P^j(0),$$

since $\rho < 1$.

Thus, for all $j < \bar{i}$, $\Delta P^j = 0$. The payoff of assets below $\bar{j}$ is not sufficiently exposed to aggregate disagreement to make pessimist investors willing to go short. Hence, even in the high-disagreement state, these assets experience no mispricing. In particular, their price is independent of the realization of aggregate disagreement. Aggregate disagreement thus creates resale price risk only for assets that experience binding short-sales constraints in high-aggregate disagreement states, that is, high $\frac{b}{\sigma^2}$ assets with $i \geq \bar{i}$.

We now turn to the assets with binding short-sales constraints in high-disagreement states, that is, assets $j > \bar{i}$. For these assets, we know that under the proposed equilibrium, $\mu_j^B(\lambda) = 0$. We thus have the following first-order conditions for hedge funds and optimist mutual funds respectively:

$$\begin{cases} d + b_j\lambda - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda} = \lambda] = \dfrac{1}{\gamma}\left(b_j\sigma_z^2\displaystyle\sum_{i\leq N}\mu_i^A(\lambda)b_i + \mu_j^A(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^A(\lambda)\Delta P_{t+1}^i\right)\right) \\ d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda} = \lambda] = \dfrac{1}{\gamma}\left(b_j\sigma_z^2\displaystyle\sum_{i\leq N}\mu_i^a(\lambda)b_i + \mu_j^a(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^a(\lambda)\Delta P_{t+1}^i\right)\right) \end{cases}.$$

Define $\Gamma = \sum_{i\geq\bar{i}}\frac{\Delta P^i}{N}$, the average price difference between high- and low-aggregate disagreement states across all assets. Summing these equations across investor types (using the weight of each investor group) and using the market-clearing condition leads to:

$$\begin{gathered} \tfrac{\alpha}{2}b_j\lambda + (1-\tfrac{\alpha}{2})\left(d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = \lambda]\right) = \\ \tfrac{1}{\gamma}\left(b_j\sigma_z^2 + \tfrac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\Gamma - \tfrac{\alpha}{2}b_j\sigma_z^2\underbrace{\sum_{i<\bar{i}}\mu_i^B(\lambda)b_i}_{S^1} - \tfrac{\alpha}{2}\rho(1-\rho)\Delta P^j\underbrace{\sum_{i<\bar{i}}\mu_i^B(\lambda)\left(\Delta P^i\right)}_{S^2=0}\right). \end{gathered}$$

In the previous equation, $S^2 = 0$ since for all $i < \bar{i}$, $\Delta P^i = 0$. To recover S^1, we use B-investors' first-order condition on assets $j < \bar{i}$, the equilibrium prices derived above for assets $j < \bar{i}$, and the fact that for all $i < \bar{j}$, $\Delta P^j = 0$. This leads to the following equation:

$$\forall j < \bar{\imath}, \quad b_j \sigma_z^2 \underbrace{\sum_{i \leq N} \mu_i^B b_i}_{S^1} + \mu_j^B \sigma_j^2 = -\lambda\gamma b_j + b_j \sigma_z^2 + \frac{\sigma_j^2}{N}.$$

Multiplying the previous expression by b_j, dividing by σ_j^2, and summing the equations over j gives the following formula for S^1:

$$S^1 = 1 - \frac{\left(\sum_{i \geq \bar{\imath}} \frac{b_i}{N}\right) + \lambda\gamma \left(\sum_{i<\bar{\imath}} \frac{b_i^2}{\sigma_i^2}\right)}{1 + \sigma_z^2 \left(\sum_{i<\bar{\imath}} \frac{b_i^2}{\sigma_i^2}\right)}.$$

This allows us to derive the excess return on assets $j \geq \bar{\imath}$:

$$\underbrace{d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = \lambda]}_{\text{Excess Return}} =$$

$$\underbrace{\frac{1}{\gamma}\left(b_j \sigma_z^2 + \frac{\sigma_j^2}{N} + (1+\theta)\rho(1-\rho)\Delta P^j \Gamma\right)}_{\text{Risk Premium}} - \underbrace{\frac{\theta}{\gamma}\left(b_j \frac{\lambda\gamma - \frac{\sigma_z^2}{N}\sum_{k \geq \bar{\imath}} b_k}{1 + \sigma_z^2\left(\sum_{i<\bar{\imath}} \frac{b_i^2}{\sigma_i^2}\right)} - \frac{\sigma_j^2}{N}\right)}_{\text{Speculative Premium}=\pi^j}.$$

Note that the risk premium embeds a term that reflects the resale price risk of high-b assets. Subtracting equation (6) from the previous equation yields, for all $j \geq \bar{\imath}$,

$$-(1+r)\Delta P^j + (2\rho - 1)\Delta P^j = -\pi^j + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma \Delta P^j \Rightarrow \left((1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma\right)\Delta P^j = \pi^j. \tag{8}$$

Remember that $\Gamma = \sum_{i \geq \bar{\imath}} \frac{\Delta P^i}{N}$. We can thus obtain a formula for Γ by adding up the previous equations for all $j \geq \bar{\imath}$ and dividing by N:

$$\left((1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma\right)\Gamma = \frac{1}{N}\sum_{j \geq \bar{\imath}} \pi^j.$$

There is a unique $\Gamma^+ > 0$ which satisfies the previous equation. Call it Γ^+:

$$\Gamma^+ = \frac{-(1+r) + (2\rho-1) + \sqrt{((1+r)-(2\rho-1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j \geq \bar{\imath}} \pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}.$$

There is also a unique $\Gamma^- < 0$ that satisfies equation (8):

$$\Gamma^- = \frac{-(1+r) + (2\rho-1) - \sqrt{((1+r)-(2\rho-1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j \geq \bar{\imath}} \pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}.$$

Let $\Gamma^\star$ be the actual value of Γ, the average price difference between high- and low-aggregate disagreement states across all assets. $\Gamma^\star \in \{\Gamma^-, \Gamma^+\}$. For $j \geq \bar{\imath}$, the price difference is simply expressed as a function of the speculative premium π^j and $\Gamma^\star$:

$$\Delta P^j = \frac{\pi^j}{1 + r - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}.$$

For the equilibrium to exist, it needs to be the case that for each asset $j \geq \bar{\imath}$, pessimists do not want to hold asset j, that is, the marginal utility of holding assets $j \geq \bar{\imath}$ at the optimal holding is zero. This is equivalent to

$$\forall j \geq \bar{\imath}, \quad d - b_j\lambda - (1+r)P^j(\lambda) + \rho P^j(\lambda) + (1-\rho)P^j(0) - \frac{1}{\gamma} b_j \sigma_z^2 \underbrace{\sum_{j<\bar{\imath}} \mu_i^B b_i}_{=S^1} < 0.$$

We have that

$$\begin{aligned}
& d - b_j\lambda - (1+r)P^j(\lambda) + \rho P^j(\lambda) + (1-\rho)P^j(0) - \frac{1}{\gamma}b_j\sigma_z^2 S^1 \\
= \quad & -b_j\lambda + \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)(1+\theta)\Delta P^j\Gamma^\star\right) - \pi^j - \frac{1}{\gamma}b_j\sigma_z^2 S^1 \\
= \quad & -\frac{\pi^j}{\theta} - \pi^j + (1+\theta)\frac{\rho(1-\rho)}{\gamma}\Gamma^\star\Delta P^j \\
= \quad & \frac{1+\theta}{\theta}\pi^j\left(\frac{\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star} - 1\right) \\
= \quad & -\frac{1+\theta}{\theta}\frac{(1+r)-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\times\pi^j.
\end{aligned}$$

Assume $\Gamma^\star = \Gamma^- < 0$. We know that

$$\theta\rho(1-\rho)\frac{\Gamma^-}{\gamma} + (1+r) - (2\rho-1) = \frac{(1+r)-(2\rho-1) - \sqrt{((1+r)-(2\rho-1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{i}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}} < 0.$$

Thus, if $\Gamma^\star = \Gamma^-$, then $-\frac{1+\theta}{\theta}\frac{(1+r)-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma} > 0$, so that it has to be the case that for all $j \geq \bar{i}$, $\pi^j < 0$. Thus $\sum_{j\geq\bar{i}}\pi^j < 0$, so that

$$\left((1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^-\right)\Gamma^- < 0.$$

However, the previous expression is strictly positive since $\Gamma^- < 0$ and $(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^- < 0$. Thus, we can't have $\Gamma^\star = \Gamma^-$ and it has to be the case that $\Gamma^\star = \Gamma^+$.

Since $\Gamma^\star > 0$, we have from the previous equilibrium condition that, for all $j \geq \bar{i}$, $\pi^j > 0$. Similarly, it is straightforward to show that for pessimists to have strictly positive holdings of assets $\bar{j}-1$, a necessary and sufficient condition is that $\pi^{\bar{j}-1} < 0$. Overall, this leads to the following equilibrium condition:

$$\frac{\sigma_z^2}{N}\left(\sum_{k\geq\bar{i}}b_k\right) + \frac{1}{N}\frac{\sigma_{\bar{j}-1}^2}{b_{\bar{j}-1}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right) \geq \lambda\gamma \geq \frac{\sigma_z^2}{N}\left(\sum_{k\geq\bar{i}}b_k\right) + \frac{1}{N}\frac{\sigma_{\bar{j}}^2}{b_{\bar{j}}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right).$$

We can define a sequence v_i, analogous to the sequence u_i defined in Theorem 1, as

$$\forall i \in [1,N], \quad v_i = \frac{\sigma_z^2}{N}\left(\sum_{k\geq i}b_k\right) + \frac{1}{N}\frac{\sigma_i^2}{b_i}\left(1+\sigma_z^2\sum_{k<i}\frac{b_k^2}{\sigma_k^2}\right), \quad v_{N+1} = 0 \text{ and } v_0 = +\infty.$$

It can be easily shown that this sequence is strictly decreasing since, for all $i \in [2,N]$,

$$v_i - v_{i-1} = \frac{1}{N}\left(\frac{\sigma_i^2}{b_i} - \frac{\sigma_{i-1}^2}{b_{i-1}}\right)\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right),$$

and assets are ranked in ascending order of $\frac{b_i}{\sigma_i^2}$.

The equilibrium condition can thus be written as $v_{\bar{i}-1} \geq \lambda\gamma \geq v_{\bar{i}}$, with $\bar{i}$ defined as the smallest $i \in [1,N]$ such that $\lambda\gamma \geq v_i$.

We now move on to the expression for expected excess returns. Since $\Delta P^j = 0$ for $j < \bar{i}$, we have that for all $j < \bar{i}$,

$$\mathbb{E}[R^j(\lambda)] = \mathbb{E}[R^j(0)] = d - rP^j(\lambda) = d - rP^j(0) = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N}\right).$$

For $j \geq \bar{i}$, however,

$$\begin{aligned}
\mathbb{E}[R^j(0)] \quad = \quad & d - (1+r)P^j(0) + \rho P^j(0) + (1-\rho)P^j(\lambda) \\
= \quad & \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right).
\end{aligned}$$

The extra term is the risk premium required by investors for holding stocks that are sensitive to disagreement and thus are exposed to changes in prices coming from changes in the aggregate disagreement state variable. Of course, in the data, since ρ is very close to one, this risk premium is going to be quantitatively small. Nevertheless, the intuition here is that high $\frac{b}{\sigma^2}$ stocks have low prices in low-disagreement states for two reasons: they are exposed to aggregate risk $\tilde{z}$, and they are exposed to changes in aggregate disagreement

$\tilde{\lambda}$. Finally,

$$\begin{aligned}
\mathbb{E}[R^j(\lambda)] &= d-(1+r)P^j(\lambda)+\rho P^j(\lambda)+(1-\rho)P^j(0) \\
&= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)-\pi^j \\
&\quad +\theta\frac{\rho(1-\rho)}{\gamma}\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j \\
&= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)-\frac{1+r-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j.
\end{aligned}$$

Thus, for assets $j \geq \bar{i}$, the expected return is strictly lower in high-disagreement states than in low-disagreement states.

The proof for the unicity of the equilibrium is similar to the proof of Theorem 1 and is thus omitted. ■

G. Proof of Corrolary 4

Proof. Part (i) is a direct consequence of the formula for expected excess returns in Theorem 2. For (ii), we do a Taylor expansion around $\rho = 1$ for $\Gamma^\star$: $\Gamma^\star \approx \frac{1}{r}\sum_{j\geq\bar{i}}\frac{\pi^j}{N} > 0$, so in the vicinity of $\rho = 1$ and for $j \geq \bar{i}$,

$$\mathbb{E}[R^j(\lambda)] \approx \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}\right)-\frac{1+r-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j.$$

The slope of the Security Market Line for assets $i < \bar{i}$ (expressed as a function of b_i – it would be equivalent as a function of β_i) is thus strictly lower for $i < \bar{i}$ than for $i \geq \bar{i}$ in the vicinity of $\rho = 1$, which proves (ii). (iii) can also be seen from the previous Taylor expansion and making λ grow to infinity. (iv) is also a direct consequence of the formula for expected excess returns in Theorem 2. ■

II. Additional Tables

Table IAI
Disagreement and Concavity of the Security Market Line: Monthly βs

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < $5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using monthly returns over the past three calendar years. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20-beta sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance, respectively.

Table IAI (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-3.69	-7.13**	-11.98***	-11.94***	3.16	13.80**	17.80**	17.44**	-2.80	-5.49*	-4.37	-4.11
	(-1.32)	(-2.46)	(-3.58)	(-3.58)	(0.66)	(2.29)	(2.42)	(2.40)	(-1.06)	(-1.75)	(-1.15)	(-1.09)
$R_{m,t}^{(12)}$		-0.32**	-0.34**	-0.34**		1.24***	1.21***	1.25***		0.12	0.17	0.15
		(-2.31)	(-2.41)	(-2.41)		(4.59)	(4.17)	(4.34)		(0.80)	(1.15)	(1.00)
$HML_t^{(12)}$		-0.34*	-0.23	-0.23		-0.18	-0.28	-0.23		0.68***	0.66***	0.63***
		(-1.79)	(-1.40)	(-1.42)		(-0.50)	(-0.77)	(-0.61)		(3.19)	(2.98)	(2.66)
$SMB_t^{(12)}$		0.98***	1.18***	1.19***		-1.45**	-1.59***	-1.63***		0.43	0.36	0.38
		(3.02)	(4.60)	(4.57)		(-2.38)	(-2.73)	(-2.84)		(1.28)	(1.04)	(1.16)
$UMD_t^{(12)}$		0.20	0.23*	0.23		-0.64**	-0.65**	-0.62**		0.40**	0.38**	0.36**
		(1.25)	(1.70)	(1.62)		(-2.21)	(-2.28)	(-2.17)		(2.54)	(2.41)	(2.39)
D/P$_{t-1}$			-4.50	-4.18			5.38	2.37			-0.80	1.31
			(-1.24)	(-1.06)			(0.71)	(0.30)			(-0.21)	(0.30)
Inflation$_{t-1}$			-4.49*	-4.30*			2.06	0.29			2.84	4.08
			(-1.93)	(-1.80)			(0.43)	(0.06)			(1.16)	(1.45)
Ted Spread$_{t-1}$				-0.62				5.79				-4.07
				(-0.24)				(1.05)				(-1.25)
Constant	-4.90	-3.37	-4.16*	-4.09	10.73*	6.92	7.83	7.18	3.94	-3.27	-3.36	-2.91
	(-1.58)	(-1.48)	(-1.70)	(-1.62)	(1.74)	(1.39)	(1.53)	(1.31)	(1.21)	(-1.13)	(-1.22)	(-0.98)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.75**	-5.99***	-9.50***	-9.35***	8.09**	10.14**	12.09**	11.52**	-2.36	-2.49	-0.13	0.25
	(-2.28)	(-2.76)	(-3.54)	(-3.51)	(2.15)	(2.51)	(2.33)	(2.30)	(-1.50)	(-1.28)	(-0.05)	(0.11)
$R_{m,t}^{(12)}$		-0.27**	-0.38***	-0.39***		1.12***	1.31***	1.36***		0.17	0.12	0.08
		(-2.04)	(-3.03)	(-3.10)		(5.14)	(5.46)	(5.68)		(1.58)	(1.04)	(0.75)
$HML_t^{(12)}$		-0.78***	-0.71***	-0.73***		0.92***	0.90***	0.97***		0.24	0.17	0.12
		(-4.34)	(-5.32)	(-5.62)		(2.75)	(3.23)	(3.73)		(1.64)	(1.29)	(1.00)
$SMB_t^{(12)}$		0.47	0.67**	0.69**		-0.07	-0.26	-0.32		0.15	0.09	0.12
		(1.52)	(2.53)	(2.58)		(-0.14)	(-0.57)	(-0.69)		(0.97)	(0.48)	(0.75)
$UMD_t^{(12)}$		0.01	0.06	0.05		-0.27	-0.35	-0.30		0.26**	0.27**	0.24***
		(0.05)	(0.58)	(0.45)		(-1.19)	(-1.62)	(-1.44)		(2.50)	(2.41)	(2.64)
D/P$_{t-1}$			0.44	1.68			-5.33	-10.05*			4.78*	7.94***
			(0.14)	(0.54)			(-0.90)	(-1.75)			(1.95)	(3.51)
Inflation$_{t-1}$			-6.87***	-6.14**			8.81*	6.03			-0.35	1.51
			(-2.61)	(-2.30)			(1.83)	(1.25)			(-0.18)	(0.73)
Ted Spread$_{t-1}$				-2.39				9.09**				-6.10***
				(-1.16)				(2.35)				(-3.44)
Constant	-9.71***	-4.85**	-4.87**	-4.60*	20.52***	9.01**	8.25*	7.23	-0.18	-4.66**	-3.88*	-3.20
	(-3.66)	(-2.36)	(-2.12)	(-1.92)	(4.35)	(2.04)	(1.73)	(1.45)	(-0.08)	(-2.08)	(-1.72)	(-1.40)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAII
Disagreement and Concavity of the Security Market Line: Different Horizons

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression:

$$r_{P,t}^{(k)} = \kappa_t^{(k)} + \pi_t^{(k)} \times \beta_P + \phi_t^{(k)} \times (\beta_P)^2 + \epsilon_{P,t}^{(k)}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t^{(k)} = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t^{(k)} = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t^{(k)} = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_3^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Panel A uses k=1 months, Panel B uses k=3 months, Panel C uses k=6 months, and Panel D uses k=18 months. Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long run EPS growth. Columns (2) and (6) add the factor $z \in Z$, where Z contains the k-month excess market return from t to $t+k-1$ and the k-month return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$; Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance, respectively.

Dep. Var:	$\phi_t^{(k)}$				$\pi_t^{(k)}$				$\kappa_t^{(k)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: k=1 months												
Agg. Disp.$_{t-1}$	-0.21	-0.31	-0.57	-0.66*	-0.01	0.53	0.89	0.96	0.04	-0.08	-0.16	-0.15
	(-0.38)	(-0.93)	(-1.58)	(-1.81)	(-0.01)	(0.75)	(1.15)	(1.22)	(0.11)	(-0.21)	(-0.42)	(-0.38)
Panel A: k=3 months												
Agg. Disp.$_{t-1}$	-0.98	-0.99	-1.89**	-2.09***	0.48	1.60	2.80*	2.99*	-0.02	-0.08	-0.30	-0.32
	(-0.81)	(-1.34)	(-2.34)	(-2.60)	(0.24)	(1.06)	(1.69)	(1.77)	(-0.02)	(-0.10)	(-0.36)	(-0.37)
Panel A: k=6 months												
Agg. Disp.$_{t-1}$	-2.81	-2.47*	-4.40***	-4.68***	2.47	4.47	6.93**	6.94**	-1.09	-0.92	-1.21	-1.04
	(-1.31)	(-1.68)	(-2.87)	(-3.08)	(0.76)	(1.60)	(2.31)	(2.30)	(-0.85)	(-0.74)	(-0.87)	(-0.74)
Panel A: k=18 months												
Agg. Disp.$_{t-1}$	-6.39**	-7.41**	-11.77***	-11.70***	7.65*	12.17**	17.55***	16.57***	-3.97*	-3.24	-3.99	-3.51
	(-2.02)	(-2.27)	(-3.49)	(-3.45)	(1.71)	(2.05)	(2.70)	(2.65)	(-1.88)	(-1.32)	(-1.47)	(-1.36)

Table IAIII

Disagreement and Concavity of the Security Market Line: Alternative Measures of Disagreement

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \qquad P = 1, ..., 20$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

In Panel A, Agg. Disp. is the monthly β-weighted average of stock level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth, where the pre-ranking β has been compressed to one ($\beta^i = 0.5\hat{\beta} + 0.5$). In Panel B, Agg. Disp. is the monthly β- and value-weighted average of stock level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. In Panel C, Agg. Disp. is the principal component of the monthly standard deviation of forecasts on GDP, IP, corporate profit and the unemployment rate in the SPF and is taken from Li and Li (2014). In Panel D, Agg. Disp is the "top-down" measure of market disagreement used in Yu (2011) and is measured as the standard deviation of analysts' forecasts of annual S&P 500 earnings, scaled by the average forecast on S&P 500 earnings. Columns (1) and (5) control only for Agg. Disp.$_{t-1}$. Columns (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5%, and 1% level of significance.

Table IAIII (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Compressed betas												
Agg. Disp.$_{t-1}$	-5.50*	-4.53	-8.71**	-8.64**	6.34	8.70*	13.52**	12.83**	-4.02*	-2.80	-2.91	-2.48
	(-1.71)	(-1.59)	(-2.49)	(-2.50)	(1.38)	(1.67)	(2.07)	(2.03)	(-1.86)	(-1.33)	(-1.12)	(-1.00)
Panel B: Value and beta-weighted												
Agg. Disp.$_{t-1}$	-5.31	-4.50	-6.90**	-6.84**	5.49	7.81	10.09*	9.31*	-3.43	-1.88	-1.51	-1.00
	(-1.58)	(-1.64)	(-2.37)	(-2.39)	(1.07)	(1.49)	(1.79)	(1.73)	(-1.50)	(-0.85)	(-0.63)	(-0.45)
Panel C: SPF disagreement												
Agg. Disp.$_{t-1}$	-2.68	-2.41	-5.13*	-4.88*	6.38	2.51	10.40*	9.41	0.90	-0.10	-4.35	-3.85
	(-1.06)	(-1.09)	(-1.90)	(-1.70)	(1.33)	(0.57)	(1.91)	(1.56)	(0.49)	(-0.05)	(-1.45)	(-1.21)
Panel D: Top-down measure												
Agg. Disp.$_{t-1}$	0.17	0.10	-3.55**	-3.72**	2.41	0.27	5.69*	6.23*	-0.11	-0.10	-1.45	-1.71
	(0.15)	(0.08)	(-2.48)	(-2.52)	(1.17)	(0.12)	(1.72)	(1.87)	(-0.11)	(-0.11)	(-0.77)	(-0.92)

Table IAIV
Disagreement and Concavity of the Security Market Line: Controlling for Stock-Level Disagreement

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market return. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full-sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \Omega_t \times \ln(\text{Disp}_{P,t}) + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio, $\text{Disp}_{P,t}$ is the value-weighted average of the stock-level dispersion in analysts' forecasts for stocks in portfolio P in month t, and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parenthesis. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAIV (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.90*	-5.49*	-9.64***	-9.67***	6.45	11.28**	17.02***	16.62***	-3.17	-2.26	-1.45	-0.51
	(-1.92)	(-1.92)	(-2.94)	(-2.98)	(1.50)	(2.16)	(2.69)	(2.68)	(-1.46)	(-0.85)	(-0.45)	(-0.17)
$R^{(12)}_{m,t}$		-0.16	-0.22*	-0.22		0.99***	1.07***	1.07***		0.29*	0.26*	0.25
		(-1.25)	(-1.65)	(-1.63)		(3.66)	(3.77)	(3.74)		(1.86)	(1.67)	(1.49)
$HML^{(12)}_t$		-0.50**	-0.38**	-0.37**		0.47	0.30	0.32		0.31*	0.28	0.22
		(-2.56)	(-2.17)	(-2.18)		(1.25)	(0.81)	(0.87)		(1.65)	(1.51)	(1.30)
$SMB^{(12)}_t$		0.34	0.52**	0.52**		-0.33	-0.58	-0.58		-0.19	-0.20	-0.20
		(1.39)	(2.33)	(2.32)		(-0.65)	(-1.10)	(-1.11)		(-0.73)	(-0.72)	(-0.75)
$UMD^{(12)}_t$		0.02	0.07	0.07		0.02	-0.06	-0.05		0.06	0.07	0.04
		(0.16)	(0.67)	(0.68)		(0.07)	(-0.24)	(-0.20)		(0.55)	(0.57)	(0.38)
D/P$_{t-1}$			-2.73	-2.79			3.49	2.43			1.76	4.27
			(-1.22)	(-1.07)			(0.71)	(0.46)			(0.60)	(1.35)
Inflation$_{t-1}$			-4.83***	-4.87***			7.00*	6.40			-0.51	0.90
			(-2.96)	(-3.13)			(1.74)	(1.50)			(-0.25)	(0.40)
Ted Spread$_{t-1}$				0.11				1.84				-4.34*
				(0.06)				(0.52)				(-1.88)
Constant	-5.69**	-2.83	-3.42	-3.43	12.95***	2.69	3.47	3.25	3.26	-0.69	-0.40	0.12
	(-2.42)	(-1.16)	(-1.46)	(-1.43)	(2.77)	(0.50)	(0.65)	(0.59)	(1.34)	(-0.24)	(-0.13)	(0.04)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-6.15**	-4.53**	-6.39***	-6.26***	8.91**	9.90**	10.72**	10.06**	-1.77	-1.83	0.86	2.20
	(-2.33)	(-2.35)	(-3.09)	(-3.12)	(2.10)	(2.41)	(2.50)	(2.46)	(-0.93)	(-0.70)	(0.31)	(0.89)
$R^{(12)}_{m,t}$		-0.22**	-0.30***	-0.30***		1.07***	1.23***	1.24***		0.14	0.14	0.12
		(-2.51)	(-3.50)	(-3.39)		(6.48)	(6.73)	(6.40)		(1.23)	(1.30)	(0.99)
$HML^{(12)}_t$		-0.61***	-0.56***	-0.57***		0.77***	0.76***	0.80***		0.41**	0.32*	0.24
		(-4.15)	(-4.80)	(-5.11)		(2.60)	(2.97)	(3.08)		(2.17)	(1.79)	(1.45)
$SMB^{(12)}_t$		-0.03	0.09	0.09		0.62**	0.48	0.48		-0.32*	-0.41**	-0.41**
		(-0.20)	(0.64)	(0.64)		(2.34)	(1.64)	(1.60)		(-1.80)	(-2.10)	(-2.07)
$UMD^{(12)}_t$		-0.09	-0.04	-0.05		0.07	-0.00	0.02		0.03	0.01	-0.02
		(-1.32)	(-0.82)	(-0.89)		(0.51)	(-0.01)	(0.11)		(0.24)	(0.11)	(-0.22)
D/P$_{t-1}$			0.89	1.24			-5.21	-6.96			3.17	6.75***
			(0.47)	(0.52)			(-1.34)	(-1.48)			(1.32)	(2.63)
Inflation$_{t-1}$			-4.64***	-4.45***			7.70**	6.71*			1.48	3.48
			(-3.38)	(-3.04)			(2.32)	(1.79)			(0.69)	(1.52)
Ted Spread$_{t-1}$				-0.61				3.03				-6.19***
				(-0.36)				(0.80)				(-2.68)
Constant	-9.11***	-4.08**	-4.05**	-3.98**	20.11***	6.35	5.66	5.30	0.23	-2.45	-1.87	-1.14
	(-4.77)	(-2.19)	(-2.20)	(-2.09)	(5.04)	(1.46)	(1.27)	(1.14)	(0.10)	(-0.89)	(-0.66)	(-0.39)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAV

Disagreement and Concavity of the Security Market Line: Controlling for Idiosyncratic Volatility

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20 beta-sorted portfolios using the same market model. We then estimate each month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \Omega_t \times \ln(\sigma_{P,t}) + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio, $\sigma_{P,t}$ is the value-weighted median of the idiosyncratic volatility of stocks in portfolio P in month t and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns(1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAV (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-7.02*	-5.26	-8.74**	-8.55**	7.78	9.44	13.51*	12.60*	-6.00*	-3.38	-3.40	-2.58
	(-1.85)	(-1.62)	(-2.36)	(-2.35)	(1.38)	(1.51)	(1.86)	(1.78)	(-1.76)	(-1.05)	(-0.92)	(-0.73)
$R_{m,t}^{(12)}$		-0.11	-0.18	-0.18		0.75***	0.91***	0.93***		0.36**	0.29*	0.28*
		(-0.83)	(-1.30)	(-1.31)		(2.77)	(3.02)	(3.01)		(2.35)	(1.82)	(1.67)
$HML_t^{(12)}$		-0.63***	-0.53***	-0.54***		0.63	0.52	0.57		0.08	0.07	0.02
		(-3.00)	(-2.78)	(-2.91)		(1.48)	(1.27)	(1.43)		(0.37)	(0.33)	(0.11)
$SMB_t^{(12)}$		0.20	0.37	0.37		0.01	-0.24	-0.24		-0.25	-0.20	-0.20
		(0.85)	(1.62)	(1.64)		(0.02)	(-0.44)	(-0.46)		(-0.87)	(-0.68)	(-0.69)
$UMD_t^{(12)}$		-0.02	0.04	0.03		0.14	0.04	0.07		0.01	0.04	0.02
		(-0.13)	(0.35)	(0.31)		(0.55)	(0.17)	(0.27)		(0.11)	(0.33)	(0.17)
D/P_{t-1}			-1.31	-0.81			-1.44	-3.87			2.54	4.72
			(-0.55)	(-0.29)			(-0.29)	(-0.75)			(0.91)	(1.58)
Inflation$_{t-1}$			-5.19***	-4.91***			9.56**	8.20*			-3.02	-1.80
			(-2.98)	(-3.01)			(2.20)	(1.81)			(-1.18)	(-0.64)
Ted Spread$_{t-1}$				-0.87				4.20				-3.77
				(-0.45)				(1.09)				(-1.60)
Constant	-6.61***	-3.30	-3.66	-3.56	13.10***	3.01	3.01	2.51	2.29	-0.98	-0.63	-0.18
	(-2.63)	(-1.39)	(-1.54)	(-1.44)	(2.75)	(0.60)	(0.59)	(0.46)	(0.90)	(-0.34)	(-0.21)	(-0.06)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.68**	-3.82*	-5.68***	-5.50***	6.03	5.38	8.22*	7.05	-1.32	-0.99	0.89	1.92
	(-2.09)	(-1.96)	(-2.65)	(-2.64)	(1.15)	(1.15)	(1.67)	(1.51)	(-0.72)	(-0.46)	(0.37)	(0.90)
$R_{m,t}^{(12)}$		-0.22***	-0.27***	-0.28***		0.77***	0.86***	0.88***		0.20**	0.23**	0.21**
		(-2.63)	(-3.05)	(-2.98)		(4.48)	(4.46)	(4.24)		(2.04)	(2.57)	(2.13)
$HML_t^{(12)}$		-0.67***	-0.62***	-0.63***		0.90***	0.82***	0.88***		0.27*	0.21	0.15
		(-4.67)	(-5.24)	(-5.73)		(3.10)	(3.17)	(3.59)		(1.83)	(1.41)	(1.06)
$SMB_t^{(12)}$		-0.01	0.09	0.09		0.37	0.21	0.21		-0.11	-0.19	-0.18
		(-0.07)	(0.60)	(0.61)		(1.16)	(0.64)	(0.61)		(-0.62)	(-1.00)	(-0.98)
$UMD_t^{(12)}$		-0.08	-0.05	-0.05		0.11	0.05	0.08		0.03	0.01	-0.02
		(-1.12)	(-0.79)	(-0.88)		(0.52)	(0.25)	(0.45)		(0.24)	(0.06)	(-0.21)
D/P_{t-1}			-0.17	0.32			-0.23	-3.33			1.33	4.07*
			(-0.09)	(0.13)			(-0.06)	(-0.75)			(0.59)	(1.84)
Inflation$_{t-1}$			-3.39**	-3.12**			5.75*	4.02			2.07	3.61
			(-2.50)	(-2.23)			(1.72)	(1.06)			(0.93)	(1.51)
Ted Spread$_{t-1}$				-0.84				5.36				-4.74**
				(-0.50)				(1.33)				(-2.31)
Constant	-8.50***	-3.27*	-3.38*	-3.28*	16.31***	4.72	4.83	4.19	1.73	-1.18	-0.89	-0.33
	(-4.24)	(-1.77)	(-1.85)	(-1.76)	(4.33)	(1.16)	(1.19)	(1.01)	(0.85)	(-0.53)	(-0.39)	(-0.14)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAVI
Disagreement and the Slope of the Security Market Line

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price $< \$5$) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and five lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20 beta-sorted portfolios using the same market model. We estimate every month the cross-sectional regression

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \epsilon_{P,t}, \qquad P = 1, ..., 20,$$

where $r_{P,t}^{(12)}$ is the 12-month excess return of the P^{th} beta-sorted portfolio and β_P is the full-sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \pi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_2^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t. \end{cases}$$

Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) add the factor $z \in Z$, where Z contains the k-month excess market return from t to $t+k-1$ and the k-month return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12 months inflation rate in $t-1$; Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAVI (Continued):

Dep. Var:	π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Value-Weighted Portfolios								
Agg. Disp.$_{t-1}$	-6.06**	-0.86	-4.90**	-5.68***	1.80	1.94	6.07***	6.62***
	(-2.11)	(-0.39)	(-2.30)	(-2.66)	(0.70)	(0.89)	(2.79)	(2.98)
$R^{(12)}_{m,t}$		0.60***	0.58***	0.59***		0.40***	0.43***	0.42***
		(5.12)	(6.59)	(6.09)		(3.55)	(4.70)	(4.24)
$HML^{(12)}_t$		-0.61***	-0.48***	-0.44***		0.77***	0.64***	0.60***
		(-3.22)	(-3.03)	(-2.88)		(4.02)	(4.07)	(4.05)
$SMB^{(12)}_t$		0.48**	0.63***	0.62***		-0.44*	-0.60***	-0.59***
		(2.13)	(3.37)	(3.54)		(-1.91)	(-3.23)	(-3.34)
$UMD^{(12)}_t$		-0.00	0.03	0.05		-0.00	-0.04	-0.05
		(-0.03)	(0.31)	(0.58)		(-0.03)	(-0.45)	(-0.65)
D/P$_{t-1}$			-3.92*	-6.01**			3.93*	5.39*
			(-1.79)	(-2.29)			(1.66)	(1.80)
Inflation$_{t-1}$			-3.22*	-4.39**			3.38**	4.20**
			(-1.90)	(-2.12)			(2.16)	(2.31)
Ted Spread$_{t-1}$				3.62*				-2.54
				(1.77)				(-1.28)
Constant	0.87	-2.58	-3.34	-3.77*	8.54***	2.50	3.26*	3.56*
	(0.31)	(-1.16)	(-1.60)	(-1.75)	(3.24)	(1.21)	(1.67)	(1.75)
Panel B: Equal-Weighted Portfolios								
Agg. Disp.$_{t-1}$	-5.11*	-0.46	-4.06**	-4.84***	3.85	2.08	6.27***	7.02***
	(-1.85)	(-0.26)	(-2.47)	(-2.99)	(1.60)	(1.05)	(3.30)	(3.60)
$R^{(12)}_{m,t}$		0.60***	0.58***	0.59***		0.40***	0.43***	0.42***
		(5.18)	(6.59)	(6.14)		(3.30)	(4.54)	(4.12)
$HML^{(12)}_t$		-0.66***	-0.54***	-0.50***		0.96***	0.83***	0.79***
		(-3.97)	(-3.72)	(-3.55)		(5.38)	(5.65)	(5.87)
$SMB^{(12)}_t$		0.70***	0.84***	0.84***		-0.11	-0.28*	-0.28*
		(3.93)	(5.37)	(5.44)		(-0.55)	(-1.70)	(-1.75)
$UMD^{(12)}_t$		-0.11	-0.08	-0.06		0.11	0.07	0.05
		(-0.93)	(-0.78)	(-0.67)		(0.95)	(0.71)	(0.57)
D/P$_{t-1}$			-3.25*	-5.32**			3.41	5.40**
			(-1.81)	(-2.56)			(1.60)	(2.03)
Inflation$_{t-1}$			-3.15*	-4.31**			4.11***	5.23***
			(-1.77)	(-2.16)			(2.62)	(3.13)
Ted Spread$_{t-1}$				3.58**				-3.45*
				(2.13)				(-1.93)
Constant	0.20	-2.61	-3.25*	-3.67*	9.50***	1.56	2.25	2.66
	(0.07)	(-1.37)	(-1.71)	(-1.91)	(3.47)	(0.83)	(1.20)	(1.41)

Table IAVII

Disagreement and Slope of the Security Market Line: Speculative versus Nonspeculative Stocks; Monthly βs

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas are estimated with a market model using monthly returns over the past three calendar years. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full-sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full-sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, nonspeculative}. We estimate each month the following cross-sectional regression, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month:

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln(\sigma_{P,s,t-1}) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the equal-weighted 12-month excess return of portfolio (P, s). We then estimate second-stage regressions in the time-series using OLS and Newey-West (1987) adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{3,s} x_{t-1} + \nu_{t,s}. \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth, where the β are the pre-ranking β computed above. Columns (2) and (6) add controls for the 12-month excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$. Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table VI (Continued):

Dep. Var:	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-6.50*	-3.26	-9.63***	-9.87***	0.29	0.33	1.17	1.13	2.68	4.55	13.08**	13.18**
	(-1.92)	(-1.64)	(-4.83)	(-4.95)	(0.24)	(0.27)	(0.77)	(0.74)	(0.52)	(1.00)	(2.28)	(2.28)
$R_{m,t}^{(12)}$		0.49***	0.46***	0.48***		0.17**	0.11	0.11		1.01***	0.87***	0.86***
		(3.71)	(5.12)	(4.72)		(2.28)	(1.39)	(1.44)		(3.50)	(2.94)	(2.87)
$HML_t^{(12)}$		-0.81***	-0.66***	-0.62***		0.16	0.13	0.13		1.32**	1.09*	1.07*
		(-4.47)	(-4.08)	(-3.57)		(1.17)	(0.84)	(0.87)		(2.58)	(1.91)	(1.84)
$SMB_t^{(12)}$		0.39	0.66***	0.64***		0.12	0.13	0.12		-0.15	-0.40	-0.39
		(1.20)	(2.80)	(2.94)		(0.84)	(0.86)	(0.82)		(-0.22)	(-0.67)	(-0.66)
$UMD_t^{(12)}$		-0.38**	-0.33**	-0.32**		0.14*	0.16**	0.16**		0.73**	0.74**	0.73**
		(-2.11)	(-2.40)	(-2.52)		(1.93)	(2.23)	(2.16)		(2.17)	(2.40)	(2.32)
D/P_{t-1}			-5.80**	-7.77**			3.20*	2.93			15.02**	15.84*
			(-2.12)	(-2.36)			(1.85)	(1.50)			(2.07)	(1.88)
Inflation$_{t-1}$			-6.00***	-7.16***			-1.61	-1.76			0.93	1.41
			(-3.03)	(-3.19)			(-1.24)	(-1.31)			(0.18)	(0.27)
Ted Spread$_{t-1}$				3.80*				0.52				-1.58
				(1.65)				(0.37)				(-0.29)
Constant	-2.93	-1.79	-2.82	-3.25	2.33	-0.94	-0.44	-0.50	18.69***	-0.47	2.00	2.18
	(-0.91)	(-0.91)	(-1.30)	(-1.45)	(1.44)	(-0.54)	(-0.26)	(-0.29)	(2.82)	(-0.08)	(0.31)	(0.33)
Panel B: Non speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ ≤ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-1.66	0.57	-5.18	-5.44	-1.44	-2.04	-1.88	-1.93	-3.65	-4.51	1.08	1.17
	(-0.65)	(0.18)	(-1.54)	(-1.58)	(-0.66)	(-0.99)	(-0.82)	(-0.84)	(-0.50)	(-0.62)	(0.13)	(0.14)
$R_{m,t}^{(12)}$		0.27	0.21	0.23		0.28***	0.31**	0.31**		1.43***	1.56***	1.55***
		(1.32)	(1.17)	(1.23)		(2.70)	(2.56)	(2.50)		(3.53)	(3.61)	(3.46)
$HML_t^{(12)}$		-1.06***	-0.92***	-0.89***		0.02	0.02	0.03		1.31**	1.19**	1.18**
		(-3.18)	(-3.35)	(-3.31)		(0.17)	(0.18)	(0.22)		(2.35)	(2.32)	(2.24)
$SMB_t^{(12)}$		0.76	1.03**	1.00**		0.55**	0.53**	0.52**		0.99	0.69	0.70
		(1.63)	(2.39)	(2.41)		(2.43)	(2.25)	(2.15)		(0.99)	(0.69)	(0.70)
$UMD_t^{(12)}$		-0.05	0.00	0.02		0.02	0.01	0.01		0.17	0.10	0.09
		(-0.23)	(0.00)	(0.12)		(0.14)	(0.06)	(0.10)		(0.39)	(0.23)	(0.22)
D/P_{t-1}			-3.76	-5.95			-0.74	-1.18			1.03	1.77
			(-1.13)	(-1.44)			(-0.27)	(-0.37)			(0.11)	(0.16)
Inflation$_{t-1}$			-6.86**	-8.15**			1.01	0.75			9.24	9.67
			(-2.03)	(-2.05)			(0.62)	(0.43)			(1.34)	(1.27)
Ted Spread$_{t-1}$				4.22				0.84				-1.42
				(1.01)				(0.35)				(-0.15)
Constant	-1.70	-0.45	-1.16	-1.63	3.61	0.23	0.12	0.02	20.30**	0.02	0.31	0.47
	(-0.41)	(-0.12)	(-0.31)	(-0.45)	(1.56)	(0.12)	(0.06)	(0.01)	(2.31)	(0.00)	(0.04)	(0.06)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table IAVIII
Disagreement and Slope of the Security Market Line: Speculative versus Nonspeculative stocks; Other Horizons

Sample Period: 12/1981 to 12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom two deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas are estimated with a market model using monthly returns over the past three calendar years. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and nonspeculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks and 20 beta-sorted portfolios for nonspeculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full-sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We estimate each month the following cross-sectional regression, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, nonspeculative} and t is a month:

$$r_{P,s,t}^{(k)} = \iota_{s,t}^{(k)} + \chi_{s,t}^{(k)} \times \beta_{P,s} + \varrho_{s,t}^{(k)} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon_{P,s,t}^{(k)},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r_{P,s,t}^{(k)}$ is the value-weighted k-month excess return of portfolio (P, s). We then estimate second-stage regressions in the time-series using OLS and Newey-West (1987) adjusted standard errors allowing for $k-1$ lags:

$$\begin{cases} \chi_{s,t}^{(k)} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta_{1,s}^{m} \cdot R_{m,t}^{(k)} + \delta_{1,s}^{HML} \cdot HML_t^{(k)} + \delta_{1,s}^{SMB} \cdot SMB_t^{(k)} + \delta_{1,s}^{UMD} \cdot UMD_t^{(k)} + \sum_{x \in X} \delta_{1,s}^{x} \cdot x_{t-1} + \zeta_{t,s} \\ \pi_{t,s}^{(k)} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta_{2,s}^{m} \cdot R_{m,t}^{(k)} + \delta_{2,s}^{HML} \cdot HML_t^{(k)} + \delta_{2,s}^{SMB} \cdot SMB_t^{(k)} + \delta_{2,s}^{UMD} \cdot UMD_t^{(k)} + \sum_{x \in X} \delta_{2,s}^{x} \cdot x_{t-1} + \omega_{t,s} \\ \kappa_{t,s}^{(k)} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta_{3,s}^{m} \cdot R_{m,t}^{(k)} + \delta_{3,s}^{HML} \cdot HML_t^{(k)} + \delta_{3,s}^{SMB} \cdot SMB_t^{(k)} + \delta_{3,s}^{UMD} \cdot UMD_t^{(k)} + \sum_{x \in X} \delta_{3,s}^{x} x_{t-1} + \nu_{t,s}. \end{cases}$$

Panel A use k=1 months, Panel B uses k=3 month, Panel C uses k=6 month, Panel D uses k=18 month. Columns (1) and (5) control for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement measured as the standard deviation of analysts' forecasts of long-run EPS growth. Columns (2) and (6) control for the k-month excess market return from t to $t+k-1$, and the k-month return on HML, SMB, and UMD from t to $t+k-1$; Columns (3) and (7) add controls for the aggregate dividend/price ratio in $t-1$ and the past-12-month inflation rate in $t-1$; Columns (4) and (8) additionally control for the TED spread in month $t-1$. t-statistics are in parentheses. *, **, and *** indicates statistically different from zero at 10%, 5% and 1% level of significance, respectively.

Table IAVIII (Continued):

Dep. Var:	$\chi_{s,t}^{(k)}$				$\varrho_{s,t}^{(k)}$				$\iota_t^{(k)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: k=1 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-1.06	-0.74	-0.78	-0.99*	0.49	0.55	0.31	0.42	0.54	0.57**	0.75**	0.89***
	(-1.39)	(-1.63)	(-1.55)	(-1.95)	(1.27)	(1.47)	(0.73)	(0.99)	(1.31)	(2.06)	(2.43)	(2.85)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	0.15	0.48*	-0.00	-0.01	-0.57	-0.68**	-0.34	-0.41	0.39	0.34	0.53*	0.64**
	(0.47)	(1.81)	(-0.01)	(-0.05)	(-1.23)	(-2.59)	(-1.07)	(-1.25)	(0.80)	(1.19)	(1.67)	(1.97)
N	396	396	396	396	396	396	396	396	396	396	396	396
Panel B: k=3 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-3.62**	-2.34**	-3.02***	-3.35***	1.51*	1.39**	1.38*	1.25	1.77*	1.74**	2.48***	2.81***
	(-2.09)	(-2.53)	(-3.04)	(-3.38)	(1.78)	(2.06)	(1.76)	(1.55)	(1.75)	(2.49)	(3.35)	(3.81)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	-0.26	0.78	-0.71	-0.57	-0.80	-0.88	0.04	-0.20	1.20	0.94	1.63**	1.72**
	(-0.42)	(1.20)	(-0.94)	(-0.78)	(-0.68)	(-1.14)	(0.05)	(-0.24)	(1.12)	(1.41)	(2.20)	(2.40)
N	394	394	394	394	394	394	394	394	394	394	394	394
Panel C: k=6 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-7.59**	-4.19**	-5.63***	-6.31***	2.53	1.70	1.28	1.04	3.55*	3.45***	5.15***	5.85***
	(-2.39)	(-2.48)	(-3.04)	(-3.49)	(1.65)	(1.39)	(0.82)	(0.67)	(1.88)	(2.64)	(3.73)	(4.36)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	-0.07	2.24	-0.96	-1.18	-2.91	-2.69**	-1.10	-1.30	2.42	1.84	3.63***	3.97***
	(-0.06)	(1.60)	(-0.64)	(-0.85)	(-1.43)	(-2.25)	(-0.82)	(-0.99)	(1.21)	(1.50)	(2.81)	(3.26)
N	391	391	391	391	391	391	391	391	391	391	391	391
Panel D: k=18 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-12.60**	-11.07***	-14.91***	-15.69***	2.90	4.42*	3.76	3.35	6.02*	8.78***	12.89***	13.49***
	(-2.35)	(-3.08)	(-3.27)	(-3.39)	(1.10)	(1.77)	(1.33)	(1.17)	(1.89)	(2.93)	(3.45)	(3.52)
Nonspeculative stocks												
Agg. Disp.$_{t-1}$	0.04	0.94	-5.59**	-6.01**	-4.21	-5.59*	-3.57	-3.98	3.05	5.37**	9.23***	9.49***
	(0.02)	(0.30)	(-2.09)	(-2.29)	(-1.12)	(-1.91)	(-1.02)	(-1.14)	(1.06)	(2.01)	(3.33)	(3.32)
N	379	379	379	379	379	379	379	379	379	379	379	379

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