Multilevel longitudinal network estimation using `sienaBayes`

principles of Bayesian inference

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Fundamentals

Take-home points

What we need to know

- Posterior distribution is
  - The distribution of the unknown parameters
  - given the known data

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  - The probability that the true parameter lies in the 95% Credibility interval **is** 0.95 (given observed data)
  - You may you use the posterior expected value ('average') of the parameter as your point estimate
  - The amount of posterior uncertainty give information in data is captured by the standard deviation of the parameter

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  - You may you use the posterior expected value (‘average’) of the parameter as your point estimate
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- Prior distribbution
  - In order to obtain a posterior distribution you need a prior distribution
  - Different priors give different posteriors for the same data

(Statistical) model

Model

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We call

$$P(\underbrace{Data}_{observable} \mid \overbrace{\theta}^{un-obs.})$$

a model for *Data* when the model allocates probability to different outcomes *Data* we can observe, indexed by some statistical **parameters** θ

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Example (ERGM)

Data are an adjacency matrix $\boldsymbol{x}$

$$\boldsymbol{x} \sim ERGM(\theta) \text{ , Model: } P(\boldsymbol{x} \mid \theta) = e^{\theta^\top z(\boldsymbol{x}) - \psi(\theta)}$$

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Example (SAOM)

Data are an adjacency matrix $\boldsymbol{x}(t_1)$, at time t_1, given $\boldsymbol{x}(t_0)$, at time t_0

$$\boldsymbol{x}(t_1) \mid \boldsymbol{x}(t_0) \sim SAOM(\theta)$$

Inference

Given data, we aim to find parameters θ that data gives most evidence for.

Likelihood

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Likelihood

We can use the model We call

$$P(Data \mid \theta)$$

as a function of θ

$$L(\theta; Data) = \underbrace{P(\overbrace{Data}^{const.} \mid \theta)}_{\text{now a function of } \theta}$$

For different choices of θ, the probability $P(Data \mid \theta)$ will be different!

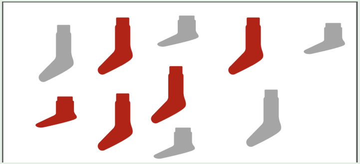
Deriving Bayes theorem

Example (Your sock drawer!)

Probability of picking BIG and red sock

$$\underbrace{P(A,B)}_{4/10=40\%=12/30} \quad =$$

$A = \{\text{sock red}\}$, $\sharp A = 5$
$B = \{\text{sock big}\}$, $\sharp B = 6$



In total 10 (single) socks

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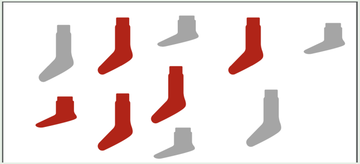

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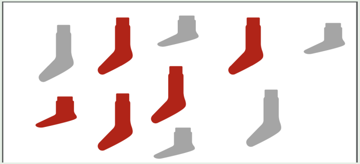

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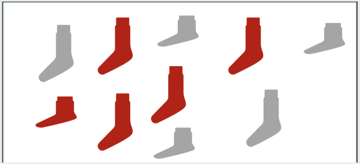

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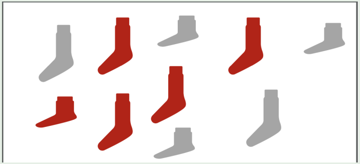

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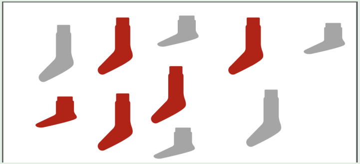

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$$\underbrace{P(A \mid B)}_{\text{red } \textit{given} \text{ BIG}} \; P(B)$$

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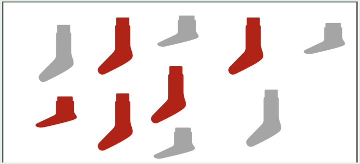

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$$\underbrace{P(A \mid B)}_{\text{red } \textit{given} \text{ BIG}}\ P(B) = \underbrace{P(B \mid A)}_{\text{red } \textit{given} \text{ BIG}}\ P(A)$$

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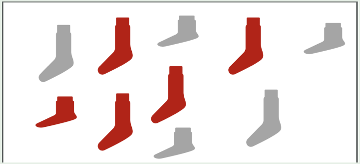

Since

$$\underbrace{P(A, B)}_{\text{both A \& B}} = P(A \mid B)P(B) = P(B \mid A)P(A)$$

we can write red *given* BIG

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

in terms of BIG *given* red

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Ergo, the ‘probability’ for each θ is the likelihood weighted by the a priori ‘probability’

Posterior of tie-probability in Bernoulli graph

Example (Posterior distribution for p in Bernoulli graph)

Assume $\boldsymbol{x} \sim BG(p, n = 5)$, i.e. independently for each $i < j$, $X_{ij} \sim Bern(p)$.

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$$\text{Likelihood} \ : L(p; \boldsymbol{x}) = p^2(1-p)^{10-2}$$

With $p \sim Beta(\alpha, \beta)$ prior

$$\text{Prior} \ : \pi(p) = cp^{\alpha-1}(1-p)^{\beta-1}$$

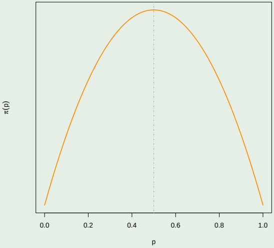
the posterior is $Beta(\alpha^*, \beta^*)$

$$\pi(p \mid \boldsymbol{x}) \propto L(p; \boldsymbol{x})\pi(p) \propto p^{\alpha^*-1}(1-p)^{\beta^*-1}$$

where $\alpha^* = \alpha + 2$ and $\beta^* = \beta + 8$

Example (Posterior distribution for p in Bernoulli graph (A))

$$p^{2-1}(1-p)^{2-1}$$



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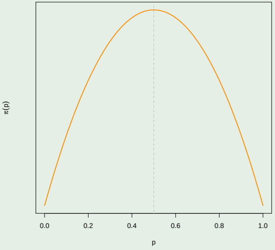

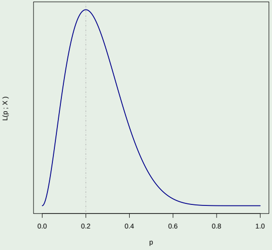

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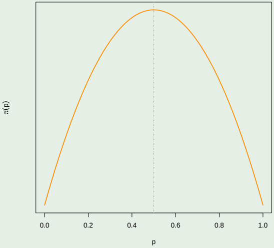

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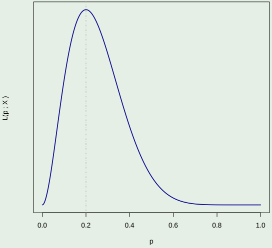
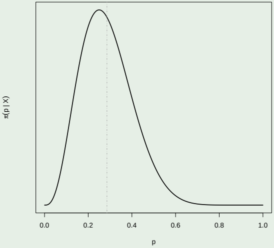


$\times$ Likelihood

$$p^{4-1}(1-p)^{10-1}$$



$\propto$ Posterior

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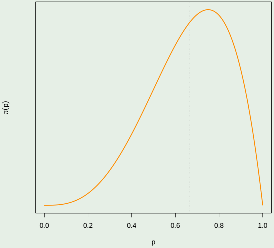
$$p^{4-1}(1-p)^{2-1}$$



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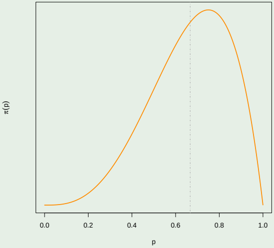

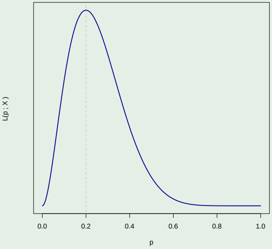

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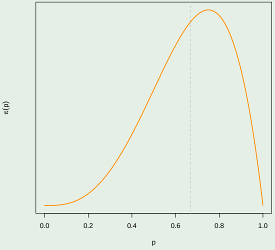
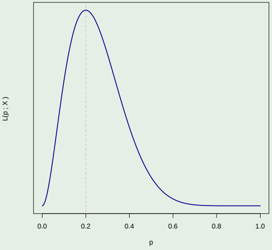
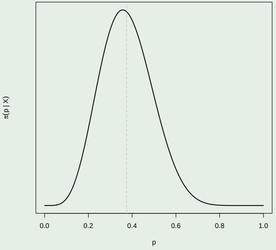
$p^{4-1}(1-p)^{2-1}$ $\qquad$ $p^2(1-p)^8$ $\qquad$ $p^{6-1}(1-p)^{10-1}$







Prior $\qquad$ $\times$Likelihood $\qquad$ $\propto$Posterior

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as **point estimates**, and standard deviations

$$SD(\theta \mid Data) = \sqrt{E\left[(\theta - E(\theta \mid Data))^2 \mid Data\right]}$$

as measures of uncertainty

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$$\hat{\theta} = E(\theta \mid Data) = \frac{\alpha^*}{\alpha^* + \beta^*} \text{, point estimate}$$

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and

$$SD(\theta \mid Data) = \sqrt{\frac{\alpha^* \beta^*}{(\alpha^* + \beta^*)^2(\alpha^* + \beta^* + 1)}} \text{, uncertainty}$$

and with $\alpha = \beta = 2$ a rough 95% credibility interval

$$\hat{\theta} \pm 2SD(\theta \mid Data) = \tfrac{2}{7} \pm 2 \times 0.12 \Rightarrow (0.052, 0.519)$$

Posterior for parameters in SAOM

For

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we cannot obtain the posterior distribution

$$\pi(\theta \mid \boldsymbol{x}(t_1), \boldsymbol{x}(t_0))$$

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Using Markov chain Monte Carlo (MCMC) we can simulate/draw from the posterior

$$\theta^0, \theta^1, \dots, \theta^M \overset{approx.iid}{\sim} \pi(\theta \mid \boldsymbol{x}(t_1), \boldsymbol{x}(t_0))$$

Convergence of MCMC

to target distribution $\pi(\theta \mid Data)$

What does ‘convergence’ look like?

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(a) update θ to $\theta^* = \theta + U$
(b) U is uniform on (-steplength,steplength)
(c) accept move with probability:

$$\frac{\pi(\theta^* \mid \boldsymbol{x})}{\pi(\theta \mid \boldsymbol{x})} = \frac{{\theta^*}^{L+\alpha-1}(1-\theta^*)^{M-L+\beta-1}}{\theta^{L+\alpha-1}(1-\theta)^{M-L+\beta-1}}$$

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(d) starting in $\theta = 1$

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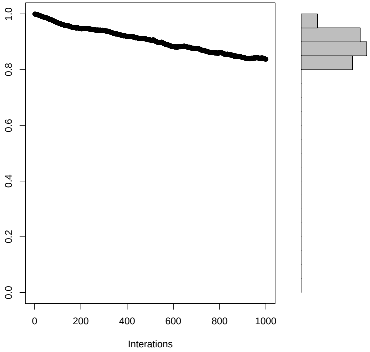

Figure: Steplength: 0.001 too small

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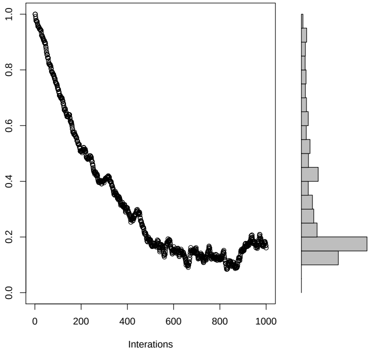

Figure: Steplength: 0.01 still too small

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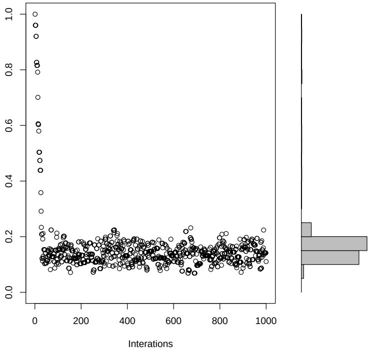

Figure: Steplength: 0.1 looking quite good

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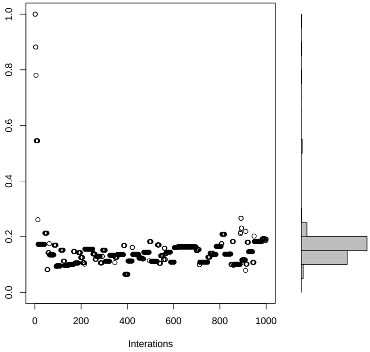
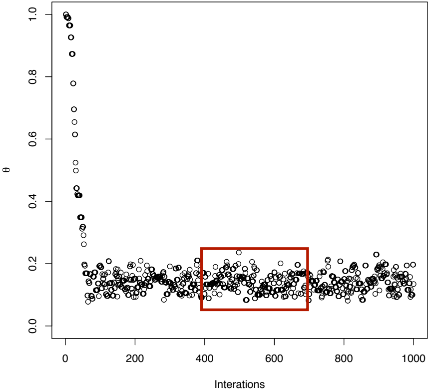


Figure: Steplength: 0.5 maybe too large

Converged to draws from the *same* distribution?



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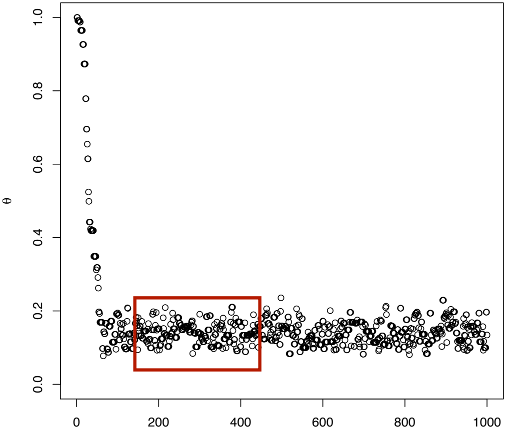

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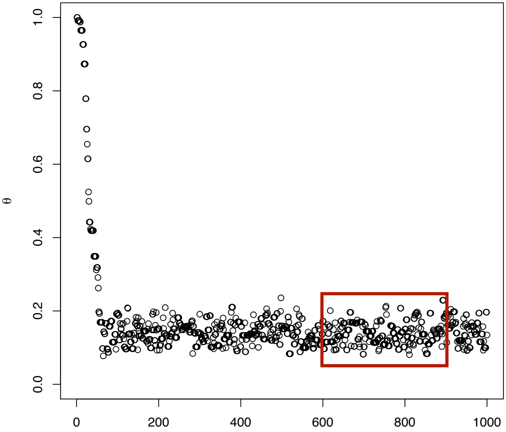

All unobservables have distributions - chain rule

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The posterior distribution for θ is

$$\pi(\theta \mid Data) = \frac{\int P(Data \mid \boldsymbol{y}) P(\boldsymbol{y} \mid \theta) \pi(\theta) d\boldsymbol{y}}{\underbrace{\int \int P(Data \mid \boldsymbol{y}) P(\boldsymbol{y} \mid \theta) \pi(\theta) d\boldsymbol{y} d\theta}}$$

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Example (Hierarchial SAOM)

Data for group j an adjacency matrix $\boldsymbol{x}^{[j]}(t_1)$, at time t_1, given $\boldsymbol{x}^{[j]}(t_0)$, at time t_0

$$\boldsymbol{x}^{[j]}(t_1) \mid \boldsymbol{x}^{[j]}(t_0) \sim SAOM(\theta_j)$$

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Interpretation: For a value on the unknown parameter μ (and Σ), we draw some unknown value θ_j, and then generate data from $\sim SAOM(\theta_j)$.

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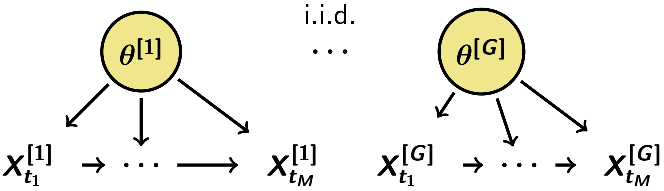
Aim: Find the 'true' value on μ

HSAOM (DAG)

groups: $g = 1, \ldots, G$



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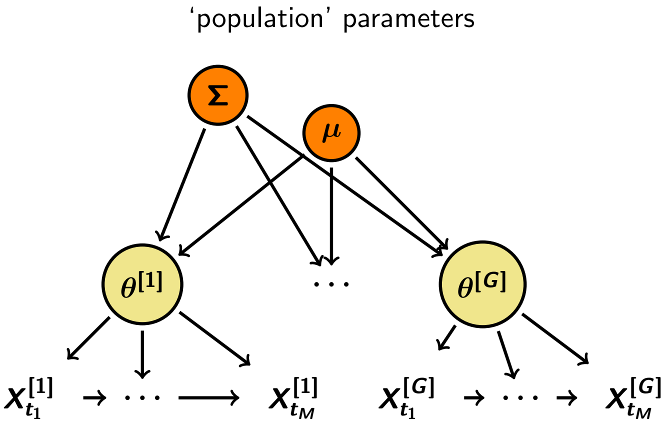

'population' parameters
Σ
μ
$\theta^{[1]}$
$\cdots$
$\theta^{[G]}$
$X_{t_1}^{[1]} \to \cdots \longrightarrow X_{t_M}^{[1]}$
$X_{t_1}^{[G]} \to \cdots \to X_{t_M}^{[G]}$

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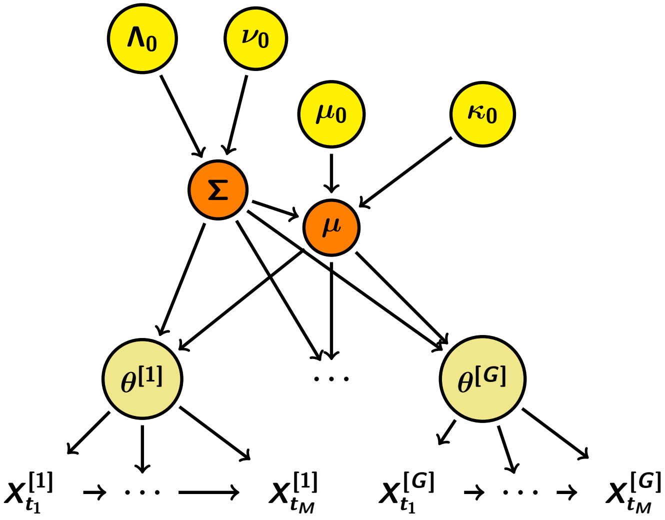

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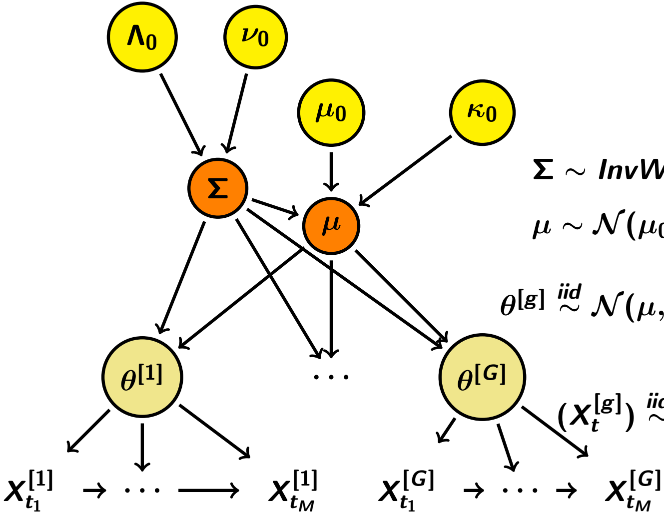

$$\boldsymbol{\Sigma} \sim InvWish_{p_1}(\Lambda_0^{-1}, \nu_0)$$

$$\mu \sim \mathcal{N}(\mu_0, \boldsymbol{\Sigma}/\kappa_0)$$

$$\theta^{[g]} \overset{iid}{\sim} \mathcal{N}(\mu, \boldsymbol{\Sigma})$$

$$(X_t^{[g]}) \overset{iid}{\sim} SAOM(\theta^{[g]})$$

Final Tips

Do you have to decide numbers in priors?

No - not everyone knows as much as Tom

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Who is right?

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e A prior so that $\theta^{[g]} \approx \eta$

When use HSAOM?

- hierarchical data
  - some groups small (borrow strength)
  - many groups with heterogeneity
- intervention: treatment on class-room-level
- network too large: can you decompose network in a natural way? C.p. settings model
- many waves: time heterogeneity potentially with time-covariate - (t_1, t_2),(t_2, t_3), etc, different 'groups'

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YES - you should!

Take-home points

What we need to know

- Posterior distribution is
  - The distribution of the unknown parameters
  - given the known data
- All uncertainty about parameters is described by the posterior distribution
  - The probability that the true parameter lies in the 95% Credibility interval **is** 0.95 (given observed data)
  - You may you use the posterior expected value (‘average’) of the parameter as your point estimate
  - The amount of posterior uncertainty give information in data is captured by the standard deviation of the parameter
- Prior distribbution
  - In order to obtain a posterior distribution you need a prior distribution
  - Different priors give different posteriors for the same data

Theorem