New Ideas
for Effective Higgs Measurements

Dissertation

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New Ideas for Effective Higgs Measurements

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Put forward by

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After years of expensive education
A car full of books and anticipation
I'm an expert on Shakespeare and that's a hell of a lot
But the world don't need scholars as much as I thought

— J. Cullum [1]

Abstract in deutscher Übersetzung

Zweihundert Wörter.

Abstract

Two hundred words.

Contents

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Preface

This thesis is based on research conducted between 20014 and 2017 at the Institute for Theoretical Physics at Heidelberg University. Chapter 3 is based on two papers, which later became part of a CERN report:

[2] J. Brehmer, A. Freitas, D. López-Val, and T. Plehn:
Pushing Higgs Effective Theory to its Limits.
Phys. Rev. D 93, 075014 (2016). `arXiv:1510.03443`.

[3] A. Biekötter, J. Brehmer, and T. Plehn:
Extending the Limits of Higgs Effective Theory.
Phys. Rev. D 94, 055032 (2016). `arXiv:1602.05202`.

[4] D. de Florian, C. Grojean, F. Maltoni, et al.:
Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector.
LHC Higgs Cross Section Working Group Yellow Report. `arXiv:1610.07922`.

Chapter 4 is based on the following publication:

[5] J. Brehmer, K. Cranmer, F. Kling, and T. Plehn:
Better Higgs Measurements Through Information Geometry.
`arXiv:1612.05261`.

In addition, it includes some original results and yet unpublished work in progress with F. Kling and T. Plehn.

Chapter 2 consists of introductory material that can be found in many textbooks and review articles, as well as on a lecture on effective field theories given by the author to fellow PhD students in Heidelberg:

[6] J. Brehmer:
Higgs Effective Field Theory.
Student lecture, research training group “Particle Physics Beyond the Standard Model”.

Finally, some of the work done during my PhD is not included in this thesis:

[7] J. Brehmer, J. Hewett, J. Kopp, T. Rizzo, and J. Tattersall:
Symmetry Restored in Dibosons at the LHC?
JHEP 1510, 182 (2015). `arXiv:1507.00013`.

[8, 9] G. Brooijmans, C. Delaunay, A. Delgado, et al.:
Les Houches 2015: Physics at TeV Colliders – New Physics Working Group Report.
`arXiv:1605.02684`.
Part of these proceedings were published separately as
J. Brehmer, G. Brooijmans, G. Cacciapaglia, et al.:
The Diboson Excess: Experimental Situation and Classification of Explanations; A Les Houches Pre-Proceeding.
`arXiv:1512.04357`.

Acknowledgements

Chapter 1

Introduction

Did you know that in my PhD defence, I didn't answer a single question correctly?
— T. Plehn [10]

The Higgs boson [11–13] is a key element of the Standard Model of particle physics (SM). Its discovery in 2012 [14, 15] completed the particle zoo of the SM. It is a triumph of a decade-old model, but it also offers us a way forward: the Higgs provides us with an unprecedented chance to understand some of the biggest unsolved mysteries of physics.

As the only known fundamental scalar, it suffers from the famous electroweak hierarchy problem: why is its mass scale (and therefore the electroweak scale) so much smaller than the Planck scale, while there is no sign of a symmetry protecting it against quantum corrections? Is the electroweak vacuum, defined by the Higgs potential, stable? Why are the Yukawa couplings, and consequently the fermion masses, spread out over so many orders of magnitude? Is the Higgs related to Dark Matter, for instance as mediator to a dark sector? Or might it even be the source of inflation, explaining the surprising level of isotropy in the Cosmic Microwave Background?

Many models of physics beyond the Standard Model have been proposed to answer at least some of these questions. Very often they predict Higgs coupling patterns different from the SM. A precise measurement of the Higgs properties thus provides a crucial probe of such models, and might be one of the most important missions for present and future runs of the Large Hadron Collider (LHC).

This poses two immediate questions:

1. Which framework should be used to parametrise the Higgs properties?
2. How can these parameters be measured efficiently at the LHC experiments?

These two questions drive the research presented in this thesis, and we will tackle them one by one.

Ideally, all Higgs measurements should use the same universal language to parametrise their results, allowing for an efficient comparison and combination. Such a framework should be general enough to describe the effects of any interesting new physics scenario without strong model assumptions. On the other hand, too large a number of parameters makes combinations of different experiments and global fits impractical.

A simple example for such a universal parametrisation is the κ framework, which was widely used during run 1 of the LHC. It is based on the SM Lagrangian, but promotes all Higgs couplings to free parameters. There are several issues with this approach: it is not gauge-invariant, and it can only describe structures that are already present in the SM. So while a measurement based on the κ framework can be useful for total rates, it will not be able to utilise information in kinematic distributions.

Instead, we work in an approach based on effective field theory (EFT) [16–18]. Based only on the assumption that new physics has a typical energy scale significantly larger than the experimental energies, all new physics effects are captured by a tower of higher-dimensional operators. The leading effects for Higgs physics should come from only a handful of operators with mass dimension 6 [19–21]. These operators are manifestly gauge-invariant and can be used beyond tree level. They describe both coupling rescalings as well as novel kinematic structures not present in the SM, allowing us to access information in distributions in addition to total rates [22, 23]. Effective operators also let us combine Higgs data with results from other experiments, including electroweak precision data or gauge boson production at the LHC [24]. However, the limited precision of the LHC Higgs measurements means that only models that are either strongly coupled or relatively light can be probed. In the latter case, the characteristic energy scale of new physics is not sufficiently separated from the momentum transfers in the experiments, casting doubt on the validity of the EFT approach.

We analyse the usefulness of higher-dimensional operators at the LHC by comparing the predictions of UV-complete scenarios of new physics to their dimension-6 approximations [2]. Our analysis covers additional scalar singlets, two-Higgs-doublet models, scalar top partners, and heavy vector bosons, focusing on parameter ranges that the LHC will be sensitive to. We take into account rates and distributions in the most important Higgs production modes and various representative decay channels as well as Higgs pair production. For this array of models, benchmark points, and observables, we ask if and where the effective description of new physics breaks down, and how it can be improved.

As it turns out, the agreement between the approaches crucially depends on the matching procedure that links the coefficients of the dimension-6 model to the full theory. We introduce v-improved matching, a procedure that improves the performance of the dimension-6 model by resumming certain terms that arise during electroweak symmetry breaking. While formally of higher order in the EFT expansion, these effects can be large under LHC conditions. With such a matching, the effective model provides a good description even in many scenarios where the EFT validity is not obvious. We then discuss a number of practical questions on the role of squared dimension-6 terms in the differential cross sections, effects on fits, and the correlation between different observables and the momentum transfer [3].

Having established that Higgs EFT works well as a largely model-independent language for Higgs physics at the LHC, the next question is how its parameters can be measured optimally. Higgs measurements are affected by many different operators, and each of them affects different couplings, often introducing non-trivial kinematic structures. This leads to a complicated relation between the high-dimensional model parameter space and often also high-dimensional phase spaces.

Traditional analyses based on selection cuts and histograms of kinematic observables are often not sensitive to such subtle signatures. At the other end of the spectrum, experiments resort more and more to high-level statistical tools, including machine learning techniques or the matrix element method [25–29]. Many of these tools are designed for the comparison between two discrete hypotheses, and applying them to high-dimensional parameter spaces such as Higgs EFT is computationally expensive. Extending machine learning techniques to such high-dimensional spaces is a current area of research [30]. These multivariate techniques are powerful, but can be non-transparent. It is therefore increasingly important to be able to characterise the information contained in LHC signatures.

We use information geometry to understand and optimise Higgs measurements [5]. The central building block is the Fisher information, which according to the Cramér-Rao bound encodes the maximal knowledge on theory parameters we can derive from an experiment. Unlike many other statistical tools, the Fisher information is intrinsically designed for continuous, high-dimensional parameter spaces, and this approach does not require any discretisation of the theory space and leads to results that do not depend on arbitrary parameter or basis choices. In addition, the Fisher information defines a metric on the model parameter space. This not only provides an intuitive geometric picture of the sensitivity of measurements, but also allows us to track the impact of higher orders in the EFT expansion.

We calculate the Fisher information for Higgs production in weak boson fusion with decays into tau pairs and four leptons, and for Higgs production in association with a single top quark. To this end, we develop an algorithm to calculate the Fisher information in particle-physics observables based on Monte-Carlo methods. Our results give the maximum precision with which dimension-6 operators can be measured in these processes, or the maximal new physics reach of these signatures. We then analyse how the differential information is distributed over phase space, which defines optimal event selections. In a next step, we calculate the information in individual kinematic distributions, and compare it to the maximal information in the full event kinematics. This provides a ranking of the most powerful production and decay observables. It allows us to compare how much we can learn from a simple fit to histograms compared to fully multivariate methods.

This is the first application of information geometry to high-energy physics. While there is no shortage of statistical tools in the field, these new methods can help to plan and optimise measurement strategies for high-dimensional continuous models in an intuitive but powerful way. While we demonstrate this approach in different Higgs channels for dimension-6 operators, our tools can easily be translated to other processes and models.

This thesis begins by recapitulating some basic ideas of Higgs physics and effective field theory in Chapter 2. In Chapter 3, we discuss the validity of effective field theory for LHC measurements and the matching between full models and effective operators. Chapter 4 presents our work on information geometry and optimal Higgs measurements. Both of these chapters will contain separate and more detailed introductions and conclusions. We summarise the results in Chapter 5.

Chapter 2

Foundations

"It is known," Irri agreed.
— G. R. R. Martin [31]

IN THIS CHAPTER we review some of the essential concepts that underlie the research presented in the next chapters. First, we briefly summarise the role of the Higgs boson in the Standard Model (SM) and its phenomenology at the LHC. Section 2.2 then presents a pedagogical introduction to the effective field theory (EFT) idea. In Section 2.3 we combine these ideas and introduce Standard Model effective field theory as a universal language for Higgs physics.

Our introduction to Higgs physics will be superficial, and the EFT part eschews mathematical rigour for a broad picture of the central ideas. For a more thorough introduction to Higgs physics, see for instance Ref. [32]. For an extensive introduction to EFTs, see Refs. [33, 34]. Note that the EFT section is almost identical to a lecture given in Heidelberg [6] and many arguments and examples are taken from Refs. [33, 34].

2.1 Higgs phenomenology recap

2.1.1 The Standard Model Higgs

In the Standard Model, the Higgs field appears as an $SU(2)$ doublet ϕ as

$$\begin{aligned}\mathcal{L}_{\text{SM}} \supset\; & (D^\mu \phi)^\dagger (D_\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \\ & - \sum_{\text{generations}} \left(y_u \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_L \tilde{\phi}\, u_R + y_d \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_L \phi\, d_R + y_\ell \begin{pmatrix} \bar{\nu} \\ \bar{\ell}^- \end{pmatrix}_L \phi\, \ell_R \right) \end{aligned} \qquad (2.1)$$

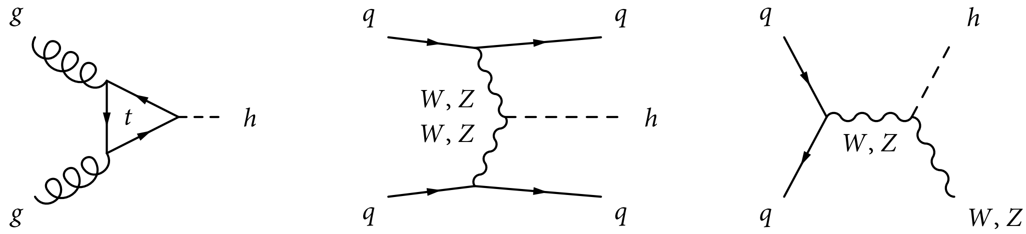


Figure 2.1: Feynman diagrams for the most important Higgs production modes considered in this thesis. Left: gluon fusion. Middle: weak boson fusion. Right: Higgs-strahlung.

with

$$D_\mu \phi = \left(\partial_\mu - \mathrm{i} g \frac{\sigma^a}{2} W_\mu^a - \mathrm{i} \frac{g'}{2} B_\mu \right) \phi \tag{2.2}$$

and $\tilde{\phi} = \mathrm{i}\tau_2 \phi^*$. For $\mu^2 < 0$, the Higgs doublet develops a non-zero vacuum expectation value (vev)

$$v^2 \equiv 2 \left| \langle \phi \rangle \right|^2 = -\frac{\mu^2}{\lambda} \,. \tag{2.3}$$

Using some of the gauge freedom, we can rotate the Higgs field such that

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -w_2 - \mathrm{i} w_1 \\ v + h + \mathrm{i} w_3 \end{pmatrix} , \tag{2.4}$$

where w_i are the would-be Goldstone bosons that give mass to the W and Z bosons, and h is the physical Higgs boson.

Plugging Eq. (2.4) into Eq. (2.1), we find the Higgs mass

$$m_h^2 = -2\mu^2 = 2\lambda v^2 \,. \tag{2.5}$$

The fermions and the massive vector bosons $W^\pm$ and Z get mass terms proportional to v, as well as couplings to the Higgs boson h. Since both terms stem from the same coupling to $\phi \sim v + h$, the Higgs couplings to other particles are always proportional to $g_{hxx} \sim m_x / v$. Finally, there are h^3 and h^4 self-couplings. The SM Higgs sector is very predictive: With the measurement of the Higgs mass $m_h = 125$ GeV [14, 15, 35], there are no more free parameters in the SM and all couplings are fixed.

2.1.2 Production and decay channels at the LHC

At the LHC, most Higgs bosons are produced in **gluon-gluon fusion** as shown in the left panel of Fig. 2.1. Due to its large Yukawa coupling, the top plays the dominant role in the loop, with small

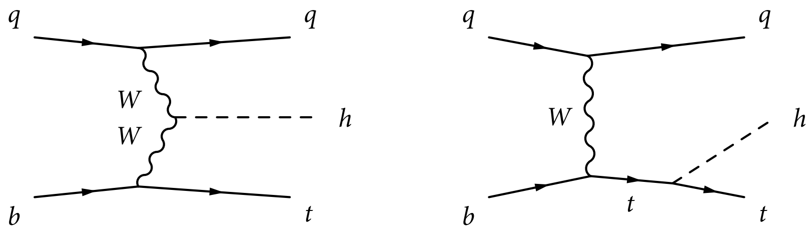


Figure 2.2: Feynman diagrams for Higgs production with a single top quark.

contributions from the bottom. The total cross section at $\sqrt{s} = 13$ TeV is approximately 49 pb [4], a large part of which comes NLO and NNLO corrections. This sizeable rate comes at the price of a lack of discerning kinematic features that could help to separate the Higgs from backgrounds.

This is certainly different for Higgs production in **weak boson fusion** (WBF)[1], as shown in the middle panel of Fig. 2.1. The production rate for this quark-initiated process is only 3.8 pb [4], but the Higgs is accompanied by two high-energetic jets that point nearly back-to-back in the two forward regions of the detector. This translates to a large invariant mass m_{jj} between them as well as a large separation in (pseudo-)rapidity $\Delta\eta_{jj}$. A second important property is provided by the colour structure of the process: at leading order, there is no colour exchange between the two quark lines, which means there is very little QCD radiation in this process. Both of these features set the WBF process apart from QCD backgrounds, which typically have many central jets. Backgrounds can therefore be reduced significantly by requiring two so-called "tagging jets" with large $\Delta\eta_{jj}$ and large m_{jj}, and vetoing any additional central jets [36].

But the tagging jets are not only useful to discriminate Higgs production from non-Higgs backgrounds. Since they recoil against the intermediate vector bosons that couple to the Higgs, they provide access to the momentum flow through the Higgs production vertex. Their properties, in particular their transverse momenta and the angular correlations between them, thus provide probes of the Higgs-gauge coupling. We will revisit this point from different perspectives, and tagging jet observables will play an important role throughout this thesis.

The right panel of Fig. 2.1 shows Higgs production in association with a vector boson, or **Higgs-strahlung**. The rate is 1.4 pb for a Wh final state plus 0.9 pb for Zh. Similarly to the tagging jets in WBF, the final-state gauge boson both helps to discriminate the Higgs from backgrounds and provides a handle to access the momentum flow through the virtual intermediate vector boson.

We will also briefly analyse **Higgs production with a single top quark**. This process exists as an s-channel and a t-channel version with very different kinematic features, and can be calculated either in the four-flavour scheme (with a gluon in the initial state) or in the five-flavour scheme (with a b quark in the initial state). We will focus on the dominant t-channel process and calculate it in the five-flavour scheme, as shown in Fig. 2.2. Diagrams where the Higgs is radiated off a

[1]The common name Vector Boson Fusion (VBF) forgets that the gluon also has spin 1.

top quark interfere destructively with amplitudes in which the Higgs couples to a W. The SM rate is small at 74 fb [4], but this interference pattern makes it very sensitive to changes in the top Yukawa coupling. This process is in fact the only direct probe of the sign or phase of the top Yukawa coupling ($t\bar{t}h$ production is only sensitive to the absolute value of the top Yukawa, while the total rate in gluon fusion can be influenced by many effects such as new particles in the loop).

Finally, we will take a look at **Higgs pair production** which provides a measurement of the Higgs self-coupling, see Fig. 2.3. It is another example of destructive interference between different amplitudes: diagrams in which the two Higgses couple to a top box loop interfere with those in which a single Higgs is produced in gluon fusion and then splits into two Higgses through the self-coupling. Close to threshold, these two contributions approximately cancel in the SM, and the total rate is very small at 33 fb. Modified Higgs sectors can spoil this cancellation and increase the rate drastically.

The Higgs decay patterns are rather simple. Since it couples to all particles proportional to their mass, it prefers to decay into the heaviest particles allowed by phase space. The dominant decay mode with a branching ratio of 58% [4] are therefore $b\bar{b}$ pairs. This signature is clearly useless for Higgs bosons produced in gluon fusion because of the overwhelming QCD $gg \to b\bar{b}$ background. WBF and Vh production provide handles to tame these backgrounds, but the channel is still difficult. Easier to detect are $\tau^+\tau^-$ pairs with a branching ratio of 6.3%. Their semi-leptonic and purely leptonic decays involve neutrinos. But if the taus are boosted enough and not exactly back-to-back, the neutrino momentum can be reconstructed for instance using a collinear approximation [32].

The decays through W^+W^- or ZZ pairs into four-lepton final states are particularly important due to their clean signatures and because they provide access to Higgs-gauge couplings. Since the Higgs mass is below the W^+W^- and ZZ thresholds, one of the vectors has to be off-shell.[2] $h \to W^+W^- \to (\ell^+\nu)(\ell^-\bar{\nu})$ with $\ell = e, \mu$ has a respectable branching fraction of 1.1% [4], but comes with two neutrinos in the final state. Still, it is one of the most important channels to

[2] This also means that branching ratios for $h \to ZZ$ and $h \to WW$ are not really well-defined. What is often quoted is in fact a term like $\mathrm{BR}(h \to 4\ell)/(\mathrm{BR}(Z \to \ell^+\ell^-))^2$.

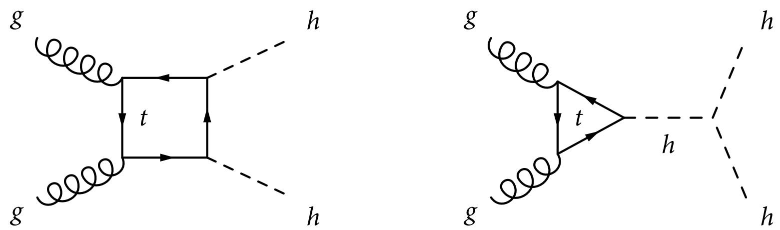


Figure 2.3: Feynman diagrams for Higgs pair production.

measure WBF Higgs production. The decay $h \to ZZ \to 4\ell$ with $\ell = e, \mu$ provides an extremely clean signal. Despite its small branching ratio of $1.3 \cdot 10^{-4}$, it was one of the main Higgs discovery channels [14, 35]. From a post-discovery perspective, its four leptons provide a rich spectrum of angular correlations and other observables that allow us to measure the Higgs behaviour in detail. We will discuss this feature in more detail later.

Finally, the small tree-level couplings of the Higgs to light particles mean that the loop-induced decay into photon pairs can compete with them. The dominant contribution comes from a W loop, which interferes destructively with the top loop, resulting in a branching ratio of 0.23% [4]. The ATLAS and CMS detectors are designed to reconstruct photons well, in fact with exactly this Higgs decay channel in mind. Together with $h \to 4\ell$ it constitutes the most important channel for the discovery.

2.1.3 How I Learned to Stop Worrying and Love the Higgs

There are several facets of the Higgs boson that make it special. From an experimental point of view, the properties of this shiny new thing in particle physics are still relatively unknown. Its couplings to vector bosons and heavy fermions are constrained at the $\mathcal{O}(10\%)$ level, while for the couplings to light fermions, invisible decays, and the total decay width of the Higgs there are only weak upper bounds [23, 35]. Many of these limits also rely on specific model assumptions. The top Yukawa coupling, for instance, is most strongly constrained from the total Higgs production rate, but only under the assumption that there no new physics plays a role in the gluon-fusion loop. The total Higgs width can be constrained indirectly from the contribution of $gg \to h \to ZZ \to 4\ell$ in the off-shell Higgs region, but again relying on strong model assumptions. All in all, the Higgs is still the least well measured elementary particle (in some sense with the exception of neutrinos), leaving plenty room for physics beyond the Standard Model.

From a theory perspective, there are several reasons to suspect manifestations of new physics in the Higgs sector. Starting with a rather general argument, the Higgs doublet is the key component of electroweak symmetry breaking (EWSB), which is often seen as the very core of the SM construction. A test of the Higgs properties therefore provides a test of the fundamental structure of Nature.

The Higgs boson is the only fundamental scalar discovered so far. This is interesting in its own right, but also leads to the famous electroweak **hierarchy problem**: in the absence of any protective symmetry, the mass of a scalar field should receive quantum corrections of the order of the largest scale in the theory. If the SM is valid all the way to the Planck scale, severe fine-tuning between the bare parameter and these quantum corrections is necessary to keep the electroweak mass scale at the observed value. Note that this argument interchangeably applies to the mass parameter of the Higgs doublet μ^2, the physical Higgs mass m_h, or the electroweak vev v. Since the strength of the weak force is suppressed by powers of $m_W \sim v$, and the gravitational force by the

Citation needed for criticism of anthropic principle.

Planck scale, the hierarchy problem is often phrased in terms of the surprising weakness of gravity compared to the weak force. This naturalness problem is of a purely aesthetic nature, but similar aesthetic problems have in the past led to new insights. Many models have been proposed to solve the hierarchy by introducing a new symmetry that protects the Higgs mass against quantum corrections.[3] Famous examples are supersymmetry, composite Higgs models in which the Higgs is the pseudo-Goldstone boson of some broken symmetry, conformal symmetries [48], or extra dimensions. To reduce tuning to an acceptable level, this new physics should reside at energy scales not too far from the electroweak scale. These models usually modify the Higgs sector in a way that translates into Higgs couplings different from their SM values.

Another hierarchy unexplained in the SM is the large difference between the **fermion masses**. There are more than five orders of magnitude between the top and the electron mass, and neutrinos are even lighter. Since the fermion masses are generated by the Yukawa couplings of the Higgs doublet, models that explain the fermion masses will usually also shift the Higgs-fermion coupling patterns.

The question of **vacuum stability** is still being discussed. The renormalisation group (RG) allows us to link this question to the running the quartic coupling λ to higher energies. Current results [49] indicate that indeed the quartic coupling becomes negative at large energies, leading to a second vacuum with lower energy at much larger values of ϕ. Fortunately for us, the tunnelling probability is very small, and in the SM "our" vacuum with $v \approx 246$ GeV seems to be metastable with a lifetime longer than the age of the universe. While this indicates there is no pressing need for physics below the Planck scale to save the electroweak vacuum from a horrible fate, this result crucially depends on the measured top and Higgs masses, higher-order corrections to the beta functions, and higher-dimensional operators stemming from UV physics [50].

In addition to these theoretical and to some degree aesthetic arguments, there is solid experimental evidence for physics beyond the SM that might be linked to the Higgs sector. First, the nature of **dark matter** (DM) [51] is still unclear. It is experimentally established that this form of matter is electrically neutral, stable over cosmological timescales, clumps (i. e. is now non-relativistic), and makes up roughly a fourth of the energy density of the universe. In many models DM is in thermal equilibrium with ordinary matter in the early universe. Interestingly,

[3] An entirely different and somewhat metaphysical argument is based on the (weak) anthropic principle that observations of the universe are conditional upon its laws of physics allowing conscious life [37, 38]. First, this explanation requires some mechanism that generates many different vacua with different values of the physics parameters, including the Higgs mass. Most of these vacua will have "natural" parameters in which the weak and gravitational scales are comparable. String theory is hypothesised to provide such a sampling mechanism (the "multiverse"). Second, there has to be a reason why larger (and thus more abundant) values of the weak scale would not allow any type of intelligent life to form and make observations. This question is difficult to answer, and the jury is still out [39–45]. Given the speculative nature of the two questions, anthropic reasoning is being criticised as unverifiable or as based on arguments from lack of imagination. Some other models such as the relaxion [46] or Nnaturalness [47] modify the cosmological evolution such that a small Higgs mass is generated dynamically during inflation or reheating.

the observed dark matter density is in good agreement with electroweak-scale masses and weak couplings. This "WIMP miracle" is one main reason behind the popularity of weakly interacting massive particles (WIMPs) as DM candidates. In this scenario, good candidates for the mediator between dark matter and the SM are the Higgs boson or other scalars in an extended Higgs sector. Such "Higgs portal" scenarios often predict signatures in Higgs physics such as modified couplings or invisible Higgs decays.

Another mystery is the **matter-antimatter asymmetry** of the universe. Assuming that the cosmos was initially perfectly symmetric, the observed excess of matter can be generated dynamically if the three Sakharov conditions are satisfied: there have to be processes with baryon-number violation as well as C and CP violation, which take place out of thermal equilibrium. In the SM, these effects are too small to account for the observed asymmetry. Models that accommodate larger effects often affect the Higgs sector. In particular, extended Higgs sectors allow for electroweak symmetry breaking to be a strong first-order phase transition, providing the required out-of-equilibrium dynamics. Again, such scenarios predict signatures in Higgs measurements.

Finally, the Higgs could play another role in the cosmological evolution of the universe. The origin of the large-scale structure of the cosmos, the surprising isotropy of the cosmic microwave background (CMB), and the flatness of the Universe are all explained by an epoch of exponential expansion of space in the early universe called **inflation**. This process is often thought to be cause by a scalar field, the inflaton, slowly rolling down a potential of a certain shape. In principle the Higgs can be the inflaton, though this scenario of Higgs inflation requires unnaturally large couplings between the Higgs and the Ricci scalar, and requires a UV completion.

The null results of the LHC searches for new particles have led to some disappointment among particle physicists. But with the discovery of the Higgs boson, the LHC might not only have completed the SM particle zoo, but rather opened the door to the unknown. The Higgs boson is not just another SM particle. Some of the big open questions of fundamental physics are deeply rooted in the Higgs sector, and many other ideas can at least be linked to the Higgs sector under some assumptions. On the other hand, the current experimental precision leaves quite some room for signatures of new physics in Higgs observables. A precise determination of the Higgs properties might be one of the most exciting measurements at the LHC, and will hopefully improve our understanding of Nature significantly. Hopefully, the Higgs boson is not just the last puzzle piece of the Standard Model, but the first sign of what lies beyond.

2.2 The effective field theory idea

This plethora of possible BSM models means that a model-independent universal theory framework is invaluable for TeV signatures of new physics. We will consider such a model based on the effective field theory (EFT) paradigm. Before discussing the specific realisation for Higgs physics in the next section, here we give a general introduction to the EFT idea.

The EFT idea is a very general tool that plays a role in many, if not all, areas of physics. Whenever phenomena are spread out over different energy or length scales, an effective description can be valuable, either to simplify calculations, or to actually allow model-independent statements that would be impossible without such a framework.

2.2.1 Different physics at different scales

Our world behaves very differently depending on which energy and length scales we look at. At extremely high energies (or short distances), Nature might be described by a quantum theory of gravity. At energies of a few hundred GeV, the Standard Model is (disappointingly) in agreement with all measurements. Going to lower energies (or larger distances), we do not have to worry about Higgs or W bosons anymore: electromagnetic interactions are described by QED, weak interactions by Fermi theory, strong physics by QCD. Below a GeV, quarks and gluons are replaced by pions and nucleons as the relevant degrees of freedom. Then by nuclei, atoms, molecules. At this point most physicists give up and let chemists (and ultimately biologists and sociologists) analyse the systems.

The important point here is that the observables at one scale are not directly sensitive to the physics at significantly different scales. This is nothing new: for molecules to stick together, the details of the Higgs sector are not relevant, just as we can calculate how an apple falls from a tree without knowing about quantum gravity. To do physics at one scale, we do not have to (and often cannot) take into account the physics from all other scales. Instead, we isolate only those features that play a role at the scale of interest.

An effective field theory is a physics model that includes all effects relevant at a given scale, but not those that only play a role at significantly different scales. In particular, EFTs ignore spatial substructures much smaller than the lengths of interest, or effects at much higher energies than the energy scale of interest.

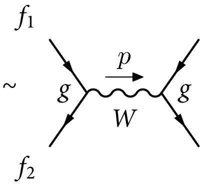
We will often use examples with one full or underlying theory and one effective theory. For simplicity, we pretend that the full theory describes physics correctly at all scales. The EFT is a simpler model than the full theory and neglects some phenomena (such as heavy particles) at energy scale Λ. However, it correctly describes the physics as long as the observables probe energy scales

$$E \ll \Lambda \,, \tag{2.6}$$

within some finite precision. This **scale hierarchy** between the energy of interest and the scale of high-energy physics not included in the EFT is the basic requirement for the EFT idea. A validity range (2.6) is a fundamental property of each EFT.

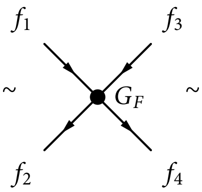
Fermi theory

The textbook example for an EFT is Fermi theory, which describes the charged current interactions between quarks (or hadrons), leptons and neutrinos at low energies. The underlying model here is the SM, in which this weak interaction is mediated by the exchange of virtual W bosons with mass m_W and coupling constant g:

$\mathcal{M}_{\text{full}} \sim$ 

$$\sim -\frac{g^2}{p^2 - m_W^2} \tag{2.7}$$

In Fermi theory, there are no W bosons, just a direct interaction between four fermions with coupling constant $G_F \propto g^2/m_W^2$:

$\mathcal{M}_{\text{EFT}} \sim$ 

$$\sim G_F \propto \frac{g^2}{m_W^2} \tag{2.8}$$

So the EFT turns the W propagator into a contact interaction between the fermions, shrinking the distance bridged by the virtual W to zero. Clearly, the two amplitudes agree as long as the momentum transfer through the vertex is small, $E^2 = p^2 \ll \Lambda^2 = m_W^2$:

$$-\frac{g^2}{p^2 - m_W^2} = \frac{g^2}{m_W^2}\left(1 + \frac{p^2}{m_W^2} + \mathcal{O}\left(p^4/m_W^4\right)\right) \approx \frac{g^2}{m_W^2}\,. \tag{2.9}$$

One process described by this interaction is muon decay. Its typical energy scale $E \approx m_\mu$ is well separated from $\Lambda = m_W$, and Fermi theory will describe the process quite accurately. The relative **EFT error**, i. e. the mistake we make when calculating an observable with the EFT rather than with the full model, should be of order $\Delta_{\text{EFT}} = \Gamma_{\text{EFT}}/\Gamma_{\text{full}} \sim E^2/\Lambda^2 \sim m_\mu^2/m_W^2 \approx 10^{-6}$.

In proton collisions at the LHC the same interaction takes place, but at potentially much larger momentum transfer $E < 13$ TeV. The EFT error increases with E. For $E \gtrsim m_W$, the full model allows on-shell W production, a feature entirely missing in the EFT. Here the two descriptions obviously diverge and Fermi theory is no longer a valid approximation of the weak interaction.

Down and up the theory ladder

In reality there are of course more than two theories, and the notion of underlying and effective model becomes relative. The SM itself is not valid up to arbitrary large energies: it does not explain dark matter, the matter-antimatter asymmetry, or gravity. It is probably also internally inconsistent since at some very large energy the quartic coupling λ and the coupling constant g' hit Landau poles. So the SM is an effective theory with validity range $E \ll \Lambda \leq M_{Pl}$ and has to be replaced by some other description at larger energies. On the other hand, going to energies lower than a few GeV, the relevant physics changes again and we should switch to a new effective theory. In this way, all theories can be thought of as a series of EFTs, where the model valid at one scale is the underlying model for the effective theory at the next lower scale.

If you think you know a theory that describes our world at sufficiently large energies, then in principle there is no need to use effective theories: you can calculate every single observable in your full model (at least if the full model is perturbative at these energies or other approximations such as lattice calculations are available). This will however make hard calculations necessary even for the simplest low-energy processes. One can save a lot of computational effort and focus on the relevant physics by dividing the phase space into regions with different appropriate effective descriptions.

Starting from a high energy scale where the parameters of the fundamental theory are defined, these parameters are run to lower energies until the physics changes substantially. At this **matching scale** an effective theory is constructed from the full model, and its coefficients are determined from, or matched to, the underlying model. Then the coefficients of this EFT are run down to the next matching scale, where a new EFT is defined and its parameters are calculated, and so on. This is the **top-down** view of EFTs. For instance, we can start from the SM and construct Fermi theory as a simpler model valid at low energies. While we can certainly use the SM to calculate the muon lifetime, it is not necessary, and a calculation in Fermi theory is quite accurate and simpler.

But often we do not know the underlying theory. As mentioned above, there has to be physics beyond the SM, and there is still hope it will appear around a few TeV. If we want to parametrise the effects of such new physics on electroweak-scale observables, we do not know how the full model looks like. But even without knowing the underlying model, we can still construct an effective field theory based on a few very general assumptions. We will go through these ingredients in the next section. For this **bottom-up** approach, an effective theory is not only useful, but actually the only way we can discuss new physics without choosing a particular model of BSM physics.

High-energy physics can be seen as the field of working ourselves up a chain of EFTs to ever higher energies. But how does this chain end? Does it end at all? Even if we one day find a consistent theory that can explain all observations to date, how would we check if it indeed describes Nature up to arbitrarily high energies? Understanding all theories as effective, these questions do not matter! The EFT framework provides us with the tools to do physics without having to worry about the far ultraviolet.

2.2.2 EFT construction and the bottom-up approach

EFTs are especially useful in the framework of quantum field theory (QFT). Before showing how to construct the effective operators of such a theory in a bottom-up approach, let us recapitulate how QFTs are organised.

Reminder: operators and power counting

The basic object describing perturbative QFTs in $d = 4$ flat space-time dimensions is the action

$$S = \int d^4x\, \mathcal{L}(x)\,. \tag{2.10}$$

The Lagrangian $\mathcal{L}(x)$ is a sum of couplings times operators, where the operators are combinations of fields and derivatives evaluated at one point x. These are either kinematic terms, mass terms or represent interactions between three or more fields. For instance, the Lagrangian

$$\mathcal{L} = \mathrm{i}\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - m_i\bar{\psi}_i\psi_i + m_V^2 V_\mu V^\mu - g\bar{\psi}_i\gamma_\mu\psi_i V^\mu \tag{2.11}$$

with implicit sum over i describes fermions ψ_i, a massive vector boson V_μ, and an interaction between them with coupling g.[4]

A key property of each coupling or operator is its **mass dimension**. In simple terms this can be formulated as the following question: if you assign a value to a quantity, which power of a mass unit such as GeV would this value carry? Since we work in units with $\hbar = c = 1$, length and distance dimensions are just the inverse of mass dimensions. We will denote the mass dimension of any object with squared brackets, where $[\mathcal{O}] = D$ means that $\mathcal{O}$ is of dimension mass^D, or mass dimension D.

In QFT, the action can appear in exponentials such as $e^{\mathrm{i}S}$, so it must be dimensionless: $[S] = 0$. The space-time integral in Eq. (2.10) then implies $[\mathcal{L}] = d = 4$, so every term in the Lagrangian has to be of mass dimension 4. Applying this to the kinetic terms, we can calculate the mass dimension of all fields. This then allows us to calculate the mass dimension of operators and couplings in the theory.

In the example in Eq. (2.11), the kinetic term for the fermions contains one space-time derivative, $[\partial] = 1$. To get $[\bar{\psi}\partial\psi] = 4$, the fermion fields must have dimension $[\psi_i] = 3/2$. Similarly, the field strength $V_{\mu\nu}$ contains a derivative, so we end up with $[V_\mu] = 1$. With these numbers we can check the other operators. In addition to the expected $[m] = [m_V] = 1$, we find $[\bar{\psi}\psi V^\mu] = 4$ or $[g] = 0$.

The mass dimension of an operator has two important consequences. First, the renormalisation group flow of a theory, i. e. the running of the couplings between different energy scales, largely depends on the mass dimensions of the operators. Operators with mass dimension $D < d$ ("relevant" operators) receive large quantum corrections when going from large energies to low energies. This is a key argument for many fine-tuning problems such as the hierarchy problem or the cosmological constant problem. On the other hand, operators with $D > d$ ("irrelevant" ones) are typically suppressed when going to lower energies. Operators with $D = d$ are called "marginal".

[4] Massive vector bosons have issues with renormalisability and unitarity, which can be solved by generating the mass in a Higgs mechanism at a higher scale, but this irrelevant for the discussion here.

The second consequence of the mass dimension affects the renormalisability of a theory. Theories with operators with $D > d$ are **non-renormalisable**:[5] particles in loops with energies $E \to \infty$ will lead to infinities in observables, and they are too many to be hidden in a renormalisation of the parameters.

Effective operators

From now on we will only consider EFTs realised as a local QFT in 4 space-time dimensions, an approach that has proven very successful in high-energy physics so far. EFTs are then defined as a sum of operators $\mathcal{O}_i$, each with a specific mass dimension D_i. We can split the coupling in front of each operator into a dimensionless constant, the **Wilson coefficient** f_i, and some powers of a mass scale, for which we use the scale of heavy physics Λ:

$$\mathcal{L}_{\text{EFT}} = (\text{kinetic and mass terms}) + \sum_i \frac{f_i}{\Lambda^{D_i - d}} \mathcal{O}_i \,. \tag{2.12}$$

Why do we force Λ to appear in front of the operators like this? If we do not know anything about the underlying model at scale Λ, our best guess (which can be motivated with arguments based on the renormalisation group flow) is that it consists of dimensionless couplings $g \sim \mathcal{O}(1)$ and mass scales $M \sim \mathcal{O}(\Lambda)$. Indirect effects mediated by this high-energy physics should therefore be proportional to a combination of these factors, as given in Eq. (2.12) with couplings $f_i \sim \mathcal{O}(1)$. This is certainly true in Fermi theory, where the effective coupling G_F is suppressed by $\Lambda^2 = m_W^2$.

Ingredients

How the operators $\mathcal{O}_i$ look like might be clear in a top-down situation where we know the underlying theory. In a bottom-up approach, however, we need a recipe to construct a list of operators in a model-independent way. It turns out that this is surprisingly straightforward, and the list of operators we need to include in the EFT is defined by three ingredients: the particle content, the symmetries, and a counting scheme that decides which operators are relevant at the scale of interest. We will go through them one by one.

1. **Particle content**: one has to define the fields that are the dynamical degrees of freedoms in the EFT, i. e. that can form either external legs or internal propagators in Feynman diagrams. At least all particles with masses $m \ll \Lambda$ should be included. The operators are then combinations of these fields and derivatives.
2. **Symmetries**: some symmetry properties of the world have been measured with high precision, and we can expect that a violation of these symmetries has to be extremely small or happens at very high energies. These can be gauge symmetries (such as the

[5] The opposite is not true: some theories contain only operators with $D \leq d$, but are still not renormalisable.

$SU(3) \times SU(2) \times U(1)$ of the SM), space-time symmetries (such as Lorentz symmetry), or other global symmetries (such as flavour symmetries). Requiring that the effective operators do not violate these symmetries is well motivated and can reduce the complexity of the theory significantly.

3. **Counting scheme**: with a set of particles and some symmetry requirements we can construct an infinite tower of different operators. We therefore need some rule to decide which of the operators we can neglect. Here the dimensionality of the operators becomes important. As argued above, we expect an operator with mass dimension $D > d$ to be suppressed by a factor of roughly $1/\Lambda^{D-d}$. Operators of higher mass dimension are therefore more strongly suppressed. Setting a maximal operator dimension is thus a way of limiting the EFT to a finite number of operators that should include the leading effects at energies $E \ll \Lambda$.

One property that is often required of theories is missing in this list: an EFT (with its intrinsic UV cutoff Λ) does not have to be renormalisable in the traditional sense. In fact, most EFTs include operators with mass dimension $D > d$ and are thus non-renormalisable. However, EFTs are still renormalisable order by order in the counting scheme, and loop effects can be calculated without any fundamental issues.

Basis choices

Usually not all operators that can be constructed in this way are independent. This can be seen from a field redefinition of the form

$$\phi(x) \to \phi'(x) = \phi(x) + \varepsilon f(x) \tag{2.13}$$

where ε is some small parameter and $f(x)$ can contain any combination of fields evaluated at x. The action in terms of the new field is then (after integration by parts)

$$\int \mathrm{d}^4x\, \mathcal{L}[\phi] \to \int \mathrm{d}^4x\, \mathcal{L}[\phi'] = \int \mathrm{d}^4x\, \left(\mathcal{L}[\phi] + \varepsilon \left[\frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right] f + \mathcal{O}(\varepsilon^2) \right) . \tag{2.14}$$

Such a transformation does not change the physics, i. e. the S-matrix elements [52–55], so we can equivalently use the new action instead of the original one. In this way, each equation of motion provides us with a degree of freedom to swap operators for a combination of other operators. Similarly, Fierz identities and integration by parts can be used to manipulate the form of operators. Together these tools reduces the number of operators and coefficients necessary in an EFT basis, and lead to some freedom to choose which operators to work with.

Why is the sky blue?

Following Ref. [34], we will demonstrate this bottom-up approach with a simple question: why is the sky blue? In other words, why is blue light coming from the sun scattered more strongly by particles in the atmosphere than red light? A full derivation of this takes some time and requires knowledge of the underlying electrodynamic interactions. Instead, we will write down an effective field theory for this process of Rayleigh scattering. The only thing we have to know are the basic scales of the process: photons with energy E_γ scatter off basically static nuclei characterised by an excitation energy ΔE, mass M and radius a_0. Looking at these numbers, we see that these scales are clearly separated:

$$E_\gamma \ll \Delta E, a_0^{-1} \ll M\,. \tag{2.15}$$

This is good news, since such a scale hierarchy is the basic requirement for an EFT. We are interested in elastic scattering, so we set the cutoff of the EFT as[6]

$$\Lambda \sim \Delta E, a_0^{-1}\,. \tag{2.16}$$

With this we can put together the building blocks for our EFT as discussed above:

1. As fields we will need photons and atoms, where we can approximate the latter as infinitely heavy.
2. The relevant symmetries are the $U(1)_{em}$ and Lorentz invariance. At these energies we will also not be able to create or destroy atoms, which you can see as another symmetry requirement on the effective Lagrangian.
3. We will include the lowest-dimensional operators that describe photon-atom scattering.

The kinetic part of such an EFT reads

$$\mathcal{L}_{\text{kin}} = \phi_v^\dagger i v^\alpha \partial_\alpha \phi_v - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\,, \tag{2.17}$$

where ϕ_v is the field operator representing an infinitely heavy atom at constant velocity v, and $F_{\mu\nu}$ is the photon field strength tensor. Boosting into the atom's rest frame, $v = (1, 0, 0, 0)$ and the first term becomes the Lagrangian of the Schrödinger equation.

The usual power counting based on $[\mathcal{L}] = 4$ gives the mass dimensions

$$[\partial] = 1\,, \quad [v] = 0\,, \quad [\phi] = \frac{3}{2} \quad \text{and} \quad [F_{\mu\nu}] = 2\,. \tag{2.18}$$

The interaction operators must be Lorentz-invariant combinations of $\phi^\dagger\phi$, $F_{\mu\nu}$, v_μ, and ∂_μ. Note that operators directly involving A_μ instead of $F_{\mu\nu}$ are forbidden by gauge invariance, and single instances of ϕ correspond to the creation or annihilation of atoms which is not possible at these energies. The first such operators appear at mass dimension 7,

$$\mathcal{L}_{\text{int}} = \frac{f_1}{\Lambda^3} \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + \frac{f_2}{\Lambda^3} \phi_v^\dagger \phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} + \mathcal{O}\left(1/\Lambda^4\right)\,, \tag{2.19}$$

[6] In reality there are two orders of magnitude between ΔE and a_0^{-1}, but this does not affect the line of argument at all and we choose to ignore this fact.

and we expect them to contain the dominant effects of Rayleigh scattering at energies $E_\gamma \ll \Lambda$.

The scattering amplitude of light off the atmospheric atoms should therefore scale as $\mathcal{M} \sim 1/\Lambda^3$, which means that the cross section scales with $\sigma \sim 1/\Lambda^6$. Since the cross section has the dimension of an area, $[\sigma] = -2$, and the only other mass scale in this low-energy process is the photon energy E_γ, we know that the effective cross section must be proportional to

$$\sigma \propto \frac{E_\gamma^4}{\Lambda^6} \left(1 + \mathcal{O}\big(E_\gamma/\Lambda\big)\right) . \tag{2.20}$$

In other words, blue light is much more strongly scattered than red light. Our effective theory, built just from a few simple assumptions, explains the colour of the sky!

Finally, we should check the validity range of our EFT. We expect it to work as long as

$$E_\gamma \ll \Lambda \sim \Delta E \sim \mathcal{O}(\mathrm{eV}) \tag{2.21}$$

which is equivalent to wavelengths above $\mathcal{O}(100\ \mathrm{nm})$. Our approximation is probably safe for visible light! In the near ultraviolet we expect deviations from the E_γ^4 proportionality and our EFT to break down.

2.2.3 Top-down approach and matching

In the top-down approach to effective field theories, we start from a known model of UV physics and calculate the corresponding effective operators and Wilson coefficients in the EFT. The defining criterion of this **matching procedure** is that at low energies the effective and underlying descriptions agree, at least up to a given order in the loop expansion (e. g. in α_s) and up to a given order in the EFT expansion in $1/\Lambda$.

This can be achieved either by functional methods or with Feynman diagrams. Here we will sketch the conceptual foundation involving functional methods, before arriving at a simple diagrammatic method. Note that the matching cannot be reversed: one cannot uniquely reconstruct a full theory only based on the EFT. Details of the matching procedure will play a crucial role in Chapter 3.

The effective action

The central object that allows us to systematically analyse the low-energy effects of heavy physics is the effective action S_{eff}. Following Ref. [56, 57], we now outline its calculation at the one-loop level. Note that this is just a conceptual sketch and not mathematically rigorous, and that we omit higher-order terms irrelevant for this thesis as well as certain cases of mixed loops with light and heavy particles [58]. For a more thorough derivation see the quantum field theory textbook of your choice.

For simplicity, let us assume that our theory $S[\phi, \Phi]$ consists of light particles ϕ and a heavy scalar Φ that should not be part of the effective theory as dynamical degree of freedom. The effective action is calculated by **integrating out** the heavy particles from the partition function,

$$e^{\mathrm{i}S_{\mathrm{eff}}[\phi]} = \int \mathcal{D}\Phi \; e^{\mathrm{i}S[\phi,\Phi]} \,. \tag{2.22}$$

While the path integral over the heavy fields is computed, the light fields are kept fixed as "background fields".

The effective action can be calculated with a saddle-point approximation. For this we expand Φ around its classical value Φ_c:

$$\Phi(x) = \Phi_c(x) + \eta(x) \,. \tag{2.23}$$

Φ_c is defined by the classical equation of motion

$$\left.\frac{\delta S[\phi, \Phi]}{\delta \Phi}\right|_{\Phi=\Phi_c} = 0 \,, \tag{2.24}$$

so expanding the action around this extremum leads to

$$S[\phi, \Phi_c + \eta] = S[\phi, \Phi_c] + \frac{1}{2} \left.\frac{\delta^2 S[\phi, \Phi]}{\delta \Phi^2}\right|_{\Phi=\Phi_c} \eta^2 + \mathcal{O}\left(\eta^3\right) \,. \tag{2.25}$$

Plugging this into Eq. (2.22), we find

$$e^{\mathrm{i}S_{\mathrm{eff}}[\phi]} \approx e^{\mathrm{i}S[\phi,\Phi_c]} \int \mathcal{D}\eta \; \exp\left(\frac{1}{2} \left.\frac{\delta^2 S[\phi, \Phi]}{\delta \Phi^2}\right|_{\Phi=\Phi_c} \eta^2 \right) . \tag{2.26}$$

The last term is a Gaussian integral with a known solution,

$$e^{\mathrm{i}S_{\mathrm{eff}}[\phi]} \approx e^{\mathrm{i}S[\phi,\Phi_c]} \left[\det\left(- \left.\frac{\delta^2 S}{\delta \Phi^2}\right|_{\Phi=\Phi_c} \right) \right]^{-1/2} \tag{2.27}$$

and finally

$$S_{\mathrm{eff}}[\phi] \approx S[\phi, \Phi_c] + \frac{\mathrm{i}}{2} \operatorname{tr} \log \left(- \left.\frac{\delta^2 S}{\delta \Phi^2}\right|_{\Phi=\Phi_c} \right) , \tag{2.28}$$

where the functional trace is defined as an integral over momentum space k together with a sum over internal states i such as spin or flavour,

$$\operatorname{tr} x \equiv \sum_i \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \; \langle k, i | x | k, i \rangle \,. \tag{2.29}$$

This result can be directly evaluated with functional methods. The first term in Eq. (2.28) can be easily calculated by solving the classical equations of motions in Eq. (2.24). Computing the functional trace is more involved, but can be simplified with a procedure called **covariant derivative expansion** [56, 59]. Universal results that can be adapted to many scenarios are available in the literature [57, 58, 60].

The effective action is in general non-local, visible as (covariant) derivatives D appearing in the denominator (formally defined as Green's functions). We expand these terms schematically as

$$\phi^\dagger \frac{1}{D^2 - M^2} \phi = -\phi^\dagger \frac{1}{M^2} \left[1 + \frac{D^2}{M^2} \right] \phi + \mathcal{O}\left(1/M^6\right) , \tag{2.30}$$

so that only a rest term of higher order in $1/\Lambda = 1/M$ remains non-local [58]. In a last step, we truncate the resulting tower of operators at some order in our counting scheme, in our case in the expansion in $1/\Lambda$. The resulting effective theory consists of a finite set of local operators up to some order in a counting scheme, compatible with our definition of effective theories in the previous section. Unlike in the bottom-up approach, not all operators have to appear, and we can calculate the Wilson coefficients based on the underlying theory.

Scalar example

As a simple example consider a theory of two real scalar fields. The light field ϕ has mass m, the heavy field Φ with mass M will be integrated out. The underlying theory is given by

$$\begin{aligned} S[\phi, \Phi] = \int \mathrm{d}^4 x \Bigg[&\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{M^2}{2} \Phi^2 \\ &- \frac{\lambda_0}{4!} \phi^4 - \frac{\lambda_2}{4} \phi^2 \Phi^2 - \frac{\lambda_4}{4!} \Phi^4 \Bigg] . \end{aligned} \tag{2.31}$$

Odd interactions and a mixing term $\phi\Phi$ are forbidden with suitable $\mathbb{Z}_2$ symmetries.

The classical equation of motion for Φ is

$$\left(\partial^2 + M^2 + \frac{\lambda_2}{2} \phi^2 + \frac{\lambda_4}{3!} \Phi_c^2 \right) \Phi_c = 0 \tag{2.32}$$

with the trivial solution $\Phi_c = 0$.

The first term in the effective action then just gives back ϕ^4 theory for the light field, without any new effective interactions:

$$S[\phi, \Phi_c] = \int \mathrm{d}^4 x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4 \right] . \tag{2.33}$$

The second term is

$$\frac{\mathrm{i}}{2}\operatorname{tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi=\Phi_c}\right)=\frac{\mathrm{i}}{2}\operatorname{tr}\log\left(\partial^2+M^2+\frac{\lambda_2}{2}\phi^2\right)$$
$$=\frac{\mathrm{i}}{2}\operatorname{tr}\log\left(\partial^2+M^2\right)+\frac{\mathrm{i}}{2}\operatorname{tr}\log\left(1+\frac{\lambda_2}{2}\frac{1}{\partial^2+M^2+\mathrm{i}\varepsilon}\phi^2\right), \quad (2.34)$$

where derivatives in the denominator are defined as Green's functions. Since $\operatorname{tr}\log\left(\partial^2+M^2\right)$ is just a constant that can be calculated for instance in dimensional regularisation, the first part does not give us any higher-dimensional operators of the light fields ϕ. Expanding the logarithm in the second term, we find

$$S_{\text{eff}}\supset\frac{\mathrm{i}\lambda_2}{4}\operatorname{tr}\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}\phi^2-\frac{\mathrm{i}\lambda_2^2}{8}\operatorname{tr}\left(\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}\phi^2\right)^2+\frac{\mathrm{i}\lambda_2^3}{12}\operatorname{tr}\left(\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}\phi^2\right)^3+\mathcal{O}\left(\lambda_2^4\right). \quad (2.35)$$

The first of these terms will renormalise the ϕ mass term, and the second will contribute to the ϕ^4 interaction. This is important for RG running, but will not create the kind of new effective interactions we are interested in here. We instead focus on the last term and evaluate the functional trace:

$$S_{\text{eff}}\supset-\frac{\mathrm{i}\lambda_2^3}{12}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\langle k|\left(\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}\phi^2\right)^3|k\rangle$$
$$\supset-\frac{\mathrm{i}\lambda_2^3}{12}\int\mathrm{d}^4x\int\mathrm{d}^4y\int\mathrm{d}^4z\int\frac{\mathrm{d}^4k}{(2\pi)^4}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\langle k|\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}|x\rangle\langle x|\phi^2|p\rangle$$
$$\times\langle p|\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}|y\rangle\langle y|\phi^2|q\rangle\langle q|\frac{1}{\partial^2+M^2-\mathrm{i}\varepsilon}|z\rangle\langle z|\phi^2|k\rangle\,. \quad (2.36)$$

Here we have used the definition of the functional trace in Eq. (2.29) and inserted unity, $1=\int\mathrm{d}^4x\;\;|x\rangle\langle x|=\int\frac{\mathrm{d}^4p}{(2\pi)^4}\;\;|p\rangle\langle p|$. $|k\rangle$, $|p\rangle$, and $|q\rangle$ are eigenstates of the derivative operator ∂, i. e. $\langle k|\,\mathrm{i}\partial_\mu=\langle k|\,k_\mu$, while $|x\rangle$, $|y\rangle$, and $|z\rangle$ denote the eigenstates of local operators, $\langle x|\,\phi^2=\langle x|\,\phi^2(x)$. Their inner product is $\langle x|k\rangle=e^{-ikx}$. Using these properties and shifting the integration variables, we get

$$S_{\text{eff}}\supset-\frac{\mathrm{i}\lambda_2^3}{12}\int\mathrm{d}^4x\int\mathrm{d}^4y\int\mathrm{d}^4z\int\frac{\mathrm{d}^4k}{(2\pi)^4}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\frac{1}{-k^2+M^2-\mathrm{i}\varepsilon}e^{\mathrm{i}kx}\phi(x)^2e^{-\mathrm{i}px}$$
$$\times\frac{1}{-p^2+M^2-\mathrm{i}\varepsilon}e^{\mathrm{i}py}\phi(y)^2e^{-\mathrm{i}(p+Q)y}\,\frac{1}{-q^2+M^2-\mathrm{i}\varepsilon}e^{\mathrm{i}qz}\phi(z)^2e^{-\mathrm{i}kz}$$
$$\supset\frac{\mathrm{i}\lambda_2^3}{12}\int\mathrm{d}^4x\int\mathrm{d}^4y\int\mathrm{d}^4z\int\frac{\mathrm{d}^4k}{(2\pi)^4}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\phi(x)^2\phi(y)^2\phi(z)^2$$
$$\times\frac{e^{\mathrm{i}p(z-x)}\,e^{\mathrm{i}q(z-y)}}{\left(k^2-M^2+\mathrm{i}\varepsilon\right)\left((k+p)^2-M^2+\mathrm{i}\varepsilon\right)\left((k+p+q)^2-M^2+\mathrm{i}\varepsilon\right)}\,. \quad (2.37)$$

We can now perform the integral over the loop momentum k with Feynman parameters:

$$\begin{aligned}
T_3(p,q) &\equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2-M^2+\mathrm{i}\varepsilon)\left((k+p)^2-M^2+\mathrm{i}\varepsilon\right)\left((k+p+q)^2-M^2+\mathrm{i}\varepsilon\right)} \\
&= 2\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \Big[x(k^2-M^2+\mathrm{i}\varepsilon) + y((k+p)^2-M^2+\mathrm{i}\varepsilon) \\
&\qquad\qquad + (1-x-y)((k+p+q)^2-M^2+\mathrm{i}\varepsilon)\Big]^{-3} \\
&= 2\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[(k+a)^2-B+\mathrm{i}\varepsilon\right]^3}
\end{aligned} \tag{2.38}$$

with $a = (1-x)p + (1-x-y)q$ and $B = M^2 - (1-x-y)(p+q)^2 - yp^2 + a^2$. Shifting the loop momentum as $k \to k+a$, we finally arrive at

$$T_3(p,q) = 2\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[k^2-B+\mathrm{i}\varepsilon\right]^3}. \tag{2.39}$$

To evaluate this, we first Wick-rotate $k^0 = \mathrm{i}k_E^0$. Formally, this means shifting the integration path in the complex plane of k^0 from along the real axis to along the imaginary axis. The Cauchy theorem assures that this does not change the value of the integral as long as we chose the contour such that the poles are not caught between the two contours. Defining $k_E^2 = (k_E^0)^2 + \boldsymbol{k}^2 = -k^2$, we find

$$I_{0,3} \equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[k^2-B+\mathrm{i}\varepsilon\right]^3} = \mathrm{i}\int \frac{\mathrm{d}^4 k_E}{(2\pi)^4} \frac{1}{\left[-k_E^2-B\right]^3}, \tag{2.40}$$

where the $+\mathrm{i}\varepsilon$ is no longer necessary. With $\bar{k} = |k_E|$ we can finally calculate the integral:

$$I_{0,3} = \frac{2\pi^2}{(2\pi)^4} \int \mathrm{d}\bar{k}\,\bar{k}^3 \frac{1}{\left[\bar{k}^2+B\right]^3} = \frac{-\mathrm{i}}{32\pi^2 B}. \tag{2.41}$$

Collecting all the pieces, we have

$$\begin{aligned}
S_{\text{eff}} \supset \frac{\lambda_2^3}{192\pi^2} & \int \mathrm{d}^4 x \int \mathrm{d}^4 y \int \mathrm{d}^4 z \, \phi(x)^2 \phi(y)^2 \phi(z)^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \, e^{\mathrm{i}p(z-x)} \, e^{\mathrm{i}q(y-z)} \\
\times & \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \left[M^2 - (1-x-y)(p+q)^2 - yp^2 + ((1-x)p + (1-x-y)q)^2\right]^{-1}.
\end{aligned} \tag{2.42}$$

At first glance, this is disappointing: this effective action looks non-local and involves highly non-trivial integrals. It turns out that these can in fact be calculated and give a finite result [61, 62].

Check discrepancy with Denner's habil!
Just like the dear proof reader...

The full expression is extremely ugly. Fortunately, we do not need it. Instead, we expand the integrand in powers of $1/M^2$. We will only calculate the leading term at $\mathcal{O}(1/M^2)$. We will show that it produces a finite result as well, so the rest term at $\mathcal{O}(1/M^4)$ also has to be finite. Even more, we can argue that the rest term at $\mathcal{O}(1/M^4)$ has to vanish: the coefficient at a given order $1/M^k$ in this expansion is an integral without any mass scales, and has to lead to a result of mass dimension $k-2$. Only $k=2$ can give a non-zero and finite result, all higher orders therefore have to vanish. In this way, we find the much simpler result

Check this argument!

$$
\begin{aligned}
S_{\text{eff}} \supset\ & \frac{\lambda_2^3}{384\pi^2 M^2} \int \mathrm{d}^4x \int \mathrm{d}^4y \int \mathrm{d}^4z\ \phi(x)^2\phi(y)^2\phi(z)^2 \int \frac{\mathrm{d}^4p}{(2\pi)^4} \int \frac{\mathrm{d}^4q}{(2\pi)^4}\ e^{\mathrm{i}p(z-x)} e^{\mathrm{i}q(y-z)} \\
& + \mathcal{O}(1/M^4) \\
\supset\ & \frac{\lambda_2^3}{384\pi^2 M^2} \int \mathrm{d}^4x \int \mathrm{d}^4y \int \mathrm{d}^4z\ \phi(x)^2\phi(y)^2\phi(z)^2\ \delta(z-x)\delta(y-z) \\
\supset\ & \frac{\lambda_2^3}{384\pi^2 M^2} \int \mathrm{d}^4x\ \phi(x)^6 .
\end{aligned}
\tag{2.43}
$$

After the expansion in $1/M$, we have finally arrived at a local theory!

What about the higher terns in Eq. (2.35)? Their calculation is analogous to the one presented here and will lead to operators like ϕ^8 and higher. They will be suppressed at least with $1/M^4$ and are thus irrelevant for our dimension-six effective theory.

Collecting the pieces in Eq. (2.33) and Eq. (2.43), up to one loop and $\mathcal{O}(1/M^2)$ the full effective action is given by

$$
S_{\text{eff}}[\phi] = \int \mathrm{d}^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4 + \frac{\lambda_2^3}{384\pi^2 M^2}\phi^6 \right] . \tag{2.44}
$$

As expected, the dimension-6 operator is suppressed by two powers of the heavy scale $\Lambda \equiv M$, and the Wilson coefficient consists of the couplings λ_2^3 times a loop factor.

Diagrammatic matching

As an alternative to this functional approach, the effective action in Eq. (2.28) can be calculated in an intuitive diagrammatic way. Since the light fields are kept fixed in Eq. (2.22), the effective action is given by all connected Feynman diagrams with only ϕ as external legs and only Φ fields as internal propagators. A more rigorous derivation than the one in the precious section in fact reveals that also certain connected loop diagrams with only ϕ as external legs and both Φ and ϕ fields as internal propagators contribute if they cannot be disconnected by cutting a single internal ϕ line [58]. The first term in Eq. (2.28) corresponds to all such tree-level diagrams, the second term describes one-loop pieces. Higher-loop corrections will play no role in this thesis and were left out.

In practice, the effective operators and their Wilson coefficients can be calculated without the need for any functional methods as follows:

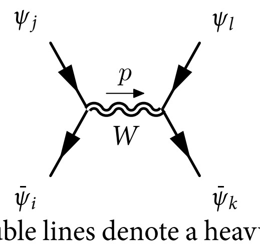
1. Start with the particle content of the full model. Choose Λ and divide the particles of the full model into light and heavy fields. Light fields, which should include at least those with masses below Λ, will make up the particle content of the effective theory. Heavy fields will be integrated out, that is, removed as dynamical degrees of freedom in the EFT.
2. Based on the particles and interactions of the full model, draw all connected Feynman diagrams in which
   - all external legs are light fields, and
   - that cannot be disconnected by cutting a single internal light field line. For tree-level diagrams this is equivalent to requiring that only heavy fields appear as internal lines.

   Using the Feynman rules of the full model, calculate the expressions for these diagrams. Do not treat the external legs as incoming or outgoing particles, but keep the field operator expressions.
3. Express quantities of the full model in terms of Λ. Truncate this infinite series of diagrams at some order in $1/\Lambda$, depending on the dimension of the operators that you want to keep. Together with kinetic and mass terms for the light fields, these form the Lagrangian of the EFT.

Fermi theory again

Let us apply this top-down procedure to our standard example of Fermi theory. For simplicity, we do not take the full SM, but just the interactions between massive W bosons and fermions as the underlying theory. The Lagrangian of these interactions is similar to that given in Eq. (2.11).

1. Our full model consists of the quarks and leptons and the W boson. We want to analyse weak interactions below the W mass, so we set $\Lambda = m_W$. So all quarks and leptons except for the top are the light particles of the EFT, while the W boson and the top quark are heavy and have to be integrated out.
2. The only diagram with the requested features that has only one heavy propagator has the form



(2.45)

   Double lines denote a heavy field. There are additional diagrams with W self-interactions or W loops, but they involve at least two W propagators, which means that all contributions from them will be of order $\mathcal{O}(1/\Lambda^4)$, which we will neglect.

This diagram evaluates to

$$\begin{aligned} &\left(\bar{\psi}_i \frac{\mathrm{i}g}{\sqrt{2}} \frac{1-\gamma_5}{2} \gamma^\mu \psi_j\right) \frac{-g_{\mu\nu}}{p^2 - m_W^2} \left(\bar{\psi}_k \frac{\mathrm{i}g}{\sqrt{2}} \frac{1-\gamma_5}{2} \gamma^\nu \psi_l\right) \\ &= \frac{g^2 \left(\bar{\psi}_i (1-\gamma_5) \gamma^\mu \psi_j\right) \left(\bar{\psi}_k (1-\gamma_5) \gamma_\mu \psi_l\right)}{8(p^2 - m_W^2)} \end{aligned} \tag{2.46}$$

3. The only dimensionful parameter is $m_W = \Lambda$, and for the EFT to be valid we assume $p^2 \ll \Lambda^2$. We can then expand this expression as

$$\frac{g^2}{8m_W^2} \left(\bar{\psi}_i (1-\gamma_5) \gamma^\mu \psi_j\right) \left(\bar{\psi}_k (1-\gamma_5) \gamma_\mu \psi_l\right) + \mathcal{O}\left(1/\Lambda^4\right) . \tag{2.47}$$

With this, we again rediscover the dimension-6 EFT matched to the weak interactions of the SM:

$$\mathcal{L} = \mathrm{i}\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - m_i \bar{\psi}_i \psi_i + \frac{c}{\Lambda^2} \left(\bar{\psi}_i (1-\gamma_5) \gamma_\mu \psi_j\right) \left(\bar{\psi}_k (1-\gamma_5) \gamma^\mu \psi_l\right) . \tag{2.48}$$

with heavy scale $\Lambda = m_W$ and Wilson coefficient $c = g^2/8$. Replacing c/Λ^2 by $G_F/\sqrt{2} = g^2/(8m_W^2)$ restores the historic form of Fermi theory.

Operator mixing

So far we have neglected that like all parameters in a QFT, the Wilson coefficients of an EFT depend on the energy scale. Running the model from one energy to a different one leads to **operator mixing**: loop effects from one operator will affect the coefficients of other operators. If the Wilson coefficients are given at the matching scale Λ (we use this symbol since the matching scale is usually chosen only slightly below the EFT cutoff), at the scale of interest E they will take on values of the form

$$f_i(E) \sim f_i(\Lambda) \pm \sum_j \frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{E^2} f_j(\Lambda) , \tag{2.49}$$

where g are the typical couplings in the loops.

If the matching scale is not too far away from the energy scale of interest and if all Wilson coefficients are already sizeable at the matching scale, this is often negligible. There is an important consequence, though: even if an operator is zero at the matching scale, operator mixing will give it a small but non-zero value at lower energies. So regardless of what the underlying model is, it can be expected that eventually all effective operators allowed by the symmetries will receive contributions from it.

2.3 Dimension-six Higgs physics

We now apply these general ideas to electroweak and in particular Higgs physics at the TeV scale and construct the Standard Model effective field theory or linear Higgs effective field theory up to dimension six. This is the framework we will use throughout this thesis. We will first argue why such an effective theory is very useful, and then construct its effective operators following the recipe laid out in Section 2.2.2. Section 2.3.2 will take a closer look at the phenomenology of these operators. Finally, in Section 2.3.3 we briefly discuss a few alternative frameworks.

As argued in Section 2.1.3, there are many reasons to suspect new physics in the Higgs sector. Some of these arguments, such as the hierarchy problem or the WIMP miracle of dark matter, point towards BSM physics close to the electroweak scale or, depending on the level of acceptable fine-tuning, up to a few TeV. Unfortunately these (purely aesthetic) arguments do not tell us how exactly such physics should look like.

This leaves us with a question highly relevant for upcoming ATLAS and CMS analyses: what is the best language to discuss indirect signs of new physics at the electroweak scale, in particular in the Higgs sector?

Directly interpreting measurements in complete models of new physics is impractical: for $n_y \gg 1$ signatures and $m_a \gg 1$ models this requires $m_a n_y$ limits to be derived.[7] Also, the parameter space of such models (think of the relatively simple MSSM) can be huge, and many of their features do not matter at the electroweak scale at all. It makes more sense to define an intermediate framework that can be linked both to measurements and to full theories, so only n_y sets of limits plus m_a translation rules from complete theories to the intermediate language have to be calculated. Such a framework should include all necessary physics, but no phenomena irrelevant at this scale…exactly the defining feature of an effective field theory.

2.3.1 Operators

Building blocks

Since we do not know what physics lays beyond the SM, we have to construct our EFT from a bottom-up perspective. As discussed above, this means we have to write down all operators based on a set of particles that are compatible with certain symmetries and are important according to some counting scheme. Let us go through these one by one:

1. As degrees of freedoms we use the SM fields. In particular, we assume the Higgs boson h and the Goldstone bosons w_i are combined in a $SU(2)_L$ doublet ϕ as in the SM, see Eq. (2.4). This is consistent with correct data and the correct choice if new physics decouples, that is, if in the limit $\Lambda \to \infty$ the SM is recovered [63]. We will discuss an alternative construction based on the physical scalar h instead of ϕ as the fundamental building block in Section 2.3.3.

[7] The weird notation is necessary because the author cannot resist a stupid pun.

2. All operators have to be invariant under Lorentz transformations and under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, and conserve lepton and baryon number.
3. We order the operators by their mass dimension and thus their suppression in powers of $1/\Lambda$. We will keep those up to mass dimension 6, i. e. $\mathcal{O}\big(1/\Lambda^2\big)$.

Simple dimensional analysis of the kinetic terms of the SM fields tells us the mass dimensions of all building blocks:

$$[f] = \frac{3}{2}\,, \quad [V_\mu] = 1\,, \quad [V_{\mu\nu}] = 2\,, \quad [\phi] = 1\,, \quad [\partial_\mu] = 1 \quad \text{and} \quad [D_\mu] = 1\,. \tag{2.50}$$

The only dimension-five operator that can be built from the SM fields is the Weinberg operator $(\bar{L}_L \tilde{\phi}*)(\tilde{\phi}^\dagger L_L)$. It generates a Majorana mass term for the neutrinos and violates lepton number, and is entirely irrelevant for Higgs physics. The leading effects in our EFT are expected to come from dimension-6 operators:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i + \mathcal{O}\big(1/\Lambda^4\big) \tag{2.51}$$

with unknown cutoff scale Λ and Wilson coefficients f_i. For convenience, we will from now on drop the higher-order terms.

As discussed in Section 2.2.2, field redefinitions (or, relatedly, equations of motions), Fierz identities and integration by parts provide equivalence relations between certain operators and give us some freedom to define a basis of operators. Taking these into account, there are 59 independent types of dimension-six operators, not counting flavour structures and Hermitian conjugation [64]. Counting all possible flavour structures, there are 2499 distinct operators. Fortunately, in practice only a small subset of these are relevant: first, the strong constraints on flavour-changing neutral currents motivates the assumption of flavour-diagonal or even flavour-universal Wilson coefficients. Second, only a small number of these operators directly affects Higgs physics. At higher orders in the EFT expansion, the number of operators increases rapidly, explaining why we stick to the leading effects at dimension 6: not counting flavour structures, there are $\mathcal{O}(1000)$ operators at dimension 8 and $\mathcal{O}(10\,000)$ dimension-10 operators [65].

Three different conventions have become popular: in addition to the complete "Warsaw" basis [64], there is the SILH convention [66] and the HISZ basis [67]. For a comparison of and conversion between these bases see Ref. [68]. Throughout this thesis we use the basis developed in Refs. [22, 69], which is strongly based on the HISZ basis and now widely used in global fits [23, 24].

More references?

We classify the operators based on their field content and on their behaviour under CP transformations. This combined charge conjugation and parity inversion is an approximate symmetry of the SM that is only violated by the complex phase of the CKM matrix. In addition, there are rather tight bounds on CP violation in many processes. This motivates many analyses to restrict

$\mathcal{O}_{\phi 1} = (D_\mu \phi)^\dagger \, (\phi \, \phi^\dagger) \, (D^\mu \phi)$	$\mathcal{O}_{GG} = (\phi^\dagger \phi) \, G^a_{\mu\nu} \, G^{\mu\nu \, a}$
$\mathcal{O}_{\phi 2} = \frac{1}{2} \partial^\mu (\phi^\dagger \phi) \, \partial_\mu (\phi^\dagger \phi)$	$\mathcal{O}_{BB} = -\frac{g'^2}{4} (\phi^\dagger \phi) \, B_{\mu\nu} \, B^{\mu\nu}$
$\mathcal{O}_{\phi 3} = \frac{1}{3} (\phi^\dagger \phi)^3$	$\mathcal{O}_{WW} = -\frac{g^2}{4} (\phi^\dagger \phi) \, W^k_{\mu\nu} \, W^{\mu\nu \, k}$
$\mathcal{O}_{\phi 4} = (\phi^\dagger \phi) \, (D_\mu \phi)^\dagger (D^\mu \phi)$	$\mathcal{O}_{BW} = -\frac{g \, g'}{4} (\phi^\dagger \sigma^k \phi) \, B_{\mu\nu} \, W^{\mu\nu \, k}$
	$\mathcal{O}_B = \frac{\mathrm{i}g}{2} (D^\mu \phi^\dagger)(D^\nu \phi) \, B_{\mu\nu}$
	$\mathcal{O}_W = \frac{\mathrm{i}g}{2} (D^\mu \phi^\dagger) \sigma^k (D^\nu \phi) \, W^k_{\mu\nu}$

Table 2.1: Bosonic CP-conserving dimension-6 operators relevant for Higgs physics.

their set of operators to the CP-conserving ones. On the other hand, new sources of CP violation are needed to explain the matter-antimatter asymmetry in the universe, and their effects at low energies could be visible as CP-violating effective operators. Both types of operators will be analysed in this thesis.

Operator basis

We begin with the CP-conserving dimension-six operators relevant for Higgs physics, following Refs. [22, 69]. In Table 2.1 we list the bosonic ones, Table 2.2 gives the Higgs-fermion operators, and the "dipole operators" made of Higgs fields, gauge bosons, and fermions are listed in Table 2.3. We use the convention for the sign in the covariant derivative given in Eq. (2.2), T^a are the $SU(3)$ generators, and

$$\phi^\dagger \overleftrightarrow{D}_\mu \phi \equiv \phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi \quad \text{and} \quad \phi^\dagger \overleftrightarrow{D}^a_\mu \phi \equiv \phi^\dagger \sigma^a D_\mu \phi - (D_\mu \phi)^\dagger \sigma^a \phi \, . \tag{2.52}$$

In addition to these CP-conserving structures, there are a number of CP-violating operators. We only list the bosonic ones relevant for Higgs physics [70, 71] in Table 2.4. They involve the dual field strength tensors

$$\widetilde{V}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} V^{\mu\nu} \, , \qquad V = B, W, G \, . \tag{2.53}$$

Finally, there are a few pure CP-even and CP-odd pure gauge operators made from field strength tensors and (covariant) derivatives, and a large number of four-fermion operators similar to the one in Eq. (2.48), which will not be important in this thesis.

$\mathcal{O}_\ell = (\phi^\dagger\phi)\,\bar{L}_L\phi\,\ell_R$	$\mathcal{O}^{(1)}_{\phi L} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{L}_L\gamma^\mu L_L)$	$\mathcal{O}^{(3)}_{\phi L} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}^a_\mu\phi)(\bar{L}_L\gamma^\mu\sigma_a L_L)$
$\mathcal{O}_u = (\phi^\dagger\phi)\,\bar{Q}_L\tilde{\phi}\,u_R$	$\mathcal{O}^{(1)}_{\phi Q} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\overline{Q}_L\gamma^\mu Q_L)$	$\mathcal{O}^{(3)}_{\phi Q} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}^a_\mu\phi)(\overline{Q}_L\gamma^\mu\sigma_a Q_L)$
$\mathcal{O}_d = (\phi^\dagger\phi)\,\bar{Q}_L\phi\,d_R$	$\mathcal{O}^{(1)}_{\phi\ell} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{\ell}_R\gamma^\mu\ell_R)$	
	$\mathcal{O}^{(1)}_{\phi u} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{u}_R\gamma^\mu u_R)$	
	$\mathcal{O}^{(1)}_{\phi d} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{d}_R\gamma^\mu d_R)$	
	$\mathcal{O}^{(1)}_{\phi ud} = \mathrm{i}(\phi^\dagger \overleftrightarrow{D}_\mu\phi)(\bar{u}_R\gamma^\mu d_R)$	

Table 2.2: *CP*-conserving dimension-6 operators relevant for the Higgs-fermion couplings. For readability, flavour indices and Hermitian conjugation are omitted.

$\mathcal{O}_{uW} = (\overline{Q}_L\sigma^{\mu\nu}u_R)\sigma^a\tilde{\phi}W^a_{\mu\nu}$	$\mathcal{O}_{uB} = (\overline{Q}_L\sigma^{\mu\nu}u_R)\tilde{\phi}B_{\mu\nu}$	$\mathcal{O}_{uG} = (\overline{Q}_L\sigma^{\mu\nu}T^a u_R)\tilde{\phi}G^a_{\mu\nu}$
$\mathcal{O}_{dW} = (\overline{Q}_L\sigma^{\mu\nu}d_R)\sigma^a\phi W^a_{\mu\nu}$	$\mathcal{O}_{dB} = (\overline{Q}_L\sigma^{\mu\nu}d_R)\phi B_{\mu\nu}$	$\mathcal{O}_{dG} = (\overline{Q}_L\sigma^{\mu\nu}T^a d_R)\phi G^a_{\mu\nu}$
$\mathcal{O}_{\ell W} = (\bar{L}_L\sigma^{\mu\nu}\ell_R)\sigma^a\phi W^a_{\mu\nu}$	$\mathcal{O}_{\ell B} = (\bar{L}_L\sigma^{\mu\nu}\ell_R)\phi B_{\mu\nu}$	

Table 2.3: Dipole operators affecting the Higgs-gauge-fermion couplings. For readability, flavour indices and Hermitian conjugation are omitted.

$\mathcal{O}_{G\widetilde{G}} = (\phi^\dagger\phi)\,G^a_{\mu\nu}\,\widetilde{G}^{\mu\nu\,a}$	$\mathcal{O}_{\widetilde{B}} = \dfrac{\mathrm{i}g}{2}(D^\mu\phi^\dagger)(D^\nu\phi)\,\widetilde{B}_{\mu\nu}$
$\mathcal{O}_{B\widetilde{B}} = -\dfrac{g'^2}{4}(\phi^\dagger\phi)\,B_{\mu\nu}\,\widetilde{B}^{\mu\nu}$	$\mathcal{O}_{B\widetilde{W}} = -\dfrac{g\,g'}{4}(\phi^\dagger\sigma^k\phi)\,B_{\mu\nu}\,\widetilde{W}^{\mu\nu\,k}$
$\mathcal{O}_{W\widetilde{W}} = -\dfrac{g^2}{4}(\phi^\dagger\phi)\,W^k_{\mu\nu}\,\widetilde{W}^{\mu\nu\,k}$	

Table 2.4: Bosonic *CP*-violating dimension-6 operators relevant for Higgs physics. The dual field strengths are defined in Eq. (2.53).

As argued above, not all of these operators are independent. The equations of motions for the Higgs field and the electroweak gauge bosons read [64]

$$D^2\phi = -\mu^2\phi - 2\lambda(\phi^\dagger\phi)\phi - \sum_f y_f \bar{f}_R f_L + \mathcal{O}\big(1/\Lambda^2\big)\,, \tag{2.54}$$

$$\partial^\rho B_{\rho\mu} = -\frac{g'}{2}\left(\mathrm{i}\phi^\dagger \overleftrightarrow{D}_\mu\phi + \sum_f Y_f \bar{f}\gamma_\mu f\right) + \mathcal{O}\big(1/\Lambda^2\big)\,, \tag{2.55}$$

$$(D^\rho W_{\rho\mu})^a = \frac{g}{2}\left(\mathrm{i}\phi^\dagger \overleftrightarrow{D}^a_\mu\phi + \bar{q}_L\gamma_\mu\sigma^a q_L + \bar{\ell}_L\gamma_\mu\sigma^a\ell_L\right) + \mathcal{O}\big(1/\Lambda^2\big)\,, \tag{2.56}$$

where Y_f are the weak hypercharges of the fermions. Following Eq. (2.14), this provides us with three equivalence relations between dimension-six operators [22, 69]:

$$\mathcal{O}_{\phi,2} + \mathcal{O}_{\phi,4} - \mu^2(\phi^\dagger\phi)^2 - 6\lambda\mathcal{O}_6 = \sum_f y_f\mathcal{O}_f + \mathcal{O}\big(1/\Lambda^4\big) \tag{2.57}$$

$$2\mathcal{O}_B + \mathcal{O}_{BW} + \mathcal{O}_{BB} + g'^2\left(\mathcal{O}_{\phi,1} - \frac{1}{2}\mathcal{O}_{\phi,2}\right) = -\frac{g'^2}{2}\sum_f Y_f\mathcal{O}^{(1)}_{\phi f} + \mathcal{O}\big(1/\Lambda^4\big) \tag{2.58}$$

$$2\mathcal{O}_W + \mathcal{O}_{BW} + \mathcal{O}_{WW} + g^2\left(\mathcal{O}_{\phi,4} - \frac{1}{2}\mathcal{O}_{\phi,2}\right) = -\frac{g^2}{4}\left(\mathcal{O}^{(3)}_{\phi L} + \mathcal{O}^{(3)}_{\phi Q}\right) + \mathcal{O}\big(1/\Lambda^4\big)\,. \tag{2.59}$$

This lets us eliminate three of the operators listed in Tables 2.1 to 2.4.

There are different strategies for picking the operators to keep. In a top-down approach, one could choose operators based on the underlying physics. In a bottom-up approach, calculations can be simplified if the operators are chosen based on their contributions to physical observables, for instance to avoid blind directions. Applied to Higgs physics, this logic suggests to discard $\mathcal{O}^{(1)}_{\phi L}$, $\mathcal{O}_{(3)}$, and $\mathcal{O}_{\phi,4}$ [22].

Constraints

Some of the remaining operators are tightly constrained experimental data. Electroweak precision measurements limit the Wilson coefficients of $\mathcal{O}_{\phi,1}$, $\mathcal{O}_{BW}$, $\mathcal{O}^{(1)}_{\phi f}$, $\mathcal{O}^{(3)}_{\phi f}$, $\mathcal{O}_{B\widetilde{W}}$, and $\mathcal{O}_{\widetilde{B}}$ to a level where their effects in Higgs physics are small. Measurements of electric diple moments put tight constrains on the dipole operators. We will ignore all these operators.[8]

Limits on flavour-changing neutral currents constrain off-diagonal fermion-Higgs couplings. Also, flavour-diagonal $\mathcal{O}_f$ involving fermions of the first and second generation will be irrelevant for many signatures considered in this thesis. We therefore only keep the Higgs-fermion operators $\mathcal{O}_f$ of the third generation.

[8] This simple argument is suitable for our rather conceptual work. In a thorough global fit, however, it should be checked carefully whether these constraints are actually strong enough to make all these operator irrelevant for Higgs physics in all cases. In particular, the increasing precision in Higgs observables means that many of these operators will become relevant again in the future.

This leaves us with a list of thirteen operators relevant for LHC Higgs physics: ten CP-even operators,

$$\mathcal{O}_{\phi,2}\,, \quad \mathcal{O}_{\phi,3}\,, \quad \mathcal{O}_{GG}\,, \quad \mathcal{O}_{BB}\,, \quad \mathcal{O}_{WW}\,, \quad \mathcal{O}_{B}\,, \quad \mathcal{O}_{W}\,, \quad \mathcal{O}_{\tau}\,, \quad \mathcal{O}_{t}\,, \quad \text{and} \quad \mathcal{O}_{b}\,; \tag{2.60}$$

and three CP-odd ones,

$$\mathcal{O}_{G\widetilde{G}}\,, \quad \mathcal{O}_{B\widetilde{B}}\,, \quad \text{and} \quad \mathcal{O}_{W\widetilde{W}}\,. \tag{2.61}$$

Renormalisation group evolution

The Wilson coefficients of these operators depend on the energy scale. In the last years, the contributions of all dimension-six operators on the running of the SM parameters, as well as the whole 59×59 anomalous dimension matrix of dimension-six operators, have been calculated at one-loop level [72–74]. This provides all necessary tools to run the EFT parameters from the matching scale Λ to the experimental scale E. Following Eq. (2.49), this shifts the Wilson coefficients by a term proportional to a loop factor and $\log \Lambda^2/E^2$.

As we will discuss in some length in Chapter 3, the LHC Higgs measurements are only sensitive to new physics scales between the electroweak scale and the TeV scale. The corresponding logarithm typically cannot compensate for the loop factor, and the RGE effects on Wilson coefficients that are already non-zero at the matching scale are small. We will therefore neglect operator running for our analyses.

2.3.2 Phenomenology

After picking a set of operators, the next question is how they affect Higgs observables. We will first discuss two examples, $\mathcal{O}_{\phi,2}$ and $\mathcal{O}_W$, in detail, before listing the effects of all operators in Eqs. (2.60) and (2.60).

$\mathcal{O}_{\phi 2}$: rescaled Higgs couplings

Our first example is the operator $\mathcal{O}_{\phi,2}$. Ignoring the Goldstones, it consists only of derivatives and Higgs fields $\phi^\dagger \phi = (v^2 + 2v\tilde{h} + \tilde{h}^2)/2$, where we use a tilde on h for reasons that will become clear soon. Its contribution to the Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\text{EFT}} &\supset \frac{f_{\phi,2}}{2\Lambda^2}\,\partial^\mu(\phi^\dagger\phi)\,\partial_\mu(\phi^\dagger\phi)\\ &= \frac{f_{\phi,2}v^2}{2\Lambda^2}\,\partial_\mu\tilde{h}\partial^\mu\tilde{h} + \frac{f_{\phi,2}\,v}{\Lambda^2}\,\tilde{h}\partial_\mu\tilde{h}\partial^\mu\tilde{h} + \frac{f_{\phi,2}\,v}{2\Lambda^2}\,\tilde{h}^2\partial_\mu\tilde{h}\partial^\mu\tilde{h}\,.\end{aligned} \tag{2.62}$$

The first term rescales the kinetic term of the Higgs boson:

$$\mathcal{L}_{\text{EFT}} \supset \left(1 + \frac{f_{\phi 2}v^2}{\Lambda^2}\right)\frac{1}{2}\,\partial_\mu\tilde{h}\partial^\mu\tilde{h}\,. \tag{2.63}$$

To restore the canonical form of the kinetic term, we have to rescale the Higgs boson $\tilde{h}$ to

$$h = \sqrt{1 + \frac{f_{\phi 2} v^2}{\Lambda^2}}\, \tilde{h} \,. \tag{2.64}$$

On the one hand, this universally shifts all Higgs couplings to other particles as

$$g_{hxx} = \frac{1}{\sqrt{1 + \frac{f_{\phi 2} v^2}{\Lambda^2}}}\, g_{hxx}^{\mathrm{SM}} \,. \tag{2.65}$$

The effect on the Higgs self-coupling is more involved. First, the rescaling in Eq. (2.64) also affects the Higgs mass term given in Eq. (2.5). For fixed v and m_h, this amounts to shifting the Higgs self-coupling λ to

$$\lambda = \frac{m_h^2}{2v^2} \left(1 + \frac{f_{\phi 2} v^2}{\Lambda^2} \right) . \tag{2.66}$$

In addition, the second term in Eq. (2.62) introduces a new Lorentz structure into the Higgs self-interaction that depends on the Higgs momenta. A non-zero Wilson coefficients $f_{\phi 2}$ will therefore have a strong impact on Higgs pair production, not only on the total rate, but also on kinematic distributions.

$\mathcal{O}_W$: new Higgs-gauge structures

Our second example is $\mathcal{O}_W$, which contracts covariant derivatives acting on Higgs doublets, defined in Eq. (2.2), with a field strength tensor $W_{\mu\nu}^k = \partial_\mu W_\nu^k - \partial_\nu W_\mu^k + g \varepsilon^{kmn} W_\mu^m W_\nu^n$. Expanding $\mathcal{O}_W$ and only keeping the pieces that will affect the hWW coupling, we find

$$\begin{aligned}
\mathcal{L}_{\mathrm{EFT}} &\supset \frac{f_W}{\Lambda^2} \frac{\mathrm{i}g}{2} (D^\mu \phi)^\dagger \sigma^k (D^\nu \phi)\, W_{\mu\nu}^k \\
&= \frac{\mathrm{i}g f_W}{2\Lambda^2} \left(\partial^\mu \phi^\dagger + \frac{\mathrm{i}g}{2} W^{m\,\mu} \phi^\dagger \sigma^m + \frac{\mathrm{i}g'}{2} B^\mu \phi^\dagger \right) \sigma^k \left(\partial^\nu \phi - \frac{\mathrm{i}g}{2} \sigma^n W^{n\,\nu} \phi - \frac{\mathrm{i}g'}{2} B^\nu \phi \right) W_{\mu\nu}^k \\
&\supset \frac{\mathrm{i}g f_W}{2\Lambda^2} \left\{ \frac{\partial^\mu h}{\sqrt{2}} \left[\sigma^k \sigma^n \right]_{22} \frac{-\mathrm{i}g}{2} W^{n\,\nu} \frac{v}{\sqrt{2}} + \frac{\mathrm{i}g}{2} W^{m\,\nu} \frac{v}{\sqrt{2}} \left[\sigma^m \sigma^k \right]_{22} \frac{\partial^\mu h}{\sqrt{2}} \right\} W_{\mu\nu}^k \\
&= \frac{f_W}{\Lambda^2} \frac{g^2 v}{8} \left[\sigma^k, \sigma^n \right]_{22} (\partial^\mu h)\, W^{n\,\nu}\, W_{\mu\nu}^k \\
&= \frac{f_W}{\Lambda^2} \frac{\mathrm{i}g^2 v}{4} \varepsilon^{nk3} (\partial^\mu h)\, W^{n\,\nu}\, W_{\mu\nu}^k \,.
\end{aligned} \tag{2.67}$$

With $m_W = gv/2$ and $W_\mu^\pm = (W_\mu^1 \mp \mathrm{i} W_\mu^2)/\sqrt{2}$ this finally fields

$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{f_W}{\Lambda^2} \frac{\mathrm{i}g m_W}{2} (\partial^\mu h) \left(W^{+\,\nu}\, W_{\mu\nu}^- + W^{-\,\nu}\, W_{\mu\nu}^+ \right) . \tag{2.68}$$

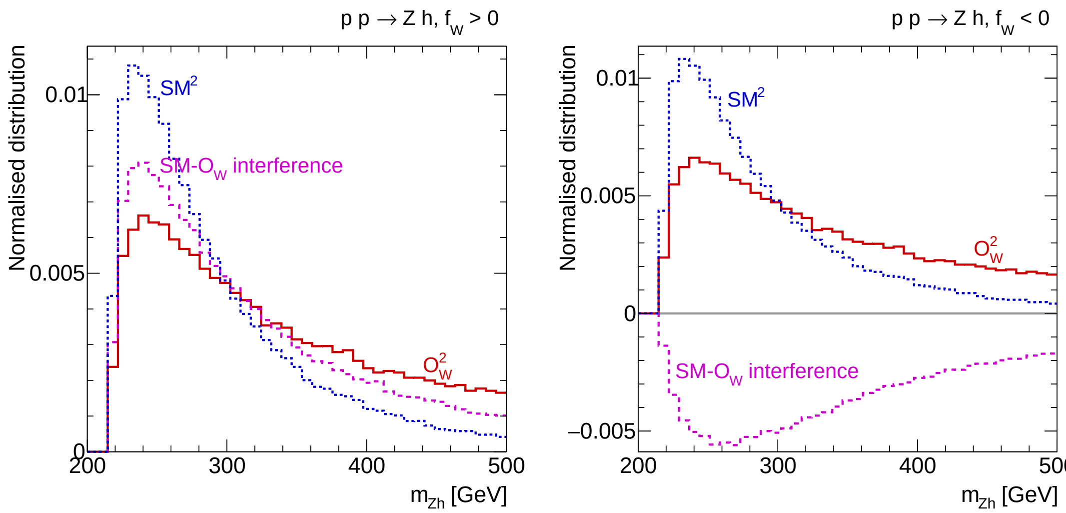
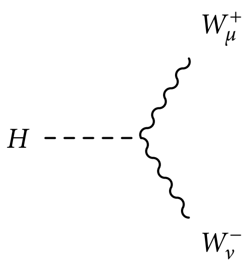


Figure 2.4: Distribution of the Zh invariant mass in the Higgs-strahlung process $pp \to Zh$ at LHC conditions. We compare the contributions from the SM, the operator $\mathcal{O}_W$ squared, and their interference, which can be constructive (left) or destructive (right).

This is another contribution to the hWW vertex. But unlike the SM-like coupling

$$\mathcal{L}_{\text{SM}} \supset g m_W \, h W^{+\mu} W^-_\mu \,, \tag{2.69}$$

the $\mathcal{O}_W$ term includes derivatives. This means that the interaction gains a momentum dependence:



$$= \mathrm{i} g m_W \left[g_{\mu\nu} + \frac{f_W}{2\Lambda^2} \, p_H^2 \, g_{\mu\nu} + \frac{f_W}{2\Lambda^2} \left(p^H_\mu p^+_\nu + p^-_\mu p^H_\nu \right) \right] , \tag{2.70}$$

where $p^\pm_\mu$ and p^H_μ are the incoming momenta of the $W^\pm$ and the H, respectively.

This operator illustrates two key features of the EFT approach. First, $\mathcal{O}_W$ does not only affect the hWW vertex, but also hZZ interactions and triple-gauge couplings such as WWZ. This means that the dimension-6 operator language allows us to combine different measurements in a global fit.

Second, $\mathcal{O}_W$ changes the shape of distributions, for instance in Higgs-strahlung at the LHC,

$$pp \to Zh \,. \tag{2.71}$$

In this process, the intermediate Z can carry arbitrary large energy and momentum, which we can measure for instance as the invariant mass of the final Zh system. From Eq. (2.70) we expect that the effects from $\mathcal{O}_W$ will grow with m_{Zh}. In Fig. **??** we demonstrate this by comparing the distribution of m_{Zh} based on only the SM couplings, on the dimension-6 operator $\mathcal{O}_W$ only, and on the interference between the two components. Indeed we see that $\mathcal{O}_W$ contributes mostly in the high-energy tail of the distribution.

All those couplings

After these two worked-out examples, we now give the complete list of single-Higgs couplings induced by the dimension-six operators of Eqs. (2.60) and (2.61), some of which were calculated in Refs. [22, 69] and cross-checked by us.

Add Higgs self-interactions?

These interactions read

$$
\begin{aligned}
\mathcal{L}_{\text{EFT}} \supset\; & g_{hgg}\, hG^a_{\mu\nu}G^{a\,\mu\nu} + g_{h\gamma\gamma}\, hA_{\mu\nu}A^{\mu\nu} \\
& + g^{(1)}_{hZ\gamma}\, A_{\mu\nu}Z^{\mu}\partial^{\nu}h + g^{(2)}_{hZ\gamma}\, hA_{\mu\nu}Z^{\mu\nu} \\
& + g^{(1)}_{hZZ}\, Z_{\mu\nu}Z^{\mu}\partial^{\nu}h + g^{(2)}_{hZZ}\, hZ_{\mu\nu}Z^{\mu\nu} + g^{(3)}_{hZZ}\, hZ_{\mu}Z^{\mu} + g^{(4)}_{hZZ}\, h\varepsilon_{\mu\nu\rho\sigma}Z^{\mu\nu}Z^{\rho\sigma} \\
& + g^{(1)}_{hWW}\, \left(W^{+}_{\mu\nu}W^{-\,\mu}\partial^{\nu}h + \text{h. c.}\right) + g^{(2)}_{hWW}\, hW^{+}_{\mu\nu}W^{-\,\mu\nu} + g^{(3)}_{hWW}\, hW^{+}_{\mu}W^{-\,\mu} \\
& \qquad\qquad\qquad\qquad + g^{(4)}_{hWW}\, h\varepsilon_{\mu\nu\rho\sigma}W^{+\,\mu\nu}W^{-\,\rho\sigma} \\
& + \sum_{f=\tau,t,b}\left(g_{hff}h\bar{f}_L f_R + \text{h. c.}\right)
\end{aligned}
\tag{2.72}
$$

with couplings

$$
\begin{aligned}
g_{hgg} &= \frac{f_{GG}v}{\Lambda^2}\,, & g^{(1)}_{hZ\gamma} &= \frac{g^2 vs(f_W - f_B)}{4c\Lambda^2}\,, \\
g_{h\gamma\gamma} &= -\frac{g^2 vs^2(f_{WW} + f_{BB})}{4\Lambda^2}\,, & g^{(2)}_{hZ\gamma} &= \frac{g^2 vs(2s^2 f_{BB} - 2c^2 f_{WW})}{4c\Lambda^2}\,, \\
g^{(1)}_{hZZ} &= \frac{g^2 v(c^2 f_W + s^2 f_B)}{4c^2\Lambda^2}\,, & g^{(1)}_{hWW} &= \frac{g^2 v f_W}{4\Lambda^2}\,, \\
g^{(2)}_{hZZ} &= -\frac{g^2 v(s^4 f_{BB} + c^4 f_{WW})}{4c^2\Lambda^2}\,, & g^{(2)}_{hWW} &= -\frac{g^2 v f_{WW}}{2\Lambda^2}\,, \\
g^{(3)}_{hZZ} &= \frac{g^2 v}{4c^2}\left(1 + \frac{v^2 f_{\phi,2}}{\Lambda^2}\right)^{-1/2}\,, & g^{(3)}_{hWW} &= \frac{g^2 v}{2}\left(1 + \frac{v^2 f_{\phi,2}}{\Lambda^2}\right)^{-1/2}\,, \\
g^{(4)}_{hZZ} &= -\frac{g^2 v(s^4 f_{B\widetilde{B}} + c^4 f_{W\widetilde{W}})}{8c^2\Lambda^2}\,, & g^{(4)}_{hWW} &= -\frac{g^2 v f_{W\widetilde{W}}}{4\Lambda^2} \\
g_{hff} &= -\frac{m_f}{v}\left(1 + \frac{v^2 f_{\phi,2}}{\Lambda^2}\right)^{-1/2} + \frac{v^2 f_f}{\sqrt{2}\Lambda^2}\,.
\end{aligned}
\tag{2.73}
$$

Check CP-odd ones.

Here $s \equiv g'/\sqrt{g^2 + g'^2}$ and $c = \sqrt{1 - s^2}$ are the sine and cosine of the weak mixing angle, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ for $V = A, W^\pm, Z$.

Note that the clear majority of the couplings in Eq. (**??**) does not exist in the SM and contains derivatives. Dimension-six operators predict a variety of novel kinematic features in Higgs interactions, making their measurement at the LHC both exciting and challenging.

To here.

2.3.3 Alternative frameworks

κ framework

One alternative is to dress the SM Lagrangian with form factors, as done for instance in the κ framework (sometimes called Δ framework) of Higgs physics. Such an approach is well suited to parametrise shifts in total rates, but unable to account for new structures that are not present in the SM, visible as changes in distributions. For better or worse, this framework is also agnostic about correlations between different couplings: there is no clear way how to combine the results from triple gauge vertices and Higgs properties, for instance. Finally, the κ framework is not gauge-invariant, which means it does not present a consistent quantum field theory and does not work reliably beyond tree level in the electroweak expansion.

See `http://indico.cern.ch/event/477407/contributions/2200060/attachments/1369793/2076900/AM_HiggsCouplings2016.pdf`

- describe; name Δ and κ conventions. Subtleties with loop effects.
- theoretical issues: not gauge invariant, not renormalisable, not unitary; so probably should not calculate electroweak loop corrections (QCD is fine). However, can be linked to UV-complete models such as 2HDMI, cite David and Til man
- practical perspective: can describe changed SM-like couplings, but not new structures. Often, but not always translates to rate vs distributions. As argued above the kinematic information will be important for general new physics models, so κ model is out.

Pseudo-Observables

- form factor approach
- same theory issues as κ framework
- can describe distributions
- conceptually, closer to experiment and farther away from theory than EFT
- lacks theoretical foundation of EFT, and something else why it's terrible

Nonlinear Higgs effective field theory

As an alternative, it is also possible to use h as the fundamental building block and to not assume the SM doublet structure of Eq. (2.4). This is called the **non-linear** Higgs EFT. The terms "linear" and "non-linear" by the way refer to the behaviour under custodial transformations.

- building blocks
- operators
- similarity to chiral perturbation theory, cite Astrid [75] if at all possible
- stress that same physics is described (with examples?), explain different c (see Claudius Krause's thesis)

Chapter 3

Higgs effective theory at its limits

3.1 Introduction

3.2 Matching intricacies

3.2.1 Default matching

3.2.2 v-improved matching

3.3 Full models vs. effective theory

3.3.1 Singlet extension

3.3.2 Two-Higgs-doublet model

3.3.3 Scalar top partners

3.3.4 Vector triplet

3.4 Practical questions

3.4.1 To square or not to square

3.4.2 Realistic tagging jets

3.4.3 Towards a simplified model

3.4.4 Which observables to study

3.5 Summary

Chapter 4

Better Higgs measurements through information geometry

4.1 Introduction

4.2 Information geometry

4.2.1 Fisher information and Cramér-Rao bound

4.2.2 Simple example

4.3 Information in LHC processes

4.3.1 Information in event counts

4.3.2 Information in histograms

4.3.3 Information in full process and differential information

4.3.4 Nuisance parameters and profiling

4.3.5 The MadFisher algorithm

4.3.6 Geometry of effective field theories

4.4 Higgs signatures from CP-even operators

4.4.1 Weak-boson-fusion Higgs to taus

4.4.2 Weak-boson-fusion Higgs to four leptons

4.4.3 Higgs plus single top

4.5 CP violation in the Higgs sector

4.6 Technical questions

4.6.1 Systematic uncertainties

4.6.2 Comparison with other tools

4.7 Conclusions

Chapter 5

Conclusions

Can we actually know the universe? My God, it's hard enough finding your way around in Chinatown.

— W. Allen [76]

Appendix

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