Introduction to Stochastic Actor-Oriented Models

Fundamentals of SAOMs

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Preamble

- All material is on the workshop repository https://github.com/johankoskinen/CHDH-SNA
  - Download the RMarkdown file CHDH-SNA-3.Rmd
- In order to run the Markdown you need
  - The R-package
  - The RStudio interface
- We will predominantly use the packages
  - sna
  - network
  - RSiena

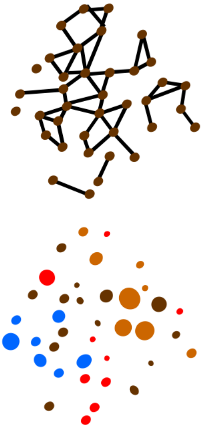
Outline of workshops

1. (Basic) Introduction to SAOM (Thursday PM)
    - SAOM as an agent-based model
    - How to estimate a SAOM
2. (Social Influence) Analysing social influence with SAOM (Friday AM)
    - Accounting for nodal attributes
    - Modelling change of nodal attributes
    - Trouble shooting and dealing with common issues
3. Advanced topics in SAOM (Friday PM)
    - Even more types of data
    - Likelihood-based estimation
    - Settings and imperfect data
    - Modelling multiple parallel networks

The two basic types of data

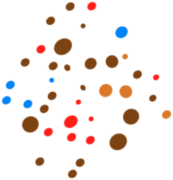
NETWORK
nodes: Andras, Per, Zsofia
have **ties:** Andras $\rightarrow$ Per

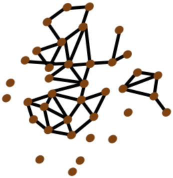
BEHAVIOUR
attributes of nodes: Andras, Per, Zsofia drink
Zsofia does not smoke



SAOM: longitudinal modelling

We have **observations** on NETWORK and BEHAVIOUR





At some fixed points in time

starting at t_0

followed by t_1

$t_0 < t_1$

inferential task: explain how t_0 change into t_1

SAOM: longitudinal modelling

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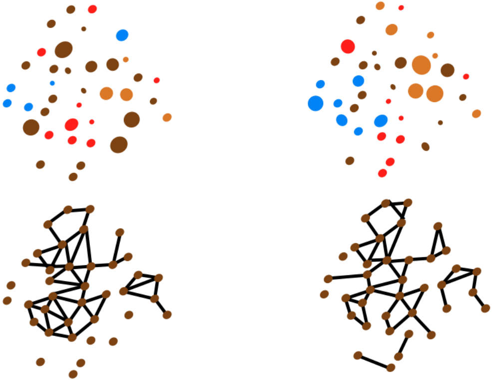

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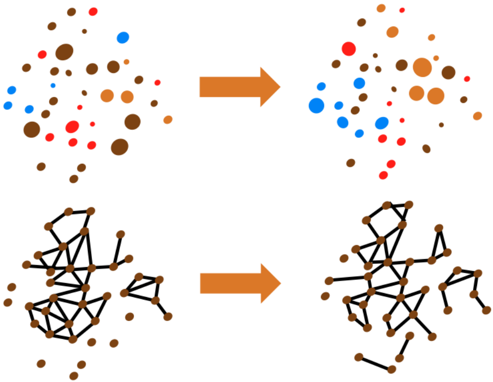

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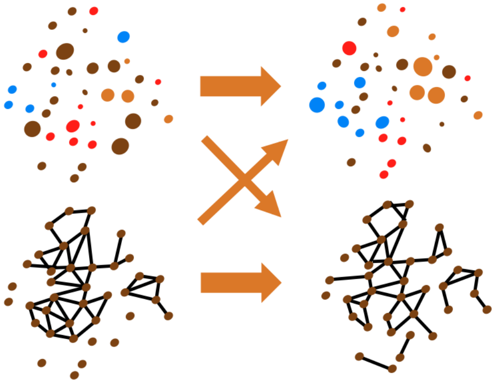
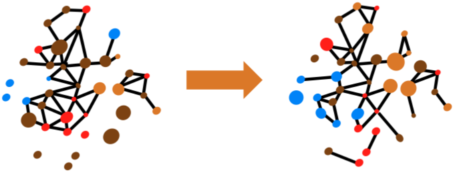


Especially the **co-evolution**:

selection
influence

inferential task: explain how t_0 change into t_1

We have **observations** on NETWORK and BEHAVIOUR



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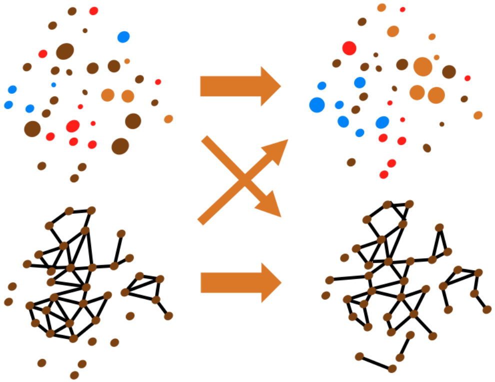

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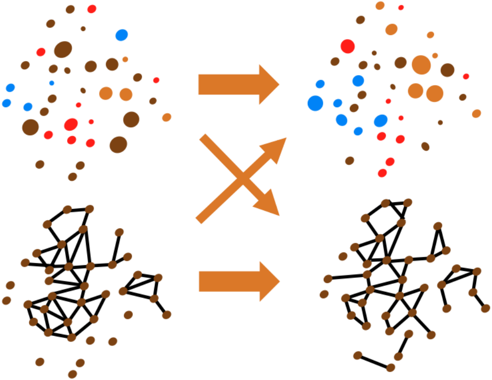
The SAO Model

We have **observations** on NETWORK and BEHAVIOUR



Especially the **co-evolution**:

selection

influence

HOW?

inferential task: explain how t_0 change into t_1

The SAO Model

Assume **PARTIAL observations** on a process

observations:
at t_0
and t_1

the rest:
missing

the process explains how t_0 change into t_1

The SAO Model

Assume **PARTIAL observations** on a process

observations:
at t_0
and t_1

the rest:
missing

the process explains how t_0 change into t_1

What type of data do we want to explain: adjacency matrix

Data represented as adjacency matrices

$$\mathbf{x} = \begin{pmatrix} . & 0 & 0 & 0 & 1 \\ 1 & . & 0 & 0 & 0 \\ 1 & 1 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 1 & 1 & . \end{pmatrix}$$

where $x_{ij} = 1$ or 0 according to wether $i \to j$ or not.

What type of data do we want to explain: longitudinal

Data represented as adjacency matrices
where elements change

$$x(t_0) = \begin{pmatrix} . & 0 & 0 & 0 & 1 \\ 1 & . & 0 & 0 & 0 \\ 1 & 1 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 1 & 1 & . \end{pmatrix}$$

What type of data do we want to explain

Data represented as adjacency matrices
where elements change

$$x(t_1) = \begin{pmatrix} . & 1 & 0 & 0 & 1 \\ 1 & . & 0 & 0 & 0 \\ 1 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 1 & 0 & 1 & 1 & . \end{pmatrix}$$

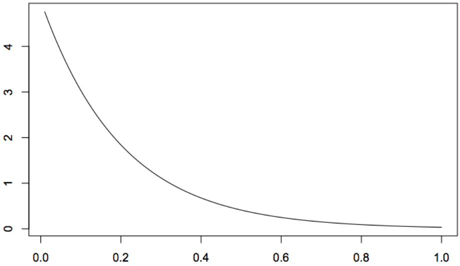
What type of data do we want to explain

Data represented as adjacency matrices
where elements change

$$x(t_2) = \begin{pmatrix} . & 1 & 0 & 1 & 1 \\ 1 & . & 0 & 0 & 1 \\ 1 & 1 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 1 & 0 & 0 & 1 & . \end{pmatrix}$$

SAOM: the rate of change

At random points in time, at rates λ_i



nodes/individuals/actors are given opportunities to change

SAOM: the *direction* of change

Conditional on an actor having an opportunity for change
the probability for each outcome

- is modelled like multinomial logistic regression
- reflects the attractiveness of the outcome to the actor

Micro-step

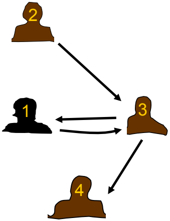
When actor i has opportunity to change
They may toggle x_{ij} to $1 - x_{ij}$
We call the new network

$$X(i \rightsquigarrow j)$$

the network X that *differs* in exactly **one** tie-variable x_{ij}

$x =$

-	0	1	0
0	-	1	0
1	0	-	1
0	0	0	-



$x =$

-	0	1	0
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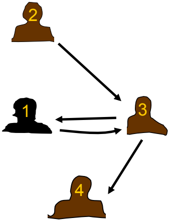
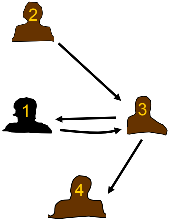
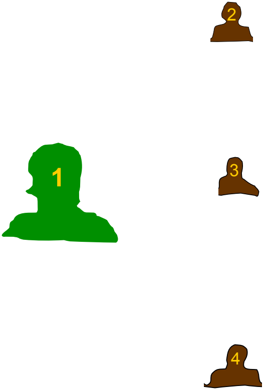


$$x = \begin{array}{|c|c|c|c|} \hline - & 0 & 1 & 0 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$





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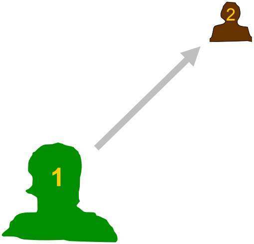

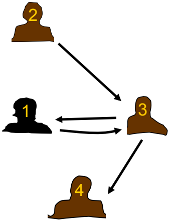

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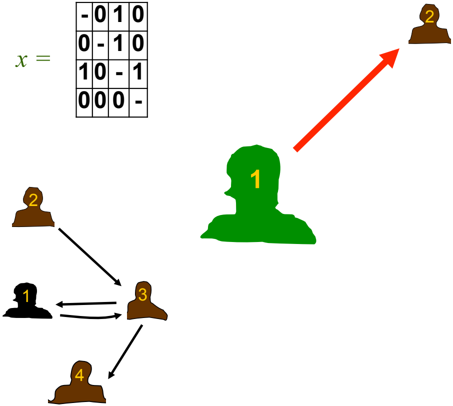

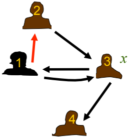

$$x\,(1\to 2) = \begin{array}{|c|c|c|c|}\hline - & 1 & 1 & 0 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$

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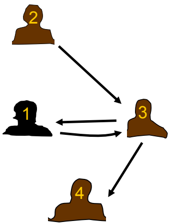

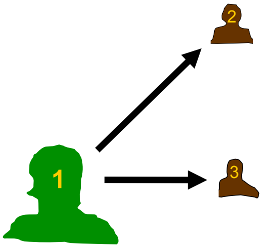

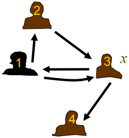

$$x\,(1 \to 2) = \begin{array}{|c|c|c|c|}\hline - & \color{red}{1} & 1 & 0 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$

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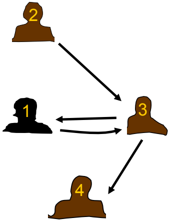

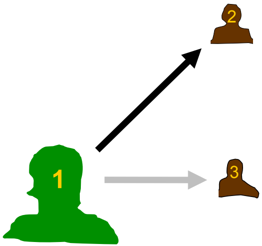

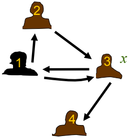

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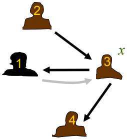
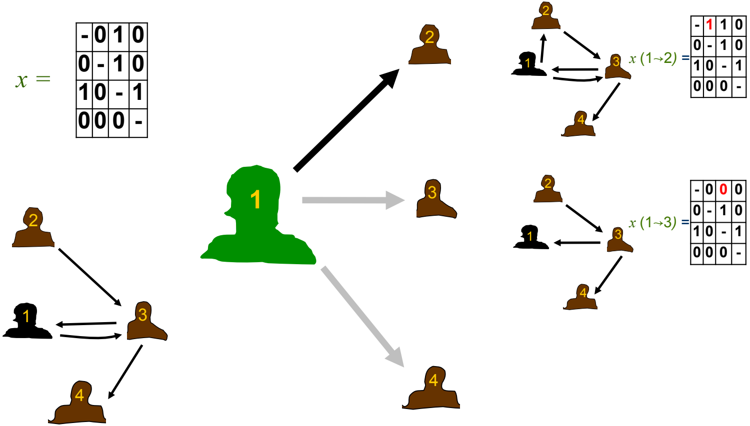


$$x\,(1 \to 3) = \begin{array}{|c|c|c|c|} \hline - & 0 & 0 & 0 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$

$$x = \begin{bmatrix} - & 0 & 1 & 0 \\ 0 & - & 1 & 0 \\ 1 & 0 & - & 1 \\ 0 & 0 & 0 & - \end{bmatrix}$$



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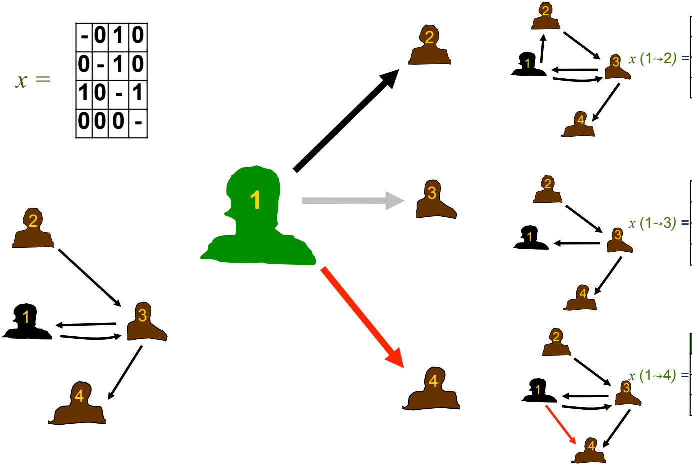

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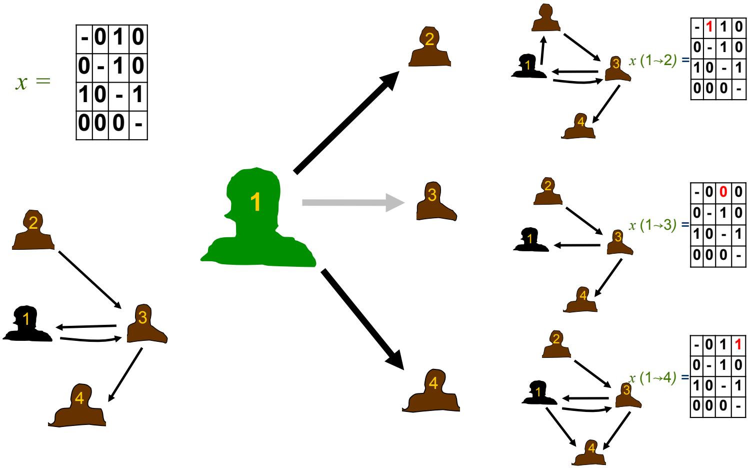
$$x\,(1\to 4) = \begin{array}{|c|c|c|c|}\hline - & 0 & 1 & 1 \\ \hline 0 & - & 1 & 0 \\ \hline 1 & 0 & - & 1 \\ \hline 0 & 0 & 0 & - \\ \hline \end{array}$$



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Probability of a change

Of the three changes (for $j = 2, 3, 4$) available to i (here 1) the probability that i toggles the tie $i \to j$ is given by

One-step jump probability

$$p_{ij}(\beta, \underbrace{X}_{\text{current network}}) = \frac{\exp\left(f_i(\beta, \underbrace{X(i \rightsquigarrow j)}_{\text{new network}})\right)}{\underbrace{\sum_{h=1}^{n} \exp\left(f_i(\beta, X(i \rightsquigarrow h))\right)}_{\text{all possible changes}}},$$

where

- $X(i \rightsquigarrow j)$ is the network resulting from the change
- β are statistical parameters
- f_i describes the attractiveness of $X(i \rightsquigarrow j)$ to i

Probability of a change: utility

One-step jump probability: *can* be derived as:

- Each network X has **untility** $U_i(X,t)$ for i
- Actor i chooses network X that maximises $U_i(X,t)$

If (random) utility has form

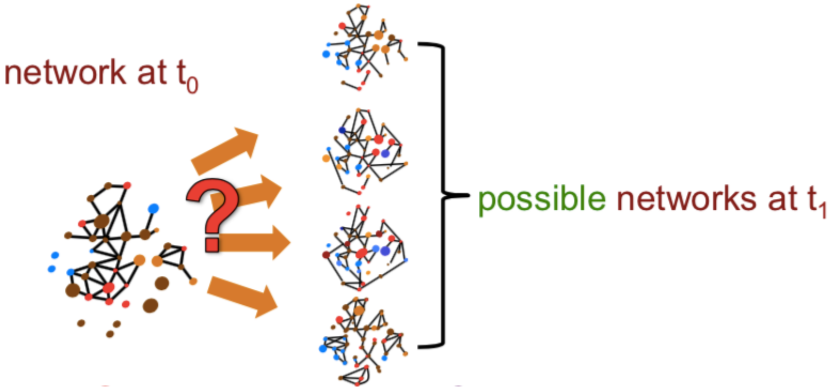
$$U_i(X,t) = \underbrace{f_i(\beta, X)}_{\text{objective function}} \quad + \quad \epsilon_{it} \,.$$

$$\Uparrow$$

random component

Actors are (*myopically*) maximising the utility of their network ties

Agent-based: Change driven by incremental updates



van de Bunt data set: R

Markdown: https://raw.githubusercontent.com/johankoskinen/CHDH-SNA/main/Markdowns/CHDH-SNA-2.Rmd

```
library('RSiena')
library('network')
library('sna')
tmp3[is.na(tmp3)] <- 0 # remove missing
tmp4[is.na(tmp4)] <- 0 # remove missing
par(mfrow = c(1,2))
coordin <- plot(as.network(tmp3))
plot(as.network(tmp4),coord=coordin)
```

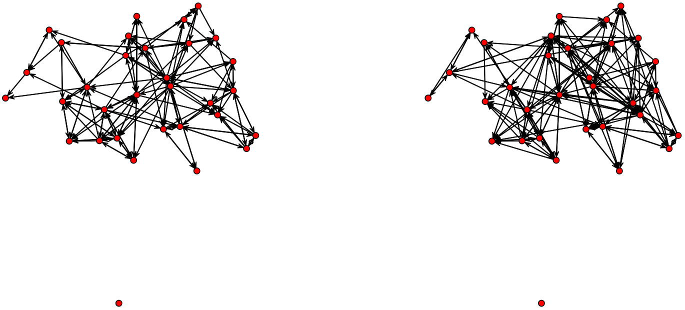


Figure: Network at t_0 and t_1

Let us assume that i ONLY cares about not having too many or two few ties:

$$f_i(\beta, X) = \beta \sum_j x_{ij}$$

meaning that

$$p_{ij}(\beta, X) = \frac{\exp\{\beta(1 - 2x_{ij})\}}{\sum_{h=1}^{n} \exp\{\beta(1 - 2x_{ih})\}} ,$$

because if

- currently $x_{ij} = 1$, then
- the number of ties for i in $X(i \rightsquigarrow j)$ will be one less (-1),
- and if currenlty $x_{ij} = 0 \ldots$

Simulation settings: actors only care about degree

Let the rate be equal for all $\lambda_i = \lambda = 5.7288$

- ✓ is each iteration, actor with shortest waiting time 'wins' (and gets to change)
- ✓ on average every actor gets 5.7 opportunities to change

and set $\beta = -0.7349$

- ✓ if $\beta = 0$ actor would not care if tie was added or deleted
- ✓ here $\beta < 0$ meaning that actor wants less than half of the possible ties

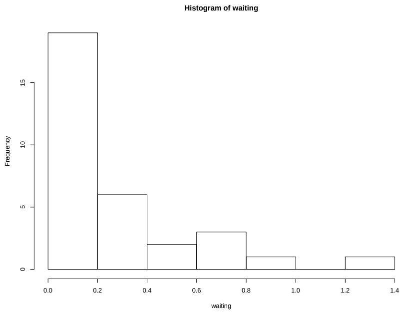
van de Bunt data set

```
mynet1 <- sienaDependent(array(c(tmp3, tmp4),
                               dim=c(32, 32,2)))
mydata <- sienaDataCreate(mynet1)
myeff <- getEffects(mydata)
myeff <- includeEffects(myeff, recip,include=FALSE)
myeff$initialValue[
             myeff$shortName == 'Rate'] <- 5.7288
myeff$initialValue[
        myeff$shortName=='density'][1] <- -0.7349
```

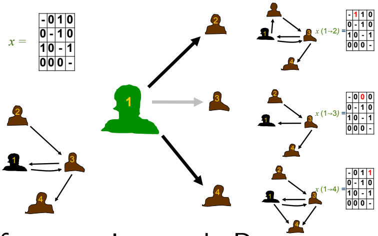
Model: rate

```
waiting <- rexp(32,5.7288)
hist(waiting)
which( waiting == min(waiting))
```

$\Longleftarrow$ the winner



Model: conditional one-step change probability



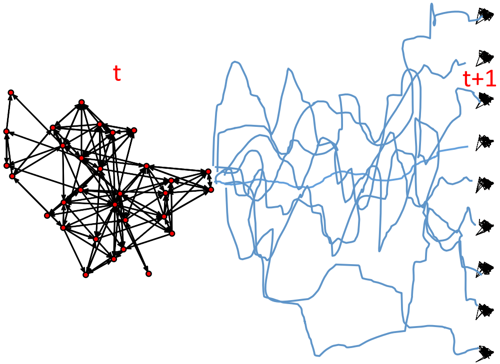
$$\Pr(1 \rightsquigarrow 2) = \frac{e^{-0.7349}}{e^{-0.7349}+e^{1.1059}+e^{-0.7349}+1} = 0.1185$$

$$\Pr(1 \rightsquigarrow 3) = \frac{e^{1.1059}}{e^{-0.7349}+e^{1.1059}+e^{-0.7349}+1} = 0.5156$$

$$\Pr(1 \rightsquigarrow 4) = \frac{e^{-0.7349}}{e^{-0.7349}+e^{1.1059}+e^{-0.7349}+1} = 0.1185$$

Of course, in van de Bunt every actor has 31+1 choices for change

Now let us simulate



Simulate from t_0 to t_1 (`simOnly = TRUE`)

```
sim_model  <-  sienaAlgorithmCreate(
                     projname = 'sim_model',
                          cond = FALSE,
                          useStdInits = FALSE, nsub = 0 ,
                          simOnly = TRUE)
sim_ans <- siena07( sim_model, data = mydata,
                          effects = myeff,
                          returnDeps = TRUE, batch=TRUE )
```

The object `sim_ans` will now contain 1000 simulated networks

Extract networks

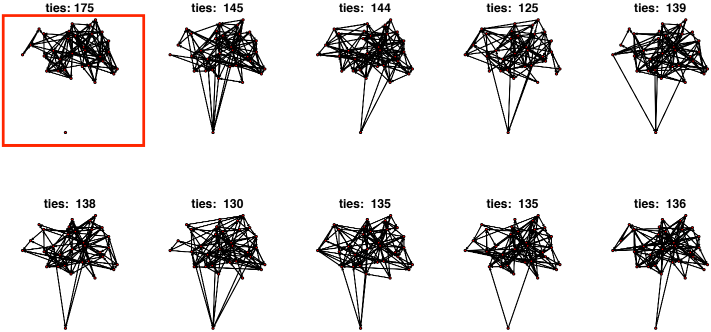
```
n <- dim(tmp4)[1]
mySimNets <- reshapeRSienaDeps( sim_ans , n )
```

The object `mySimNets` is a 1000 by n by n array of adjacency matrices

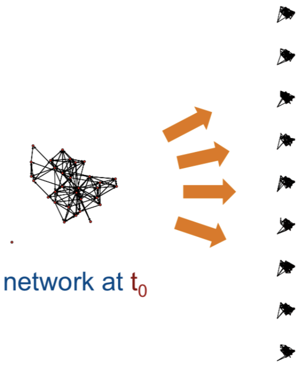
Plot observed network and 9 simulated

```
pdf(file='simnets1.pdf', width = 9,height =4.5)
par(mfrow=c(2,5),  oma = c(0,4,0,0) + 0.1,
    mar = c(5,0,1,1) + 0.1)
plot(as.network(tmp4),coord=coordin,
              main=paste('ties:',sum(tmp4) ) )
apply(mySimNets[1:9,,],1,function(x)
                  plot(as.network(x),
                       coord=coordin,
                       main=paste('ties: ',sum(x))) )
dev.off()
```

The observed at t_1 and possible netowrks at t_1



Simulated networks v t_1 obs



Dyad Census

```
> dyad.census(tmp4)
     Mut Asym Null
[1,]  46   83  367
> dyad.census(mySimNets[1:9,,])
      Mut Asym Null
 [1,]  23  128  345
 [2,]  25  133  338
 [3,]  17  136  343
 [4,]  20  134  342
 [5,]  16  143  337
 [6,]  21  136  339
 [7,]  26  118  352
 [8,]  23  128  345
 [9,]  30  122  344
```

Conclusions

A process where i ONLY cares about not having too many or two few ties does to replicate the reciprocity at t_1

Assume that i ALSO cares about

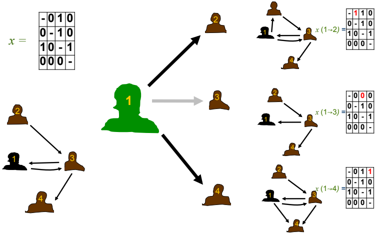
having ties $i \to j$ reciprocated $j \to i$

$$f_i(\beta, X) = \beta_d \sum_j x_{ij} + \beta_r \sum_j x_{ij} x_{ji}$$

meaning that probability that i toggles relationship to j

$$p_{ij}(\beta, X) = \frac{\exp\{\beta_d(1-2x_{ij}) + \beta_r(1-2x_{ij})x_{ji}\}}{\sum_{h=1}^{n} \exp\{\beta_d(1-2x_{ih}) + \beta_r(1-2x_{ih})x_{hi}\}},$$

Model



Objective function:
$\beta_d \sum_j x_{ij} + \beta_r \sum_j x_{ij}x_{ji}$

- adding $1 \to 2$: β_d
- deleting $1 \to 3$: $-\beta_d - \beta_r$
- adding $1 \to 4$: β_d

Our simulated networks had too few reciprocated dyads so we need to set $\beta_r \ldots$

Simulation settings: actors care about degree and reciprocity

Let the rate be equal for all $\lambda_i = \lambda = 6.3477$

✓ on average every actor gets 6.3 opportunities to change

and set $\beta_d = -1.1046$

✓ here $\beta_d < 0$ - actors do not want too many ties

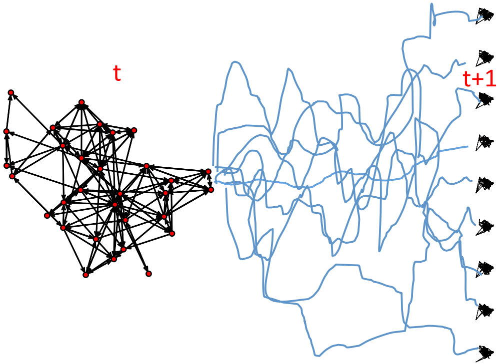
and set $\beta_r = 1.2608$

✓ here $\beta_r > 0$ - actors prefer reciprocated to assymetric ties

van de Bunt data set

```
myeff <- includeEffects(myeff, recip,include=TRUE)
myeff$initialValue[
             myeff$shortName == 'Rate'] <- 6.3477
myeff$initialValue[
        myeff$shortName =='density'][1] <- -1.1046
myeff$initialValue[
          myeff$shortName =='recip'][1] <- 1.2608
```

Now let us simulate



Simulate from t_0 to t_1 now with reciprocity (`simOnly = TRUE`)

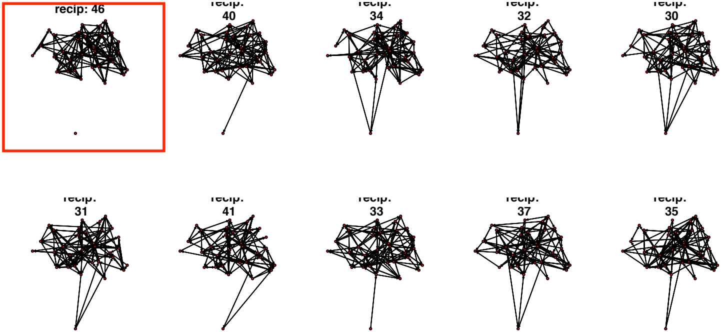
```
sim_model  <-  sienaAlgorithmCreate(
                         projname = 'sim_model',
                              cond = FALSE,
                              useStdInits = FALSE, nsub = 0 ,
                              simOnly = TRUE)
sim_ans <- siena07( sim_model, data = mydata,
                              effects = myeff,
                              returnDeps = TRUE, batch=TRUE )
```

The object `sim_ans` will now contain 1000 simulated networks
NOTE: this piece of code is unchanged

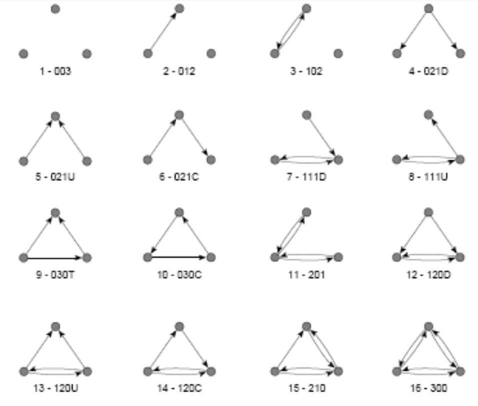
Plot observed network and 9 simulated

```
mySimNets <- reshapeRSienaDeps(sim_ans,n)
plot(as.network(tmp4),coord=coordin,
              main=paste('recip:',dyad.census(tmp4)[1] )  )
apply(mySimNets[1:9,,],1,function(x)
                  plot(as.network(x),
                       coord=coordin,
                       main=paste('recip:
   ',dyad.census(x)[1] ) ) )
```

The observed at t_1 and possible netowrks at t_1



Simulated networks v t_1 obs: triad census



Simulated networks v t_1 obs: triad census

```
> triad.census(tmp4)
      003  012 102 021D 021U 021C 111D 111U 030T 030C 201 120D 120U 120C 210 300
[1,] 2078 1329 745  146   80   52   65  217   37    0  68   16   65   10  30  22
> triad.census(mySimNets[1:9,,])
       003  012 102 021D 021U 021C 111D 111U 030T 030C 201 120D 120U 120C 210 300
 [1,] 1968 1381 718   95   84  160  148  224   16    4  77   13   13   17  33   9
 [2,] 2067 1348 703   85   82  154  150  191   16    8  75    9   11   25  27   9
 [3,] 2073 1397 687  102   75  158  129  181   18    2  68   14   13   23  13   7
 [4,] 2185 1313 733   78   60  132  102  172   20    7  89    7   10   20  28   4
 [5,] 2040 1340 766   89   64  155  129  189   18    7  82   12   11   19  27  12
 [6,] 2206 1403 669   76   68  135  122  143   17    6  64    8   12    9  17   5
 [7,] 1788 1357 760  113   89  169  195  238   30   11  98   14   16   34  37  11
 [8,] 2164 1301 681   70   65  136  168  174   12    5  91   15    8   30  28  12
 [9,] 1988 1383 729  111   67  151  148  173   26   10  82   13   22   19  32   6
```

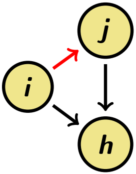
Reciprocity is clearly not enough to explain the incidence of *transitive triangles* and *simmelian ties* (3 Mutual 0, Assymetric, 0 Null)

Assume that i ALSO cares about *closure*

$$f_i(\beta, X) = \exp\left\{ \beta_d \sum_j x_{ij} + \beta_r \sum_j x_{ij} x_{ji} + \beta_t s_{i,t}(x) \right\}$$

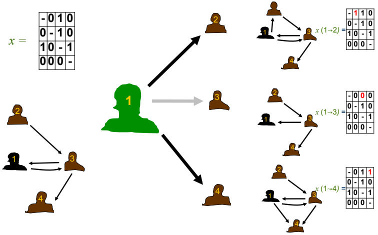
Modelled through, e.g.

transitive triplets effect,
number of transitive patterns in i's ties
$(i \to j,\ j \to h,\ i \to h)$
$s_{i,t}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$



transitive triplet

Model



Objective function including $s_{i,t}(x)$:

- adding $1 \to 2$: $\ldots + \beta_t$
- deleting $1 \to 3$: no change in closure
- adding $1 \to 4$: $\ldots + \beta_t$

Our simulated networks had too few 030T and 300 so we need to set $\beta_t \ldots$

Assume actors care about degree, reciprocity, and closure

Let the rate be equal for all $\lambda_i = \lambda = 7.0959$

- ✓ on average every actor gets 7 opportunities to change

and set $\beta_d = -1.6468$

- ✓ here $\beta_d < 0$ - actors do not want too many ties

and set $\beta_r = 0.8932$

- ✓ here $\beta_r > 0$ - actors prefer reciprocated to assymetric ties

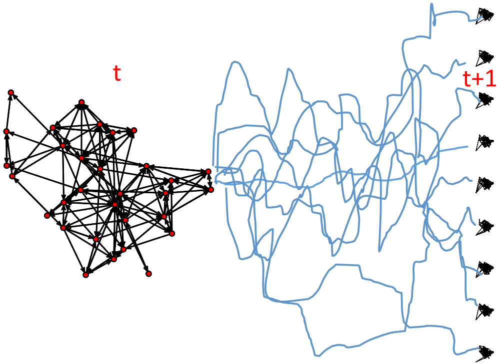
and set $\beta_t = 0.2772$

- ✓ here $\beta_t > 0$ - actors prefer ties that close open triads

van de Bunt data set

```
myeff <- includeEffects(myeff, recip,include=TRUE)
myeff <- includeEffects(myeff, transTrip,include=TRUE)
myeff$initialValue[
             myeff$shortName == 'Rate'] <- 7.0959
myeff$initialValue[
        myeff$shortName =='density'][1] <- 1.6468
myeff$initialValue[
          myeff$shortName =='recip'][1] <-  0.8932
myeff$initialValue[
          myeff$shortName =='transTrip'][1] <-  0.2772
```

Now let us simulate



Simulate from t_0 to t_1 now with transitivity (simOnly = TRUE)

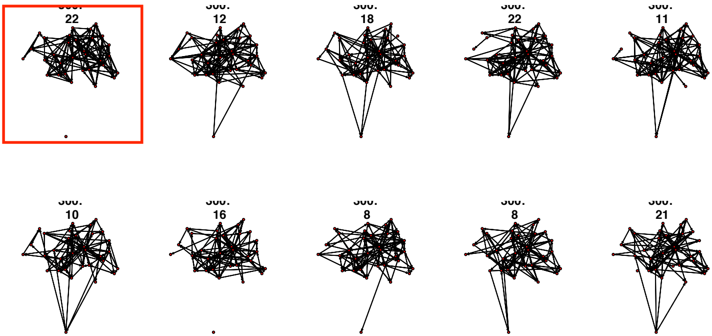
```
sim_model  <-  sienaAlgorithmCreate(
                     projname = 'sim_model',
                          cond = FALSE,
                          useStdInits = FALSE, nsub = 0 ,
                          simOnly = TRUE)
sim_ans <- siena07( sim_model, data = mydata,
                          effects = myeff,
                          returnDeps = TRUE, batch=TRUE )
```

NOTE: this piece of code is unchanged

Plot observed network and 9 simulated

```
mySimNets <- reshapeRSienaDeps(sim_ans,n)
plot(as.network(tmp4),coord=coordin,
              main=paste('recip:',triad.census(tmp4)[16] )  )
apply(mySimNets[1:9,,],1,function(x)
                  plot(as.network(x),
                       coord=coordin,
                       main=paste('300:
   ',triad.census(x)[16] ) ) )
```

The observed at t_1 and possible netowrks at t_1



Simulated networks v t_1 obs: triad census

```
> triad.census(tmp4)
      003  012 102 021D 021U 021C 111D 111U 030T 030C 201 120D 120U 120C 210 300
[1,] 2078 1329 745  146   80   52   65  217   37    0  68   16   65   10  30  22
> triad.census(mySimNets[1:9,,])
       003  012 102 021D 021U 021C 111D 111U 030T 030C 201 120D 120U 120C 210 300
 [1,] 2029 1223 662  108   79  159  138  257   36    6  82   19   35   29  68  30
 [2,] 2163 1292 727   89   62  125  136  160   15    4  61    9   23   25  29  40
 [3,] 2551 1191 657   44   34   84   88  143    5    0  78    6   16   14  27  22
 [4,] 1990 1430 576  100   84  173  128  200   23   10 103   12   38   22  45  26
 [5,] 2219 1390 631  103   83  121   84  175   25    5  45    8   21    9  31  10
 [6,] 2031 1218 799   75   56  109  136  213   15    9 114   15   25   25  61  59
 [7,] 2079 1443 597  104   72  160   97  206   24    4  54   17   24   24  38  17
 [8,] 2105 1256 764   77   55  114   99  212   12    5  98   11   33   16  50  53
 [9,] 2405 1260 569  103   55  126   78  184   22    6  41   12   25    8  46  20
```

Reciprocity togehter with transitivity seems enough to explain the incidence of *transitive triangles* and *simmelian ties* (3 Mutual 0, Assymetric, 0 Null)

Computer simulation algorithm for arbitrary rate function $\lambda_i(\alpha, \rho, x)$

1. Set $t = 0$ and $x = X(0)$.
2. Generate S according to the exponential distribution with mean $1/\lambda_+(\alpha, \rho, x)$ where $\lambda_+(\alpha, \rho, x) = \sum_i \lambda_i(\alpha, \rho, x)$.
3. Select $i \in \{1, ..., n\}$ using probabilities $\frac{\lambda_i(\alpha,\rho,x)}{\lambda_+(\alpha,\rho,x)}$.

4. Select $j \in \{1, ..., n\}$, $j \neq i$ using probabilities $p_{ij}(\beta, x)$.
5. Set $t = t + S$ and $x = x(i \rightsquigarrow j)$.
6. Go to step 2
   (unless stopping criterion is satisfied).

What effects are there?

- RSiena Manual http://www.stats.ox.ac.uk/~snijders/siena/RSiena_Manual.pdf - check for `shortName`
- scroll through the effects available to you for your data `myeff` - check for `shortName`
- also `effectsDocumentation(myeff)`

Where did I get these numbers?

Estimation by Method of Moments: data

Basics for data

- You need at least 2 observations on $X(t)$ for waves $t_0,\ t_1$
- First observations is fixed and contains no information about θ
- No assumption of a stationary network distribution

Estimation by Method of Moments: procedure

How to estimate $\theta = (\lambda, \beta)$*?*

- pick starting values for θ
- simulate from $X(t_0)$ until t_1 - call the simulated network (-s) $X_{\rm rep}$
- if statistic $Z_k(X_{\rm rep})$ for parameter k is different to $Z_k(X_{\rm obs})$, adjust accordingly

Estimation by Method of Moments: aim

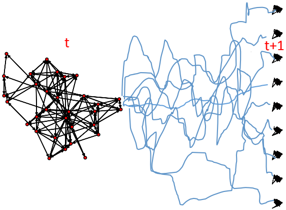
For suitable statistic $Z = (Z_1, \ldots, Z_K)$,
i.e., K variables which can be calculated from the network;
the statistic Z_k must be *sensitive* to the parameter θ_k
e.g. number of mutual dyads is sensitive to the reciprocity parameter (as we have seen)

The MoM estimate is a value: $\hat{\theta}$ of θ such that for

- observed stats $Z(X_{\mathrm{obs}})$
- and the the expected value $\mathrm{E}_\theta(Z(X_{\mathrm{rep}}))$

$$E_{\hat{\theta}}\left\{Z(X_{\mathrm{rep}})\right\} \;=\; Z(X_{\mathrm{obs}}) \; .$$

Method of Moments matches the moments



Do we have to do this for every update of the parameter θ?

Robbins-Monro algorithm

The moment equation $E_{\hat{\theta}}\{Z\} = z$ cannot be solved by analytical or the usual numerical procedures

Stochastic approximation (Robbins-Monro, 1951)

Iteration step:

$$\hat{\theta}_{N+1} = \hat{\theta}_N - a_N \, D^{-1}(z_N - z) \; , \tag{1}$$

where z_N is a simulation of Z with parameter $\hat{\theta}_N$,

D is a suitable matrix, and $a_N \to 0$.

Computer algorithm has 3 phases:

1. brief phase for preliminary estimation of $\partial \mathrm{E}_\theta \{Z\}/\partial\theta$ for defining D;
2. estimation phase with Robbins-Monro updates, where a_N remains constant in *subphases* and decreases between subphases;
3. final phase where θ remains constant at estimated value; this phase is for checking that

$$\mathrm{E}_{\hat{\theta}} \{Z\} \approx z \ ,$$

and for estimating D_θ and Σ_θ to calculate standard errors.

Convergence

We say that $E_{\hat{\theta}}\{Z\} = z$ is approximately satisfied if, for each statistic $Z_k(X_{\rm obs})$ is within 0.1 standard deviation of $\mathrm{E}_{\theta}(Z(X_{\rm rep}))$.
This is provided in the output as the *convergence t-ratio* (and the overall maximum convergence ratio is less than 0.25)

Summary

What is the purpose of having the embedded Markov Chain in continuous time?
DYNAMICS
can model change of *tie* as dependent on current ties AND behaviour
can model change in *behaviour* as dependent on current behaviour AND the behavior of those you are tied to

Summary

What is the purpose of having the embedded Markov Chain in continuous time?

STATISTICAL

This is a statistical model that has estimable parameters for selection and influence

This is a generative model from which we can also generate replicate data AND assess GOF

Summary

What is the purpose of having the embedded Markov Chain in continuous time?

STATISTICAL

This is a statistical model that has estimable parameters for selection and influence

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Compare

- Generalized Estimation Equations
- Regressing behaviour wave 1 on wave 0

Is the model any good?

How do we choose parameters and how do we know that we have a good model?

- CHDH-SNA-4: more extensive suit of possible effects.
- We have already simulated to investigate goodness-of-fit
- . . . but there are ready-made routines in RSiena

Everything you need to know (including scipts for all kinds of data) is avaiable at http://www.stats.ox.ac.uk/~snijders/siena/