Intertemporal Choices and Markets

Part 2: Investment

Johan Hombert (HEC Paris)

Questions Today

How does investment shape and is shaped by economic fundamentals and financial conditions?

Road Map

Production Economy Without Uncertainty

Evidence

Production Economy

- Exchange economy + agents have access to a production technology
  - Invest k units of good at time 0 $\rightarrow$ generate $f(k)$ units at time 1
  - $f(.)$ is the production function: increasing and concave (decreasing return to scale)
- Some agents have better technologies than others: $f_i(.) = z_i f(.)$

Equilibrium Definition

- Equilibrium: all agents maximize and markets clear
- Agent i chooses $(c_{i0}, c_{i1}, k_i, b_i)$ to max $u(c_{i0}) + \beta u(c_{i1})$ subject to

  time 0 budget constraint: $c_{i0} + k_i + b_i \leq e_{i0}$

  time 1 budget constraint: $c_{i1} \leq e_{i1} + z_i f(k_i) + (1 + r) b_i$
- Market clearing for bond: $\sum_i b_i = 0$

  Market clearing for good at time 0: $\sum_i c_{i0} + \sum_i k_i = \sum_i e_{i0}$

  Market clearing for good at time 1: $\sum_i c_{i1} = \sum_i e_{i1} + \sum_i z_i f(k_i)$

Building the Intuition

Q1. Agents with high time 0 endowment e_{i0} tend to:

a. invest more at time 0 (higher k_i)

b. lend more at time 0 (higher b_i)

c. both

Building the Intuition

Q1. Agents with high time 0 endowment e_{i0} tend to:

a. invest more at time 0 (higher k_i) no

b. lend more at time 0 (higher b_i) yes

c. both

Building the Intuition

Q1. Agents with high time 0 endowment e_{i0} tend to:

a. invest more at time 0 (higher k_i) no

b. lend more at time 0 (higher b_i) yes

c. both

Q2. Agents with high productivity z_i tend to:

a. invest more at time 0

b. lend more at time 0

c. borrow more at time 0

d. consume less at time 0

Building the Intuition

Q1. Agents with high time 0 endowment e_{i0} tend to:

a. invest more at time 0 (higher k_i) no

b. lend more at time 0 (higher b_i) yes

c. both

Q2. Agents with high productivity z_i tend to:

a. invest more at time 0 yes

b. lend more at time 0 no

c. borrow more at time 0 yes

d. consume less at time 0 no

Solving for Equilibrium

Step 1: Determine agents' choice given r

Step 2: Clear markets to determine r

Step 1: Agents' Choices

- Consolidate budget constraint to eliminate b_i

$$\max_{c_{i0}, c_{i1}, k_i} \; u(c_{i0}) + \beta u(c_{i1})$$

$$\text{s.t. } c_{i0} + \frac{c_{i1}}{1+r} \leq e_{i0} + \frac{e_{i1}}{1+r} + \left(-k_i + \frac{z_i f(k_i)}{1+r} \right)$$

- FOCs w.r.t. c_{i0} and c_{i1}: $u'(c_{i0}) = (1+r)\beta u'(c_{i1})$
  - Euler equation: same as in the exchange economy
- FOC w.r.t. k_i: $z_i f'(k_i) = 1 + r$
  - Left: marginal productivity of capital
  - Right: marginal cost of capital

Investment

$$\Rightarrow k_i = f'^{-1}\left(\frac{1+r}{z_i}\right)$$

- k_i is increasing in z_i
  - Proof: $f(.)$ is concave, so $f'^{-1}(.)$ is decreasing
  - More productive agents invest more
- k_i is decreasing in r
  - Higher cost of capital lowers investment
- k_i does not depend on e_{i0} and e_{i1}
  - Investment depends on productivity not wealth
- Cobb-Douglas production function: $f(k) = k^{\alpha}$, $\alpha \in (0,1)$

$$\Rightarrow k_i = \left(\frac{\alpha z_i}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Step 2: Market Clearing

- CRRA utility + Cobb-Douglas production function
- Market clearing for good

$$\text{at time 0: } \sum_i c_{i0} = \sum_i e_{i0} - \sum_i \left(\frac{\alpha z_i}{1+r} \right)^{\frac{1}{1-\alpha}} \tag{C0}$$

$$\text{at time 1: } \sum_i c_{i1} = \sum_i e_{i1} + \sum_i \left(\frac{\alpha}{1+r} \right)^{\frac{\alpha}{1-\alpha}} z_i^{\frac{1}{1-\alpha}} \tag{C1}$$

- Sum the Euler equation over agents and rearrange

$$1 + r = \beta^{-1} \left(\frac{\sum_i c_{i1}}{\sum_i c_{i0}} \right)^{\gamma} \tag{FOC}$$

- Substitute agg consump in (FOC) using (C0) and (C1) $\rightarrow$ unique solution $r \rightarrow$ determine $k_i \rightarrow$ determine c_{i0} and c_{i1}

Road Map

Production Economy Without Uncertainty

Evidence

Predictions of Complete Markets

- Prediction 1: More productive agents invest more

  With Cobb-Douglas production function, the cross-sectional correlation between log investment and log productivity is $Cor(\log(k_i), \log(z_i)) = 1$

- Prediction 2: ... up to the point where the marginal product of capital is equalized across agents

  The cross-sectional dispersion of marginal product of capital is $Var(z_i f'(k_i)) = 0$

- Prediction 3: Investment does not depend on wealth

Q. How would you test predictions 1 and 2?

Estimating Productivity

- Estimating productivity is challenging
- How to recover z_i in $y_i = z_i k_i^{\alpha}$ using data on production y_i and capital k_i?
- Note that $\log(y_i) = \log(z_i) + \alpha \log(k_i)$ (1)

Q. Does OLS regression $\log(y_i) = b_0 + b_1 \log(k_i) + \varepsilon_i$ in the cross-section of firms identify $\alpha = \hat{b}_1^{ols}$ and $\log(z_i) = \hat{b}_0^{ols} + \hat{\varepsilon}_i^{ols}$?

Estimating Productivity

- Estimating productivity is challenging
- How to recover z_i in $y_i = z_i k_i^{\alpha}$ using data on production y_i and capital k_i?
- Note that $\log(y_i) = \log(z_i) + \alpha \log(k_i)$ (1)

Q. Does OLS regression $\log(y_i) = b_0 + b_1 \log(k_i) + \varepsilon_i$ in the cross-section of firms identify $\alpha = \hat{b}_1^{ols}$ and $\log(z_i) = \hat{b}_0^{ols} + \hat{\varepsilon}_i^{ols}$?

- No, because k_i is correlated with z_i (simultaneity bias)

FOC w.r.t. k_i: $\log(k_i) = \frac{1}{1-\alpha} \log\left(\frac{\alpha}{1+r}\right) + \frac{1}{1-\alpha} \log(z_i)$ (2)

Substitute $\log(z_i)$ in (1) using (2): $\log(y_i) = \log\left(\frac{1+r}{\alpha}\right) + \log(k_i)$

$\hat{b}_1^{ols} = 1 \Rightarrow$ OLS over-estimate α

$\hat{b}_0^{ols} + \hat{\varepsilon}_i^{ols} = cste \Rightarrow$ OLS under-estimate dispersion in z_i

Estimating Productivity

- Other challenges:
  - Mis-measurement in y_i and k_i $\Rightarrow$ inflate error term $\Rightarrow$ over-estimate dispersion in z_i
  - Mis-specification of production function (not Cobb-Douglas)

- Important for debates on misallocation and on rise of market power[1]

[1] De Loecker and Syverson 2021. An Industrial Organization Perspective on Productivity. *Handbook of Industrial Organization* [pdf]

Investment and Productivity

- Empirically, the correlation between firm productivity (z_i) and firm size (k_i) is:

  1. Positive
  2. Below 1
  3. Higher in US than in developing countries

  Interpretation?

Investment and Productivity

- Empirically, the correlation between firm productivity (z_i) and firm size (k_i) is:

  1. Positive
  2. Below 1
  3. Higher in US than in developing countries

  Interpretation?

- Empirically, the distribution of marginal productivity ($z_i f'(k_i)$) exhibit:

  4. Significant dispersion
  5. Lower dispersion in US than in developing countries

  Interpretation?

Investment and Productivity

- Dispersion in marginal productivity suggests misallocation of capital
- Methodological challenge: estimation error in z_i and other mismeasurement/misspecification $\Rightarrow$ overstate dispersion in marginal productivity
- Research agenda: what causes capital misallocation?[2]

[2] Restuccia & Rogerson 2017. The Causes and Costs of Misallocation. *Journal of Economic Perspectives* [pdf]

Investment and Wealth

Q. How would you test if investment depends on wealth?

Investment and Wealth

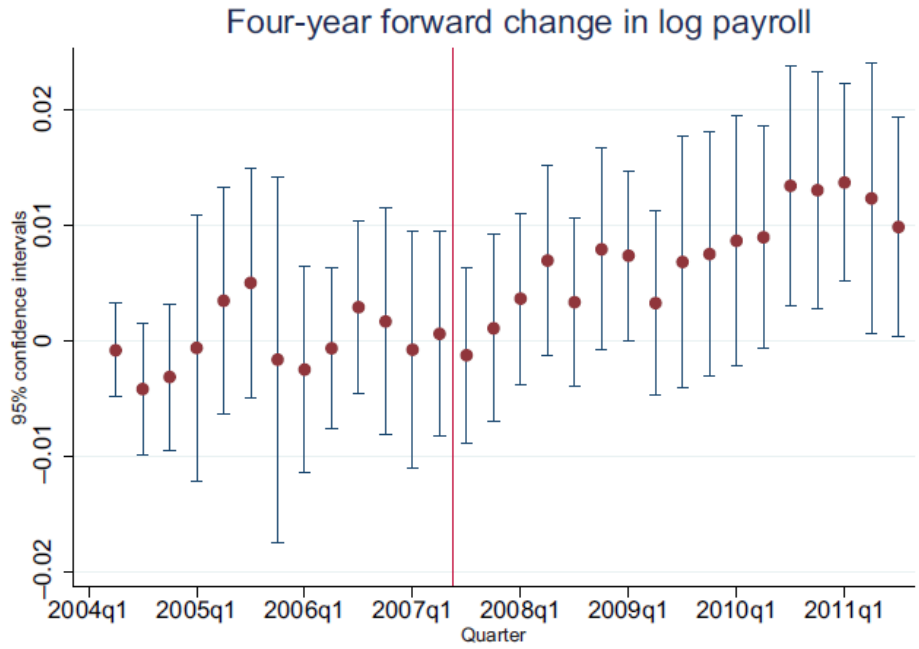
Q. How would you test if investment depends on wealth?

- Identification challenge: wealth is correlated with productivity

- Idea: investment response to cash windfall exogenous to productivity

Investment Response to Cash Windfall

- Barrot and Nanda 2020. The Employment Effects of Faster Payment: Evidence from the Federal Quickpay Reform. *Journal of Finance* [pdf]
  - Faster payment to small business gov't contractors: $e_{i0} \uparrow$ but no change in intertemporal income
  - Firm size $\uparrow$ after the reform



$\Rightarrow$ Investment depends on cash, inconsistent with complete markets

Misallocation

- Both deviations from complete markets are related: if cash affects investment, marginal productivity isn't equalized across firms
- Example
  - Economy with two firms $i = A, B$, which have wealth e_i at time 0, invest $k_i \in [0, 1]$ at time 0, produce $z_i k_i$ at time 1, and consume at time 1
  - A is rich and unproductive: $e_A = 1$, $z_A = 0.5$
  - B is poor and productive: $e_B = 0$, $z_B = 1.5$
  - Suppose no external financing: $k_A = e_A = 1$ and $k_B = e_B = 0$

Q1. How could productive efficiency be improved?

Q2. Which arrangement could implement this improvement?

Q3. What may prevent this from happening?

Misallocation

- Both deviations from complete markets are related: if cash affects investment, marginal productivity isn't equalized across firms
- Example
  - Economy with two firms $i = A, B$, which have wealth e_i at time 0, invest $k_i \in [0, 1]$ at time 0, produce $z_i k_i$ at time 1, and consume at time 1
  - A is rich and unproductive: $e_A = 1$, $z_A = 0.5$
  - B is poor and productive: $e_B = 0$, $z_B = 1.5$
  - Suppose no external financing: $k_A = e_A = 1$ and $k_B = e_B = 0$

Q1. How could productive efficiency be improved? $k_A = 0$ and $k_B = 1$

Q2. Which arrangement could implement this improvement?

Q3. What may prevent this from happening?

Misallocation

- Both deviations from complete markets are related: if cash affects investment, marginal productivity isn't equalized across firms
- Example
  - Economy with two firms $i = A, B$, which have wealth e_i at time 0, invest $k_i \in [0, 1]$ at time 0, produce $z_i k_i$ at time 1, and consume at time 1
  - A is rich and unproductive: $e_A = 1$, $z_A = 0.5$
  - B is poor and productive: $e_B = 0$, $z_B = 1.5$
  - Suppose no external financing: $k_A = e_A = 1$ and $k_B = e_B = 0$

Q1. How could productive efficiency be improved? $k_A = 0$ and $k_B = 1$

Q2. Which arrangement could implement this improvement? A lends to B

Q3. What may prevent this from happening?

Misallocation

- Both deviations from complete markets are related: if cash affects investment, marginal productivity isn't equalized across firms
- Example
  - Economy with two firms $i = A, B$, which have wealth e_i at time 0, invest $k_i \in [0, 1]$ at time 0, produce $z_i k_i$ at time 1, and consume at time 1
  - A is rich and unproductive: $e_A = 1$, $z_A = 0.5$
  - B is poor and productive: $e_B = 0$, $z_B = 1.5$
  - Suppose no external financing: $k_A = e_A = 1$ and $k_B = e_B = 0$

Q1. How could productive efficiency be improved? $k_A = 0$ and $k_B = 1$

Q2. Which arrangement could implement this improvement? A lends to B

Q3. What may prevent this from happening? borrowing constraints

Bonus Slides

Borrowing Constraints

- Borrowing constraints may arise from moral hazard[3]

- Idea: if B borrows from A, B may be tempted to take actions that reduce repayment to A. Anticipating this, A refuses to lend

[3] Tirole, *The Theory of Corporate Finance,* chapter 3

Borrowing Constraints

- Suppose, after investing $k_B = 1$, B can choose to
  - "behave", i.e., run the firm as expected

    $\Rightarrow$ produce $\begin{cases} 2z_B & \text{with probability } 0.5 \\ 0 & \text{with probability } 0.5 \end{cases}$
  - or "shirk", e.g., engage in side projects

    $\Rightarrow$ produce 0 for sure, but B enjoys private benefits b
- Suppose
  - risk neutrality over time 1 consumption
  - $z_B > z_A > b$: B invests & behaves $>$ A invests $>$ B invests & shirks
- When B borrows from A and invests, B's output ($2z_B$ or 0) is verifiable (e.g., by a court) but its action (behave or shirk) is not

Q. Can B borrow from A?

Borrowing Constraints

- Let's try to design a financing contract that both parties agree to
  - A lends 1 to B. B invests
  - B repays $\begin{cases} D \leq 2z_B & \text{if production succeeds} \\ 0 & \text{if production fails} \end{cases}$
- A agrees to this contract if and only if she obtains more than if she invests in her own firm:

  (1) $\quad 0.5D \geq z_A$

- ...which also requires that B has incentives to behave:

  (2) $\quad 0.5(2z_B - D) \geq b$

- (1) and (2) imply: $\quad z_B \geq b + z_A$

Borrowing Constraints

- If $z_B \geq b + z_A$
  - There exists an incentive-compatible financing arrangement
  - First-best is achieved
  - This happens when temptation to shirk (b) is mild and B is sufficiently productive relative to A
- If $z_B < b + z_A$
  - There is no incentive-compatible financing arrangement
  - Investment is determined by wealth not productivity. Capital is misallocated

Borrowing Constraints

- Non-verifiability of action is called moral hazard
- Moral hazard is a type of asymmetric information (the other type is adverse selection)
- Asymmetric information leads to deviations from First Welfare Theorem. See Micro 2 course in second semester

Research Areas

- Corporate finance: credit frictions, implications for corporate policies and real effects (innovation, competition, employment)
- Financial intermediaries (banks, PE, etc.), credit cycles, banking crises
- Macro-finance: implications of credit frictions for business cycle, development, productivity, inequality, etc.
- See M2 courses!