

Solution to Problem Set 3

Problem 3.1: Glosten-Milgrom Model

- a. Less-than-rational traders and liquidity traders.

Less-than-rational traders may trade because they are overconfident. For example, the trader believes she has relevant information but in reality she doesn't or the information is already incorporated in the price.

A liquidity sell order may come from an individual who buys a house and needs to liquidate her stock portfolio; or from an individual who takes a job at company ABC and sells her stocks of ABC to diversify.

- b. Profit of 10 cents. The situation is symmetric for buy and sell orders. Consider for instance a market sell order executed against a limit buy order at the bid price €49.90. Since the order is uninformed, the price will remain €50 on average, so the market maker's average profit is $50 - 49.90 = +0.10$.
- c. Loss of 20 cents. Again, the situation is symmetric for buys and sells. Consider a market sell order. Since the order is informed, the price will subsequently decrease to €49.70 on average, so the market maker's average profit is $49.70 - 49.90 = -0.20$.
- d. Consider the case of limit sell orders. Denote the ask price by A . The average profit from trading against an uninformed buy market order is $A - 50$. The average profit from trading against an informed buy market order is $A - 50.30$. Therefore, the average profit over informed and uninformed orders when the fraction of informed orders is z , is equal to:

$$z \times (A - 50.30) + (1 - z) \times (A - 50) = A - 50 - 0.30 \times z$$

Using the fact that competition between market makers bring this average profit per trade to 0.05, we obtain that the ask price is $A = 50.05 + 0.30 \times z$

Following the same reasoning, the bid price is $B = 49.95 - 0.30 \times z$

NB: The bid-ask spread is $A - B = 0.10 + 0.60 \times z$ and is equal to 70 cents when there are no noise traders ($z = 1$).

- e. Informed traders create adverse selection for market makers. When the fraction of informed traders is higher, the fraction of losing trades for market makers is higher, therefore market makers must charge a higher bid-ask spread to recoup these losses.

f. Noise traders make no trading profits but incur transaction costs coming from the bid-ask spread: each trade costs half the spread. The bid-ask spread is an increasing function of z . Therefore, noise traders suffer from the presence of informed traders in the form of higher transaction costs.

g. Using the value for A from question d, the profit of an informed buy order is:

$$50.30 - A = 0.25 - 0.30 \times z$$

The profit of an informed sell order is:

$$B - 49.70 = 0.25 - 0.30 \times z$$

h. Informed traders incur transaction costs coming from the bid-ask spread, which reduces their trading profits. The bid-ask spread is an increasing function of z . Therefore, informed traders suffer from the presence of other informed traders in the form of higher transaction costs.

Problem 3.2: Payment For Order Flow

- a. RobinDesBois users are uninformed, so the market maker can earn the bid-ask spread on their orders without facing adverse selection (i.e., without incurring losses due to trading against an informed investor).
- b. The profit per uninformed trade is equal to half the bid-ask spread, i.e., 20 cents. The market maker is ready to pay up to 20 cents per order to execute RobinDesBois users' orders.
- c. 12 cents for RobinDesBois and $20 - 12 = 8$ cents for the market maker.
- d. It is not free in the sense that users still incur the bid-ask spread: they buy at €50.20 and sell at €49.80. However, it is free in the sense that they don't incur any additional costs on top of the bid-ask spread, which all investors (even institutional investors) face.

Problem 3.3: Price Impact and the Value of Information

- a. The fund can buy 20k shares at €50.20, 20k shares at €50.30, and 20k shares at €50.40. After that, the ask is €50.50 and buying is no longer profitable. The trading profit is:

$$20k \times 0.30 + 20k \times 0.20 + 20k \times 0.10 = 12 \text{ k€}$$
- b. After each fund has purchased 5k shares, the ask moves to €50.30, and so on. Therefore, each fund can buy 5k shares at €50.20, 5k shares at €50.30, and 5k shares at €50.40. The trading profit is 3 k€ for each fund.

- c. We observe from answers a. and b. that the total trading profit generated by this piece of data is 12k€, shared between the number of funds which trade on this information. Therefore, you can sell the data for up to 12k€ to one fund, or 6k€ to two funds, or 4k€ to three funds, and so on. In any case, you can earn up to 12k€ in total and your profit does not depend on the number of funds to which you sell the data. The unit price of the data is inversely proportional to the number of buyers.
- d. A reason that could make it profitable to sell to very few funds is that hedge funds try to trade slowly not to be detected by market makers, because market makers will adjust their quotes if they realize that an informed investor is trading. If many hedge funds trade on the same information at the same time, they may be detected more quickly by market makers, reducing the overall profit that can be earned out of a given piece of information.¹

A reason that could make it more profitable to sell to several funds is that different funds may use the same piece of information in different markets. For example, if you have information about oil prices, a commodity fund may trade oil ETFs while an equity fund may trade stocks of oil majors. In this case, higher trading profits may be generated out of the same piece of information.²

¹This logic also highlights a difference between investment firms' internal research vs. research purchased from market intelligence firms. The former may be more unique whereas the latter may be sold to competing investment firms.

²This insight has an implication for market makers. A market maker in one market should also monitor the order flow in other markets because the order flow in other markets can contain information about the first market.