Minimal intro to siena Bayes

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Preamble

This takes you through the very basics of Bayesian estimation forstochastic actor-oriented models (SAOMs). We refer to the multiSiena workshop page https://www.stats.ox.ac.uk/~snijders/siena/Workshop_sienaBayes_2024.htm for further resources. Here we will

- Provide you with a minimal example for sienaBayes to take home with you
- Illustrate the difference between 'random' and 'fixed' effects
- Provide some very brief pointers for Bayesian inference
- Explore group-level parameters

It is appropriate that we **simulate** the model as Siena stands for

Simulation Investigation for Empirical Network Analysis

Packages

We will use functionality from the network packages sna and network (see https://raw.githubusercontent.com/johankoskinen/CHDH-SNA/main/Markdowns/CHDH-SNA-2.Rmd). The main packages for SAOM is

RSiena.

You will only be able to run the code in here if you have already installed the latest version of multiSiena.

```
require(RSiena)

## Loading required package: RSiena
require(multiSiena)

## Loading required package: multiSiena
```

Load mock dataset

Use a routine for creating synthetic dataset with 6 networks

 $\textbf{source("https://raw.githubusercontent.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setupnoordinate.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setupnoordinate.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setupnoordinate.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setupnoordinate.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setupnoordinate.com/johankoskinen/Sunbelt2024/main/multiSiena/data.set.enzo.setupnoordinate.setupnoordin$

This dataset, enzo,

```
enzo <- create.enzo()</pre>
```

is a sienaGroup object.

What data

Plot the 6 networks for the two waves. For each group $j = 1, \ldots, 6$, we have adjacency matrices

$$x^{(j)}(t_0)$$
, and $x^{(j)}(t_1)$

For example, for group j = 1, $\boldsymbol{x}^{(j)}(t_0)$

```
enzo$Data1$depvars[[1]][,,1]
```

```
##
         [,1] [,2] [,3] [,4] [,5]
## [1,]
                  1
                        0
## [2,]
                                   0
            1
                  0
                        0
## [3,]
                                   0
            0
                  0
                        0
                             1
## [4,]
                                   1
            0
                  0
                        0
## [5,]
                        0
and x^{(j)}(t_1)
```

enzo\$Data1\$depvars[[1]][,,2]

```
##
        [,1] [,2] [,3] [,4] [,5]
## [1,]
                            0
           0
                 1
                       0
                                  0
## [2,]
           1
## [3,]
                       0
                            0
           0
                 0
                                 1
## [4,]
                       1
                            0
                                 1
## [5,]
```

For

```
require(sna)
coord <- matrix(c(1,1,
1,2,
2,2,
2,1,
.5,.5),5,2,byrow=TRUE)
# === plot</pre>
```

```
par(mfrow = c(6,2), oma = c(0,1,0,0) + 0.1,
         mar = c(1,0,1,1) + 0.1)
gplot( enzo$Data1$depvars[[1]][,,1] ,coord = coord, vertex.col = enzo$Data1$depvars[[2]][,1,1], label=c
gplot( enzo$Data1$depvars[[1]][,,2] , coord = coord, vertex.col = enzo$Data1$depvars[[2]][,1,2], label=
gplot( enzo$Data2$depvars[[1]][,,1] ,coord = coord, vertex.col = enzo$Data2$depvars[[2]][,1,1], label=c
gplot( enzo$Data2$depvars[[1]][,,2] , coord = coord, vertex.col = enzo$Data2$depvars[[2]][,1,2], label=
# qrp3
gplot( enzo$Data3$depvars[[1]][,,1] ,coord = coord, vertex.col = enzo$Data3$depvars[[2]][,1,1], label=c
gplot( enzo$Data3$depvars[[1]][,,2] , coord = coord, vertex.col = enzo$Data3$depvars[[2]][,1,2], label=
gplot( enzo$Data4$depvars[[1]][,,1] ,coord = coord, vertex.col = enzo$Data4$depvars[[2]][,1,1], label=c
gplot( enzo$Data4$depvars[[1]][,,2] , coord = coord, vertex.col = enzo$Data4$depvars[[2]][,1,2], label=
gplot( enzo$Data5$depvars[[1]][,,1] ,coord = coord, vertex.col = enzo$Data5$depvars[[2]][,1,1], label=c
gplot( enzo$Data5$depvars[[1]][,,2] , coord = coord, vertex.col = enzo$Data5$depvars[[2]][,1,2], label=
gplot( enzo$Data6$depvars[[1]][,,1] ,coord = coord, vertex.col = enzo$Data6$depvars[[2]][,1,1], label=c
gplot( enzo$Data6$depvars[[1]][,,2] , coord = coord, vertex.col = enzo$Data6$depvars[[2]][,1,2], label=
```

Basic Model

For each group j = 1, ..., 6 we define a stochastic actor-oriented model

$$\boldsymbol{x}^{(j)}(t_1) \mid \boldsymbol{x}^{(j)}(t_0) \sim SAOM(\theta_j)$$

The $SAOM(\theta)$ is determined by its **objective** and **rate** functions

$$f_i(\boldsymbol{x}, \theta)$$
, and $\lambda_i(\theta)$

that are assumed to be the same for all groups j = 1, ..., N.

Define model

You define the *effects* of the objective function of the model in the standard way. The effects structure is obtained from getEffects()

```
seed <- 1234
GroupEffects <- getEffects(enzo)
GroupsModel <- sienaAlgorithmCreate(projname = 'Enzo', seed=seed)</pre>
```

If you use this algorithm object, siena07 will create/use an output file Enzo.txt .

Hierarchial model

All groups have the same effects and model, $SAOM(\theta_j)$, but some of the parameters, γ_j , vary across groups, and some, η , are the same for all groups.

$$\theta_j = \begin{bmatrix} \gamma_j \\ \eta \end{bmatrix}$$

Group-varying and fixed

By default

```
summary(GroupEffects)$random
              [1] FALSE FA
## [13] FALSE FALSE FALSE
summary(GroupEffects)$effectName[summary(GroupEffects)$random]
## [1] "outdegree (density)"
summary(GroupEffects)$effectName[summary(GroupEffects)$random==FALSE]
##
                [1] "constant net rate (period 1)" "constant net rate (period 3)"
              [3] "constant net rate (period 5)"
                                                                                                                                                                    "constant net rate (period 7)"
              [5] "constant net rate (period 9)" "constant net rate (period 11)"
              [7] "reciprocity"
                                                                                                                                                                     "rate beh (period 1)"
## [9] "rate beh (period 3)"
                                                                                                                                                                     "rate beh (period 5)"
## [11] "rate beh (period 7)"
                                                                                                                                                                     "rate beh (period 9)"
```

Model for group-varying

[13] "rate beh (period 11)"

The model assumes that the group-varying parameters follow a multivariate normal distribution

$$\gamma_j \sim N(\mu, \Sigma)$$

"beh linear shape"

Target of inference

We want to estimate the population mean μ and the non-varying parameter η .

Running default

To estimate the model with default settings

Results

What did we get?

Check objects returned

```
names(groupModel.e)
```

Non-varying

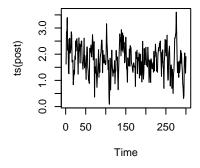
The nmain: 300 posterior draws of η are in ThinPosteriorEta

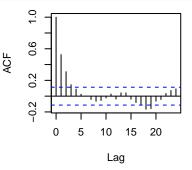
```
dim(groupModel.e$ThinPosteriorEta)
```

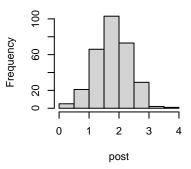
```
## [1] 300 2
```

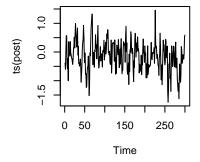
This corresponds to reciprocity and the linear shape for behaviour

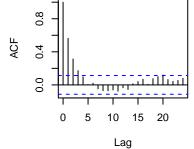
```
par(mfrow=c(2,3))
for (k in c(1:2)){
  post <- groupModel.e$ThinPosteriorEta[,k]
plot(ts(post))# draws in each iteration
  acf(post,main='')# the autocorrelation function - how much dependence
  hist(post,main='')# (empirical) posterior distribution
}</pre>
```

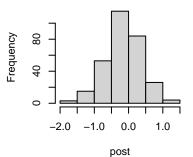






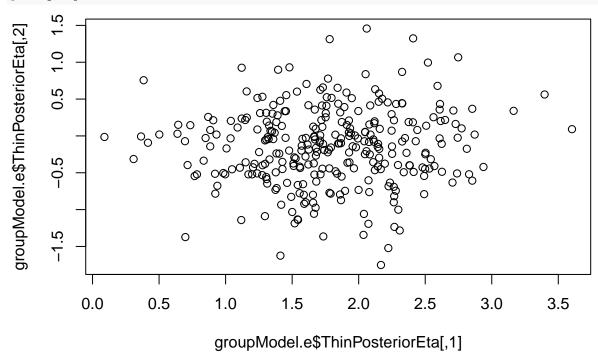






Note that these are draws from a bivariate distribution

plot(groupModel.e\$ThinPosteriorEta)



We can calculate probabilites for the parameters using the values simulated from the posterior For example, we can calculate the probability that the reciprocity parameter is positive **given data** as mean(groupModel.e\$ThinPosteriorEta[,1]>0)

[1] 1

Given these 6 (synthetic) networks, the reciprocity parameter is positive with a posterior probability of 1

Population parameters

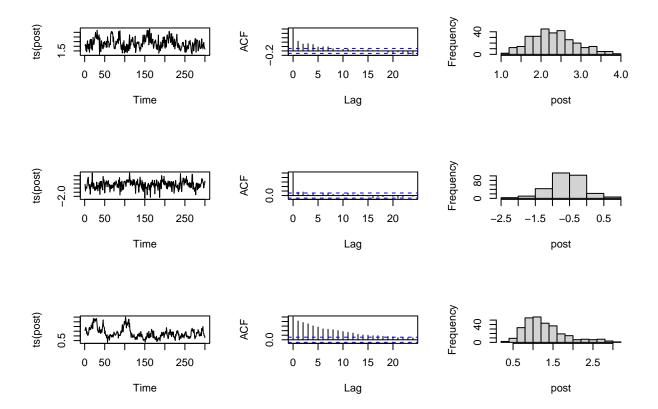
The posteriors for μ are found in ThinPosteriorMu

```
dim(groupModel.e$ThinPosteriorMu)
```

[1] 300 3

Corresponding to the rate for the network, density, and rate for behaviour

```
par(mfrow=c(3,3))
for (k in c(1:3)){
  post <- groupModel.e$ThinPosteriorMu[,k]
plot(ts(post))
acf(post,main='')
hist(post,main='')
}</pre>
```



Summary of results

summary(groupModel.e)

To get a table to numerical summaries of the posteriors

```
## Bayesian estimation.
## Prior distribution:
##
## Mu
           basic rate parameter net
                                         2.5161
##
           outdegree (density)
                                         0.0000
##
           rate beh period 1
                                         0.7323
##
             0.5812
                       0.0000
                                -0.1585
## Sigma
             0.0000
                       1.0000
                                 0.0000
##
            -0.1585
                       0.0000
                                 0.1310
##
##
            5
## Prior Df
##
## Kappa
             0.0000
##
## Eta
## For the fixed parameters, constant prior.
```

##
Algorithm specifications were nprewarm = 50 , nwarm = 50 , nmain = 250 , nrunMHBatches = 40 , nImpr
nSampVarying = 1 , nSampConst = 1 , mult = 5 .
Posterior means and standard deviations are averages over the last 250 runs.
##
Proportion of acceptances in MCMC proposals after warming up:
0.19 0.20 0.23 0.24 0.26 0.31 0.25 0.25

```
##
## This should ideally be between 0.15 and 0.50.
## Note: this summary does not contain a convergence check.
## Note: the print function for sienaBayesFit objects can also use a parameter nfirst,
        indicating the first run from which convergence is assumed.
##
## Groups:
## Data1
                                   Data5
           Data2
                   Data3
                           Data4
                                           Data6
##
## Posterior means and standard deviations for global mean parameters
\#\# Total number of runs in the results is 300 .
## Posterior means and standard deviations are averages over 250 MCMC runs (excluding warming, after th
##
##
                                           Post.
                                                      Post.
                                                                cred.
                                                                         cred.
                                                                                        varying
                                                                                                  Post
##
                                           mean
                                                      s.d.m.
                                                                from
                                                                                                  s.d.
                                                                         to
## Network Dynamics
     1. rate constant net rate (period 1)
                                            2.1459 ( 0.7326 )
                                                                0.8441
     2. rate constant net rate (period 3)
                                            2.1719 ( 0.7819 )
##
                                                               0.9366 3.9436
     3. rate constant net rate (period 5)
##
                                            2.3410 ( 0.7160 )
                                                               1.2694 3.7795
##
     4. rate constant net rate (period 7)
                                            2.3381 (0.7119) 1.1655 3.7801
     5. rate constant net rate (period 9)
                                            2.5540 (0.7121) 1.2057 3.9846
##
     6. rate constant net rate (period 11) 2.2758 ( 0.6683 ) 1.1175 3.7375
##
     7. eval outdegree (density)
                                           -0.5602 ( 0.4986 ) -1.6841 0.4112 0.09
                                                                                                  0.97
##
     8. eval reciprocity
                                                   (0.5568) 0.6394 2.7458 1.00
##
                                            1.7251
## Behavior Dynamics
     9. rate rate beh (period 1)
                                                    (0.5262)
                                                               0.4060 2.4141
##
                                            1.2257
    10. rate rate beh (period 3)
                                                    (0.5480)
                                                               0.2743 2.6184
##
                                            1.1665
##
    11. rate rate beh (period 5)
                                            1.1340 ( 0.5262 )
                                                               0.3071 2.4207
##
    12. rate rate beh (period 7)
                                            1.1159
                                                    (0.5089)
                                                               0.3590
                                                                        2.4234
##
    13. rate rate beh (period 9)
                                            1.0906 (0.5191)
                                                               0.2869
                                                                        2.3592
##
    14. rate rate beh (period 11)
                                            1.2065
                                                   (0.5260) 0.3976 2.5006
    15. eval beh linear shape
                                           -0.1760 ( 0.5260 ) -1.2722  0.8481  0.36
##
## Posterior mean of global covariance matrix (varying parameters)
    0.6371 -0.0077 -0.1424
##
  -0.0077
             0.9590
                      0.0054
##
   -0.1424
             0.0054
                      0.1591
##
## Posterior standard deviations of elements of global covariance matrix
##
    0.3928
             0.3808
                     0.1483
    0.3808
             0.8117
                      0.1823
##
##
    0.1483 0.1823
                     0.0921
## For the rate parameters across all groups:
                           Post.mean Post.sd
## basic rate parameter net
                             2.30241 0.55871
## rate beh period 1
                             1.16753 0.44662
```

Changing hierarchial model

In the previous model, reciprocity was assumed to have the same parameter $\theta_{j,rec} = \eta_{rec}$, for all j = 1, ..., 6. To allow the reciprocity parameter to vary across groups j, set

```
GroupEffects <- setEffect( GroupEffects, recip,random=TRUE)</pre>
```

```
## effectName shortName include fix test initialValue parm randomEffects
## 1 reciprocity recip TRUE FALSE FALSE 0 0 TRUE
so that now
```

$$\theta_{j} = \begin{bmatrix} \gamma_{j,rate_{x}} \\ \gamma_{j,dens} \\ \gamma_{j,rec} \\ \gamma_{j,rate_{z}} \\ \eta_{shape,z} \end{bmatrix}, \text{ and } \gamma_{j} \sim \mathcal{N}_{4}(\mu, \Sigma)$$

Running new model

To estimate the model with one more random effect

Output

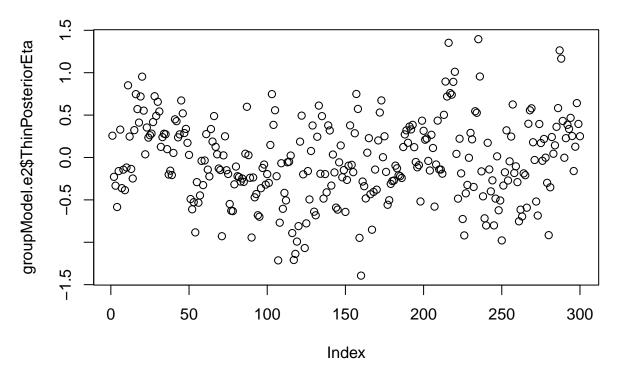
To highlight the fact that our posterior inference is represented by *simulated draws of random variables*, we will now examine and plot these random parameters using standard R functions.

The script BayesPlots.R contains functions for plots that are **custom made** for exaxamining the estimation results from sienaBayes

You will find BayesPlots.R here: https://www.stats.ox.ac.uk/~snijders/siena/BayesPlots.r; and explanations and examples of their usage are found here: https://www.stats.ox.ac.uk/~snijders/siena/SienaBayesExample 5_s.pdf.

Now we only have one common parameter, one η

```
plot(groupModel.e2$ThinPosteriorEta)
```



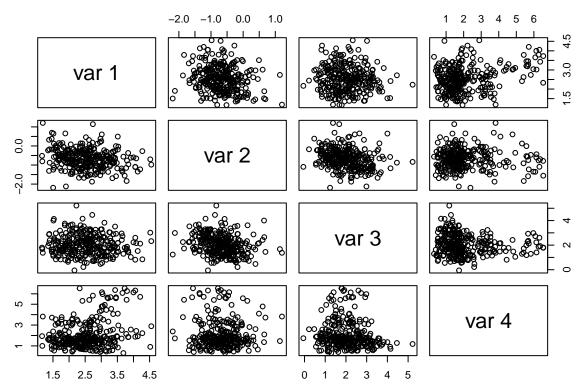
In BayesPlots.R, the function GlobalNonRateParameterPlots, provides custom-made plots of these posteriors

In the plot we see the 300 simulated values of η that are drawn using MCMC from the posterior distribution of η given data.

Given these 6 (synthetic) networks, the reciprocity parameter is positive with a posterior probability of 1

But we have μ has four dimentions (parameters)

pairs(groupModel.e2\$ThinPosteriorMu)



We can calculate the probability that the reciprocity parameter is positive given data as

mean(groupModel.e2\$ThinPosteriorMu[,3]>0)

[1] 0.9966667

Group-level analysis

For reciprocity, we saw in the previous example that

$$\Pr(\mu_{rec} > 0 \mid Data) \approx 1$$

but what about the group-level parameters $\gamma_{1,rec}, \gamma_{2,rec}, \dots, \gamma_{6,rec}$ - are they always positive for all of the groups?

Group and population-level

The posterior (predictive) distributions of the γ_j 's are stored in ThinParameters as iteration by group by parameter

```
dim(groupModel.e2$ThinParameters)
```

[1] 300 6 5

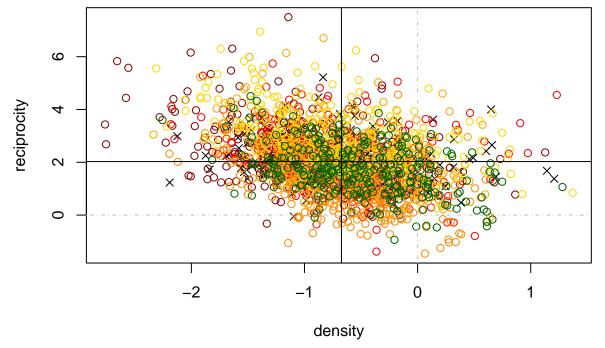
and within each group

head(groupModel.e2\$ThinParameters[,1,])

```
## constant net rate outdegree (density) reciprocity rate beh
## [1,] 3.084515 -1.8143328 2.38747125 1.188040
## [2,] 2.534277 -0.8095603 2.10497307 1.392641
## [3,] 3.319430 -0.5882144 0.66770252 1.857220
## [4,] 2.482389 -1.6900750 1.84137145 2.622582
```

```
## [5,]
                 3.122526
                                    -1.6681452 1.25374655 1.728405
##
   [6,]
                 3.727048
                                     -0.9129924 0.06870421 1.269398
##
        beh linear shape
               0.2569260
## [1,]
## [2,]
              -0.2283378
## [3,]
              -0.3325189
## [4,]
              -0.5843580
## [5,]
              -0.1605018
## [6,]
               0.3287635
```

To illustrate how we have inference for μ for density and reciprocity, but also for γ_j for density and reciprocity for each group, let us plot density against reciprocity for both levels

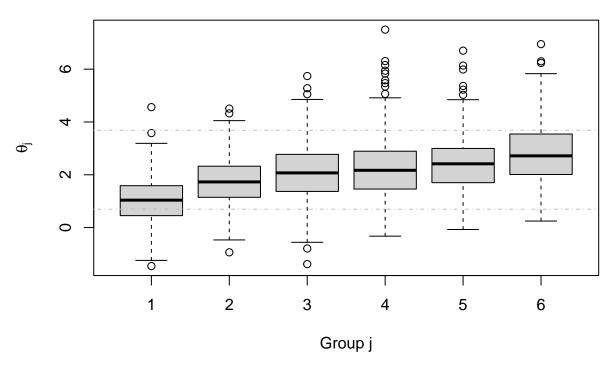


For an individual parameter it is common to look at the catepilar plot

```
require(HDInterval)
```

Loading required package: HDInterval

Posteriors for group-level reciprocity (ordered)



In the catepillar plot, we have the (posterior) predictive distributions for the group-level reciprocity parameters, ordered accounding to the group-mean. Grey lines represent the credibility interval for the population-level reciprocity parameter μ_{rec}

Prior and posterior

To get a posterior distribution for μ and η (and Σ), we need to have **prior distributions** for these parameters

A prior distribution for a parameter quantifies the uncertainty that we have about a parameter before we collect and observe DATA

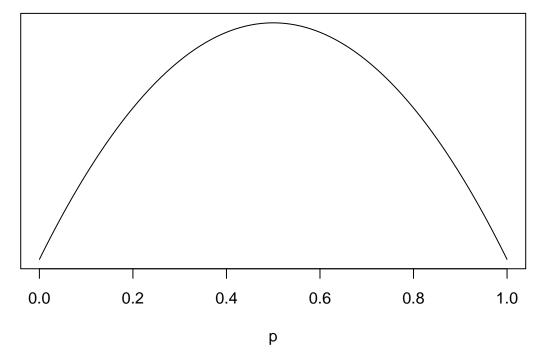
Example: Bernoulli graph

For a cross-section n=5 network, a Bernoulli graph says that each of the 5(5-1)/2=10 possible ties all have a probability of p of being present, independently of each other. The parameter $0 \le p \le 1$ and hence a prior for p might be $p \sim Beta(\alpha, \beta)$

```
alpha.p <- 2
beta.p <- 2

p <- seq(from=.0001,to =.9999, length.out = 1000)
plot( p , dbeta(p, alpha.p, beta.p) , type = 'l', yaxt='n',ylab=expression(pi(p)))</pre>
```

(g)



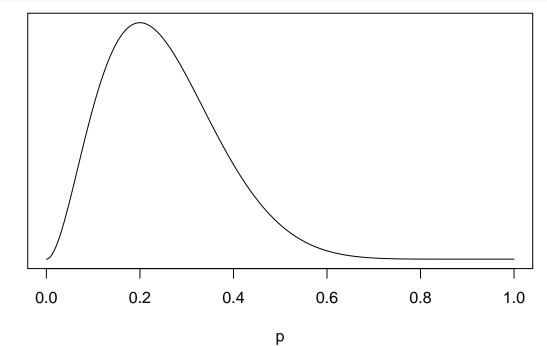
Assume that we observe the network

$$\mathbf{X} = \begin{bmatrix} - & 1 & 0 & 0 & 0 \\ - & - & 0 & 1 & 0 \\ - & - & - & 0 & 0 \\ - & - & - & - & 0 \\ - & - & - & - & - \end{bmatrix}$$

This has $L = \sum_{i < j} x_{ij} = 2$, which gives the **likelihood** function for p

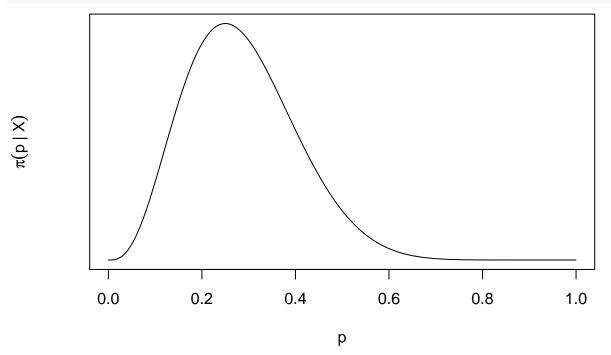
 $plot(p,p^2*(1-p)^8, type = 'l', yaxt='n', ylab='L(p; X)')$





The posterior distribution will be $p \mid \mathbf{X} \sim Beta(2 + \alpha, 8 + \beta)$

```
plot( p , dbeta(p, alpha.p+2, beta.p+8) , type = 'l', yaxt='n',ylab=expression(pi(p ~"|"~ X)))
```



Try with different values of the hyperparameters α and β in the prior for p

Even better, try Mattias Villani's widget that illustates Bayesian inference https://observablehq.com/@mat tiasvillani/bayesian-inference-for-bernoulli-iid-data?collection=@mattiasvillani/bayesian-learning[Beta-Binomial]

Example: SAOM

To understand the hierarchical SAOM, have a look at what group-level parameters our model assumed for a given μ . You may investigate this using simulation.

Drawing θ_i

If we knew μ and Σ , we can draw $\theta_i \sim \mathcal{N}_4(\mu, \Sigma)$. The (implied) model in our estimated model is given by mu <- groupModel.e2\$priorMu

```
Sigma <- (1/groupModel.e2$priorDf)*groupModel.e2$priorSigma
```

For Σ , we plug in the prior expected value $E(\Sigma) = \nu^{-1}\Lambda_0$.

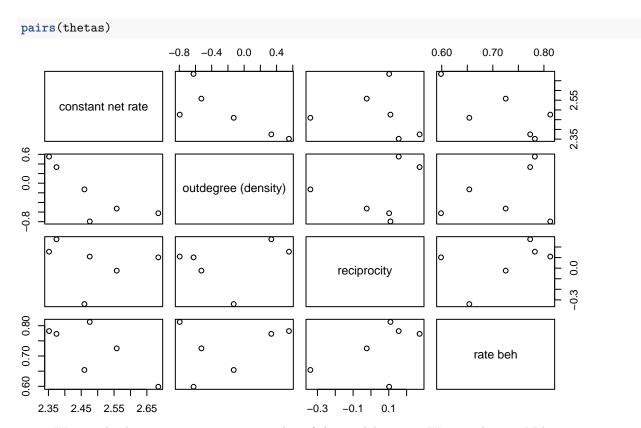
We can draw M sets of parameters

```
require(mvtnorm)
## Loading required package: mvtnorm
```

```
M <- 6
thetas <- rmvnorm(M, mean =mu, sigma = Sigma)
```

So, these are M vectors, each with 4 parameters.

```
colnames(thetas) <- colnames(groupModel.e2$ThinParameters[,1,1:4])</pre>
```

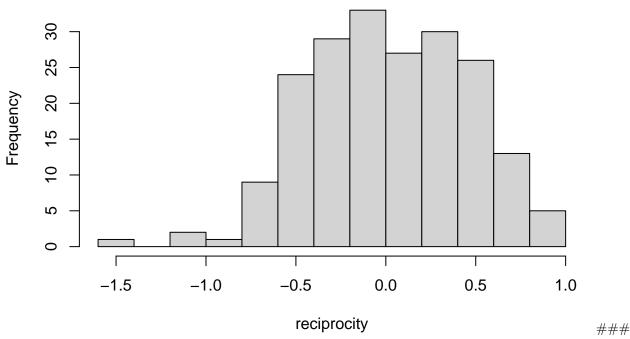


We can do this many times to get an idea of the model means. We can also set M large, to get an idea of the distribution

For a given value of μ and Σ , each $\theta_{j,k}$ will be normally distributed. For example, here, the distribution for reciprocity

```
M <- 200
thetas <- rmvnorm(M, mean =mu, sigma = Sigma)
colnames(thetas) <- colnames(groupModel.e2$ThinParameters[,1,1:4])
k <- 3
hist(thetas[,k],main='Prior (conditional) model',xlab=colnames(thetas)[k])</pre>
```

Prior (conditional) model



Drawing μ

When we estimate the model, μ and Σ are not fixed and, recall, we are not necessarily interested in the θ_j 's, but the parameters μ and Σ . Our prior for Σ is

$$\Sigma \sim InvWish(\Lambda_0, \nu)$$

and

$$\mu \mid \Sigma \sim \mathcal{N}_4(\mu_0, \Sigma/\kappa_0)$$

NB: the current default for κ_0 is

groupModel.e2\$priorKappa

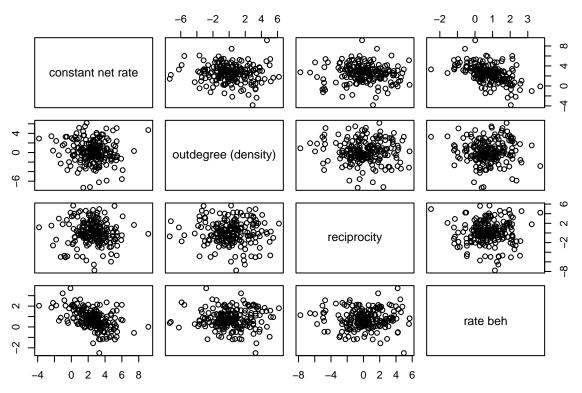
[1] 0

For now, let us set it to 1 for the purpose of illustration.

```
priorMuDraws <- matrix(0,M,length(groupModel.e2$priorMu))
colnames(priorMuDraws) <- colnames(groupModel.e2$ThinParameters[,1,1:4])
for (k in c(1:M))
{
    priorMuDraws[k,] <- rmvnorm(1, mean =groupModel.e2$priorMu, sigma = priorSigDraws[,,k])
}</pre>
```

A plot of our prior distribution of μ is given by

```
pairs(priorMuDraws)
```



In practice, people do not investigate their prior but you may in order to get an idea of what you are assuming when estimating the model

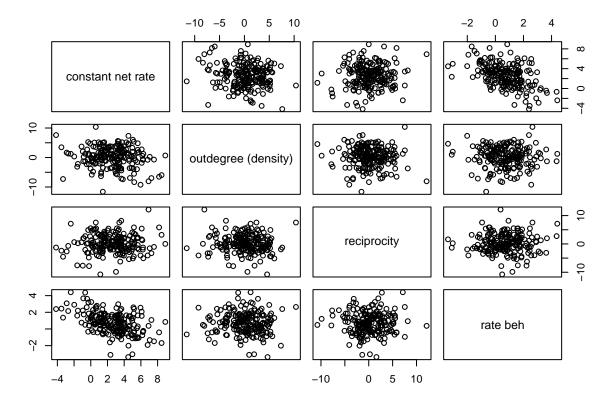
Drawing θ_j unconditionally

Previously, we drew from the distribution of $\theta_j \mid \mu, \Sigma$, i.e. conditionally on the unknown parameters. Taking our prior uncertainty about μ and Σ into account, we can now draw θ_j , un-conditionally on the unknown parameters

```
thetas <- matrix(0,M,length(groupModel.e2$priorMu))
colnames(thetas) <- colnames(groupModel.e2$ThinParameters[,1,1:4])
for (k in c(1:M))
{
    thetas[k,] <- rmvnorm(1, mean = priorMuDraws[k,], sigma = priorSigDraws[,,k])
}</pre>
```

The uncertainty about μ will propagate into the uncertainty about θ_i

```
pairs(thetas)
```



More and bigger

Load routies

Create two waves for M networks, all of size n

```
n< 10 # you could make this larger or smaller M < \!- 5 # this is very few groups and you could try to produce many
```

The Karen dataset has as many groups as you like

The networks are simulated from a model with effects, that in addition to the defaults have

- transTrip
- simX with interaction1 = "beh"

Now, figure out what inputs you need. The sienaGroup object is Karen\$my.Karen

Karen\$my.Karen

```
## Dependent variables:
## net : oneMode
## beh : behavior
## Total number of groups: 5
## Total number of periods: 5
```

For different model specifications, think of what priors you need to specify. Which ones should be random and which fixed?

With the same prior, how does increasing M affect inference?

Increasing the prior variance (Σ) , how does that affect inference?