Introduction to Stochastic Actor-Oriented Models

Fundamentals of SAOMs

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Preamble

- All material is on the workshop repository https://github.com/johankoskinen/CHDH-SNA
  - Download the RMarkdown file CHDH-SNA-2.Rmd
- In order to run the Markdown you need
  - The R-package
  - The RStudio interface
- We will predominantly use the packages
  - `sna`
  - `network`
  - `RSiena`

Outline of workshops

1. (Basic) Introduction to SAOM (Thursday AM)
    - SAOM as an agent-based model
    - How to estimate a SAOM
2. Analysing data with SAOM
    - Different model specifications
    - Different types of data
    - Trouble shooting and dealing with common issues
3. Extensions to SAOM
    - Even more types of data
    - Likelihood-based estimation
    - Settings and imperfect data

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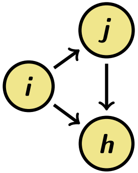
   $s_{i1}(x) = x_{i+} = \sum_j x_{ij}$
2. *reciprocity effect*, number of reciprocated ties
   $s_{i2}(x) = \sum_j x_{ij} \, x_{ji}$

Four ways of closure (1)

Four potential effects representing network closure:

3. *transitive triplets effect*,
   number of transitive patterns in i's ties
   $(i \to j,\ j \to h,\ i \to h)$
   $s_{i3}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$



transitive triplet

4. *transitive ties effect*,
   number of actors j to whom i is tied indirectly
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   and also directly $x_{ij} = 1$),
   $s_{i4}(x) = \#\{j \mid x_{ij} = 1,\ \max_h(x_{ih}\, x_{hj}) > 0\}$

Four ways of closure (2)

5. *indirect ties effect*,
   number of actors j to whom i is tied indirectly
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   but not directly $x_{ij} = 0$),
   $=$ number of geodesic distances equal to 2,
   $s_{i5}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih}\, x_{hj}) > 0\}$

Four ways of closure (3)

6. *balance* or structural equivalence, similarity between outgoing ties of i with outgoing ties of his friends,

$$s_{i6}(x) = \sum_{j=1}^{n} x_{ij} \sum_{\substack{h=1 \\ h \neq i,j}}^{g} \left(1 - |x_{ih} - x_{jh}|\right) ,$$

[note that $(1 - |x_{ih} - x_{jh}|) = 1$ if $x_{ih} = x_{jh}$, and 0 if $x_{ih} \neq x_{jh}$, so that

$$\sum_{\substack{h=1 \\ h \neq i,j}}^{g} \left(1 - |x_{ih} - x_{jh}|\right)$$

measures agreement between i and j .]

Four ways of closure (4)

Differences between these three network closure effects:

- transitive triplets effect: i more attracted to j if there are *more* indirect ties $i \rightarrow h \rightarrow j$;

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- transitive ties effect: i more attracted to j if there is *at least one* such indirect connection ;

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Differences between these three network closure effects:

- transitive triplets effect: i more attracted to j
  if there are *more* indirect ties $i \rightarrow h \rightarrow j$;
- transitive ties effect: i more attracted to j
  if there is *at least one* such indirect connection ;
- balance effect:
  i prefers others j who make same choices as i.

One way of closure: GWESP

Nowadays, we often use GWESP (geometrically weighted edgewise shared partners) - combines `transTrip` and `transTies`:

$$GWESP(i, \alpha)$$

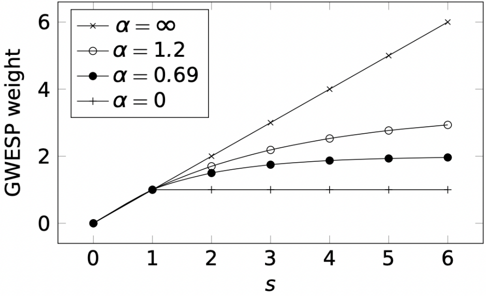
One way of closure: GWESP

Nowadays, we often use GWESP (geometrically weighted edgewise shared partners) - combines `transTrip` and `transTies`:

$$GWESP(i,\alpha) = \sum_j x_{ij} e^{\alpha} \left[1 - (1 - e^{-\alpha})^{\overbrace{\sum_h x_{ih} x_{jh}}^{\sharp\text{com. partn.}}} \right]$$

- for $\alpha \geq 0$ (effect parameter $= 100 \times \alpha$).
- Default $\alpha = \log(2)$, `parameter = 69`

One way of closure: GWESP



Weight tie $i \to j$ for $s = \sum_h x_{ih} x_{jh}$

Degree-based effects

7. *in-degree related popularity effect*, sum friends' in-degrees
$s_{i7}(x) = \sum_j x_{ij} \sqrt{x_{+j}} = \sum_j x_{ij} \sqrt{\sum_h x_{hj}}$
related to dispersion of in-degrees
(can also be defined without the $\sqrt{}$ sign);

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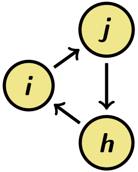
   related to dispersion of in-degrees
   (can also be defined without the $\sqrt{\ }$ sign);
8. *out-degree related popularity effect*,
   sum friends' out-degrees
   $s_{i8}(x) = \sum_j x_{ij}\,\sqrt{x_{j+}} = \sum_j x_{ij}\sqrt{\sum_h x_{jh}}$
   related to association in-degrees — out-degrees;
9. *Outdegree-related activity effect* ,
   $s_{i9}(x) = \sum_j x_{ij}\,\sqrt{x_{i+}} = x_{i+}^{1.5}$
   related to dispersion of out-degrees;
10. *Indegree-related activity effect* ,
   $s_{i10}(x) = \sum_j x_{ij}\,\sqrt{x_{+i}} = x_{i+}\,\sqrt{x_{+i}}$
   related to association in-degrees — out-degrees;

Four ways of closure (5)

11. *three-cycle effect*,
    number of three-cycles in i's ties
    $(i \to j,\ j \to h,\ h \to i)$
    $s_{i11}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{hi}$



three-cycle

This represents a kind of generalized reciprocity, and absence of hierarchy.

12. ... and potentially many others ...

Selection effects

Preferences of actors dependent on their degrees:

- out ego - out alter degrees
- out ego - in alter degrees
- in ego - out alter degrees
- in ego - in alter degrees

All these are product interactions between the two degrees (or square roots).

Selection effects: types of evaluations

Four kinds of evaluation function effect associated with actor covariate v_i.

This applies also to behavior variables Z_h.

13. *covariate-related popularity*, ‘alter’
    sum of covariate over all of i ’s friends
    $s_{i13}(x) = \sum_j x_{ij} v_j$;

Selection effects: types of evaluations

Four kinds of evaluation function effect associated with actor covariate v_i.

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13. *covariate-related popularity*, 'alter'
    sum of covariate over all of i 's friends
    $s_{i13}(x) = \sum_j x_{ij} v_j$;
14. *covariate-related activity*, 'ego'
    i's out-degree weighted by covariate
    $s_{i14}(x) = v_i x_{i+}$;

Selection effects: similarity

15. *covariate-related similarity*,
    sum of measure of covariate similarity
    between i and his friends,
    $s_{i15}(x) = \sum_j x_{ij}\,\mathrm{sim}(v_i, v_j)$
    where $\mathrm{sim}(v_i, v_j)$ is the similarity between v_i and v_j,

$$\mathrm{sim}(v_i, v_j) = 1 - \frac{|v_i - v_j|}{R_V} ,$$

R_V being the range of V;

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    R_V being the range of V;
16. *covariate-related interaction*, 'ego $\times$ alter'
    $s_{i16}(x) = v_i \sum_j x_{ij}\, v_j$;

Selection effects: similarity

Snijders and Lomi (2019) *Beyond homophily: Incorporating actor variables in statistical network models*:

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  - combination of tendencies of
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- for (non-binary) variables v_i
  - combination of tendencies of
    - homophily,
    - aspiration, and
    - social norm
  - yields 5 effects:
    1. ego $x_{ij} v_i$
    2. alter $x_{ij} v_j$
    3. ego-squared $x_{ij} v_i^2$
    4. ego-alter difference squared $x_{ij}(v_i - v_j)^2$ and
    5. alter squared $x_{ij} v_j^2$

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    5. alter squared $x_{ij} v_j^2$

Do we really *have* to use this?

Example van de Bunt

Example (Gerhard van de Bunt)

Data

- 32 university freshmen (24 fem and 8 male)
- (here) 3 obs. $(t_1,\ t_2,\ t_3)$ at 6, 9, and 12 weeks
- The relation: 'friendly relation'.

Missing entries $x_{ij}(t_m)$ set to 0 and not used in calculations of statistics.
Densities increase from 0.15 at t_1 via 0.18 to 0.22 at t_3 .

Example van de Bunt (2)

Example (Gerhard van de Bunt (cont.))

Very simple model: only out-degree and reciprocity effects

	Model 1	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.51	(0.54)
Rate $t_2 - t_3$	3.09	(0.49)
Out-degree	-1.10	(0.15)
Reciprocity	1.79	(0.27)

rate parameters:
per actor about 3 opportunities for change between observations;
out-degree parameter negative:
on average, cost of friendship ties higher than their benefits;
reciprocity effect strong and highly significant ($t = 1.79/0.27 = 6.6$).

Example van de Bunt (3)

Example (Gerhard van de Bunt (cont.))

Evaluation function is

$$f_i(x) = \sum_j \Big(-1.10\, x_{ij} \; + \; 1.79\, x_{ij}\, x_{ji} \Big).$$

This expresses ‘how much actor i likes the network’.

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Adding a reciprocated tie (i.e., for which $x_{ji} = 1$) gives

$$-1.10 + 1.79 = 0.69.$$

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Adding a non-reciprocated tie (i.e., for which $x_{ji} = 0$) gives

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i.e., this has negative benefits.

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i.e., this has negative benefits.
Gumbel distributed disturbances are added:
these have variance $\pi^2/6 = 1.645$ and s.d. 1.28.

Example van de Bunt (4)

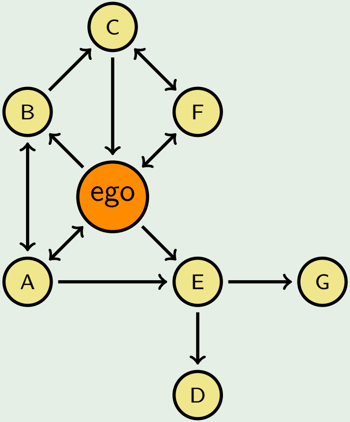
Example (Gerhard van de Bunt: with simple closure)

The estimates with only transitive ties:

Structural model with one network closure effect

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.89	(0.60)
Rate $t_2 - t_3$	3.06	(0.47)
Out-degree	-2.14	(0.38)
Reciprocity	1.55	(0.28)
Transitive ties	1.30	(0.41)

Example (Gerhard van de Bunt: with simple closure (cont.))



for ego:

out-degree $x_{i+} = 4$
$\#\{\text{recipr. ties}\} = 2,$
$\#\{\text{trans. ties }\} = 3.$

Example van de Bunt (5)

Example (Gerhard van de Bunt: with simple closure (cont.))

The evaluation function is

$$f_i(x) = \sum_j \Big(-2.14\, x_{ij} \;+\; 1.55\, x_{ij}\, x_{ji} \;+\; 1.30\, x_{ij} \;\max_h \big(x_{ih}\, x_{hj} \big) \Big)$$

$\Big($ note: $\sum_j x_{ij}\ \max_h \big(x_{ih}\, x_{hj} \big)$ is $\#\{$trans. ties $\}$ $\Big)$

Example van de Bunt (5)

Example (Gerhard van de Bunt: with simple closure (cont.))

The evaluation function is

$$f_i(x) = \sum_j \Big(-2.14\, x_{ij} \;+\; 1.55\, x_{ij}\, x_{ji} \;+\; 1.30\, x_{ij} \,\max_h \big(x_{ih}\, x_{hj} \big) \Big)$$

$\Big($ note: $\sum_j x_{ij} \,\max_h \big(x_{ih}\, x_{hj}\big)$ is $\#\{$trans. ties $\}$ $\Big)$
so its current value for this actor is

$$f_i(x) = -2.14 \times 4 \;+\; 1.55 \times 2 \;+\; 1.30 \times 3 \;=\; -1.56.$$

Example van de Bunt (6)

Example (Gerhard van de Bunt: with simple closure (cont.))

Options when ‘ego’ has opportunity for change:

	out-degr.	recipr.	trans. ties	gain	prob.
current	4	2	3	0.00	0.061
new tie to C	5	3	5	+2.01	0.455
new tie to D	5	2	4	+0.46	0.096
new tie to G	5	2	4	+0.46	0.096
drop tie to A	3	1	0	–3.31	0.002
drop tie to B	3	2	1	–0.46	0.038
drop tie to E	3	2	2	+0.84	0.141
drop tie to F	3	1	3	+0.59	0.110

The actor adds random influences to the gain (with s.d. 1.28), and chooses the change with the highest total ‘value’.

Example van de Bunt (7)

Example (Gerhard van de Bunt: with more closure)

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.64	(0.80)
Rate $t_2 - t_3$	3.53	(0.57)
Out-degree	–0.90	(0.58)
Reciprocity	2.27	(0.41)
Transitive triplets	0.35	(0.06)
Transitive ties	0.75	(0.45)
Three-cycles	–0.72	(0.21)
In-degree popularity ($\surd$)	–0.71	(0.27)

Conclusions:
Reciprocity, transitivity;
negative 3-cycle effect;
negative
popularity effect.

Example van de Bunt (8)

Example (Gerhard van de Bunt: Add effects of gender & program, smoking similarity)

	Model 4	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.71	(0.80)
Rate $t_2 - t_3$	3.54	(0.59)
Out-degree	–0.81	(0.61)
Reciprocity	2.14	(0.45)
Transitive triplets	0.33	(0.06)
Transitive ties	0.67	(0.46)
Three-cycles	–0.64	(0.22)
In-degree popularity ($\sqrt{}$)	–0.72	(0.28)
Sex (M) alter	0.52	(0.27)
Sex (M) ego	–0.15	(0.27)
Sex similarity	0.21	(0.22)
Program similarity	0.65	(0.26)
Smoking similarity	0.25	(0.18)

Conclusions:
Trans. ties now not needed any more to represent transitivity;
men more popular;
program similarity.

Example van de Bunt (8)

Example (Gerhard van de Bunt: selection table)

We may do the calculations with $F = 0,\ M = 1$ (even if centered) The joint effect of the gender-related effects
for the tie variable x_{ij} from i to j is

$$-0.15\, z_i \,+\, 0.52\, z_j \,+\, 0.21\, I\{z_i = z_j\}\ .$$

$i \ \backslash \ j$	F	M
F	0.21	0.52
M	−0.15	0.58

Conclusion:
men seem not to like female friends...?

Influence effects

Many different reasons why networks
are important for behavior:

1. *imitation* :
   individuals imitate others
   (basic drive; uncertainty reduction).
2. *social capital* :
   individuals may use resources of others;
3. *coordination* :
   individuals can achieve some goals
   only by concerted behavior;

In this presentation, only imitation is considered,
but the other two reasons are also of eminent importance.

Influence effects

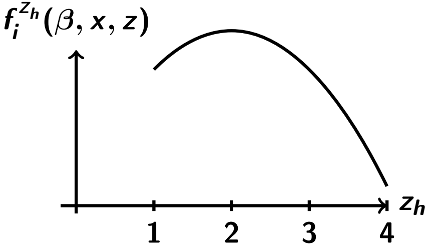
Basic effects for dynamics of behavior f_i^Z:

$$f_i^Z(\beta, x, z) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x, z)\,,$$

1. *tendency* ,
   $s_{i1}^Z(x, z) = z_{ih}$
2. *quadratic tendency*, ‘effect behavior on itself’,
   $s_{i2}^Z(x, z) = z_{ih}^2$
   Quadratic tendency effect important for model fit.

Influence effects

For a negative quadratic tendency parameter,
the model for behavior is a unimodal preference model.



For positive quadratic tendency parameters ,
the behavior objective function can be bimodal
('positive feedback').

Influence effects

3. *behavior-related average similarity*,
average of behavior similarities between i and friends
$s_{i3}(x) = \frac{1}{x_{i+}} \sum_j x_{ij} \, \mathrm{sim}(z_{ih}, z_{jh})$
where $\mathrm{sim}(z_{ih}, z_{jh})$ is the similarity between v_i and v_j,

$$\mathrm{sim}(z_{ih}, z_{jh}) \; = \; 1 - \frac{|z_{ih} - z_{jh}|}{R_{Z^h}} \; ,$$

R_{Z^h} being the range of Z^h;

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   $$\mathrm{sim}(z_{ih}, z_{jh}) = 1 - \frac{|z_{ih} - z_{jh}|}{R_{Z^h}} ,$$
   R_{Z^h} being the range of Z^h;
4. *average behavior alter* — an alternative to similarity:
   $s_{i4}(x, z) = z_{ih} \frac{1}{x_{i+}} \sum_j x_{ij} z_{jh}$

Effects 3 and 4 are alternatives for each other:
they express the same theoretical idea of influence
in mathematically different ways.
The data will have to differentiate between them.

Influence effects

Network position can also have influence on behavior dynamics
e.g. through degrees rather than through behavior
of those to whom one is tied:

5. *popularity-related tendency*, (in-degree)
   $s_{i5}(x, z) = z_{ih} \, x_{+i}$

Influence effects

Network position can also have influence on behavior dynamics
e.g. through degrees rather than through behavior
of those to whom one is tied:

7. *popularity-related tendency*, (in-degree)
   $s_{i7}(x, z) = z_{ih}\, x_{+i}$
8. *activity-related tendency*, (out-degree)
   $s_{i8}(x, z) = z_{ih}\, x_{i+}$

Influence effects

7. *dependence on other behaviors* $(h \neq \ell)$,
   $s_{i7}(x, z) = z_{ih}\, z_{i\ell}$

For both the network and the behavior dynamics,
extensions are possible depending on the network position.

Influence effects

The *similarity effect* in evaluation function :

sum of absolute behavior differences between i and his friends
$s_{i2}(x,z) = \sum_j x_{ij} \, \mathrm{sim}(z_{ih}, z_{jh})$.

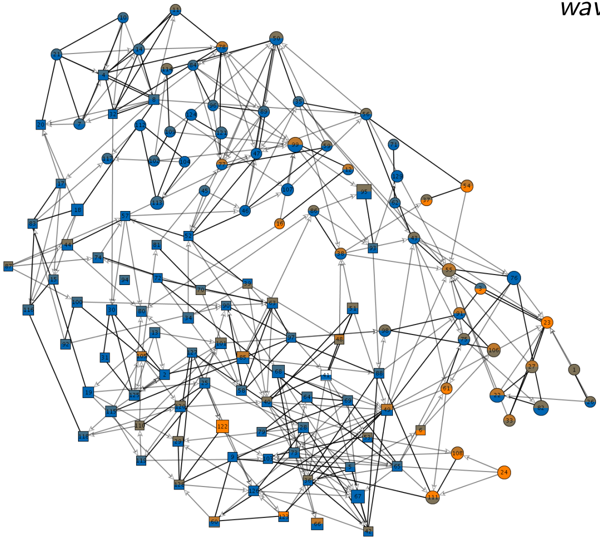
This is fundamental both
to network selection based on behavior,
and to behavior change based on network position.

Influence effects: Example

Example (Smoke rings)

Study of smoking initiation and friendship
(following up on earlier work by P. West, M. Pearson & others).
One school year group from a Scottish secondary school starting at age 12-13 years, was monitored over 3 years;
total of 160 pupils, of which 129 pupils present at all 3 observations; with sociometric & behavior questionnaires at three moments, at appr. 1 year intervals.
Smoking: values 1–3;
drinking: values 1–5;

covariates:
gender, smoking of parents and siblings (binary),
money available (range 0–40 pounds/week).

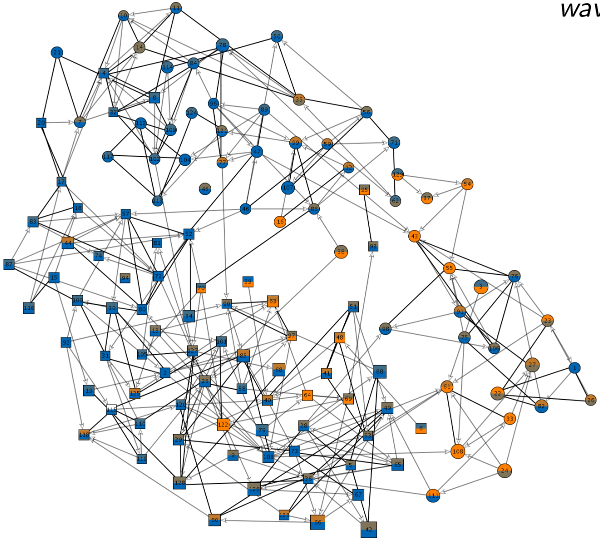


wave 1

girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)

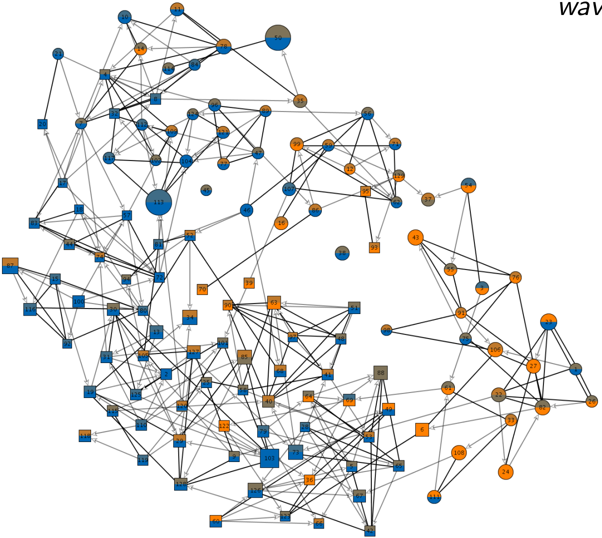


wave 2

girls: circles
boys: squares

node size: pocket money

color: top = drinking
bottom = smoking
(orange = high)



wave 3

girls: circles
boys: squares

node size: pocket money

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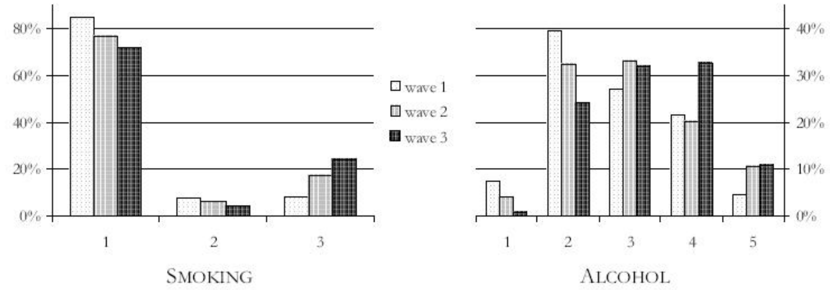
bottom = smoking
(orange = high)

FIGURE 2. — OBSERVED DISTRIBUTION OF SUBSTANCE USE IN THE THREE WAVES.



More realistic model

Friendship dynamics	Rate 1	18.67	(2.17)
	Rate 2	12.42	(1.30)
	Outdegree	–1.57	(0.27)
	Reciprocity	2.04	(0.13)
	Transitive triplets	0.35	(0.04)
	Transitive ties	0.84	(0.09)
	Three-cycles	–0.41	(0.10)
	In-degree based popularity ($\surd$)	0.05	(0.07)
	Out-degree based popularity ($\surd$)	–0.45	(0.16)
	Out-degree based activity ($\surd$)	–0.39	(0.07)
	Sex alter	–0.14	(0.08)
	Sex ego	0.08	(0.10)
	Sex similarity	0.66	(0.08)
	Romantic exp. similarity	0.10	(0.06)
	Money alter (unit: 10 pounds/w)	0.11	(0.05)
	Money ego	–0.06	(0.06)
	Money similarity	0.98	(0.27)

More realistic model (continued)

Friendship dynamics	Drinking alter	–0.01	(0.07)
	Drinking ego	0.09	(0.09)
	Drinking ego × drinking alter	0.14	(0.06)
	Smoking alter	–0.08	(0.08)
	Smoking ego	–0.14	(0.09)
	Smoking ego × smoking alter	0.03	(0.08)

Smoking dynamics	Rate 1	4.74	(1.88)
	Rate 2	3.41	(1.29)
	Linear tendency	–3.39	(0.45)
	Quadratic tendency	2.71	(0.40)
	Ave. alter	2.00	(0.95)
	Drinking	–0.11	(0.24)
	Sex (F)	–0.12	(0.35)
	Money	0.10	(0.20)
	Smoking at home	–0.05	(0.29)
	Romantic experience	0.09	(0.33)

Alcohol consumption dynamics	Rate 1	1.60	(0.32)
	Rate 2	2.50	(0.42)
	Linear tendency	0.44	(0.17)
	Quadratic tendency	–0.64	(0.22)
	Ave. alter	1.34	(0.61)
	Smoking	0.01	(0.21)
	Sex (F)	0.04	(0.22)
	Money	0.17	(0.16)
	Romantic experience	–0.19	(0.27)

Conclusion:

In this case, the conclusions from a more elaborate model
– i.e., with better control for alternative explanations –
are similar to the conclusions from the simple model.

There is evidence for friendship selection based on drinking,
and for social influence with respect to smoking and drinking.

Parameter interpretation for behavior change

Omitting the non-significant parameters yields
the following objective functions.
For smoking

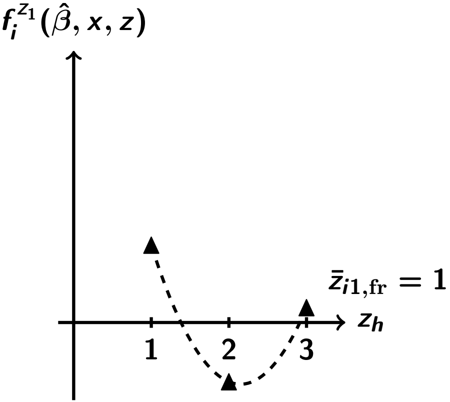
$$f_i^{z_1}(\hat{\beta}, x, z) =$$

$$-3.39\,(z_{i1} - \bar{z}_1) + 2.71\,(z_{i1} - \bar{z}_1)^2 + 2.00\,(z_{i1} - \bar{z}_1)\,(\bar{z}_{i1,\mathrm{fr}} - \bar{z}_1)\ ,$$

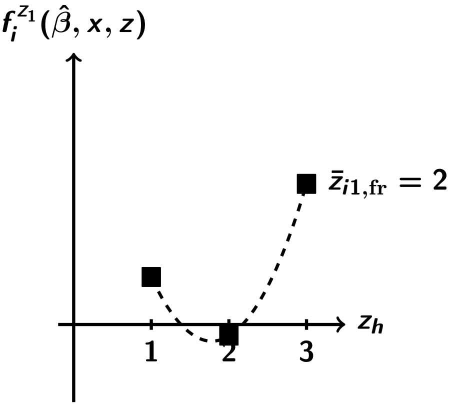
where z_{i1} is smoking of actor i: values 1–3, mean 1.4.
$\bar{z}_{i1,\mathrm{fr}}$ is the average smoking behavior of i's friends.

Convex function – consonant with addictive behavior.

$$-3.39\,(z_{i1}-\bar{z}_1)\;+\;2.71\,(z_{i1}-\bar{z}_1)^2\;+\;2.00\,(z_{i1}-\bar{z}_1)\,(\bar{z}_{i1,\mathrm{fr}}-\bar{z}_1)$$



$$-3.39\,(z_{i1}-\bar{z}_1) + 2.71\,(z_{i1}-\bar{z}_1)^2 + 2.00\,(z_{i1}-\bar{z}_1)\,(\bar{z}_{i1,\mathrm{fr}}-\bar{z}_1)$$



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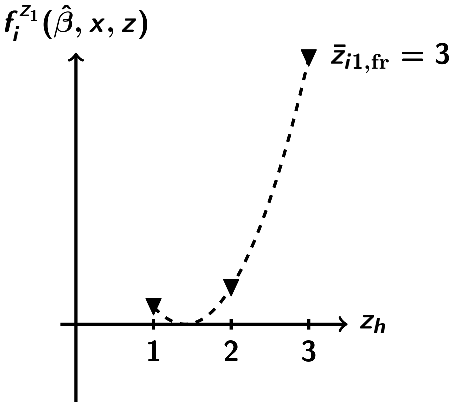
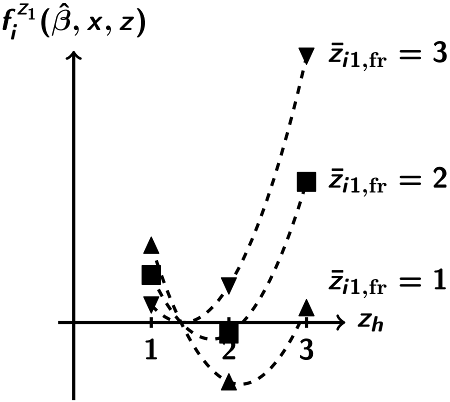


$$-3.39\,(z_{i1} - \bar{z}_1) + 2.71\,(z_{i1} - \bar{z}_1)^2 + 2.00\,(z_{i1} - \bar{z}_1)\,(\bar{z}_{i1,\mathrm{fr}} - \bar{z}_1)$$



For drinking the objective function (significant terms only) is

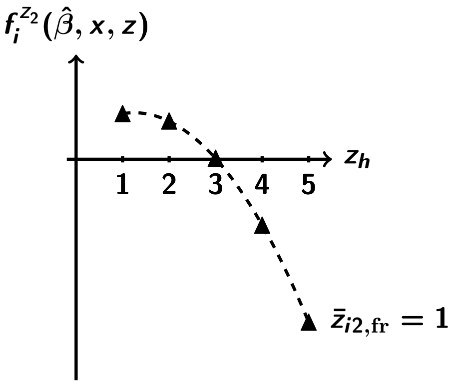
$$f_i^{z_2}(\hat{\beta}, x, z) =$$

$$0.44\,(z_{i2} - \bar{z}_2) - 0.64\,(z_{i2} - \bar{z}_2)^2 + 1.34\,(z_{i2} - \bar{z}_2)\,(\bar{z}_{i2,\mathrm{fr}} - \bar{z}_2)\ ,$$

where z_{i2} is drinking of actor i: values 1–5, mean 3.0.

Unimodal function – consonant with non-addictive behavior.

$$0.44\,(z_{i2}-\bar{z}_2)\;-\;0.64\,(z_{i2}-\bar{z}_2)^2\;+\;1.34\,(z_{i2}-\bar{z}_2)\,(\bar{z}_{i2,\mathrm{fr}}-\bar{z}_2)$$



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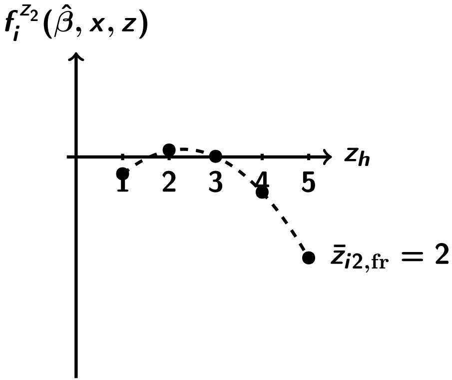

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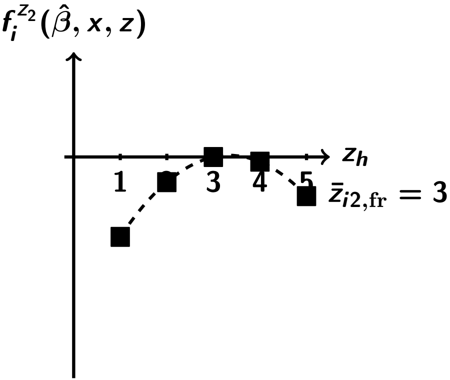

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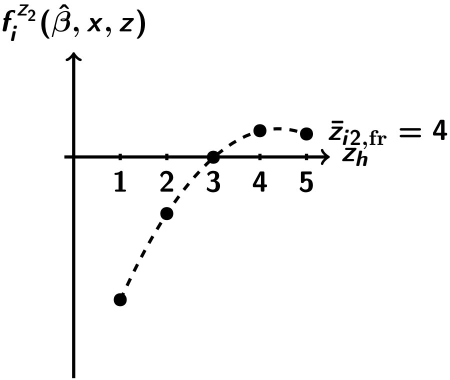

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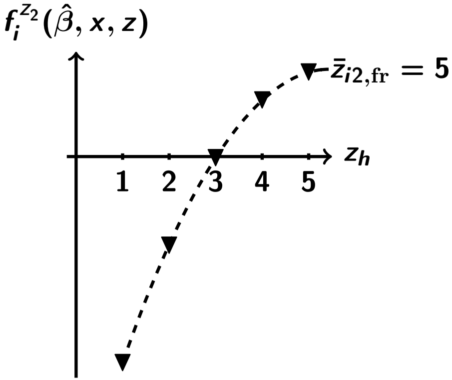

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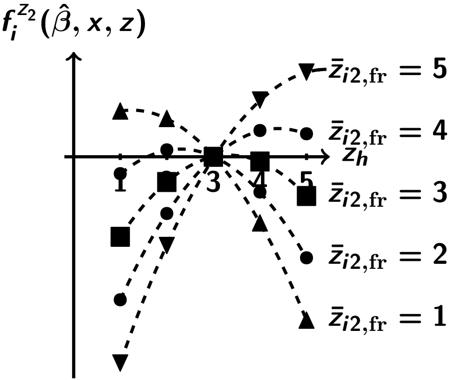

Testing assumptions:Goodness-of-fit (GOF)

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but model is very complex

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We can (almost) always get estimates
but model is very complex
so how do we know that it is realistic?

Two routines for goodness-of-fit

- `sienaTimeTest()`
  for testing time heterogeneity
- `sienaGOF()`
  for checking that the model reproduces the features of the observed networks (that were not modelled).

Time-test

Standard assumptions M waves, the $M-1$ periods follow the same model with the same parameters.
Use

- `sienaTimeTest()`
  to test if some parameters differ across any of the periods
- if test 'positive'
  include interactions with time using
  `includeTimeDummy()`

see `RscriptSienaTimeTest.r`

Extension 2: Is model homogenous over time

Example time test (Lospinoso et al., 2010)
vdb_tt.R:

```
vdb.ans1 <- siena07(vdb.model, data=vdb.data,
                        effects=vdb.eff,
                       useCluster=FALSE, initC=TRUE)
        timetest.1 <- sienaTimeTest(vdb.ans1)
        summary(timetest.1)
        plot(timetest.1, effects=c(1,3))
```

Goodness of fit

Principle: simulate replicate data
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This is exactly what we did in 'Simulating SAOM'
What are we looking for?

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This is exactly what we did in 'Simulating SAOM'
What are we looking for?
does model capture features that we have not modelled?

built in GOF-function

Siena has function `sienaGOF()`
This operates on your siena-object
generated from `siena07()` with option `returnDeps = TRUE`

choosing features for GOF

Some preprogrammed 'auxiliary' functions
that can be passed to `sienaGOF` are:
`OutdegreeDistribution()`
`IndegreeDistribution()`
`BehaviorDistribution()`
you can also create custom functions

More help on GOF

Use ? function and `sienaGOF_new.R`

```
results1 <- siena07(myalg, data=mydata,
                 effects=myeff, returnDeps=TRUE)
gof1.od <- sienaGOF(results1, verbose=TRUE,
              varName="friendship",
              OutdegreeDistribution,
              cumulative=TRUE, levls=0:10)
gof1.od
plot(gof1.od)
```

Trouble shooting: non-convergence

What stochastic approximation algorithm does

1. Gauging sensitivity of (estimation) statistics Z to parameters θ;
2. Robbins-Monro updates for θ
   - *nsub* subphases (usually 4)
   - decreasing step sizes, determined by *firstg*
3. Final: *n3* runs, θ constant at $\hat{\theta}$
   - Check deviations from targets

$$E_{\hat{\theta}}\{Z\} - z$$

   - estimating standard errors

Trouble shooting: non-convergence - bad start!

Initial values:

1. `sienaAlgorithmCreate`

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     - can be modified by functions `setEffect` and `updateTheta`
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2. With arg `prevAns` passed to `siena07`
   - initial values used from existing `sienaFit` object,
   - Skipping Phase 1 if mods identical

Trouble shooting: non-convergence - when?

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You may try

- start with only rate and density (-effects)
- `updateTheta` $\Rightarrow$ restart

Trouble shooting: non-convergence - normal

Typically, for `tconv.max > 0.25`,

- repeat estimation,
- using the `prevAns` parameter in `siena07`,
- until `tconv.max < 0.25`

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Warning sign

- estimation *diverges right away*
  - check data and model specification;
  - perhaps use a simpler model.
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NB: `siena07` will **tell you** if effects *co-linear* - so don't worry about that

Trouble shooting: non-convergence - brute force

If model resits converging (`tconv.max > 0.25` after many restarts)

- Brute force: increase e.g. n2start and/or n3, with smaller `firstg`
- Better model
- Check for time-heterogeneity
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  - Do you miss important covariates?

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