Integrating Wildfire Risk in Forest Road Network Planning

A Linear Optimization Approach to Select a Cost-Minimizing and Fire Response Enhancing Road Network in Timber Forests

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Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

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This work was created using LaTeX typesetting language in the Overleaf environment.

Abstract

WILDFIRES are an increasingly severe threat to Mediterranean forests, where climate, vegetation, and topography amplify fire risk. Forest road networks play a dual role in these landscapes: they are indispensable for timber harvesting operations and provide critical access for firefighting crews. Yet, current forest road planning approaches focus almost exclusively on minimizing construction and maintenance costs for timber extraction, without accounting for wildfire resilience.

This dissertation develops a linear optimization model that integrates wildfire risk considerations into multiperiod forest road network planning. The model minimizes total road construction and maintenance costs across five ten-year periods, while ensuring both mechanized harvesting access and ground-based fire vehicle access to stands with low fire resistance. Constraints are based on stand-level wildfire resistance indicators, enabling managers to adjust thresholds that determine the extent of protection. Two road width options are incorporated: narrow roads sufficient for timber extraction and wider roads allowing fire response operations.

The model is applied to the Paiva forest intervention zone in northern Portugal, a fire-prone landscape managed collectively by smallholders. Results demonstrate how the framework generates cost-minimal road networks that also guarantee firefighting access to the least resilient areas. While fire-blind networks would be cheaper in the short term, the integrated approach highlights the long-term value of strategic investment in infrastructure that protects both timber resources and broader landscape resilience.

This work contributes decision support for forest managers and can be generalized to other fire-prone regions. By aligning infrastructure planning with climate adaptation needs, the model offers a practical framework for enhancing forest protection.

Keywords

Forest Road Network Planning; Wildfire Resilience; Operational Research; Decision Support; Linear Programming

Resumo

Os incêndios florestais constituem uma ameaça cada vez mais grave para as florestas mediterrânicas, onde o clima, a vegetação e a topografia amplificam o risco de ignição e propagação. As redes de estradas florestais desempenham um papel duplo nestas paisagens: são indispensáveis para as operações de extração de madeira e fornecem acesso essencial às equipas de combate a incêndios. No entanto, as abordagens atuais de planeamento de estradas florestais concentram-se quase exclusivamente na minimização dos custos de construção e manutenção associados à exploração madeireira, sem integrar a resiliência face ao fogo.

Esta dissertação apresenta um modelo de otimização linear que incorpora o risco de incêndio no planeamento multiperíodo da rede viária florestal. O modelo minimiza os custos totais de construção e manutenção ao longo de cinco períodos de dez anos, assegurando simultaneamente o acesso mecanizado para a exploração florestal e o acesso de veículos de combate a incêndios a áreas de baixa resistência ao fogo. As restrições adaptadas ao risco baseiam-se em indicadores de resistência ao incêndio a nível do povoamento, permitindo aos gestores ajustar os limiares de proteção. Foram consideradas duas tipologias de estrada: vias estreitas adequadas à extração de madeira e vias mais largas aptas a suportar operações de combate a incêndios.

A aplicação do modelo à Zona de Intervenção Florestal do Paiva, no norte de Portugal — uma paisagem propensa a incêndios e gerida coletivamente por pequenos proprietários — demonstra como a abordagem proposta gera redes viárias de custo mínimo que garantem também o acesso dos meios de combate às áreas mais vulneráveis. Embora as redes concebidas sem considerar o risco fossem, a curto prazo, mais económicas, a integração da dimensão de resiliência evidencia o valor estratégico do investimento em infraestruturas que simultaneamente protegem os recursos madeireiros e reforçam a resiliência da paisagem.

Este trabalho contribui com uma ferramenta integrada de apoio à decisão para os gestores florestais, articulando a logística da madeira com a gestão do risco de incêndio. Ao alinhar o planeamento de infraestruturas com as necessidades de adaptação às alterações climáticas,

o modelo oferece uma abordagem prática para reforçar a proteção florestal e pode ser generalizado a outras regiões suscetíveis a incêndios.

Palavras Chave

Planeamento de Redes Viárias Florestais; Resiliência face a Incêndios Florestais; Investigação Operacional; Apoio à Decisão; Programação Linear

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Acronyms

AFVS Associação Florestal de Vale do Sousa

bwd backward

CAOF Comissão de Acompanhamento para as Operações Florestais

CRS Coordinate Reference System

EWE Extreme Wildfire Event

fwd forward

FFPR Forest Fire Prevention Road

GIS Geographic Information System

IP Integer Programming

LP Linear Programming

ZIF Zona de Intervenção Florestal

Glossary

CPLEX Python API
"A Python interface to the CPLEX Callable Library, Community Edition. The CPLEX Python API is a Python package named cplex that allows the Callable Library to be accessed from the Python programming language. It is equally suitable for interactive use through the Python interpreter or for writing scripts or full-fledged applications." (reference `https://pypi.org/project/cplex/` 59, 66

digraph
A **digraph!** (**digraph!**) is a Graph in which edges have orientations. It is written as a pair $G = (V, A)$, where V is a set of elements called *vertices*, *nodes*, or *points*, and A is a set consisting of ordered pairs of vertices (v_i, v_j) called *arcs*, *directed edges*, *arrows*, or *directed lines*. 14

FIRE-RES
4-year project (2021-2025) dedicated to developing an integrated forest management strategy to address the challenges posed by Extreme Wildfire Events (EWEs) in Europe. It is led by the Forest Science and Technology Centre of Catalonia, Spain, and funded under the European Union's H2020 research and innovation program. See also appendix A .... xvii, 9, 85

Graph
Mathematical structure used to model pairwise relationships between objects. It consists of a set of vertices (also called nodes) and a set of edges that connect pairs of vertices. Graphs can be directed (edges have a direction) or undirected (edges have no direction), and they are fundamental in fields such as computer science, network analysis, and combinatorics. .... xvi, 12

IBM ILOG CPLEX Optimization Studio
IBM® ILOG® CPLEX® Optimization Studio is a commercial decision optimization software developed by IBM for building and solving complex optimization models. Analytical decision support toolkit for rapid development and deployment of optimization models using mathemati-

cal and constraint programming. It combines an integrated development environment with the powerful Optimization Programming Language and high-performance CPLEX and CP Optimizer solvers. 66

Integer Programming

In Integer Programming (IP), a subset of Linear Programming, the decision variables are restricted to integer values. . . . . xv, xvii, 12

Linear Programming

mathematical optimization method used to achieve the best outcome in a mathematical model, where the objective and constraints are expressed by linear relationships. By systematically selecting values for the decision variables using techniques such as branch-and-bound or the simplex method, the goal is to find the values that optimize the objective function while satisfying all the constraints. . . . . xv, xvii, 29

linear relaxation

Linear relaxation allows solutions to take on any value within a given range (e.g. in the intervall between 0 and 1), instead of strictly adhering to binary or integer values (in this example, either 0 or 1). To interpret solution values from a linearly relaxed binary problem, standard methods include rounding or applying a threshold (e.g., converting values above 0.5 to 1 and those below 0.5 to 0). 48

Living Lab

name for case study regions within the FIRE-RES project 9, 85

nonindustrial private forest

Privately owned forests that are not managed for industrial purposes but for other objectives such as recreation, conservation, or personal use. 9

QGIS

Quantum Geographic Information System, an open-source GIS software for spatial data visualization and analysis . . . . vii, 38, 39

1

Introduction

Contents

1.1 Background and Motivation

FIRE occurrences play a significant role in the dynamics of forest ecosystems, particularly in the Mediterranean region. Although they can have beneficial ecological effects (Keeley et al. 2011; Pereira et al. 2021; Davim et al. 2022), each year, some extreme wildfires occur globally that exceed suppression capabilities, cause extensive damage, and frequently result in fatalities (Tedim et al. 2018; Schwörer et al. 2024). Large wildfires release large quantities of greenhouse gases and air pollutants, thereby deteriorating air quality and compromising ecosystems, their provisioning services, and public health (Wang, Faloona, and Houlton 2023; Bolan et al. 2025; Carreras-Sospedra et al. 2024). They lead to widespread vegetation loss, which accelerates soil erosion, disrupts water and nutrient cycles, and can increase the risks of floods and landslides (Shakesby and Doerr 2006; Francos, Colino-Prieto, and Sánchez-García 2024; Roces-Díaz et al. 2022; Sutanto et al. 2024; Culler et al. 2023). Hence, preventing wildfires from escalating into such extreme events is a major environmental and societal concern.

However, both the frequency and intensity of the most extreme wildfire events worldwide have increased in recent decades (Cunningham, Williamson, and Bowman 2024). A growing consensus among scientists, fire managers, and agencies recognizes that wildland areas are increasingly exposed to challenging fire weather, longer fire seasons, and more extensive fires — all exacerbated by climate change (San-Miguel-Ayanz et al. 2017; Jolly et al. 2015; Bowman et al. 2017; Robinne et al. 2018; Ellis et al. 2022). In 2017, Portugal experienced an unprecedented wildfire season, with approximately 500,000 hectares burned and more than 120 fatalities reported (Turco, Jerez, et al. 2019). Climate change is expected to further intensify fire-conducive conditions, thereby increasing the risk of large wildfires (Dupuy et al. 2020; Turco, Rosa-Cánovas, et al. 2018). Projections for Europe suggest that, by 2090, the annual area burned could rise by 120–270% compared to the 2000–2010 average (Robinne et al. 2018; FAO 2020). Specifically, in the Mediterranean Basin, the frequency of heat-induced fire-weather types — closely associated with large wildfire activity — is projected to increase by 14% under the RCP4.5 scenario and by 30% under the RCP8.5 scenario by the end of the century, significantly raising the likelihood of extreme wildfire events (Ruffault et al. 2020).

As wildfires are projected to become more frequent and intense, particularly in fire-prone regions such as the Mediterranean basin, it is increasingly important to adopt a holistic approach to wildfire management. This includes moving away from zero-burn policies[1] and prioritizing

[1]The suppression of low-intensity fires can lead to fuel accumulation, increasing the severity of subsequent wildfires (Mateus and Fernandes 2014; Collins et al. 2013; Kreider et al. 2024).

preventive measures and fuel management, such as prescribed burning and agro-silvo-pastoral practices (Pais et al. 2023; Oliveras Menor et al. 2025; Mauri et al. 2023). Within the broader context of integrated forest management, it is both reasonable and necessary to incorporate wildfire response considerations into forest road network planning. As Stefanović, Stojnić, and Danilović (2016) noted, forest roads are the backbone of ground-based firefighting access:

> *"The importance of forest roads in fire protection is great, because they represent the basic infrastructure necessary for effective fire prevention and suppression."*

Developed forest road networks facilitate faster fire response and are associated with reduced fire size (burned area) – but they also increase the risk of anthropogenic wildfire ignitions due to improved public access (Viegas and Ribeiro 2022; Thompson, Gannon, and Caggiano 2021; Narayanaraj and Wimberly 2012; Pinto et al. 2020). In a study on the influence of forest roads on the spatial patterns of human- and lightning-caused wildfire ignitions, Narayanaraj and Wimberly (2012) found that although most fires occurred in roaded areas, they accounted for only a small proportion of the total burned area, whereas a few large fires, typically lightning-induced in roadless areas, accounted for most of the burned area. Similarly, in an analysis on the impact of several landscape features on the occurrence and size of forest fires, Pinto et al. (2020) observed that while higher road density and more firebreaks were associated with increased fire occurrence, these features also correlated with smaller fire sizes. The authors suggested a link to the improved fire suppression: Shorter travel distances between ignition points and fire brigade locations can reduce response time and enable earlier control, whereas sparse road networks lead to a longer period of uncontrolled spread, resulting in larger burned areas.

In areas without road access and beyond the operational reach of fire vehicle pumps, fire suppression at the front line requires either aerial deployment, ground crews advancing on foot[2], or a strategic "awaiting"[3] of the fireline until it reaches accessible terrain -— at which point roads can act as natural control lines and support corridors. Where such infrastructure is lacking or distant, this strategy allows fires to spread unhindered until ground-based firefighting becomes feasible — significantly increasing the risk of large-scale ecological and economic loss. While aerial suppression can effectively support containment efforts — and enjoys broad public and media appeal — it has notable limitations, including high costs, limited availability, and operational constraints (Peña et al. 2022; Thompson, Calkin, et al. 2013) such as dependence on

[2]This reliance on manual methods limits the amount and type of equipment deployable and increases physical risks for personnel.

[3]This does not mean inaction by fire responders; it refers to indirect tactics such as preparing containment zones, conducting backburns, creating defensible space, staging resources, and evacuating at-risk areas.

weather conditions and is not a sufficient substitute for accessible, strategically located roads. Yocom et al. (2019) identified distance to roads as the most influential variable in predicting fire perimeters, noting that approximately 26% of observed perimeters aligned with existing roads. This suggests that responders routinely utilize roads as containment boundaries in areas with adequate road infrastructure.

> *"The first thought as forest engineers is to use constructions that already exist (road network, water springs, etc) so as to become useful for forest protection and fire fighting."* (Stergiadou 2014)

Building on this idea, the importance of incorporating wildfire protection and response considerations into the early stages of forest infrastructure planning becomes evident. In fire-prone landscapes, road infrastructure is not only essential for access, but also serves as a strategic asset for wildfire control. When road networks are planned and developed with dual purposes in mind, they can significantly enhance the effectiveness of suppression efforts. Roads designed to support wildfire response enable fire crews to intervene earlier and more safely, reducing the risk of fires escalating beyond containment capacity and causing extensive ecological or economic damage. Given the increasing frequency and severity of wildfires, the question is no longer whether roads can serve multiple purposes — but why they are not routinely designed to do so.

1.2 Integrating Wildfire Risk in Forest Road Network Planning – An Overview of Related Work

Integrating wildfire risk into forest road network planning represents a small, yet existing field of research. The crucial role of forest roads in wildfire management is well documented: there is broad agreement that early suppression efforts are essential for containing wildfire spread and that forest roads provide the critical access needed for both prevention and emergency response (Viegas and Ribeiro 2022; Thompson, Gannon, and Caggiano 2021; Majlingova 2012; Stefanović, Stojnić, and Danilović 2016; Stergiadou 2014; Laschi et al. 2019). To the best of the author's knowledge, no existing method directly addresses the selection of road segments for construction from a predefined set of alternatives while explicitly accounting for wildfire risk. Nonetheless, a variety of related research efforts — ranging from Geographic Information System (GIS)-based spatial analyses to multi-criteria decision-making tools and spatial decision support systems — incorporates wildfire considerations in forest road network planning or offers valuable conceptual foundations. These approaches, while not directly solving the constrained selection problem, inform how spatial wildfire vulnerability and operational objectives can be jointly considered. Some of these contributions are presented below.

Liampas, Stamatiou, and Drosos (2019) observed that while forest opening up was initially driven by the need for timber transportation and connectivity between forest settlements, it is increasingly acknowledged as a vital preventive measure for forest protection, particularly within the context of multipurpose forestry in the Greek Mediterranean mountains. In response to this shift in perspective, they proposed a method for forest road planning that employs least cost path analysis and multi-criteria analysis within a GIS environment. Although wildfire risk was not explicitly integrated into their model, the criteria and indicators used in the multi-criteria analysis were informed by a previous work developing a forest fire risk zone map based on satellite imagery and GIS data (Dong et al. 2005).

Psilovikos, Doukas, and Drosos (2009) examined the role of forest roads in enhancing wildfire suppression effectiveness in the suburban forests of Thessaloniki, Greece. Focusing on two major forest roads that had been the subject of environmental and functional debates, the authors assessed their contribution to fire protection in terms of spatial coverage and critical response time. Specifically, they evaluated whether fire-fighting vehicles traveling along these roads could cover the entire forest area and reach ignition points within the crucial 15-minute window considered essential for successful early fire suppression. Their findings emphasized the significance of strategically placed roads in achieving rapid response and effective protection while also addressing the need to balance technical efficiency with environmental compatibility, and offers practical and theoretical guidance for improving forest road networks as an integral component of wildfire defense planning.

Kravanja and Potočnik (2007) examined the role of Forest Fire Prevention Roads (FFPRs) as a proactive fire protection measure in the Kras Forest Management Region in Slovenia, an area characterized by high wildfire risk due to climatic, geological, and anthropogenic factors. They found that although FFPRs are critical for enabling access for emergency interventions, the existing and even the planned FFPR network in the Kras I Forest Management Unit falls short of providing sufficient openness to support modern fire suppression techniques. Their work highlights the importance of aligning road density and layout with fire-prone zones to ensure effective protection, underscoring the spatial mismatch that can exist between planned infrastructure and operational firefighting needs.

Akay et al. (2017) delineated “fire-access zones” in Turkey by combining terrain slope, road position, and firetruck hose-reach capacities. Their GIS analysis highlighted areas beyond current access that warrant new road construction. Eastaugh and Molina (2012) introduced a GIS metric comparing reference fire intensities at roads and fuelbreaks to forest averages. They found that roads generally coincide with average-risk zones, whereas fuelbreaks often intersect higher-intensity areas, guiding targeted siting and maintenance. Zhang et al. (2020) proposed a new firefighting distance criterion to evaluate the actual firefighting coverage of the road network and an approach that aims at mapping the optimal forest road network based on the firefighting distance criterion.

Drosos et al. (2014) developed an optimization model for the W. Nestos forest (Greece) that uses uphill/downhill buffer zones (150 m/250 m, 200 m/400 m, 300 m/500 m) around firetruck positions to quantify protected area given hose-length limits. Their GIS analysis showed that, despite effective coverage by 4×4 patrols with 500 m hoses and early warning systems, expanding road density is economically warranted to ensure comprehensive fire protection and efficient timber operations.

Stefanović, Stojnić, and Danilović (2016) evaluated four forest road network variants in Serbia using a multi-criteria decision-making framework that incorporated fire frequency data, the behavior of a recent large wildfire, and seven criteria weighted through the entropy weight coefficient method. These criteria were selected to reflect trade-offs between construction costs, forest accessibility, and the effectiveness of firefighting operations. Their findings revealed that although one road layout was more expensive from a construction cost perspective, its superior capacity to reduce fire-related tree losses justified the investment as the avoided damage costs would outweigh the initial expenditure.

These contributions are highly relevant; yet there remains a need to further support decision-making in forest road network planning in a more practical manner, like already done in other areas: For instance, Sakellariou et al. (2020) presented a spatial decision support system that allocates fire vehicles based on critical response times and dynamically reroutes them around evolving hazards, which provides a practical decision-making tool for emergency services. By contrast, no comparable tool exists that integrates wildfire resistance considerations into forest road network planning. What is still lacking is a decision support tool targeted at timber forest managers and addressing an earlier, fundamental step: guiding construction and maintenance decisions for dual-use forest roads that serve both timber harvesting and fire vehicle access. Such a tool would assist in prioritizing which roads should be built or upgraded to a wider standard, thereby integrating fire vehicle access to critical low-resilience areas directly into the road planning process.

1.3 Problem Statement, Research Objective and Contributions

Although integrated forest management practices include fuel reduction, ecosystem restoration, and other preventive measures, forest road network design does not typically account for access requirements of ground-based fire suppression resources. For timber forests, no comprehensive framework currently integrates multiperiod economic cost minimization with timber and fire vehicle accessibility requirements, nor offers a decision support tool for dual-purpose road planning. Existing optimization models for forest road network planning focus primarily on minimizing construction and maintenance costs or maximizing timber-harvesting efficiency (e.g., Mesquita et al. 2022). In contrast, this master's dissertation develops a decision support tool for forest managers that, over multiple harvesting periods, identifies road segments for construction and maintenance, thereby recommending a cost-efficient road network that supports timber harvesting when and where needed, while also ensuring fire vehicle access to the least fire-resistant areas.

Problem Statement

Mediterranean-type timber forests, where climate and topography amplify fire risk, require road infrastructure that simultaneously supports timber extraction and wildfire containment. This work addresses a case study in the fire-prone Vale do Sousa region (described in detail in section 1.4), where private forest owners engage in collaborative planning of timber harvests over ten-year periods. In each period, some forest stands are scheduled for harvesting and therefore require access roads wide enough to accommodate timber extraction operations. These roads may either be newly constructed in that period or maintained if built in earlier periods. At the same time, certain stands are less wildfire-resistant than others, and the least fire-resistant zones require wide access roads to enable effective ground-based firefighting in the event of a nearby fire.

With the goal of ensuring equitable cost distribution among landowners, road construction is limited to stand boundaries. The selected road network should be optimized to minimize total construction and maintenance costs.

Research Objective

The aim of this Master's dissertation is to develop a linear optimization model for selecting road segments along stand boundaries for construction and maintenance across five ten-year periods, which:

- minimizes total economic costs of road construction and maintenance over all periods,
- ensures mechanized harvesting access according to the harvesting schedule,
- provides ground-based fire vehicle access to stands with low fire resistance,
- while taking into account two different road width options (small ones suitable for timber harvest and transport, and wider ones suitable for fire-fighting operations).

Contributions

This work introduces a novel linear programming model that integrates fire vehicle accessibility into cost-optimized forest road network planning, bridging timber logistics and wildfire risk management. Building upon traditional models focused solely on timber harvest and transport cost minimization, this approach explicitly incorporates constraints ensuring fire vehicle access to stands with low fire resistance[4].

The model supports multi-period planning, capturing the dynamic interplay between forest growth and fire risk. Its application to the study region Paiva demonstrates how stand-boundary constraints and collaborative management can be aligned with wildfire response goals. In the context of increasing wildfire frequency and intensity in Mediterranean-type ecosystems, this framework offers a practical tool for climate-resilient forest infrastructure planning.

In light of prior research, key contributions include:

- **Integrated Modeling Framework**: Combining cost efficiency, multi-period timber harvest accessibility and firefighting accessibility in a single optimization model.
- **Fire-Adapted Constraints**: Novel inclusion of fire vehicle access requirements for vulnerable stands, based on stand-level fire resistance indicators.
- **Real-World Application**: Case study in Castelo da Paiva illustrating the integration of timber road planning with wildfire resilience considerations.

[4] The designation of these high-risk zones is based on the stand-level fire wildfire resistance indicator developed by Ferreira et al. (2015) and implemented in form of a quantile threshold. The level of this threshold is to be adjusted by forest managers to determine the degree of emphasis placed on wildfire protection (i.e., how many areas are classified as high risk and therefore prioritized for wider road access).

1.4 Description of the Study Area

Study area for this research is the Zona de Intervenção Florestal (ZIF)[5] Paiva[6], located in Portugal's northwestern Aveiro district near the Douro River. The forest intervention zone Paiva encompasses approximately 7,620 hectares. Its administrative scope includes the entirety of the municipality Castelo de Paiva with all its parishes. The Associação Florestal de Vale do Sousa (AFVS)[7] serves as the managing entity, overseeing the participation of 185 registered members. Paiva lies within the broader region of Vale do Sousa, which serves as a Living Lab for the Horizon 2020 project FIRE-RES (2021–2025), aimed at increasing wildfire resilience across European landscapes. (Ministério da Agricultura, do Desenvolvimento Rural e das Pescas 2008; AFVS – Associação Florestal do Vale do Sousa 2024)

Vale do Sousa extends over 14,316 hectares and comprises small-scale properties. The current dominant tree species include *Eucalyptus globulus* (blue gum) and *Pinus pinaster* (maritime pine) – both of which are fire-prone and managed primarily for timber production. Apart from forestland, the landscape composition features agricultural plots, urban regions, as well as shrubland and abandoned agricultural areas. Stakeholders include nonindustrial private forest owners, forest industry actors, investment funds, and municipal entities - all coordinated under the umbrella of the AFVS and collaboratively managed by over 360 landowners.

The region is highly vulnerable to wildfires: Approximately 46% of the ZIF has burned between 2014-2023, highlighting the urgent need for more strategic and integrated risk-reduction strategies. In response, stakeholders are implementing a landscape-scale joint management plan that seeks to balance wildfire mitigation with sustained forest productivity and ecosystem services. (FIRE-RES 2023)

Despite its wildfire exposure, Paiva is a touristically attractive region known for its scenic landscapes and cultural heritage.[8]

1.5 About the Structure of this Document

This document comes in seven chapters: Following the introductory chapter which introduces the reader to the research context, problem statement and proposed solution, in chapter 2

[5] For details on the concept of forest intervention zones in Portugal, see `https://www.icnf.pt/florestas/zif`

[6] For details on the forest intervention zone 075/07 — Paiva, see Portaria n.º 1515/2008

[7] Association of Forest Producers of Vale do Sousa

[8] To access a tourist map that showcases key points of interest in Castelo da Paiva, see appendix E.

a single-objective integer linear programming model is developed and the solution approach is described. Chapter 3 provides a detailed overview of the data used, including characteristics, preprocessing steps and challenges. Chapter 4 outlines the implementation details of that model, while chapter 5 presents and discusses the resulting solution, i.e. the forest road network design considered optimal under given constraints for the case study area, as well as potential limitations of this work. Concluding with a summary of the main findings and contributions of the work and some indications on potential further research, chapter 6 constitutes the final part of this work.

2

The Model

Contents

THIS chapter presents and explains the model developed to address the problem outlined in the introduction and the necessary fundamentals. A single-objective Integer Programming (IP) approach is adopted, allowing the usage of an efficient solver to find the cost-minimal solution to the problem defined. In section 2.1, fundamental definitions and basic terminology are introduced. In section 2.2, the model is formulated, to cover all aspects of the real life problem which are relevant to the optimization. Section 2.3 presents the model-strengthening methods used to enhance convergence toward a viable solution.

2.1 Basic Definitions & Terminology

The network of road segments that can be selected for construction, maintenance and upgrade will be modeled as a Graph $G = (V, E)$, where a junction is represented by a vertex $v \in V$, and road segments are represented by so called "edges" (pairs of vertices $\{u, v\} \in E$). Throughout this chapter, we will work with the following sets and indices:

Sets

T	set of harvesting periods t,
S	set of forest stands,
$B(s)$	the *Boundary* of a stand $s \in S$,
V_{source}	set of *Source Nodes*, representing the forest stands,
V_{exit}	set of *Exit Nodes*, representing connection points to the existing road network,
V_{transit}	set of *Transit Nodes*, representing all other nodes (neither exit nor source),
V	set of **all** *Vertices/Nodes*, $V := V_{\text{source}} \dot{\cup} V_{\text{exit}} \dot{\cup} V_{\text{transit}}$,
E	set of *Edges* $\{u, v\}$,
A_{fwd}	set of *Forward arcs* (u, v) starting at u and ending at v,
A_{bwd}	set of *Backward arcs* (v, u) starting at v and ending at u,
A	set of **all** *Arcs* (u, v) and (v, u), $A := A_{\text{fwd}} \dot{\cup} A_{\text{bwd}}$,
$N(v)$	Neighborhood of vertex $v \in V$ (nodes adjacent to v)
$B(s)$	*Boundary* of stand s (nodes on the stand's geographical border).

Indices

w	the *road width* (either *timber* or *wide*)
t	a *harvesting period* (from 1 to 5)
s	a forest stand (identified via its associated source node)
v	a *node* (junction in the road network)
u	a *node* (junction in the road network)
$e := \{u, v\}$	an *edge*, denoting the road segment connecting the nodes u and v
$a := (u, v)$	an *arc* (directed edge), denoting the road segment from start u to end v

Before defining the decision variables, objective function and constraints, we must establish the framework for accurately representing the network of forest road segments that can be constructed. In this context, we differentiate between various types of vertices (nodes):

Definition 2.1 (Source Node)**.** A *source node (root)* is a point located at the center of a forest stand. Each forest stand has exactly one source node, and each source node uniquely identifies the forest stand surrounding it.

Definition 2.2 (Exit Node)**.** An *exit node (sink)* is a point that connects a forest road segment to the public road infrastructure.

Definition 2.3 (Transit Node)**.** A *transit node* is a point that is neither a source node nor an exit node.

By definition, each node can be only one option of the three above. With all node types defined, we can represent the network of potential roads segments formally.

Definition 2.4 (Vertices/Nodes)**.** Let the set of vertices

$$V := V_{\text{source}} \dot{\cup} V_{\text{exit}} \dot{\cup} V_{\text{transit}} = \Big\{ (x, y) \text{ coordinate pairs)} \Big\},$$

be the disjoint union of all source nodes (V_{source}), exit nodes (V_{exit}), and transit nodes (V_{transit}).

Definition 2.5 (Edges)**.** Let

$$E := \Big\{ \{u, v\} \mid u \in V, v \in V \text{and } u \neq v \Big\}$$

be the set of *edges* (undirected road segments) that are linking the vertices.

Definition 2.6 (Network of potential road segments)**.** With the sets V and E as defined above, the graph

$$G = (V, E)$$

represents the *network of potential road segments*.

To formulate certain constraints, it is necessary to incorporate information about the direction of *flow* along each edge. For this purpose, we define a set of directed edges, referred to as *arcs*.

While an edge simply denotes a connection between two nodes without implying direction, an arc explicitly specifies the direction by indicating a *start* node and an *end* node. This distinction allows us to represent the movement of entities along a segment in a directed manner.

Definition 2.7 (Arcs)**.** Given the set of vertices V, as defined in definition 2.6, we define the set of **arcs** (directed edges) as the set of ordered pairs

$$A := \{(u, v) \in V \times V \mid u \neq v\},$$

where each pair (u, v) represents a directed connection (arc) from node u to node v.

We additionally define a digraph $D = (V, A)$ that captures the directionality of movement. For each undirected edge representing a potential road segment, the digraph contains two directed arcs (one in each direction), rather than a single undirected edge.

Definition 2.8 (Directed network of potential road segments)**.** The digraph

$$D = (V, A)$$

represents the *directed network of potential road segments*.

The set of arcs A can be partitioned into two equally sized subsets representing the two possible directions along each undirected edge. Specifically, if $\{u, v\} \in E$, then $(u, v) \in A_{\text{fwd}}$ and $(v, u) \in A_{\text{bwd}}$ represent the forward- and backward-directed arcs, respectively.

Definition 2.9 (Forward- and backward-directed arcs)**.** Let A be the set of directed arcs corresponding to an undirected edge set E. We define two subsets of A as follows:

$$A_{\text{fwd}} = \{(u, v) \in A \mid \{u, v\} \in E\},$$
$$A_{\text{bwd}} = \{(v, u) \in A \mid \{u, v\} \in E\},$$

where A_{fwd} contains the forward-directed arcs and A_{bwd} contains the corresponding backward-directed arcs. These subsets satisfy:

$$A_{\text{fwd}} \cap A_{\text{bwd}} = \emptyset, \quad A_{\text{fwd}} \cup A_{\text{bwd}} = A.$$

Let us now introduce the concept of a *neighborhood*:

Definition 2.10 (Neighborhood)**.** Let $G = (V, E)$ be a graph and let $v \in V$ be a vertex. The **neighborhood** of v, denoted $N(v)$, is the set of all vertices adjacent to v:

$$N(v) = \{u \in V \mid (v, u) \in E\}.$$

We refer to the neighborhood $N(s)$ of a source node $s \in V_{\text{source}}$ as the *boundary* $B(s)$. It contains all the nodes situated along the geographical border of stand s:

Definition 2.11 (Boundary)**.** The **boundary** $B(s)$ is the neighborhood of a source node $s \in V_{\text{source}}$, representing the nodes located along the geographical border of the forest stand s.

Let us further agree on some basic terminology:

Definition 2.12 (Road Existence)**.** A road segment is considered to *exist* in period t at width w if it undergoes one of the following actions during that period:

- construction at width w,
- maintenance at width w,
- upgrade to width w.

The attentive reader may have noticed that the defined road network includes segments connecting the interior of a stand s to points on its boundary $B(s)$. However, we do not intend to construct such segments within the stands. Why, then, are they included in the model?

These segments serve a conceptual purpose: they are *virtual* road segments, introduced to facilitate the formulation of certain constraints that ensure the existence of required access paths. These virtual segments are assigned zero cost and are never intended to be physically constructed, maintained, or upgraded.

While the solution to the linear model may formally select some of these segments for construction, maintenance, or upgrade, this selection is purely auxiliary and does not correspond to real-world implementation. These virtual segments primarily support the representation of real-world requirements, as reflected in Constraints 2.6 and 2.5, where decision variables associated with the virtual arcs are used to model the initiation of ”flow” in stands that require road access. Together with the flow conservation constraints 2.10 and 2.11, this formulation ensures that the set of road segments selected for construction connects each stand requiring road access to an exit point.

2.2 Model Formulation

In this section, we define the decision variables representing the choices to be made (see section 2.2.1) and the necessary parameters (see section 2.2.2), before formulating the objective function (see section 2.2.3). The objective function is subject to several constraints, which are explained in section 2.2.4.

While developing the cost-minimization objective function is relatively straightforward, the more complex challenge lies in formulating appropriate constraints that ensure the model accurately reflects real-world requirements. For the scenario under study, these constraints must, on the one hand, guarantee the existence of roads that facilitate timber harvesting and transport during each period where necessary, and on the other hand, ensure fire truck accessibility to stands with low wildfire resistance.

2.2.1 Decision Variables

A variety of decision variables is necessary to accurately represent the problem. Each additional distinction from the ”real world” increases the number of variables in the model, as will be demonstrated below. We begin by defining the binary decision variables.

2.2.1.A Binary Decision Variables

For each road segment, binary decisions must be made regarding five possible actions: construction at timber or wide width[1], maintenance at timber or wide width[2], and upgrade[3]:

[1]Construct the road? (Y/N)

[2]Maintain the existing road? (Y/N)

[3]Upgrade the existing timber road to a wide road? (Y/N)

Definition 2.13 (Decision variables, binary)**.** For $e \in E$, $w \in \{\text{timber}, \text{wide}\}$, and $t \in \{1, 2, 3, 4, 5\}$, define:

$$C^w_{e,t} := \begin{cases} 1 & \text{if edge } e \text{ is } \textbf{selected for construction} \text{ at width } w \text{ in period } t, \\ 0 & \text{otherwise;} \end{cases}$$

$$M^w_{e,t} := \begin{cases} 1 & \text{if edge } e \text{ is } \textbf{selected for maintenance} \text{ at width } w \text{ in period } t, \\ 0 & \text{otherwise;} \end{cases}$$

$$U_{e,t} := \begin{cases} 1 & \text{if edge } e \text{ is } \textbf{selected for upgrade} \text{ to wide width in period } t, \\ 0 & \text{otherwise.} \end{cases}$$

Considering a harvesting schedule spanning multiple time periods, these decisions must be made iteratively for each period – leading to five decisions per possible action, per road segment. Additionally, the construction and maintenance decisions must be made for both possible road widths, which increases the number of corresponding decision variables. In summary, we have

$$\underbrace{5 \cdot 2}_{\text{construction}} + \underbrace{5 \cdot 2}_{\text{maintenance}} + \underbrace{5}_{\text{upgrade}} = 25 \tag{2.1}$$

decisions to take per road segment e, leading to a total of $25 \cdot |E|$ binary decision variables, represented by $C^w_{e,t}, M^w_{e,t}$, and $U_{e,t}$ – see Table C.1.

2.2.1.B Integer Decision Variables

To model access feasibility, we introduce auxiliary integer decision variables, commonly referred to as *flow variables*. While such variables in network optimization typically represent the movement of goods or traffic along arcs, here they only indicate whether an arc will be used for traffic and for which type of traffic (see constraints in section 2.2.4.B). Specifically, the *timber flow* and *wide flow variables* on each directed arc a, listed in table C.2, count the number of forest stands whose access path uses that arc during a given time period.

Definition 2.14 (Auxiliary variables, integer)**.** For $a \in A$, $w \in \{\text{timber}, \text{wide}\}$ and $t \in \{1, 2, 3, 4, 5\}$:

$$\mathit{flow}_{a,t}^{\text{timber}} := \text{number of stands whose timber traffic traverses arc } a \text{ in period } t;$$
$$\mathit{flow}_{a,t}^{\text{firefighting}} := \text{number of stands whose fire vehicle traffic traverses arc } a \text{ in period } t.$$

These variables allow us to ensure connectivity: Via the flow variables, we can create constraints (as described in section 2.2.4.C) that encode stand-level access requirements by limiting the outgoing flow from a Source Node s to either 0 (stand does not need access) or 1 (needs access) via the access needs parameters (see definition in definition 2.16), and then ensure that each stand requiring access gets linked to an exit point via a valid sequence of road segments selected for construction (see constraints in section 2.2.4.E).

Since flow may occur in both directions — forward (fwd) and backward (bwd) — and since each edge yields two directed arcs, we have 20 flow variables per undirected edge e: two types of flow (timber and wide), two directions per flow type, across five time periods. This results in a total of:

$$\underbrace{5 \cdot 2}_{\text{timber (fwd + bwd)}} + \underbrace{5 \cdot 2}_{\text{wide (fwd + bwd)}} = 20 \cdot |E| = 10 \cdot |A| \tag{2.2}$$

integer variables, as illustrated in Table C.2.

Remark. While the flow variables refer to directed arcs a, with $A_{\text{fwd}} \ni (u,v) \neq (v,u) \in A_{\text{bwd}}$, they are by definition direction-sensitive, so it is $flow_{(u,v),t} \neq flow_{(v,u),t}$. In contrast, the binary decision variables refer to undirected edges e, where $\{u,v\} = \{v,u\}$, so it is $C^w_{\{u,v\},t} = C^w_{\{v,u\},t}$; $M^w_{\{u,v\},t} = M^w_{\{v,u\},t}$; and $U_{\{u,v\},t} = U_{\{v,u\},t}$.

2.2.2 Parameters

To represent the costs associated with the construction, maintenance, and upgrade of each road segment, we use the parameters $CostC^w_{e,t}, CostM^w_{e,t}, CostU^w_{e,t}$.

Definition 2.15 (Parameters to represent costs)**.**

$$CostC^{w}_{e,t} := \text{Cost of constructing road segment } e \text{ at width } w,$$
$$CostM^{w}_{e,t} := \text{Cost of maintaining road segment } e \text{ at width } w,$$
$$CostU_{e,t} := \text{Cost of upgrading road segment } e \text{ to wider width.}$$

Secondly, we introduce the parameters $need\_road^{\text{timber}}_{s,t}$ and $need\_road^{\text{wide}}_{s,t}$ to represent the varying access requirements of forest stands across time periods. If a stand s has timber to harvest in period t — that is, it requires an access path — we set $need\_road^{\text{timber}}_{s,t} = 1$. Similarly, if stand s has low fire resistance and requires a wide access path in period t, we set $need\_road^{\text{wide}}_{s,t} = 1$.

Definition 2.16 (Parameters to represent stand access needs)**.**

$$need\_road^{\text{timber}}_{s,t} := \begin{cases} 1 & \text{if stand } s \text{ has timber to harvest during period } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$need\_road^{\text{wide}}_{s,t} := \begin{cases} 1 & \text{if the fire resistance in } s \text{ is below the defined limit} \\ 0 & \text{otherwise.} \end{cases}$$

2.2.3 Objective Function

To identify the cost-minimal configuration among all feasible road networks, we define the objective function Z to represent the total costs of road construction, maintenance, and upgrades across all road segments, time periods, and road widths. The goal is to determine values for the decision variables such that the total cost Z is minimized.

Definition 2.17 (Objective Function)**.** Minimize

$$Z(C^{w}_{e,t}, M^{w}_{e,t}, U_{e,t}) = \sum_{t\in T}\left(\sum_{w\in W} \left(\sum_{e\in E} (C^{w}_{e,t} \cdot CostC^{w}_{e,t} + M^{w}_{e,t} \cdot CostM^{w}_{e,t} + U_{e,t} \cdot CostU_{e,t}) \right)\right)$$

2.2.4 Constraints

The objective function is subject to a set of constraints that reflect the real-world requirements of the problem. These include logical relationships between road construction, maintenance, and upgrades, as well as the necessity of ensuring access to all forest stands. Since we distinguish between timber and wide roads, each requirement is formulated through two corresponding constraints. As previously explained, fire vehicle access requires wide roads, while timber transport can be handled by narrower ones. Because fire access roads can also accommodate timber transport, the two constraints derived from each real-world requirement are not necessarily identical in form.

2.2.4.A Maintenance or Upgrade Only When Road Already Exists

Any maintenance or upgrade action must build on an existing road. Specifically, constraint (2.3) ensures that a road segment can only be maintained at timber width or upgraded to wide width in period t if, in the previous period $t-1$, it was either constructed or maintained at timber width:

$$\forall t \in T,\ \forall e \in E : \quad M_{e,t}^{\text{timber}} + U_{e,t} \leq C_{e,t-1}^{\text{timber}} + M_{e,t-1}^{\text{timber}} \tag{2.3}$$

Similarly, constraint (2.4) ensures that a road segment can only be maintained at wide width in period t if, in the previous period $t-1$, it was either constructed or maintained at wide width, or upgraded from timber to wide width:

$$\forall t \in T,\ \forall e \in E : \quad M_{e,t}^{\text{wide}} \leq C_{e,t-1}^{\text{wide}} + M_{e,t-1}^{\text{wide}} + U_{e,t-1} \tag{2.4}$$

The defined set T contains the harvesting periods 1 through 5. In constraints (2.3) and (2.4), the time index t appears together with $t-1$. To ensure that these constraints are well-defined in the first period ($t = 1$), we need to **initialize all variables indexed with $t = 0$ to zero**, representing the non-existence of the road segments previous to the first period.

Road Maintenance and Upgrade Constraints

For all $t \in T$ and all $e \in E$:

$$M_{e,t}^{\text{timber}} + U_{e,t} \leq C_{e,t-1}^{\text{timber}} + M_{e,t-1}^{\text{timber}} \tag{2.3}$$

$$M_{e,t}^{\text{wide}} \leq C_{e,t-1}^{\text{wide}} + M_{e,t-1}^{\text{wide}} + U_{e,t-1} \tag{2.4}$$

with Initialization for $t = 0$:

$$C_{e,0}^{w} = 0, \quad M_{e,0}^{w} = 0, \quad U_{e,0} = 0 \qquad \forall e \in E, \ \forall w \in W$$

2.2.4.B Flow only on Existing Roads

The flow of fire suppression forces or timber through a road segment is only permitted if two conditions are met:

1. The road segment exists.
2. The road segment meets the minimum required width.

Accordingly, we must establish dependencies between the relevant decision variables. This model focuses on whether *any* flow occurs from a given source. This introduces a binary decision framework: either timber flow (firetruck flow, respectively) from a specific stand s occurs (flow = 1), or it does not occur (flow = 0).

Consequently, for each road segment $e = \{u, v\} \in E$ and each period $t \in T$, the total timber flow (fire truck flow, respectively) in both directions, $flow_{(u,v),t} + flow_{(v,u),t}$, is bounded by the number of stands requiring road access (wide-road access, respectively) during that period. If, in period t, the road segment e is to be used for timber harvest/transport, then it must exist (be constructed, maintained or upgraded) during that period. Conversely, if no such timber road exists between u and v in period t, then the timber flow along that segment must be zero, see (2.5):

$\forall t \in T \quad \forall e \in E$

$$\underbrace{flow_{(u,v),t}^{\text{timber}} + flow_{(v,u),t}^{\text{timber}}}_{\in \mathbb{N}_0} \leq \underbrace{\overbrace{\left(\sum_{s \in S} need\_road_{s,t}^{\text{timber}} \right)}^{\text{\# of stands needing timber road}}}_{\in \mathbb{N}} \cdot \underbrace{\overbrace{\left(C_{e,t}^{\text{timber}} + M_{e,t}^{\text{timber}} + C_{e,t}^{\text{wide}} + M_{e,t}^{\text{wide}} + U_{e,t} \right)}^{\text{Existence of timber or wide road segment } e}}_{\in \{0,1\}} \tag{2.5}$$

where e denotes the edge $\{u, v\}$, while $(u, v) \in A_{\text{fwd}}$ and $(v, u) \in A_{\text{bwd}}$ denote the corresponding directed arcs. This constraint is formulated as an inequality because not every stand requiring firetruck access must use the same road segment. The number of stands with low fire resistance may exceed the actual firetruck flow over any given segment, as access routes can differ across the network.

Similarly, if, in period t, the road segment between u and v shall enable firetruck flow, the road segment e must exist at wide width during that period. And if in period t no road segment between u and v exists, the firetruck flow between u and v must be zero during that period, enforced by (2.5).

$\forall t \in T, \quad \forall e \in E:$

$$flow^{\text{firefighting}}_{(u,v),t} + flow^{\text{firefighting}}_{(v,u),t} \leq \overset{\text{\# of stands requiring wide-road access}}{\left(\sum_{s \in S} need\_road^{\text{wide}}_{s,t}\right)} \cdot \overset{\text{existence of wide road segment } e}{\left(C^{\text{wide}}_{e,t} + M^{\text{wide}}_{e,t} + U_{e,t}\right)} \tag{2.6}$$

Let us summarize:

Maximum Capacity Constraints (Flow only on existing roads)

$\forall t \in T \quad \forall e \in E$

$$flow^{\text{timber}}_{(u,v),t} + flow^{\text{timber}}_{(v,u),t} \leq \left(\sum_{s \in S} need\_road^{\text{timber}}_{s,t}\right)\left(C^{\text{timber}}_{e,t} + M^{\text{timber}}_{e,t} + C^{\text{wide}}_{e,t} + M^{\text{wide}}_{e,t} + U_{e,t}\right) \tag{2.5}$$

$$flow^{\text{firefighting}}_{(u,v),t} + flow^{\text{firefighting}}_{(v,u),t} \leq \left(\sum_{s \in S} need\_road^{\text{wide}}_{s,t}\right)\left(C^{\text{wide}}_{e,t} + M^{\text{wide}}_{e,t} + U_{e,t}\right) \tag{2.6}$$

2.2.4.C Access Points for Stands That Need Road Access

Now we benefit from the imaginary road segments (explained at the end of section 2.1). Each stand's access road (and its timber/firetruck flow) must begin somewhere —- specifically, at a point along the boundary of that stand. Thus, we must enforce the existence of a "stand access point": During period t, for each stand s requiring an access road, at least one imaginary outgoing arc from inside the stand to a node in its boundary $B(s)$ must have flow. This boundary node then becomes the entrance to stand s, because non-zero flow already enforces the existence of the corresponding road segment in that period (see 2.2.4.B; thanks to (2.5) and (2.6), it suffices to enforce flow from stands when road existence is needed).

If stand s has timber to harvest in period t ($need\_road^{\text{timber}}_{s,t} = 1$), then one of the outgoing arcs from s must carry a flow of 1 during that period. If s has no timber to harvest, then all the outgoing timber flow from s (on all segments) must be zero., as represented via equation (2.7):

$$\forall t \in T \quad \forall s \in V_{\text{source}} : \quad \overbrace{\sum_{v \in B(s)} flow^{\text{timber}}_{(s,v),t}}^{\text{outgoing timber flow from } s} = need\_road^{\text{timber}}_{s,t} \tag{2.7}$$

Similarly, if stand s has low fire resistance in period t ($need\_road^{\text{wide}}_{s,t} = 1$), we ensure that some "firetruck flow" exits s through one of its boundary arcs. If the stand's fire resistance is not particularly low, then no such flow is permitted; represented by equation (2.8).

$$\forall t \in T \quad \forall s \in V_{\text{source}} : \quad \overbrace{\sum_{v \in B(s)} flow^{\text{firefighting}}_{(s,v),t}}^{\text{outgoing firetruck flow from } s} = need\_road^{\text{wide}}_{s,t} \tag{2.8}$$

Remark. In contrast to the general flow variables, which allow integer values, the outgoing flow on the stand-internal auxiliary road segments is binary due to constraints (2.7) and (2.8): The flow from a source node to a boundary node can only take values of either 0 or 1.

Constraints ensuring that stands needing access have an access point

$\forall t \in T, \quad \forall s \in V_{\text{source}}$:

$$\sum_{v \in B(s)} flow^{\text{timber}}_{(s,v),t} = need\_road^{\text{timber}}_{s,t} \tag{2.7}$$

$$\sum_{v \in B(s)} flow^{\text{firefighting}}_{(s,v),t} = need\_road^{\text{wide}}_{s,t} \tag{2.8}$$

2.2.4.D Avoiding In-flow into Stands

Would any forest owner be pleased if another stand's access road cut right through their own stand? Clearly not. However, to build access paths for other stands, the model would "prefer" to select the imaginary road segments located inside the other stands, since they have zero cost assigned – "minimum cost". We need to forbid the selection any of the imaginary road segments for construction, except when the stand containing that segment has outgoing flow —- in which case the access point is genuinely needed.[4]

[4]As mentioned in section 2.1, the imaginary segment from the access point in the boundary into the stand will never actually be built, which is why the assigned costs are zero. They are purely theoretical and very useful to represent our real world constraints in mathematical terms (as seen in section 2.2.4.C).

What we must ensure is that the model cannot incorporate such segments into the access path of another stand. To prevent these stand-crossing access roads, we require that the total incoming flow into each Source Node s be zero (2.9):

Constraints forbidding in-flow into stands

$$\forall t \in T \quad \forall s \in V_{\text{source}} \qquad \overbrace{\sum_{u \in B(s)} \left(flow^{\text{firefighting}}_{(u,s),t} + flow^{\text{timber}}_{(u,s),t} \right)}^{\text{total incoming flow to stand} s} = 0 \tag{2.9}$$

2.2.4.E Flow Conservation

To ensure that each access path forms a complete and uninterrupted connection to the public road network, the model must preserve flow continuity at all internal points. In other words, the total *incoming flow* to each Transit Node must equal the total *outgoing flow*.

Specifically, for every transit node $v \in V_{\text{transit}}$, the total incoming firetruck flow must match the total outgoing firetruck flow. This guarantees that a complete route from a stand access point to an exit point in the public road system is possible, see equation (2.10). Similarly, the total incoming timber flow to each transit node must equal the total outgoing timber flow (2.11):

$$\forall t \in T \quad \forall v \in V_{\text{transit}} \qquad \overbrace{\sum_{u \in N(v)} flow^{\text{firefighting}}_{(u,v),t}}^{\text{incoming "firetruck flow" to } v} = \overbrace{\sum_{u \in N(v)} flow^{\text{firefighting}}_{(v,u),t}}^{\text{outgoing "firetruck flow" from } v} \tag{2.10}$$

$$\forall t \in T \quad \forall v \in V_{\text{transit}} \qquad \overbrace{\sum_{u \in N(v)} flow^{\text{timber}}_{(u,v),t}}^{\text{incoming "timber flow" to } v} = \overbrace{\sum_{u \in N(v)} flow^{\text{timber}}_{(v,u),t}}^{\text{outgoing "timber flow" from } v} \tag{2.11}$$

These constraints ensure that any road built — initiated by an outgoing flow from a stand access point via (2.6) and (2.5) — will ultimately connect to the public road network. This prevents access roads from ending in dead-ends and guarantees that each constructed path forms a valid and continuous connection.

Remark (Transit nodes versus Exit nodes)**.** The effectiveness of the above constraints relies on the clear separation between transit nodes and exit nodes. Since flow conservation applies exclusively to transit nodes, the model is naturally incentivized to route flows to exit nodes in the most cost-effective manner. At exit nodes, flow conservation is not enforced, allowing paths to terminate there without incurring additional costs, thereby ensuring efficient and realistic network solutions.

Flow conservation

$\forall t \in T \quad \forall v \in V_{\text{transit}}$

$$\sum_{u \in N(v)} flow_{(u,v),t}^{\text{firefighting}} = \sum_{u \in N(v)} flow_{(v,u),t}^{\text{firefighting}} \tag{2.10}$$

$$\sum_{u \in N(v)} flow_{(u,v),t}^{\text{timber}} = \sum_{u \in N(v)} flow_{(v,u),t}^{\text{timber}} \tag{2.11}$$

2.3 Model Strengthening Strategies

To improve computational efficiency, the problem was decomposed into smaller, logically distinct subproblems by geographically separating areas whose decisions are unlikely to influence each other. Each subproblem corresponds to a geographically cohesive cluster of inaccessible forest stands, separated by already-accessible areas. Since road network decisions in one cluster do not affect others, the subproblems can be solved independently without compromising global optimality. This segmentation reduces the number of decision variables and constraints per instance, leading to faster solution times, while ensuring realistic network designs that respect practical road-building constraints and enabling parallel or sequential optimization.

In addition to geographical decomposition, further model strengthening was achieved by simplifying the underlying graph representation. Only the minimal set of nodes necessary to represent each inaccessible component and its neighboring accessible stands was retained, reducing computational complexity without affecting the logical structure of the optimization problem. This approach further enhances solving efficiency while preserving the integrity of the decision-making process.

The implementation of these strategies is described in chapter 4.

2.4 Model Summary

Sets

S	Inaccessible forest stands
$T := \{1, 2, 3, 4, 5\}$	Harvesting periods
$W := \{\text{timber, wide}\}$	Road width options
V_{source}	set of *Source Nodes*, representing the forest stands,
V_{exit}	set of *Exit Nodes*, representing connection points to the existing road network,
V_{transit}	set of *Transit Nodes*, representing all other nodes (neither exit nor source),
V	set of all *Vertices/Nodes*, $V := V_{\text{source}} \dot{\cup} V_{\text{exit}} \dot{\cup} V_{\text{transit}}$,
E	set of undirected *Edges* $\{\{u, v\} \vert u \in V, v \in V\}$ (road segments)
A	set of all directed *Arcs* (u, v) and (v, u)
$N(v)$	*Neighborhood* of vertex $v \in V$ (nodes adjacent to v)
$B(s)$	nodes that are part of the *Boundary* of stand s

Decision Variables

$C^w_{e,t}, M^w_{e,t}, U_{e,t}$	$\in \{0, 1\}$	Decision variables (road construction, maintenance, upgrade)
$flow^{\text{timber}}_{a,t}, flow^{\text{firefighting}}_{a,t}$	$\in \mathbb{N}_0$	Auxiliary variables (”timber flow”, ”firefighting flow”)

Parameters

$CostC^w_{e,t}, CostM^w_{e,t}, CostU_{e,t}$	$\in \mathbb{R}_{\geq 0}$	Cost parameters for road construction, maintenance, upgrade
$need\_road^{\text{timber}}_{s,t}, need\_road^{\text{wide}}_{s,t}$	$\in \{0, 1\}$	Parameters for access requirements per stand s and period t

Objective function

Minimize

$$Z(C^w_{e,t}, M^w_{e,t}, U^t_{e,t}) = \sum_{t \in T} \left(\sum_{w \in W} \left(\sum_{e \in V} (C^w_{e,t} \cdot CostC^w_{e,t} + M^w_{e,t} \cdot CostM^w_{e,t} + U^t_{e,t} \cdot CostU_{e,t}) \right) \right)$$

subject to

Constraints

- Maintenance or Upgrade Only When Road Already Exists

$\forall t \in T,\ \forall e \in E :$

$$M^{\text{timber}}_{e,t} + U_{e,t} \leq C^{\text{timber}}_{e,t-1} + M^{\text{timber}}_{e,t-1} \tag{2.3}$$

$$M^{\text{wide}}_{e,t} \leq C^{\text{wide}}_{e,t-1} + M^{\text{wide}}_{e,t-1} + U_{e,t-1} \tag{2.4}$$

- Flow only on Existing Roads

$\forall t \in T \quad \forall e \in E:$

$$flow^{\text{timber}}_{(u,v),t} + flow^{\text{timber}}_{(v,u),t} \leq \left(\sum_{s \in S} need\_road^{\text{timber}}_{s,t} \right) \left(C^{\text{timber}}_{e,t} + M^{\text{timber}}_{e,t} + C^{\text{wide}}_{e,t} + M^{\text{wide}}_{e,t} + U_{e,t} \right) \tag{2.5}$$

$$flow^{\text{firefighting}}_{(u,v),t} + flow^{\text{firefighting}}_{(v,u),t} \leq \left(\sum_{s \in S} need\_road^{\text{wide}}_{s,t} \right) \left(C^{\text{wide}}_{e,t} + M^{\text{wide}}_{e,t} + U_{e,t} \right) \tag{2.6}$$

- Access Points for Stands That Need Road Access

$\forall t \in T,\ \forall w \in W,\ \forall s \in V_{\text{source}}:$

$$\sum_{v \in B(s)} flow^{\text{timber}}_{(s,v),t} = need\_road^{\text{timber}}_{s,t} \tag{2.7}$$

$$\sum_{v \in B(s)} flow^{\text{firefighting}}_{(s,v),t} = need\_road^{\text{wide}}_{s,t} \tag{2.8}$$

- Avoiding In-flow into Stands

$$\forall t \in T \quad \forall s \in V_{source}: \quad \sum_{u \in B(s)} \left(flow^{\text{firefighting}}_{(u,s),t} + flow^{\text{timber}}_{(u,s),t} \right) = 0 \tag{2.9}$$

- Flow Conservation

$\forall t \in T \quad \forall v \in V_{\text{transit}}:$

$$\sum_{u \in N(v)} flow^{\text{firefighting}}_{(u,v),t} = \sum_{u \in N(v)} flow^{\text{firefighting}}_{(v,u),t} \tag{2.10}$$

$$\sum_{u \in N(v)} flow^{\text{timber}}_{(u,v),t} = \sum_{u \in N(v)} flow^{\text{timber}}_{(v,u),t} \tag{2.11}$$

Initialization

$$C^{w}_{e,0} = 0, \quad M^{w}_{e,0} = 0, \quad U_{e,0} = 0 \qquad \forall e \in E,\ \forall w \in W$$

3

Data

Contents

THIS chapter presents the data used in this work and describes the preprocessing steps necessary to transform it into a suitable input for the Linear Programming (LP) model. Each section within this chapter covers a different dataset, all of which are essential to addressing the problem. These sections are further organized into subsections, beginning with an introduction that presents the data source and a general data description, followed by subsections on preprocessing performed. Each section ends with a brief summary of the most relevant preprocessing results.

As a foundation for this study, we begin by analyzing the geospatial data relevant to the problem. This includes an examination of the existing public road infrastructure (section 3.1) and the forest management units (section 3.2) within the study area. A key aspect of this analysis is assessing the spatial relationships between roads and forest stands to determine which stands are already accessible and can therefore be excluded from the optimization model. Additionally, we review the harvesting schedule data (section 3.3), identifying the relevant information for this study and ensuring its compatibility with the subsequent modeling steps. Finally, we estimate the costs associated with constructing, maintaining, and upgrading forest roads on a per-kilometer basis (section 3.4). This structured approach transforms the data provided into the necessary foundation for this project.

Before delving into the data and the preprocessing steps undertaken to make it usable, some preliminary remarks regarding the scope and relevance of the data discussed might help the reader's understanding of the presented extracts: In all datasets used for this study, a wide range of attributes is available, as the data were originally collected for various purposes beyond the scope of this work. Many of these attributes pertain to other domain-specific aspects which, while valuable in their original context, are not directly relevant to the problem addressed here. Given the breadth of available information, it is neither practical nor necessary to present and discuss every attribute in detail. Instead, the following sections focus solely on the attributes that are essential for this study — those that contribute directly or indirectly to the analysis and modeling performed.

Regarding the preprocessing presented, only the key steps are outlined here for the sake of brevity and readability. Standard cleaning operations, validation checks, and other routine procedures have been omitted but can be found in the accompanying Jupyter notebooks, along with the full list of libraries used. Data manipulation and geospatial analysis were primarily conducted using the Pandas and GeoPandas libraries, with visualizations generated through Matplotlib. Supplementary libraries were incorporated as required throughout the process.

3.1 Public Road Infrastructure (Geospatial Data)

The study area is intersected by public roads which serve as a critical component of the transportation infrastructure for timber and ground-based firefighting forces within the region. This data is particularly relevant to our problem for two reasons: First, because some forest stands are already accessible via existing roads, which reduces the complexity of the problem by limiting the need for new road construction; and second, because any future road network must connect to this existing infrastructure to ensure continuity and accessibility.

This section provides a brief description of the road dataset, followed by an overview of the preprocessing steps applied. Key steps performed on this data included the removal of roads marked as "inoperable" in the dataset (see 3.1.2), filling relevant data gaps (see 3.1.3), and distinguishing between two different road networks (see 3.1.4): one consisting of standard timber roads, and the other consisting exclusively of wider roads deemed suitable for fire vehicle access.

3.1.1 Introduction to the Road Dataset

The data on the roads considered in this study originates[1] from a geospatial dataset provided as a shapefile `RVF_CPv_clip.shp`, projected in the Lisboa Hayford Gauss IGeoE (ESRI:102164) coordinate system.[2] The dataset comprises 311 road segments in the municipality of Castelo de Paiva, each represented as a vector linestring. In total, the road network spans approximately 390 kilometers. Figure 3.1 visualizes the geometries of these segments.

Apart from the spatial geometries, the following attributes were used in this work:

Attribute	**Description**
`Id`	Unique identifier for the road
`COMPRIM`	Length of the road
`OPERAC`	Operational status of the road
`LARGURA`	Width of the road
`DESIGNACAO`	Designation or name of the road

[1] According to its provider (Susete Marques, PhD), this dataset was — prior to its inclusion in this work — clipped from a bigger dataset originally obtained from the Municipal Forest Office.

[2] A projected coordinate system typically used for mapping in mainland Portugal; see `https://epsg.io/102164`.

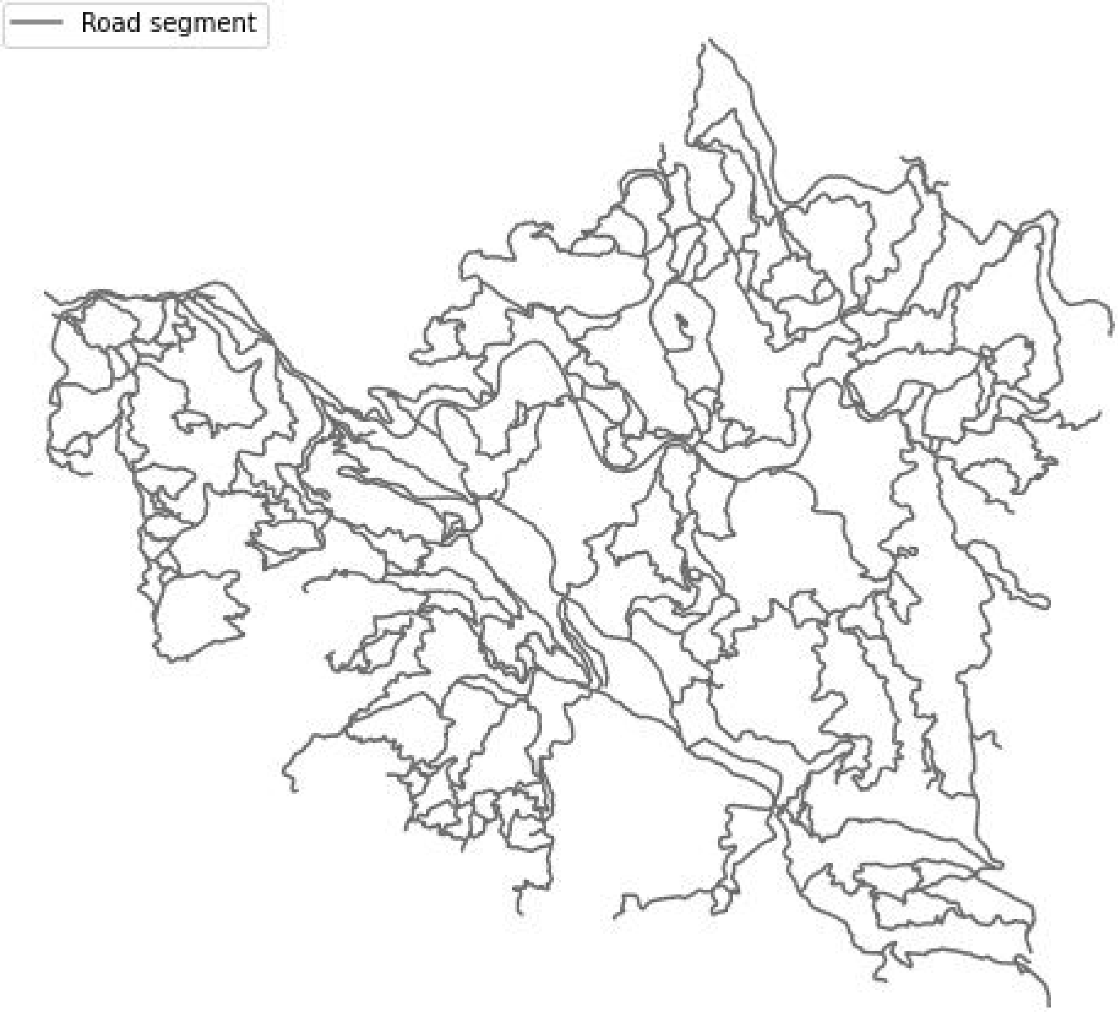

Figure 3.1: Road network in the study area of Paiva, prior to preprocessing

3.1.2 Removal of Inoperable Roads

Some roads in the dataset are marked as inoperable and can not be used. Only operable roads are relevant to this study. An assessment of road operability based on the `OPERAC` column revealed that 13 road segments are inoperable (visualized as red dotted lines in fig. 3.2). As these roads do not contribute to the problem under investigation, they were removed from the dataset.

→ After this adjustment, 298 operable road segments remain with a total length of approximately 376 km.

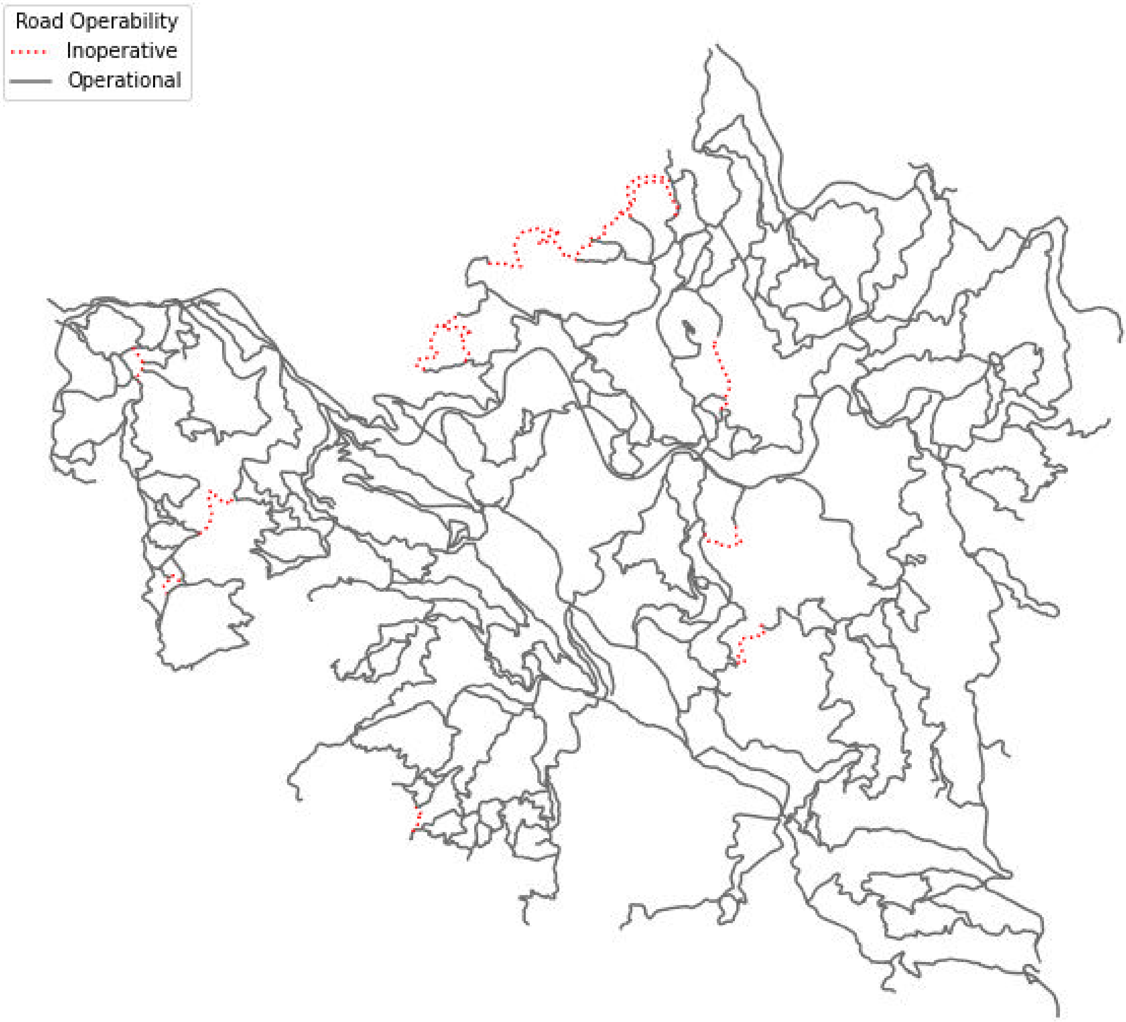

Figure 3.2: Operability status of the road network in Paiva; inoperative roads shown as dotted red lines.

3.1.3 Addressing Data Deficiencies: Road Width Values Reported as Zero

Upon examining the frequency distribution of the road width values (column `LARGURA`, see table 3.1 and a corresponding plot in fig. 3.3), it becomes evident that there are some "holes" in the data. Out of 298 operable road segments, 70 segments have a reported width of 0 meters, which is clearly implausible. Expected road widths typically fall between 3 meters and 10 meters, and they should always be greater than zero. After further data exploration, these zero values were deemed erroneous and attributed to data quality deficiencies requiring correction.

Given the high proportion of missing values (70 out of 298 segments), the corresponding features could not be removed from the dataset. Instead, a new column `ROADWIDTH` was created,

Table 3.1: Frequency table for `LARGURA` with implausible values highlighted in orange

LARGURA [m]	0	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6.5	7	7.5
Count	70	1	54	38	45	22	54	5	5	1	1	1	1

Figure 3.3: Visualization of implausible road width information on operable road segments, segments with road width of zero shown as dashed lines in orange

and new road width values were imputed for segments with missing data. An estimate for the missing values can be derived from the `REDE_DFCI` attribute, following the classification system defined in Categorias de classificação da rede viária florestal (Ministério da Agricultura 2014). According to these guidelines, **first-order roads** (*1.ª ordem*) should have a usable lane width of at least 6 m; **second-order roads** (*2.ª ordem*) should range between 4 m and 6 m; and

complementary roads (*complementar*) can be narrower. Based on this classification:

- 67 segments (`REDE_DFCI` = 1) were assigned a width of 6 m;
- 1 segment (`REDE_DFCI` = 2) was assigned a width of 4 m;
- 2 segments (`REDE_DFCI` = 3) were assigned a width of 3 m.

This approach ensures a consistent and interpretable road width variable across the entire dataset, enabling more reliable analysis in subsequent modeling steps. The resulting dataset includes a complete `ROADWIDTH` column, as illustrated in fig. 3.4.

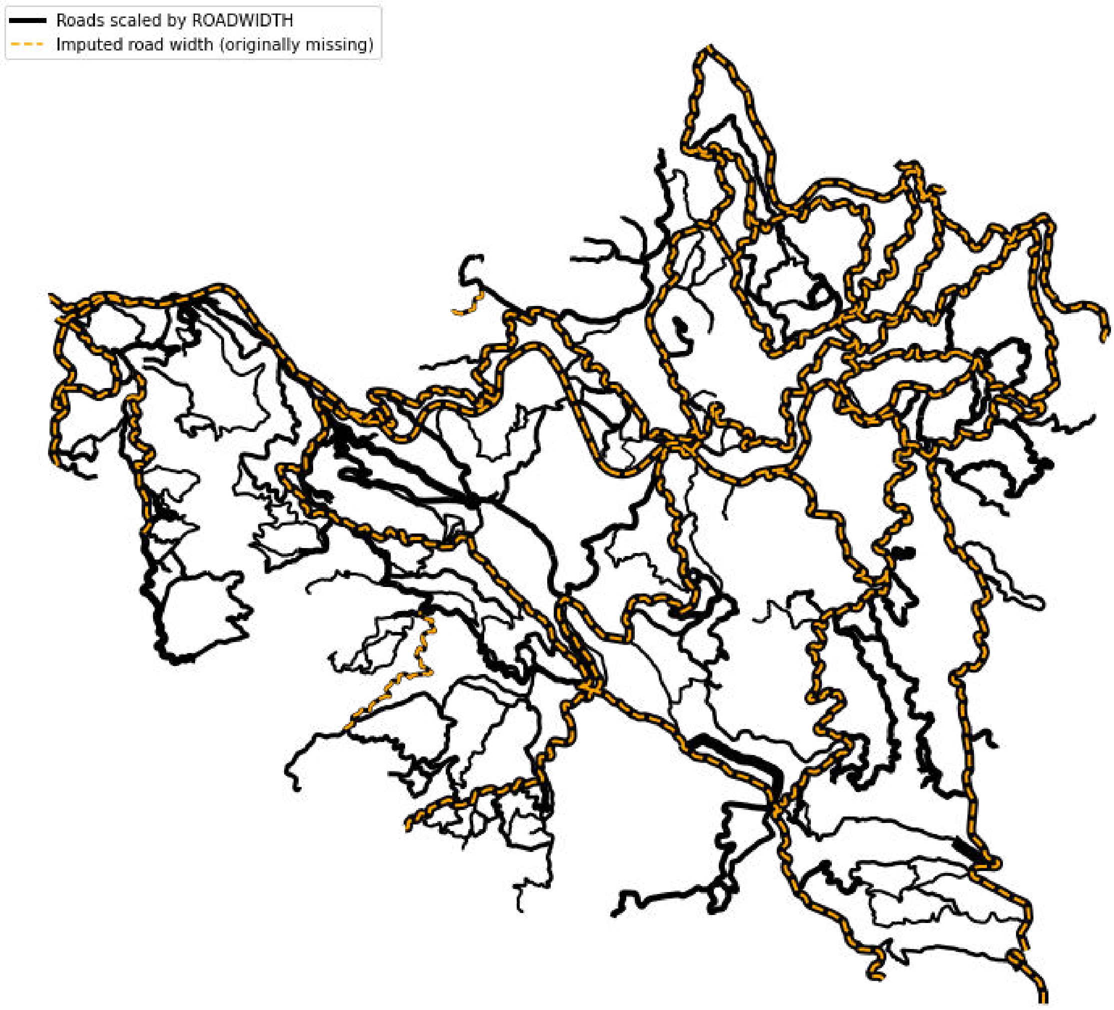

Figure 3.4: Operable roads displayed with line widths proportional to road widths; imputed road width values highlighted as dashed lines in orange

3.1.4 Distinguishing Fire Roads from the Full Set of Roads by Width

To enable timber transport, roads must have a minimum width of 3 m. To accommodate two-way traffic for firefighting vehicles, a wider minimum width of 6 m is required, which calls for the creation of a separate subset of wide roads. While fig. 3.5 shows the entire network of *"timber roads"*, fig. 3.6 displays only the wider *"fire roads"*.

Note that forest stands adjacent to roads wider than 6 meters are fully accessible for both timber extraction and firefighting activities, and can therefore be excluded from the scope of this study. This will be addressed in the following section.

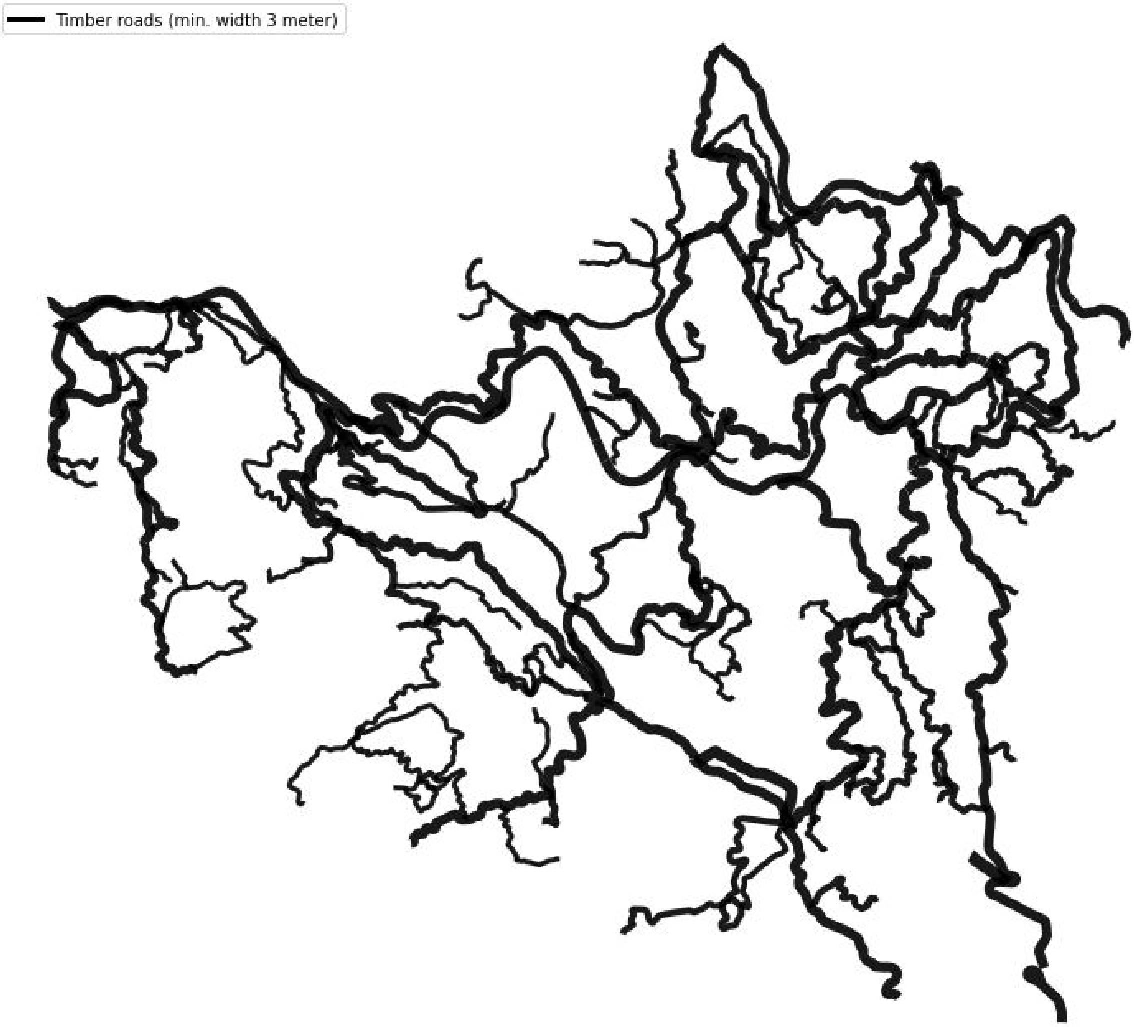

Figure 3.5: Network of operable roads with widths greater than 3 m ("timber roads"), visualized with line width proportional to assigned road width values and colored by road type.

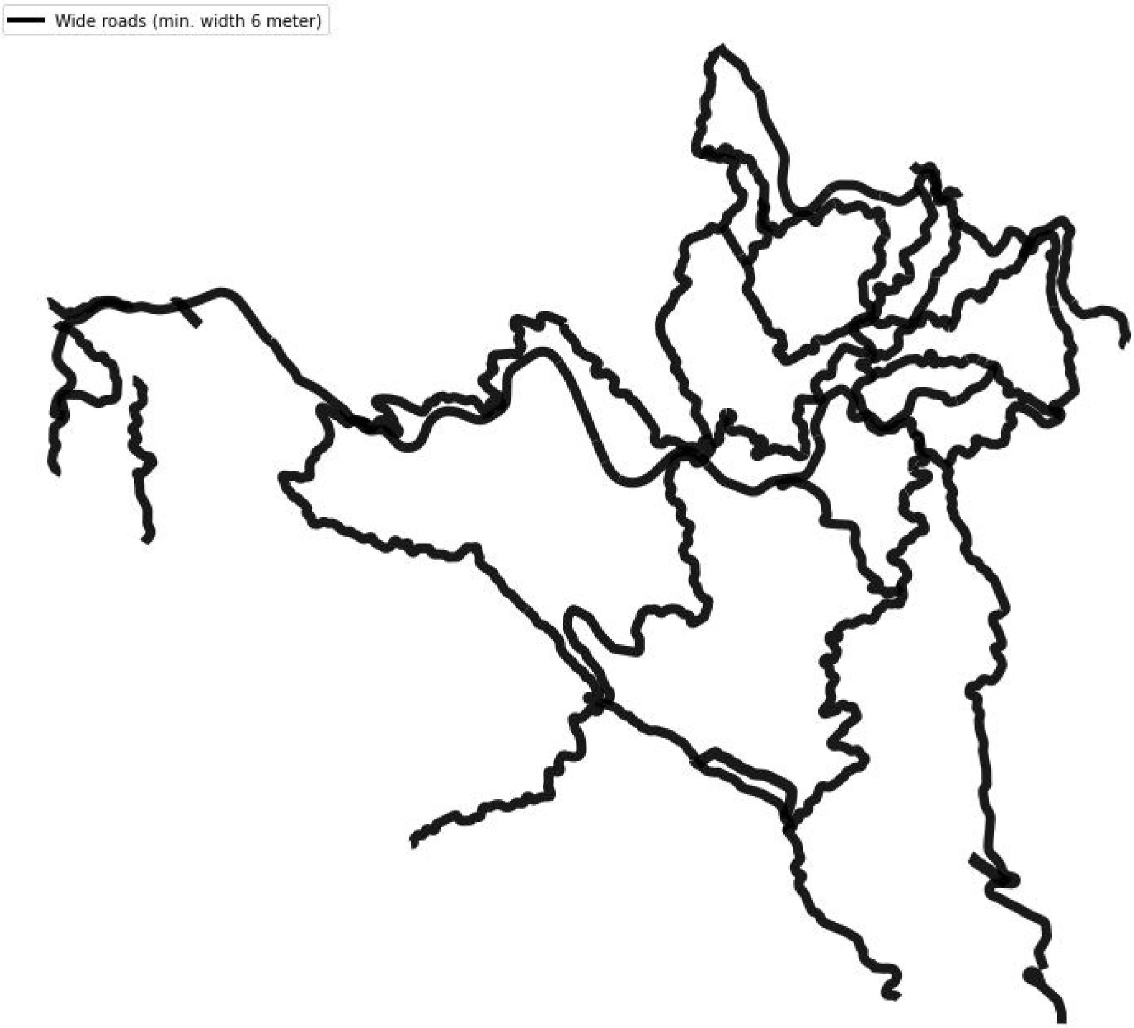

Figure 3.6: Network of operable roads with widths greater than 6 m (”fire roads”), visualized with line width proportional to road width values.

3.1.5 Summary of Road Data for Further Usage

Summarizing the relevant results from this section, in the area under study, we have:

- a **timber road network**, consisting of 205 roads wider than at least 3 meters suitable for timber transport (see fig. 3.5),
- a **wider road network**, subset of the above timber roads, consisting of 70 roads wider than at least 6 meters suitable for both timber transport and firefighting traffic (see fig. 3.6).

3.2 Forest Management Units (Geospatial Data)

Following the preparation of the existing road network, we now turn our attention to the forest management units (stands) within the study area — equally essential elements in the problem under investigation. A forest management unit is a defined area of forest land that is managed as a unit. These units are central to the planning of forest road infrastructure.

The significance of forest stands in the modeling framework for this problem stems from both their access requirements and spatial boundaries. First, the characteristics of each stand determine its specific needs for road access. These needs are incorporated as constraints in the linear programming model (see chapter 2, section 2.2.4.C), ensuring that any proposed road construction accounts for the stand-level factors of timber availability and fire resilience. Second, due to the fragmented ownership of stands across different entities, the design of future road infrastructure must respect stand boundaries to uphold property divisions and support coordinated management. In particular, this approach allows for a fair distribution of the area losses associated with road placement among different landowners, ensuring that no single entity bears a disproportionate impact on timber production or economic returns.

This section begins with a description of the dataset used (section 3.2.1) and the corrections necessary to make it usable (section 3.2.2), followed by an analysis of road accessibility of the forest stands (section 3.2.3) - because only inaccessible stands are part of the decision problem under study.[3]

3.2.1 Introduction to the Forest Management Units Dataset

The data on forest management units used in this study originates from a geospatial dataset provided in the form of a shapefile `CasteloPaiva.shp`, made available by Susete Marques, PhD. According to its provider, the dataset was derived from Pléiades satellite imagery with a spatial resolution of 0.5 meters. Each forest management unit is represented as a polygon (vector data). The dataset is projected in ETRS89 / Portugal TM06 (EPSG:3763)[4]. It comprises **687 forest management units** that vary in shape and size (see fig. 3.7). Each unit (stand) is homogeneous with respect to species composition. For the purposes of this study, the only attribute used — aside from the spatial geometry — is `ID_UG`, the unique identifier for each management unit.

[3] Nevertheless, all stands are relevant for the creation of the road network.

[4] Standard projected Coordinate Reference System (CRS) for mainland Portugal, for details see https://epsg.io/3763.

Figure 3.7: Castelo da Paiva: Forest stands

3.2.2 Spatial Correction and Topology Cleaning of Stands Data in QGIS

During verification in QGIS using multiple baselayers (e.g., Google Maps, OpenStreetMap), a noticeable spatial misalignment in the dataset became evident. The geometries of the forest stands appeared systematically shifted relative to recognizable geographic features such as rivers and roads. In the absence of accompanying metadata and after unsuccessful reprojection attempts, the issue was ultimately resolved by **manually shifting** the entire layer. This correction was particularly important, as subsequent analyses involve spatial relationships between forest stands and the road network, for which accurate alignment is essential.

Following this spatial realignment, additional topological inconsistencies were identified. Visual inspection and the QGIS Topology Checker tool revealed numerous small gaps and overlaps between adjacent polygons - errors that could compromise spatial operations such as intersection and buffering. Specifically, the tool flagged **165 overlaps** and **709 gaps**, a quantity impractical to address manually. To resolve these issues, the following steps were undertaken:

1. **Snapping:** The dataset was snapped to itself using a 1-meter tolerance to improve the alignment of adjacent polygon boundaries.

2. **Geometry Validation:** Invalid geometries were identified and repaired using QGIS's Fix Geometry.

3. **Second Topology Check:** A second run of the Topology Checker reported **84 gaps** and **32 overlaps**.

4. **Manual Inspection and Cleaning:** Upon review, **65 gaps** were retained as legitimate features (e.g., clearings or administrative boundaries), while **19 gaps** and **all 32 overlaps** were manually corrected using the Vertex Tool to ensure topological consistency.

5. **Multipolygon Correction:** During the previous steps, **two multipolygon features** were inadvertently created. These were manually inspected and compared with the original geometries, then corrected back into simple polygons using the Vertex Tool to ensure consistency across the dataset and simplify further data processing.

These corrections were essential for preparing the data to be suitable for the spatial operations and optimization modeling conducted in later stages of this work - particularly for the creation of the network of potential roads along the stand boundaries (see section 4.2).

3.2.3 Classification of Forest Stands by Road Accessibility

In this work, a road is considered suitable for timber logging and transport operations if it has a minimum width of 3 meters, hereafter referred to as a "*timber road*".[5] A stand that is adjacent to a timber road (i.e., one with a minimum width of 3 meters) is considered "*timber-accessible*".[6] A stand that is adjacent to a wide road (i.e., one with a minimum width of 6 meters) is considered "*fully accessible*". The decision problem addressed in this study includes only those stands that are currently not timber-accessible, hereafter referred to as *"inaccessible"*.[7]

[5] This definition does not account for legal ownership constraints, nor for other physical or environmental limitations that may affect actual usability. All roads in the provided dataset that are marked as operable and have a width of at least 3 meters are considered suitable for timber logging.

[6] Since wider fire roads are a subset of timber roads, stands adjacent to them are also timber-accessible.

[7] Determining which narrow roads should be widened to improve firefighting access constitutes a separate planning problem and falls outside the scope of this study (discussed in chapter 5).

A corresponding spatial analysis of the relationships between forest stands and the surrounding road network was performed using the `geometry.intersects` method from GeoPandas, which determines whether two geometric objects share any points or areas. To ensure spatial accuracy and address initial data imprecision, preprocessing steps included aligning the CRSs of the datasets and applying a 6-meter spatial buffer around the stands. In a first classification step, a set of **203 fully accessible stands** (fig. 3.8, shown in green) was identified.[8]

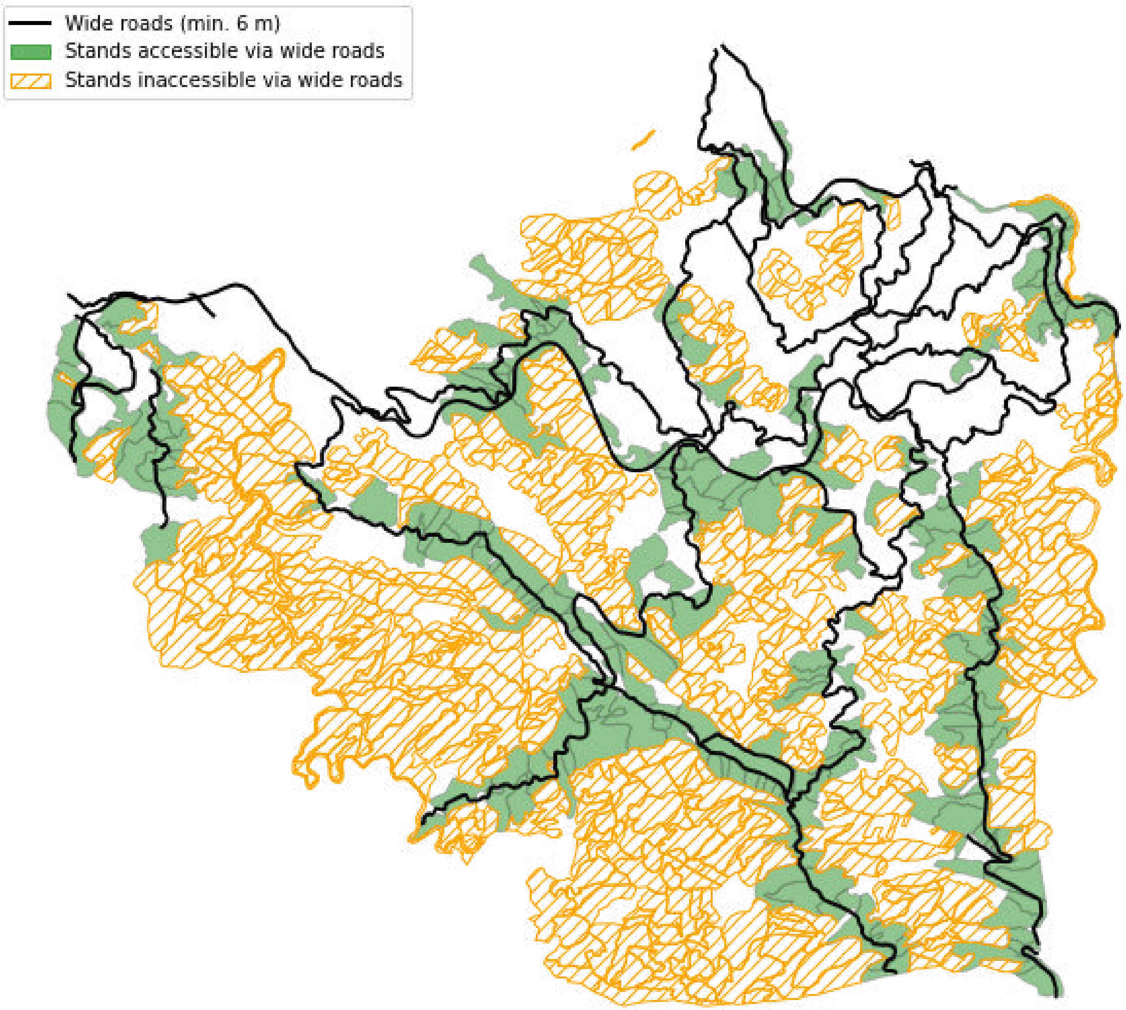

Figure 3.8: Stand accessibility via the existing network of wide roads

[8]Since both timber extraction and firefighting access to these stands are already supported by the existing road network, their access needs are considered fulfilled and must be excluded from the decision problem. This was done by filtering out the corresponding prescription data (see section 3.3). This exclusion reduces the required road construction, thereby lowering overall costs and simplifying the optimization by reducing its size and complexity.

In a second step, the remaining 484 stands not accessible via wide roads (orange-hatched in fig. 3.8) were further classified. Figure 3.9 shows the resulting sets: There are **169 timber-accessible stands** (light green) and **315 inaccessible stands** (orange cross-hatched).

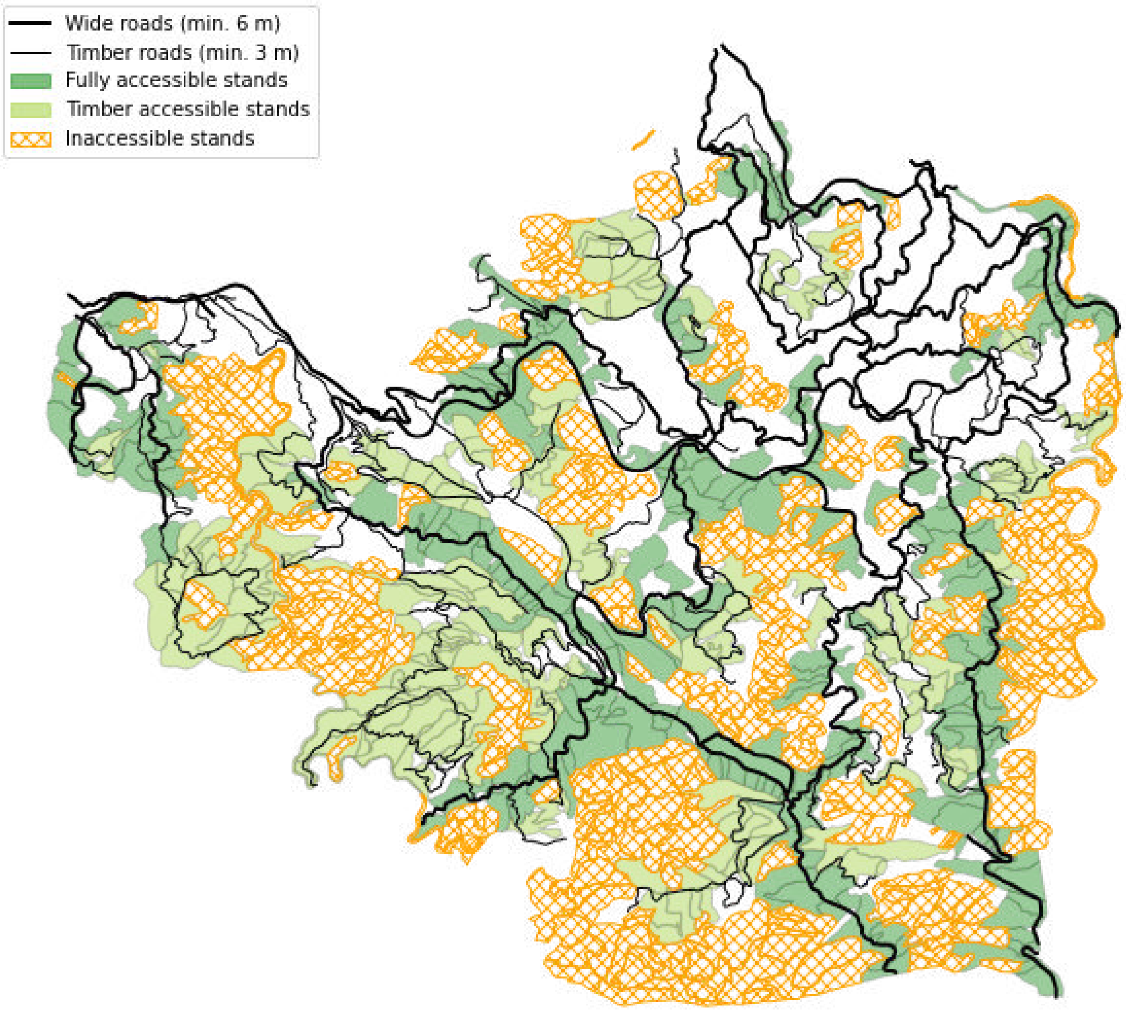

Figure 3.9: Stand accessibility via the existing timber road network

Only the 315 inaccessible stands (orange cross-hatched in fig. 3.9 and solid orange in fig. 3.10) are considered in the optimization. However, all stands remain relevant for the creation of the road network, as roads should follow stand boundaries.

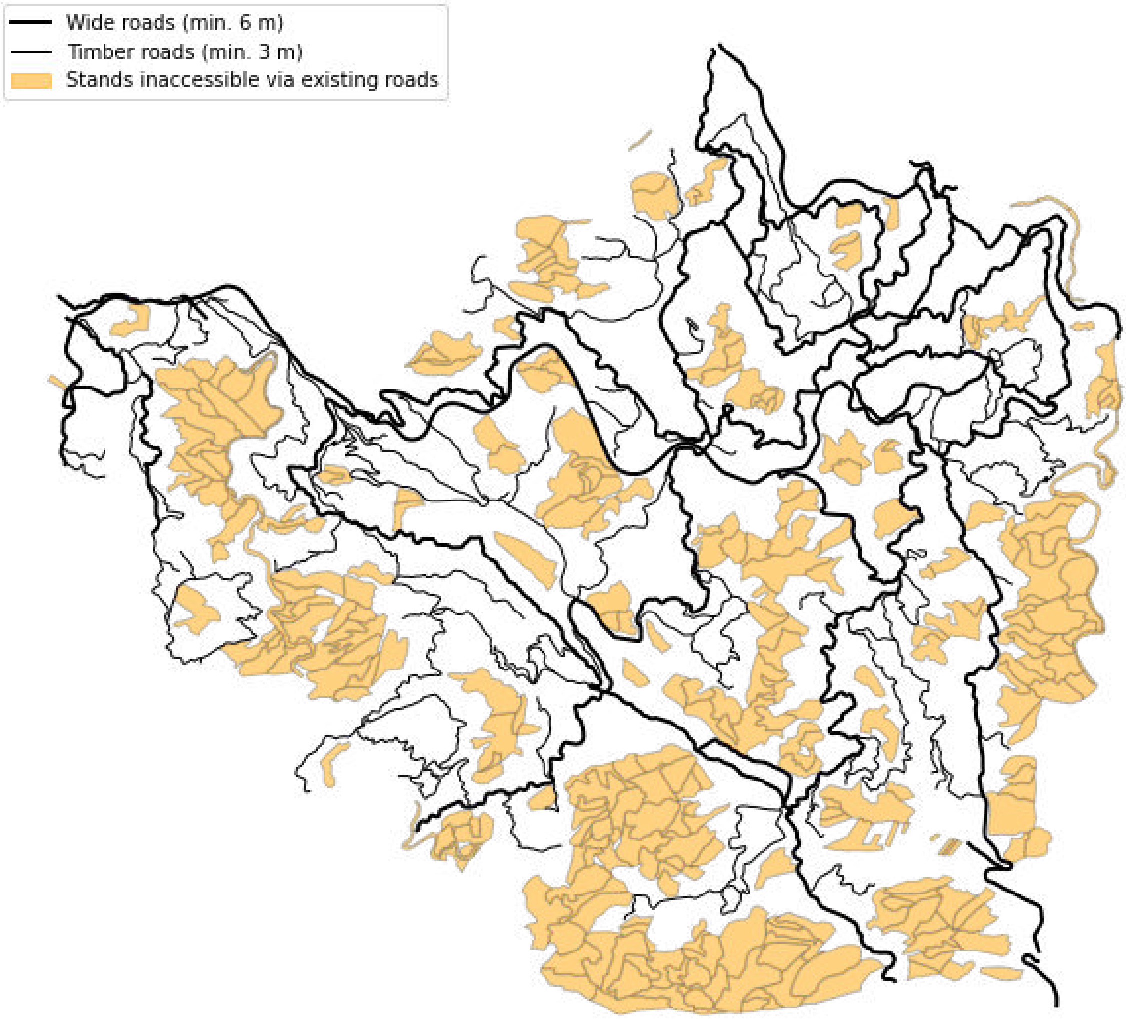

Figure 3.10: Sets of stands whose access needs are considered in the decision problem

3.2.4 Summary of Stand Accessibility Classification

The accessibility analysis yields the following classification:

- **315 inaccessible stands** (see fig. 3.10)
  → These are included in the optimization model; their access needs are to be extracted from the harvesting schedule data (see section 3.3).
- **372 stands that are already accessible**
  → Excluded from the optimization, but relevant for geometry since stand boundaries constrain road placement.

3.3 Harvesting Schedules (Tabular Data)

This section outlines how the provided harvesting schedules - detailing when, where, and how much timber is to be removed - inform the road access requirements used in the optimization model. Starting from three pre-defined harvesting schedules (referred to as "solutions"),[9] we determine, for each period, which stands require road access and what type (timber road vs. wider fire road). Although this information is not explicitly included in the original datasets, it can be systematically inferred through a sequence of processing steps.

The data preparation began by reformatting the stakeholder solution to match the structure of the other two (section 3.3.1). Each solution was then merged with the full prescription dataset to incorporate the key attributes `Vremovido`, `rait0/rait5/rait10`, for each `period` (see section 3.3.2). Fire resistance values were assigned based on shrub cleaning periodicity (`ShPer`), which may occur every 1, 5, or 10 years (section 3.3.3). Special cases and anomalies in the data were identified and analyzed (section 3.3.4). Finally, the dataset was filtered to include only stands that are part of the decision problem, based on the results of the accessibility classification from section 3.2, and the road access requirements for each stand and planning period were derived (section 3.3.5).

3.3.1 Introduction to the Harvesting Schedules Dataset

The harvesting schedules used in this study were developed by Susete Marques, PhD, Olha Nahorna, and Felipe Silva, based on data collected in the fall of 2019 and processed in 2020. These schedules represent the outcome of simulations of multiple forest management alternatives, developed in collaboration with stakeholders from the AFVS. Each management alternative is referred to as a "solution," with every solution assigning a specific prescription to each forest stand.

The dataset includes **three alternative harvesting schedules**, each representing a distinct management strategy:

- **MaxWood solution** — optimized for maximum wood production,
- **MaxRes solution** — optimized for maximum fire resistance,
- **Stakeholder solution** — selected by stakeholders[10].

[9] These were generated through a prior optimization process conducted independently of this study.

[10] According to the data provider, this selection was based on a Pareto front involving four optimization criteria.

All three schedules cover the same set of forest management units and share a 50-year planning horizon, divided into five 10-year periods. The dataset was provided in the form of an Excel workbook `Presc_dataset21_missingstakeholders_sol.xlsx` containing three worksheets, along with a separate log file:

- `Presc` — includes the full set of possible prescriptions (section 3.3.1.A);
- `MaxWood` — details the MaxWood solution (section 3.3.1.B);
- `MaxFireRES` — details the MaxRes solution (section 3.3.1.C);
- `Stakeholders_Solution.log` — logfile containing the stakeholder-selected solution (section 3.3.1.D).

Please note that the harvesting schedule data includes stands not only within Paiva (the study area for this work, introduced in section 1.4 and with corresponding stands detailed in section 3.2) but also from the neighboring ZIF Entre Douro e Sousa. Since this Master's dissertation focuses exclusively on Paiva, stands outside the study area must be excluded. However, to ensure a comprehensive understanding of the dataset and maintain consistency during preprocessing, the harvesting schedule data were initially analyzed in full. Most preprocessing steps were applied to the complete dataset, with filtering of out-of-scope stands performed only at the final stage.

3.3.1.A Prescription data

The prescription data `Presc` is extensive, containing 41 attributes (columns) and 317,883 entries (rows), where each row corresponds to a possible prescription for a forest stand in a given period. Most of the columns include information irrelevant to the current problem, and a large portion of the rows represent prescriptions not selected in any solution. Therefore, only a subset of this data was used in this project. The attributes relevant to the problem under study are detailed in the grey box below and exemplified by the table excerpt in table C.3 in the appendix.

Attribute	Description
`UG`	Unique identifier for the forest stand
`presc`	Unique identifier for the prescription
`species`	Species prescribed for the stand
`area`	Area in hectares occupied by the species
`ug_Sp`	Control column for data consistency (`UG_species`)
`Control`	Control column for data consistency (`UG.presc`)
`Na sol MaxRES`	Indicator whether the prescription is included in MaxRES solution
`Na sol MaxWood`	Indicator whether the prescription is included in MaxWood solution
`period`	Harvesting period
`vthin (ton)`	Volume of the stand thinned in tons
`vharv (ton)`	Volume of the stand harvested in tons
`Vremovido (ton)`	Total biomass removed (in tons)
`rait0`	Fire resistance indicator for shrub cleaning interval of 0 years
`rait5`	Fire resistance indicator for shrub cleaning interval of 5 years
`rait10`	Fire resistance indicator for shrub cleaning interval of 10 years

Remark. The attentive reader may notice that some attribute names contain spaces, such as `Na sol MaxRES`. While this formatting is not ideal for programmatic handling and such spaces should ideally be removed, the names are shown here as-is to reflect the original dataset and to improve readability in the accompanying exemplary table extracts.

3.3.1.B MaxWood Solution

The ⊞ `MaxWood` solution was provided in tabular format, comprising 1,512 entries across the seven columns described in the grey box below. This dataset, illustrated in table 3.2, specifies the prescriptions selected under the MaxWood scenario — that is, the set of management actions aimed at maximizing wood production.

Attribute	Description
`Decision_Variable_Name`	Identifier of the decision variable, formed by concatenating `Presc`, `UG`, `Sp`, and `ShPer`
`Control`	Control attribute for data consistency (`UG.Presc`)
`Presc`	Unique identifier of the selected prescription
`UG`	Unique identifier of the forest stand
`Sp`	Prescribed species
`ShPer`	The prescribed periodicity of shrub cleaning (0, 5 or 10)
`Sol`	Indicator if prescription is included in the solution (`Yes`/`No`)

Table 3.2: Excerpt from MaxWood Solution sheet

Decision Variable Name	Control	Presc	UG	Sp	ShPer	Sol
Presc3502_Pa1_Ec_0	1.3502	3502	1	Ec	0	Yes
Presc19_Pa2_Ec_0	2.19	19	2	Ec	0	Yes
Presc21_Pa3_Ec_10	3.21	21	3	Ec	10	Yes
...	...	...	...	...	...	Yes

Full table length: 1512 rows

Remark. The length of the MaxWood solution table (1512 rows) does not correspond with the number of stands contained in it (1406 unique values in column `UG`), indicating the presence of stands that have two different prescriptions within one and the same harvesting plan. This will be further addressed in section 3.3.4.

3.3.1.C MaxRES solution

Similarly, the sheet ⊞ `MaxFireRES` identifies the selected prescriptions for the MaxRes solution — that is, the prescriptions recommended to maximize the fire resistance of the forest. It was also provided in tabular format (1,512×7), and an excerpt is shown in table 3.3. The columns are identical to those in the MaxWood Solution sheet; for a description, please refer to the grey box above.

Table 3.3: Excerpt from MaxRES Solution sheet

Decision Variable Name	Control	Presc	UG	Sp	ShPer	Sol
Presc1601_Pa1_Ec_5	1.1601	1601	1	Ec	5	Yes
...	...	...	...	...	...	Yes

Full table length: 1512 rows

Remark. As with the previous solution, the number of stands does not match the row count: while there are 1,512 rows, the solution contains prescriptions for 1,406 stands, of which 106 have two prescriptions instead of one. The reason for this discrepancy is explained in section 3.3.4.

3.3.1.D Stakeholder Solution

The stakeholder solution was provided in a different format — essentially as raw output in the form of a logfile generated directly by the optimizer. To make the data usable, the decision variable names and their corresponding solution values were extracted from the file and converted into a tabular format (see table 3.4 below at the left). At this stage, the dataset contained only the two columns described in the grey box below.

Attribute	**Description**
`Variable Name`	name of the decision variable, concatenating `Presc, UG, Sp, ShPer`
`Solution Value`	indicator if this prescription is included in the solution (0 or 1)

The data in the `Variable Name` column are not immediately usable, as they contain concatenated identifiers — specifically the prescription ID, forest stand ID, species, and shrub cleaning periodicity (`Presc`, `UG`, `Sp`, and `ShPer`). To enable further processing, these components had to be parsed and restructured into separate attributes, matching the format used in the MaxWood and MaxRes solutions. This was accomplished using regular expressions to extract the relevant elements: the numeric values following the prefixes "Presc" and "Pa" were assigned to the `Presc` and `UG` columns, respectively, and the shrub cleaning periodicity (`ShPer`) was also extracted. A species column was not created at this stage, as that information already exists in the complete prescription dataset, which would later be merged with each solution. An excerpt of the resulting structured data is shown in table 3.5 at the right as an illustrative example.

Table 3.4: Stakeholder solution (from logfile)

Variable Name	Solution Value
Presc503_Pa101_Pb_10	1.000000
Presc503_Pa1015_Pb_5	1.000000
Presc500602_Pa1016_Pb_0	1.000000
...	...

Full table length: 1514 rows

Table 3.5: Stakeholder solution (processed)

Presc	UG	ShPer	Solution Value
503	101	10	1.000000
503	1015	5	1.000000
500602	1016	0	1.000000
...	...	...	...

Full table length: 1514 rows

Correction of Linear Relaxation Artifacts

During the transformation of the stakeholder solution, two management units were identified that contained multiple prescriptions with non-binary solution values — i.e., fractional values between 0 and 1 (see table 3.6). These values are anomalous, as the solution space in this context is defined over binary decision variables: a value of 1 indicates that a prescription is selected, while 0 denotes exclusion. Notably, the fractional values within each affected management unit (`UG`) sum to 1, suggesting that the solution was obtained from a linear relaxation of the original optimization problem, of which our so-called stakeholder solution is the result. To produce a valid binary solution, a standard post-processing approach was applied: values greater than 0.5 were set to 1, and values less than or equal to 0.5 were set to 0, as shown in table 3.6. This step reduced the stakeholder solution by two entries, as prescriptions assigned a zero value were removed. The final stakeholder solution thus consists of 1,512 rows, with all decision values strictly binary.

Table 3.6: Anomalies in stakeholder solution, highlighted in yellow with corrected values

UG	Presc	ShPer	Solution Value	Corrected Solution Value
1253	36503	10	0.494295	0
1253	36	5	0.505705	1
1339	19700	0	0.567668	1
1339	19318	0	0.432332	0

Remark. At this stage, it is worth noting that all three solutions now contain the same number of prescriptions — specifically, 1,512 entries. However, this total does not correspond to the number of unique forest stands: each solution includes prescriptions for 1,406 distinct stands. The discrepancy arises from 106 stands being assigned two prescriptions each, indicating the presence of mixed stands. This aspect will be discussed in more detail in section 3.3.4.

Before delving deeper into the further processing, let us define the intended outcome:

3.3.1.E Objective of Preprocessing the Harvesting Schedule Data

The goal is to identify, for each period, the **set of stands that require road access** and determine the **appropriate road width**. This relies on two key stand-level attributes:

- **Timber removal volume (`Vremovido`)** – Any stand scheduled for timber harvesting or thinning necessitates a harvesting path connected to the road network.
- **Fire resistance indicator (`rait0/rait5/rait10`)**[a] – Stands with fire resistance below a predefined threshold need a wider road to ensure access for firefighting.

[a]The variable to be used is the one corresponding to the shrub cleaning periodicity prescribed.

3.3.2 Integrating Prescription Data into the Solutions

Each of the three solutions must be integrated with the prescription data to extract essential attributes: `Vremovido`, `rait0`, `rait5`, `rait10`, and the corresponding harvesting `period`. This was accomplished by merging the relevant prescription data with each solution.

For the MaxRes and MaxWood solutions, the integration process began by filtering the prescription dataset to retain only those rows where the columns `Na sol MaxRES` or `Na sol MaxWood`, respectively, indicated inclusion in the solution. Then, `pandas.merge` was used with an outer join to combine the filtered prescription data with the corresponding solution, matching rows based on the `UG` and `Presc` columns. This outer join ensured that all rows from both tables were retained. The results of these merges are exemplarily illustrated in table 3.7 and table 3.8, with only the relevant columns shown.

Unlike the other solutions, the stakeholder solution lacks an explicit indicator in the prescription data identifying which prescriptions are included. As a result, no filtering was performed prior to merging. Instead, an outer join was applied directly, and any rows containing *NaN* values in the merged result were removed in a subsequent step. The resulting merged stakeholder solution is exemplarily illustrated in table 3.9.

Table 3.7: MaxRes solution merged with prescription data

UG	Presc	species	period	Vremovido (ton)	rait0	rait5	rait10	ShPer
0	1	1601	Ec	2999.2720	110.7646	115.0248	85.2036	5
...	...	...	...	...	...	...	...	...

Length: 8212 rows

Table 3.8: MaxWood solution merged with prescription data

UG	Presc	species	period	Vremovido (ton)	rait0	rait5	rait10	ShPer
1	3502	Ec	1	3968.3560	136.3257	149.1062	119.285	0
...	...	...	...	...	...	...	...	...

Length: 7670 rows

Table 3.9: Stakeholder solution merged with prescription data

UG	Presc	species	period	Vremovido (ton)	rait0	rait5	rait10	ShPer
69	1	Ec	1	2999.2719	110.7646	115.0248	85.2036	5
...	...	...	...	...	...	...	...	...

Length: 8391 rows

The attentive reader might wonder why, starting from 1,512 selected prescriptions in each solution, the integration with the prescription data results in tables of different lengths — none of which equal the product of the number of prescriptions times the number of periods:

$$\underbrace{1512}_{\text{\# prescriptions}} \times \underbrace{5}_{\text{\# periods}} = \underbrace{7560}_{\text{expected length of integrated solution/prescription data}}$$

This discrepancy arises because the harvesting schedule data includes special cases where, after a total harvest, a different species is planted. In these cases, two rows of data are generated for the period of change instead of one, which explains the increased number of rows. A detailed explanation of the impact of such special cases on the data is provided in section 3.3.4.

With these nuances accounted for, the integration process ensures that all necessary information is now in place, setting the stage for the next analytical step and allowing us to confidently assign the appropriate fire resistance values based on the harvesting schedules data.

3.3.3 Assigning the Appropriate Fire Resistance Value

Each prescription specifies a shrub cleaning periodicity (`ShPer`) — either 0, 5, or 10 years — which directly affects the stand's fire resistance. Correspondingly, the dataset includes three fire resistance indicators: `rait0`, `rait5`, and `rait10`. Since decisions on road access requirements depend on a single fire resistance value per stand, only one of these indicators can be retained and used for each record.

To address this, a new column, `Rait`, was created. For each record, the appropriate fire resistance value was assigned by selecting the fire resistance indicator that corresponds to its specified `ShPer`, as illustrated in Table 3.10.

Table 3.10: Visualization of assigned fire resistance values (`Rait`) based on shrub cleaning periodicity (`ShPer`), selected values highlighted in yellow

ShPer	rait0	rait5	rait10	Rait
...	...	...	...	...
5	6.5650	7.8357	6.7768	7.8357
0	136.3257	149.1062	119.2850	136.3257
...	...	...	...	...

With all relevant attributes now available in the dataset for each stand and period, the data is nearly ready for use in the model. However, before moving on to formatting and storage considerations (see section 3.3.5), it is important to address a few special cases present in the data — namely, species changes and mixed stands.

3.3.4 Notable Special Cases: Species Changes and Mixed Stands

In preparing the dataset for model input, two noteworthy structural patterns emerged that require to be addressed: **species changes** and **mixed stands**. Although they may appear similar at first glance — both involving more than one species in a stand — their data structure and implications are fundamentally different.

Each case is described in detail below, followed by an assessment of its impact on the size of the merged datasets.

Species Changes

A species change occurs when, within a single prescription and stand, different species are listed for the same period. This reflects a harvest-replant sequence within the 10-year period: the original species (e.g., *Ec*) is harvested and then replaced by a new species (e.g., *Ct*). These rows represent consecutive steps within the same prescription.

Table 3.11 illustrates an example: During period 1, prescription no. 1312 in stand no. 1 prescribes a change from *Ec* to *Ct*. Characteristically for such species changes is that the "old" species shows a high `Vremovido` value, while the newly planted species has a `Vremovido` value of zero. Additionally, the space previously occupied by the "old" species is fully allocated to the "new" species (see `Area`).

Table 3.11: Example of a species change

UG	Presc	Species	Area	Period	Vremovido (ton)
1	1312	Ct	42.602	1	0.0
1	1312	Ec	42.602	1	2999.27
1	1312	Ct	42.602	2	0.0
1	1312	Ct	42.602	3	50.84

Mixed Stands

A mixed stand refers to a case where, in the same stand and period, *two different prescriptions* with distinct species are applied simultaneously. In contrast to species changes, both species are present together — each covering a different portion of the stand's area.

As illustrated in table 3.12, during the first two periods, stand no. 1015 contains the species *Ec* along with the second species *Pb*, each applied via a different prescription (no. 3 and no. 503) on only a fraction of the total area.

Table 3.12: Example of a mixed stand

UG	Presc	Species	Area	Period
1015	3	Ec	2.934	1
1015	503	Pb	8.036	1
1015	3	Ec	2.934	2
1015	503	Pb	8.036	2

Impact on Row Counts

These structural variations explain the peculiarities mentioned previously: Each solution's prescriptions apply only to a total of 1,406 stands (see table 3.2, table 3.3 and the corresponding remarks below). Still, the solutions consist of 1,512 rows - which is caused by the presence of mixed stands: 106 stands in the solution have two prescription instead of one.

Secondly, the species changes affect the number of rows in the merged solution datasets (table 3.7, table 3.8, table 3.9). Each species change results in two entries per `[UG, Period,` `Presc]` combination, thus inflating row counts beyond the baseline of $1,512 \times 5 = 7,560$ expected for five periods per prescription. The numbers of species change cases contained in each solution dataset are as follows: 652 in the MaxRes solution, 110 in the MaxWood solution, and 831 in the Stakeholder solution - which explains the table lengths:

MaxRes solution: $7,560 + 652 = 8,212$ rows,

MaxWood solution: $7,560 + 110 = 7,670$ rows,

Stakeholder solution: $7,560 + 831 = 8,391$ rows.

Remark. We observe that the Stakeholder and MaxRes solutions — both prioritizing fire resistance — feature significantly more species changes than the MaxWood solution, which instead optimizes for timber yield. This difference arises from variations in growth rates among species. In particular, species changes away from eucalyptus, which is currently dominant and grows rapidly, are rare in the MaxWood solution due to its emphasis on maximizing productivity. Notably, eucalyptus replanting is restricted under current regulations[11].

3.3.5 Determining and Storing Access Requirements by Stand and Period

At this stage, preprocessing of the harvesting schedule data is essentially complete. Data pertaining to stands that fall outside the scope of the study can therefore be removed. As outlined in section 3.2.1, the study area is only Castelo de Paiva. However, no separate filtering step was needed for that: As stated in section 3.2.4, of the 687 stands located in Castelo de Paiva, only the 315 that are classified as inaccessible are relevant for the road access analysis, while all accessible stands shall be excluded from the problem. Consequently, the preprocessed harvesting schedule data were filtered to retain only data on the 315 inaccessible stands (see illustration in fig. 3.10). The final datasets contain 1898 rows for the MaxRes solution, 1754 rows for the MaxWood solution, and 1952 rows for the stakeholder solution.

In a final step, the access needs of each stand in each period were identified. For each harvesting schedule (i.e., each solution), a binary matrix was constructed to indicate, for every stand and period, whether road access was required. This was done separately for each road type – timber road and wider fire road – with 1 denoting access required and 0 denoting no access required.

[11] Following the devastating wildfires in 2017, the Portuguese government enacted legislation (Decreto-Lei n.º 11/2019) restricting eucalyptus planting, particularly its reintroduction in areas previously occupied by other species, with the aim to protect native forests and reduce fire risk.

Access Determination Rules

Road access requirements for each stand and period were determined based on the following two key variables:

- `Vremovido`: Indicates the volume of timber removed during a given period. If the value is greater than zero, timber road access (i.e., the construction of at least a 3-meter road) is required.
- `Rait`: Indicates the level of fire resistance in a given period. If the value falls within the lowest 5% quantile, wide firefighting road access (i.e., the construction of a 6-meter road) is required.[12]

Remark. The `Rait` values used in this step are static and reflect current infrastructure. Dynamic recalculation of `Rait` in response to network changes over time is **not** implemented in this work.

Handling Stands with Multiple Prescriptions

In some cases, a stand may have more than one prescription per period due to special conditions such as species changes or mixed stands (as discussed in section 3.3.4), leading to multiple `Vremovido` and `Rait` values. These cases were handled as follows:

- `Vremovido`: If multiple values exist for a given stand and period and any prescription of them has a value greater than zero, access is considered required.
- `Rait`: If multiple values exist for a given stand and period, the minimum `Rait` value is used.[13]

Following these rules, a road access needs matrix was constructed for each of the three harvesting schedules (see section 3.3.6). In each matrix, the first column identifies the stand. The next five columns correspond to periods $t = 1, \ldots, 5$, indicating whether general (timber) road access is required. The final five columns indicate whether wide firefighting road access is required during each respective period.

[12] This threshold is illustrative and adjustable. Final selection should reflect expert and stakeholder judgment.

[13] This conservative approach was chosen to avoid overestimating fire resilience.

3.3.6 Summary of Preprocessed Harvesting Schedule Data

The completed preprocessing of the harvesting schedule data results eventually in the road access requirements by stand and period, which were stored in form of matrices.

MaxRes solution: access requirements matrix (315x11)

$$\begin{array}{c}
\begin{array}{ccccccccccc} \text{UG} & t_1^{\text{timber}} & t_2^{\text{timber}} & t_3^{\text{timber}} & t_4^{\text{timber}} & t_5^{\text{timber}} & t_1^{\text{wide}} & t_2^{\text{wide}} & t_3^{\text{wide}} & t_4^{\text{wide}} & t_5^{\text{wide}} \end{array} \\
\begin{pmatrix}
840 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
841 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1620 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}$$

MaxWood solution: access requirements matrix (315x11)

$$\begin{array}{c}
\begin{array}{ccccccccccc} \text{UG} & t_1^{\text{timber}} & t_2^{\text{timber}} & t_3^{\text{timber}} & t_4^{\text{timber}} & t_5^{\text{timber}} & t_1^{\text{wide}} & t_2^{\text{wide}} & t_3^{\text{wide}} & t_4^{\text{wide}} & t_5^{\text{wide}} \end{array} \\
\begin{pmatrix}
840 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
841 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1620 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}$$

Stakeholder solution: access requirements matrix (315x11)

$$\begin{array}{c}
\begin{array}{ccccccccccc} \text{UG} & t_1^{\text{timber}} & t_2^{\text{timber}} & t_3^{\text{timber}} & t_4^{\text{timber}} & t_5^{\text{timber}} & t_1^{\text{wide}} & t_2^{\text{wide}} & t_3^{\text{wide}} & t_4^{\text{wide}} & t_5^{\text{wide}} \end{array} \\
\begin{pmatrix}
840 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
841 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1620 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}$$

3.4 Costs for Road Construction and Maintenance

The final pillar of information required to address the problem at hand concerns the costs associated with constructing and maintaining forest roads. As outlined before, the model is designed with the objective is to minimize these costs. Accordingly, a preparatory step involves quantifying the cost per kilometer for each of the following actions:

- Constructing a timber road,
- Constructing a wider fire road road,
- Maintaining an existing timber road,

- Maintaining an existing wider fire road,
- Upgrading (Widening) a timber harvesting path to a wider fire road.

This section begins by outlining the available cost data (section 3.4.1), followed by the assumptions used to fill data gaps (section 3.4.2), and concludes with the derived cost estimates applied in the optimization model (section 3.4.3).

3.4.1 Available Cost data

The Portuguese Monitoring Commission for Forest Operations (Comissão de Acompanhamento para as Operações Florestais (CAOF)) provides a reference matrix detailing minimum and maximum costs for key forestry activities, including manual, mechanical, and mixed afforestation operations; reforestation; infrastructure development; and maintenance within forest territories. This matriz de (re)arborisação (CAOF 2022) includes cost estimates for the construction and the maintenance of one kilometer of timber road, categorized by three slope classes.

The CAOF values refer to forest paths of standard width suitable for mechanized timber harvesting operations. However, this study also accounts for wider fire roads, assumed to be twice the width of standard timber roads. As not all actions considered in this work are explicitly covered in the cost listings of CAOF's matriz de (re)arborisação, several assumptions[14] were necessary to estimate the remaining costs for the additional configurations.

3.4.2 Cost Estimation Assumptions

(i) The cost of constructing a wider fire road is assumed to be 150% of the cost of constructing a standard timber road:

$$\text{CostC}_{\text{wide}} = 1.5 \cdot \text{CostC}_{\text{timber}} \tag{3.1}$$

(ii) The cost of maintaining a wider fire road is assumed to be the same as maintaining a standard timber road:

$$\text{CostM}_{\text{wide}} = \text{CostM}_{\text{timber}} \tag{3.2}$$

[14] These assumptions were developed in consultation with the thesis supervisors who are considered subject matter experts.

(iii) The cost of upgrading a standard timber road to a wider fire road is assumed to be 75% of the cost of constructing a standard timber road:

$$\text{CostsU} = 0.75 \times \text{CostC}_{\text{timber}} \tag{3.3}$$

(iv) The cost of operating with a cross slope between 5% and 25% is approximated by averaging the minimum and maximum slope category costs:

$$\text{Cost}_{(5<\text{slope}<25\%)} = \frac{\text{Cost}_{(\text{slope}\leq 5\%)} + \text{Cost}_{(\text{slope}\geq 25\%)}}{2} \tag{3.4}$$

3.4.3 Estimated Costs per Action per Road kilometer

Table 3.13 presents the estimated costs in EUR, based on the values provided by CAOF and the assumptions outlined above. The color coding in the table corresponds to the colors used in the assumptions, indicating which assumption was applied to derive each value.

Table 3.13: Estimated costs; with highlighted entries color-coded to their respective assumptions.

Slope [%]	$\text{CostC}_{\text{timber}}$	$\text{CostM}_{\text{timber}}$	$\text{CostC}_{\text{wide}}$	$\text{CostM}_{\text{wide}}$	CostU
≤ 5	2147	1073.5	3220.5	1073.5	1610.25
$5 < \cdots < 25$	4830.75	1878.625	7246.125	1878.625	3623.0625
≥ 25	7514.5	2683.75	11271.75	2683.75	5635.875

Using this data, we can determine the construction, maintenance, and upgrade costs for all potential road segments considered in this study, as detailed in section 4.2. Specifically, for each segment, once the appropriate slope category has been identified, its length is multiplied by the corresponding cost per kilometer to obtain the total cost. These values are then incorporated into the LP model as input parameters.

Summary: Estimated Costs (EUR) per Action per Kilometer

Slope [%]	$\text{CostC}_{\text{timber}}$	$\text{CostM}_{\text{timber}}$	$\text{CostC}_{\text{wide}}$	$\text{CostM}_{\text{wide}}$	CostU
≤ 5	2147	1073.5	3220.50	1073.5	1610.25
$5 < \cdots < 25$	4830.75	1878.625	7246.125	1878.625	3623.06
≥ 25	7514.50	2683.75	11271.75	2683.75	5635.875

4

Implementation and Intermediate Results

Contents

THIS chapter describes the key steps of implementation of the proposed model. First, the problem was decomposed into smaller, more manageable subproblems through spatial segmentation (section 4.1). Next, the graph structure representing the network potential forest road segments (mathematically introduced in chapter 2, definition 2.6) was created (section 4.2). The model was then implemented and solved using Python and the CPLEX Python API (section 4.3): Each of the subproblems was solved independently, and their solutions were subsequently aggregated to form a complete road network for the entire study area, with the total cost obtained by summing the results across all components. This was performed for each of the three different harvesting schedules, but due to page limitations in this Master's dissertation, only the results of the stakeholder-based solution are presented in this document. The results for all three schedules are available in the accompanying GitHub repository.

4.1 Segmentation of the Problem into Independent Subproblems

The segmentation approach mentioned conceptually in section 2.3 was implemented using the `NetworkX` library in Python. Each inaccessible forest stand was represented as a node in an undirected graph[1], with spatial adjacency relationships between the stands forming the edges of the graph. Connected component analysis was used to partition this graph into geographically cohesive components. Each component corresponds to a spatial cluster of adjacent inaccessible stands that can be treated as an independent optimization subproblem. To maintain consistency in the modeling, spatially adjacent components with potentially shared access roads were combined. This allows that infrastructure can be shared, where geographically possible.

Remark. Note that natural geographical barriers, if existing, (e.g., rivers, streams, or steep terrain) should be additionally incorporated in a previous segmentation step. Such barriers may prevent connectivity between stands even if they are spatially adjacent, and therefore must be accounted for to ensure a realistic decomposition.

4.1.1 Identification of Clusters of Inaccessible Stands

To systematically identify clusters of inaccessible stands, a graph-based method using the Python library `NetworkX` was used. Each inaccessible stand in the dataset was represented

[1] This graph is not to be confused with the graph representing potential future roads, they are entirely different, just the same mathematical concept is applied.

as a node in an undirected graph, uniquely identified by its `ID_UG` attribute. An edge was created between two nodes if the stands they represented were spatially adjacent. Adjacency was determined by applying a 1-meter buffer to each stand polygon and checking for overlaps between these buffered geometries. Stands with overlapping buffers were considered neighbors and connected by an edge in the graph. This procedure resulted in a graph capturing the spatial relationships among inaccessible stands (see fig. 4.1). Connected components of the graph – subsets of nodes connected to each other but disconnected from other such subsets – were then computed. Each connected component corresponds to a contiguous cluster of inaccessible stands. Using this method, **51 clusters of inaccessible stands** were identified.

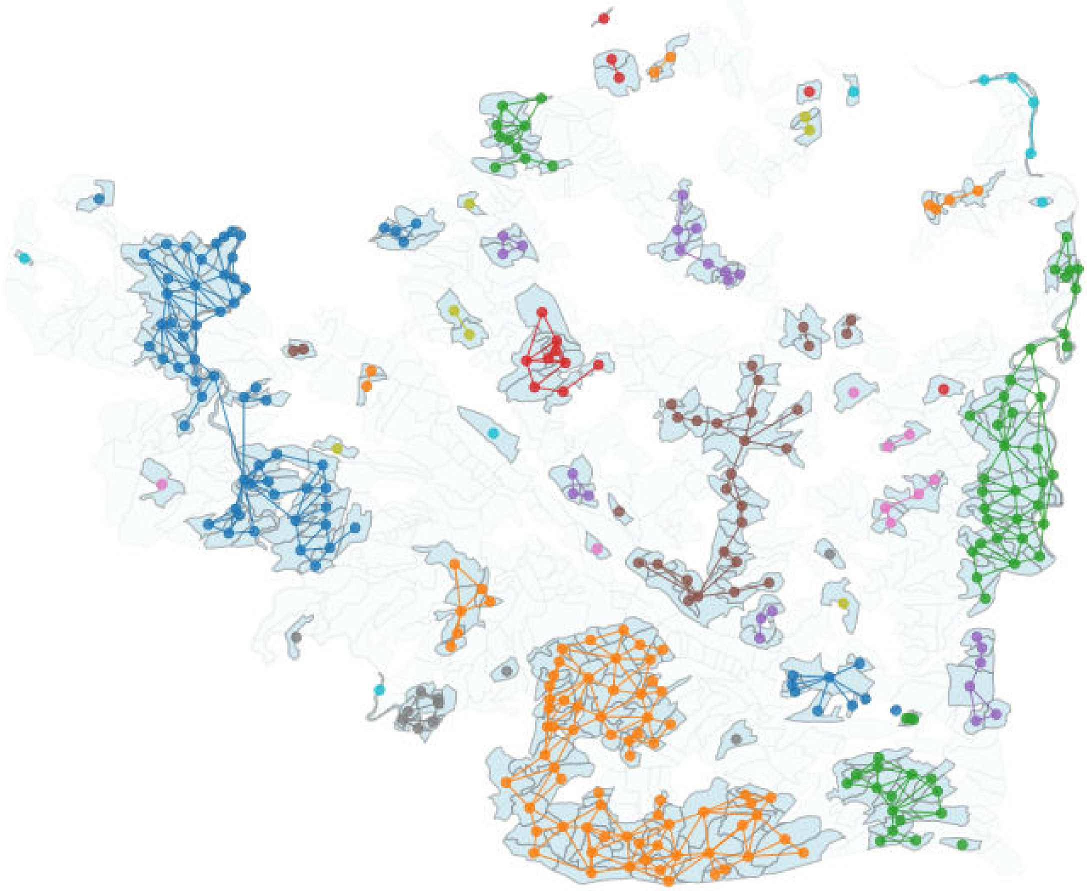

Figure 4.1: Inaccessible stands with their graph representation, forming 51 clusters

4.1.2 Merging Inaccessible Clusters with Surrounding Accessible Neighbors

The inaccessible patches had to be combined with their neighboring stands already connected to the road network, as new access roads would inevitably follow the boundaries of these accessible stands. To achieve this, each inaccessible cluster was dissolved into a single geometry, and a 1-meter buffer was applied to identify neighboring stands (their "ring of neighbors," see fig. 4.2 in green). These neighbors form the immediate interface through which future access roads can connect to the existing infrastructure. After combining each inaccessible component with its neighboring stands, the boundaries of this newly created components were stored for further analysis.

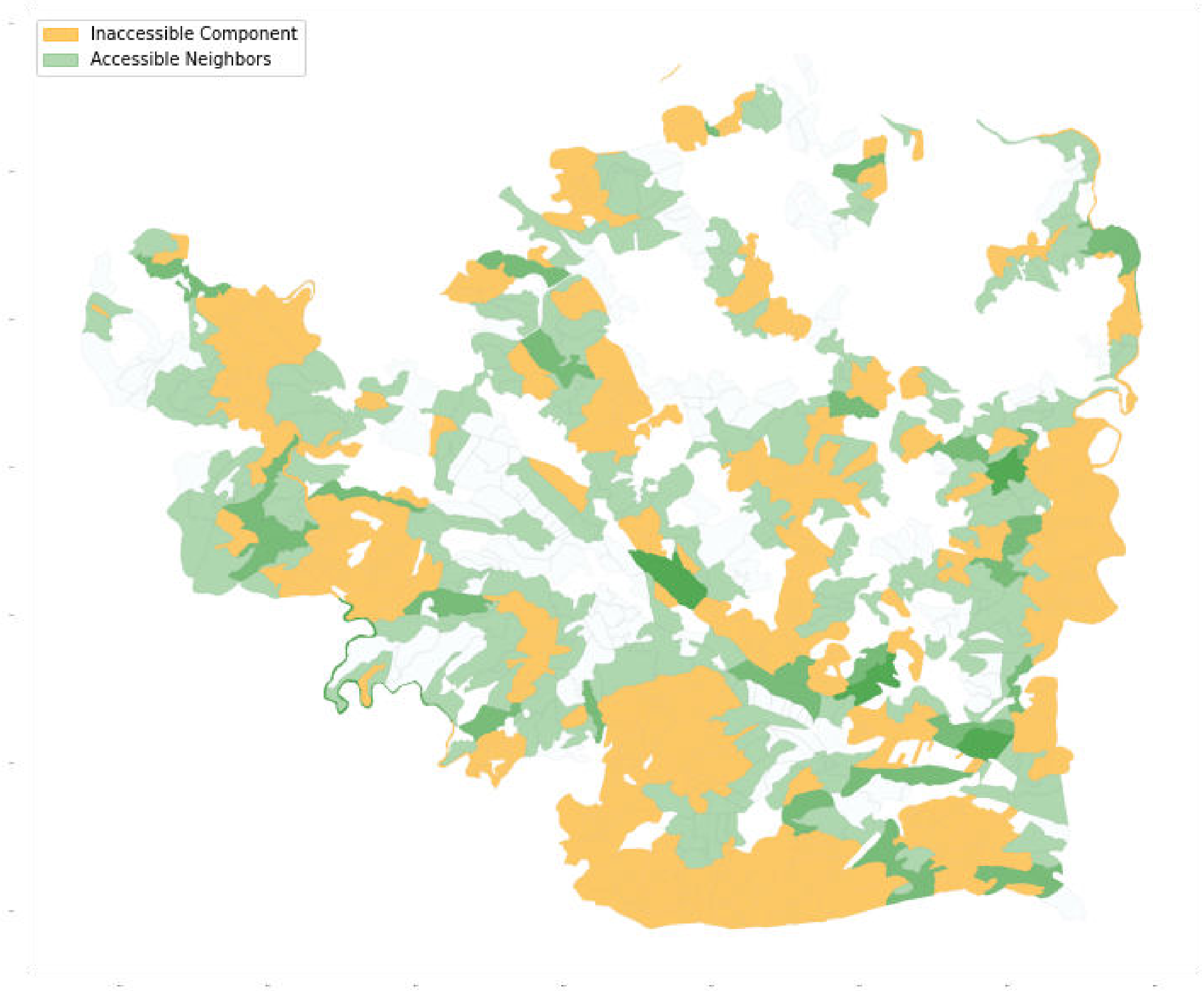

Figure 4.2: Inaccessible stands surrounded by their neighboring already accessible stands

As indicated by the darker green tones in fig. 4.2, several accessible stands are adjacent to more than one inaccessible component, which requires particular attention.

4.1.3 Handling Overlaps Between Rings of Accessible Neighboring Stands

Overlaps occur when an accessible stand is adjacent to more than one cluster of inaccessible stands, thereby belonging to multiple of the components created in section 4.1.2. To avoid redundant infrastructure planning, such overlapping components must be merged whenever access roads to one cluster could also serve another. In contrast, if two clusters share an accessible stand but lie on opposite sides of the existing road network – so that their future access roads cannot realistically overlap – their planning can remain independent, since access to one does not affect access to the other. Similarly, if two inaccessible clusters touch along their boundaries but connecting them would require a significantly longer detour than linking each directly to the network, they are treated as independent "opposite-side" cases.

Applying this rule, overlapping components were merged into joint units for access planning. This procedure yielded a decomposition into **15 independent areas**, which are presented in appendix D.

4.1.4 Intermediate Result: Independent Subproblems for Road Network Planning

Figures D.2 to D.35 illustrate the segmentation of the study area into 35 independent components. Each segment consists of at least one of the components created in section 4.1.2 – inaccessible stands combined with their accessible neighboring stands – along whose boundaries the inaccessible areas are to be connected to the pre-existing road network. The resulting segments provide the basis for the subsequent optimization in CPLEX, with each segment treated as a separate decision problem whose solutions will later be integrated.

Remark. One component (fig. D.13) can not be processed within the current study using the chosen approach, as it is not adjacent to any existing roads and has no neighboring stands whose boundaries could guide the modeling of future access. For road construction decisions in this area, stakeholders need to coordinate with the owners of surrounding lands through which future connections to the road network could be established.

4.2 Creation of the Network of Potential Road Segments

For each area segment determined in section 4.1.4, the network of potential roads was designed. What are the key requirements on the design of that road network?

- Each road segment must align with the borders between forest stands to ensure that the impact of road construction is distributed among different forest stands and their respective owners.
- The network must guarantee that every forest stand has at least one viable connection to the existing public road infrastructure.[2]

To construct the graph for this network, each component determined in section 4.1.1 was processed to extract stand boundaries and first generate an initial graph representing the road network via nodes and edges (section 4.2.1), assign respective costs (section 4.2.3), and then simplify the graph by retaining only the necessary nodes to improve computational efficiency.

In the following sections the key steps from the workflow are outlined. Additionally, data storage, visualization, and verification steps were performed which will not be explicitly detailed here, but can be reviewed in the accompanying notebooks. Throughout the construction and modification of the graph, special attention was given to the integrity of the spatial and cost-related attributes. At each major step — such as the addition of exit nodes, the splitting of edges, and the introduction of centroid-to-boundary connections — cumulative metrics (total edge length and aggregated costs) were tracked. This continuous monitoring served both as a validation mechanism to detect unintended topological distortions and as a verification tool to ensure that cost manipulations (e.g., assigning zero-cost to imaginary edges) were correctly applied.

4.2.1 Extracting Nodes and Edges from Stand Boundaries to Create a Road Network Graph

To construct the road network graph, nodes and edges were extracted directly from the boundaries of forest stands. The graph nodes were identified, accessed and stored in form of the coordinate pairs they represent. The process starts by parsing the polygonal geometries within the `GeoDataFrame` of stands of the component. Each polygon's exterior boundary is analyzed to derive graph vertices (nodes) and the linear segments between them (edges), along with associated attributes such as edge length and slope. It iterates through each feature's geometry and checks whether it is a `Polygon`.[3] Each exterior coordinate is snapped to a fixed spatial grid

[2]Without this, the model would not be able to generate feasible solutions, as certain stands would remain inaccessible regardless of the selected road segments.

[3]In this study's dataset, all geometries were simple `Polygons`. If your data includes `MultiPolygons`, ensure only the outer boundaries are used. Interior holes are enclosed within a single stand and do not connect to neighboring stands or the existing road network, making them irrelevant for graph construction.

to improve precision and ensure consistency across adjacent stands. These snapped coordinates form the graph's nodes. Edges are constructed by connecting each consecutive pair of exterior points. For each edge, two main attributes are calculated:

- **Edge length**, computed as the Euclidean distance between the two vertices.
- **Slope**, inherited from the "`Declive`" attribute of the associated stand polygon.[4]

This returns five key elements: A list of all unique boundary vertices (nodes), a list of edges (pairs of nodes), a list of dictionaries storing edge attributes (length, slope, etc.), a list of exit points, where stand boundaries intersect the existing road network, and a mapping of nodes to the stands they belong to. These are then used to construct an undirected graph using the `networkx` library, where nodes represent snapped boundary coordinates, and edges represent the physical path segments along stand boundaries. An illustration of one resulting graph component is shown in fig. D.36.

Remark. It is critical for this step that the polygon geometries in the shapefile are complete and not corrupted. This requirement enforced the extensive preprocessing steps described in section 3.2.2, whose completion ensured valid, clean geometries such that the boundary extraction process was eventually working without weird things happening.

4.2.2 Creating exit nodes

To integrate potential exit points into the network model, a series of spatial procedures were applied to ensure correct mapping to the graph topology. Each candidate exit point is associated with the closest graph node based on Euclidean distance. If the nearest node lies within a 10-meter threshold, the exit point is merged with that node; otherwise, it is added as a new node. If the exit point does not coincide with an existing node, the graph is modified to introduce a new vertex at the closest edge by splitting it into two segments connected via the new exit node. The new edges retain key attributes like slope and are flagged with a `has_exit` attribute for downstream use. See fig. D.37 as an example.

[4] While this per-polygon slope approximation is coarse, it is the only slope information available. See further discussion in chapter 5.

4.2.3 Assigning Cost Attributes to Network Edges

The costs for each road segment were calculated using the base costs per action performed on road kilometer as estimated in section 3.4 and their individual length and slope. The respective costs were assigned as attribute to the edges. Based on the distinction made by CAOF, three slope categories were distinguished:

- **Flat Terrain** ($\leq 5°$)
- **Moderate Slope** (between 5 and $25\circ$)
- **Steep Terrain** ($\geq 25°$)

The function created to assign construction, maintenance, and upgrade costs to each edge in the road network graph G based on its slope and length iterates over all edges. Each edge is classified into one of three slope categories, and the costs to build and maintain for both widths, and upgrade of the road are computed multiplying the predefined base cost for each action performed on a kilometer of road (see table 3.13) with the length of each edge. The resulting costs are stored as edge's attributes.

4.2.4 Improve the graph by Merging Short Edges

As shown in fig. D.36, some edges in the graph created are quite short, resulting in unnecessarily fragmented road segments. There is no justification to keep this over-fragmentation, it occurs purely due to the shape of the geometries. To simplify our model and improve efficiency (again, model strenghtening by reducing the number of decision variables), we merge "short" edges, which reduces the number of decision variables.[5]

Remark. Keep in mind that after this step, the edges in the graph no longer represent the stands' borders with geographical accuracy, however, the relevant information (costs) is retained. Furthermore, the original length of the edges is retained, to be able to verify the results of the merge. Please note that the order the steps are done matters: The assignment of costs and determining the exit nodes needs to be done before abstracting the graph further.

The function created iteratively merges "short" edges in the graph G based on the specified threshold. In this case, the threshold was set to 50 meters. It identifies edges shorter than the

[5]This could also be mentioned in section 2.3, but to better understand the process it is explained here.

threshold and attempts to merge them by connecting two neighboring nodes that both have exactly two neighbors[6]. For each such pair, the function:

- Calculates the new merged edge's length, slope, and cost attributes (e.g., `Build5m`, `Maintain10m`).
- Removes the original edges and the central node.
- Adds a new merged edge between the remaining nodes.

This process continues iteratively until no further merges are possible. The result is an optimized graph with fewer road segments. This reduces the overall network complexity and ensures a more efficient representation of the road network.

After merging edges, the integrity of the network was verified by visually comparing pre- and post-merged graphs, and especially the relevant information (like, road intersections and exit nodes are maintained). Furthermore, key attributes, such as network length, and total costs, were checked to ensure that no inconsistencies were introduced.

Remark. Please note that after this step of merging edges, the edges in the graph no longer represent the stands' borders with geographical accuracy. However, this was the exact goal and is not a concern, as the essential information (edge length for verification purposes and cost attributes for optimization) remain preserved. This aligns with the fundamental principle of modeling: simplifying the representation while retaining only the necessary elements.

The resulting graph with all the created additional information was then stored to be used in the next step.

4.3 Model Implementation and Solution using CPLEX Python API

The model was implemented and solved using Python and the CPLEX Python API, provided via the package cplex. This package serves as an interface to the CPLEX solver engine, which must be installed on the system.

The theoretical model, as formulated in chapter 2 and summarized in 2.4, was translated into a computational representation compatible with IBM ILOG CPLEX Optimization Studio. Each

[6]This condition is essential because if a node has three or more neighbors, it represents a road intersection, which must be preserved.

variable, constraint, and parameter was defined programmatically so that the solver could correctly interpret the problem. The original model structure – including capacity constraints, flow conservation, and connectivity rules – was preserved while feeding the formulation to the CPLEX Python API. This enabled automated solution of the optimization problem for each segment.

The optimization model was implemented to evaluate forest road planning across multiple harvesting schedules and spatial components. The model loops over the three provided harvesting schedules, and for each schedule, it iterates over all problem area components. For each component, nodes, arcs, and associated attributes are loaded, and identifiers are mapped and categorized into source, transit, and exit nodes. Edge attributes, costs, and access requirements are prepared to reflect operational constraints. Decision variables representing road construction, maintenance, upgrades, and material flows are defined, forming the basis of a CPLEX optimization model. The model incorporates binary and flow variables, an objective function to minimize total costs, and a comprehensive set of constraints, including flow enforcement, flow conservation at transit nodes, source and exit handling, and maintenance or upgrade requirements for different road types. After constructing the model, variables and constraints are verified, the model is exported in standard formats (LP, SAV, MPS), and solved using CPLEX. The resulting solutions are post-processed, with non-zero flows remapped to the original node coordinates, and stored for each component. Component-level solutions are then aggregated.

5

Results and Discussion

Contents

5.1 Overview of the Optimal Road Network

Present the key numerical and visual outputs of the optimization model

The implemented model produces an optimized forest road network that minimizes construction and maintenance costs while maximizing its contribution to forest fire resilience.

...

5.2 Model Scope and Limitations

Discuss the intended use-case of the model and where its predictions are most relevant or limited.

While the study provides valuable insights, several limitations should be acknowledged regarding data quality, model applicability, and forest type considerations.

Forest Type and Management Objective

This work was developed for an area where forests are primarily managed for timber harvesting. The model is therefore limited to timber-oriented forests and may not directly apply to national parks or forests with other objectives. Wildfires are part of natural dynamics; not all fires are harmful, and the model assumes management prioritizes economically valuable stands.

Targeting Inaccessible Stands

Only currently inaccessible stands are targeted for new roads due to practical and methodological reasons. First, there is significant data uncertainty, as the width and usability of existing small roads are unclear (see section 3.1.3). Second, forest owners' priorities focus on timber harvesting, which makes creating new access more important than upgrading existing roads. Third, inaccessible stands represent the most urgent connectivity gaps for both timber extraction and firefighting operations. Fourth, constructing new roads in isolated areas is generally more cost-efficient than upgrading small existing roads. Fifth, many existing roads are publicly owned, which complicates coordination for upgrades, whereas new roads on privately managed land allow more straightforward implementation. Finally, limiting the scope to inaccessible stands improves model tractability and simplifies the optimization problem.

Assumptions and Practical Constraints

Outline simplifying assumptions and limitations that affect practical implementation.

Minor streams are seasonal and generally do not hinder forestry operations. The Douro and Paiva rivers form study area boundaries and are not crossed by roads. The segmentation algorithm accounts for disconnected components caused by unbridgeable terrain.

Consideration of Natural Boundaries

Natural barriers such as rivers or streams were not considered in this study. In practice, such features may require splitting components along their course if roads cannot feasibly cross them without specialized infrastructure.

A hydrographic dataset from the Copernicus EU-Hydro River Network Database was identified, but it lacked key attributes such as depth, permanence, and seasonal variation. Moreover, as discussed in Chapter **??** (Section **??**), many mapped streams may not exist in practice or may only carry water intermittently.

To avoid introducing misleading constraints, natural boundaries were therefore not used for segmentation. Instead, it was assumed that these features either do not present substantial obstacles or can be crossed without specialized infrastructure. Nonetheless, a spatial overlay showed that several components intersect with mapped hydrographic features (see fig. 5.1). While excluded here, these could provide a basis for refinement if more reliable hydrological data becomes available.

5.3 Data Quality

What could have been in ideal world with ideal data

Different segmentation of problem would have been possible in ideal world with ideal data. Might have better represented this problem.

I would suggest to start by segmenting the problem in segments first that are not accessible by wide roads, then find the ring of wide road accessible stands around it, and treat each of these components as one seperate piece of forest.

Reflect on how data uncertainties may influence the robustness of the results.

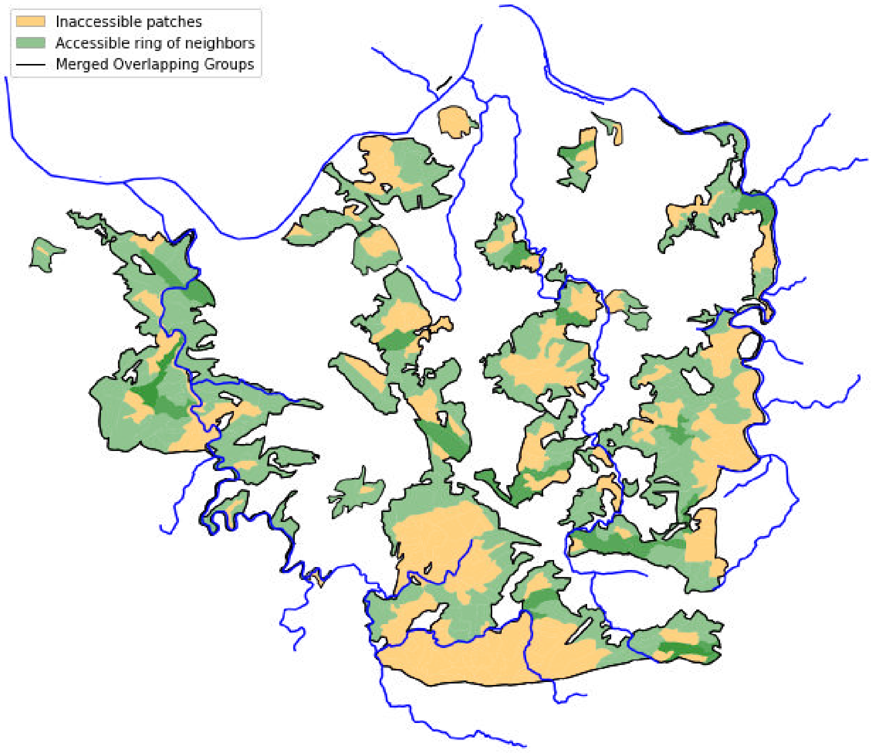

Figure 5.1: ???

Results depend heavily on data quality, and poor input data may affect real-world applicability.

The accuracy of cost estimates directly affects the optimality of the solution. If input costs are imprecise, the model may select road segments based on flawed assumptions, potentially identifying a cost-minimal solution that does not correspond to the real-world optimum.

5.3.1 Dataset Provenance and Limitations

The dataset was originally collected for other purposes. For information beyond the description in this work, please its provider[1]. While supplementary sources were reviewed, granularity limitations persist.

Road width and slope values were estimated due to incomplete data, which may affect feasibility and connectivity predictions. Misestimation of wide roads could result in ineffective firefighting access in practice.

[1] smarques@isa.ulisboa.pt

Road Width Estimation

Road widths were estimated according to *DESIGNACAO*:

- **ENxxx / Variante ENxxx:** 10 m
- **REM:** 6 m
- **RVF:** 3 m

Measured road widths varied significantly.

Slope Data Limitations

Slope data were available only as stand-averaged values, which may not reflect local variations critical for road feasibility. Future analyses should use high-resolution DEMs for more accurate modeling.

Impact of Estimations (Road Width, Slope)

Highlight which estimated values had the most critical impact on model behavior.

—

5.4 Methodological Considerations

Review the reasoning behind selected modeling techniques and consider possible alternatives.

Solver and Algorithm Choices: Why CPLEX

The Simplex algorithm was chosen for its efficiency, robustness, interpretability, and suitability for moderate-size structured problems.

IBM CPLEX was chosen for its performance, numerical stability, and strong handling of MILP constraints.

Methodology: Mixed-Integer Linear Programming (MILP)

MILP was preferred over AI-based methods for several reasons:

- **Interpretability:** MILP provides transparent and explainable results essential for communication with stakeholders, whereas AI models often act as black boxes.
- **Optimality and Feasibility:** MILP guarantees optimal solutions within strict constraints, which is critical for infrastructure investments and regulatory compliance. AI heuristics provide approximate solutions without firm guarantees.
- **Computational Efficiency:** MILP solves the problem effectively without the extensive training and tuning required by AI methods.
- **Data Requirements:** MILP relies on mathematically defined parameters rather than large historical datasets needed for AI.
- **Problem Context:** This study involves a static, one-time optimization rather than dynamic real-time decision-making, making MILP more suitable.

Using IBM CPLEX assed via its python API combined with **MILP!** (**MILP!**) offered the best balance of performance, reliability, interpretability, and practical applicability for this problem. While AI-based approaches have merits in dynamic or data-driven settings, the structured nature and policy-driven requirements of this study favored a deterministic, exact optimization framework

Alternative Ideas

Potential for a Two-Stage Decision Process – Given the distinct purposes of small and wide roads, the decision process could be structured in two stages. The first stage focuses on constructing new roads where access is entirely absent, ensuring timber harvesting and fire resilience where needed. The second stage, which could be explored in future research, would evaluate whether existing small roads require widening based on fire risk assessments.

6

Conclusion

Contents

6.1 Practical Implications

6.1.1 Real-World Application Scenarios

Describe how the model might be used in actual planning and decision-making environments.

6.1.2 Challenges in Stakeholder Adoption

Anticipate resistance or barriers from stakeholders, and suggest ways to build trust and usability.

6.1.3 Use in Decision-Making Frameworks

Explain how the model outputs could feed into multi-criteria or participatory planning processes.

6.2 Potential Future Work

6.2.1 Data Improvement Needs

Identify key data gaps and propose strategies for obtaining more accurate, high-resolution, or dynamic inputs.

-Future analyses should use high-resolution DEMs for more accurate modeling of slope data.

-Road Width Data needs to be improved

6.2.2 Consideration of Natural Boundaries

Natural barriers such as rivers or streams were not considered in this study. In practice, such features may require splitting components along their course if roads cannot feasibly cross them without specialized infrastructure.

6.2.3 Integration of Firebreaks and Fire Behavior in the Model

Discuss the value and challenges of incorporating fire spread models and preventive infrastructure.

6.2.4 Inclusion of Broader Ecological and Social Factors

Consider adding biodiversity, habitat fragmentation, or community access as factors in optimization.

The proposed road network may have ecological impacts, including increased soil erosion, local biodiversity disturbance, and habitat fragmentation. Special attention is warranted for areas near rivers and streams, which are biodiversity hotspots. Future models could include penalties for roads near sensitive areas to reduce environmental impacts.

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A

The FIRE-RES project

FIRE-RES is a four-year European project (2021–2025) dedicated to developing integrated forest management strategies to address the challenges posed by Extreme Wildfire Events (EWEs). It aims to harmonize wildfire emergency response with landscape-level planning and evaluate the economic feasibility of resilience-enhancing measures. The project's overarching goal is to develop tools, processes, and methodologies that promote integrated approaches to managing EWEs. It further advocates for proactive governance to improve societal resilience, raise risk awareness, and support effective communication—ultimately fostering fire-resilient landscapes and communities.

FIRE-RES is implemented through eleven Living Labs, which serve as open innovation ecosystems located in fire-prone regions across Europe and beyond. These include: Bulgaria, the Canary Islands, Catalonia and Galicia (Spain), Chile, Germany/The Netherlands, Greece, Norway/Sweden, Nouvelle-Aquitaine (France), Portugal, and Sardinia (Italy). Measures and approaches developed within these contexts are intended for broader replication and scaling.

For more information, see: `https://fire-res.eu`.

B

Quantifying wildfire risk: the wildfire resistance indicator

A stand-specific fire resistance indicator was built to assess resistance to wildfires based on stand-level wildfire occurrence and damage probabilities (Garcia- Gonzalo et al. 2012, Marques et al. 2012), taking also into account stand geometric (e.g., shape and size) and topological (e.g., spatial context) features. In a following step, the authors adjusted this stand-specific indicator according to the neighboring relations and the features of adjacent stands, also incorporating the risk of wildfire spread from one stand to another.

In a first step, the stand-specific fire resistance indicator was defined to reflect the environmental and biometrical features (e.g., stand density, shrubs biomass) of a forest stand during one period.

Definition B.1 (Specific Wildfire Resistance Indicator)**.** The *specific wildfire resistance indicator* $R_{s,t} \in [0;1]$ of a stand s in period t reflects the proportion of trees in stand s that are prone[a] to survive a wildfire occurring in period t.

[a]according to wildfire occurrence and postfire mortality probability models from Garcia-Gonzalo et al. 2011, Marques et al. 2012

Secondly, the resistance indicator was adjusted using via parameters. One parameter reflects the overall impact of neighboring stands on the wildfire resistance of a stand s, while the other explains the contribution of each individual neighboring stand x (in terms of fire spread likelihood). Before continuing to the adjusted indicator, let's define those parameters:

Definition B.2 (Overall Impact of Neighboring Stands)**.** The *overall impact of neighboring stands* w_s on the fire resistance of stand s is defined as

$$w_s := \theta \frac{\sqrt{\pi}\sqrt{Area_s}}{Perimeter_s}, \qquad \text{(Overall Impact)}$$

with $\theta := \frac{Area_s}{\rho_s + Area_s}$, where $\rho_s \geq 0$ may be another scale-related parameter.

Definition B.3 (Individual Impact of a Neighbouring Stand)**.** The *individual impact of a neighboring stand* x on the wildfire resistance of stand s is defined by

$$\alpha_{sx} := f_{sx} u_{sx} p_{sx}, \qquad \text{(Individual Impact of } x \text{ on } s\text{)}$$

with

$f_{sx} \in [0,1]$: the *fraction of the border* of stand s that is *shared* with stand x;

$u_{sx} \in [0,1]$: the *likelihood of fire spread from x to s, based on the relative positions* of x and s;

$p_{sx} \in [0,1]$: the *likelihood of fire spread from x to s, based on the existence of barriers* between the two stands.

While the overall impact of neighboring stands on wildfire resistance of a stand s is assumed to decrease with its area and increase with its perimeter, the likelihood of a wildfire occurring in a stand x to spread to stand s is assumed to increase with the length of the shared edge between the two stands. This likelihood also depends on the relative position of the two stands, especially the slope, relative altitude and aspect, and the existence of barriers between them (e.g., roads).

Remark. Let $V(s)$ be the set of *neighbors of stand* s. Then the sum of the individual impacts of all neighboring stands $x \in V(s)$ is bounded by 1, i.e. it is

$$\sum_{x \in V(s)} \alpha_{sx} \leq 1. \tag{B.1}$$

Proof. It is

$$\sum_{x \in V(s)} f_{sx} \leq 1 \quad \text{and} \quad \forall s \forall x : \quad \alpha_{sx} := f_{sx} \underbrace{u_{sx} p_{sx}}_{\in[0,1]} \leq f_{sx}$$

by definition (see definition B.3). The upper bound B.1 directly follows.

□

Now, using the previously defined parameters, the adjusted wildfire resistance can be defined as per following:

Definition B.4 (Adjusted Wildfire Resistance Indicator)**.** The *adjusted resistance level* RA_{st} of stand s in period t is defined as the unique[a] solution for all $s \in S, \quad t \in T$ of the following equations

$$RA_{st} = R_{st} + (1 - w_s) \sum_{x \in V(s)} \alpha_{sx} (RA_{xt} - R_{st}), \tag{B.2}$$

with $(1 - w_s)$ as a weight to reflect the overall impact of neighboring stands on the wildfire resistance of s, and $\alpha_{sx} \in [0; 1]$ reflecting the likelihood of a fire spreading from stand x to s.

[a]For the assumptions described here, it can be shown that the equations B.2 have exactly one solution (Ferreira 2011).

C

Large Tables

Action	Road Width	Period t	Decision Variable
Construction	timber	1	$C_{e,1}^{\text{timber}}$
Construction	timber	2	$C_{e,2}^{\text{timber}}$
Construction	timber	3	$C_{e,3}^{\text{timber}}$
Construction	timber	4	$C_{e,4}^{\text{timber}}$
Construction	timber	5	$C_{e,5}^{\text{timber}}$
Construction	wide	1	$C_{e,1}^{\text{wide}}$
Construction	wide	2	$C_{e,2}^{\text{wide}}$
Construction	wide	3	$C_{e,3}^{\text{wide}}$
Construction	wide	4	$C_{e,4}^{\text{wide}}$
Construction	wide	5	$C_{e,5}^{\text{wide}}$
Maintenance	timber	1	$M_{e,1}^{\text{timber}}$
Maintenance	timber	2	$M_{e,2}^{\text{timber}}$
Maintenance	timber	3	$M_{e,3}^{\text{timber}}$
Maintenance	timber	4	$M_{e,4}^{\text{timber}}$
Maintenance	timber	5	$M_{e,5}^{\text{timber}}$
Maintenance	wide	1	$M_{e,1}^{\text{wide}}$
Maintenance	wide	2	$M_{e,2}^{\text{wide}}$
Maintenance	wide	3	$M_{e,3}^{\text{wide}}$
Maintenance	wide	4	$M_{e,4}^{\text{wide}}$
Maintenance	wide	5	$M_{e,5}^{\text{wide}}$
Upgrade	wide	1	$U_{e,1}$
Upgrade	wide	2	$U_{e,2}$
Upgrade	wide	3	$U_{e,3}$
Upgrade	wide	4	$U_{e,4}$
Upgrade	wide	5	$U_{e,5}$

Table C.1: Binary Decision Variables for Construction, Maintenance, and Upgrade of road segment e

Flow type	Flow direction	Period	Decision Variable
timber	backwards	1	$\text{flow}^{\text{timber}}_{(v,u),1}$
timber	backwards	2	$\text{flow}^{\text{timber}}_{(v,u),2}$
timber	backwards	3	$\text{flow}^{\text{timber}}_{(v,u),3}$
timber	backwards	4	$\text{flow}^{\text{timber}}_{(v,u),4}$
timber	backwards	5	$\text{flow}^{\text{timber}}_{(v,u),5}$
timber	forwards	1	$\text{flow}^{\text{timber}}_{(u,v),1}$
timber	forwards	2	$\text{flow}^{\text{timber}}_{(u,v),2}$
timber	forwards	3	$\text{flow}^{\text{timber}}_{(u,v),3}$
timber	forwards	4	$\text{flow}^{\text{timber}}_{(u,v),4}$
timber	forwards	5	$\text{flow}^{\text{timber}}_{(u,v),5}$
wide	backwards	1	$\text{flow}^{\text{wide}}_{(v,u),1}$
wide	backwards	2	$\text{flow}^{\text{wide}}_{(v,u),2}$
wide	backwards	3	$\text{flow}^{\text{wide}}_{(v,u),3}$
wide	backwards	4	$\text{flow}^{\text{wide}}_{(v,u),4}$
wide	backwards	5	$\text{flow}^{\text{wide}}_{(v,u),5}$
wide	forwards	1	$\text{flow}^{\text{wide}}_{(u,v),1}$
wide	forwards	2	$\text{flow}^{\text{wide}}_{(u,v),2}$
wide	forwards	3	$\text{flow}^{\text{wide}}_{(u,v),3}$
wide	forwards	4	$\text{flow}^{\text{wide}}_{(u,v),4}$
wide	forwards	5	$\text{flow}^{\text{wide}}_{(u,v),5}$

Table C.2: Flow variables defined for road segment $e = \{u, v\}$

Table C.3: Illustration of relevant columns from the prescription data sheet

UG	ug_Sp	area	presc	Na sol MaxRES	Na sol MaxWood	period	species	vthin (ton)	vharv (ton)	Vremovido (ton)	rait0	rait5	rait10
1	1_Ec	42.6018	1	#N/A	#N/A	1	Ec	0	2999.2720	2999.2720	132.0655	166.147	136.3257
1	1_Ec	42.6018	1	#N/A	#N/A	2	Ec	0	5575.8278	5575.8278	140.5859	166.147	140.5859

Full table size: 317,883 rows across 41 columns.

D

Visualizations

D.1 Segmentation Results

Figure D.1: Component 1

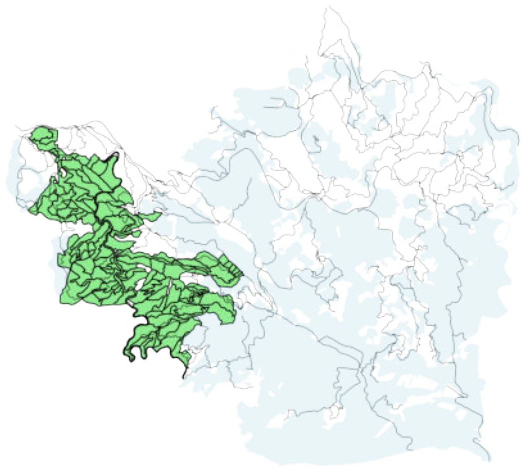

Figure D.2: Component 2_44 merged

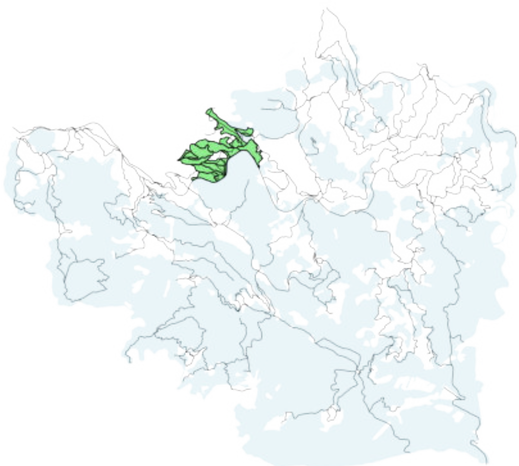

Figure D.3: Component 3_43 merged

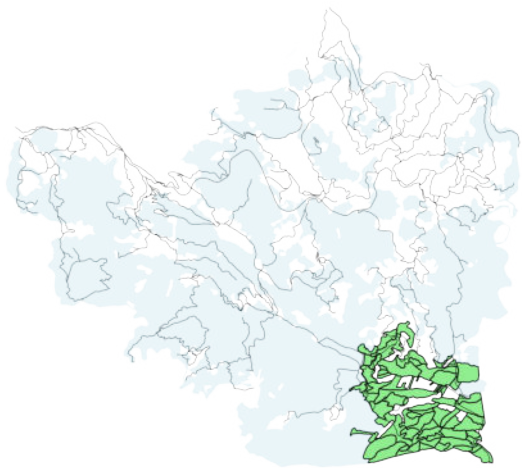

Figure D.4: Component 4_11_12_13_51 merged

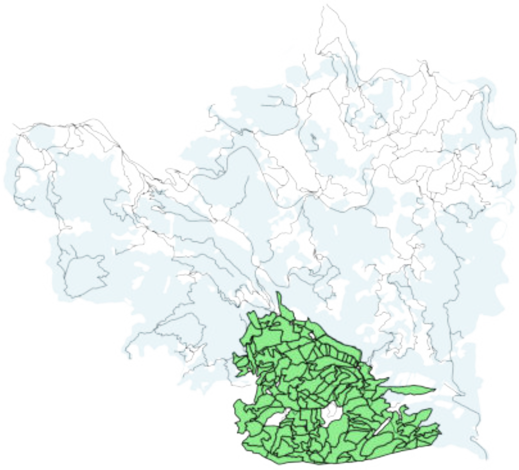

Figure D.5: Component 5_39 merged

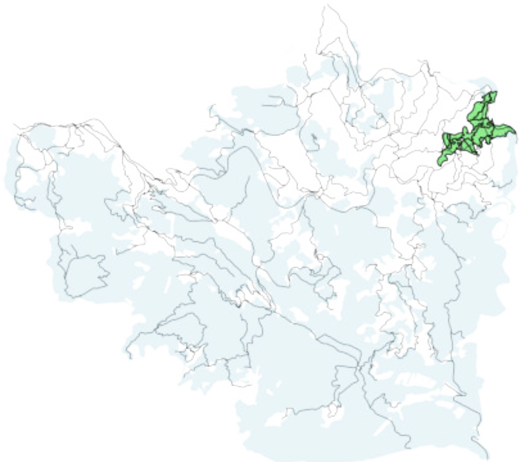

Figure D.6: Component 6

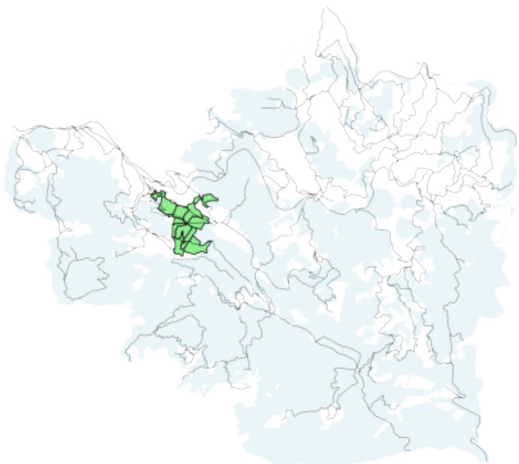

Figure D.7: Component 7

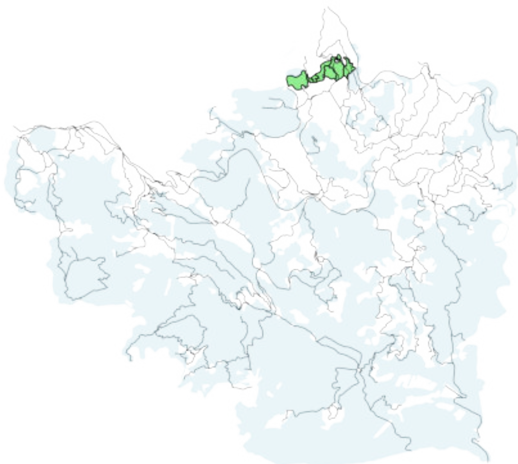

Figure D.8: Component 8

Figure D.9: Component 9

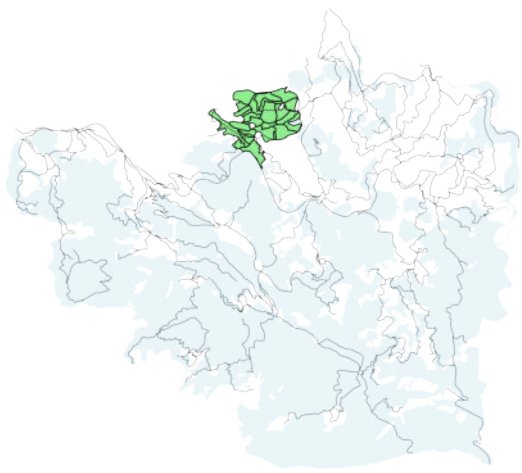

Figure D.10: Component 10

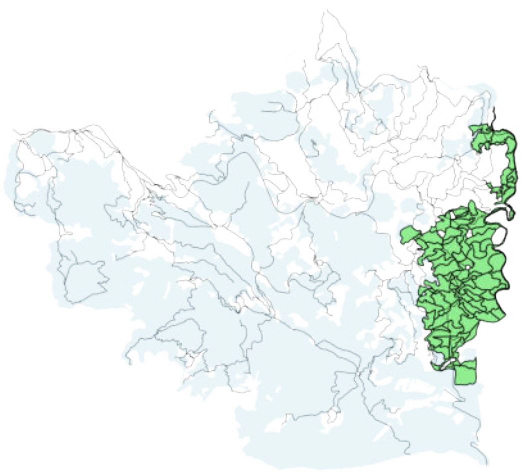

Figure D.11: Component 14_15_48 merged

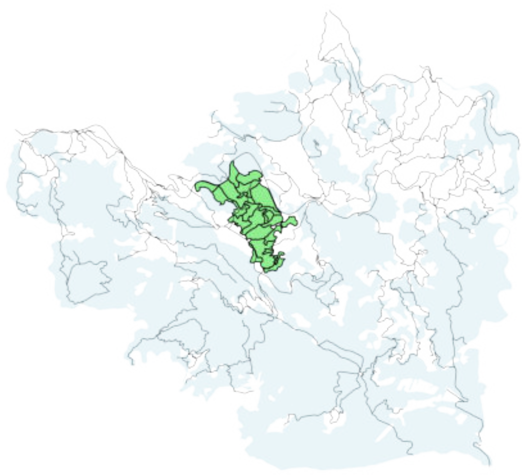

Figure D.12: Component 16

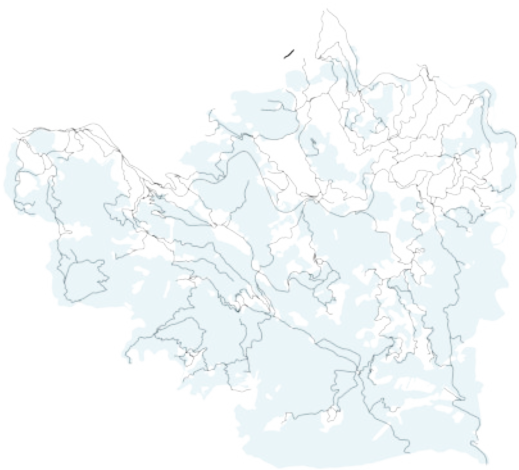

Figure D.13: Component 17

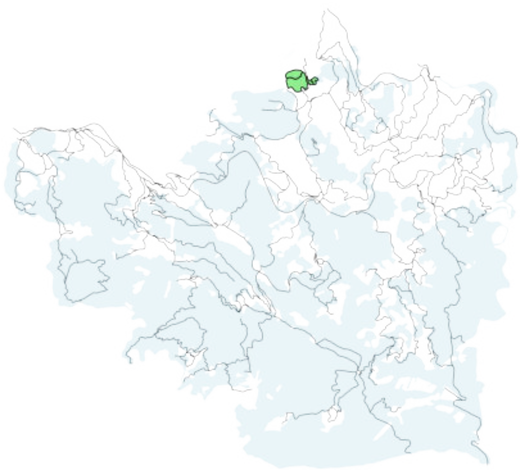

Figure D.14: Component 18

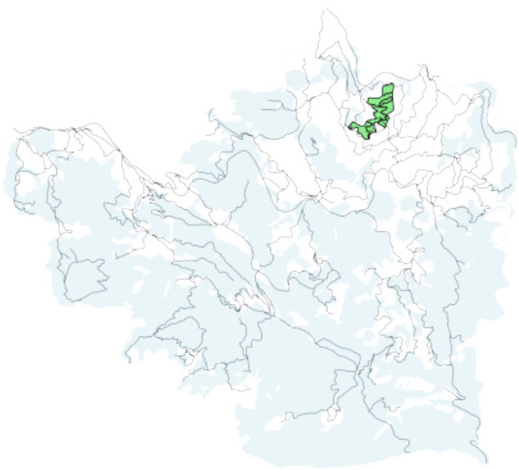

Figure D.15: Component 19_41 merged

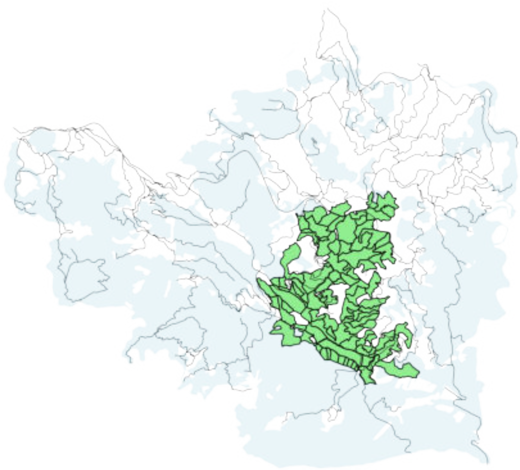

Figure D.16: Component 20_25_28_33 merged

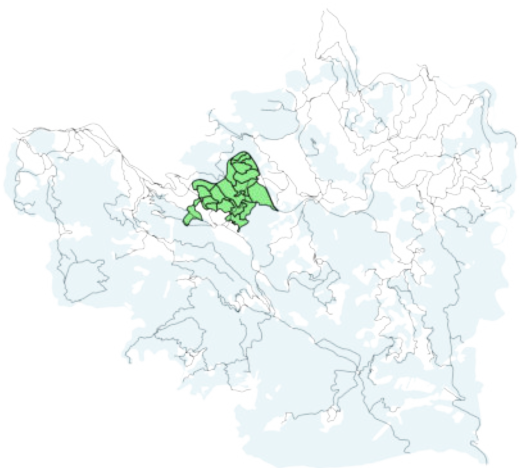

Figure D.17: Component 21_40 merged

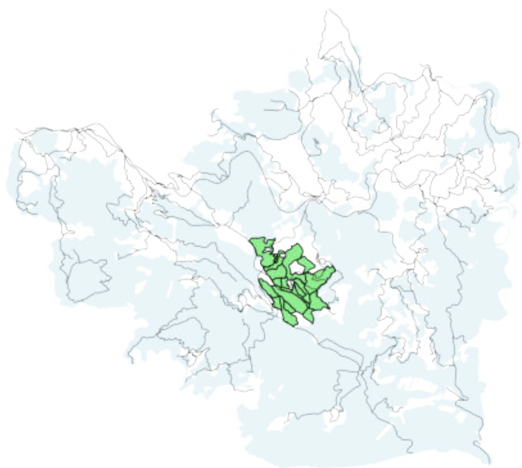

Figure D.18: Component 22

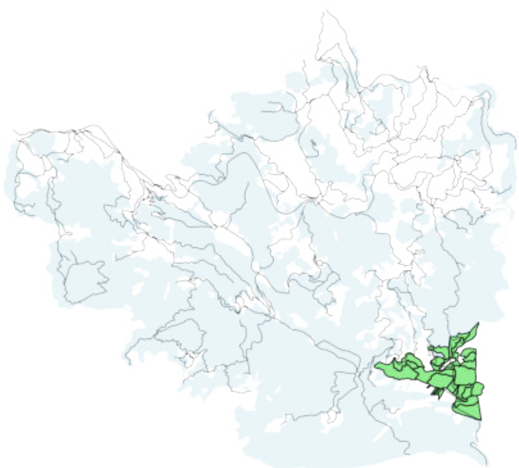

Figure D.19: Component 23

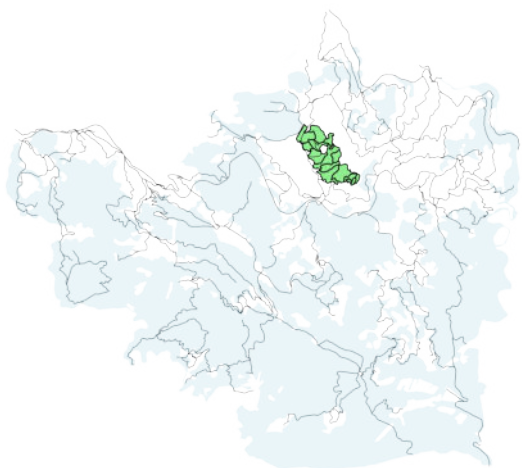

Figure D.20: Component 24

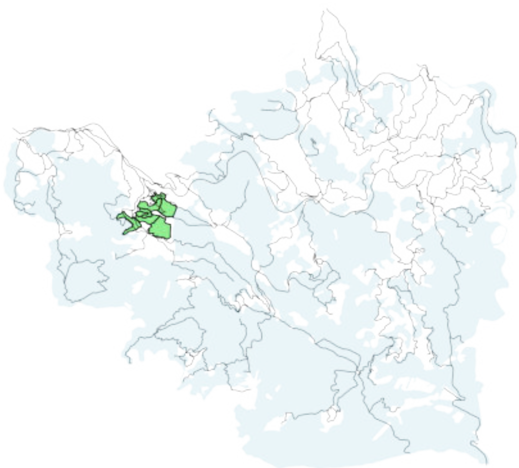

Figure D.21: Component 26

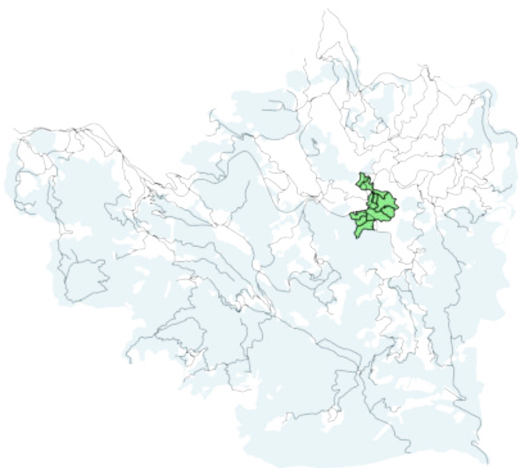

Figure D.22: Component 27

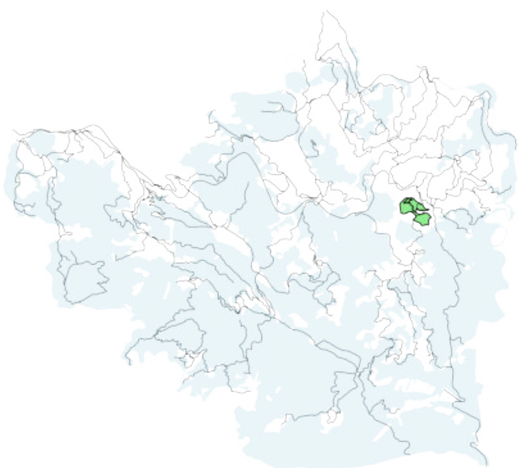

Figure D.23: Component 29

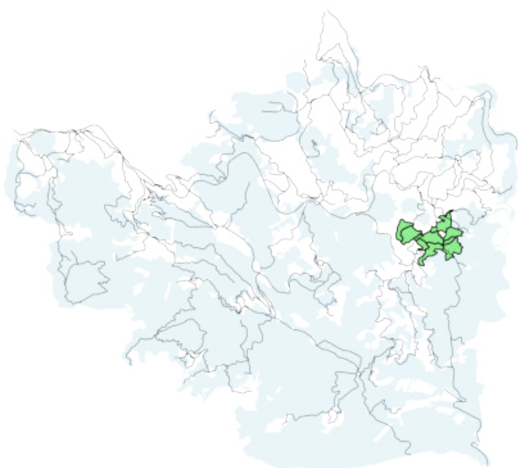

Figure D.24: Component 30

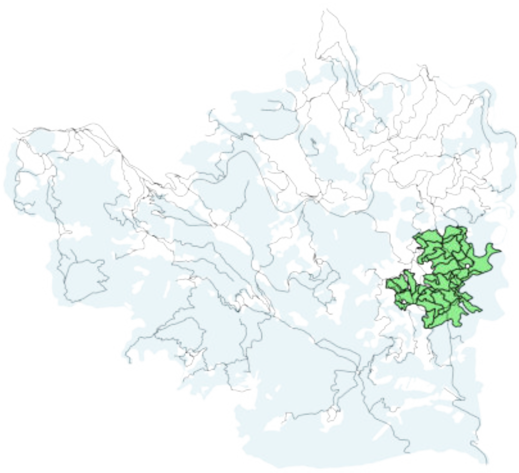

Figure D.25: Component 31_34 merged

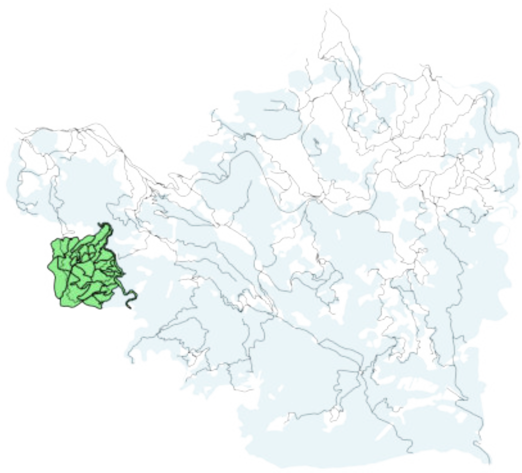

Figure D.26: Component 32

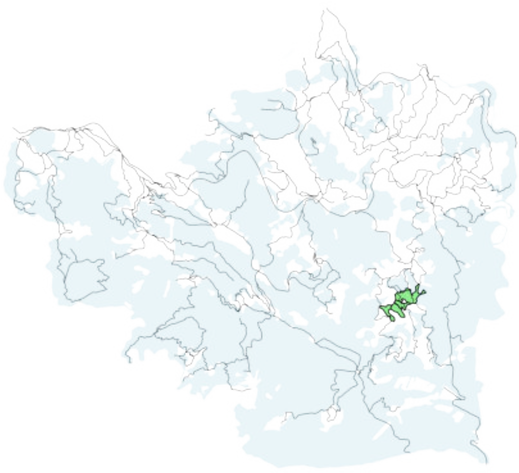

Figure D.27: Component 35

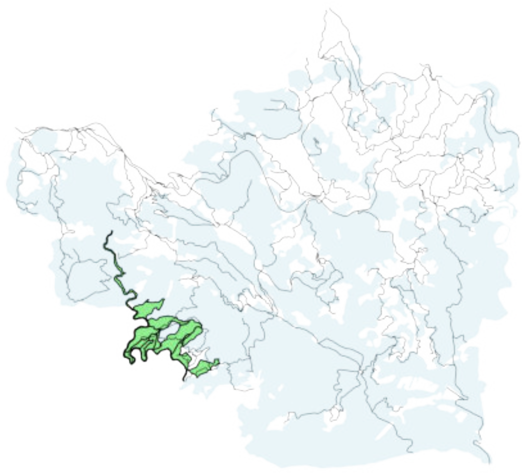

Figure D.28: Component 36_50 merged

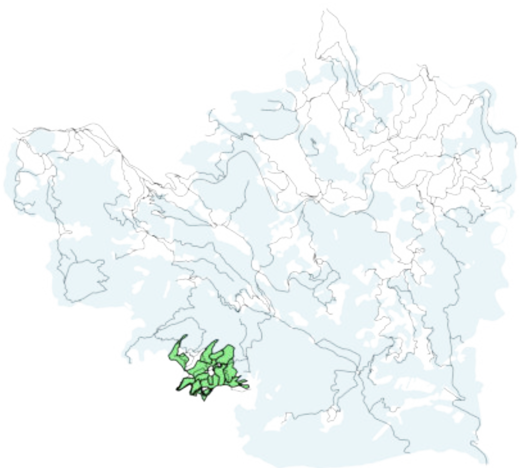

Figure D.29: Component 37

Figure D.30: Component 38

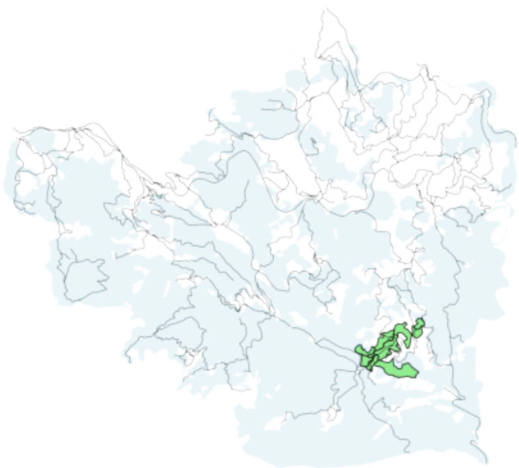

Figure D.31: Component 42

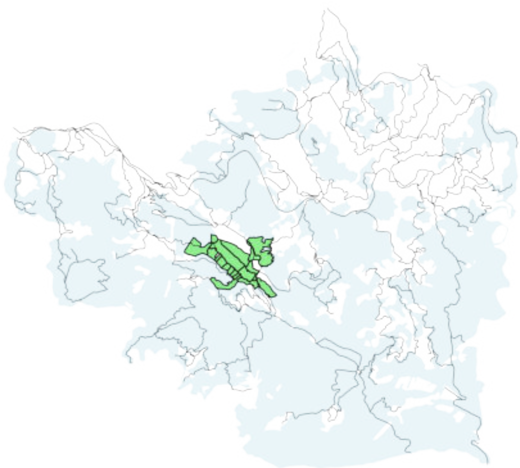

Figure D.32: Component 45

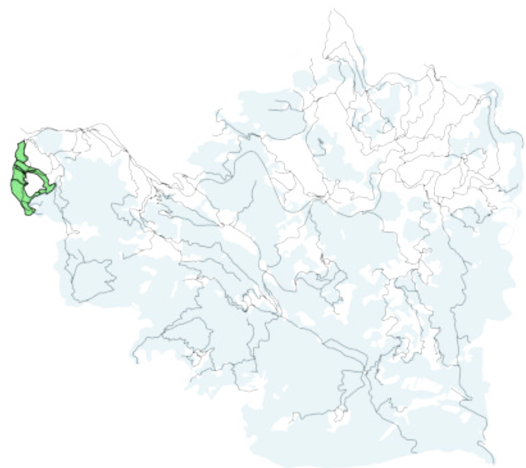

Figure D.33: Component 46

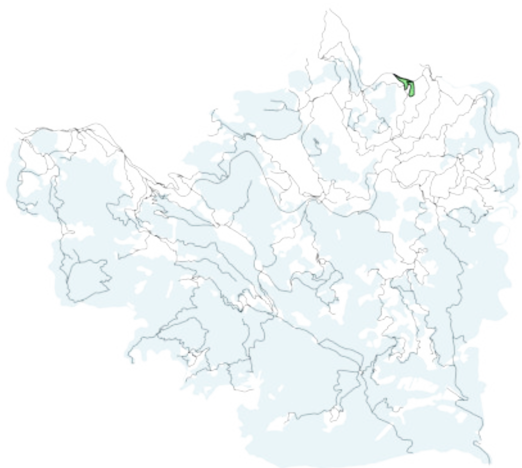

Figure D.34: Component 47

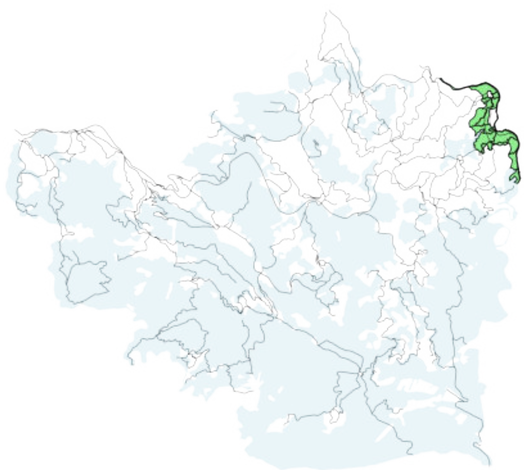

Figure D.35: Component 49

D.2 Road Network Graphs - Component 1 Example

Figure D.36: Road network graph for component 1

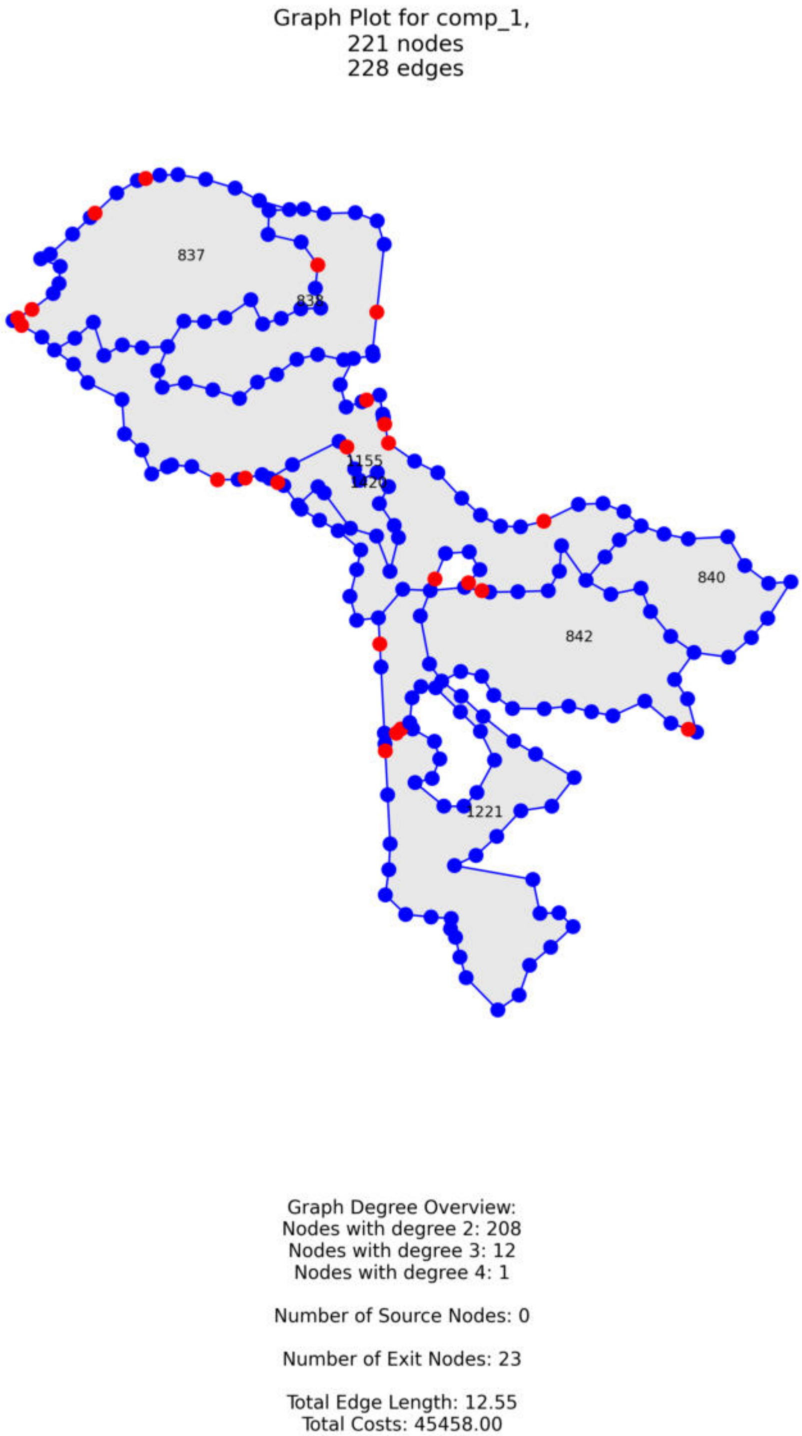

Figure D.37: Road network graph for component 1 with exit nodes

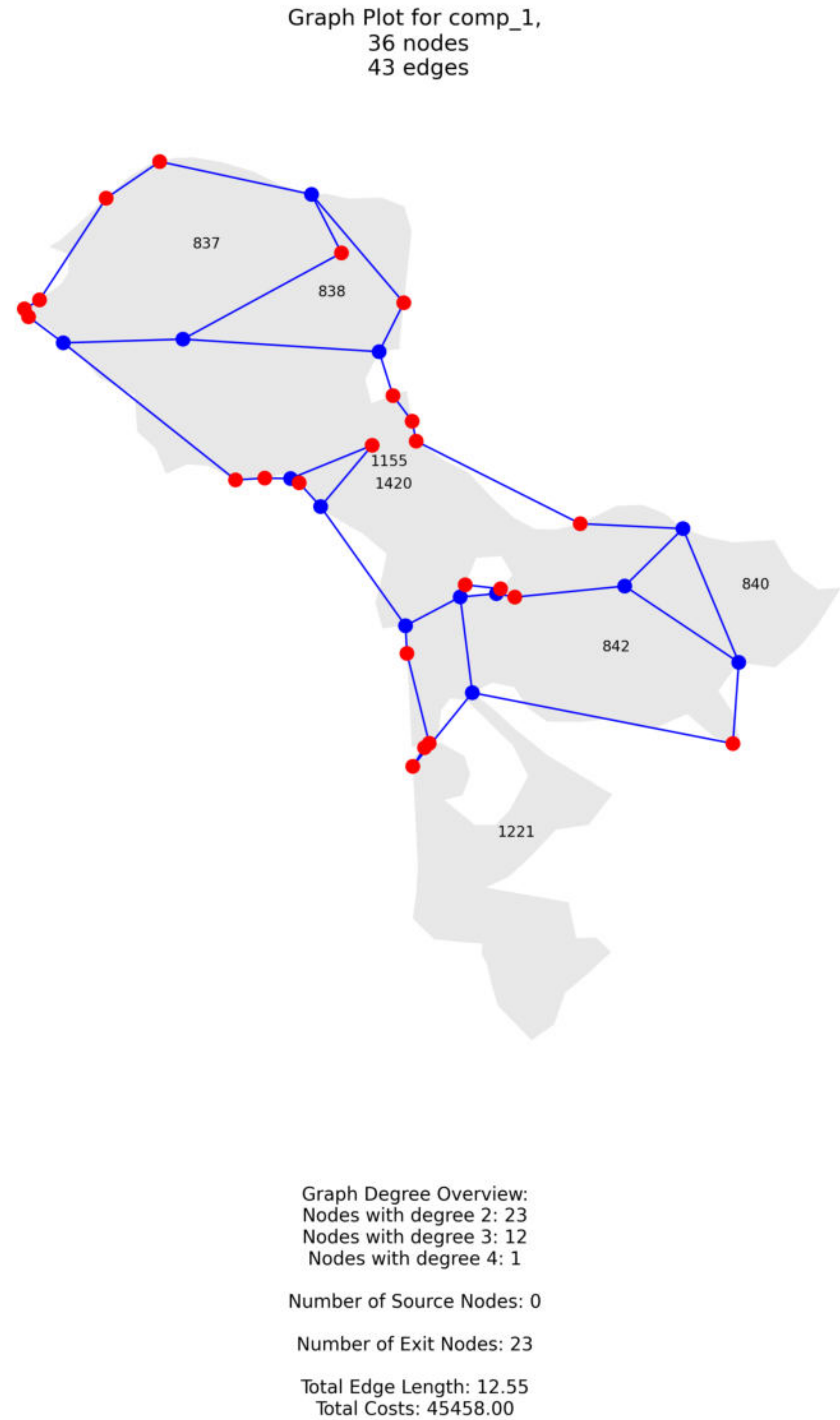

Figure D.38: Abstracted road network graph of component 1

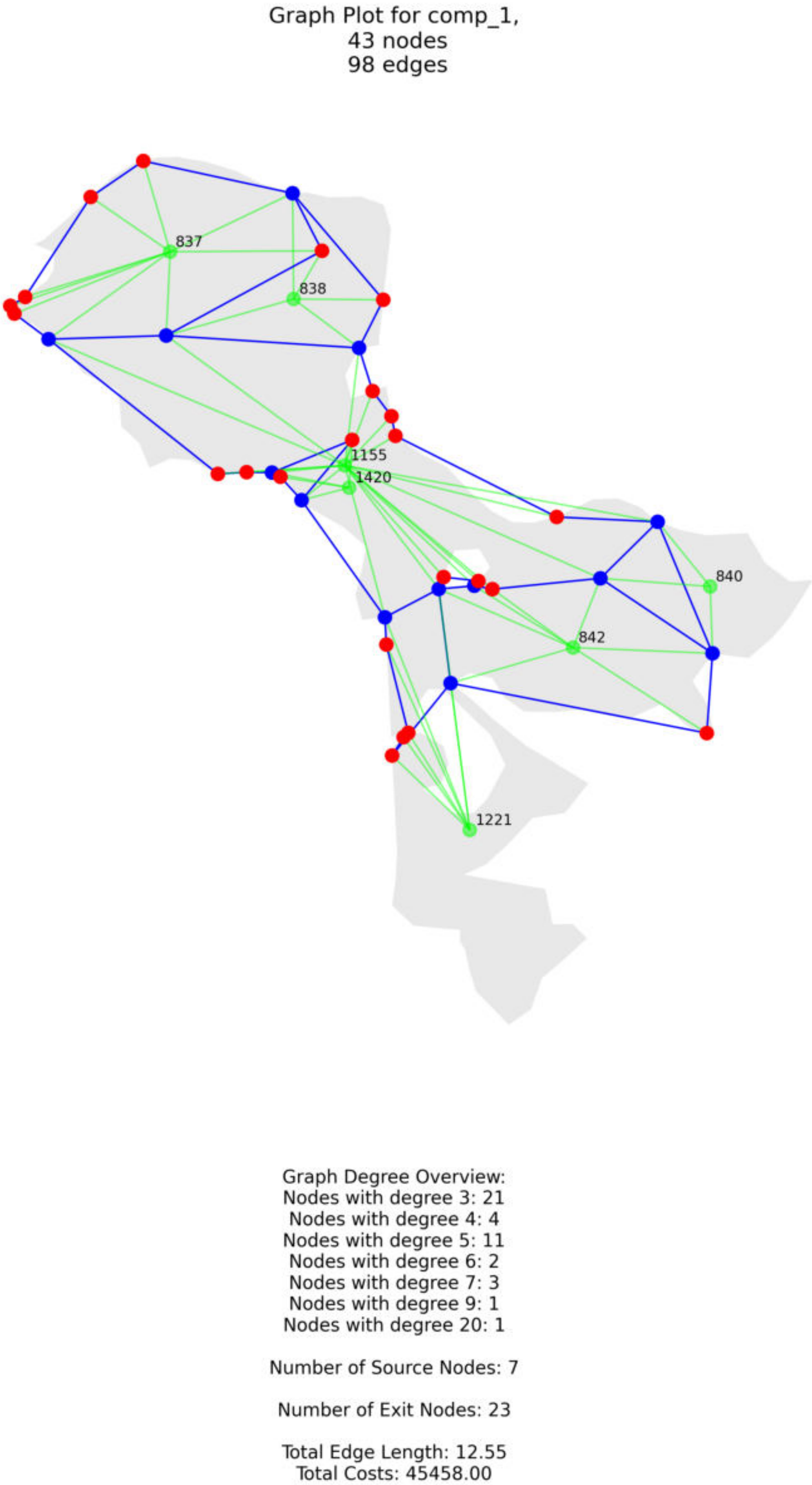

Figure D.39: Abstracted road network graph of component 1 with source nodes

E

Bonus Material

Figure E.1: Tourist map of Castelo da Paiva