

# Minimal intro to sienaBayes

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## Preamble

### Package

```
require(RSiena)
```

```
## Loading required package: RSiena
```

```
require(multiSiena)
```

```
## Loading required package: multiSiena
```

### Load mock dataset

Use a routine for creating synthetic dataset with 6 networks

```
source("https://raw.githubusercontent.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setup")
```

This dataset, `enzo`,

```
enzo <- create.enzo()
```

is a `sienaGroup` object.

## What data

Plot the 6 networks for the two waves. For each group  $j = 1, \dots, 6$ , we have adjacency matrices

$$\mathbf{x}^{(j)}(t_0), \text{ and } \mathbf{x}^{(j)}(t_1)$$

For example, for group  $j = 1$ ,  $\mathbf{x}^{(j)}(t_0)$

```
enzo$Data1$depvars[[1]][,1]
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    0    0    0
## [2,]    1    0    0    0    0
## [3,]    0    0    0    1    0
## [4,]    0    0    0    0    1
## [5,]    0    0    0    0    0
```

and  $\mathbf{x}^{(j)}(t_1)$

```
enzo$Data1$depvars[[1]][,2]
```

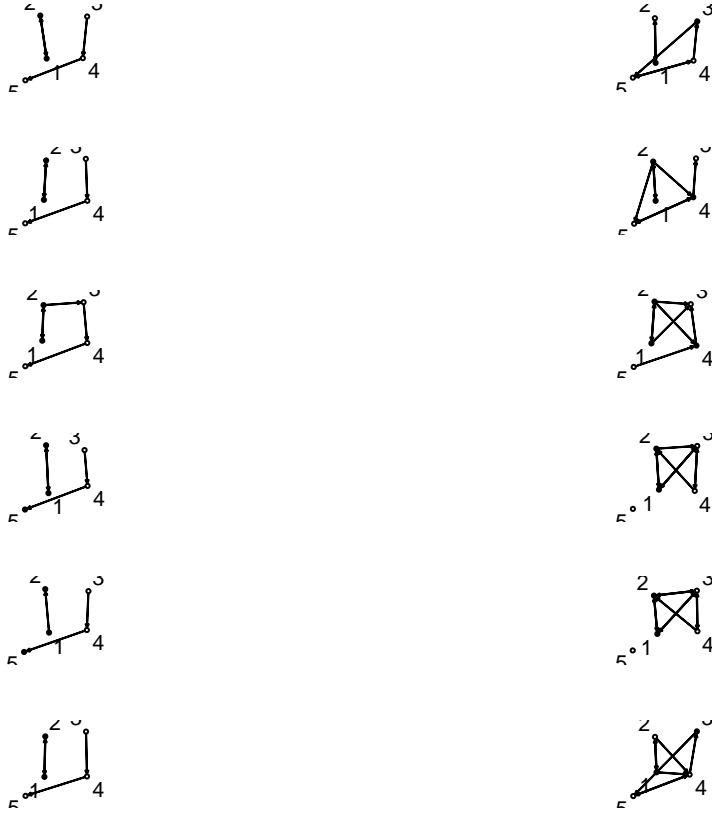
```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    0    0    0
## [2,]    1    0    0    0    0
## [3,]    0    0    0    0    1
## [4,]    0    0    1    0    1
## [5,]    0    0    0    1    0
```

For

```
require(sna)
coord <- matrix(c(1,1,
1,2,
2,2,
2,1,
.5,.5),5,2,byrow=TRUE)
# === plot
par( mfrow = c(6,2) ,oma = c(0,1,0,0) + 0.1,
      mar = c(1,0,1,1) + 0.1)

# grp1
gplot( enzo$Data1$depvars[[1]][,1] ,coord = coord, vertex.col = enzo$Data1$depvars[[2]][,1,1], label=c
gplot( enzo$Data1$depvars[[1]][,2] ,coord = coord, vertex.col = enzo$Data1$depvars[[2]][,1,2], label=c
# grp2
gplot( enzo$Data2$depvars[[1]][,1] ,coord = coord, vertex.col = enzo$Data2$depvars[[2]][,1,1], label=c
gplot( enzo$Data2$depvars[[1]][,2] ,coord = coord, vertex.col = enzo$Data2$depvars[[2]][,1,2], label=c
# grp3
gplot( enzo$Data3$depvars[[1]][,1] ,coord = coord, vertex.col = enzo$Data3$depvars[[2]][,1,1], label=c
gplot( enzo$Data3$depvars[[1]][,2] ,coord = coord, vertex.col = enzo$Data3$depvars[[2]][,1,2], label=c
# grp4
gplot( enzo$Data4$depvars[[1]][,1] ,coord = coord, vertex.col = enzo$Data4$depvars[[2]][,1,1], label=c
gplot( enzo$Data4$depvars[[1]][,2] ,coord = coord, vertex.col = enzo$Data4$depvars[[2]][,1,2], label=c
# grp5
gplot( enzo$Data5$depvars[[1]][,1] ,coord = coord, vertex.col = enzo$Data5$depvars[[2]][,1,1], label=c
```

```
gplot( enzo$Data5$depvars[[1]][,2] , coord = coord, vertex.col = enzo$Data5$depvars[[2]][,1,2] , label=
# grp6
gplot( enzo$Data6$depvars[[1]][,1] , coord = coord, vertex.col = enzo$Data6$depvars[[2]][,1,1] , label=c
gplot( enzo$Data6$depvars[[1]][,2] , coord = coord, vertex.col = enzo$Data6$depvars[[2]][,1,2] , label=c
```



## Basic Model

For each group  $j = 1, \dots, 6$  we define a **stochastic actor-oriented model**

$$\mathbf{x}^{(j)}(t_1) \mid \mathbf{x}^{(j)}(t_0) \sim SAOM(\theta_j)$$

The  $SAOM(\theta)$  is determined by its **objective** and **rate** functions

$$f_i(\mathbf{x}, \theta), \text{ and } \lambda_i(\theta)$$

that are assumed to be the same for all groups  $j = 1, \dots, N$ .

## Define model

You define the *effects* of the objective function of the model in the standard way. The effects structure is obtained from `getEffects()`

```
seed <- 1234
GroupEffects <- getEffects(enzo)
GroupsModel <- sienaAlgorithmCreate(projname = 'Enzo', seed=seed)
```

## If you use this algorithm object, siena07 will create/use an output file Enzo.txt .

## Hierarchical model

All groups have the same effects and model,  $SAOM(\theta_j)$ , but some of the parameters,  $\gamma_j$ , vary across groups, and some,  $\eta$ , are the same for all groups.

$$\theta_j = \begin{bmatrix} \gamma_j \\ \eta \end{bmatrix}$$

### Group-varying and fixed

By default

```
summary(GroupEffects)$random
```

```
## [1] FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE FALSE FALSE FALSE
## [13] FALSE FALSE FALSE
```

```
summary(GroupEffects)$effectName[summary(GroupEffects)$random]
```

```
## [1] "outdegree (density)"
```

```
summary(GroupEffects)$effectName[summary(GroupEffects)$random==FALSE]
```

```
## [1] "constant net rate (period 1)" "constant net rate (period 3)"
## [3] "constant net rate (period 5)" "constant net rate (period 7)"
## [5] "constant net rate (period 9)" "constant net rate (period 11)"
## [7] "reciprocity"                  "rate beh (period 1)"
## [9] "rate beh (period 3)"          "rate beh (period 5)"
## [11] "rate beh (period 7)"         "rate beh (period 9)"
## [13] "rate beh (period 11)"        "beh linear shape"
```

### Model for group-varying

The model assumes that the group-varying parameters follow a **multivariate normal distribution**

$$\gamma_j \sim N(\mu, \Sigma)$$

## Target of inference

We want to estimate the population mean  $\mu$  and the non-varying parameter  $\eta$ .

### Running default

To estimate the model with default settings

```
groupModel.e <- sienaBayes(GroupsModel, data = enzo,
  initgainGlobal=0.1, initgainGroupwise = 0.001,
  effects = GroupEffects,
  nrunMHBatches=40, silentstart=FALSE)
```

## Results

What did we get?

Check objects returned

```
names(groupModel.e)
```

## Non-varying

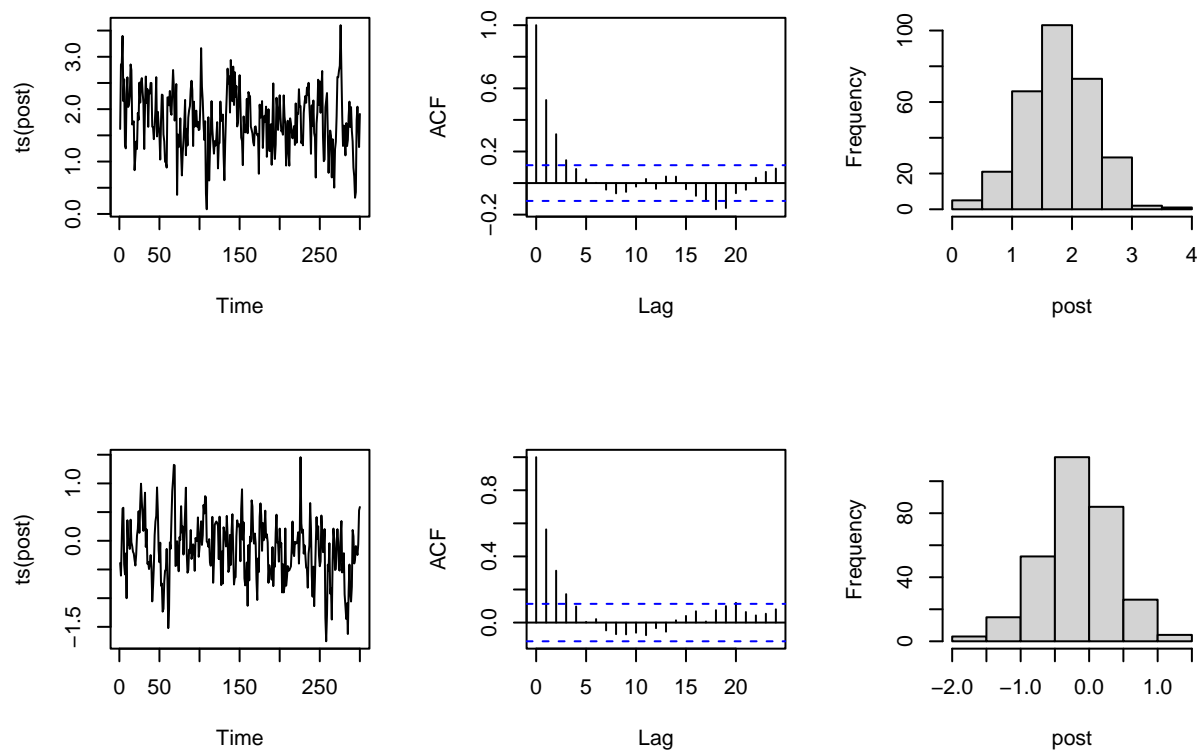
The `nmain`: 300 posterior draws of  $\eta$  are in `ThinPosteriorEta`

```
dim(groupModel.e$ThinPosteriorEta)
```

```
## [1] 300  2
```

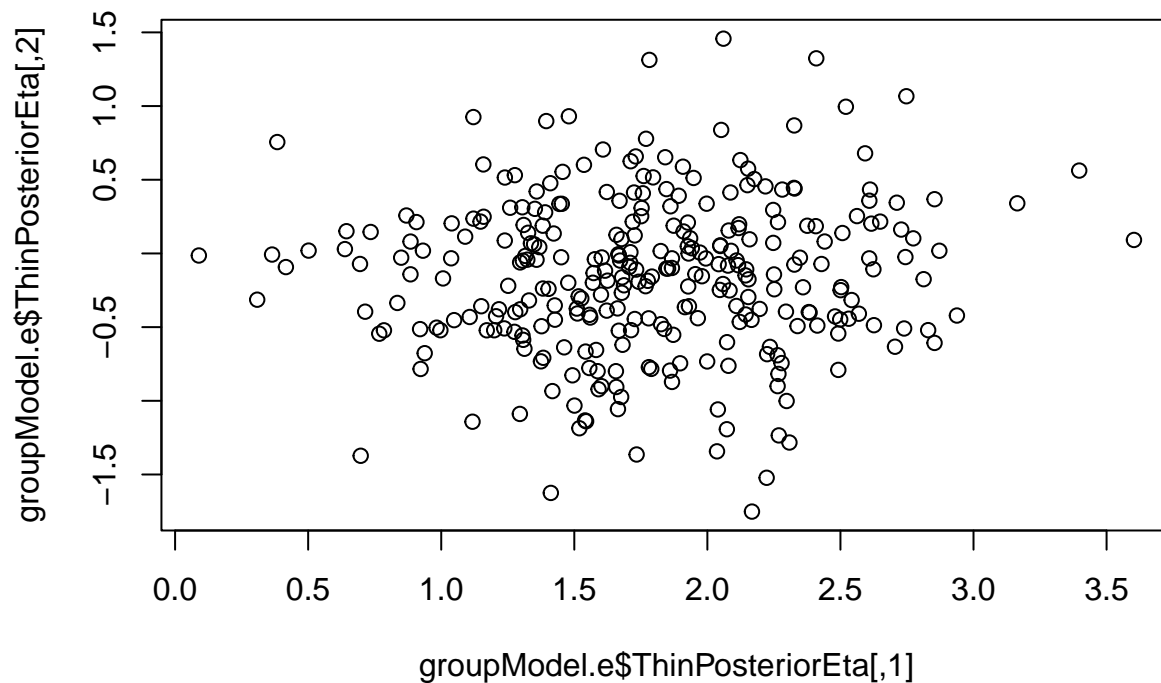
This corresponds to reciprocity and the linear shape for behaviour

```
par(mfrow=c(2,3))
for (k in c(1:2)){
  post <- groupModel.e$ThinPosteriorEta[,k]
  plot(ts(post))# draws in each iteration
  acf(post,main='')# the autocorrelation function - how much dependence
  hist(post,main='')# (empirical) posterior distribution
}
```



Note that these are draws from a bivariate distribution

```
plot(groupModel.e$ThinPosteriorEta)
```



We can calculate probabilities for the parameters using the values simulated from the posterior. For example, we can calculate the probability that the reciprocity parameter is positive **given data** as

```
mean(groupModel.e$ThinPosteriorEta[,1]>0)
```

```
## [1] 1
```

Given these 6 (synthetic) networks, the reciprocity parameter is positive with a posterior probability of 1

### Population parameters

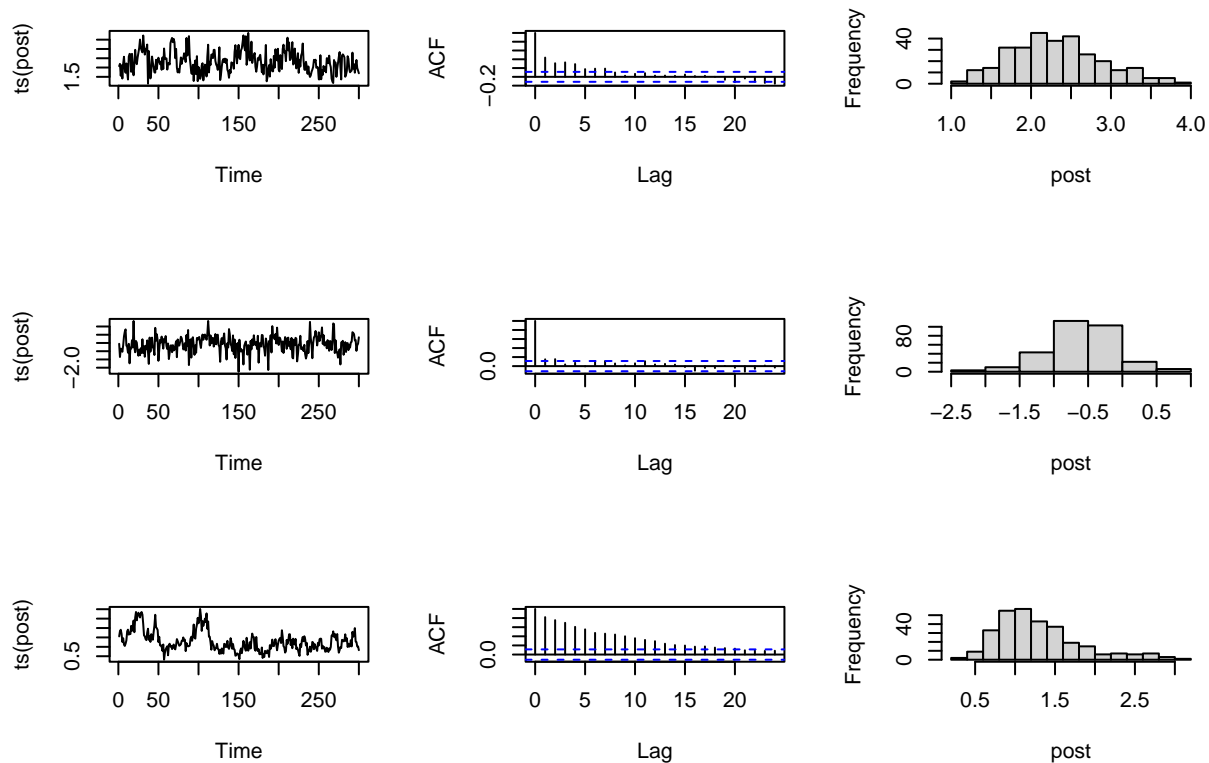
The posteriors for  $\mu$  are found in ThinPosteriorMu

```
dim(groupModel.e$ThinPosteriorMu)
```

```
## [1] 300 3
```

Corresponding to the rate for the network, density, and rate for behaviour

```
par(mfrow=c(3,3))
for (k in c(1:3)){
  post <- groupModel.e$ThinPosteriorMu[,k]
  plot(ts(post))
  acf(post,main='')
  hist(post,main='')
}
```



## Summary of results

To get a table to numerical summaries of the posteriors

```
summary(groupModel.e)
```

```
## Bayesian estimation.
## Prior distribution:
##
## Mu      basic rate parameter net    2.5161
##         outdegree (density)         0.0000
##         rate beh period 1           0.7323
##
## Sigma   0.5812  0.0000 -0.1585
##         0.0000  1.0000  0.0000
##         -0.1585  0.0000  0.1310
##
## Prior Df  5
##
## Kappa     0.0000
##
## Eta
## For the fixed parameters, constant prior.
##
## Algorithm specifications were  nprewarm = 50 , nwarm = 50 , nmain = 250 , nrunMHBatches = 40 , nImpr
## nSampVarying = 1 , nSampConst = 1 , mult = 5 .
## Posterior means and standard deviations are averages over the last 250 runs.
##
## Proportion of acceptances in MCMC proposals after warming up:
## 0.19 0.20 0.23 0.24 0.26 0.31 0.25 0.25
```

```

##
## This should ideally be between 0.15 and 0.50.
## Note: this summary does not contain a convergence check.
## Note: the print function for sienaBayesFit objects can also use a parameter nfirst,
##       indicating the first run from which convergence is assumed.
##
## Groups:
##   Data1   Data2   Data3   Data4   Data5   Data6
##
## Posterior means and standard deviations for global mean parameters
##
## Total number of runs in the results is 300 .
## Posterior means and standard deviations are averages over 250 MCMC runs (excluding warming, after th
##
##           Post.      Post.      cred.      cred.      p      varying      Post
##           mean      s.d.m.      from      to
##
## Network Dynamics
##   1. rate constant net rate (period 1)  2.1459 ( 0.7326 )  0.8441  3.4185
##   2. rate constant net rate (period 3)  2.1719 ( 0.7819 )  0.9366  3.9436
##   3. rate constant net rate (period 5)  2.3410 ( 0.7160 )  1.2694  3.7795
##   4. rate constant net rate (period 7)  2.3381 ( 0.7119 )  1.1655  3.7801
##   5. rate constant net rate (period 9)  2.5540 ( 0.7121 )  1.2057  3.9846
##   6. rate constant net rate (period 11) 2.2758 ( 0.6683 )  1.1175  3.7375
##   7. eval outdegree (density)          -0.5602 ( 0.4986 ) -1.6841  0.4112  0.09      +      0.97
##   8. eval reciprocity                  1.7251 ( 0.5568 )  0.6394  2.7458  1.00      -
##
## Behavior Dynamics
##   9. rate rate beh (period 1)          1.2257 ( 0.5262 )  0.4060  2.4141
##  10. rate rate beh (period 3)          1.1665 ( 0.5480 )  0.2743  2.6184
##  11. rate rate beh (period 5)          1.1340 ( 0.5262 )  0.3071  2.4207
##  12. rate rate beh (period 7)          1.1159 ( 0.5089 )  0.3590  2.4234
##  13. rate rate beh (period 9)          1.0906 ( 0.5191 )  0.2869  2.3592
##  14. rate rate beh (period 11)         1.2065 ( 0.5260 )  0.3976  2.5006
##  15. eval beh linear shape             -0.1760 ( 0.5260 ) -1.2722  0.8481  0.36      -
##
## Posterior mean of global covariance matrix (varying parameters)
##   0.6371 -0.0077 -0.1424
##  -0.0077  0.9590  0.0054
##  -0.1424  0.0054  0.1591
##
## Posterior standard deviations of elements of global covariance matrix
##   0.3928  0.3808  0.1483
##   0.3808  0.8117  0.1823
##   0.1483  0.1823  0.0921
##
## For the rate parameters across all groups:
##           Post.mean Post.sd
## basic rate parameter net  2.30241 0.55871
## rate beh period 1        1.16753 0.44662

```



## Changing hierarchial model

In the previous model, `reciprocity` was assumed to have the same parameter  $\theta_{j,rec} = \eta_{rec}$ , for all  $j = 1, \dots, 6$ . To allow the `reciprocity` parameter to *vary* across groups  $j$ , set

```
GroupEffects <- setEffect( GroupEffects, recip, random=TRUE)
```

```
## effectName shortName include fix test initialValue parm randomEffects
## 1 reciprocity recip TRUE FALSE FALSE 0 0 TRUE
```

so that now

$$\theta_j = \begin{bmatrix} \gamma_{j,rate_x} \\ \gamma_{j,dens} \\ \gamma_{j,rec} \\ \gamma_{j,rate_z} \\ \eta_{shape,z} \end{bmatrix}, \text{ and } \gamma_j \sim \mathcal{N}_4(\mu, \Sigma)$$

### Running new model

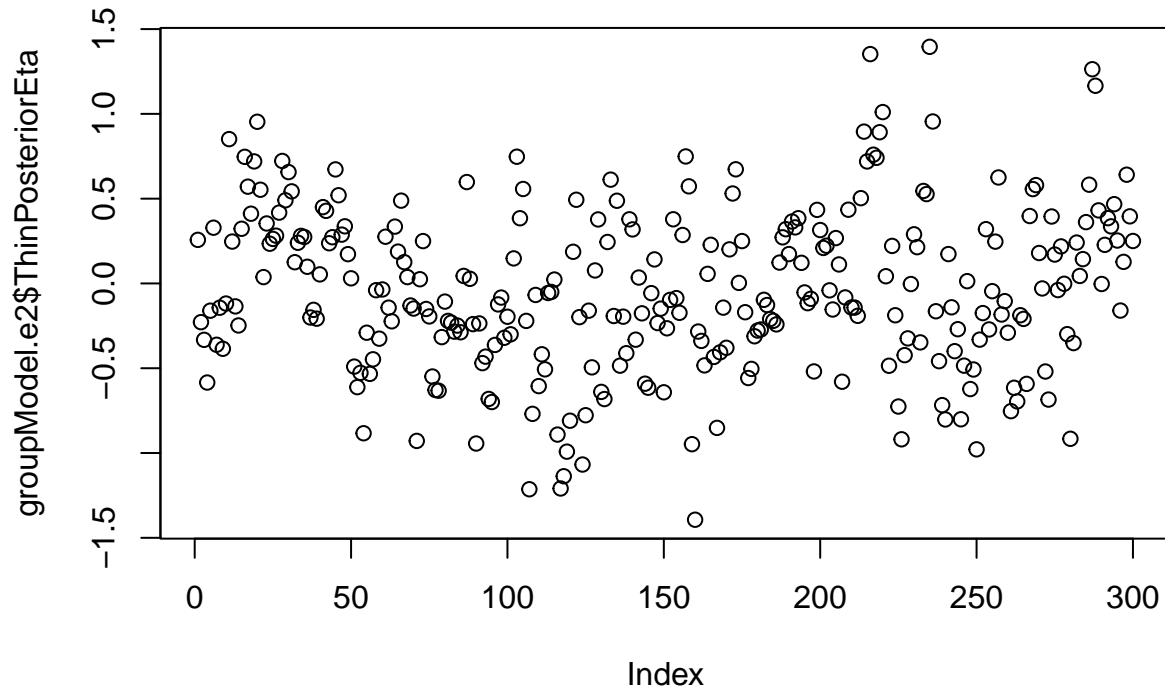
To estimate the model with one more random effect

```
groupModel.e2 <- sienaBayes(GroupsModel, data = enzo,
  initgainGlobal=0.1, initgainGroupwise = 0.001,
  effects = GroupEffects,
  nrunMHBatches=40, silentstart=FALSE)
```

### Output

Now we only have one common parameter, one  $\eta$

```
plot(groupModel.e2$ThinPosteriorEta)
```

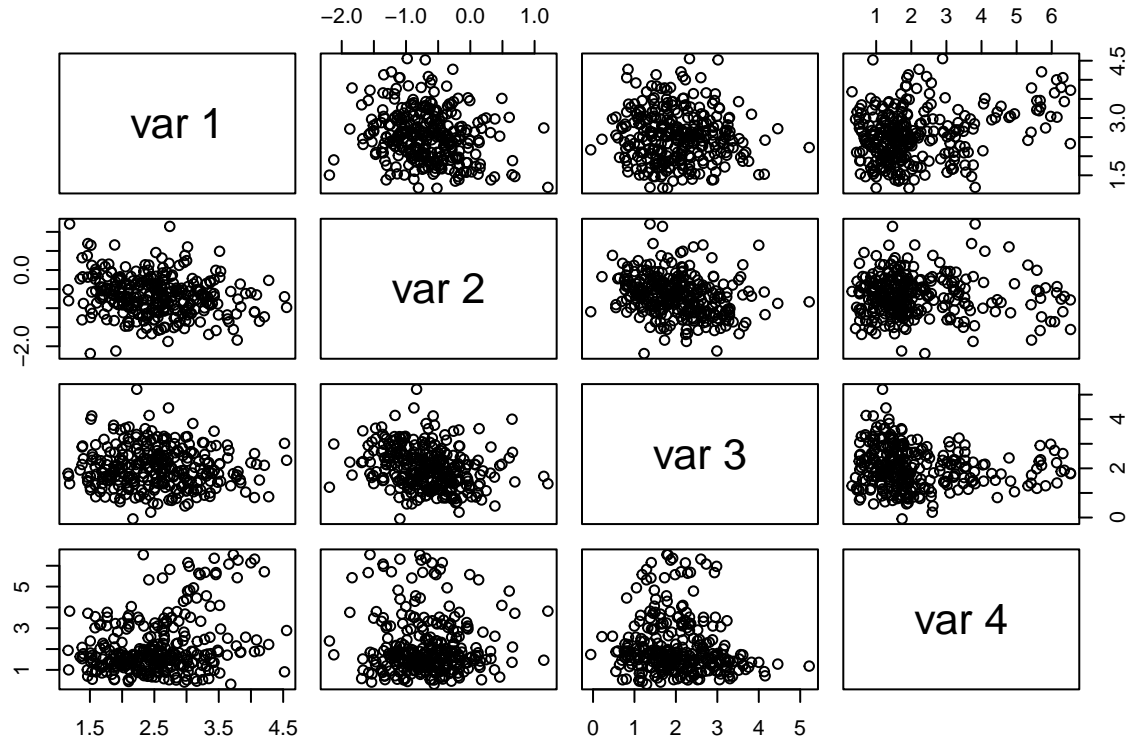


In the plot we see the 300 simulated values of  $\eta$  that are drawn using MCMC from the posterior distribution of  $\eta$  given data.

Given these 6 (synthetic) networks, the reciprocity parameter is positive with a posterior probability of 1

But we have  $\mu$  has four dimensions (parameters)

```
pairs(groupModel.e2$ThinPosteriorMu)
```



We can calculate the probability that the reciprocity parameter is positive **given data** as

```
mean(groupModel.e2$ThinPosteriorMu[,3]>0)
```

```
## [1] 0.9966667
```

## Group-level analysis

For reciprocity, we saw in the previous example that

$$\Pr(\mu_{rec} > 0 \mid Data) \approx 1$$

but what about the group-level parameters  $\gamma_{1,rec}, \gamma_{2,rec}, \dots, \gamma_{6,rec}$  - are they always positive for all of the groups?

## Group and population-level

The posterior (predictive) distributions of the  $\gamma_j$ 's are stored in `ThinParameters` as **iteration** by **group** by **parameter**

```
dim(groupModel.e2$ThinParameters)
```

```
## [1] 300 6 5
```

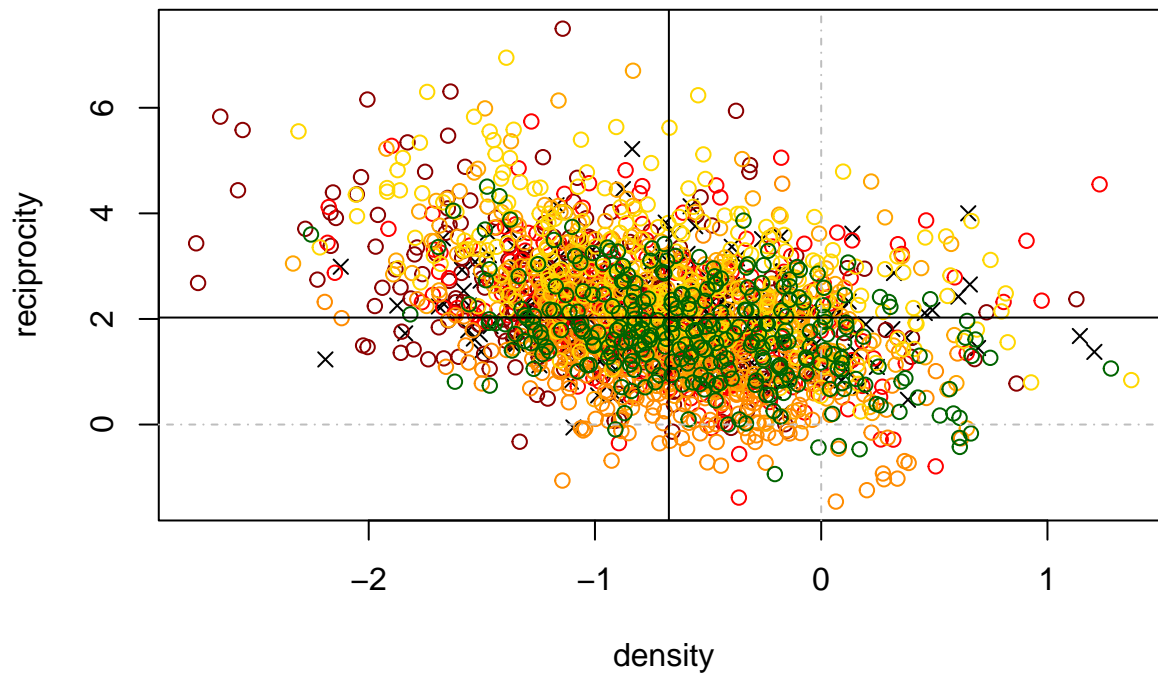
and within each group

```
head(groupModel.e2$ThinParameters[,1,])
```

```
##      constant net rate outdegree (density) reciprocity rate beh
## [1,]      3.084515          -1.8143328  2.38747125 1.188040
## [2,]      2.534277          -0.8095603  2.10497307 1.392641
## [3,]      3.319430          -0.5882144  0.66770252 1.857220
## [4,]      2.482389          -1.6900750  1.84137145 2.622582
## [5,]      3.122526          -1.6681452  1.25374655 1.728405
## [6,]      3.727048          -0.9129924  0.06870421 1.269398
##      beh linear shape
## [1,]      0.2569260
## [2,]     -0.2283378
## [3,]     -0.3325189
## [4,]     -0.5843580
## [5,]     -0.1605018
## [6,]      0.3287635
```

To illustrate how we have inference for  $\mu$  for density and reciprocity, but also for  $\gamma_j$  for density and reciprocity for each group, let us plot density against reciprocity for both levels

```
plot(groupModel.e2$ThinPosteriorMu[,2],
      groupModel.e2$ThinPosteriorMu[,3],
      xlim=range(groupModel.e2$ThinParameters[,2]),
      ylim=range(groupModel.e2$ThinParameters[,3]),
      pch=4, xlab='density', ylab='reciprocity')
grp.cols <- c('darkred', 'red', 'darkorange', 'orange', 'gold', 'darkgreen')
for (j in c(1:6))
{
  lines( groupModel.e2$ThinParameters[,j,2], groupModel.e2$ThinParameters[,j,3], type='p', pch=1, col=grp.cols[j])
}
abline(v=mean(groupModel.e2$ThinPosteriorMu[,2]))
abline(h=mean(groupModel.e2$ThinPosteriorMu[,3]))
abline(v=0, col='grey', lty=4)
abline(h=0, col='grey', lty=4)
```

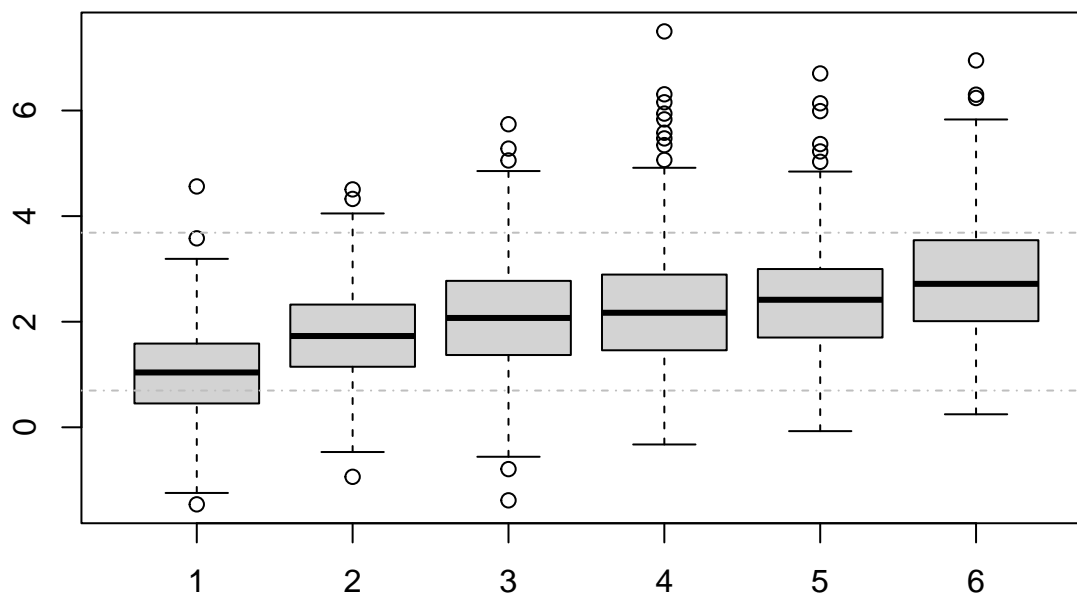


For an individual parameter it is common to look at the caterpillar plot

```
require(HDInterval)
```

```
## Loading required package: HDInterval
```

```
grp.means <- colMeans(groupModel.e2$ThinParameters[, ,3])
boxplot(groupModel.e2$ThinParameters[, ,order(grp.means),3])
CI <- hdi(groupModel.e2$ThinPosteriorMu[,3],1-.05)
abline(h=CI[1],col='grey',lty=4)
abline(h=CI[2],col='grey',lty=4)
```



## Prior and posterior

To get a posterior distribution for  $\mu$  and  $\eta$  (and  $\Sigma$ ), we need to have **prior distributions** for these parameters

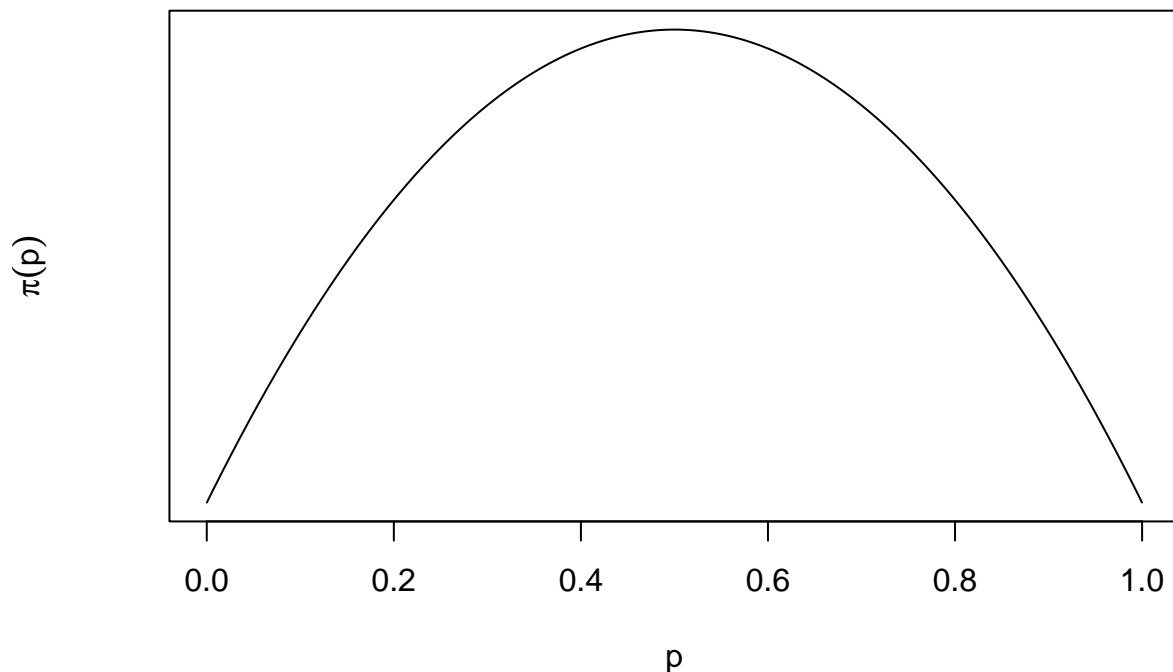
A prior distribution for a parameter quantifies the uncertainty that we have about a parameter before we collect and observe DATA

### Example: Bernoulli graph

For a cross-section  $n = 5$  network, a Bernoulli graph says that each of the  $5(5 - 1)/2 = 10$  possible ties all have a probability of  $p$  of being present, independently of each other. The parameter  $0 \leq p \leq 1$  and hence a prior for  $p$  might be  $p \sim \text{Beta}(\alpha, \beta)$

```
alpha.p <- 2  
beta.p <- 2
```

```
p <- seq(from=.0001,to=.9999, length.out = 1000)  
plot( p , dbeta(p, alpha.p, beta.p) , type = 'l', yaxt='n',ylab=expression(pi(p)))
```

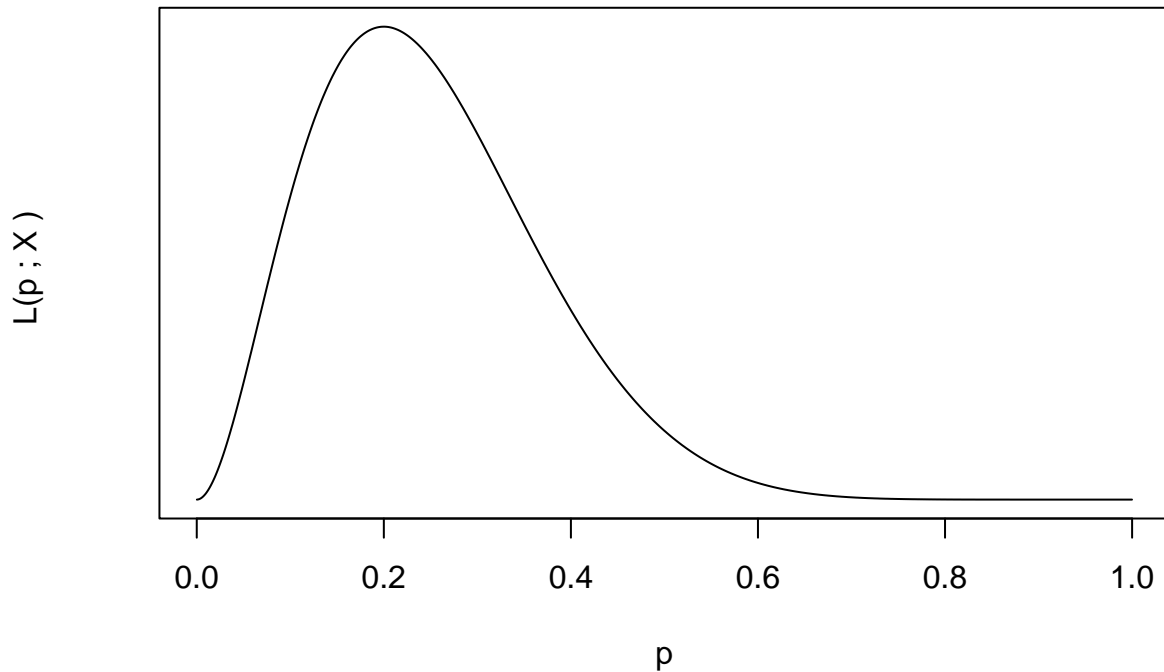


Assume that we observe the network

$$\mathbf{X} = \begin{bmatrix} - & 1 & 0 & 0 & 0 \\ - & - & 0 & 1 & 0 \\ - & - & - & 0 & 0 \\ - & - & - & - & 0 \\ - & - & - & - & - \end{bmatrix}$$

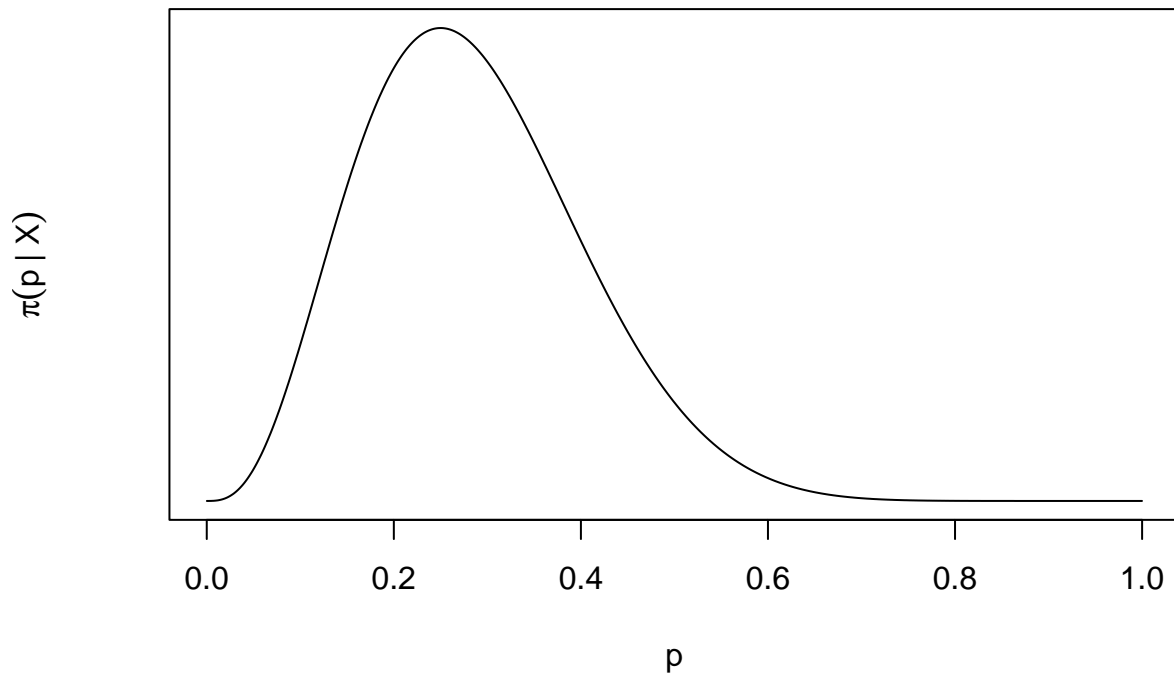
This has  $L = \sum_{i < j} x_{ij} = 2$ , which gives the **likelihood** function for  $p$

```
plot(p,p^2*(1-p)^8, type = 'l', yaxt='n',ylab='L(p ; X ) ' )
```



The posterior distribution will be  $p \mid \mathbf{X} \sim \text{Beta}(2 + \alpha, 8 + \beta)$

```
plot( p , dbeta(p, alpha.p+2, beta.p+8) , type = 'l', yaxt='n',ylab=expression(pi(p ~"|"~ X)))
```



Try with different values of the hyperparameters  $\alpha$  and  $\beta$  in the prior for  $p$

Even better, try Mattias Villani's widget that illustrates Bayesian inference [https://observablehq.com/@mattiasvillani/bayesian-inference-for-bernoulli-iid-data?collection=@mattiasvillani/bayesian-learning\[Beta-Binomial\]](https://observablehq.com/@mattiasvillani/bayesian-inference-for-bernoulli-iid-data?collection=@mattiasvillani/bayesian-learning[Beta-Binomial])

## More and bigger

Load routines

Create two waves for  $M$  networks, all of size  $n$

```
n <- 10 # you could make this larger or smaller
M <- 5 # this is very few groups and you could try to produce many
```

The Karen dataset has as many groups as you like

```
Karen <- data.set.karen.set.up(n=10, # number of nodes
                               M=5, # number of networks
                               seed=123, nbrNodes=1, nmain=20, nwarm=20)
```

The networks are simulated from a model with effects, that in addition to the defaults have

- transTrip
- simX with interaction1 = "beh"

Now, figure out what inputs you need. The `sienaGroup` object is `Karen$my.Karen`

```
Karen$my.Karen
```

```
## Dependent variables:
## net : oneMode
## beh : behavior
## Total number of groups: 5
## Total number of periods: 5
```

For different model specifications, think of what priors you need to specify. Which ones should be random and which fixed?

With the same prior, how does increasing  $M$  affect inference?

Increasing the prior variance ( $\Sigma$ ), how does that affect inference?

```
groupModel.e <- sienaBayes(Karen$GroupsModel, data = Karen$my.Karen,
                           initgainGlobal=0.1, initgainGroupwise = 0.001,
                           effects = Karen$effects, priorMu = Karen$priorMu, priorSigma = Karen$priorS,
                           priorKappa = 0.01,
                           nwarm=Karen$nwarm, nmain=Karen$nmain, nrunMHBatches=40,
                           nbrNodes=Karen$nbrNodes, silentstart=FALSE)
```