

New Ideas for Effective Higgs Measurements

Dissertation

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New Ideas for Effective Higgs Measurements

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*After years of expensive education
A car full of books and anticipation
I'm an expert on Shakespeare and that's a hell of a lot
But the world don't need scholars as much as I thought*

— J. Cullum [1]

Abstract in deutscher Übersetzung

Abstract

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Preface

This thesis is based on research conducted between 2004 and 2017 at the Institute for Theoretical Physics at Heidelberg University. Chapter 3 is based on two articles, which later became part of a CERN report:

- [2] J. Brehmer, A. Freitas, D. López-Val, and T. Plehn:
Pushing Higgs Effective Theory to its Limits.
Phys. Rev. D 93, 075014 (2016). [arXiv:1510.03443](#).
- [3] A. Biekötter, J. Brehmer, and T. Plehn:
Extending the Limits of Higgs Effective Theory.
Phys. Rev. D 94, 055032 (2016). [arXiv:1602.05202](#).
- [4] D. de Florian, C. Grojean, F. Maltoni, et al.:
Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector.
LHC Higgs Cross Section Working Group Yellow Report. [arXiv:1610.07922](#).

Chapter 4 is based on the following publication:

- [5] J. Brehmer, K. Cranmer, F. Kling, and T. Plehn:
Better Higgs Measurements Through Information Geometry.
[arXiv:1612.05261](#).

In addition, it includes some original results and yet unpublished work in progress with F. Kling and T. Plehn.

Chapter 2 consists of introductory material that can be found in many textbooks and review articles, as well as on a lecture on effective field theories given by the author to fellow PhD students in Heidelberg:

- [6] J. Brehmer:
Higgs Effective Field Theory.
Student lecture, research training group “Particle Physics Beyond the Standard Model”.

Finally, some of the work done during my PhD is not included in this thesis:

- [7] J. Brehmer, J. Hewett, J. Kopp, T. Rizzo, and J. Tattersall:
Symmetry Restored in Dibosons at the LHC?
JHEP 1510, 182 (2015). [arXiv:1507.00013](#).
- [8, 9] G. Brooijmans, C. Delaunay, A. Delgado, et al.:
Les Houches 2015: Physics at TeV Colliders – New Physics Working Group Report.
[arXiv:1605.02684](#).
Part of these proceedings were published separately as
J. Brehmer, G. Brooijmans, G. Cacciapaglia, et al.:
The Diboson Excess: Experimental Situation and Classification of Explanations; A Les Houches Pre-Proceeding.
[arXiv:1512.04357](#).

Chapter 1

Introduction

THE HIGGS BOSON [10–12] is a key element of the Standard Model of particle physics (SM). Its discovery in 2012 [13, 14] completed the particle zoo of the SM. It is a triumph of a decade-old model, but it also offers us a way forward: the Higgs provides us with an unprecedented chance to understand some of the biggest unsolved mysteries of physics.

As the only known fundamental scalar, it suffers from the famous electroweak hierarchy problem: why is its mass scale (and therefore the electroweak scale) so much smaller than the Planck scale, while there is no sign of a symmetry protecting it against quantum corrections? Is the electroweak vacuum, defined by the Higgs potential, stable? Why are the Yukawa couplings, and consequently the fermion masses, spread out over so many orders of magnitude? Is the Higgs related to Dark Matter, for instance as mediator to a dark sector? Or might it even be the source of inflation, explaining the surprising level of isotropy in the Cosmic Microwave Background?

Many models of physics beyond the Standard Model have been proposed to answer at least some of these questions. Very often they predict Higgs coupling patterns different from the SM. A precise measurement of the Higgs properties thus provides a crucial probe of such models, and might be one of the most important missions for present and future runs of the Large Hadron Collider (LHC).

This poses two immediate questions:

1. Which framework should be used to parametrise the Higgs properties?
2. How can these parameters be measured efficiently at the LHC experiments?

These two questions drive the research presented in this thesis, and we will tackle them one by one.

Ideally, all Higgs measurements should use the same universal language to parametrise their results, allowing for an efficient comparison and combination. Such a framework should be general enough to describe the effects of any interesting new physics scenario without strong

model assumptions. On the other hand, too large a number of parameters makes combinations of different experiments and global fits impractical.

A simple example for such a universal parametrisation that was widely used during run 1 of the LHC is the κ framework. It is based on the SM Lagrangian, but promotes all Higgs couplings to free parameters. The main limitation of this approach is that it can only describe structures that are already present in the SM. So while a measurement based on the κ framework can be useful for total rates, it will not be able to utilise information in kinematic distributions.

Instead, we work in an approach based on effective field theory (EFT) [15–17]. Based only on the assumption that new physics has a typical energy scale significantly larger than the experimental energies, all new physics effects are captured by a tower of higher-dimensional operators. The leading effects for Higgs physics should come from only a handful of operators with mass dimension 6 [18–20]. These operators are manifestly gauge-invariant and can be used beyond tree level. They describe both coupling rescalings as well as novel kinematic structures not present in the SM, allowing us to access information in distributions in addition to total rates [21, 22]. Effective operators also let us combine Higgs data with results from other experiments, including electroweak precision data or gauge boson production at the LHC [23]. However, the limited precision of the LHC Higgs measurements means that only models that are either strongly coupled or relatively light can be probed. In the latter case, the characteristic energy scale of new physics is not sufficiently separated from the momentum transfers in the experiments, casting doubt on the validity of the EFT approach.

We analyse the usefulness of higher-dimensional operators at the LHC by comparing the predictions of UV-complete scenarios of new physics to their dimension-six approximations [2]. Our analysis covers additional scalar singlets, two-Higgs-doublet models, scalar top partners, and heavy vector bosons, focusing on parameter ranges that the LHC will be sensitive to. We take into account rates and distributions in the most important Higgs production modes and various representative decay channels as well as Higgs pair production. For this array of models, benchmark points, and observables, we ask if and where the effective description of new physics breaks down, and how it can be improved.

As it turns out, the agreement between the approaches crucially depends on the matching procedure that links the coefficients of the dimension-six model to the full theory. We introduce v -improved matching, a procedure that improves the performance of the dimension-six model by resumming certain terms that arise during electroweak symmetry breaking. While formally of subleading order in the EFT expansion, these effects can be large under LHC conditions. With such a matching, the effective model provides a good description even in many scenarios where the EFT validity is not obvious. We then discuss a number of practical questions on the role of squared dimension-six terms in the differential cross sections, effects on fits, and the correlation between different observables and the momentum transfer [3].

Having established that Higgs EFT works well as a largely model-independent language for

Higgs physics at the LHC, the next question is how its parameters can be measured optimally. Higgs measurements are affected by many different operators, and each of them affects different couplings, typically introducing non-trivial kinematic structures. This leads to a complicated relation between the high-dimensional model parameter space and often also high-dimensional phase spaces.

Traditional analyses based on selection cuts and histograms of kinematic observables are in many cases not sensitive to such subtle signatures. At the other end of the spectrum, experiments resort more and more to high-level statistical tools, including machine learning techniques or the matrix element method [24–28]. Many of these tools are designed for the comparison between two discrete hypotheses, and applying them to high-dimensional parameter spaces such as Higgs EFT is computationally expensive. Extending machine learning techniques to such high-dimensional spaces is a current area of research [29]. Also, while these multivariate techniques are powerful, it is often not transparent which physical properties exactly they probe. It is therefore increasingly important to be able to characterise the information contained in LHC signatures.

We use information geometry to understand and optimise Higgs measurements [5]. The central building block is the Fisher information, which according to the Cramér-Rao bound encodes the maximal knowledge on theory parameters we can derive from an experiment. Unlike many other statistical tools, the Fisher information is intrinsically designed for continuous, high-dimensional parameter spaces, and this approach does not require any discretisation of the theory space and leads to results that do not depend on arbitrary parameter or basis choices. In addition, the Fisher information defines a metric on the model parameter space. This not only provides an intuitive geometric picture of the sensitivity of measurements, but also allows us to track the impact of higher orders in the EFT expansion.

We calculate the Fisher information for Higgs production in weak boson fusion with decays into tau pairs and four leptons, and for Higgs production in association with a single top quark. To this end, we develop an algorithm to calculate the Fisher information in particle-physics observables based on Monte-Carlo methods. Our results give the maximum precision with which dimension-six operators can be measured in these processes, or the maximal new physics reach of these signatures. We then analyse how the differential information is distributed over phase space, which defines optimal event selections. In a next step, we calculate the information in individual kinematic distributions, and compare it to the maximal information in the full event kinematics. This provides a ranking of the most powerful production and decay observables. It allows us to compare how much we can learn from a simple fit to histograms compared to fully multivariate methods.

This is the first application of information geometry to high-energy physics. While there is no shortage of statistical tools in the field, these new methods can help to plan and optimise measurement strategies for high-dimensional continuous models in an intuitive but powerful way. While we demonstrate this approach in different Higgs channels for dimension-six operators, our

tools can easily be translated to other processes and models.

This thesis begins by recapitulating some basic ideas of Higgs physics and effective field theory in Chapter 2. In Chapter 3, we discuss the validity of effective field theory for LHC measurements and the matching between full models and effective operators. Chapter 4 presents our work on information geometry and optimal Higgs measurements. Both of these chapters will contain separate and more detailed introductions and conclusions. We summarise the results in Chapter 5.

Chapter 2

Foundations

IN THIS CHAPTER we review some of the essential concepts that underlie the research presented in the rest of this thesis. First, we briefly summarise the role of the Higgs boson in the Standard Model (SM) and its phenomenology at the LHC. Section 2.2 then presents a pedagogical introduction to the effective field theory (EFT) concept. In Section 2.3 we combine these ideas and construct an effective field theory for the Higgs sector.

Our introduction to Higgs physics is superficial, and the EFT part eschews mathematical rigour in favour of a broad picture of the central ideas. For a more thorough introduction to Higgs physics, see for instance Reference [30]. For an extensive introduction to EFTs, see References [31, 32].

The EFT section is based on a lecture given by the author and largely identical to the lecture notes [6]. Some of the examples are taken from References [31, 32].

2.1 The Higgs boson at the LHC

2.1.1 The Standard Model Higgs

In the Standard Model, the Higgs boson is part of a scalar $SU(2)_L$ doublet ϕ . The relevant terms in the Lagrangian are

$$\begin{aligned} \mathcal{L}_{\text{SM}} \supset & (D^\mu \phi)^\dagger (D_\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \\ & - \sum_{\text{generations}} \left(y_u \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_L \tilde{\phi} u_R + y_d \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_L \phi d_R + y_\ell \begin{pmatrix} \bar{\nu} \\ \bar{\ell}^- \end{pmatrix}_L \phi \ell_R + \text{h. c.} \right). \end{aligned} \quad (2.1)$$

Here u , d , and ℓ are the up-type quarks, down-type quarks, and leptons of the three generations, all appearing in a left-handed and right-handed version marked by the subscripts L and R . μ^2 and λ are real parameters, and the Yukawa couplings y_i are complex-valued matrices in flavour

space. The covariant derivatives are defined as

$$D_\mu \phi = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a - ig' \frac{B_\mu}{2} \right) \phi \quad (2.2)$$

and

$$\tilde{\phi} \equiv i\sigma_2 \phi^*, \quad (2.3)$$

with $SU(2)_L$ gauge bosons W_μ^a and $U(1)_Y$ gauge boson B_μ . g and g' are the two corresponding coupling constants, and σ_i the Pauli matrices. We give the full Lagrangian of the Standard Model including all definitions and conventions in Appendix A.1.1.

For $\mu^2 < 0$, the Higgs doublet develops a non-zero vacuum expectation value (VEV)

$$v^2 \equiv 2 |\langle \phi \rangle|^2 = -\frac{\mu^2}{\lambda}. \quad (2.4)$$

Using some of the gauge freedom, we can rotate the scalar doublet such that

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -w_2 - iw_1 \\ v + h + iw_3 \end{pmatrix}. \quad (2.5)$$

Here w_i are the would-be Goldstone bosons resulting from the spontaneous breaking of the electroweak symmetry. They combine with the gauge bosons W^a and B to the mass eigenstates γ , W^\pm , and Z . The remaining degree of freedom, the scalar field h , is the Higgs boson.

Plugging Equation (2.5) into Equation (2.1), we find its mass

$$m_h^2 = -2\mu^2 = 2\lambda v^2. \quad (2.6)$$

In addition, the fermions and the vector bosons W^\pm and Z get mass terms proportional to v , as well as couplings to the Higgs boson h . Since both terms stem from the same coupling to $\phi \sim v + h$, the Higgs couplings to other particles x are always proportional to $g_{hxx} \sim m_x/v$. Finally, there are h^3 and h^4 self-couplings. The SM Higgs sector is very predictive: with the measurement of the Higgs mass $m_h = 125$ GeV [13, 14, 33], there are no more free parameters in the SM and all couplings are fixed.

2.1.2 Production and decay

At the LHC, most Higgs bosons are produced in **gluon-gluon fusion** (ggF) as shown in the left panel of Figure 2.1. Due to its large Yukawa coupling, the top plays the dominant role in the loop, with small contributions from the bottom. The total cross section at $\sqrt{s} = 13$ TeV is approximately 49 pb [4], a large part of which comes from NLO and NNLO corrections. This sizeable rate comes

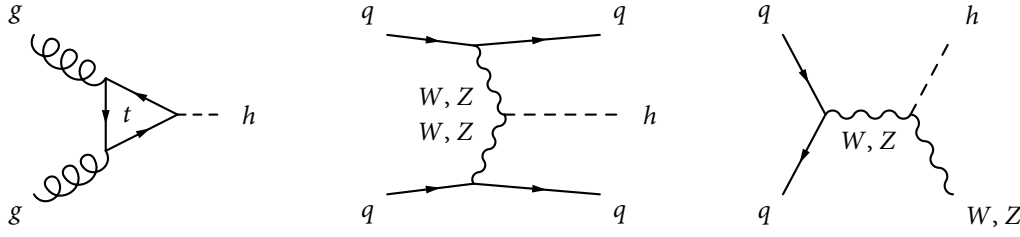


Figure 2.1: Feynman diagrams for the most important Higgs production modes considered in this thesis. Left: gluon fusion. Middle: weak boson fusion. Right: Higgs-strahlung.

at the price of a lack of discerning kinematic features that could help to separate the Higgs signal from QCD backgrounds.

This is certainly different for Higgs production in **weak boson fusion** (WBF)¹, as shown in the middle panel of Figure 2.1. The production rate for this quark-initiated process is only 3.8 pb [4], but the Higgs is accompanied by two highly energetic jets that point nearly back-to-back into the two forward regions of the detector. This translates to a large invariant mass m_{jj} between them as well as a large separation in (pseudo-)rapidity $\Delta\eta_{jj}$. A second important property is provided by the colour structure of the process: at leading order, there is no colour exchange between the two quark lines, which means there is very little QCD radiation in this process. Both of these features set the WBF process apart from QCD backgrounds, which typically have many central jets. Such backgrounds can therefore be reduced significantly by requiring two so-called “tagging jets” with large $\Delta\eta_{jj}$ and large m_{jj} , and vetoing any additional central jets [34].

But the tagging jets are not only useful to discriminate Higgs production from non-Higgs backgrounds. Since they recoil against the intermediate vector bosons that couple to the Higgs, they provide access to the momentum flow through the Higgs production vertex. Their properties, in particular their transverse momenta and the angular correlations between them, thus provide probes of the Higgs-gauge coupling [35–41]. We revisit this important feature from different perspectives in this thesis.

The right panel of Figure 2.1 shows Higgs production in association with a vector boson, or **Higgs-strahlung**. The rate is 1.4 pb for a Wh final state plus 0.9 pb for Zh . Similarly to the tagging jets in WBF, the final-state gauge boson both helps to discriminate the Higgs from backgrounds and provides a handle on the momentum flow through the virtual intermediate vector boson.

Higgs production in association with a $t\bar{t}$ pair is another production mode relevant for the LHC, but do not discuss it in this thesis. Instead, we briefly analyse **Higgs production with a single top quark**. This process exists as an s -channel and a t -channel version with very different kinematic features, and can be calculated either in the four-flavour scheme (with a gluon in the

¹This channel is also known as Vector Boson Fusion or VBF. But this is slightly misleading since the gluon also has spin 1.

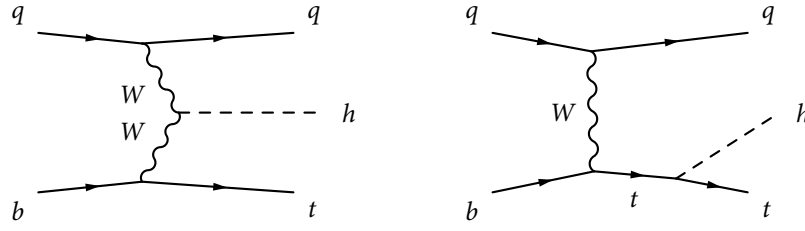


Figure 2.2: Feynman diagrams for Higgs production with a single top quark.

initial state) or in the five-flavour scheme (with a b quark in the initial state). We focus on the dominant t -channel process and calculate it in the five-flavour scheme, as shown in Figure 2.2. Diagrams where the Higgs is radiated off a top quark interfere destructively with amplitudes in which the Higgs couples to a W . The SM rate is small at 74 fb [4], but this interference pattern makes it very sensitive to modified Higgs couplings. In fact, this process is the only direct probe of the sign or complex phase of the top Yukawa coupling ($t\bar{t}h$ production is only sensitive to the absolute value of the top Yukawa, while the total rate in gluon fusion can be influenced by many effects such as new particles in the loop).

Our final channel is **Higgs pair production**, showing in Figure 2.3, which allows to measure the Higgs self-coupling [42, 43]. It is another example of destructive interference between different amplitudes: diagrams in which the two Higgses couple to a top box loop interfere with those in which a single Higgs is produced in gluon fusion and then splits into two Higgses through the self-coupling. Close to threshold, these two contributions approximately cancel in the SM [42, 44], and the total rate is very small at 33 fb. But modified Higgs sectors can spoil this cancellation and increase the rate drastically, as we demonstrate in the next chapter.

The Higgs decay patterns are rather simple. Since it couples to all particles proportional to their mass, it prefers to decay into the heaviest particles allowed by phase space. The dominant decay mode with a branching ratio of 58% [4] is therefore $h \rightarrow b\bar{b}$. This signature is clearly useless for Higgs bosons produced in gluon fusion because of the overwhelming QCD $gg \rightarrow b\bar{b}$

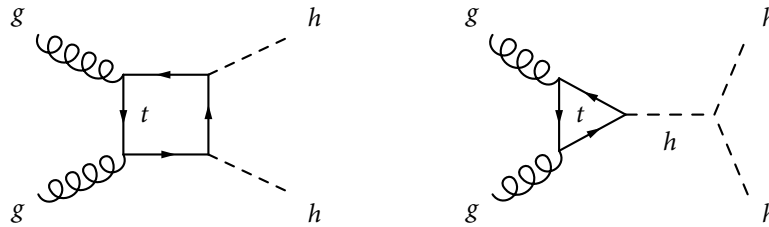


Figure 2.3: Feynman diagrams for Higgs pair production.

background. WBF and Vh production provide handles to tame these backgrounds, but the channel is still difficult. Easier to detect are $\tau^+\tau^-$ pairs with a branching ratio of 6.3%. Their semi-leptonic and purely leptonic decays involve neutrinos. But if the taus are boosted enough and not exactly back-to-back, the neutrino momentum can be reconstructed for instance using a collinear approximation [30].

The decays through W^+W^- or ZZ pairs into four-lepton final states are particularly important due to their clean signatures and because they provide access to Higgs-gauge couplings. Since the Higgs mass is below the W^+W^- and ZZ thresholds, one of the vectors has to be off-shell.² The channel $h \rightarrow W^+W^- \rightarrow (\ell^+\nu)(\ell^-\bar{\nu})$ with $\ell = e, \mu$ has a respectable branching fraction of 1.1% [4], but comes with two neutrinos in the final state. Still, it is one of the most important channels to measure WBF Higgs production. The decay $h \rightarrow ZZ \rightarrow 4\ell$ with $\ell = e, \mu$ provides an extremely clean signal. Despite its small branching ratio of $1.3 \cdot 10^{-4}$, it was one of the most important channels for the discovery of the Higgs boson [13, 33]. From a post-discovery perspective, its four leptons provide a rich spectrum of angular correlations and other observables that allow us to measure the Higgs behaviour in detail. We discuss this feature in more depth in Chapter 4.

Finally, the small tree-level couplings of the Higgs to light particles mean that the loop-induced decay into photon pairs can compete with them. The dominant contribution comes from a W loop, which interferes destructively with the top loop, resulting in a branching ratio of 0.23% [4]. The ATLAS and CMS detectors are designed to reconstruct photons well, in fact with exactly this Higgs decay channel in mind. Together with $h \rightarrow 4\ell$ it constitutes the most important channel for the discovery.

2.1.3 How I Learned to Stop Worrying and Love the Higgs

There are several facets of the Higgs boson that make it special. From an experimental point of view, the properties of this shiny new thing in particle physics are still relatively unknown. Its couplings to vector bosons and heavy fermions are constrained at the $\mathcal{O}(10\%)$ level, while for the couplings to light fermions, invisible decays, and the total decay width of the Higgs there are only weak upper bounds [22, 33]. Many of these limits also rely on specific model assumptions. The top Yukawa coupling, for instance, is most strongly constrained from the total Higgs production rate, but only under the assumption that there no new physics plays a role in the gluon-fusion loop. The total Higgs width can be constrained indirectly from the contribution of $gg \rightarrow h \rightarrow ZZ \rightarrow 4\ell$ in the off-shell Higgs region, but again relying on strong model assumptions. All in all, the Higgs is still the least well measured elementary particle (in some sense with the exception of neutrinos), leaving plenty of room for physics beyond the Standard Model.

From a theory perspective, there are several reasons to suspect manifestations of new physics in

²This also means that the branching ratios for $h \rightarrow ZZ$ and $h \rightarrow WW$ are not really well-defined. What is often quoted is in fact a term like $\text{BR}(h \rightarrow 4\ell)/(\text{BR}(Z \rightarrow \ell^+\ell^-))^2$.

the Higgs sector. Starting with a rather general argument, the Higgs doublet is the key component of electroweak symmetry breaking (EWSB), which is often seen as the very core of the SM construction. A test of the Higgs properties therefore provides a test of the fundamental structure of Nature.

The Higgs boson is the only fundamental scalar discovered so far. This is interesting in its own right, but also leads to the famous electroweak **hierarchy problem**: in the absence of any protective symmetry, the mass of a scalar field should receive quantum corrections of the order of the largest scale in the theory. If the SM is valid all the way to the Planck scale, severe fine-tuning between the bare parameter and these quantum corrections is necessary to keep the electroweak mass scale at the observed value. Note that this argument interchangeably applies to the mass parameter of the Higgs doublet μ^2 , the physical Higgs mass m_h , or the electroweak VEV v . Since the strength of the weak force is suppressed by powers of $m_W \sim v$, and the gravitational force by the Planck scale, the hierarchy problem is often phrased in terms of the surprising weakness of gravity compared to the weak force. This naturalness problem is of a purely aesthetic nature, but similar aesthetic problems have in the past led to new insights. Many models have been proposed to solve the hierarchy by introducing a new symmetry that protects the Higgs mass against quantum corrections.³⁴ Famous examples are supersymmetry, composite Higgs models in which the Higgs is the pseudo-Goldstone boson of some broken symmetry, conformal symmetries [56], or extra dimensions. To reduce tuning to an acceptable level, this new physics should reside at energy scales not too far from the electroweak scale. These models usually modify the Higgs sector in a way that translates into Higgs couplings different from their SM values.

Citation needed for criticism of anthropic principle.

Another hierarchy unexplained in the SM is the large difference between the **fermion masses**. There are more than five orders of magnitude between the top and the electron mass, and neutrinos are even lighter. Since the fermion masses are generated by the Yukawa couplings of the Higgs doublet, models that explain the fermion masses usually also shift the Higgs-fermion coupling patterns.

The question of **vacuum stability** is still being discussed. The renormalisation group (RG) allows us to link this question to the running of the quartic coupling λ to higher energies. Current results [57] indicate that indeed the quartic coupling becomes negative at large energies, leading to

³An entirely different and somewhat metaphysical argument is based on the (weak) anthropic principle that observations of the universe are conditional upon its laws of physics allowing conscious life [45, 46]. First, this explanation requires some mechanism that generates many different vacua with different values of the physics parameters, including the Higgs mass. Most of these vacua have “natural” parameters in which the weak and gravitational scales are comparable. String theory is hypothesised to provide such a sampling mechanism (the “multiverse”). Second, there has to be a reason why larger (and thus more abundant) values of the weak scale would not allow any type of intelligent life to form and make observations. This question is difficult to answer, and the jury is still out [47–53]. Given the speculative nature of the two questions, anthropic reasoning is being criticised as unverifiable or as based on arguments from lack of imagination.

⁴Finally, other models such as the relaxion [54] or Naturalness [55] modify the cosmological evolution such that a small Higgs mass is generated dynamically during inflation or reheating.

a second vacuum with lower energy at much larger values of ϕ . Fortunately for us, the tunnelling probability is very small, and in the SM “our” vacuum with $v \approx 246$ GeV seems to be metastable with a lifetime longer than the age of the universe. While this indicates there is no pressing need for physics below the Planck scale to save the electroweak vacuum from a horrible fate, this result crucially depends on the measured top and Higgs masses, higher-order corrections to the beta functions, and higher-dimensional operators stemming from ultra-violet (UV) physics [58].

In addition to these theoretical and to some degree aesthetic arguments, there is solid experimental evidence for physics beyond the SM that might be linked to the Higgs sector. First, the nature of **dark matter** (DM) [59] is still unclear. It is experimentally established that this form of matter is electrically neutral, stable over cosmological timescales, clumps (i. e. is now non-relativistic), and makes up roughly a fourth of the energy density of the universe. In many models DM is in thermal equilibrium with ordinary matter in the early universe. Interestingly, the observed dark matter density is in good agreement with electroweak-scale masses and weak couplings. This “WIMP miracle” is one main reason behind the popularity of weakly interacting massive particles (WIMPs) as DM candidates. In this scenario, good candidates for the mediator between dark matter and the SM are the Higgs boson or other scalars in an extended Higgs sector. Such “Higgs portal” scenarios often predict signatures in Higgs physics such as modified couplings or invisible Higgs decays.

Another mystery is the **matter-antimatter asymmetry** of the universe. Assuming that the cosmos was initially perfectly symmetric, the observed excess of matter can be generated dynamically if the three Sakharov conditions are satisfied: there have to be processes with baryon-number violation as well as C and CP violation, which take place out of thermal equilibrium. In the SM, these effects are too small to account for the observed asymmetry. Models that accommodate larger effects often affect the Higgs sector. In particular, extended Higgs sectors allow for electroweak symmetry breaking to be a strong first-order phase transition, providing the required out-of-equilibrium dynamics. Again, such scenarios predict signatures in Higgs measurements.

Finally, the Higgs could play another role in the cosmological evolution of the universe. The origin of the large-scale structure of the cosmos, the surprising isotropy of the cosmic microwave background (CMB), and the flatness of the Universe are all explained by an epoch of exponential expansion of space in the early universe called **inflation**. This process is often thought to be caused by a scalar field, the inflaton, slowly rolling down a potential of a certain shape. In principle the Higgs can be the inflaton, though this scenario of Higgs inflation requires unnaturally large couplings between the Higgs and the Ricci scalar, and requires a UV completion.

The null results of the LHC searches for new particles have led to some disappointment among particle physicists. But with the discovery of the Higgs boson, the LHC might not only have completed the SM particle zoo, but rather opened the door to the unknown. The Higgs boson is not just another SM particle. Some of the big open questions of fundamental physics are deeply rooted in the Higgs sector, and many other ideas can at least be linked to the Higgs sector under

some assumptions. On the other hand, the current experimental precision leaves quite some room for signatures of new physics in Higgs observables. A precise determination of the Higgs properties might be one of the most exciting measurements at the LHC, and hopefully improve our understanding of Nature significantly. Hopefully, the Higgs boson is not just the last puzzle piece of the Standard Model, but the first sign of what lies beyond.

2.2 The effective field theory idea

This plethora of possible BSM models means that a model-independent universal theory framework is invaluable for TeV signatures of new physics. We consider such a model based on the effective field theory (EFT) paradigm. Before discussing the specific realisation for Higgs physics in the next section, here we introduce the general EFT idea.

Effective field theories are a powerful tool that play a role in many, if not all, areas of physics. Whenever phenomena are spread out over different energy or length scales, an effective description can be valuable, either to simplify calculations, or to actually allow model-independent statements that would be impossible without such a framework.

2.2.1 Different physics at different scales

Our world behaves very differently depending on which energy and length scales we look at. At extremely high energies (or short distances), Nature might be described by a quantum theory of gravity. At energies of a few hundred GeV, the Standard Model is (maybe disappointingly) in agreement with all measurements. Going to lower energies (or larger distances), we do not have to worry about the Higgs or W bosons anymore: electromagnetic interactions are described by QED, weak interactions by Fermi theory, strong physics by QCD. Below a GeV, quarks and gluons are replaced by pions and nucleons as the relevant degrees of freedom. Then by nuclei, atoms, molecules. At this point most physicists give up and let chemists (and ultimately biologists and sociologists) analyse the systems.

The important point here is that the observables at one scale are not directly sensitive to the physics at significantly different scales. This is nothing new: for molecules to stick together, the details of the Higgs sector are not relevant, just as we can calculate how an apple falls from a tree without knowing about quantum gravity. To do physics at one scale, we do not have to (and often cannot) take into account the physics from all other scales. Instead, we isolate only those features that play a role at the scale of interest.

An effective field theory is a physics model that includes all effects relevant at a given scale, but not those that only play a role at significantly different scales. In particular, EFTs ignore spatial substructures much smaller than the lengths of interest, or effects at much higher energies than the energy scale considered.

Check that all arguments of http://indico.cern.ch/event/477407/contributions/2200060/attachments/1369793/2076900/AM_HiggsCouplings2016.pdf are in this section. In particular slide 5.

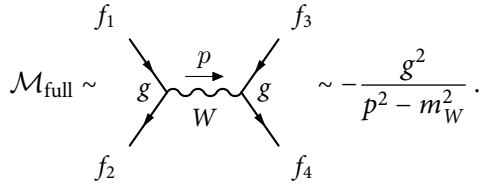
We often use examples with one full or underlying theory and one effective theory. For simplicity, we pretend that the full theory describes physics correctly at all scales. The EFT is a simpler model than the full theory and neglects some phenomena (such as heavy particles) at an energy scale Λ . However, it correctly describes the physics as long as the observables probe energy scales

$$E \ll \Lambda, \quad (2.7)$$

within some finite precision. This **scale hierarchy** between the energy of interest and the scale of high-energy physics not included in the EFT is the basic requirement for the EFT idea. A validity range (2.7) is a fundamental property of each EFT.

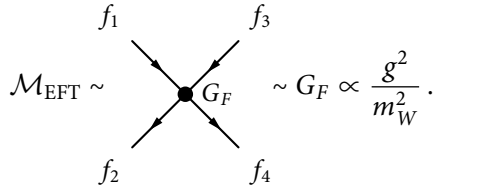
Fermi theory

The textbook example for an EFT is Fermi theory, which describes the charged current interactions between quarks (or hadrons), leptons and neutrinos at low energies. The underlying model here is the SM, in which this weak interaction is mediated by the exchange of virtual W bosons with mass m_W and coupling constant g :



$$\mathcal{M}_{\text{full}} \sim \sim -\frac{g^2}{p^2 - m_W^2}. \quad (2.8)$$

In Fermi theory, there are no W bosons, just a direct interaction between four fermions with coupling constant $G_F \propto g^2/m_W^2$:



$$\mathcal{M}_{\text{EFT}} \sim \sim G_F \propto \frac{g^2}{m_W^2}. \quad (2.9)$$

So the EFT turns the W propagator into a contact interaction between the fermions, shrinking the distance bridged by the virtual W to zero. Clearly, the two amplitudes agree as long as the momentum transfer through the vertex is small, $E^2 = p^2 \ll \Lambda^2 = m_W^2$.

One process described by this interaction is muon decay. Its typical energy scale $E \approx m_\mu$ is well separated from $\Lambda = m_W$, and Fermi theory describes the process quite accurately. The relative **EFT error**, i. e. the mistake we make when calculating an observable with the EFT rather than with the full model, should be of order $\Delta_{\text{EFT}} = \Gamma_{\text{EFT}}/\Gamma_{\text{full}} \sim E^2/\Lambda^2 \sim m_\mu^2/m_W^2 \approx 10^{-6}$.

In proton collisions at the LHC the same interaction takes place, but at potentially much larger momentum transfer $E \lesssim 13$ TeV. The EFT error increases with E . For $E \gtrsim m_W$, the full model allows on-shell W production, a feature entirely missing in the EFT. Here the two descriptions obviously diverge and Fermi theory is no longer a valid approximation of the weak interaction.

Down and up the theory ladder

In reality there are of course more than two theories, and the notion of underlying and effective model becomes relative. The SM itself is not valid up to arbitrary large energies: it does not explain dark matter, the matter-antimatter asymmetry, or gravity. It is probably also internally inconsistent since at some very large energy the quartic coupling λ and the coupling constant g' hit Landau poles. So the SM is an effective theory with validity range $E \ll \Lambda \leq M_{Pl}$ and has to be replaced by some other description at larger energies. On the other hand, going to energies lower than a few GeV, the relevant physics changes again and we should switch to a new effective theory. In this way, all theories can be thought of as a series of EFTs, where the model valid at one scale is the underlying model for the effective theory at the next lower scale.

If you think you know a theory that describes our world at sufficiently large energies, then in principle there is no need to use effective theories: you can calculate every single observable in your full model (at least if the full model is perturbative at these energies or other approximations such as lattice calculations are available). This however makes hard calculations necessary even for the simplest low-energy processes. One can save a lot of computational effort and focus on the relevant physics by dividing the phase space into regions with different appropriate effective descriptions.

Starting from a high energy scale where the parameters of the fundamental theory are defined, these parameters are run to lower energies until the physics changes substantially. At this **matching scale** an effective theory is constructed from the full model, and its coefficients are determined from, or matched to, the underlying model. Then the coefficients of this EFT are run down to the next matching scale, where a new EFT is defined and its parameters are calculated, and so on. This is the **top-down** view of EFTs. For instance, we can start from the SM and construct Fermi theory as a simpler model valid at low energies. While we can certainly use the SM to calculate the muon lifetime, it is not necessary, and a calculation in Fermi theory is quite accurate and simpler.

But often we do not know the underlying theory. As mentioned above, there has to be physics beyond the SM, and there is still hope it will appear around a few TeV. If we want to parametrise the effects of such new physics on electroweak-scale observables, we do not know how the full model looks like. But even without knowing the underlying model, we can still construct an effective field theory based on a few very general assumptions. We go through these ingredients in the next section. For this **bottom-up** approach, an effective theory is not only useful, but actually the only way we can discuss new physics without choosing a particular model of BSM physics.

High-energy physics can be seen as the field of working ourselves up a chain of EFTs to ever higher energies. But how does this chain end? Does it end at all? Even if we one day find a consistent theory that can explain all observations to date, how would we check if it indeed describes Nature up to arbitrarily high energies? Understanding all theories as effective, these questions do not matter. The EFT framework provides us with the tools to do physics without having to worry about the far ultraviolet.

2.2.2 EFT construction and the bottom-up approach

EFTs are especially useful in the framework of quantum field theory (QFT). Before showing how to construct the effective operators of such a theory in a bottom-up approach, let us recapitulate how QFTs are organised.

Reminder: operators and power counting

The basic object describing perturbative QFTs in $d = 4$ flat space-time dimensions is the action

$$S = \int d^4x \mathcal{L}(x). \quad (2.10)$$

The Lagrangian $\mathcal{L}(x)$ is a sum of couplings times operators, where the operators are combinations of fields and derivatives evaluated at one point x . These are either kinematic terms, mass terms or represent interactions between three or more fields. For instance, the Lagrangian

$$\mathcal{L} = i\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} - m_i \bar{\psi}_i \psi_i + m_V^2 V_\mu V^\mu - g \bar{\psi}_i \gamma_\mu \psi_i V^\mu \quad (2.11)$$

with implicit sum over i describes fermions ψ_i , a vector boson V_μ of mass m_V , and an interaction between them governed by a coupling g .⁵

A key property of each coupling or operator is its **canonical dimension** or mass dimension. In simple terms this can be formulated as the following question: if you assign a value to a quantity, which power of a mass unit such as GeV would this value carry? Since we work in units with $\hbar = c = 1$, length and distance dimensions are just the inverse of mass dimensions. We denote the mass dimension of any object with squared brackets, where $[\mathcal{O}] = D$ means that \mathcal{O} is of dimension mass^D , or mass dimension D .

In QFT, the action can appear in exponentials such as e^{iS} , so it must be dimensionless: $[S] = 0$. The space-time integral in Equation (2.10) then implies $[\mathcal{L}] = d = 4$, so every term in the Lagrangian has to be of mass dimension 4. Applying this to the kinetic terms, we can calculate the mass dimension of all fields. This then allows us to calculate the mass dimension or canonical dimension of operators and couplings in the theory.

In the example in Equation (2.11), the kinetic term for the fermions contains one space-time derivative, $[\partial] = 1$. To get $[\bar{\psi}\partial\psi] = 4$, the fermion fields must have dimension $[\psi_i] = 3/2$. Similarly, the field strength $V_{\mu\nu}$ contains a derivative, so we end up with $[V_\mu] = 1$. With these numbers we can check the other operators. In addition to the expected $[m] = [m_V] = 1$, we find $[\bar{\psi}\psi V^\mu] = 4$ or $[g] = 0$.

The canonical dimension of an operator has two important consequences. First, the renormalisation group flow of a theory, i. e. the running of the couplings between different energy

⁵Massive vector bosons have issues with renormalisability and unitarity, which can be solved by generating the mass in a Higgs mechanism at a higher scale, but this is irrelevant for the discussion here.

scales, largely depends on the mass dimensions of the operators. Operators with mass dimension $D < d$ (“relevant” operators) receive substantial quantum corrections when going from high to low energies. This is a key argument for many fine-tuning problems such as the hierarchy problem or the cosmological constant problem. On the other hand, operators with $D > d$ (“irrelevant” ones) are typically suppressed when going to lower energies. Operators with $D = d$ are called “marginal”.

The second consequence of the mass dimension affects the renormalisability of a theory. Theories with operators with $D > d$ are **non-renormalisable**:⁶ particles in loops with energies $E \rightarrow \infty$ lead to infinities in observables, and they are too many to be hidden in a renormalisation of the parameters.

Effective operators

From now on we only consider EFTs realised as a local QFT in 4 space-time dimensions, an approach that has proven very successful in high-energy physics so far. EFTs are then defined as a sum of operators \mathcal{O}_i , each with a specific canonical dimension D_i . We can split the coupling in front of each operator into a dimensionless constant, the **Wilson coefficient** f_i , and some powers of a mass scale, for which we use the scale of heavy physics Λ :

$$\mathcal{L}_{\text{EFT}} = (\text{kinetic and mass terms}) + \sum_i \frac{f_i}{\Lambda^{D_i-d}} \mathcal{O}_i. \quad (2.12)$$

Why do we force Λ to appear in front of the operators like this? If we do not know anything about the underlying model at scale Λ , our best guess (which can be motivated with arguments based on the renormalisation group flow) is that it consists of dimensionless couplings $g \sim \mathcal{O}(1)$ and mass scales $M \sim \mathcal{O}(\Lambda)$. Indirect effects mediated by this high-energy physics should therefore be proportional to a combination of these factors, as given in Equation (2.12) with couplings $f_i \sim \mathcal{O}(1)$. This is certainly true in Fermi theory, where the effective coupling G_F is suppressed by $\Lambda^2 = m_W^2$.

Ingredients

How the operators \mathcal{O}_i look like might be clear in a top-down situation where we know the underlying theory. In a bottom-up approach, however, we need a recipe to construct a list of operators in a model-independent way. It turns out that this is surprisingly straightforward, and the list of operators we need to include in the EFT is defined by three ingredients: the particle content, the symmetries, and a counting scheme that decides which operators are relevant at the scale of interest. We go through them one by one.

⁶The opposite is not true: some theories contain only operators with $D \leq d$, but are still not renormalisable.

1. **Particle content:** one has to define the fields that are the dynamical degrees of freedoms in the EFT, i. e. that can form either external legs or internal propagators in Feynman diagrams. At least all particles with masses $m \ll \Lambda$ should be included. The operators are then combinations of these fields and derivatives.
2. **Symmetries:** some symmetry properties of the world have been measured with high precision, and we can expect that a violation of these symmetries has to be extremely small or happens at very high energies. These can be gauge symmetries (such as the $SU(3) \times SU(2) \times U(1)$ of the SM), space-time symmetries (such as Lorentz symmetry), or other global symmetries (such as flavour symmetries). Requiring that the effective operators do not violate these symmetries is well motivated and can reduce the complexity of the theory significantly.
3. **Counting scheme:** with a set of particles and some symmetry requirements we can construct an infinite tower of different operators. We therefore need some rule to decide which of the operators we can neglect. Here the dimensionality of the operators becomes important. As argued above, we expect an operator with canonical dimension $D > d$ to be suppressed by a factor of roughly $1/\Lambda^{D-d}$. Operators of higher mass dimension are therefore more strongly suppressed. Setting a maximal operator dimension is thus a way of limiting the EFT to a finite number of operators that should include the leading effects at energies $E \ll \Lambda$.

One property that is often required of theories is missing in this list: an EFT (with its intrinsic UV cutoff Λ) does not have to be renormalisable in the traditional sense. In fact, most EFTs include operators with mass dimension $D > d$ and are thus non-renormalisable. However, EFTs are still renormalisable order by order in the counting scheme, and loop effects can be calculated without any fundamental issues.

Basis choices

Usually not all operators that can be constructed in this way are independent. This can be seen from a field redefinition of the form

$$\phi(x) \rightarrow \phi'(x) = \phi(x) + \varepsilon f(x) \quad (2.13)$$

where ε is some small parameter and $f(x)$ can contain any combination of fields evaluated at x . The action in terms of the new field is then (after integration by parts)

$$\int d^4x \mathcal{L}[\phi] \rightarrow \int d^4x \mathcal{L}[\phi'] = \int d^4x \left(\mathcal{L}[\phi] + \varepsilon \left[\frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right] f + \mathcal{O}(\varepsilon^2) \right). \quad (2.14)$$

Such a transformation does not change the physics, i. e. the S-matrix elements [60–63], so we can equivalently use the new action instead of the original one. In this way, each equation of motion

provides us with a degree of freedom to swap operators for a combination of other operators. Similarly, Fierz identities and integration by parts can be used to manipulate the form of operators. Together these tools reduce the number of operators and coefficients necessary in an EFT basis, and lead to some freedom to choose which operators to work with.

Why is the sky blue?

Following Reference [32], we demonstrate this bottom-up approach with a simple question: why is the sky blue? In other words, why is blue light coming from the sun scattered more strongly by particles in the atmosphere than red light? A full derivation of this takes some time and requires knowledge of the underlying electrodynamic interactions. Instead, we write down an effective field theory for this process of Rayleigh scattering. The only thing we have to know are the basic scales of the process: photons with energy E_γ scatter off basically static nuclei characterised by an excitation energy ΔE , mass M and radius a_0 . Looking at these numbers, we see that these scales are clearly separated:

$$E_\gamma \ll \Delta E, a_0^{-1} \ll M. \quad (2.15)$$

This is good news, since such a scale hierarchy is the basic requirement for an EFT. We are interested in elastic scattering, so we set the cutoff of the EFT as⁷

$$\Lambda \sim \Delta E, a_0^{-1}. \quad (2.16)$$

With this we can put together the building blocks for our EFT as discussed above:

1. As fields we need photons and atoms, where we can approximate the latter as infinitely heavy.
2. The relevant symmetries are the $U(1)_{\text{em}}$ and Lorentz invariance. At these energies atoms can also not be created or destroyed, which one can see as another symmetry requirement on the effective Lagrangian.
3. We include the lowest-dimensional operators that describe photon-atom scattering.

The kinetic part of such an EFT reads

$$\mathcal{L}_{\text{kin}} = \phi_v^\dagger i v^\alpha \partial_\alpha \phi_v - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (2.17)$$

where ϕ_v is the field operator representing an infinitely heavy atom at constant velocity v , and $F_{\mu\nu}$ is the photon field strength tensor. Boosting into the atom's rest frame, $v = (1, 0, 0, 0)$ and the first term becomes the Lagrangian of the Schrödinger equation.

⁷In reality there are two orders of magnitude between ΔE and a_0^{-1} , but this does not affect the line of argument at all and we choose to ignore this fact.

The usual power counting based on $[\mathcal{L}] = 4$ gives the mass dimensions

$$[\partial] = 1, \quad [v] = 0, \quad [\phi] = \frac{3}{2} \quad \text{and} \quad [F_{\mu\nu}] = 2. \quad (2.18)$$

The interaction operators must be Lorentz-invariant combinations of $\phi^\dagger \phi$, $F_{\mu\nu}$, v_μ , and ∂_μ . Note that operators directly involving A_μ instead of $F_{\mu\nu}$ are forbidden by gauge invariance, and single instances of ϕ correspond to the creation or annihilation of atoms, which is not possible at these energies. The first such operators appear at mass dimension 7:

$$\mathcal{L}_{\text{int}} = \frac{f_1}{\Lambda^3} \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + \frac{f_2}{\Lambda^3} \phi_v^\dagger \phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} + \mathcal{O}(1/\Lambda^4), \quad (2.19)$$

with Wilson coefficients f_1 and f_2 . These two operators should capture the dominant effects of Rayleigh scattering at energies $E_\gamma \ll \Lambda$.

The scattering amplitude of light off the atmospheric atoms should therefore scale as $\mathcal{M} \sim 1/\Lambda^3$, which means that the cross section scales with $\sigma \sim 1/\Lambda^6$. Since the cross section has the dimension of an area, $[\sigma] = -2$, and the only other mass scale in this low-energy process is the photon energy E_γ , we know that the effective cross section must be proportional to

$$\sigma \propto \frac{E_\gamma^4}{\Lambda^6} (1 + \mathcal{O}(E_\gamma/\Lambda)). \quad (2.20)$$

In other words, blue light is much more strongly scattered than red light. Our effective theory, built just from a few simple assumptions, explains the colour of the sky!

Finally, we should check the validity range of our EFT. We expect it to work as long as

$$E_\gamma \ll \Lambda \sim \Delta E \sim \mathcal{O}(\text{eV}), \quad (2.21)$$

equivalent to wavelengths above $\mathcal{O}(100 \text{ nm})$. Our approximation is probably safe for visible light. In the near ultraviolet we expect deviations from the E_γ^4 proportionality and the EFT to lose its validity.

2.2.3 Top-down approach and matching

In the top-down approach to effective field theories, we start from a known model of UV physics and calculate the corresponding effective operators and Wilson coefficients in the EFT. The defining criterion of this **matching procedure** is that at low energies the effective and underlying descriptions agree, at least up to a given order in the loop expansion (e. g. in α_s) and up to a given order in the EFT expansion in $1/\Lambda$.

This can be achieved either by functional methods or with Feynman diagrams. Here we sketch the conceptual foundation involving functional methods, before arriving at a simple diagrammatic method. Note that the matching cannot be reversed: one cannot uniquely reconstruct a full theory only based on the EFT. Details of the matching procedure play a crucial role in Chapter 3.

The effective action

The central object that allows us to systematically analyse the low-energy effects of heavy physics is the effective action S_{eff} . Following Reference [64, 65], we now outline its calculation at the one-loop level. Note that this is just a conceptual sketch and not mathematically rigorous, and that we omit higher-order terms irrelevant for this thesis as well as certain cases of mixed loops with light and heavy particles [66]. For a more thorough derivation see the quantum field theory textbook of your choice.

For simplicity, let us assume that our theory $S[\phi, \Phi]$ consists of light particles ϕ and a heavy scalar Φ that should not be part of the effective theory as dynamical degree of freedom. The effective action is calculated by **integrating out** the heavy particles from the partition function,

$$e^{iS_{\text{eff}}[\phi]} = \int \mathcal{D}\Phi e^{iS[\phi, \Phi]}. \quad (2.22)$$

While the path integral over the heavy fields is computed, the light fields are kept fixed as “background fields”.

The effective action can be calculated with a saddle-point approximation. For this we expand Φ around its classical value Φ_c :

$$\Phi(x) = \Phi_c(x) + \eta(x). \quad (2.23)$$

Φ_c is defined by the classical equation of motion

$$\left. \frac{\delta S[\phi, \Phi]}{\delta \Phi} \right|_{\Phi=\Phi_c} = 0, \quad (2.24)$$

so expanding the action around this extremum leads to

$$S[\phi, \Phi_c + \eta] = S[\phi, \Phi_c] + \frac{1}{2} \left. \frac{\delta^2 S[\phi, \Phi]}{\delta \Phi^2} \right|_{\Phi=\Phi_c} \eta^2 + \mathcal{O}(\eta^3). \quad (2.25)$$

Plugging this into Equation (2.22), we find

$$e^{iS_{\text{eff}}[\phi]} \approx e^{iS[\phi, \Phi_c]} \int \mathcal{D}\eta \exp \left(\frac{1}{2} \left. \frac{\delta^2 S[\phi, \Phi]}{\delta \Phi^2} \right|_{\Phi=\Phi_c} \eta^2 \right). \quad (2.26)$$

The last term is a Gaussian integral with a known solution,

$$e^{iS_{\text{eff}}[\phi]} \approx e^{iS[\phi, \Phi_c]} \left[\det \left(- \left. \frac{\delta^2 S}{\delta \Phi^2} \right|_{\Phi=\Phi_c} \right) \right]^{-1/2} \quad (2.27)$$

and finally

$$S_{\text{eff}}[\phi] \approx S[\phi, \Phi_c] + \frac{i}{2} \text{tr} \log \left(- \frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_c} \right), \quad (2.28)$$

where the functional trace is defined as an integral over momentum space k together with a sum over internal states i such as spin or flavour,

$$\text{tr } x \equiv \sum_i \int \frac{d^4 k}{(2\pi)^4} \langle k, i | x | k, i \rangle. \quad (2.29)$$

This result can be directly evaluated with functional methods. The first term in Equation (2.28) can be easily calculated by solving the classical equations of motion in Equation (2.24). Computing the functional trace is more involved, but can be simplified with a procedure called **covariant derivative expansion** [64, 67, 68]. Universal results that can be adapted to many scenarios are available in the literature [65, 66, 69].

The effective action is in general non-local, visible as (covariant) derivatives D appearing in the denominator (formally defined as Green's functions). We expand these terms schematically as

$$\phi^\dagger \frac{1}{D^2 - M^2} \phi = -\phi^\dagger \frac{1}{M^2} \left[1 + \frac{D^2}{M^2} \right] \phi + \mathcal{O}(1/M^6), \quad (2.30)$$

so that only a rest term of higher order in $1/\Lambda = 1/M$ remains non-local [66]. In a last step, we truncate the resulting tower of operators at some order in our counting scheme, in our case in the expansion in $1/\Lambda$. The resulting effective theory consists of a finite set of local operators up to some order in a counting scheme, compatible with our definition of effective theories in the previous section. Unlike in the bottom-up approach, not all operators have to appear, and we can calculate the Wilson coefficients based on the underlying theory.

Scalar example

As a simple example consider a theory of two real scalar fields. The light field ϕ has mass m , the heavy field Φ with mass M is being integrated out. The underlying theory is given by

$$S[\phi, \Phi] = \int d^4 x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{M^2}{2} \Phi^2 - \frac{\lambda_0}{4!} \phi^4 - \frac{\lambda_2}{4} \phi^2 \Phi^2 - \frac{\lambda_4}{4!} \Phi^4 \right]. \quad (2.31)$$

Odd interactions and a mixing term $\phi\Phi$ are forbidden with suitable \mathbb{Z}_2 symmetries.

The classical equation of motion for Φ is

$$\left(\partial^2 + M^2 + \frac{\lambda_2}{2} \phi^2 + \frac{\lambda_4}{3!} \Phi_c^2 \right) \Phi_c = 0 \quad (2.32)$$

with the trivial solution $\Phi_c = 0$.

The first term in the effective action then just gives back ϕ^4 theory for the light field, without any new effective interactions:

$$S[\phi, \Phi_c] = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4 \right]. \quad (2.33)$$

The second term is

$$\begin{aligned} \frac{i}{2} \text{tr} \log \left(- \frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_c} \right) &= \frac{i}{2} \text{tr} \log \left(\partial^2 + M^2 + \frac{\lambda_2}{2} \phi^2 \right) \\ &= \frac{i}{2} \text{tr} \log (\partial^2 + M^2) + \frac{i}{2} \text{tr} \log \left(1 + \frac{\lambda_2}{2} \frac{1}{\partial^2 + M^2 + i\epsilon} \phi^2 \right), \end{aligned} \quad (2.34)$$

where derivatives in the denominator are defined as Green's functions. Since $\text{tr} \log (\partial^2 + M^2)$ is just a constant that can be calculated for instance in dimensional regularisation, the first part does not give us any higher-dimensional operators of the light fields ϕ . Expanding the logarithm in the second term, we find

$$S_{\text{eff}} \supset \frac{i\lambda_2}{4} \text{tr} \frac{1}{\partial^2 + M^2 - i\epsilon} \phi^2 - \frac{i\lambda_2^2}{8} \text{tr} \left(\frac{1}{\partial^2 + M^2 - i\epsilon} \phi^2 \right)^2 + \frac{i\lambda_2^3}{12} \text{tr} \left(\frac{1}{\partial^2 + M^2 - i\epsilon} \phi^2 \right)^3 + \mathcal{O}(\lambda_2^4). \quad (2.35)$$

The first of these terms renormalises the ϕ mass term, and the second contributes to the ϕ^4 interaction. This is important for RG running, but does not create the kind of new effective interactions we are interested in here. We instead focus on the last term and evaluate the functional trace:

$$\begin{aligned} S_{\text{eff}} &\supset -\frac{i\lambda_2^3}{12} \int \frac{d^4k}{(2\pi)^4} \langle k | \left(\frac{1}{\partial^2 + M^2 - i\epsilon} \phi^2 \right)^3 | k \rangle \\ &\supset -\frac{i\lambda_2^3}{12} \int d^4x \int d^4y \int d^4z \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \langle k | \frac{1}{\partial^2 + M^2 - i\epsilon} | x \rangle \langle x | \phi^2 | p \rangle \\ &\quad \times \langle p | \frac{1}{\partial^2 + M^2 - i\epsilon} | y \rangle \langle y | \phi^2 | q \rangle \langle q | \frac{1}{\partial^2 + M^2 - i\epsilon} | z \rangle \langle z | \phi^2 | k \rangle. \end{aligned} \quad (2.36)$$

Here we have used the definition of the functional trace in Equation (2.29) and inserted unity, $1 = \int d^4x |x\rangle \langle x| = \int \frac{d^4p}{(2\pi)^4} |p\rangle \langle p|$. $|k\rangle$, $|p\rangle$, and $|q\rangle$ are eigenstates of the derivative operator

∂ , i. e. $\langle k | i\partial_\mu = \langle k | k_\mu$, while $|x\rangle$, $|y\rangle$, and $|z\rangle$ denote the eigenstates of local operators, $\langle x | \phi^2 = \langle x | \phi^2(x)$. Their inner product is $\langle x | k \rangle = e^{-ikx}$. Using these properties and shifting the integration variables, we get

$$\begin{aligned} S_{\text{eff}} &\supset -\frac{i\lambda_2^3}{12} \int d^4x \int d^4y \int d^4z \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{-k^2 + M^2 - i\epsilon} e^{ikx} \phi(x)^2 e^{-ipx} \\ &\quad \times \frac{1}{-p^2 + M^2 - i\epsilon} e^{ipy} \phi(y)^2 e^{-iqy} \frac{1}{-q^2 + M^2 - i\epsilon} e^{iqz} \phi(z)^2 e^{-ikz} \\ &\supset \frac{i\lambda_2^3}{12} \int d^4x \int d^4y \int d^4z \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \phi(x)^2 \phi(y)^2 \phi(z)^2 \\ &\quad \times \frac{e^{ip(z-x)} e^{iq(z-y)}}{(k^2 - M^2 + i\epsilon) ((k+p)^2 - M^2 + i\epsilon) ((k+p+q)^2 - M^2 + i\epsilon)}. \end{aligned} \quad (2.37)$$

We can now perform the integral over the loop momentum k with Feynman parameters:

$$\begin{aligned} &\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2 + i\epsilon) ((k+p)^2 - M^2 + i\epsilon) ((k+p+q)^2 - M^2 + i\epsilon)} \\ &= 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int \frac{d^4k}{(2\pi)^4} \left[x_1(k^2 - M^2 + i\epsilon) + x_2((k+p)^2 - M^2 + i\epsilon) \right. \\ &\quad \left. + (1-x_1-x_2)((k+p+q)^2 - M^2 + i\epsilon) \right]^{-3} \\ &= 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k+a)^2 - B + i\epsilon]^3} \end{aligned} \quad (2.38)$$

with $a = (1-x_1)p + (1-x_1-x_2)q$ and $B = M^2 - (1-x_1-x_2)(p+q)^2 - x_2p^2 + a^2$. Shifting the loop momentum as $k \rightarrow k+a$, we finally arrive at

$$T_3(p, q) = 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - B + i\epsilon]^3}. \quad (2.39)$$

To evaluate this, we first Wick-rotate $k^0 = ik_E^0$. Formally, this means shifting the integration path in the complex plane of k^0 from along the real axis to along the imaginary axis. The Cauchy theorem assures that this does not change the value of the integral as long as we chose the contour such that the poles are not caught between the two contours. Defining $k_E^2 = (k_E^0)^2 + \mathbf{k}^2 = -k^2$, we find

$$I_{0,3} \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - B + i\epsilon]^3} = i \int \frac{d^4k_E}{(2\pi)^4} \frac{1}{[-k_E^2 - B]^3}, \quad (2.40)$$

where the $+i\epsilon$ is no longer necessary. With $\bar{k} = |k_E|$ we can finally calculate the integral:

$$I_{0,3} = \frac{2\pi^2}{(2\pi)^4} \int d\bar{k} \bar{k}^{-3} \frac{1}{[\bar{k}^2 + B]^3} = \frac{-i}{32\pi^2 B}. \quad (2.41)$$

Collecting all the pieces, we have

$$S_{\text{eff}} \supset \frac{\lambda_2^3}{192\pi^2} \int d^4x \int d^4y \int d^4z \phi(x)^2 \phi(y)^2 \phi(z)^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{ip(z-x)} e^{iq(z-y)} \\ \times \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left[M^2 - (1-x_1-x_2)(p+q)^2 - x_2 p^2 + ((1-x_1)p + (1-x_1-x_2)q)^2 \right]^{-1}. \quad (2.42)$$

At first glance, this is disappointing: this effective action looks non-local and involves highly non-trivial integrals. It turns out that these can in fact be calculated and give a finite result [70, 71]. The full expression is quite ugly, but we fortunately do not need it. Instead, we expand the integrand in powers of $1/M^2$. We only calculate the leading term at $\mathcal{O}(1/M^2)$. Since it produces a finite result as well, the rest term at $\mathcal{O}(1/M^4)$ also has to be finite. Even more, we can argue that the rest term at $\mathcal{O}(1/M^4)$ has to vanish: the coefficient at a given order $1/M^k$ in this expansion is an integral without any mass scales, and has to lead to a result of mass dimension $k-2$. Only $k=2$ can give a non-zero and finite result, all higher orders therefore have to vanish. In this way, we find the much simpler result

$$S_{\text{eff}} \supset \frac{\lambda_2^3}{192\pi^2 M^2} \int d^4x \int d^4y \int d^4z \phi(x)^2 \phi(y)^2 \phi(z)^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{ip(z-x)} e^{iq(z-y)} \\ + \mathcal{O}(1/M^4) \\ \supset \frac{\lambda_2^3}{192\pi^2 M^2} \int d^4x \int d^4y \int d^4z \phi(x)^2 \phi(y)^2 \phi(z)^2 \delta(z-x) \delta(z-y) \\ \supset \frac{\lambda_2^3}{192\pi^2 M^2} \int d^4x \phi(x)^6. \quad (2.43)$$

After the expansion in $1/M$, we have finally arrived at a local theory!

What about the higher terms in Equation (2.35)? Their calculation is analogous to the one presented here and leads to operators like ϕ^8 and higher. They are suppressed at least with $1/M^4$ and are thus irrelevant for our dimension-six effective theory.

Collecting the pieces in Equation (2.33) and Equation (2.43), up to one loop and $\mathcal{O}(1/M^2)$ the full effective action is given by

$$S_{\text{eff}}[\phi] = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4 + \frac{\lambda_2^3}{12(4\pi)^2 M^2} \phi^6 \right]. \quad (2.44)$$

As expected, the dimension-six operator is suppressed by two powers of the heavy scale $\Lambda \equiv M$, and the Wilson coefficient consists of the couplings λ_2^3 times a loop factor.

Diagrammatic matching

As an alternative to this functional approach, the effective action in Equation (2.28) can be calculated in an intuitive diagrammatic way. Since the light fields are kept fixed in Equation (2.22),

Something is fishy.
Check this argument,
and check discrepancy
with Denner's habil!

the effective action is given by all connected Feynman diagrams with only ϕ as external legs and only Φ fields as internal propagators. A more rigorous derivation than the one in the previous section in fact reveals that also certain connected loop diagrams with only ϕ as external legs and both Φ and ϕ fields as internal propagators contribute if they cannot be disconnected by cutting a single internal ϕ line [66]. The first term in Equation (2.28) corresponds to all such tree-level diagrams, the second term describes one-loop pieces. Higher-loop corrections play no role in this thesis and were left out.

In practice, the effective operators and their Wilson coefficients can be calculated without the need for any functional methods as follows:

1. Start with the particle content of the full model. Choose Λ and divide the particles of the full model into light and heavy fields. Light fields, which should include at least those with masses below Λ , make up the particle content of the effective theory. Heavy fields are integrated out, i. e. removed as dynamical degrees of freedom in the EFT.
2. Based on the particles and interactions of the full model, draw all connected Feynman diagrams that satisfy two conditions:
 - all external legs are light fields; and
 - the diagram cannot be disconnected by cutting a single internal light-field line. For tree-level diagrams this is equivalent to requiring that only heavy fields appear as internal lines.

Using the Feynman rules of the full model, calculate the expressions for these diagrams. Do not treat the external legs as incoming or outgoing particles, but keep the field operator expressions.

3. Express quantities of the full model in terms of Λ . Truncate this infinite series of diagrams at some order in $1/\Lambda$, depending on the dimension of the operators that you want to keep. Together with kinetic and mass terms for the light fields, these form the Lagrangian of the EFT.

Fermi theory again

Let us apply this top-down procedure to our standard example of Fermi theory. For simplicity, we do not take the full SM, but just the interactions between massive W bosons and fermions as the underlying theory. The Lagrangian of these interactions is similar to that given in Equation (2.11).

1. Our full model consists of the quarks and leptons and the W boson. We want to analyse weak interactions below the W mass, so we set $\Lambda = m_W$. The light particles of the EFT thus consist of the quarks and leptons except for the top, while the W boson and the top quark are heavy and are integrated out.

2. The only diagram with the requested features that has only one heavy propagator has the form



$$(2.45)$$

Double lines denote a heavy field. There are additional diagrams with W self-interactions or W loops, but they involve at least two W propagators, which means that all contributions from them are of order $\mathcal{O}(1/\Lambda^4)$, which we neglect.

Applying the SM Feynman rules, this diagram evaluates to

$$\begin{aligned} & \left(\bar{\psi}_i \frac{ig}{\sqrt{2}} \frac{1-\gamma_5}{2} \gamma^\mu \psi_j \right) \frac{-g_{\mu\nu}}{p^2 - m_W^2} \left(\bar{\psi}_k \frac{ig}{\sqrt{2}} \frac{1-\gamma_5}{2} \gamma^\nu \psi_l \right) \\ &= \frac{g^2 (\bar{\psi}_i (1-\gamma_5) \gamma^\mu \psi_j) (\bar{\psi}_k (1-\gamma_5) \gamma_\mu \psi_l)}{8(p^2 - m_W^2)}. \end{aligned} \quad (2.46)$$

3. The only dimensionful parameter is $m_W = \Lambda$, and for the EFT to be valid we assume $p^2 \ll \Lambda^2$. We can then expand this expression as

$$\frac{g^2}{8m_W^2} (\bar{\psi}_i (1-\gamma_5) \gamma^\mu \psi_j) (\bar{\psi}_k (1-\gamma_5) \gamma_\mu \psi_l) + \mathcal{O}(1/\Lambda^4). \quad (2.47)$$

With this, we again rediscover the dimension-six EFT matched to the weak interactions of the SM:

$$\mathcal{L} = i\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - m_i \bar{\psi}_i \psi_i + \frac{c}{\Lambda^2} (\bar{\psi}_i (1-\gamma_5) \gamma_\mu \psi_j) (\bar{\psi}_k (1-\gamma_5) \gamma^\mu \psi_l), \quad (2.48)$$

with heavy scale $\Lambda = m_W$ and Wilson coefficient $c = g^2/8$. Replacing c/Λ^2 by $G_F/\sqrt{2} = g^2/(8m_W^2)$ restores the historic form of Fermi theory.

Operator mixing

So far we have neglected that like all parameters in a QFT, the Wilson coefficients of an EFT depend on the energy scale. Running the model from one energy to a different one leads to **operator mixing**: loop effects from one operator affect the coefficients of other operators. If the Wilson coefficients are given at the matching scale Λ (we use this symbol since the matching scale is usually chosen only slightly below the EFT cutoff), at the scale of interest E they take on values of the form

$$f_i(E) \sim f_i(\Lambda) \pm \sum_j \frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{E^2} f_j(\Lambda), \quad (2.49)$$

where g are the typical couplings in the loops.

If the matching scale is not too far away from the energy scale of interest and if all Wilson coefficients are already sizeable at the matching scale, this is often negligible. There is an important consequence, though: even if an operator is zero at the matching scale, operator mixing will give it a small but non-zero value at lower energies. So regardless of what the underlying model is, it can be expected that eventually all effective operators allowed by the symmetries receive contributions from it.

2.3 Dimension-six Higgs physics

We now apply these general ideas to electroweak and in particular Higgs physics at the TeV scale and construct the Standard Model effective field theory (interchangeably called linear Higgs effective field theory) up to dimension six [18–20, 72]. This is the framework we use throughout this thesis. We first argue why such an effective theory is very useful, and then construct its effective operators following the recipe laid out in Section 2.2.2. Section 2.3.3 takes a closer look at the phenomenology of these operators. Finally, in Section 2.3.4 we briefly discuss a few alternative frameworks.

2.3.1 Motivation

As argued in Section 2.1.3, there are many reasons to suspect new physics in the Higgs sector. Some of these arguments, such as the hierarchy problem or the WIMP miracle of dark matter, point towards BSM physics close to the electroweak scale or, depending on the level of acceptable fine-tuning, up to a few TeV. Unfortunately these (purely aesthetic) arguments do not tell us how exactly such physics should look like.

This leaves us with a question highly relevant for upcoming ATLAS and CMS analyses: what is the best language to discuss indirect signs of new physics at the electroweak scale, in particular in the Higgs sector? Which parametrisation of Higgs properties provides a good interface between different experiments, and between experiment and theory?

Directly interpreting measurements in complete models of new physics is impractical: for $n_\gamma \gg 1$ signatures and $m_a \gg 1$ models this requires $m_a n_\gamma$ limits to be derived.⁸ Also, the parameter space of such models (think of the relatively simple MSSM) can be huge, and many of their features do not matter at the electroweak scale at all. It makes more sense to define an intermediate framework that can be linked both to measurements and to full theories, so only n_γ sets of limits plus m_a translation rules from complete theories to the intermediate language have to be calculated. Such a framework should include all necessary physics, but no phenomena irrelevant at this scale. This is exactly the defining feature of an effective field theory.

⁸The weird notation is necessary because the author cannot resist a stupid pun.

2.3.2 Operators

Building blocks

Since we do not know what physics lays beyond the SM, we have to construct our EFT from a bottom-up perspective. As discussed above, this means we have to write down all operators based on a set of particles that are compatible with certain symmetries and are important according to some counting scheme. Let us go through these one by one:

1. As degrees of freedoms we use the SM fields. In particular, we assume the Higgs boson h and the Goldstone bosons w_i are combined in an $SU(2)_L$ doublet ϕ as in the SM, see Equation (2.5). This is consistent with data and presents the correct choice if new physics decouples, that is, if in the limit $\Lambda \rightarrow \infty$ the SM is recovered [73]. We discuss an alternative construction based on the physical scalar h instead of ϕ as the fundamental building block in Section 2.3.4.
2. All operators have to be invariant under Lorentz transformations and under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, and conserve lepton and baryon number.
3. We order the operators by their mass dimension and thus their suppression in powers of $1/\Lambda$. We keep those up to mass dimension 6, i. e. $\mathcal{O}(1/\Lambda^2)$.

Simple dimensional analysis of the kinetic terms of the SM fields tells us the mass dimensions of all building blocks:

$$[f] = \frac{3}{2}, \quad [V_\mu] = 1, \quad [V_{\mu\nu}] = 2, \quad [\phi] = 1, \quad [\partial_\mu] = 1 \quad \text{and} \quad [D_\mu] = 1. \quad (2.50)$$

The only dimension-five operator that can be built from the SM fields is the “Weinberg operator” $(\bar{L}_L \tilde{\phi}^*)(\tilde{\phi}^\dagger L_L)$. It generates a Majorana mass term for the neutrinos, violates lepton number, and is entirely irrelevant for Higgs physics. The leading effects in our EFT are expected to come from dimension-six operators:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(1/\Lambda^4) \quad (2.51)$$

with the unknown cutoff scale Λ and Wilson coefficients f_i . For convenience, from now on we drop the higher-order terms.

As discussed in Section 2.2.2, field redefinitions (or, relatedly, equations of motion), Fierz identities and integration by parts provide equivalence relations between certain operators and give us some freedom to define a basis of operators. Taking these into account, there are 59 independent types of dimension-six operators, not counting flavour structures and Hermitian conjugation [74]. Counting all possible flavour structures, there are 2499 distinct operators. Fortunately, in practice only a small subset of these are relevant: first, the strong constraints on

$\mathcal{O}_{\phi 1} = (D_\mu \phi)^\dagger (\phi \phi^\dagger) (D^\mu \phi)$	$\mathcal{O}_{GG} = (\phi^\dagger \phi) G_{\mu\nu}^a G^{\mu\nu a}$
$\mathcal{O}_{\phi 2} = \frac{1}{2} \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi)$	$\mathcal{O}_{BB} = -\frac{g'^2}{4} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\phi 3} = \frac{1}{3} (\phi^\dagger \phi)^3$	$\mathcal{O}_{WW} = -\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^k W^{\mu\nu k}$
$\mathcal{O}_{\phi 4} = (\phi^\dagger \phi) (D_\mu \phi)^\dagger (D^\mu \phi)$	$\mathcal{O}_{BW} = -\frac{g g'}{4} (\phi^\dagger \sigma^k \phi) B_{\mu\nu} W^{\mu\nu k}$
	$\mathcal{O}_B = \frac{ig}{2} (D^\mu \phi^\dagger) (D^\nu \phi) B_{\mu\nu}$
	$\mathcal{O}_W = \frac{ig}{2} (D^\mu \phi^\dagger) \sigma^k (D^\nu \phi) W_{\mu\nu}^k$

Table 2.1: Bosonic CP -conserving dimension-six operators relevant for Higgs physics.

flavour-changing neutral currents motivate the assumption of flavour-diagonal or even flavour-universal Wilson coefficients. Second, only a small number of these operators directly affects Higgs physics. At higher orders in the EFT expansion, the number of operators increases rapidly, explaining why we stick to the leading effects at dimension 6: not counting flavour structures, there are $\mathcal{O}(10^3)$ operators at dimension eight and $\mathcal{O}(10^4)$ dimension-ten operators [75].

Three different conventions have become popular: the complete “Warsaw” basis [74], the “Strongly Interacting Light Higgs” convention (SILH) [76, 77] and the Hagiwara-Ishihara-Szalapski-Zeppenfeld basis (HISZ) [78]. All three maximise the use of bosonic operators to describe Higgs and electroweak observables. For a comparison of and conversion between these bases see Appendix A.1 and References [2, 79]. Throughout this thesis we use the basis developed in References [21, 80, 81], which is strongly based on the HISZ basis and now widely used in global fits [22, 23].

More references?

We classify the operators based on their field content and on their behaviour under CP transformations. This combined charge conjugation and parity inversion is an approximate symmetry of the SM that is only violated by the complex phase of the CKM matrix. In addition, there are rather tight bounds on CP violation in many processes. This motivates many analyses to restrict their set of operators to the CP -conserving ones. On the other hand, new sources of CP violation are needed to explain the matter-antimatter asymmetry in the universe, and their effects at low energies could be visible as CP -violating effective operators. Both types of operators are analysed in this thesis.

$\mathcal{O}_\ell = (\phi^\dagger \phi) \bar{L}_L \phi \ell_R$	$\mathcal{O}_{\phi L}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{L}_L \gamma^\mu L_L)$	$\mathcal{O}_{\phi L}^{(3)} = i(\phi^\dagger \overleftrightarrow{D}_\mu^a \phi)(\bar{L}_L \gamma^\mu \sigma_a L_L)$
$\mathcal{O}_u = (\phi^\dagger \phi) \bar{Q}_L \tilde{\phi} u_R$	$\mathcal{O}_{\phi Q}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{\phi Q}^{(3)} = i(\phi^\dagger \overleftrightarrow{D}_\mu^a \phi)(\bar{Q}_L \gamma^\mu \sigma_a Q_L)$
$\mathcal{O}_d = (\phi^\dagger \phi) \bar{Q}_L \phi d_R$	$\mathcal{O}_{\phi \ell}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{\ell}_R \gamma^\mu \ell_R)$	
	$\mathcal{O}_{\phi u}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{u}_R \gamma^\mu u_R)$	
	$\mathcal{O}_{\phi d}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{d}_R \gamma^\mu d_R)$	
	$\mathcal{O}_{\phi ud}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{u}_R \gamma^\mu d_R)$	

Table 2.2: *CP*-conserving dimension-six operators relevant for the Higgs-fermion couplings. All operators contain an implicit Hermitian conjugation. For readability, flavour indices are omitted.

$\mathcal{O}_{uW} = (\bar{Q}_L \sigma^{\mu\nu} u_R) \sigma^a \tilde{\phi} W_{\mu\nu}^a$	$\mathcal{O}_{uB} = (\bar{Q}_L \sigma^{\mu\nu} u_R) \tilde{\phi} B_{\mu\nu}$	$\mathcal{O}_{uG} = (\bar{Q}_L \sigma^{\mu\nu} T^a u_R) \tilde{\phi} G_{\mu\nu}^a$
$\mathcal{O}_{dW} = (\bar{Q}_L \sigma^{\mu\nu} d_R) \sigma^a \phi W_{\mu\nu}^a$	$\mathcal{O}_{dB} = (\bar{Q}_L \sigma^{\mu\nu} d_R) \phi B_{\mu\nu}$	$\mathcal{O}_{dG} = (\bar{Q}_L \sigma^{\mu\nu} T^a d_R) \phi G_{\mu\nu}^a$
$\mathcal{O}_{\ell W} = (\bar{L}_L \sigma^{\mu\nu} \ell_R) \sigma^a \phi W_{\mu\nu}^a$	$\mathcal{O}_{\ell B} = (\bar{L}_L \sigma^{\mu\nu} \ell_R) \phi B_{\mu\nu}$	

Table 2.3: Dipole operators affecting the Higgs-gauge-fermion couplings. All operators contain an implicit Hermitian conjugation. For readability, flavour indices are omitted.

$\mathcal{O}_{G\tilde{G}} = (\phi^\dagger \phi) G_{\mu\nu}^a \tilde{G}^{\mu\nu a}$	$\mathcal{O}_{\tilde{B}} = \frac{ig}{2} (D^\mu \phi^\dagger) (D^\nu \phi) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{B\tilde{B}} = -\frac{g'^2}{4} (\phi^\dagger \phi) B_{\mu\nu} \tilde{B}^{\mu\nu}$	$\mathcal{O}_{B\tilde{W}} = -\frac{gg'}{4} (\phi^\dagger \sigma^k \phi) B_{\mu\nu} \tilde{W}^{\mu\nu k}$
$\mathcal{O}_{W\tilde{W}} = -\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^k \tilde{W}^{\mu\nu k}$	

Table 2.4: Bosonic CP -violating dimension-six operators relevant for Higgs physics. The dual field strengths $\tilde{V}_{\mu\nu}$ with $V = G, B, W$ are defined in Equation (2.53).

Operator basis

We begin with the CP -conserving dimension-six operators relevant for Higgs physics, following References [21, 80, 81]. In Table 2.1 we list the bosonic ones, Table 2.2 gives the Higgs-fermion operators, and the “dipole operators” made of Higgs fields, gauge bosons, and fermions are listed in Table 2.3. We use the convention for the sign in the covariant derivative given in Equation (2.2). T^a are the $SU(3)$ generators, and

$$\phi^\dagger \overleftrightarrow{D}_\mu \phi \equiv \phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi \quad \text{and} \quad \phi^\dagger \overleftrightarrow{D}_\mu^a \phi \equiv \phi^\dagger \sigma^a D_\mu \phi - (D_\mu \phi)^\dagger \sigma^a \phi. \quad (2.52)$$

All other symbols appearing in these operators are defined in Appendix A.1.1.

In addition to these CP -conserving structures, there are a number of CP -violating operators. We only list the bosonic ones relevant for Higgs physics [37, 82] in Table 2.4. They involve the dual field strength tensors

$$\tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} V^{\rho\sigma}, \quad V = B, W, G. \quad (2.53)$$

Finally, there are a few CP -even and CP -odd pure gauge operators made from field strength tensors and (covariant) derivatives, and a large number of four-fermion operators similar to the one in Equation (2.48), which are not important in this thesis.

As argued above, not all of these operators are independent. The equations of motion for the Higgs field and the electroweak gauge bosons read [74]

$$D^2 \phi = -\mu^2 \phi - 2\lambda (\phi^\dagger \phi) \phi - \sum_f y_f \bar{f}_R f_L + \mathcal{O}(1/\Lambda^2), \quad (2.54)$$

$$\partial^\rho B_{\rho\mu} = -\frac{ig'}{2} \phi^\dagger \overleftrightarrow{D}_\mu \phi - \sum_f g' Y_f \bar{f} \gamma_\mu f + \mathcal{O}(1/\Lambda^2), \quad (2.55)$$

$$(D^\rho W_{\rho\mu})^a = -\frac{ig}{2} \phi^\dagger \overleftrightarrow{D}_\mu^a \phi - \frac{g}{2} \sum_f \bar{f}_L \gamma_\mu \sigma^a f_L + \mathcal{O}(1/\Lambda^2), \quad (2.56)$$

where Y_f are the weak hypercharges of the fermions. Following Equation (2.14), this provides us with three equivalence relations between dimension-six operators [21, 80, 81]:

$$2\mathcal{O}_{\phi,2} + 2\mathcal{O}_{\phi,4} + 2\mu^2(\phi^\dagger\phi)^2 + 12\lambda\mathcal{O}_6 = \sum_f y_f \mathcal{O}_f + \mathcal{O}(1/\Lambda^4) \quad (2.57)$$

$$\mathcal{O}_{BB} + \mathcal{O}_{BW} - 2\mathcal{O}_B + g'^2 \left(\mathcal{O}_{\phi,1} - \frac{1}{2}\mathcal{O}_{\phi,2} \right) = -\frac{g'^2}{2} \sum_f Y_f \mathcal{O}_{\phi f}^{(1)} + \mathcal{O}(1/\Lambda^4) \quad (2.58)$$

$$\mathcal{O}_{WW} + \mathcal{O}_{BW} - 2\mathcal{O}_W + g^2 \left(\mathcal{O}_{\phi,4} - \frac{1}{2}\mathcal{O}_{\phi,2} \right) = -\frac{g^2}{4} \sum_{F=L,Q} \mathcal{O}_{\phi F}^{(3)} + \mathcal{O}(1/\Lambda^4). \quad (2.59)$$

This lets us eliminate three of the operators listed in Tables 2.1 to 2.4.

There are different strategies for picking the operators to keep. In a top-down approach, one could choose operators based on the underlying physics. In a bottom-up approach, calculations can be simplified if the operators are chosen based on their contributions to physical observables, for instance to avoid non-trivial blind directions. Following Reference [21], we choose to discard $\mathcal{O}_{\phi L}^{(1)}$, $\mathcal{O}_{\phi L}^{(3)}$, and $\mathcal{O}_{\phi,4}$.

Constraints

Some of the remaining operators are tightly constrained from experimental data. Electroweak precision measurements limit the Wilson coefficients of $\mathcal{O}_{\phi,1}$, \mathcal{O}_{BW} , $\mathcal{O}_{B\widetilde{W}}$, $\mathcal{O}_{\widetilde{B}}$, $\mathcal{O}_{\phi L}^{(3)}$, and the remaining $\mathcal{O}_{\phi f}^{(1)}$ to a level where their effects in Higgs physics are small. Measurements of electric dipole moments put tight constraints on the dipole operators. Therefore we mostly ignore these operators in this thesis.⁹

Limits on flavour-changing neutral currents constrain off-diagonal fermion-Higgs couplings. Also, flavour-diagonal \mathcal{O}_f involving fermions of the first and second generation are irrelevant for many signatures considered in this thesis. We therefore only keep the Higgs-fermion operators \mathcal{O}_f of the third generation.

This leaves us with a list of thirteen operators relevant for LHC Higgs physics: ten CP -even operators,

$$\mathcal{O}_{\phi,2}, \quad \mathcal{O}_{\phi,3}, \quad \mathcal{O}_{GG}, \quad \mathcal{O}_{BB}, \quad \mathcal{O}_{WW}, \quad \mathcal{O}_B, \quad \mathcal{O}_W, \quad \mathcal{O}_\tau, \quad \mathcal{O}_t, \quad \text{and} \quad \mathcal{O}_b; \quad (2.60)$$

and three CP -odd ones,

$$\mathcal{O}_{G\widetilde{G}}, \quad \mathcal{O}_{B\widetilde{B}}, \quad \text{and} \quad \mathcal{O}_{W\widetilde{W}}. \quad (2.61)$$

⁹This simple argument is suitable for our rather conceptual work. In a thorough global fit, however, it should be checked carefully whether these constraints are actually strong enough to make all these operator irrelevant for Higgs physics in all cases. Such a check should include RGE effects when comparing constraints from different scales. The increasing precision in Higgs observables means that many of these operators will become relevant again in the future.

Renormalisation group evolution

The Wilson coefficients of these operators depend on the energy scale. In the last years, the contributions of all dimension-six operators on the running of the SM parameters, as well as the whole 59×59 anomalous dimension matrix of dimension-six operators, have been calculated at one-loop level [83–85]. This provides all necessary tools to run the EFT parameters from the matching scale Λ to the experimental scale E . Following Equation (2.49), this shifts the Wilson coefficients by a term proportional to a loop factor and $\log \Lambda^2/E^2$.

As we discuss at some length in Chapter 3, the LHC Higgs measurements are only sensitive to new physics scales between the electroweak scale and the TeV scale. The corresponding logarithm typically cannot compensate for the loop factor, and the RGE effects on Wilson coefficients that are already non-zero at the matching scale are small. We therefore neglect operator running for our analyses.

2.3.3 Phenomenology

After picking a set of operators, the next question is how they affect Higgs observables. We first discuss two examples, $\mathcal{O}_{\phi,2}$ and \mathcal{O}_W , in detail, before listing the effects of all operators in Equations (2.60) and (2.61).

$\mathcal{O}_{\phi,2}$: rescaled Higgs couplings

Our first example is the operator $\mathcal{O}_{\phi,2}$. Ignoring the Goldstones, it consists only of derivatives and Higgs fields $\phi^\dagger \phi = (v^2 + 2v\tilde{h} + \tilde{h}^2)/2$, where we use a tilde on h because this field is not yet a mass eigenstate. Its contribution to the Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\text{EFT}} \supset & \frac{f_{\phi,2}}{2\Lambda^2} \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi) \\ & = \frac{f_{\phi,2} v^2}{2\Lambda^2} \partial_\mu \tilde{h} \partial^\mu \tilde{h} + \frac{f_{\phi,2} v}{\Lambda^2} \tilde{h} \partial_\mu \tilde{h} \partial^\mu \tilde{h} + \frac{f_{\phi,2} v}{2\Lambda^2} \tilde{h}^2 \partial_\mu \tilde{h} \partial^\mu \tilde{h}. \end{aligned} \quad (2.62)$$

The first term rescales the kinetic term of the Higgs boson:

$$\mathcal{L}_{\text{EFT}} \supset \left(1 + \frac{f_{\phi,2} v^2}{\Lambda^2} \right) \frac{1}{2} \partial_\mu \tilde{h} \partial^\mu \tilde{h}. \quad (2.63)$$

To restore the canonical form of the kinetic term, we have to rescale the Higgs boson \tilde{h} to

$$h = \sqrt{1 + \frac{f_{\phi,2} v^2}{\Lambda^2}} \tilde{h}. \quad (2.64)$$

This universally shifts all Higgs couplings to other particles as

$$g_{hxx} = \frac{1}{\sqrt{1 + \frac{f_{\phi 2} v^2}{\Lambda^2}}} g_{hxx}^{\text{SM}}. \quad (2.65)$$

There is a second, more involved effect on the Higgs self-coupling. First, the rescaling in Equation (2.64) also affects the Higgs mass term given in Equation (2.6). For fixed v and m_h , this amounts to shifting the Higgs self-coupling λ to

$$\lambda = \frac{m_h^2}{2v^2} \left(1 + \frac{f_{\phi 2} v^2}{\Lambda^2} \right). \quad (2.66)$$

In addition, the second term in Equation (2.62) introduces a new Lorentz structure into the Higgs self-interaction that depends on the Higgs momenta. A non-zero Wilson coefficient $f_{\phi 2}$ therefore has a strong impact on Higgs pair production, changing not only the total rate, but also kinematic shapes.

\mathcal{O}_W : new Higgs-gauge structures

Our second example is \mathcal{O}_W , $vvvv$ which contracts covariant derivatives acting on Higgs doublets, defined in Equation (2.2), with a field strength tensor $W_{\mu\nu}^k = \partial_\mu W_\nu^k - \partial_\nu W_\mu^k + g\epsilon^{kmn} W_\mu^m W_\nu^n$. Expanding \mathcal{O}_W and only keeping the pieces that affect the hWW coupling, we find

$$\begin{aligned} \mathcal{L}_{\text{EFT}} &\supset \frac{f_W}{\Lambda^2} \frac{ig}{2} (D^\mu \phi)^\dagger \sigma^k (D^\nu \phi) W_{\mu\nu}^k \\ &= \frac{ig f_W}{2\Lambda^2} \left(\partial^\mu \phi^\dagger + \frac{ig}{2} W^{m\mu} \phi^\dagger \sigma^m + \frac{ig'}{2} B^\mu \phi^\dagger \right) \sigma^k \left(\partial^\nu \phi - \frac{ig}{2} \sigma^n W^{n\nu} \phi - \frac{ig'}{2} B^\nu \phi \right) W_{\mu\nu}^k \\ &\supset \frac{ig f_W}{2\Lambda^2} \left\{ \frac{\partial^\mu h}{\sqrt{2}} [\sigma^k \sigma^n]_{22} \frac{-ig}{2} W^{n\nu} \frac{v}{\sqrt{2}} + \frac{ig}{2} W^{m\nu} \frac{v}{\sqrt{2}} [\sigma^m \sigma^k]_{22} \frac{\partial^\mu h}{\sqrt{2}} \right\} W_{\mu\nu}^k \\ &= \frac{f_W}{\Lambda^2} \frac{g^2 v}{8} [\sigma^k, \sigma^n]_{22} (\partial^\mu h) W^{n\nu} W_{\mu\nu}^k \\ &= \frac{f_W}{\Lambda^2} \frac{ig^2 v}{4} \epsilon^{nk3} (\partial^\mu h) W^{n\nu} W_{\mu\nu}^k. \end{aligned} \quad (2.67)$$

With $m_W = gv/2$ and $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ this finally yields

$$\mathcal{L}_{\text{EFT}} \supset \frac{f_W}{\Lambda^2} \frac{igm_W}{2} (\partial^\mu h) (W^{+\nu} W_{\mu\nu}^- + W^{-\nu} W_{\mu\nu}^+). \quad (2.68)$$

This is another contribution to the hWW vertex. But unlike the SM-like coupling

$$\mathcal{L}_{\text{SM}} \supset gm_W h W^{+\mu} W_\mu^-, \quad (2.69)$$

expect that the effects from \mathcal{O}_W grow with m_{Zh} . In Figure 2.4 we demonstrate this by comparing the distribution of m_{Zh} based on the SM diagrams alone, on the contributions from only the dimension-six operator \mathcal{O}_W only, and on the interference between the two components. Indeed we see that \mathcal{O}_W contributes mostly in the high-energy tail of the distribution.

All those couplings

After these two worked-out examples, we now give the complete list of single-Higgs couplings induced by the dimension-six operators of Equations (2.60) and (2.61) [21, 80, 81].¹⁰

These interactions read

$$\begin{aligned}
\mathcal{L}_{\text{EFT}} \supset & g_{hgg}^{(1)} h G_{\mu\nu}^a G^{a\mu\nu} + g_{hgg}^{(2)} \varepsilon_{\mu\nu\rho\sigma} h G^{a\mu\nu} G^{a\rho\sigma} + g_{h\gamma\gamma} h A_{\mu\nu} A^{\mu\nu} \\
& + g_{hZ\gamma}^{(1)} A_{\mu\nu} Z^\mu \partial^\nu h + g_{hZ\gamma}^{(2)} h A_{\mu\nu} Z^{\mu\nu} \\
& + g_{hZZ}^{(1)} Z_{\mu\nu} Z^\mu \partial^\nu h + g_{hZZ}^{(2)} h Z_{\mu\nu} Z^{\mu\nu} + g_{hZZ}^{(3)} h Z_\mu Z^\mu + g_{hZZ}^{(4)} h \varepsilon_{\mu\nu\rho\sigma} Z^{\mu\nu} Z^{\rho\sigma} \\
& + g_{hWW}^{(1)} (W_{\mu\nu}^+ W^{-\mu} \partial^\nu h + \text{h. c.}) + g_{hWW}^{(2)} h W_{\mu\nu}^+ W^{-\mu\nu} + g_{hWW}^{(3)} h W_\mu^+ W^{-\mu} \\
& + g_{hWW}^{(4)} h \varepsilon_{\mu\nu\rho\sigma} W^{+\mu\nu} W^{-\rho\sigma} \\
& + \sum_{f=\tau,t,b} (g_{hff} h \bar{f}_L f_R + \text{h. c.})
\end{aligned} \tag{2.72}$$

with couplings

$$\begin{aligned}
g_{hgg}^{(1)} &= \frac{f_{GG} v}{\Lambda^2}, & g_{hZ\gamma}^{(1)} &= -\frac{g^2 v s_W (f_W - f_B)}{4 c_W \Lambda^2}, \\
g_{hgg}^{(2)} &= \frac{f_{G\tilde{G}} v}{2 \Lambda^2}, & g_{hZ\gamma}^{(2)} &= \frac{g^2 v s_W (2 s_W^2 f_{BB} - 2 c_W^2 f_{WW})}{4 c_W \Lambda^2}, \\
g_{h\gamma\gamma} &= -\frac{g^2 v s_W^2 (f_{WW} + f_{BB})}{4 \Lambda^2}, & g_{hff} &= -\frac{m_f}{v} \left(1 + \frac{v^2 f_{\phi,2}}{\Lambda^2} \right)^{-1/2} + \frac{v^2 f_f}{\sqrt{2} \Lambda^2}, \\
g_{hZZ}^{(1)} &= -\frac{g^2 v (c_W^2 f_W + s_W^2 f_B)}{4 c_W^2 \Lambda^2}, & g_{hWW}^{(1)} &= -\frac{g^2 v f_W}{4 \Lambda^2}, \\
g_{hZZ}^{(2)} &= -\frac{g^2 v (s_W^4 f_{BB} + c_W^4 f_{WW})}{4 c_W^2 \Lambda^2}, & g_{hWW}^{(2)} &= -\frac{g^2 v f_{WW}}{2 \Lambda^2}, \\
g_{hZZ}^{(3)} &= \frac{g^2 v}{4 c_W^2} \left(1 + \frac{v^2 f_{\phi,2}}{\Lambda^2} \right)^{-1/2}, & g_{hWW}^{(3)} &= \frac{g^2 v}{2} \left(1 + \frac{v^2 f_{\phi,2}}{\Lambda^2} \right)^{-1/2}, \\
g_{hZZ}^{(4)} &= -\frac{g^2 v (s_W^4 f_{B\tilde{B}} + c_W^4 f_{W\tilde{W}})}{8 c_W^2 \Lambda^2}, & g_{hWW}^{(4)} &= -\frac{g^2 v f_{W\tilde{W}}}{4 \Lambda^2}.
\end{aligned} \tag{2.73}$$

¹⁰When comparing with References [21, 80, 81], note the different sign conventions in the covariant derivative.

Here $s_W = g'/\sqrt{g^2 + g'^2}$ and $c_W = g/\sqrt{g^2 + g'^2}$ are the sine and cosine of the weak mixing angle, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ for $V = A, W^\pm, Z$.

Note that the clear majority of the couplings in Equation (2.72) does not exist in the SM and contains derivatives. Dimension-six operators predict a variety of novel kinematic features in Higgs interactions, making their measurement at the LHC both exciting and challenging.

2.3.4 Alternative frameworks

The linear Higgs EFT discussed above is not the only useful parametrisation of Higgs properties. We briefly go through some of the alternative frameworks and explain their main properties, before finishing this chapter with a comparison between the different approaches.

Non-linear Higgs effective field theory

In the SM EFT (or linear Higgs EFT), constructed in Section 2.3.2, the Higgs boson h and the Goldstone bosons w_a form an $SU(2)$ doublet ϕ as given in Equation (2.5). But in some models of new physics the Higgs is not part of an elementary $SU(2)$ doublet. Typical examples are composite Higgs models in which the Higgs boson is a pseudo-Goldstone from some strongly interacting dynamics [86–90]. Non-linear Higgs EFT, sometimes simply called “Higgs EFT”, is an effective theory designed for these scenarios [82, 91–100]. This model is also often referred to as “chiral Lagrangian”, and indeed its structure is similar to that of chiral perturbation theory, for an introduction see for instance References [101, 102]. Again, we begin by going through the ingredients to the effective theory before constructing the Lagrangian.

The **particle content** of the non-linear Higgs EFT constitutes the main difference to the linear Lagrangian: the physical scalar h is separated from the Goldstones w_a , both are included as independent degrees of freedom rather than as part of the doublet ϕ . The Higgs boson h is now a singlet under the SM gauge symmetry. The Goldstone bosons w_a , no longer a part of a Higgs doublet, are organised in the exponential form

$$U = e^{i\sigma_a w_a}, \quad (2.74)$$

where σ_a are the Pauli matrices. The Goldstones transform non-linearly under the (approximate) global custodial symmetry $SU(2)_L \times SU(2)_R$, giving the EFT its name.

The **symmetries** are the same as in the linear case. We require invariance under Lorentz transformations as well as under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and baryon and lepton number conservation. For simplicity, we also restrict our brief discussion to CP -even operators that conserve lepton flavour.

Choosing the **counting scheme** is a little more involved. To account for strongly interacting scenarios, we now have to distinguish three different scales [97]:

- The electroweak scale $v = 246$ GeV, which defines the W and Z mass, but is not necessarily the Higgs VEV.
- The scale f associated to the Goldstone bosons w_a and the Higgs boson h due to some breaking of the underlying dynamics, in analogy to the pion decay constant f_π .¹¹
- The cut-off Λ of the theory. For weakly coupled physics this can be arbitrary. But one can calculate that the low-energy effective theories from spontaneously broken strongly coupled dynamics break down around $\Lambda \approx 4\pi f$ [101]. A cut-off of this size guarantees that the EFT is renormalisable order by order.

The existence of three scales means there are two dimensionless parameters, so in general the EFT terms are organised in a double expansion [97]. The first is

$$\xi \equiv \frac{v^2}{f^2}. \quad (2.75)$$

The value of ξ defines the non-linearity of the model: the limit $\xi \rightarrow 0$ restores the linear Lagrangian. An expansion in ξ exactly corresponds to the power-counting scheme of the linear EFT, i. e. it orders operators by their canonical dimension.

The second dimensionless parameter is

$$\frac{f^2}{\Lambda^2} \approx \frac{1}{16\pi^2} \quad (2.76)$$

for strongly coupled scenarios. Expanding in f^2/Λ^2 corresponds to a loop expansion similar to that in chiral perturbation theory. Equivalently, one can define a **chiral dimension** $\chi = [\mathcal{O}]_c$ for each operator \mathcal{O} with the assignments [97]

$$\begin{aligned} [f]_c &= 1, & [V_\mu]_c &= 0, & [V_{\mu\nu}]_c &= 1, & [U]_c &= 0, & [h]_c &= 0, \\ [\partial_\mu]_c &= 1, & [D_\mu]_c &= 1, & [g]_c &= 1, & [y_f]_c &= 1. \end{aligned} \quad (2.77)$$

The loop order L of an operator is equivalent to the chiral dimension $\chi = 2L + 2$. This chiral counting can also be linked to an expansion in \hbar [103].

The correct expansion scheme depends on the value of ξ . For $\xi \gg 1/16\pi^2$ or $f \ll 3$ TeV, the chiral expansion is more appropriate. For $\xi \ll 1/16\pi^2$ or $f \gg 3$ TeV, the canonical expansion is correct. In the intermediate region, a combined expansion gives the best results. Since LHC Higgs physics is mostly sensitive to new physics scenarios with $f \ll 3$ TeV, the chiral expansion can be considered phenomenologically more relevant. For a more thorough discussion of power counting in this framework, see Reference [73].

¹¹In general, the scales associated with w_a and h , f_w and f_h , can be different, making the power-counting even more complicated.

At the leading chiral order $\chi = 2$ or $L = 0$, the Higgs sector of the Lagrangian is given by [95]¹²

$$\begin{aligned} \mathcal{L}_{\text{non-linear EFT}} \supset & \frac{1}{2} \partial_\mu h \partial^\mu h (1 + c_H \xi \mathcal{F}_H(h)) - V(h) \\ & - \frac{v^2}{4} \text{tr}[V_\mu V^\mu] \mathcal{F}_C(h) + c_T \xi \frac{v^2}{4} \text{tr}[T V^\mu] \text{tr}[T V_\mu] \mathcal{F}_T(h) \\ & - \frac{v}{\sqrt{2}} \left[\sum_f \bar{f}_L U y_f \mathcal{F}_Y^f(h) P_f f_R + \text{h. c.} \right] \end{aligned} \quad (2.78)$$

with $V_\mu \equiv (D_\mu U) U^\dagger$, $T \equiv U \sigma_3 U^\dagger$ and projectors $P_u = (\mathbb{1} + \sigma_3)/2$, $P_d = P_\ell = (\mathbb{1} - \sigma_3)/2$. The functions $\mathcal{F}_C(h)$, $V(h)$, $\mathcal{F}_T(h)$, $\mathcal{F}_T(h)$, $\mathcal{F}_Y^u(h)$, $\mathcal{F}_Y^d(h)$, and $\mathcal{F}_Y^\ell(h)$ encode the coupling of the Higgs h and are arbitrary functions. They can be expanded as a power series in h/v , or to simplify the expressions in h/v , for instance

$$\mathcal{F}_C(h) = 1 + 2a_C \frac{h}{v} + b_C \left(\frac{h}{v} \right)^2 + \dots \quad (2.79)$$

At next-to-leading order in the chiral expansion many more terms relevant for Higgs physics appear. We do not list them here and refer the interested reader for instance to Reference [98].

Finally, the relationship between the linear and non-linear effective theories deserves some discussion. The two approaches in principle provide different parametrisations of the same physics, as can be seen by expanding the non-linear Lagrangian in ξ rather than χ . The difference is the ordering of the operators in the EFT expansion and, equivalently, the expected size of different effects. Operators that appear at one order in the $1/\Lambda$ expansion of the linear EFT may appear at a very different order in the chiral expansion of the non-linear EFT.

Since the symmetry requirements on the non-linear setup are smaller, we expect it to be more general than the linear Lagrangian at a comparable order in the expansions. This is exactly what is found when comparing the linear dimension-six operators to the NLO chiral Lagrangian: the dimension-six operators predict certain correlations, while the non-linear description has more operators that can break these correlations [98]. A straightforward example is the relation between hxx and $hhxx$ couplings. For dimension-six operators, it is fixed due to the appearance of $\phi^\dagger \phi \sim (v^2 + 2vh + h^2)/2$, while these couplings are always independent in the chiral approach, as can be seen in Equation (2.78).

The current experimental limits leave room for both strongly or weakly coupled new physics, for ξ smaller or larger than $1/(16\pi^2)$, for scenarios in which the linear or non-linear effective

¹²Two comments on this Lagrangian are in order. First, in principle there could be further functions of h coupling to the kinetic terms of the gauge bosons. Such interactions arise in typical strongly coupled theories at one-loop level with a coefficient $\sim 1/(16\pi^2)$ and are therefore usually classified as NLO operators [95, 96]. Second, the function $\mathcal{F}_C(h)$ (and a corresponding one for the fermion kinetic terms) can be removed with field redefinitions, shifting its physics into the other couplings [96, 100].

theories work better. Only a precise measurement of the Higgs properties and a global analysis of correlations will tell us which approach is correct. As a general rule, more SM-like results favour the linear approach that we follow throughout this thesis [73]. On the other hand, certain deviations that do not follow the correlations predicted by dimension-six operators point towards non-linear physics [98].

κ framework

Effective field theories are of course not the only way to describe the Higgs sector. During run 1 of the LHC, the most widely used parametrisation was the κ framework [104] or the closely related Δ framework [105]. Its construction is remarkably simple: starting from the SM Higgs sector, all Higgs couplings are dressed with form factors,

$$g_{hxx} = \kappa_x g_{hxx}^{\text{SM}} = (1 + \Delta_x) g_{hxx}^{\text{SM}}. \quad (2.80)$$

Some care has to be taken to treat the Higgs-gluon and Higgs-photon couplings consistently, where indirect effects of shifted Higgs-top or Higgs- W couplings compete with direct effects from new physics [105].

From a theoretical point of view, the κ framework is not gauge-invariant and does not present a consistent quantum field theory. In practice, this means that electroweak loop effects may introduce divergences that cannot be renormalised. This problem can be solved by embedding the κ framework in a UV completion [106].

From a more phenomenological point of view, this approach is well suited to parametrise measurements of total rates. Simple shifts of SM-like Higgs coupling structures are expected in some scenarios of new physics, for instance in many scalar extensions of the Higgs sector [106]. But many other models predict new kinematic features, visible as changed kinematic shapes, and the κ framework is unable to describe these. For better or worse, it is also agnostic about correlations between different Higgs couplings, and about correlations between Higgs observables and triple gauge vertices or electroweak precision measurements.

The strength of the κ framework is clearly not its theoretical foundation. Its allure comes from its simplicity and the fact that it is designed around the simple question of measuring the couplings of the (SM-like) Higgs boson. This parametrisation made sense as a common denominator for the first Higgs measurements with limited statistics of run 1 of the LHC. But the increased amount of data and crucial kinematic information collected during run 2 require a different, more sophisticated language.

Pseudo-Observables

Higgs pseudo-observables (POs) [107–109] are designed as a generalisation of the κ framework to include BSM kinematic features. In a very broad sense, this term encompasses any number that is

Citation?

field theoretically defined and can be experimentally accessed [73]. Signal strengths, cross sections, partial widths, total widths, and individual form factors or couplings all fall under this umbrella term. Here we follow the more narrow definition of effective-coupling PO [4]. Pseudo-observables are then constructed process by process by writing down all contributing amplitudes under some broad assumptions on new physics. These expressions are then decomposed in a pole expansion, and the resulting residues are identified as pseudo-observables. This procedure also requires an expansion in the inverse of the new physics scale Λ . This means that just as the EFT approach, pseudo-observables rely on new physics being heavy, $E \ll \Lambda$. So far, this framework has been developed for Higgs production in WBF or Higgs-strahlung, as well as for all phenomenologically relevant Higgs decays.

Phenomenologically, pseudo-observables can describe shifts in SM couplings as well as kinematic shapes. Like the EFT approach, their construction requires certain minimal assumptions on the symmetries of new physics as well as an expansion in $1/\Lambda$. In fact, at tree level pseudo-observables can be mapped linearly to an EFT constructed with the same ingredients (which is the non-linear Higgs EFT discussed below).

The main difference between pseudo-observables and the EFT approach is a conceptual one. Pseudo-observables are designed from the perspective of a given process: they describe the coefficients of the different contributing amplitudes. They are not parameters of a Lagrangian and do not define a consistent quantum field theory. In particular, the values of POs measured in one process have no meaning for other processes. Effective operators on the other hand are a proper, gauge-invariant quantum field theory that universally describes any physics below the cutoff scale, and the same Wilson coefficients predict the behaviour of very different processes.

Proponents of the PO approach favour a multi-layer interpretation of LHC data, where the data is first presented in terms of pseudo-observables, which can then be interpreted within EFTs or specific models of UV physics. They argue that this approach provides a clear separation between measurement and interpretation [4]. On the other hand, proponents of the “direct EFT approach” argue that there is no need for such an intermediate layer, and suggest to directly fit effective operators. The debate about which approach is better is still ongoing [4]. Ultimately, both effective operators and POs are well-defined frameworks that can describe all relevant kinematic effects, and thus present a suitable interface between experiment and theory.

Simplified models

All these approach have in common that they assume the absence of new light particles. Simplified models are designed to close this gap and to describe kinematic effects from new light resonances. In addition to resonance peaks these include threshold effects in loops and Higgs decays into (invisible) new light degrees of freedom. Simplified models also allow to combine information from direct searches with indirect measurements. Except for the key element of adding new light propagating degrees of freedom to the SM, the term is not particularly well-defined, and there is a

lot of freedom to construct such models.

The simplest version of a simplified model consists of the SM supplemented with another particle, with ad-hoc coupling structures based on phenomenological requirements [3]. Such a setup might even be not gauge-invariant and thus inconsistent beyond tree level. At the other end of the spectrum, simplified models can be consistently defined quantum field theories, potentially involving higher-dimensional operators. The only difference to the linear and non-linear EFT approaches discussed above is the extended particle content. The additional flexibility of course comes at the price of an increased number of parameters.

Examples of such models for Higgs physics include an extended Higgs sector with an additional singlet and a doublet, which offers great flexibility to tune the Higgs couplings [106]. The authors of Reference [110] develop a model with an additional singlet and vector-like quarks. Finally, References [111, 112] discuss additional scalar singlets supplemented with higher-dimensional operators.

Comparison

All these approaches define parametrisations of the Higgs properties that can be used as interfaces between different measurements and between experiment and theory. In Table 2.5 we summarise and compare their different properties.

The different frameworks can be classified into consistent quantum field theories, which include the EFTs, and process-based parametrisations of amplitudes through form factors. The κ framework and pseudo-observables belong to the latter category. Simplified models can fall into either category. The QFT formalism allows to link different processes and to incorporate any loop effects.

More important for practical purposes is the range of phenomena that can be described. The κ framework is limited to rescalings of the SM Higgs couplings and is not able to incorporate kinematic information. Pseudo-observables and the two EFT approaches are much more flexible and can describe a large number of kinematic features. However, they rely on new physics being substantially heavier than the experimentally probed energies around the weak scale. Features from light new particles, for instance resonances or loop thresholds, are only covered by appropriate simplified models. The dimension-six operators of linear Higgs EFT and to a lesser extent the leading operators of the chiral EFT also predict certain correlations between different couplings and measurements, while by definition the pseudo-observables are only valid for a given process.

To summarise, in the absence of new light particles, Higgs properties can be adequately parametrised by pseudo-observables, non-linear Higgs EFT, and linear Higgs EFT. The linear EFT approach is theoretically well-motivated and phenomenologically powerful, and we focus on this framework during this thesis.

	κ framework	POs	Non-linear EFT	Linear EFT	Simplified models
Motivation	experiment: simplest Higgs parametrisation	experiment: amplitude decomposition for a given process	theory: complete low-energy effects of NP with singlet h	theory: complete low-energy effects of NP with doublet ϕ	exp. / theory: new light particles
Input	SM Higgs couplings	process amplitudes, pole expansion, NP expansion ($1/\Lambda$)	SM particles (h), symmetries, counting scheme (loops)	SM particles (ϕ), symmetries, counting scheme ($1/\Lambda$)	new particles (masses, charges, interactions)
Parameters	coefficients of SM amplitude	coefficients of amplitudes	Lagrangian parameters of consistent QFT	Lagrangian parameters of consistent QFT	depends
Validity conditions	SM-like	NP heavy, symmetries	NP heavy, symmetries	NP heavy, symmetries	single light new particles, other NP decouples
Shifted SM couplings	yes	yes	yes	yes	depends
Kinematic effects	no	yes	yes	yes	depends
New resonances, loop thresholds, invisible decays	no	no	no	no	yes
Correlations	no	no	some	many	depends

Table 2.5: Comparison between different parametrisations of Higgs properties. The upper part of the table focuses on the theoretical foundation, the lower on the phenomenology. Since “simplified models” describe a rather general idea, many details depend on the specific realisation.

Chapter 3

Higgs effective theory at its limits

All models are wrong, but some are useful.

— G. Box [113]

HIGGS EFFECTIVE THEORIES can universally describe all signatures expected from new physics as long as its mass scale Λ is sufficiently separated from the experimentally accessible scale. However, the limited precision of the LHC means that Higgs measurements are often only sensitive to models with scales not much higher than the electroweak scale. In this chapter we discuss if and when the EFT approach is nevertheless useful, and how its validity can be improved.

In Section 3.1 we discuss the LHC sensitivity and its implications for effective theories, and formulate our strategy to test the dimension-six approach. In Section 3.2 we discuss some crucial details of the matching between full models and EFTs, and introduce the novel “ v -improved” matching procedure. Section 3.3 contains the bulk of our results, the comparison of full models to their EFT approximations for a variety of models and observables. We go into more detail and analyse some practical questions in a specific example case in Section 3.4, and finally present our conclusions in Section 3.5.

Most of the work presented in this chapter was previously published in Reference [2], while the content of Section 3.4 was published in Reference [3]. A part of the content of this chapter was also included in Reference [4]. Nearly all of the results and most of their presentation in this chapter — including all plots, most of the tables, and a significant part of the text — are identical to that in these three publications.

3.1 Introduction

3.1.1 Energy scales in Higgs measurements

There is no doubt that effective field theories work extraordinarily well as long as there is a scale hierarchy between the experimentally probed energy and the new physics scale, $E \ll \Lambda$. On the other hand, it is clear that the EFT expansion breaks down at $E \geq \Lambda$, where an infinite number of operators contributes at the same size and the model is neither predictive nor renormalisable. The validity of the effective theory in the intermediate region,

$$E \lesssim \Lambda, \quad (3.1)$$

is less obvious and will depend on the specific underlying model as well as on the observable studied. So in which of these categories do LHC Higgs measurements belong?

Higgs production at hadron colliders does not probe a single experimental energy scale over the full relevant phase space. The momentum transfer is bounded from below by the Higgs mass. Selection criteria necessary to separate a signal from the QCD backgrounds often require a higher momentum transfer $E > m_h$. More importantly, most of the information on operators with derivatives comes from high-energy tails, as demonstrated in Section 2.3.3. During run 1, significant event numbers are recorded within the range

$$m_h \leq E \lesssim \mathcal{O}(400 \text{ GeV}), \quad (3.2)$$

depending on the process, observable, collected amount of data, and analysis methods.

On the other hand, we can roughly estimate the new physics scale Λ that LHC Higgs measurements are able to probe. Assuming a 10% precision on total Higgs rate measurements and no loop suppression of new physics effects, such a signature lies within the experimental reach of the LHC if

$$\left| \frac{\sigma \times \text{BR}}{(\sigma \times \text{BR})_{\text{SM}}} - 1 \right| = \frac{g^2 m_h^2}{\Lambda^2} \gtrsim 10\% \quad \Leftrightarrow \quad \Lambda < \frac{g m_h}{\sqrt{10\%}} \approx g \, 400 \text{ GeV}. \quad (3.3)$$

where g are the typical couplings of the underlying theory.

This simple estimation shows that the scale separation E/Λ is limited by the experimental precision and crucially depends on the size of the couplings of underlying physics. For very weakly coupled theories, $g^2 < 1/2$, only new physics models with new particles at or below the electroweak scale can leave measurable signatures in Higgs observables, and the EFT approach clearly does not make sense. For truly strongly coupled theories, $1 < g \lesssim 4\pi$, new physics scenarios up to $\Lambda \lesssim 5 \text{ TeV}$ are relevant, and the EFT expansion converges flawlessly. In fact, the EFT approach to Higgs observables has largely been motivated by the desire to describe models with strongly

interacting electroweak symmetry breaking [76]. For moderately weakly to moderately strongly coupled theories, $1/2 \lesssim g^2 \lesssim 2$, the LHC Higgs programme is sensitive to scales

$$280 \text{ GeV} \lesssim \Lambda \lesssim 560 \text{ GeV} \quad (3.4)$$

This corresponds exactly to the intermediate region defined in Equation (3.1).

In this simple argument we ignored that new physics might also change distributions and especially affect the high-energy tails or off-shell regions. A thorough global fit of Higgs results including kinematic information confirms the rough estimate given in Equation (3.4) [22].

We conclude that for moderately weakly coupled scenarios of new physics, the limited precision of LHC Higgs measurements cannot guarantee a clear scale hierarchy, and the effective field theory approach cannot be trusted blindly. But this does not mean that an analysis of LHC data in terms of a truncated dimension-six Lagrangian cannot be useful. Instead, the applicability of the dimension-six model now depends on the nature of the underlying physics as well as on the process and observable, and has to be carefully checked for each situation.

From a practical perspective, in Reference [22] it has been shown that a fit of dimension-six operators to the Higgs data at Run I is a sensible and practicable extension of the usual Higgs couplings fit. As discussed at length in the previous chapter, dimension-six operators including derivatives complement the Higgs coupling modifications and allow us to extract information from kinematic distributions. Even if the LHC constraints do not induce a hierarchy of scales, and the EFT approach is not formally well defined, there appears to be no problem in using the truncated dimension-six Lagrangian as a phenomenological model to describe the LHC Higgs data. This description induces theory uncertainties if we want to interpret the LHC results in terms of an effective field theory [114].

3.1.2 Questions for the EFT approach

In this chapter we analyse the usefulness of the dimension-six description of the Higgs sector with a comprehensive comparison of full models and their dimension-six description. As argued above, the EFT validity has to be tested on a process-to-process as well as model-to-model basis. We therefore select four representative models of new physics, map them onto dimension-six operators, and compare the predictions of the full and the effective model in various Higgs observables.

We select moderately weakly interacting extensions of the Higgs sector of the Standard Model by

1. a scalar singlet,
2. a scalar doublet,
3. a coloured top-partner scalar, and
4. a massive vector triplet.

For each of these models we pick a number of parameter benchmark points, designed to highlight phenomenological features of the model, and to be within the experimental reach of the LHC Higgs programme.

The corresponding EFT descriptions are constructed by integrating out the heavy fields and expanding the effective action to $\mathcal{O}(1/\Lambda^2)$. In other words, we match the theories to the dimension-six operators of the linear Higgs EFT introduced in Section 2.3.2. The details of this matching procedure are a key element of our analysis and will be discussed in detail in the next section.

For all these scenarios, we calculate the Higgs couplings, and in a next step rates and distributions for selected Higgs production modes and decay channels. The key questions we aim to address are:

- Which masses and coupling predict Higgs signatures relevant for the LHC? Is the corresponding new physics scale sufficiently separated from v ?
- Which observables are correctly described by dimension-six model? Where does the EFT description break down, and why?
- Does this breakdown pose a problem for LHC analyses?

In this way, we analyse what problems the lack of a clear hierarchy of scales leads to in practice, and discuss how these might affect global LHC-Higgs fits including kinematic distributions. Turning the argument around, we ask whether and when the analysis of a UV-complete model offers an advantage compared to the effective theory.

After this broad survey of the applicability of dimension-six operators, we focus on the vector triplet scenario and WBF Higgs production and briefly discuss some practical aspects. First, we analyse which observables are particularly useful to characterise the validity of the EFT approach in WBF Higgs production. We then discuss whether the square of dimension-six operators should be included in calculations while neglecting dimension-eight operators interfering with the SM. Both contribute at the same order $\mathcal{O}(1/\Lambda^4)$ to the squared amplitude. Finally, we show how the EFT validity and its breakdown affect the limit setting procedure when the EFT is used as an intermediate parametrisation.

In addition and partly simultaneously to our work, published in References [2, 3], the applicability of EFTs to Higgs physics at the LHC was studied in a range of different situations [4, 115–127]. The differences to our work lies in the considered new physics scenarios and observables. A lot of attention focused on Higgs production in weak boson fusion and its sensitivity to UV physics [38, 39, 41, 128]. Similar points were discussed for Higgs-strahlung [115], in the production of (potentially off-shell) Higgs bosons in gluon fusion [120, 124, 129–131], and in electroweak precision observables as well as Higgs decays to photons [127]. With the notable exception of Reference [127], these other studies generally do not discuss ambiguities in the matching procedure, a central aspect of the research presented here.

The problem of a lack of scale hierarchy is not unique to the EFT approach. As discussed at the end of Section 2.3.4, pseudo-observables rely on the same expansion in $1/\Lambda$, and the breakdown of this expansion has been studied in Reference [109].

As an aside, similar concerns have fuelled an intense investigation in the context of dark matter searches [132–137]. While in that field EFT-based predictions are usually robust for early-universe and late-time annihilation rates as well as for dark matter-nucleon scattering, the required hierarchy of scales often breaks down for dark matter signals at colliders.

3.2 Matching in the Time of LHC Run 2

3.2.1 Ambiguities

Before discussing the individual models and presenting our results, we have to define how we construct the effective models. Matching the dimension-six Lagrangian to a full model is a three-step procedure. Its starting point is the definition of a heavy mass scale Λ . Second, we integrate out the degrees of freedom above Λ as described in Section 2.2.3, which leads to an infinite tower of higher-dimensional operators. Finally, this effective action is truncated so that only the dimension-six terms, suppressed by $1/\Lambda^2$, remain.

The matching is not unambiguous: on the one hand, Λ is usually not uniquely defined. A typical case is a new physics scenario with only one heavy mass scale M in the Lagrangian, but also some mixing terms of the new fields with the SM Higgs doublet. In the unbroken electroweak phase the only new physics scale is then

$$\Lambda_1 = M. \quad (3.5)$$

But after electroweak symmetry breaking, the electroweak VEV contributes through the mixing term to the actual physical masses m of the new particles. Even if there is only one dimensionful scale of new physics in the Lagrangian, this defines additional scales of the form

$$\Lambda_2^2 = m^2 = M^2 \pm g^2 v^2, \quad (3.6)$$

where g is a combination of couplings or mixing angles. Of course there can be many such scales.

Further ambiguities arise in the third step since we can choose which parameters to keep constant while expanding in $1/\Lambda$. For instance we can often choose to define Wilson coefficients in terms of Lagrangian couplings or in terms of mixing angles. Again, the first choice corresponds to the natural choice in the unbroken phase of the electroweak symmetry, while the latter is often only defined in the broken phase.

In both scenarios, switching from one choice to another corresponds to including additional contributions suppressed by v^2/Λ^2 to the Wilson coefficients of the dimension-six operators. In

the first example,

$$\begin{aligned} \frac{f_x}{\Lambda_2^2} \mathcal{O}_x &= \frac{f_x}{M^2 \pm g^2 v^2} \mathcal{O}_x = \frac{f_x}{M^2} \left(1 \mp \frac{g^2 v^2}{M^2} + \mathcal{O}(1/M^4) \right) \mathcal{O}_x \\ &= \left(\frac{f_x}{\Lambda_1^2} \mp \frac{f_x g^2 v^2}{\Lambda_1^4} \right) \mathcal{O}_x + \mathcal{O}(1/\Lambda_1^6) . \end{aligned} \quad (3.7)$$

It should be stressed that these effects always contribute to observables at $\mathcal{O}(1/\Lambda^4)$, the same order in the EFT expansion as the leading effects from dimension-eight operators, which we always neglect. From a purely theoretical point of view these terms are subleading, and indeed in the obvious validity regime of the EFT they are irrelevant. But this is not the situation we find at the LHC. In practice, these formally suppressed terms may be important even if the dimension-eight terms are small.

These ambiguities in the matching procedure raise the question if we can improve the agreement between full model and dimension-six Lagrangian by incorporating effects of the non-zero electroweak VEV in the matching. To answer this question we now define two different matching prescriptions.

3.2.2 Default vs. v -improved matching

Our **default matching** follows a purely theoretical motivation and represents the conventional approach to matching. The linear Higgs EFT is formulated in terms of the doublet ϕ and based on the assumption $\Lambda \gg v$. It should therefore be matched to the full theory in the unbroken phase of the electroweak symmetry. An obvious choice for the matching scale is then the mass scale of new particles in the limit of $v \rightarrow 0$, which as in our simple example above we denote

$$\Lambda_{\text{default}} = M . \quad (3.8)$$

For simplicity we assume there is only one such scale, i. e. that all new particles are mass-degenerate in the unbroken electroweak phase. Otherwise the new particles would have to be integrated out consecutively at different scales M_i . We then expand the effective action, expressed in parameters of the Lagrangian, and drop all terms of $\mathcal{O}(\Lambda^{-4})$.

Alternatively, we define a **v -improved matching** procedure that accounts for additional terms suppressed by v^2/Λ^2 in the Wilson coefficients of the dimension-six Lagrangian. This corresponds to matching the linear EFT in the broken electroweak phase. In the first matching step, we define Λ as the physical mass m of the new particles in the broken phase including contributions from v ,

$$\Lambda_{v\text{-improved}} = m , \quad (3.9)$$

rather than the mass scale in the unbroken phase. Again, multiple particles with substantial mass splittings will require a multi-step matching procedure, but there is no fundamental problem to

describe them. The Wilson coefficients are expressed in terms of phenomenologically relevant quantities such as mixing angles and physical masses, again defined in the broken phase. This is a somewhat subjective criterion that depends on model and process: the v -improved matching procedure is not a unique definition, but rather a general guideline. We will demonstrate an example for a natural choice in the singlet scenario.

3.2.3 Making sense of v -improvement

We can interpret the two matching schemes from two different perspectives. First we analyse it from the unbroken electroweak phase. Some of the operators with dimension 8 or higher are of the form

$$\mathcal{O}_i^{(d=6+2n)} = (\phi^\dagger \phi)^n \mathcal{O}_i^{(6)}, \quad (3.10)$$

where $\mathcal{O}_i^{(6)}$ is a dimension-six operator. The untruncated, infinite tower of higher-dimensional operators generated from the full theory can be re-organised as

$$\begin{aligned} \mathcal{L}_{\text{EFT}} &\equiv \mathcal{L}_{\text{SM}} + \sum_{d=6}^{\infty} \sum_i \frac{f_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} \\ &= \mathcal{L}_{\text{SM}} + \sum_i \sum_{n=0}^{\infty} \frac{f_i^{(6+2n)}}{\Lambda^{2+2n}} (\phi^\dagger \phi)^n \mathcal{O}_i^{(6)} + \sum_{d=8}^{\infty} \sum_k \frac{f_k^{(d)}}{\Lambda^{d-4}} \mathcal{O}_k^{(d)}, \end{aligned} \quad (3.11)$$

where $\mathcal{O}_k^{(d)}$ are the dimension-eight and higher operators of a different form than Equation (3.10). A v -improved matching corresponds to replacing $\phi^\dagger \phi \rightarrow v^2/2$ in (part of) the first sum:¹

$$\mathcal{L}_{v\text{-improved dim-6}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{\overbrace{f_i^{(6), v\text{-improved}}}^{\sum_{n=0}^{\infty} f_i^{(6+2n)} \left(\frac{v^2}{2\Lambda^2} \right)^n}}{\Lambda^2} \mathcal{O}_i^{(6)}. \quad (3.12)$$

So from the perspective of the unbroken phase of the electroweak symmetry, v -improvement corresponds to a **partial resummation of dimension-eight and higher operators** into the Wilson coefficients of the dimension-six operators.

From an experimental point of view, or in the broken electroweak phase, physical masses and mixing angles are simply the natural choices to describe a model. We then expect that the v -improved matching procedure can improve the validity of the dimension-six model. For a

¹The argument is slightly more complicated if additional powers of ϕ appear in $\mathcal{O}_i^{(6)}$. One can then also take into account terms where instances of ϕ in $\mathcal{O}_i^{(6)}$ are replaced by the VEV and fields in the prefactor are left alone. This effectively adds a combinatorial factor to Equation (3.12).

more precise statement we have to distinguish between an expansion in v/Λ and \mathbf{p}/Λ . We expect the v -improved matching prescription can lead to a better agreement with the full models in situations where the expansion in v/Λ is relevant, while it cannot help with the expansion in \mathbf{p}/Λ , corresponding to genuine new dimension-eight operators not of the form in Equation (3.10). We will come back to this difference later and demonstrate it in the vector triplet scenario.

Again, the truncation of the EFT Lagrangian is formally justified as long as $v \ll \Lambda$ and we only probe energies $E \ll \Lambda$. In this limit the dimension-eight operators as well as the Λ -suppressed terms in the Wilson coefficients are negligible; our two matching procedures then give identical results. In the absence of a large enough scale separation, our bottom-up approach allows us to treat them independently. This way we can use the v -improved matching to enhance the validity of the dimension-six Lagrangian.

3.3 Full models vs. effective theory

The main aim of this chapter is to compare a comprehensive set of LHC predictions from specific new physics models to their corresponding effective field theory predictions. In this way we test the applicability of the dimension-six model for four different, more or less UV-complete, scenarios of underlying physics:

1. a scalar singlet extension with mixing effects and a second scalar resonance;
2. a two-Higgs doublet model, adding a variable Yukawa structure, a CP-odd, and a charged Higgs;
3. scalar top partners, contributing to Higgs couplings at one loop; and
4. a vector triplet with gauge boson mixing.

This ensemble of models covers a wide range of CP -even new physics signatures in the Higgs sector.

After describing our technical setup, we analyse these four scenarios one by one. For each model we first define the theory and introduce the main phenomenological features at the LHC. We discuss the decoupling in the Higgs sector, and derive the dimension-six setup. Finally, we define a number of benchmark points and give a detailed account of the full and dimension-six phenomenology at the LHC.

Effects in the SM-like Higgs couplings will be parametrised with the relative shifts from the SM values

$$\Delta_x \equiv \frac{g_{hxx}}{g_{hxx}^{\text{SM}}} - 1, \quad (3.13)$$

as defined in Equation (2.80). Unlike in the published version [2], we express the effective Lagrangian in the HISZ basis with the ten dimension-six operators of Equation (2.60).

3.3.1 Setup

Our comparison covers the most relevant observables for LHC Higgs physics. Acceptance and background rejections cuts are kept to a minimum to be able to test the effective field theory approach over as much of the phase space as possible.

In the case of Higgs production through gluon fusion, we analyse the production process with a Higgs decay to four leptons or to photons,

$$\begin{aligned} gg &\rightarrow h \rightarrow 4\ell, \\ gg &\rightarrow h \rightarrow \gamma\gamma. \end{aligned} \quad (3.14)$$

For the photons we do not apply any cuts, while for $\ell = e, \mu$ we require

$$m_{4\ell} > 100 \text{ GeV} \quad \text{and} \quad m_{\ell^+\ell^-}^{\text{same flavour}} > 10 \text{ GeV} \quad (3.15)$$

to avoid too large contributions from the Z peak and bremsstrahlung.

For Higgs production in weak boson fusion (WBF), we first evaluate the pure production process

$$ud \rightarrow h ud, \quad (3.16)$$

which is the dominant partonic contribution at the LHC, without any cuts. We also consider WBF production with a decay

$$ud \rightarrow W^+ W^- ud \rightarrow (\ell^+ \nu) (\ell^- \bar{\nu}) ud, \quad (3.17)$$

where we require the standard WBF cuts

$$\begin{aligned} p_{T,j} &> 20 \text{ GeV}, & \Delta\eta_{jj} &> 3.6, & m_{jj} &> 500 \text{ GeV}, \\ p_{T,\ell} &> 10 \text{ GeV}, & \cancel{E}_T &> 10 \text{ GeV}. \end{aligned} \quad (3.18)$$

Unlike for gluon fusion, the kinematics of the final state can now introduce new scales and a dependence on the UV structure of the model. The process is particularly interesting in the context of perturbative unitarity [138–144]. While the latter is satisfied in a UV-complete model by construction, deviations from the SM Higgs-gauge couplings in the EFT may lead to an increasing rate at very large energies [41, 145], well outside the EFT validity range $E/\Lambda \ll 1$. To look for such signatures, we focus on the high-energy tail of the transverse mass distribution,

$$m_T^2 = (E_{T,\ell\ell} + E_{T,\nu\nu})^2 - (\mathbf{p}_{T,\ell\ell} + \mathbf{p}_T^{\text{miss}})^2 \quad (3.19)$$

with

$$E_{T,\ell\ell} = \sqrt{\mathbf{p}_{T,\ell\ell}^2 + m_{\ell\ell}^2}, \quad E_{T,\nu\nu} = \sqrt{\cancel{E}_T^2 + m_{\ell\ell}^2}. \quad (3.20)$$

As the last single Higgs production process we evaluate Higgs-strahlung

$$qq \rightarrow Vh \quad (3.21)$$

with $V = W^\pm, Z$. We do not simulate the Higgs and gauge boson decays, assuming that we can always reconstruct for example the full $Zh \rightarrow \ell^+ \ell^- b\bar{b}$ final state. No cuts are applied.

Finally, Higgs pair production,

$$gg \rightarrow hh, \quad (3.22)$$

is well known to be problematic when it comes to the effective theory description [43, 146, 147]. Again, neither Higgs decays nor kinematic cuts are expected to affect our analysis, so we leave them out.

We test all these channels for the singlet and doublet Higgs sector extensions. For the top partner and vector triplet models we focus on the WBF and Higgs-strahlung modes.

In the dimension-six simulations we always include the square of the dimension-six operator contributions. While these terms are technically of the same mass dimension as dimension-eight operators, which we neglect, we keep them to avoid negative values of the squared matrix element in extreme phase-space regions. Notice that these situations do not necessarily imply a breakdown of the EFT expansion. On the contrary, they may appear in scenarios where new physics contributions dominate over the SM part, while the EFT expansion is fully valid (with $E/\Lambda \ll 1$). In such cases, the bulk effects stem from the squared dimension-six terms instead of the interference with the SM, while the effects from dimension-eight operators are smaller and can be safely neglected. We will discuss this aspect in more detail in the next section.

Most amplitudes are calculated at leading order in α_s and α_{ew} , which is sufficient given the size of new physics effects that the LHC is sensitive to. We always take into account interference terms between Higgs and gauge amplitudes.

For tree-level processes we generate event samples with MadGraph 5 [148], using publicly available our own model files implemented in FeynRules [149], which provides the corresponding UFO files [150]. For the dimension-six predictions we resort to a modified version of the HEL model file [151].

The Higgs-gluon and Higgs-photon couplings are evaluated with the full one-loop form factors [152], including top, bottom and W loops as well as new particles present in the respective models. For Higgs pair production, we use a modified version of Reference [153], see also Reference [154].

Other loop effects are analysed using reweighting: we generate event samples using appropriate general couplings. Next, we compute the one-loop matrix element for each phase space point and reweight the events with the ratio of the renormalised one-loop matrix element squared to the tree-level model. For the one-loop matrix elements we utilise FeynArts and FormCalc [155]

with our own model files that include the necessary counterterms. The loop form factors are handled with dimensional regularisation in the 't Hooft-Veltman scheme, and written in terms of standard loop integrals. These are further reduced via Passarino-Veltman decomposition and evaluated with the help of LoopTools [156].

Generally we create event samples of at least 100 000 events per benchmark point and process for pp collisions at $\sqrt{s} = 13$ TeV. We use the CTEQ6L pdf [157] and the default dynamical choices of the factorisation and renormalisation scale implemented in MadGraph. For this broad survey of EFT validity we limit ourselves to parton level and do not apply a detector simulation, a more realistic simulation will be discussed in the next section. The mass of the SM-like Higgs is fixed to $m_h = 125$ GeV [158]. For the top mass we take $m_t = 173.2$ GeV [159, 160]. The Higgs width in each model is based on calculations with Hdecay [161], which we rescale with the appropriate coupling modifiers and complement with additional decay channels where applicable.

3.3.2 Singlet extension

Model setup

The simplest extension of the minimal Higgs sector of the Standard Model is by a real scalar singlet S [162–168]. For the sake of simplicity we consider a minimal version of the singlet model, in which a discrete \mathbb{Z}_2 parity forbids additional (e. g. cubic) terms in the potential. The theory is then given by

$$\mathcal{L}_{\text{singlet}} \supset (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2} \partial_\mu S \partial^\mu S - V(\phi, S), \quad (3.23)$$

where the scalar potential has the form

$$V(\phi, S) = \mu_1^2 \phi^\dagger \phi + \lambda_1 (\phi^\dagger \phi)^2 + \mu_2^2 S^2 + \lambda_2 S^4 + \lambda_3 (\phi^\dagger \phi) S^2. \quad (3.24)$$

The new scalar S can mix with the SM doublet ϕ provided the singlet develops a VEV,

$$\langle S \rangle = \frac{v_s}{\sqrt{2}}. \quad (3.25)$$

The mixing angle is given by

$$\tan(2\alpha) = \frac{\lambda_3 v v_s}{\lambda_2 v_s^2 - \lambda_1 v^2}. \quad (3.26)$$

Details on the parametrisation and Higgs mass spectrum are given in Appendix A.2.1.

Signatures and decoupling patterns

The additional scalar singlet affects Higgs physics in three ways. First, it mixes with the Higgs via the mixing angle α , which leads to a universal rescaling of all Higgs couplings to fermions and vectors. Second, it modifies the Higgs self-coupling. Finally, it introduces a new, heavy resonance H coupled to the Standard Model through mixing.

The key parameter is the portal interaction between the doublet and the singlet fields $\lambda_3(\phi^\dagger\phi)S^2$, which is responsible for the mixed mass eigenstates. The mixing reduces the coupling of the SM-like Higgs h to all Standard Model particles universally,

$$\Delta_x = \cos \alpha - 1 \quad (3.27)$$

for $x = W, Z, t, b, \tau, g, \gamma, \dots$. It also affects the self-coupling of the light Higgs, which takes on the form

$$g_{hhh} = 6 \cos^3 \alpha \lambda_1 v - 3 \cos^2 \alpha \sin \alpha \lambda_3 v_s + 3 \cos \alpha \sin^2 \alpha \lambda_3 v - 6 \sin^3 \alpha \lambda_2 v_s. \quad (3.28)$$

The parameter $\sin \alpha$ quantifies the departure from the SM limit $\alpha \rightarrow 0$. This limit can be attained in two ways: first, a small mixing angle can be caused by a weak portal interaction,

$$|\tan(2\alpha)| = \left| \frac{\lambda_3 v v_s}{\lambda_2 v_s^2 - \lambda_1 v^2} \right| \ll 1 \quad \text{if} \quad \lambda_3 \ll 1. \quad (3.29)$$

The Higgs couplings to SM particles approach their SM values, but there is no large mass scale associated with this limit. In the extreme case of $\lambda_2, \lambda_3 \ll \lambda_1$ we find small $\alpha \approx -\lambda_3/\lambda_1 \times v_s/(2v)$ even for $v_s \lesssim v$. This situation is to some extent the singlet model counterpart of the “alignment without decoupling” scenario in the Two-Higgs-doublet model (2HDM) [169, 170] or the MSSM [171, 172]. It relies on a weak portal coupling and a small scale separation, which cannot be properly described by an effective field theory.

As a second possibility, the additional singlet can introduce a large mass scale $v_s \gg v$, giving us

$$\tan \alpha \approx \frac{\lambda_3}{2\lambda_2} \frac{v}{v_s} \ll 1 \quad \text{if} \quad v \ll v_s, \quad (3.30)$$

where $\lambda_3/(2\lambda_2)$ is an effective coupling of up to order one. In this limit the heavy Higgs mass, which we identify as the heavy mass scale, is given by

$$m_H \approx \sqrt{2\lambda_2} v_s. \quad (3.31)$$

In terms of m_H , the Higgs couplings scale like

$$\Delta_x = -\frac{\alpha^2}{2} + \mathcal{O}(\alpha^3) \approx -\frac{\lambda_3^2}{4\lambda_2} \left(\frac{v}{m_H} \right)^2. \quad (3.32)$$

This is a dimension-six effect. If we require $|\Delta_x| \gtrsim 10\%$ to keep our discussion relevant for the LHC, this implies

$$m_H \approx \sqrt{2\lambda_2} v_s < \frac{\sqrt{5}\lambda_3}{\sqrt{2\lambda_2}} v = 390 \text{ GeV} \times \frac{\lambda_3}{\sqrt{\lambda_2}}. \quad (3.33)$$

If we also assume that the ratio of quartic couplings is of the order of a perturbative coupling, $\lambda_3/\sqrt{\lambda_2} \lesssim 0.5$, the LHC reach in the Higgs coupling analysis translates into heavy Higgs masses below 200 GeV. For strongly coupled scenarios, $\lambda_3/\sqrt{\lambda_2} \lesssim 1 \dots \sqrt{4\pi}$, the heavy mass reach increases to $m_H \lesssim 0.4 \dots 1.5$ TeV. This suggests that a weakly coupled Higgs portal will fail to produce a sizeable separation of scales when looking at realistic Higgs coupling analyses. The question becomes if and where this lack of scale separation hampers our LHC analyses.

Dimension-six description

In the EFT approach the singlet model only generates $\mathcal{O}_{\phi,2}$ at dimension six [122]. Before electroweak symmetry breaking, the only mass scale in the Lagrangian that describes the new physics is $\mu_2^2 < 0$. Defining the Wilson coefficients suppressed by this new physics scale gives clearly wrong results, as we will discuss in the analogous case for the two-Higgs-doublet model in the next section. Instead identify the leading contribution to the heavy Higgs mass as the new physics scale in our default matching, in agreement with the logic Section 3.2. Following the discussion of decoupling patterns above this means

$$\Lambda_{\text{default}} = \sqrt{2\lambda_2} v_s \approx m_H. \quad (3.34)$$

The corresponding Wilson coefficient, expressed in terms of Lagrangian parameters, is

$$\bar{f}_{\phi,2}^{\text{default}} = \frac{\lambda_3^2}{2\lambda_2}. \quad (3.35)$$

For the v -improved matching, we instead use the actual physical mass

$$\Lambda_{v\text{-improved}} = m_H. \quad (3.36)$$

In the broken phase the Higgs couplings are fully expressed through the mixing angle α as given in Equation (3.27). We define

$$f_{\phi,2}^{v\text{-improved}} = 2(1 - \cos \alpha) \frac{m_H^2}{v^2}. \quad (3.37)$$

which ensures that the Higgs couplings exactly agree between the full model and the v -improved dimension-six description.

	Singlet				Default EFT			ν -improved EFT		
	m_H	$\sin \alpha$	v_s/v	Δ_x	Λ	$f_{\phi,2}$	Δ_x	Λ	$f_{\phi,2}$	Δ_x
S ₁	500	0.2	10	−0.020	491	0.14	−0.018	500	0.15	−0.020
S ₂	350	0.3	10	−0.046	336	0.14	−0.037	350	0.16	−0.046
S ₃	200	0.4	10	−0.083	190	0.04	−0.031	200	0.06	−0.083
S ₄	1000	0.4	10	−0.083	918	2.60	−0.092	1000	3.13	−0.083
S ₅	500	0.6	10	−0.200	407	1.26	−0.231	500	1.24	−0.200

Table 3.1: Benchmarks for the singlet extension. We show the model parameters and the universal coupling modification for the complete model, as well as the cutoff scales Λ , Wilson coefficients $\tilde{f}_{\phi,2}$, and the universal coupling modification in the EFT approach for the default and ν -improved matching schemes. All mass scales are given in GeV.

Benchmark points

We start our numerical analysis by defining five singlet benchmark points in Table 3.1, with heavy Higgses ranging from 200 to 1000 GeV. The first three scenarios are in agreement with current experimental and theoretical constraints. This includes direct mass bounds from heavy Higgs searches at colliders, Higgs coupling measurements, electroweak precision observables, perturbative unitarity and vacuum stability [165–167]. Note that for S₄ and S₅ the combination of large heavy Higgs masses together with large mixing angles is incompatible with perturbative unitarity and electroweak precision constraints. We nevertheless keep such benchmarks for illustration purposes.

Higgs couplings and total production rates

Table 3.1 also shows the universal shift Δ_x of the light Higgs couplings, both for the full singlet model and its dimension-six EFT descriptions. For the default matching, we find reasonable agreement with the full model for the scenarios with a heavy additional Higgs, while large discrepancies appear when the new physics is lighter. In particular, note the difference between S₃ and S₄. Both describe the same coupling shift $\Delta_x = -0.08$. But while S₃ realises this with a weakly coupled light Higgs, which the default EFT cannot describe, S₄ has a heavier, more strongly coupled Higgs, and the EFT description works. In all cases, the ν -improved EFT by construction predicts the Higgs couplings correctly.

In Table 3.2 we show how well the effective models describe the total Higgs production cross sections in gluon fusion, WBF and Higgs-strahlung. These numbers confirm what we expect from the coupling modifications: while the default dimension-six model predict qualitatively similar shifts in the total rates, there are rate deviations of up to 10%. In the ν -improved EFT we

	$\sigma_{\text{default EFT}}/\sigma_{\text{singlet}}$			$\sigma_{\nu\text{-improved EFT}}/\sigma_{\text{singlet}}$		
	ggF	WBF	Vh	ggF	WBF	Vh
S1	1.006	1.006	1.004	1.001	1.001	1.000
S2	1.019	1.021	1.019	1.000	1.001	1.000
S3	1.119	1.118	1.118	1.000	0.999	1.000
S4	0.982	0.982	0.982	0.999	0.999	1.000
S5	0.925	0.925	0.925	0.999	0.999	1.000

Table 3.2: Cross section ratios of the matched dimension-six EFT approximation to the full singlet model at the LHC. We show the leading Higgs production channels for all singlet benchmark points. The statistical uncertainties on these ratios are below 0.4%.

find that the Higgs couplings and total rates agree exactly with the full model predictions. The dimension-six operators are entirely sufficient to capture the coupling shifts, but a significant part of their coefficients are formally of $\mathcal{O}(v^4/\Lambda^4)$.

Distributions

The most obvious source of discrepancy between the full model and the EFT is the heavy resonance H . It can for example be produced in gluon fusion and then observed as a peak in the $m_{4\ell}$ distribution. By construction, it will not be captured by the dimension-six model. We illustrate this in the upper left panel of Figure 3.1. For Higgs-strahlung production (Figure 3.1, upper right panel), where the novel H resonance does not appear in an intermediate Born-level propagator and hence has no impact, we find instead excellent agreement between both descriptions over the entire phase space.

The second Higgs has a second, more subtle effect. In the full model, both Higgs exchange diagrams are needed to unitarise WW scattering. Correspondingly, the EFT description breaks perturbative unitarity roughly at the scale [145]

$$m_{WW}^2 \sim \frac{16\pi}{\frac{f_{\phi,2}}{\Lambda^2} \left(1 - \frac{f_{\phi,2}v^2/\Lambda^2}{4(1+f_{\phi,2}v^2/\Lambda^2)}\right)} \approx \left(\frac{1.7 \text{ TeV}}{\sin \alpha}\right)^2. \quad (3.38)$$

In our benchmark point S5, this is around 2.8 TeV. The incomplete cancellations between Higgs and gauge amplitudes means that the dimension-six model tends to have a larger rate at energies already below this scale. For this specific benchmark choice, this can be seen in the lower left panel of Figure 3.1, where we show the distribution of the transverse mass defined in Equation (3.19) in the process $ud \rightarrow W^+ W^- ud \rightarrow (\ell^+ \nu) (\ell^- \bar{\nu}) ud$, to which WBF production of both h and H contributes. We observe that the dimension-six model predicts a slightly higher rate at large m_T

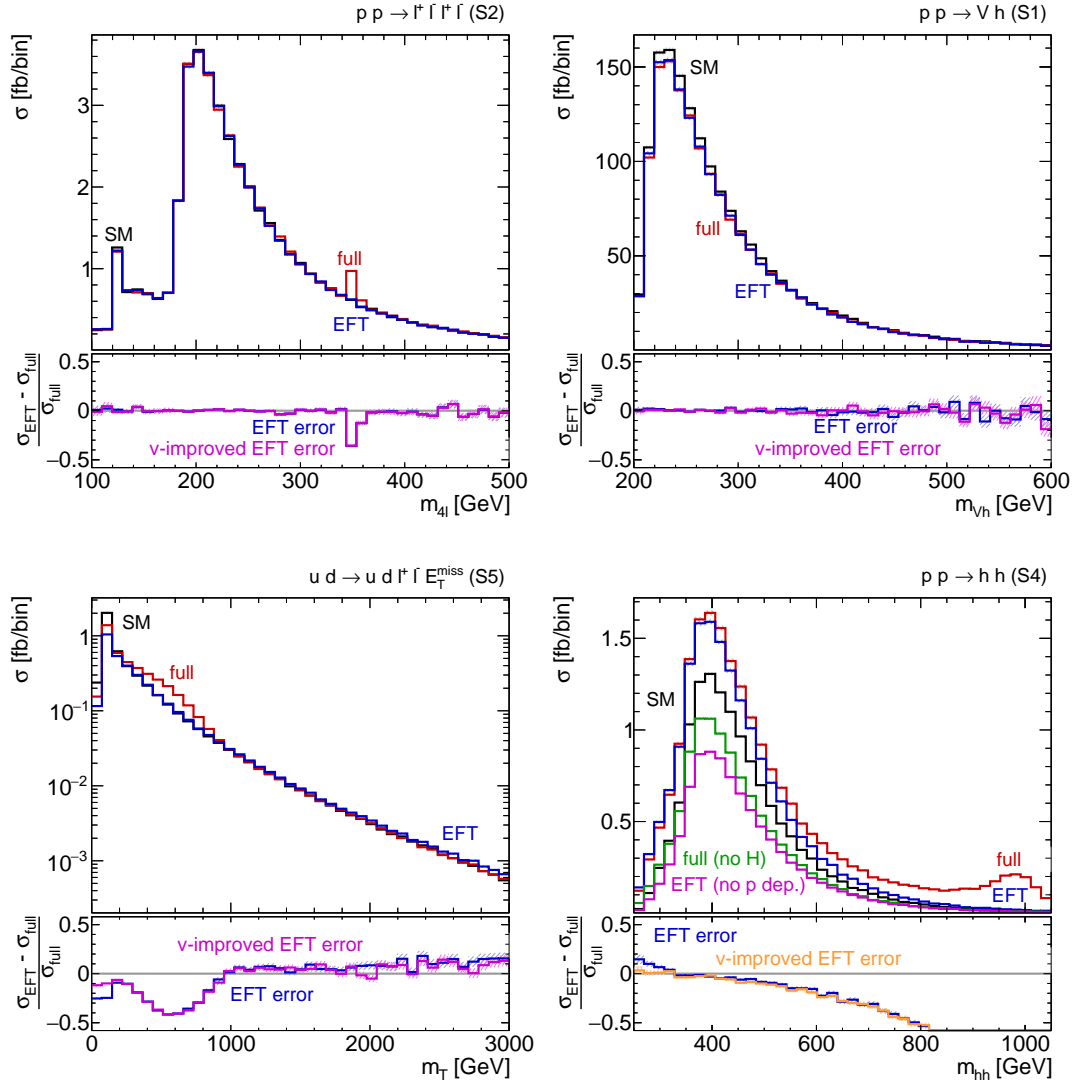


Figure 3.1: Selected kinematic distributions in the singlet model. The different curves show the SM, the full singlet model, and the dimension-six model. Top left: $m_{4\ell}$ distribution in the $gg \rightarrow h \rightarrow 4\ell$ channel for S2. Top right: m_{Vh} distribution in Vh production for S1. Bottom left: m_T distribution in the WBF $h \rightarrow \ell^+ \ell^- \cancel{E}_T$ channel for S5. Bottom right: m_{hh} distribution in Higgs pair production for S4. For m_{hh} we show several contributions in the full theory and the dimension-six approach. In all plots, the error bars give the statistical uncertainties.

than both the full singlet model and the SM. Given the very mild signal, which results from the fast decrease in the parton densities and the small mixing angle for realistic scenarios, such effect is likely of no relevance for LHC physics.

A more interesting channel to study in the singlet model is Higgs pair production. The Higgs self-coupling is the only Higgs coupling which gains a momentum dependence in the matched EFT. In addition, there exists an approximate cancellation between the two leading amplitudes in the SM at threshold [42, 44]. This induces a second relevant scale and with it a sensitivity to small deviations in the Higgs couplings.

In the bottom right panel of Figure 3.1 we give the m_{hh} distribution in the full and dimension-six models. In addition, we show how the distributions would look in the full model without a H state, and in the EFT without the momentum-dependent (derivative) terms from $\mathcal{O}_{\phi,2}$. Already at threshold and far away from the H resonance, the interference of the SM-like terms with the H diagrams makes up a significant part of the amplitude. In the EFT, the derivative terms are similarly relevant already at low energies. Close to threshold, the (v -improved) dimension-six model approximates the full theory well. But this agreement becomes worse already at moderately larger energies, and clearly breaks down towards the H pole.

Summary

If we limit ourselves to Higgs properties relevant for single Higgs production at the LHC, the modifications from a singlet extension are very simple: all Standard Model couplings acquire a common scaling factor, and no relevant new Lorentz structures appear at tree-level. The dimension-six setup reproduces this effect correctly: the reduced couplings to all SM fields alone do not require a large hierarchy of scales. A standard matching procedure that expands the coefficients to leading order in v^2/Λ^2 may lead to sizeable deviations from the full model. However, a v -improved EFT construction that takes into account higher orders in v^2/Λ^2 gives perfect agreement with the full theory. In other words, many of the dimension-eight and higher operators generated from the singlet model are of the form $(\phi^\dagger \phi)^n \mathcal{O}_i^{(6)}$. After EWSB they can be resummed into the Wilson coefficients of the dimension-six operators as given by Eq. (3.12).

Higgs pair production is different. There is a large contribution from off-shell H , while in the EFT the h self-coupling involves a derivative. These different structures lead to discrepancies between full and effective description that increase with momentum transfer. Of course, the effective theory by definition does not include the second resonance, so it fails whenever a heavy Higgs appears on-shell in the full theory.

3.3.3 Two-Higgs-doublet model

Model setup

The two-Higgs-doublet model (2HDM) [173, 174] adds a second scalar $SU(2)_L$ doublet ϕ_2 with hypercharge $Y_{\phi_2} = 1/2$ to the SM Higgs sector. The combined potential reads

$$V(\phi_1, \phi_2) = m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 |\phi_1^\dagger \phi_2|^2 + \left[-m_{12}^2 \phi_1^\dagger \phi_2 + \frac{\lambda_5}{2} (\phi_1^\dagger \phi_2)^2 + \text{h. c.} \right]. \quad (3.39)$$

The mass terms m_{ij}^2 and the dimensionless self-couplings λ_i are real parameters. The doublet VEVs $\langle \phi_j^0 \rangle = v_j/\sqrt{2}$ are parametrised by their ratio $\tan \beta = v_2/v_1$. For the Yukawa couplings, there are four possible scenarios that satisfy the SM flavour symmetry and preclude tree-level flavour-changing neutral currents (FCNCs) [175]:

- type I, where all fermions couple to just one Higgs doublet ϕ_2 ;
- type II, where up-type (down-type) fermions couple exclusively to ϕ_2 (ϕ_1);
- lepton-specific, with a type-I quark sector and a type-II lepton sector; and
- flipped, with a type-II quark sector and a type-I lepton sector.

For simplicity, we restrict our discussion to type I and type II. In all four cases, the absence of tree-level FCNCs is protected by a global \mathbb{Z}_2 discrete symmetry $\phi_i \rightarrow (-1)^i \phi_i$ (for $i = 1, 2$). This symmetry is softly broken by the mixed mass term m_{12} .

The physical degrees of freedom are two neutral CP -even scalars h^0, H^0 , one neutral CP -odd scalar A^0 , and a set of charged scalars H^\pm , parametrised by the mixing angle between the CP -even scalars α . For a detailed account of the model setup, see Appendix A.2.2.

Signatures and decoupling patterns

Just as the singlet extension, the 2HDM predicts two types of LHC signatures: first, scalar and VEV mixing lead to modified light Higgs couplings. Unlike for the singlet extension, these coupling modifications are not universal and reflect the more flexible flavour structure as well as the multiple scales of the model. Second, there are three new heavy resonances H^0, A^0 , and H^\pm , which should have near-degenerate masses to avoid custodial symmetry breaking.

The light Higgs coupling to weak bosons $V = W, Z$ always scales like

$$\Delta_V = \sin(\beta - \alpha) - 1 = -\frac{\cos^2(\beta - \alpha)}{2} + \mathcal{O}(\cos^4(\beta - \alpha)). \quad (3.40)$$

We can insert the leading contribution of a mass-degenerate heavy Higgs sector and find

$$\Delta_V \approx \frac{\sin^2(2\beta)}{8} \left(\frac{v}{m_{A^0}} \right)^4. \quad (3.41)$$

While in the singlet model the light Higgs coupling to gauge bosons is shifted at $\mathcal{O}(v^2/m_H^2)$, see Equation (3.32), the same coupling is now affected at $\mathcal{O}(v^4/m_{A^0}^4)$, corresponding to a dimension-eight effect.

The couplings to the fermions on the other hand are modified at $\mathcal{O}(v^2/m_{A^0}^2)$. For up-type quarks, we find

$$1 + \Delta_t = \frac{\cos \alpha}{\sin \beta}. \quad (3.42)$$

The couplings to down-type quarks and leptons are

$$1 + \Delta_b = 1 + \Delta_\tau = \frac{\cos \alpha}{\sin \beta} \quad (3.43)$$

in a type-I model and

$$1 + \Delta_b = 1 + \Delta_\tau = -\frac{\sin \alpha}{\cos \beta} \quad (3.44)$$

in a type-II 2HDM.

Finally, a H^\pm loop contributes to the Higgs-photon coupling. The expression for this coupling shift is given in Equation (A.32) in Appendix A.2.2.

Two aspects turn the decoupling in the general 2HDM into a challenge: first, delayed decoupling effects appear after electroweak symmetry breaking [176]. For example, in type-II models we find [106]

$$\Delta_b \approx -\tan \beta \frac{\sin(2\beta)}{2} \left(\frac{v}{m_{A^0}} \right)^2. \quad (3.45)$$

This correction to the bottom Yukawa coupling corresponds to a dimension-six effect, and already moderate values of $\tan \beta$ significantly delay the decoupling of the heavy 2HDM states in the Yukawa sector.

Second, unlike in the MSSM, the Higgs self-couplings $\lambda_1 \dots \lambda_5$ and m_{12} are not bounded from above. In combinations like $\lambda_j v^2$, potentially enhanced with factors of $\tan' \beta$, they contribute to the masses of the heavy Higgs bosons and to the interactions of the SM-like Higgs state, effectively inducing new energy scales.

Such additional mass scales driven by v leads to problems with any EFT derived from and matched to the full theory assuming only one new physics scale. While this should not be viewed as a problem of the EFT approach in general, it will require a v -improved matching procedure.

Dimension-six description

We first follow our default procedure and match the effective theory to the 2HDM in the unbroken electroweak phase. To this end, we first rotate ϕ_1 and ϕ_2 into the so-called Higgs basis, where only one Higgs doublet obtains a vacuum expectation value, $\langle \phi_l \rangle = v/\sqrt{2}$, $\langle \phi_h \rangle = 0$ [175, 177]. This doublet ϕ_l is then identified with the SM-like Higgs doublet, while the other doublet ϕ_h is integrated out. This doublet ϕ_l is then identified with the SM-like Higgs doublet, while the other doublet ϕ_h is integrated out. Its decoupling is described by the mass scale

$$\Lambda_{\text{default}}^2 = m_{11}^2 \sin^2 \beta + m_{22}^2 \cos^2 \beta + m_{12}^2 \sin(2\beta). \quad (3.46)$$

The 2HDM generates a number of dimension-six operators at tree level, for which the Wilson coefficients depend on the flavour structure. While the up-type Yukawa coupling is always modified the same way, the down-type and lepton couplings are different for type-I and type-II. With the definition

$$\bar{\lambda} \equiv \frac{\sin(2\beta)}{2} \left[\frac{\lambda_1}{2} - \frac{\lambda_2}{2} + \left(\frac{\lambda_1}{2} + \frac{\lambda_2}{2} - \lambda_3 - \lambda_4 - \lambda_5 \right) \cos(2\beta) \right] \quad (3.47)$$

we find

$$\begin{aligned} f_t &= -\frac{\bar{\lambda} y_t}{\tan \beta}, \\ f_b &= \begin{cases} -\frac{\bar{\lambda} y_b}{\tan \beta} & \text{type I,} \\ \bar{\lambda} y_b \tan \beta & \text{type II,} \end{cases} \\ f_\tau &= \begin{cases} -\frac{\bar{\lambda} y_\tau}{\tan \beta} & \text{type I,} \\ \bar{\lambda} y_\tau \tan \beta & \text{type II.} \end{cases} \end{aligned} \quad (3.48)$$

Here y_t , y_b , y_τ refer to the SM values of the respective Yukawa couplings, $y_f = \sqrt{2}m_f/v$.

The contribution of the H^\pm loop to the Higgs-photon coupling is mapped onto \mathcal{O}_{BB} with a Wilson coefficient

$$\begin{aligned} f_{BB} &= \frac{(\tan \beta + \cot \beta)}{3072 \pi^2} \left[\left(\lambda_1 + \lambda_2 - 2\lambda_3 + 6\lambda_4 + 6\lambda_5 - 8\frac{m_{h^0}^2}{v^2} \right) \sin(2\beta) \right. \\ &\quad \left. + 2(\lambda_1 - \lambda_2) \sin(4\beta) + (\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4 - 2\lambda_5) \sin(6\beta) \right]. \end{aligned} \quad (3.49)$$

In the effective Lagrangian there are no non-decoupling term of $\mathcal{O}(\Lambda^0)$ because the charged Higgs loop decouples in the limit $m_{A^0} \rightarrow \infty$ with finite λ_i . If instead we keep m_{12} fixed and let

2HDM							
	Type	$\tan \beta$	α/π	m_{12}	m_{H^0}	m_{A^0}	m_{H^\pm}
D1	I	1.5	-0.086	45	230	300	350
D2	II	15	-0.023	116	449	450	457
D3	II	10	0.032	157	500	500	500
D4	I	20	0	45	200	500	500

Table 3.3: Benchmarks for the 2HDM extension. We show the model parameters and the heavy Higgs masses. All masses are in GeV.

one of the couplings λ_i grow with m_{A^0} , the charged Higgs does not decouple. For a derivation of these results see Appendix A.2.2.

Upon electroweak symmetry breaking, the physical heavy Higgs masses m_{H^0} , m_{A^0} , and m_{H^\pm} acquire contributions $\sim \lambda_i v^2$ from the electroweak VEV in addition to the heavy scale Λ_{default} defined above. We therefore again consider an alternative v -improved matching where the matching scale is

$$\Lambda_{v\text{-improved}} = m_{A^0}. \quad (3.50)$$

In this setup, the Wilson coefficients in Equations (3.48) and (3.49) remain unchanged. The two matching schemes can exhibit significant differences in the 2HDM since the pseudoscalar mass $m_{A^0}^2 = m_{12}^2/(\sin \beta \cos \beta) - \lambda_5 v^2$ does not coincide with Λ_{default} over wide ranges of the parameter space.

Benchmark points

In Table 3.3 we define four benchmark points for the 2HDM. They are in agreement with all constraints at the time of publication of Reference [2], implemented with the help of 2HDMC [178], HiggsBounds [179, 180], SuperIso [181], and HiggsSignals [182]. To better illustrate certain model features, in some scenarios we tolerate deviations between 1σ and 2σ in the Higgs couplings measurements.

The key physics properties of the different 2HDM scenarios can be summarised as:

- D1 **Moderate decoupling:** with Higgs couplings shifts of up to 2σ in terms of the LHC constraints. This generates $\Delta_{\tau,b,t} \approx \mathcal{O}(15\%)$ as well as a large $h^0 H^+ H^-$ coupling. Additional Higgs masses around 250 . . . 350 GeV can leave visible imprints.
- D2 **Supersymmetric:** reproducing the characteristic mass splittings and Higgs self-couplings of the MSSM with light stops [183].

	$ \Lambda $ [GeV]	Default EFT			
		f_t	f_b	f_τ	f_{BB}
D1	100	0.12	0.003	0.001	0.009
D2	448	0.00	-0.006	-0.002	-0.001
D3	99	-0.07	0.206	0.077	-0.016
D4	142	0.00	0.000	0.000	-0.023

Table 3.4: Matching scales and Wilson coefficients for the effective theory matched to the 2HDM, based on the default matching in the unbroken phase.

	Λ [GeV]	ν -improved EFT			
		f_t	f_b	f_τ	f_{BB}
D1	300	-0.12	-0.003	-0.001	-0.009
D2	450	0.00	-0.006	-0.002	-0.001
D3	500	-0.07	0.206	0.077	-0.016
D4	500	0.00	0.000	0.000	-0.023

Table 3.5: Matching scales and Wilson coefficients for the effective theory matched to the 2HDM, based on the ν -improved matching in the unbroken phase with $\Lambda = m_{A^0}$.

D3 **Sign-flipped bottom Yukawa**: this is possible in type-II models at large $\tan \beta$, as shown in Equation (3.45) [184]. This can be viewed as a manifestation of a delayed decoupling [176].

D4 **Fermiophobic heavy Higgs**: possible only in type-I models for $\sin \alpha = 0$. The heavy Higgs H^0 is relatively light, but essentially impossible to observe at the LHC [154].

In Tables 3.4 and 3.5 we show the heavy scales Λ and the Wilson coefficients for the EFT in the two matching schemes. With the exception of benchmark D2, the suppression scales are drastically different. The matching in the unbroken phase is particularly pathological in benchmark D1, where $\Lambda_{\text{default}}^2$ is negative and the signs of the Wilson coefficients are switched compared to the ν -improved matching.

Higgs couplings and total production rates

Table 3.6 shows the tree-level coupling shifts of the light Higgs in the three models. The results confirm that the default matching defined in the unbroken phase does not reproduce the coupling patterns of the full model at all. We conclude that an EFT matched to the 2HDM in the unbroken electroweak phase is essentially useless, and we have to rely on ν -improved matching. For simplicity,

	Δ_V		Δ_t			$\Delta_b = \Delta_\tau$		
	2HDM	EFT	2HDM	dEFT	ν EFT	2HDM	dEFT	ν EFT
D1	-0.05	0.00	0.16	-0.74	0.08	0.16	-0.74	0.08
D2	0.00	0.00	0.00	0.00	0.00	0.07	0.07	0.07
D3	-0.02	0.00	0.00	0.46	0.02	-2.02	-46.5	-1.84
D4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.6: Normalised tree-level couplings of the light Higgs in our 2HDM benchmarks, comparing the full 2HDM model to the EFT based on the default matching (“dEFT”) and that based on ν -improved matching (“ ν EFT”).

we will from now on leave out the results based on the default matching, which only confirm these initial results.

The ν -improved matching, on the other hand, essentially captures most of the coupling shifts. It still fails to describe shifts in the couplings to weak bosons, which correspond to a dimension-eight operator as discussed above.² Unlike in the singlet model, the ν -improved EFT also struggles with scenarios of very light new physics such as D1. But all in all, it performs well in situations with a modest scale hierarchy such as benchmarks D2.

A particularly interesting scenario is described by benchmark D3. In the full model, the bottom Yukawa is exactly sign-flipped, a signature hardly visible at the LHC. Generating such a signature from higher-dimensional operators requires their contributions to be twice as large as the SM Yukawa coupling due to the enhancement of ν/Λ by a large coupling. The EFT fails to capture this coupling shift fully, leading to a significantly different prediction for the Higgs decay into bottom quarks.

In Table 3.7 we repeat this analysis for the loop-induced couplings of an on-shell Higgs to gluons and photons. A large part of these coupling shifts stems from the modified Higgs-top and Higgs- W couplings, leading to good agreement in the same scenarios where the tree-level couplings were described accurately. Separating the H^\pm contribution to the Higgs-photon coupling, we find that \mathcal{O}_{BB} captures its effect very well.

Table 3.8 compares total production rates at the LHC. Depending on the benchmark, the dimension-six truncation leads to up to 10% departures, in agreement with the coupling deviations.

²Note that the operator \mathcal{O}_{BB} does contribute to the $h^0 VV$ coupling, representing the effect of a charged Higgs loop. But as our results show, this effect is negligible.

	Δ_g		Δ_γ			
	2HDM	ν -improved EFT	2HDM	(-0.05)	ν -improved EFT	(-0.07)
D1	$0.16 + 0.00i$	$0.08 + 0.00i$	-0.16	(-0.05)	-0.10	(-0.07)
D2	$0.00 + 0.00i$	$0.00 + 0.00i$	0.00	(0.00)	0.00	(0.00)
D3	$0.07 - 0.09i$	$0.02 + 0.00i$	-0.08	(-0.05)	-0.05	(-0.05)
D4	$0.00 + 0.00i$	$0.00 + 0.00i$	-0.05	(-0.05)	-0.05	(-0.05)

Table 3.7: Normalised couplings of the light Higgs to gluons and photons in our 2HDM benchmarks. The bottom loop leads to small imaginary parts of Δ_g and Δ_γ . For the Higgs-photon coupling, these imaginary parts are always smaller than 1% of the real part of the amplitude and neglected here. The numbers in parentheses ignore the modification of the Higgs-fermion couplings, allowing us to separately analyse how well the H^\pm loop is captured by \mathcal{O}_{BB} .

	$\sigma_{\nu\text{-improved EFT}}/\sigma_{2\text{HDM}}$		
	ggF	WBF	Vh
D1	0.872	1.109	1.108
D2	1.001	1.000	1.000
D3	1.022	1.042	1.042
D4	1.001	1.001	1.003

Table 3.8: Cross section ratios of the ν -improved dimension-six approximation to the full 2HDM at the LHC. The statistical uncertainties on these ratios are below 0.4%.

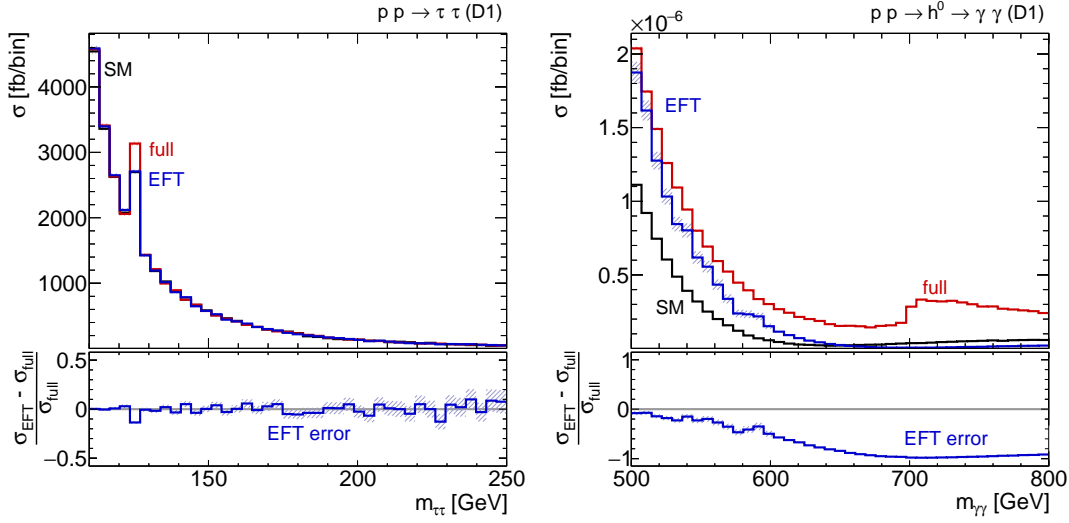


Figure 3.2: Left: $m_{\tau\tau}$ distribution in the $gg \rightarrow \tau^+ \tau^-$ channel. Right: off-shell behaviour of the process $gg \rightarrow h^0 \rightarrow \gamma\gamma$ in 2HDM benchmark D1, only taking into account the Higgs diagrams. At $m_{\gamma\gamma} \gtrsim 2m_{H^\pm} = 700$ GeV, the charged Higgs threshold is visible.

Distributions

As for the singlet model, the phenomenology of the 2HDM is mostly reflected in the coupling patterns discussed above, with little new kinematic effects. In the left panel of Figure 3.2 we illustrate the coupling deviations in gluon-fusion Higgs production with a decay $h \rightarrow \tau^+ \tau^-$, showing how the full 2HDM and the (ν -improved) EFT give substantially different predictions for the size of the Higgs signal.

While on-shell Higgs decays to photons are generally well described by the EFT, this changes for off-shell Higgs production. At $m_{\gamma\gamma} \gtrsim 2m_{H^\pm}$, the H^\pm in the loop can resolve the charged Higgs, enhancing the size of its contribution significantly. This effect is not captured by the effective operator and leads to a different behaviour of the amplitude $gg \rightarrow h^0 \rightarrow \gamma\gamma$ between the full and effective model, as shown in the right panel of Figure 3.2. However, the tiny rate and the large combinatorial background mean that this discrepancy will be irrelevant for LHC phenomenology. Similar threshold effects have been computed for the top-induced Higgs-gluon coupling and appear to be similarly irrelevant in practice [185].

The situation in Higgs pair production resembles the observations in the singlet model [154, 186, 187]. The agreement can be worse already at threshold if Higgs-top coupling shifts are not correctly captured by the effective model.

Summary

Eventually, the 2HDM discussion leads us to a similar conclusion as the singlet model: as long as the mixing is small and Higgs-gauge coupling shifts are negligible, all the LHC probes in single Higgs production is a set of three coupling modifications Δ_t , Δ_b , Δ_τ , and the charged Higgs loop contribution to the Higgs-photon coupling. These aspects of Higgs phenomenology are generally well captured by an appropriately defined EFT. Problems arise in scenarios with very light new Higgs bosons; when the Higgs- W and Higgs- Z couplings are modified, which requires dimension-eight operators; or in the special case of Higgs pair production.

A naive construction of the EFT by matching the effective dimension-six Lagrangian to the 2HDM in the gauge symmetric phase fails to correctly describe the modified Higgs boson dynamics in typical 2HDM scenarios, since terms formally suppressed by powers of v^2/Λ^2 can be crucial for phenomenologically relevant scenarios.

3.3.4 Scalar top partners

Model setup

New coloured scalar particles are, strictly speaking, not an extension of the SM Higgs sector, but they can lead to interesting modifications of the LHC observables. We consider a scalar top-partner sector mimicking the stop and sbottom sector of the MSSM. Its Lagrangian has the form

$$\mathcal{L}_{\text{top partners}} \supset (D_\mu \tilde{Q})^\dagger D^\mu \tilde{Q} + (D_\mu \tilde{t}_R)^* D^\mu \tilde{t}_R - M^2 \tilde{Q}^\dagger \tilde{Q} - M^2 \tilde{t}_R^* \tilde{t}_R - \kappa_{LL} (\phi \cdot \tilde{Q})^\dagger (\phi \cdot \tilde{Q}) - \kappa_{RR} (\tilde{t}_R^* \tilde{t}_R) (\phi^\dagger \phi) - [\kappa_{LR} M \tilde{t}_R^* (\phi \cdot \tilde{Q}) + \text{h. c.}] . \quad (3.51)$$

Here, \tilde{Q} and \tilde{t}_R are the additional isospin doublet and singlet in the fundamental representation of $SU(3)_C$. Their mass terms can be different, but for the sake of simplicity we unify them to a single heavy mass scale M . The singlet state \tilde{b}_R is assumed to be heavier and integrated out.

This leaves us with three physical degrees of freedom, the scalars \tilde{t}_1 , \tilde{t}_2 and $\tilde{b}_2 = \tilde{b}_L$. The eigenvalues of the stop mass matrix

$$\mathcal{M}_{\tilde{t}} = \begin{pmatrix} M^2 + \kappa_{LL} \frac{v^2}{2} & \kappa_{LR} \frac{Mv}{\sqrt{2}} \\ \kappa_{LR} \frac{Mv}{\sqrt{2}} & M^2 + \kappa_{RR} \frac{v^2}{2} \end{pmatrix} \quad (3.52)$$

define two masses $m_{\tilde{t}_1} < m_{\tilde{t}_2}$ and a mixing angle $\theta_{\tilde{t}}$. Again, we provide a detailed description of the model setup in Appendix A.2.3.

Signatures and decoupling patterns

The main new physics effects in the Higgs sector are loop-induced modifications of the Higgs interactions, most notably to Δ_g , Δ_γ , Δ_W , and Δ_Z , possibly including new Lorentz structures. The Yukawa couplings do not change at one loop because we do not include gauge boson partners. As a side remark, the 2HDM described in Section 3.3.3 combined with the scalar top partners given here corresponds to an effective description of the Minimal Supersymmetric Standard Model in the limit of infinitely heavy gauginos, sleptons, and light-flavour squarks.

In the limit of small $\theta_{\tilde{t}}$, the leading correction to the hVV coupling scales like

$$\Delta_V \approx \frac{\kappa_{LL}^2}{16\pi^2} \left(\frac{v}{m_{\tilde{t}_1}} \right)^2. \quad (3.53)$$

This shift can be sizeable for relatively low stop and sbottom masses combined with large couplings κ_{ij} to the Higgs sector.

As already noted for the 2HDM, the decoupling of the heavy scalars becomes non-trivial in the presence of a Higgs VEV. Following Equation (3.52), the masses of the heavy scalars $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ are not only controlled by the mass scale in the symmetric phase of the electroweak symmetry M , but they receive additional contributions of the type $\kappa_{LR} vM$, $\kappa_{LL} v^2$, or $\kappa_{RR} v^2$ after electroweak symmetry breaking. This leads to a mass splitting of order v between masses of order M , which is increased by large values of the couplings κ_i .

Dimension-six description

This motivates us to again define two different matching schemes. First, we stick to our default prescription and carry out the matching of the linear EFT Lagrangian to the full model in the unbroken phase. The matching scale is then dictated by the intrinsic mass scale of the heavy fields,

$$\Lambda_{\text{default}} = M, \quad (3.54)$$

completely oblivious to contributions to the masses from the electroweak VEV. The suppression scale of loop effects in the complete model and this matching scale in the EFT only agree in the limit $M - m_{\tilde{t}_1} \sim v \ll M$.

In this dimension-six approach the stop loops generate a number of operators [65, 69, 124],

$$f_{\phi,1} = -\frac{1}{2(4\pi)^2} \left[\kappa_{LL}^2 - \frac{\kappa_{LL} \kappa_{LR}^2}{2} + \frac{\kappa_{LR}^4}{10} \right],$$

$$f_{\phi,2} = \frac{1}{4(4\pi)^2} \left[2\kappa_{RR}^2 - \kappa_{RR} \kappa_{LR}^2 + \frac{\kappa_{LR}^4}{5} \right],$$

	Scalar top-partner model						
	M	κ_{LL}	κ_{RR}	κ_{LR}	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$\theta_{\tilde{t}}$
P1	500	-1.16	2.85	0.147	500	580	-0.15
P2	350	-3.16	-2.82	0.017	173	200	-0.10
P3	500	-7.51	-7.17	0.012	173	200	-0.10

Table 3.9: Benchmarks for the scalar top-partner scenario. We show Lagrangian parameters (left) and physical parameters (right). All masses are given in GeV.

$$\begin{aligned}
f_{GG} &= \frac{g_s^2}{24(4\pi)^2} [\kappa_{LL} + \kappa_{RR} - \kappa_{LR}^2], \\
f_{BB} &= -\frac{1}{36(4\pi)^2} \left[\kappa_{LL} + 16\kappa_{RR} - \frac{67}{10}\kappa_{LR}^2 \right], \\
f_{WW} &= -\frac{1}{4(4\pi)^2} \left[\kappa_{LL} - \frac{3}{10}\kappa_{LR}^2 \right], \\
f_{BW} &= \frac{1}{12(4\pi)^2} \left[2\kappa_{LL} - \frac{11}{5}\kappa_{LR}^2 \right], \\
f_B &= \frac{1}{20(4\pi)^2} \kappa_{LR}^2, \\
f_W &= \frac{1}{20(4\pi)^2} \kappa_{LR}^2.
\end{aligned} \tag{3.55}$$

Unlike the tree-level effects in the previous two models, the top partner loops do not only induce modifications to the SM Higgs couplings, but induce new Lorentz structures. Note that some of these operators are tightly constrained from electroweak precision data, see Section 2.3.2. We will ignore these constraints for our discussion of Higgs physics.

In addition, we define a v -improved matching. Like in the 2HDM we pick the matching scale as a physical mass in the broken phase,

$$\Lambda = m_{\tilde{t}_1}. \tag{3.56}$$

The Wilson are the same as in Equation (3.55).

Benchmark points

As Equation (3.53) suggests, sizeable loop corrections to the hVV coupling require light top partners with unrealistically strong couplings to the Higgs sector [188]. In Table 3.9 we define a set of benchmark points with this aim in mind. The corresponding Wilson coefficients in our two matching schemes are given in Table 3.10.

	EFT				EFT (ν -improved)			
	Λ	$f_{\phi,2}$	f_{WW}	f_W	Λ	$f_{\phi,2}$	f_{WW}	f_W
P ₁	500	0.026	0.000	0.000	500	0.026	0.000	0.000
P ₂	350	0.023	0.005	0.000	173	0.023	0.005	0.000
P ₃	500	0.152	0.115	-0.207	173	0.152	0.115	-0.207

Table 3.10: Matching scales (in GeV) and selected Wilson coefficient for the top-partner benchmarks, both for default and ν -improved matching.

Benchmark	$\sigma_{\text{EFT}}/\sigma_{\text{triplet}}$		$\sigma_{\nu\text{-improved EFT}}/\sigma_{\text{triplet}}$	
	WBF	Vh	WBF	Vh
P ₁	1.000	0.999	1.000	0.999
P ₂	1.095	1.100	1.074	1.049
P ₃	2.081	1.904	1.749	1.363

Table 3.11: Cross section ratios of the matched dimension-six EFT approximation to the full scalar top-partner model at the LHC. We give the results both for the default matching scheme with matching scale $\Lambda = M$ as well as for the ν -improved matching at $\Lambda = m_{\tilde{t}_1}$. The statistical uncertainties on these ratios are below 0.4%.

Higgs production rates and distributions

The contributions from scalar top partners to Higgs production in gluon fusion are well known [189–193] and the validity of the EFT approach for this process has been thoroughly scrutinised [120, 124]. We therefore focus on corrections to the hVV coupling in WBF and Higgs-strahlung.

The total Higgs production rates in these channels are given in Table 3.11. In benchmark P₁ the WBF cross section is reduced by about 0.6% compared to the Standard Model, with good agreement between effective and full description. Clearly, such a scenario is not relevant for LHC measurements in the foreseeable future. In more extreme corners of the parameter space the loop effects in the full model grow, higher-dimensional terms in the EFT become larger, the validity of the latter worsens, and discrepancies between both increase. In benchmarks P₂ and P₃ the WBF rate is reduced by 9.1% and 43.5% with respect to the Standard Model. By construction, the EFT based on the default matching captures only the formally leading term at $\mathcal{O}(v^2/\Lambda^2)$, only giving a reduction of 0.5% and 2.0%. The corresponding difference is again independent for example of the tagging jet’s transverse momentum. With the ν -improved matching, the cross section is reduced by 2.4% and 17.7%, still far from the result of the full model.

The results for Higgs-strahlung look similar: in the moderate benchmark P₁ the predictions

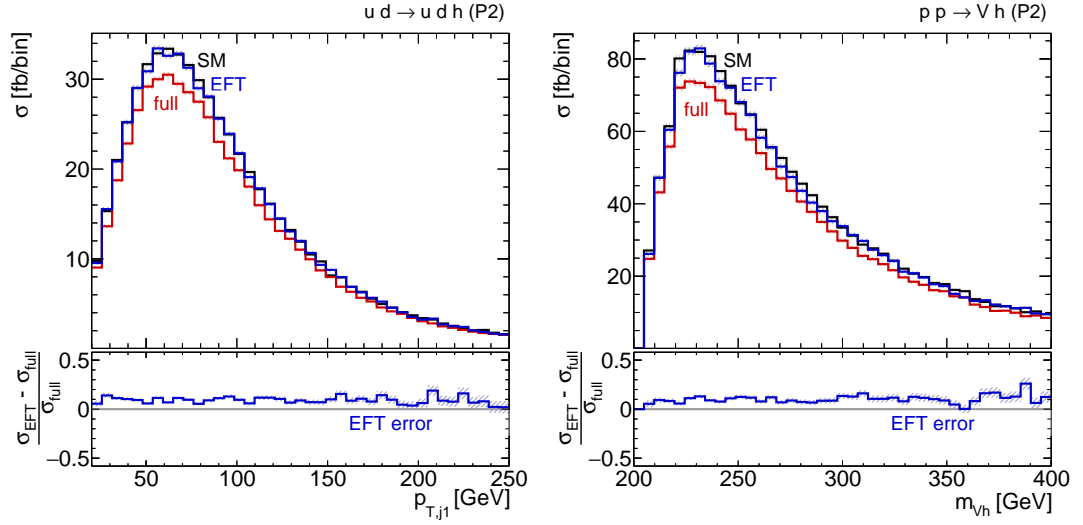


Figure 3.3: Kinematic distributions for the top partner model in benchmark P2. Left: tagging jet properties in WBF Higgs production. Right: m_{Vh} distribution in Higgs-strahlung.

of the full model and the dimension-six Lagrangian agree within 0.1%, but in this scenario the overall deviation from the Standard Model is negligible. In scenarios with larger loop effects, the dimension-six predictions fails to capture most of the full top partner loops, with numerical results similar to those given for WBF Higgs production. Again switching to the ν -improved matching does not improve the EFT approximation significantly.

In Figure 3.3 we finally show that these changes in the total rates do not have a dramatic effect on the kinematic distributions. This is not surprising, since the largest Wilson coefficient generated in our benchmark points is consistently that of $\mathcal{O}_{\phi,2}$, see Table 3.10, which corresponds to a universal rescaling of the SM Higgs couplings.

Summary

Scalar top partners generate a large set of dimension-six operators through electroweak loops. However, in realistic scenarios with a large scale separation the loop corrections for example to the hVV vertex are tiny. Pushing for loop effects that are large enough to leave a visible imprint in WBF and Higgs-strahlung requires breaking the scale separation between the observed Higgs scalar and the top partners dramatically. In that case the EFT fails already for the total rates, kinematic distributions hardly add to this discrepancy.

3.3.5 Vector triplet

Model setup

Heavy vector bosons appear in many new physics scenarios, for instance when a larger gauge group is spontaneously broken down to the SM gauge group at higher energies. Often such particles are connected to the gauge-Higgs sector of the SM, predicting signatures in Higgs measurements [115, 194, 195]. As an example, we study a massive vector field³ V_μ^a which is a triplet under $SU(2)_L$ and uncharged under $SU(3)_c$ and $U(1)_Y$. These charges allow it to mix with the W bosons of the Standard Model and to couple to Higgs and fermion currents [115, 195]. For simplicity, we assume CP invariance and a flavour-universal coupling to the fermion current. Following the conventions of Reference [195], the Lagrangian then reads

$$\begin{aligned} \mathcal{L}_{\text{vector triplet}} \supset & -\frac{1}{4} V_{\mu\nu}^a V^{\mu\nu a} + \frac{M_V^2}{2} V_\mu^a V^{\mu a} \\ & + i \frac{g_V}{2} c_H V_\mu^a \left[\phi^\dagger \sigma^a \overleftrightarrow{D}^\mu \phi \right] + \frac{g_W^2}{2g_V} V_\mu^a \sum_f c_F \bar{f}_L \gamma^\mu \sigma^a f_L \\ & + g_V^2 c_{VVHH} V_\mu^a V^{\mu a} \phi^\dagger \phi \\ & + \frac{g_V}{2} c_{VVV} \varepsilon_{abc} V_\mu^a V_\nu^b D^{[\mu} V^{\nu]c} - \frac{g_W}{2} c_{VWW} \varepsilon_{abc} W^{\mu\nu} V_\mu^b V_\nu^c \end{aligned} \quad (3.57)$$

with the field strength $V_{\mu\nu}^a = D_\mu V_\nu^a - D_\nu V_\mu^a$ and where $D_\mu V_\nu^a = \partial_\mu V_\nu^a + g_V \varepsilon^{abc} V_\mu^b V_\nu^c$.

The coupling g_V characterises the interactions of the heavy vector, while g_W is the $SU(2)_L$ coupling constant. After mixing, the original fields V^a and W^a combine to the mass eigenstates W^\pm , ξ^\pm , and ξ^0 , while a combination of the couplings g_w and g_V becomes the observed weak gauge coupling g . c_{VWW} and c_{VVV} are irrelevant for Higgs phenomenology at the LHC. For more details, see Appendix A.2.4.

Signatures and decoupling patterns

In addition to the new heavy resonances ξ^0 and ξ^\pm , the signature feature of the vector triplet is that the mixing of the new states with the W and Z bosons affects the properties of the electroweak gauge bosons at tree level. In particular, the shift from the Lagrangian parameter g_w to the observable weak coupling g combined with the direct heavy vector coupling to the Higgs doublet

³Note that such a model is not UV-complete: as fundamental particles, massive vector bosons are not renormalisable. However, such a mass can easily be generated from a consistent gauge theory with a Higgs mechanism at a higher scale [195]. The details of such an embedding do not matter for the discussion here.

modify the Higgs couplings as

$$\begin{aligned}\Delta_V &\approx \frac{g^2 c_F c_H}{4} \left(\frac{v}{M_V} \right)^2 - \frac{3g_V^2 c_H^2}{8} \left(\frac{v}{M_V} \right)^2 \\ \Delta_f &\approx \frac{g^2 c_F c_H}{4} \left(\frac{v}{M_V} \right)^2 - \frac{g_V^2 c_H^2}{8} \left(\frac{v}{M_V} \right)^2.\end{aligned}\quad (3.58)$$

In addition, contributions from virtual heavy states ξ modify the phase-space behaviour of Higgs signals in many ways.

Just as for the 2HDM and the top partners, the mass matrix for the massive vectors contains both the intrinsic mass scale M_V and terms proportional to some power of v multiplied by a combination of couplings. The new vector states have roughly degenerate masses

$$m_\xi^2 \approx M_V^2 \left(1 + g_V^2 c_{VVHH} \frac{v^2}{M_V^2} + \frac{g_V^2 c_H^2}{4} \frac{v^2}{M_V^2} + \mathcal{O} \left(\frac{v^4}{M_V^4} \right) \right). \quad (3.59)$$

Even if there appears to be a clear scale separation $M_V \gg v$, large values of g_V , c_{VVHH} , or c_H can change m_ξ significantly and effectively induce a second mass scale.

Dimension-six description

The obvious choice of the matching scale in the unbroken electroweak phase is the heavy mass scale in the Lagrangian

$$\Lambda_{\text{default}} = M_V, \quad (3.60)$$

which we use for our default matching scheme.

Integrating out the heavy vector triplet at tree level generates a number of operators with Wilson coefficients

$$\begin{aligned}f_{WW} &= c_F c_H, & f_{\phi 2} &= \frac{3}{4} c_H (c_H g_V^2 - 2 c_F g^2), \\ f_{BB} &= c_F c_H, & f_{\phi 3} &= -3 \lambda c_H (c_H g_V^2 - 2 c_F g^2), \\ f_{BW} &= c_F c_H, & f_f &= -\frac{1}{4} y_f c_H (c_H g_V^2 - 2 c_F g^2), \\ f_W &= -2 c_F c_H.\end{aligned}\quad (3.61)$$

Additional four-fermion contributions are irrelevant for Higgs physics. Loop-induced contributions will be further suppressed and do not add qualitatively new features, so we neglect them here.

Once again, we compare this default matching to an alternative v -improved matching, where as a cutoff we now use the physical mass

$$\Lambda_{v\text{-improved}} = m_{\xi^0}. \quad (3.62)$$

	Triplet model					
	M_V	g_V	c_H	c_F	c_{VVHH}	m_ξ
T1	591	3.0	-0.47	-5.0	2.0	1200
T2	946	3.0	-0.47	-5.0	1.0	1200
T3	941	3.0	-0.28	3.0	1.0	1200
T4	1246	3.0	-0.50	3.0	-0.2	1200
T5	846	1.0	-0.56	-1.32	0.08	849

Table 3.12: Benchmark points for the vector triplet model. The masses are given in GeV.

The coefficients in Equation (3.61) remain unchanged.⁴

Unlike the previous models, the vector triplet generates \mathcal{O}_W , \mathcal{O}_{WW} , and \mathcal{O}_{BB} at tree level. As discussed in Section 2.3.3, these induce new kinematic structures in the hWW and hZZ couplings.

Benchmark points

As for the other models we study a set of benchmark points, defined in Table 3.12 and Table 3.13. Unlike additional scalars, light new vector triplets with masses just above the electroweak scale are unrealistic given the constraints from electroweak precision measurements and direct searches. We therefore focus on new vector bosons around the TeV scale. The different setups are motivated phenomenologically, from experimental constraints, or based on specific UV completions:

T1-2 Higgs-gauge dynamics: designed for large momentum-dependent effects in the hVV couplings. \mathcal{O}_W and \mathcal{O}_{WW} receive large Wilson coefficients, while $\mathcal{O}_{\phi,2}$, $\mathcal{O}_{\phi,3}$, and \mathcal{O}_f vanish along the line

$$\frac{c_H}{c_F} = 2 \frac{g^2}{g_V^2}. \quad (3.63)$$

The large couplings also imply a large difference between M_V and m_ξ .

- T3 Interference patterns:** changes the sign of c_W or the Wilson coefficients compared to T1 and T2. This allows us to compare constructive and destructive interference patterns between SM amplitudes and new physics contributions.
- T4 Realistic:** the vector triplet couplings and masses satisfy the leading constraints from direct collider searches at the time of publication of Reference [2]. The most stringent bounds come from di-lepton and di-boson channels [195, 196].

⁴In the spirit of v -improvement we could alternatively parametrise the Wilson coefficients with the physical mixing angles between the W , Z and V bosons, but this does not significantly change the results.

	Default EFT					ν -improved EFT				
	Λ	$f_{\phi 2}$	f_{WW}	f_W	f_t	Λ	$f_{\phi 2}$	f_{WW}	f_W	f_t
T1	591	0.00	2.45	-4.90	0.00	1200	0.00	2.45	-4.90	0.00
T2	946	0.00	2.35	-4.71	0.00	1200	0.00	2.35	-4.71	0.00
T3	941	1.09	-0.82	1.64	-0.36	1200	1.09	-0.82	1.64	-0.36
T4	1246	2.64	-1.56	3.12	-0.87	1200	2.64	-1.56	3.12	-0.87
T5	846	-0.24	0.78	-1.55	0.08	849	-0.24	0.78	-1.55	0.08

Table 3.13: Matching scales (in GeV) and selected Wilson coefficients for the effective theory matched to the vector triplet model. We give these results both for the EFT matching in the unbroken phase as well as for the ν -improved matching with $\Lambda = m_{\xi^0}$.

T5 **UV completion:** typical coupling patterns from a weakly coupled UV completion based on the extended gauge group $SU(3) \times SU(2) \times SU(2) \times U(1)$ [197]. Such a scenario could for instance arise from deconstructed extra dimensions [198]. The vector triplet phenomenology is effectively described by the parameter $\alpha = g_V / \sqrt{g_V^2 - g_W^2}$ together with the symmetry breaking scale f [195], with the couplings

$$\begin{aligned}
M_V^2 &= \alpha^2 g_V^2 f^2, & c_H &= -\alpha \frac{g_W^2}{g_V^2}, & c_{VVHH} &= \alpha^2 \left[\frac{g_W^4}{4g_V^4} \right], \\
c_F &= -\alpha, & c_{VW} &= 1, & c_{VVV} &= -\frac{\alpha^3}{g_V} \left[1 - \frac{3g_W^2}{g_V^2} + \frac{2g_W^2}{g_V^4} \right].
\end{aligned} \quad (3.64)$$

Benchmarks T1 to T3 are meant to emphasise the phenomenological possibilities of the vector triplet model, ignoring experimental constraints or parameter correlations from an underlying UV completion.

Higgs production rates and distributions

As shown in Figure 3.4, virtual heavy vector bosons contribute to as intermediate t -channel mediators to WBF Higgs production and in the s -channel to Higgs-strahlung, promising non-trivial kinematic features in these channels. In the EFT these effects are mapped onto large Wilson coefficients like \mathcal{O}_W and \mathcal{O}_{WW} , adding a momentum dependence to the hVV couplings. Therefore our analysis focuses on these electroweak Higgs production modes.

Table 3.14 shows the agreement between EFT and full model for the total Higgs production rates in WBF Higgs production and Higgs-strahlung. The default dimension-six term model matched in the unbroken phase, oblivious to the difference between the Lagrangian mass term M_V and the actual physical mass m_{ξ} , struggles with the first three benchmark points, in which this splitting is large. The discrepancies to the full model are particularly evident in Vh production.

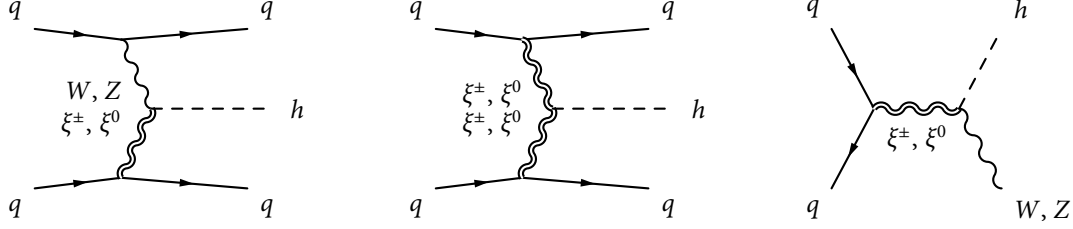


Figure 3.4: Example Feynman diagrams with contributions from virtual heavy vector bosons ξ to Higgs production in weak boson fusion (left, middle) or Higgs-strahlung (right).

	$\sigma_{\text{default EFT}}/\sigma_{\text{triplet}}$		$\sigma_{v\text{-improved EFT}}/\sigma_{\text{triplet}}$	
	WBF	Vh	WBF	Vh
T ₁	1.299	0.299	0.977	0.794
T ₂	1.045	0.737	0.992	0.907
T ₃	0.921	1.066	0.966	1.024
T ₄	1.026	0.970	1.012	0.978
T ₅	1.001	1.043	1.002	1.043

Table 3.14: Cross section ratios of the matched dimension-six EFT approximation to the full vector triplet at the LHC. To avoid large contributions from the ξ resonance in the Vh channel, we only take into account the region $m_{Vh} < 600$ GeV. The statistical uncertainties on these ratios are below 0.4%.

The v -improved EFT, on the other hand, performs better and describes the rate accurately in most of the scenarios. Only in Higgs-strahlung and only in the extreme scenarios T1 and T2 we find significant deviations.

To better understand these differences, we have to look at kinematic distributions. Figure 3.5 shows different properties of the tagging jets in WBF Higgs production. In addition to the predictions of the full vector triplet model and the default and v -improved EFT, we show distributions of the vector triplet model where we have artificially removed all contributions from virtual ξ propagators.

We find that the vector triplet significantly modifies the WBF rate with respect to the Standard Model. Its effect increases with momentum transfer, measured as transverse jet momentum. This modification can be traced to contributions from ξ fusion and mixed W - ξ fusion diagrams as given in Figure 3.4. These contributions from ξ propagators can become relevant already at energy scales well below m_ξ and further increase with the energy flow. In addition to the high-energy tails of the transverse momenta, large effects are visible in the azimuthal angle between the tagging jets, as shown in the bottom left panel of Figure 3.5. This angular correlation is well known to be sensitive to the modified Lorentz structure of the hWW vertex [35–41].

The EFT approach qualitatively captures these features of the full model, now parametrised by momentum-dependent operators such as \mathcal{O}_W and \mathcal{O}_{WW} . The signs of the Wilson coefficients in benchmarks T1 and T2 yields a non-linear increase of the cross section with energy. Conversely, the switched signs in T3 reduces the rate with energy, eventually driving the combined amplitude through zero.

Comparing full and effective model for the more realistic benchmark points T4 and T5, we find good agreement in the bulk of the distribution. The deviations from the Standard Model are entirely captured by the dimension-six operators, including the momentum dependence coming from the ξ diagrams. Only at very large momentum transfer, likely beyond the sensitivity of the LHC, the validity of the EFT breaks down.

In the more strongly coupled benchmark points T1 to T3, the full model predicts shifts in the jet distributions that are large enough to be relevant for the upcoming LHC run. We find good agreement between the full model and the default EFT only at low momentum transfer, where the effects of new physics are small. This naive dimension-six model fails to reproduce the full model results already at energy scales $p_{T,j} \gtrsim 80$ GeV, a phase space region highly relevant for constraints on new physics [22]. Perhaps counter-intuitively, this discrepancy does not signal a breakdown of the E/Λ expansion, but is linked to the difference between the physical mass m_ξ , which suppresses the ξ fusion diagrams, and the matching scale $\Lambda_{\text{default}} = M_V$, which suppresses the dimension-six operators. This can be seen by comparing the results to those based on v -improved matching, where the EFT cutoff scale matches the physical mass. Here the agreement is significantly better, and the dimension-six description successfully describes the momentum dependence up to large momentum transfer. Only at very high energies, $p_{T,j1} \gtrsim 300$ GeV, even the v -improved EFT

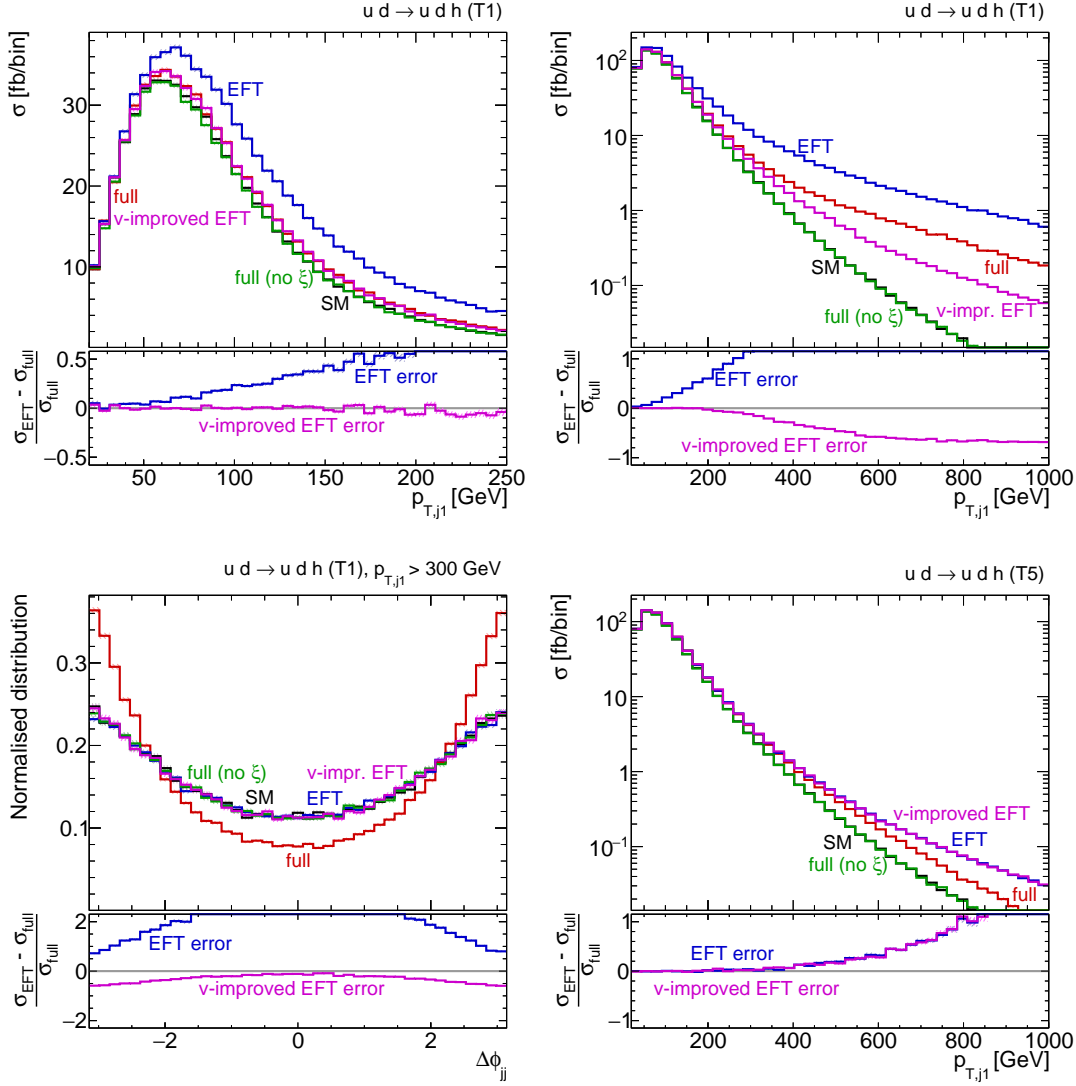


Figure 3.5: Tagging jet distributions in WBF Higgs production in the vector triplet model. Top: $p_{T,j1}$ distribution in benchmark T1, focusing on the low (left) and high (right) transverse momentum regions. Bottom left: $\Delta\phi_{jj}$ distribution above a certain $p_{T,j1}$ threshold for T1. Bottom right: $p_{T,j1}$ distribution for scenario T5.

breaks down.

The situation is similar in Higgs-strahlung, shown in Figure 3.6. Again, the dominant new physics effect is the interference with ξ -mediated diagrams rather than the modified hWW interaction. Not only does this lead to a significant change of the rate, it also introduces a strong dependence on the momentum transfer, probed by either the invariant mass of the gauge-Higgs system or the transverse momentum of the final vector boson. The relative sign of the interference between ξ amplitudes and SM-like diagrams is opposite to that in WBF: in T3 and T4 we find a non-linear increase of the cross section with the energy scale. The other benchmarks predict a decrease of the amplitude with energy, eventually including a sign flip when the amplitude is driven through zero.

In the more weakly coupled benchmarks T4 and T5, the full and effective models agree well over most of the phase space, and the dimension-six operators successfully capture how the ξ contributions affect the Higgs-strahlung kinematics. At larger momentum transfer, higher-order terms in the EFT expansion become important, and dimension-six operators alone cannot describe the kinematics accurately any more. Ultimately, the ξ resonance in the full model marks the obvious failure of the effective theory.

For benchmarks T1 to T3, the default EFT has a more limited validity range. The large couplings lead to a failure of this dimension-six model already at low energies $m_{Vh} \gtrsim 220$ GeV, even though the actual ξ resonances only appear at $m_\xi = 1.2$ TeV. The EFT approximation can again be significantly improved by switching to the v -improved matching. But even then, there is a pronounced mismatch between full and effective model. This EFT error is larger in Higgs-strahlung than in WBF, showing how ξ contributions play a larger role in this s -channel process than in the t -channel WBF diagrams.

Summary

Heavy vector bosons can induce large kinematic effects in Higgs-gauge interactions, providing a perfect test case for the EFT approach. For realistic scenarios, the EFT works up to large momentum transfer. Operators such as \mathcal{O}_W and \mathcal{O}_{WW} successfully capture the effects from virtual ξ contributions to WBF Higgs production, including non-trivial momentum dependencies. In Higgs-strahlung, s -channel ξ contributions prove more renitent to be mapped onto effective operators, but we still find good agreement for most of the phase space for realistic parameter choices.

Again, this remarkable performance of the dimension-six model requires particular care in the matching to the full theory. When the mixing with the SM gauge bosons is large, a naive matching procedure defined in the unbroken electroweak phase can lead to substantial errors already in the bulk of the WBF distributions. A v -improved dimension-six description, however, improves the EFT accuracy such that large deviations only occur in the high-energy tails of distributions.

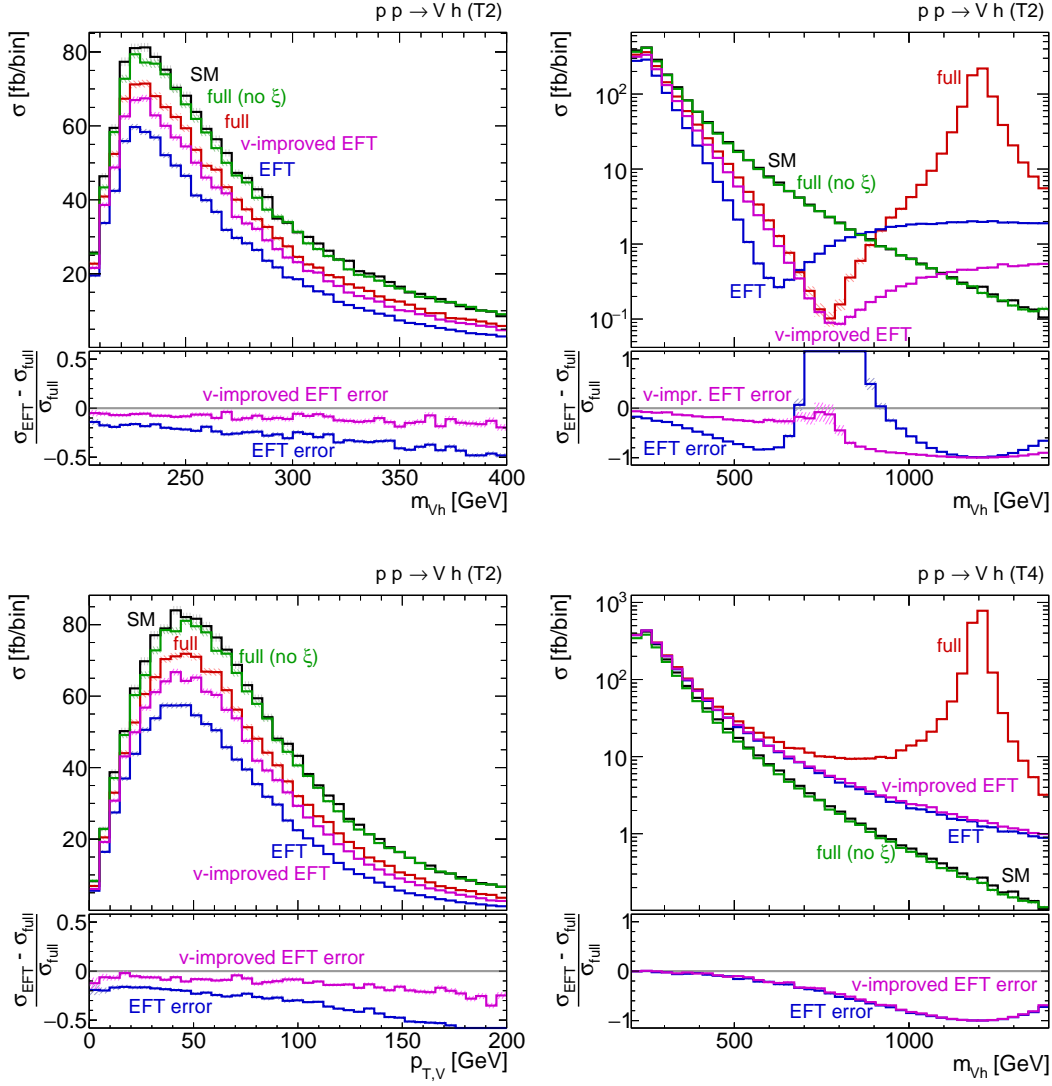


Figure 3.6: Higgs-strahlung distributions in the vector triplet model. Top: m_{Vh} distribution for benchmark T2, focusing on the low (left) and high (right) invariant mass regions. Bottom left: $p_{T,V}$ distribution for the same benchmark. Bottom right: m_{Vh} distribution for T4.

3.4 Practical questions

We now focus on the example of WBF Higgs production in the vector triplet model to discuss some practical aspects. The first is the question which observables are most sensitive to the momentum transfer in this production mode, and therefore provide a handle on the validity regions of the EFT. In Section 3.4.2 we discuss the role of squared dimension-six terms in the differential cross section. Finally, Section 3.4.3 illustrates how limits on the vector triplet model can be achieved directly in the full model or indirectly via EFT measurements, and how this is affected by the breakdown of the EFT description.

In the previous section we established that the ν -improved matching, defined in Equations (3.61) and (3.62), improves the agreement between the effective model and the full theory, so we now use it exclusively.

3.4.1 Weak boson fusion observables

We have repeatedly argued that the validity of the effective field theory depends on the energy scale or momentum flow in a process. In a $2 \rightarrow 2$ process where all final-state particles can be measured precisely, the momentum transfer can be easily calculated, for instance with the Mandelstam variables s , t , and u . For WBF Higgs production, both the definition and the measurement of the momentum transfer are more involved. The definition has to take into account that this is a $2 \rightarrow 3$ process even without adding a Higgs decay. The measurement is problematic since many phenomenologically relevant decay modes do not allow a precise reconstruction of the Higgs momentum.

To define the momentum transfer in weak boson fusion, we first note that W -mediated and Z -mediated diagrams contribute to this process. Quark-vector splittings are more likely to happen in the collinear direction [30], and the direction of the final-state quarks is typically only slightly deflected from that of the corresponding initial partons. The W -mediated and Z -mediated diagrams thus dominate in very different regions of phase space and their interference is negligible. This allows us to define “ W -like” and “ Z -like” phase-space regions and to assign four-momenta p_{V1} and p_{V2} of the intermediate vector bosons [109]. For the dominant amplitude $ud \rightarrow u'd' h$,

$$\begin{aligned} p_{V1} &= p_{u'} - p_d, & p_{V2} &= p_{d'} - p_u & \text{for } W\text{-like phase-space points,} \\ p_{V1} &= p_{u'} - p_u, & p_{V2} &= p_{d'} - p_d & \text{for } Z\text{-like phase-space points.} \end{aligned} \quad (3.65)$$

The validity of the EFT is limited by the largest momentum transfer in the process, so we define it as the maximum virtuality of the intermediate bosons,

$$q = \sqrt{\max(p_{V1}^2, p_{V2}^2)}. \quad (3.66)$$

Important Higgs boson decay modes such as $h \rightarrow WW^* \rightarrow \ell^+ \nu \ell^- \bar{\nu}$ do not allow us to reconstruct the Higgs four-momentum, and q cannot be calculated. We therefore have to resort to

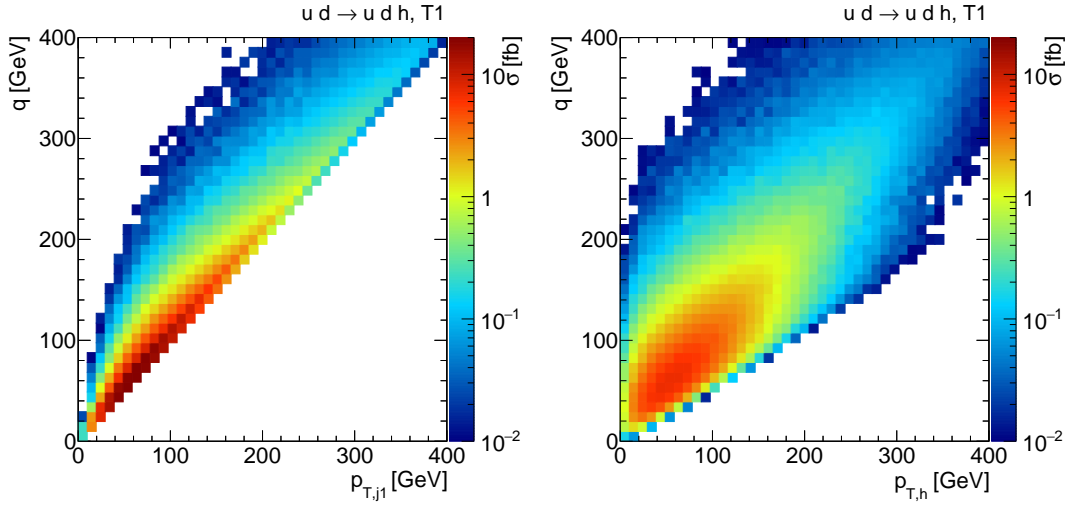


Figure 3.7: Correlations between the WBF momentum transfer q , defined in Equation (3.66), and the observable transverse momenta $p_{T,j1}$ (left) and $p_{T,h}$ (right).

an experimentally accessible proxy for the true momentum transfer. The transverse momenta of the two tagging jets, $p_{T,j1}$ and $p_{T,j2}$, are two good candidates. As can be seen from the Feynman diagram in Figure 2.1, these two jets directly recoil against the intermediate vector bosons, so $p_{T,ji} = p_{T,Vi}$. We order them such that $p_{T,j1} > p_{T,j2}$. As an alternative to the two jets, we can consider the transverse momentum of the Higgs, $p_{T,h}$, which can be reconstructed even if the Higgs decay involves neutrinos.

Figure 3.7 shows the correlations of the transverse momenta of the leading tagging jet or the Higgs boson with the virtuality q . Both are visibly correlated with the momentum transfer, but the correspondence is particularly clear for the jet p_T .

From the previous section we know that an EFT analysis of kinematic distributions faces a trade-off: on the one hand, new physics signatures often grow with the energy scale. On the other hand, the EFT validity is on a more secure footing at lower energy scales. However, we have never defined the “energy scale” in these statements quantitatively. We now discuss this choice for WBF Higgs production. More precisely, we ask which observable x is suitable to

1. isolate phase-space regions with interesting NP signatures with a cut $x > x_{\min}$, and simultaneously
2. ensure the EFT validity with a cut $x < x_{\max}$.

We compare the observables

$$x \in \{q, p_{T,j1}, p_{T,j2}, p_{T,h}\}. \quad (3.67)$$

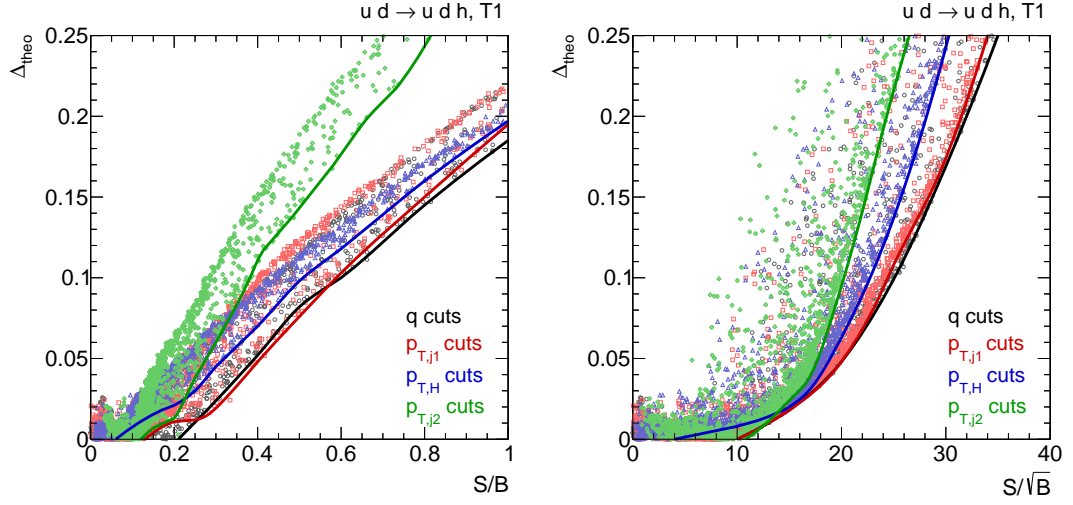


Figure 3.8: Significance of expected vector triplet signals vs. theoretical uncertainties dimension-six description. Each point corresponds to a selection window $x_{\min} < x < x_{\max}$ in one of the four momentum observables $x \in \{q, p_{T,j1}, p_{T,j2}, p_{T,h}\}$. See text for more details.

As discussed above, they all provide probes of the momentum transfer through the intermediate vector bosons.

For each observable x we scan over values of x_{\min} and x_{\max} . For each window (x_{\min}, x_{\max}) we calculate the predictions for the parton-level WBF process defined in Equation (3.16) for the SM, vector triplet benchmark point T1, and the corresponding v -improved EFT description. We reject windows that only leave a signal cross section of less than 20 fb before Higgs decays. We then estimate the significance of the vector triplet signal over the SM background, disregarding non-Higgs backgrounds or detector effects for our toy study. In a measurement limited by systematic uncertainties, the relevant quantity is

$$\frac{S}{B}(x_{\min}, x_{\max}) = \left| \frac{\sigma_{\text{vector triplet}} - \sigma_{\text{SM}}}{\sigma_{\text{SM}}} \right|, \quad (3.68)$$

while for a statistics-limited analysis we have to consider

$$\frac{S}{\sqrt{B}}(x_{\min}, x_{\max}) = \sqrt{L} \left| \frac{\sigma_{\text{vector triplet}} - \sigma_{\text{SM}}}{\sqrt{\sigma_{\text{SM}}}} \right|, \quad (3.69)$$

For our simple illustration we pick an integrated luminosity times Higgs branching ratio times efficiencies of $L \times \text{BR} \times \varepsilon = 30 \text{ fb}^{-1}$. For each window (x_{\min}, x_{\max}) we also calculate the theory

uncertainty that quantifies the EFT error in this kinematic region as

$$\Delta_{\text{theo}}(x_{\min}, x_{\max}) = \left| \frac{\sigma_{\text{EFT}} - \sigma_{\text{vector triplet}}}{\sigma_{\text{vector triplet}}} \right|, \quad (3.70)$$

The question is for which observable x we expect significant new physics signatures, i. e. large values of S/B and S/\sqrt{B} , while keeping the EFT error Δ_{theo} small. We show the results of our scan in Figure 3.8. The momentum transfer q as well as the leading tagging jet's p_{T,j_1} define kinematic regions with the highest significance for a given theoretical uncertainty Δ_{theo} . This indicates that as long as q is not directly accessible, the transverse momentum of the leading tagging jet indeed provides the best probe of the momentum flow through the WBF process, and justifies our choice of observables in the previous section.

3.4.2 To Square or Not to Square

Differential cross sections are proportional to the squared matrix element. In our EFT approach, we have

$$|\mathcal{M}_{\text{EFT}}|^2 = \underbrace{|\mathcal{M}_4|^2}_{\mathcal{O}(1)} + \underbrace{2 \text{Re} \mathcal{M}_4^* \mathcal{M}_6}_{\mathcal{O}(1/\Lambda^2)} + \underbrace{|\mathcal{M}_6|^2 + 2 \text{Re} \mathcal{M}_4^* \mathcal{M}_8}_{\mathcal{O}(1/\Lambda^4)} + \mathcal{O}(1/\Lambda^6), \quad (3.71)$$

where the subscripts in \mathcal{M}_i denote the dimension of the operators in the amplitude, i. e. \mathcal{M}_4 is the SM amplitude and \mathcal{M}_6 contains one dimension-six interaction. The squared amplitudes from dimension-six operators contribute at the same order in the EFT expansion in $1/\Lambda$ as the leading contributions from dimension-eight operators. This raises the question whether these squared terms should be included in calculations when dimension-eight operators are ignored [109, 114, 125, 126, 199–201].

In a top-down perspective, i. e. knowing the underlying physics, the answer depends on the typical coupling strengths of the underlying physics. At least for tree-level effects, the Wilson coefficients of both dimension-six and dimension-eight operators will generally contain two couplings g of the heavy field, $f_i \sim g_{UV}^2$, and we have to compare the terms

$$|\mathcal{M}_{d=6}|^2 \sim \frac{g_{UV}^4}{\Lambda^4} \quad \text{vs.} \quad 2 \text{Re} \mathcal{M}_{d=4}^* \mathcal{M}_{d=8} \sim \frac{g_{SM}^2 g_{UV}^2}{\Lambda^4}, \quad (3.72)$$

where g_{SM} denotes a typical SM coupling. In strongly coupled scenarios we therefore expect the squared dimension-six terms to dominate over the dimension-eight contributions, and it is perfectly justified to include the squared dimension-six term, but no dimension-eight operators. In more weakly coupled models with $g_{UV} \sim g_{SM}$, the two should contribute approximately equally. The size of the squared dimension-six terms can then be seen as an estimate for the size of the

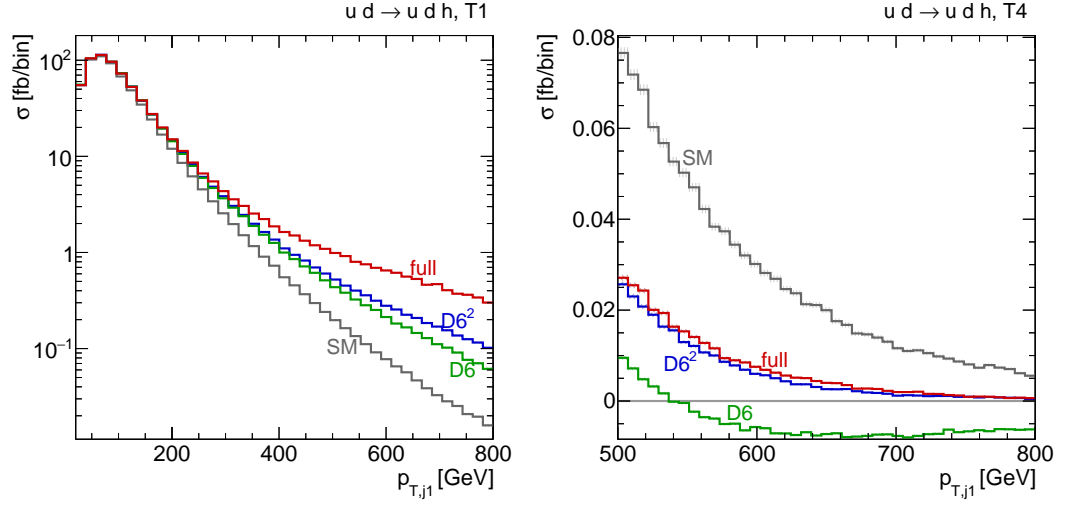


Figure 3.9: WBF distributions with (“D6²”) and without (“D6”) the squared amplitudes from dimension-six operators. The right panels zooms in on the region where leaving out the squared dimension-six terms leads to a negative cross section.

missing higher orders in the EFT expansion, and therefore as an indicator for the validity of the EFT approach [125].

Of course, in practice we do not know the coupling strength of UV physics. Generally discarding the squared dimension-six terms will degrade the EFT performance at least for strongly coupled scenarios. Similarly, universally using the size of the squared dimension-six amplitude as a theory error in global fits introduces a theory dependence in the results.

Note also that the argument in Equation (3.72) relies on the assumption that except for the typical couplings and the suppression scale Λ all amplitudes have approximately the same size. But in phase-space regions where the dimension-four contribution is suppressed, the dimension-six squared term can easily be larger than the interference between SM and dimension-eight operator, even if the EFT expansion in v/Λ holds and higher-dimensional operators are negligible. One example is Higgs pair production with its accidental cancellation between the two SM contributions, as discussed in Section 2.1.2 and demonstrated in 3.3.2.

If we consider the dimension-six Lagrangian not as the leading term of a consistent effective field theory, but rather as a phenomenological parametrisation describing a vast range of LHC Higgs signatures, then the counting argument becomes irrelevant and the square of dimension-six terms should always be included.

Finally, from a technical perspective the squared dimension-six terms are necessary to guarantee positive cross sections. Without them, the expected rate can become negative in high-energy tails

or in other regions of the phase space where the SM predictions are small.

Figure 3.9 demonstrates these considerations, again with WBF Higgs production in the vector triplet model. The squared dimension-six terms improve the agreement with the full model, and in one of the two benchmarks shown they are also necessary to avoid negative rates in the high-energy tail of the distribution.

To conclude, a simple expansion in $1/\Lambda$ suggests that the square of dimension-six amplitudes should not be taken into account in calculations within the dimension-six framework. But this argument is only valid if the underlying physics is weakly coupled and in phase-space regions with sizeable contributions from the SM. In these situations no harm is done by including the squared dimension-six terms, and they can improve the EFT validity in many other scenarios and are necessary to guarantee positive cross sections. In most situations these terms should therefore be included in calculations.

3.4.3 Limit setting

An important function of the dimension-six Lagrangian is to act as an intermediate parametrisation in the process of using Higgs measurements to set exclusion limits on the parameter space of specific models. We now test explicitly if limits derived in this way agree with constraints directly calculated in the full model.

We follow a simplified limit setting procedure. Expected exclusion limits on the vector triplet in the absence of a signal are calculated, either directly based on the full model, or first on the dimension-six Wilson coefficients and then translated onto the full model. Motivated by the discussion in the previous section, we also calculate limits on the dimension-six Wilson coefficients without taking into account the squared dimension-six contributions, and again translate the results to the vector triplet parameters. As a process we consider WBF Higgs production as given in Equation (3.16) and multiply the total Higgs production cross sections with a branching ratio $\text{BR}(h \rightarrow 2\ell 2\nu) \approx 0.01$. We disregard non-Higgs backgrounds as well as parton-shower or detector effects. Our limits are based on expected event counts in two high-energy bins of the p_{T,j_1} distributions. We define a parameter point to be excluded if $S/\sqrt{S+B} > 2$. While this statistical analysis is not designed to be realistic, it illustrates how the validity of our dimension-six approach affects possible limits.

We consider a two-dimensional plane in the parameter space of the vector triplet model. It is spanned by the mass m_ξ and a universal coupling rescaling c , and we choose the couplings as

$$g_V = 1, \quad c_H = c, \quad c_F = \frac{g_V^2}{2g^2} c, \quad c_{HHVV} = c^2. \quad (3.73)$$

such that Wilson coefficients $f_{\phi,2}$, $f_{\phi,3}$, and f_t vanish,

$$f_{WW} = f_{BW} = \frac{c^2}{2g^2} \quad \text{and} \quad f_W = -\frac{c^2}{g^2}. \quad (3.74)$$

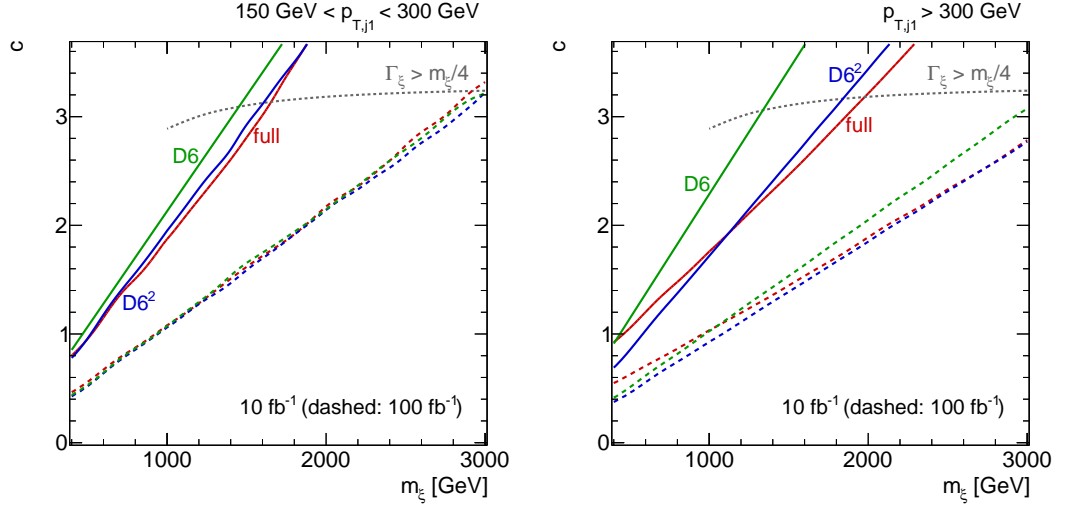


Figure 3.10: Toy limits on a two-dimensional slice of the vector triplet parameter space. We show the analysis based on the event numbers in $150 \text{ GeV} < p_{T,j_1} < 300 \text{ GeV}$ (left) and based on the tail $p_{T,j_1} > 300 \text{ GeV}$ (right).

In this parameter plane, all amplitudes from dimension-six operators scale with c^2/m_ξ^2 . Our perturbative calculation does makes sense for very strongly interacting systems, so we limit our analysis to $\Gamma_\xi/m_\xi < 1/4$.

The resulting toy limits are shown in Figure 3.10. Based on event numbers in the range $150 \text{ GeV} < p_{T,j_1} < 300 \text{ GeV}$, constraints calculated directly in the full model and EFT limits translated to the vector triplet agree very well. Including the dimension-six squared terms, however, can be important for this agreement. With more statistics, the differences between the different procedures become smaller, and ultimately the question of whether the squared dimension-six amplitudes should be taken into account is rendered irrelevant.

Larger differences appear when we use the information from the high-energy tail $p_{T,j_1} > 300 \text{ GeV}$ to constrain vector bosons with masses down to $m_\xi \gtrsim 500 \text{ GeV}$. This lack of a scale hierarchy does not improve with more statistics. The high-energy tail is also more sensitive to the square of dimension-six terms.

3.5 Conclusions

The dimension-six operators of linear Higgs effective field theory provide a theoretically well-defined, largely model-independent, and phenomenologically powerful framework to parametrise deviations from the Standard Model. But the validity of the EFT approach relies on a gap between

the experimental momentum transfer and the mass scales of the probed models of new physics. The limited precision of the LHC can guarantee such a clear scale hierarchy only under the additional assumption that the underlying physics is strongly coupled. For moderately weakly to moderately strongly coupled models, LHC Higgs measurements are sensitive to new physics scales ranging from the electroweak scale to approximately a TeV, casting doubt on the validity of the EFT approximation.

In this chapter we have studied the usefulness of the effective theory for LHC Higgs measurements by comparing the predictions of different complete models of new physics to those of their descriptions in terms of dimension-six operators. Our comparison included a singlet extension of the Higgs sector, a two-Higgs doublet model, scalar top partners, and a heavy vector triplet, focusing on parameter ranges relevant for the LHC. We have analysed Higgs couplings, total production rates, and kinematic distributions for the main Higgs production modes and representative decay channels.

A naive construction of the dimension-six model, where the effective theory is matched to the full model in the unbroken phase of the electroweak symmetry, confirms the simple analysis of energy scales: such a dimension-six approximation does not describe the phenomenology of most of the models adequately. In Higgs couplings and total production rates the discrepancies between the full model and its effective counterpart is of the order of v^2/Λ^2 , where Λ is the new physics scale. In the high-energy tails of distributions the EFT error even scales with E^2/Λ^2 . For many weakly coupled scenarios relevant for LHC Higgs measurements, this implies an unacceptable large error.

But this is not the end of the story. As we discussed at lengths in Section 3.2, matching the dimension-six model to the full theory is not unambiguous. First, instead of setting the matching scale Λ to the intrinsic new physics scale in the Lagrangian, we can use the actual physical masses after electroweak symmetry breaking, including contributions from the electroweak VEV. Second, we can express the Wilson coefficients of the dimension-six operators in terms of physical quantities such as mixing angles instead of Lagrangian parameters. These alternative choices, which we collectively call “ v -improved matching”, amount to matching the EFT in the broken phase of the electroweak symmetry.

While a v -improved EFT construction may be unconventional from a purely theoretical perspective, expressing quantities in physical masses and mixing angles is a natural choice from a practical point of view. It can be interpreted as a partial absorption of the dimension-eight and higher operators of the form $(\phi^\dagger \phi)^n \mathcal{O}_i$ into the Wilson coefficients of dimension-six operators with the replacement $\phi^\dagger \phi \rightarrow v^2/2$. In this way, it can improve the effective description where the expansion in v/Λ converges slowly for the default matching; it cannot help in high-energy tails where E/Λ becomes large. Note that v -improvement does not change the form of the effective operators, nor does it affect their phenomenology, or the way that experimental collaborations should set limits on operators. It purely affects the interpretation of Wilson coefficients in terms

Model	Process	EFT failure		
		rates	kinematics	matching
singlet	on-shell $h \rightarrow 4\ell$, WBF, Vh , ...			×
	off-shell WBF, ...		(×)	×
	hh	×	×	×
2HDM	on-shell $h \rightarrow 4\ell$, WBF, Vh , ...	(×)		×
	off-shell $h \rightarrow \gamma\gamma$, ...		(×)	×
	hh	×	×	×
top partners	WBF, Vh	(×)		(×)
vector triplet	WBF		(×)	×
	Vh		(×)	×

Table 3.15: Possible sources of failure of the dimension-six Lagrangian in Higgs observables. We use parentheses where deviations appear, but are unlikely to be observed in realistic scenarios. In addition, new light resonances that contribute to the same final states present an obvious breakdown of the effective description.

of model parameters.

We find that with a v -improved matching procedure the dimension-six operators provide an adequate description in almost all scenarios. The exceptions that confirm the rule are summarised in Table 3.15. The singlet and doublet extensions of the Higgs sector lead to simple shifts of the SM Higgs couplings, effects well captured by the corresponding dimension-six operators. The effective description only struggles with shifts of the Higgs-gauge couplings in the 2HDM, which only appear at dimension eight in the EFT, and with Higgs pair production, where two types of SM diagrams approximately cancel and off-shell contributions from new resonances have a dramatic impact. With the scalar top partners we illustrate that loop effects in Higgs-gauge couplings are either too small to be relevant for the LHC, or require light new particles and large couplings, in which case the effective description clearly breaks down. Finally, the vector triplet model generates interesting kinematic effects in Vh and WBF Higgs production at tree level. A v -improved dimension-six model can describe these effects over a large part of the phase space for realistic scenarios, breaking down only in the high-energy tails of certain kinematic distributions. Finally, the effective theory clearly fails to describe new light resonances. Such a signature at the LHC would be an obvious signal to switch to an appropriate simplified model.

Focusing on Higgs production in weak boson fusion in the vector triplet model, we proceeded with a set of practical questions. First we studied how the momentum transfer in weak boson fusion can be defined and measured, confirming the established notion that the transverse momenta of the tagging jets provide the most useful probe of the energy flow. We discussed the role of

squared dimension-six amplitudes, which contribute to cross sections at the same order in the EFT expansion as the leading effects from the neglected dimension-eight operators. Nevertheless, they should in general be included in calculations from a bottom-up perspective. We concluded our discussion of the vector triplet model with a brief demonstration of how the validity of the dimension-six model affects setting limits on a model in practice. The effective theory is less reliable when very high-energy events are taken into account, suggesting that experiments should constrain Wilson coefficients not only with the full event samples, but also with additional upper bounds on the momentum transfer.

With the limited precision of the LHC, the EFT approach is not guaranteed to accurately describe all potential signatures of new physics in Higgs measurements. Nevertheless, we have demonstrated that the dimension-six model works remarkably well for a wide range of models, parameter choices, and observables in single Higgs production. Key to this good performance is a suitable matching procedure, which takes into account subleading contributions from the Higgs VEV. This does not present a complication for an experimental fit of dimension-six operators to LHC Higgs data, it is a purely theoretical issue for the interpretation of the results.

Chapter 4

Better Higgs measurements through information geometry

Anything that is in the world when you're born is normal and ordinary and is just a natural part of the way the world works.

Anything that's invented between when you're fifteen and thirty-five is new and exciting and revolutionary and you can probably get a career in it.

Anything invented after you're thirty-five is against the natural order of things.

— D. Adams [202]

Q IS A PARTICULAR NICE LETTER to start a chapter with. Unfortunately, there is a distinct lack of suitable words beginning with a Q. Also, for a pretty initial to impress, the opening paragraph needs to have a certain minimum length, otherwise it just looks sad. And now I am unfortunately already running out of things to write. So here is something stolen: “You don’t know about me without you have read a book by the name of ‘The Adventures of Tom Sawyer’ but that ain’t no matter. That book was made by a Mr Mark Twain, and he told the truth, mainly.”

4.1 Introduction

After its experimental discovery [1] the Higgs boson and its properties have immediately become one of the most important and active fields of searches for physics beyond the Standard Model at the LHC. In the Lagrangian language of fundamental physics, the Higgs properties can be described by a continuous and high-dimensional parameter space, for instance in terms of Wilson coefficients in an effective field theory (EFT) [2]. One of the main features of moving from simple coupling modifications to higher-dimensional operators is that we can now include kinematic distributions in these searches [3]. A common challenge of all Higgs analyses is how to navigate the vast family of phase-space distributions.

Responding to the overwhelming amount of search strategies, we expect the LHC collaborations to focus more and more on high-level statistical tools, including hypothesis tests based on multivariate analysis with machine learning or the matrix element method [4]. Historically, these tools compare two discrete hypotheses, and applying them to continuous, high-dimensional parameter spaces is computationally expensive. Only recently, machine learning techniques have been extended to include inference on such continuous high-dimensional parameter spaces [5]. With these capabilities, it becomes increasingly important to be able to effectively characterize the information contained in these distributions. We present an approach based on information geometry [6], intrinsically designed to study continuous parameter spaces of arbitrary dimensionality without the need for any discretization of the hypothesis. We use this to compare and improve Higgs measurement strategies.

Our central object is the Fisher information matrix. Through the Cramér-Rao bound it determines the maximum knowledge on model parameters that we can derive from an observation [7]. In that sense the Cramér-Rao bound for the Fisher information plays a similar role as the Neyman-Pearson lemma [8] plays for a discrete hypothesis test and the log-likelihood ratio: it allows us to define and to compute the best possible outcome of any multivariate black-box analysis [9]. In addition, the Fisher information matrix defines a metric in the space of model parameters, which not only provides an intuitive geometric picture, but also gives us a handle on the linearization of the observable in terms of new physics effects.

When we apply our information geometry framework to Higgs physics, in particular to analyses of the dimension-6 Higgs Lagrangian, we can tackle questions of the kind:

- What is the maximum precision with which we can measure continuous model parameters?
- How is the information distributed over phase space?
- How much of the full information is contained in a given set of distributions?
- Which role do higher-dimensional corrections in the EFT expansion play?

We demonstrate our approach using three examples: Higgs production in weak boson fusion (WBF) [10] with its tagging-jet kinematics [11] is well known to probe many aspects of the Higgs-gauge coupling structure [12]. Focusing on the WBF production kinematics we first analyze

its combination with a Higgs decay to tau leptons [?]. This will for example allow us to estimate how much of the entire information on higher-dimensional operators is typically included in the leading tagging jet distributions. Combining WBF production with a Higgs decay to ZZ^* pairs we can test how much additional information is included in the decay distributions. Conceptually, this contrasts two ways to constrain the same effective Lagrangian via large momentum flow through the relevant vertices or via precision observables [?]. Finally, we will test how useful Higgs production in association with a single top [?] is for a dimension-6 operator analysis.

In a set of appendices we give a worked-out simple example for our approach, explain how we compute the Fisher information, show more information on our example processes, indicate how systematic or theory uncertainties can be included, and discuss the relation of our approach to standard log-likelihood ratios.

4.2 Information geometry

4.2.1 Fisher information and Cramér-Rao bound

Before jumping into Higgs physics, we have to recapitulate how a measurement is described statistically, and what makes it “optimal”. The basic principle of a measurement is to use some observed data \mathbf{x} to provide an estimate $\hat{\theta}$ of the unknown true value of some parameters θ , the outcome of which is described by a probability distribution $f(\hat{\theta}|\theta_0)$. The most important properties of this distribution are its expectation value and its variance. The former measures the bias of the measurement: the expectation value of an unbiased estimator is always equal to the true value,

$$\bar{\theta} \equiv E[\hat{\theta}|\theta_0] = \theta_0. \quad (4.1)$$

On the other hand, the variance of the estimator, or for more than one parameter its covariance matrix

$$\text{cov}[\hat{\theta}|\theta_0]_{ij} \equiv E[(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)|\theta_0], \quad (4.2)$$

provides a measure of its precision. In practice there is often a trade-off between minimizing the bias of an experiment and the variance.

But how good can an estimator be? Clearly, any given experiment does not allow the measurement of parameters with arbitrary precision. We can make this statement more quantitatively. If the relation between parameters and data is given by a probability distribution function $f(\mathbf{x}|\theta)$ that is twice differentiable in continuous parameters θ_i and fulfills certain regularity conditions, we can calculate the **Fisher information**

$$I_{ij}(\theta) = E\left[\frac{\partial \log f(\mathbf{x}|\theta)}{\partial \theta_i} \frac{\partial \log f(\mathbf{x}|\theta)}{\partial \theta_j} \middle| \theta\right] = -E\left[\frac{\partial^2 \log f(\mathbf{x}|\theta)}{\partial \theta_i \partial \theta_j} \middle| \theta\right]. \quad (4.3)$$

The Cramér-Rao bound [?] then states that the covariance matrix of any estimator $\hat{\theta}$ is bounded from below by the inverse Fisher information:

$$\text{cov}[\hat{\theta}|\theta_0]_{ab} \geq \frac{\partial \bar{\theta}_a}{\partial \theta_i} I_{ij}^{-1}(\theta_0) \frac{\partial \bar{\theta}_b}{\partial \theta_j}. \quad (4.4)$$

In particular, for an unbiased estimator $\bar{\theta} = \theta$ and

$$\text{cov}[\hat{\theta}|\theta_0]_{ij} \geq I_{ij}^{-1}(\theta_0). \quad (4.5)$$

In the one-dimensional case, this corresponds to a typical measurement error of

$$\Delta\theta = \sqrt{\text{var}[\hat{\theta}|\theta_0]} \geq 1/\sqrt{I(\theta_0)}. \quad (4.6)$$

The Fisher information thus encodes the maximal precision with which parameters can be measured at an experiment. Large entries in this matrix correspond to directions that can be measured particularly precisely. On the other hand, an eigenvector of the information matrix with corresponding eigenvalue zero is a blind direction that can never be probed by the experiment.

I_{ij} has several useful properties. Unlike many other statistical tools, it summarizes the sensitivity to **all** directions in theory space in one matrix, making it particularly useful for high-dimensional parameter spaces. It is additive between different measurements or between different phase-space regions in the same experiment. This will later allow us to calculate the distribution of information over phase space. Furthermore, describing experiments in terms of the Fisher information is independent of arbitrary basis choices: it is invariant under the parameterization of the observables \mathbf{x} , and transforms covariantly under a reparameterization of the theory parameters $\theta \rightarrow \Theta(\theta)$,

$$I_{ab}(\Theta) = \frac{\partial \theta_i}{\partial \Theta_a} I_{ij}(\theta) \frac{\partial \theta_j}{\partial \Theta_b} \quad (4.7)$$

with an implicit summation over repeated parameter indices i, j .

As a symmetric and positive definite rank-two tensor, the Fisher information defines a Riemannian metric on the theory space [?].¹ This allows us to use methods from differential geometry and provides a distance measure between parameter points. We can differentiate between local distance in the tangent space at some point θ_a ,

$$d_{\text{local}}(\theta_b; \theta_a) = \sqrt{I_{ij}(\theta_a)(\theta_{bi} - \theta_{ai})(\theta_{bj} - \theta_{aj})}, \quad (4.8)$$

and global distances. Contours of local distances translate the Fisher information in one point into the minimal error ellipsoids according to the Cramér-Rao bound. The distance values have

¹This is only true in the absence of blind directions, otherwise the information is only positive semi-definite and defines a pseudo-metric.

an intuitive interpretation: if the estimator is distributed according to a Gaussian around the true value, $d_{\text{local}}(\boldsymbol{\theta}_b; \boldsymbol{\theta}_a)$ corresponds to how unlikely it is to measure $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_b$ if the true value is $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_a$, expressed in standard deviations or “sigmas”. In other words, in this limit the distance measure is equivalent to the maximal expected significance with which $\boldsymbol{\theta}_a$ can be excluded if $\boldsymbol{\theta}_b$ is true.

Goying beyond the tangent space, global distances can be defined along geodesics,

$$d(\boldsymbol{\theta}_a, \boldsymbol{\theta}_b) = \min_{\boldsymbol{\theta}(s)} \int_{s_a}^{s_b} ds \sqrt{I_{ij} \frac{d\theta_i(s)}{ds} \frac{d\theta_j(s)}{ds}}, \quad (4.9)$$

providing a more general and symmetric notion of discrimination power between two points without having to pick the true value of the parameter.

As an aside, the Fisher information also plays an important role in Bayesian inference. In Jeffrey’s prior it is used to define an “objective” prior that is invariant under reparameterization, and according to the Bernstein-von Mises theorem it describes the posterior mode in the asymptotic limit.

4.2.2 Simple example

As a simple example we study the Fisher information in a number of rates n_c measured in various Higgs channels c . In the absence of systematic uncertainties they follow Poisson statistics,

$$f(\mathbf{n}|\boldsymbol{\nu}) = \prod_c \text{Pois}(n_c|\nu_c) = \prod_c \frac{\nu_c^{n_c} e^{-\nu_c}}{n_c!}. \quad (4.10)$$

We can calculate the Fisher information in terms of the Poisson mean $\boldsymbol{\nu}$ as

$$\frac{\partial \log f}{\partial \nu_c} = \frac{n_c}{\nu_c} - 1 \quad \frac{\partial^2 \log f}{\partial \nu_c \partial \nu_{c'}} = -\frac{\delta_{cc'} n_c}{\nu_c^2} \quad I_{cc'} \equiv -E \left[\frac{\partial^2 \log f}{\partial \nu_c \partial \nu_{c'}} \Big| \boldsymbol{\nu} \right] = \frac{\delta_{cc'}}{\nu_c}. \quad (4.11)$$

If we express the expected count rates in terms of model parameters g_i , the Fisher information becomes

$$I_{ij} = \sum_c \frac{1}{\nu_c} \frac{\partial \nu_c}{\partial g_i} \frac{\partial \nu_c}{\partial g_j}. \quad (4.12)$$

The matrix $\partial \nu_c / \partial g_i$ is determined by selection requirements, detector acceptance, and efficiencies. In the κ framework that only scales cross sections and branching ratios, the matrix $\partial \nu_c / \partial g_i$ is trivial to calculate in closed form. For each channel this matrix is singular, which means it measures one direction in parameter space and is blind to all others. At least as many channels as parameters are required to make the combined information in Equation (4.12) non-singular

and remove all blind directions (assuming the channels do not provide degenerate information, i. e. the same eigenvectors in the Fisher information).

For illustration, we consider the case where we want to measure one coupling g in one channel with the expected number of events

$$\nu = L (\sigma_S + \sigma_B) = L g^2 \sigma_0 + L \sigma_B . \quad (4.13)$$

The Fisher information is then

$$I = 4L \frac{g^2 \sigma_0^2}{g^2 \sigma_0 + \sigma_B} = \frac{4L}{g^2} \frac{\sigma_S^2}{\sigma_S + \sigma_B} . \quad (4.14)$$

According to the Cramér-Rao bound, the standard deviation of any unbiased estimator \hat{g} is at least

$$\frac{\Delta \hat{g}}{g} \geq \frac{1}{g \sqrt{I}} = \frac{1}{2 \sqrt{L}} \frac{\sqrt{\sigma_S + \sigma_B}}{\sigma_S} . \quad (4.15)$$

The three terms show how the sensitivity to g profits from the square in the cross section, the square-root dependence on the statistics, and the dependence on the signal-to-background ratio.

4.3 Information in LHC processes

4.3.1 Information in event counts

The simplest LHC experiment is an event count in some fiducial region described by Poisson statistics. Analogous to the example in the previous section, the Fisher information for such a measurement and arbitrary theory parameters θ reads

$$I_{ij}^{\text{xsec}}(\theta) = L \frac{\partial \sigma(\theta)}{\partial \theta_i} \frac{\partial \sigma(\theta)}{\partial \theta_j} \frac{1}{\sigma(\theta)} . \quad (4.16)$$

Here L is the integrated luminosity and $\sigma(\theta)$ the total cross section as a function of the theory parameters.

4.3.2 Information in histograms

The simplest way to add kinematic features is measuring single or double differential cross sections. Leaving aside certain systematic uncertainties, such histograms are just a set of independent counting experiments, and the Fisher information is given by

$$I_{ij}^{\text{histogram}}(\theta) = L \sum_{\text{bins } b} \frac{\partial \sigma_b(\theta)}{\partial \theta_i} \frac{\partial \sigma_b(\theta)}{\partial \theta_j} \frac{1}{\sigma_b(\theta)} . \quad (4.17)$$

4.3.3 Information in full process and differential information

To describe the full high-dimensional kinematics of LHC processes without relying on a choice of observables or binning, we use an extended likelihood ansatz. The observables in LHC physics are then split into a total number of events n , each of which has some kinematic properties x_i , with a probability distribution

$$f(\mathbf{x}|\boldsymbol{\theta}) = \text{Pois}(n|L\sigma(\boldsymbol{\theta})) \prod_{i=1}^n f_1(x_i|\boldsymbol{\theta}). \quad (4.18)$$

Here $f_1(x_i|\boldsymbol{\theta})$ is the probability distribution function for a single event. The Fisher information is then

$$I_{ij}^{\text{full}}(\boldsymbol{\theta}) = L \frac{\partial \sigma(\boldsymbol{\theta})}{\partial \theta_i} \frac{\partial \sigma(\boldsymbol{\theta})}{\partial \theta_j} \frac{1}{\sigma(\boldsymbol{\theta})} + L \sigma(\boldsymbol{\theta}) E \left[\frac{\partial \log f_1(x|\boldsymbol{\theta})}{\partial \theta_i} \frac{\partial \log f_1(x|\boldsymbol{\theta})}{\partial \theta_j} \middle| \boldsymbol{\theta} \right], \quad (4.19)$$

adding a distribution term to the Poisson part. Typically, the single-event likelihood functions $f_1(x|\boldsymbol{\theta})$ are complex expressions. Instead of calculating their derivatives analytically, we rely on a Monte-Carlo approach, approximating the phase-space integral with a sum over events:

$$\int dx f_1(x) \rightarrow \sum_{\text{events } k} \frac{\Delta \sigma_k}{\sigma} \quad (4.20)$$

This turns the Fisher information into the familiar form of a sum of Poisson terms for the individual events,

$$I_{ij}^{\text{full}}(\boldsymbol{\theta}) = L \sum_{\text{events } k} \frac{\partial \Delta \sigma_k(\boldsymbol{\theta})}{\partial \theta_i} \frac{\partial \Delta \sigma_k(\boldsymbol{\theta})}{\partial \theta_j} \frac{1}{\Delta \sigma_k(\boldsymbol{\theta})}. \quad (4.21)$$

This can be calculated from standard Monte-Carlo samples. We use a combination of MadGraph 5 [?], MadMax [?], and our own code, which we call MadFisher. It is described in detail in Appendix ??.

We calculate the Fisher information in ?? with Monte-Carlo methods. With

$$\int dx f^{(1)}(x) \rightarrow \sum_{\text{events } k} \Delta \sigma_k / \sigma \quad (4.22)$$

we find

$$I_{ij}(\boldsymbol{\theta}) = \sum_{\text{events } k} \frac{L}{\Delta \sigma_k(\boldsymbol{\theta})} \frac{\partial \Delta \sigma_k(\boldsymbol{\theta})}{\partial g_i} \frac{\partial \Delta \sigma_k(\boldsymbol{\theta})}{\partial g_j}, \quad (4.23)$$

requiring the differential cross sections and their derivatives as input.

4.3.4 Nuisance parameters and profiling

Our information geometry approach can easily be extended to include systematic and theory uncertainties. The parameter space then consists of nuisance parameters v_i in addition to the Wilson coefficients, and constraint terms are added to the likelihood. If for instance the k th parameter is a nuisance parameter with a Gaussian constraint term with width σ_k , the additional term in the Fisher information reads

$$I_{ij}(\boldsymbol{\theta}, \mathbf{v}) = \dots + \frac{\delta_{ik}\delta_{jk}}{\sigma_k^2}. \quad (4.24)$$

This also applies to a log-normal constraint through reparametrization of v . Local and global distances now refer to combined theory and nuisance parameters $(\boldsymbol{\theta}, \mathbf{v})$.

We define a profiled local distance between two points $\boldsymbol{\theta}_b$ and $\boldsymbol{\theta}_a$

$$d_{\text{profiled}}(\boldsymbol{\theta}_b, \boldsymbol{\theta}_a) = \min_{\mathbf{v}} d_{\text{local}}((\boldsymbol{\theta}_b, \mathbf{v}); (\boldsymbol{\theta}_a, \mathbf{0})). \quad (4.25)$$

Equivalently, we can define a profiled Fisher information matrix. Assuming the last parameter to be the only nuisance parameter, the Fisher information matrix has the form

$$I_{ij} = \begin{pmatrix} I^{\text{theory}} & \mathbf{m} \\ \mathbf{m}^T & n \end{pmatrix} \quad (4.26)$$

where I^{theory} is the information matrix restricted to the theory parameters, the vector \mathbf{m} describes the mixing between theory and nuisance parameter, and n is the component that only affects the nuisance parameters. Technically described by the parallel projection of an ellipsoid, the projected Fisher information is given by

$$I_{\text{profiled } ij} = I_{ij}^{\text{theory}} - \frac{m_i m_j}{n}. \quad (4.27)$$

4.3.5 The MadFisher algorithm

We first generate event samples for a number of benchmark parameters with MadMax [?]. This add-on to MadGraph 5 [?] allows us to simultaneously calculate differential rates for different parameter points using the same phase-space grid. Our FeynRules [?] model file of the relevant dimension-6 operators does not truncate operator effects at $\mathcal{O}(1/\Lambda^2)$. MadMax requires fixed renormalization and factorization scales, which we set following Reference [?]. To keep the calculation times manageable, we restrict some processes to the dominant sub-processes, for instance to initial-state u and d quarks in the WBF case. We then normalize the Higgs rates to the LHC HXS WG recommendations for the total cross section [?], calculating the effect of the different acceptance regions with MadGraph 5. Background processes are simply rescaled to MadGraph predictions.

A morphing technique allows us to calculate the differential cross sections and their derivatives at arbitrary positions in parameter space [?]. The effect of $\mathcal{O}_{\phi,2}$ and systematic rate uncertainties (see ??) is taken into account analytically. The contributions from the other operators are decomposed into a number of basis components and can be exactly reconstructed from a set of simulated benchmark points. Our example processes require up to 70 such basis components (in the WBF $H \rightarrow 4\ell$ case).

We can then easily calculate the Fisher information according to Equation (4.23). Finally, global distances as in ?? are calculated in analogy to free fall in general relativity: a starting point and a set of directions in parameter space define the initial conditions, from which we numerically calculate distances along curves defined by the geodesic equation.

4.3.6 Geometry of effective field theories

Motivated by the lack of any direct signs of new physics and by the similarity of the observed scalar with the SM agent of electroweak symmetry breaking, we parametrize the properties of the Higgs boson with dimension-6 operators of the SM effective field theory [? ? ?],

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{f_i}{\Lambda^2} \mathcal{O}_i. \quad (4.28)$$

These operators model-independently describe the effects of new physics at energy scales far below the new physics scale, $E \ll \Lambda$. The LHC cannot guarantee such a scale separation, but the effective description still works for a wide range of new physics scenarios [?].

In the cross section, amplitudes from dimension-6 operators squared appear at $\mathcal{O}(1/\Lambda^4)$, i. e. at the same order as the leading effects from dimension-8 operators, which we neglect. If the new physics couplings are of the same size as the relevant SM couplings, and if the dimension-4 amplitudes are not accidentally suppressed, the size of the dimension-6 squared term can therefore be used as an estimate for the error of the EFT approximation. In general, the dimension-6 squared terms can be larger than the dimension-8 effects, and are necessary from a technical point of view to guarantee positive cross sections [?].

In this paper we will calculate the information geometry of the dimension-6 model for various processes and operators. As theory parameters we pick a dimensionless rescaling of the Wilson coefficients,

$$\theta_i = \frac{f_i}{\Lambda^2} v^2, \quad (4.29)$$

such that the SM corresponds to the origin $\theta = \mathbf{0}$. As can be seen from Equation (4.23), the Fisher information at the SM $I_{ij}(\mathbf{0})$ only measures the $\mathcal{O}(\theta)$ terms in the differential cross sections and is insensitive to higher orders. In other words, information distances in the tangent space at the SM only take into account the leading dimension-6 operator effects at $\mathcal{O}(1/\Lambda^2)$. On the

other hand, the dimension-6 squared contributions do appear in the Fisher information away from the SM and therefore in global distances as given in Equation (4.9). The difference between linearized distances based on the SM Fisher information metric and global distances measured along geodesics, or the curvature of the information geometry, therefore provides an intuitive measure of the role of $\mathcal{O}(1/\Lambda^4)$ contributions.

Following Equation (4.6), the Fisher information in a given direction I defines a maximal precision $\Delta\theta_{\min} = 1/\sqrt{I}$, with which this direction in theory space can be measured. For an EFT this can directly be translated into a maximal new physics reach

$$\frac{\Lambda_{\max}}{\sqrt{f}} = \frac{v}{\sqrt{\Delta\theta_{\min}}} = vI^{1/4}. \quad (4.30)$$

4.4 Higgs signatures from CP -even operators

4.4.1 Weak-boson-fusion Higgs to taus

The first question we tackle with our information geometry approach is what we can learn about higher-dimensional operators from the non-trivial kinematics of weak-boson-fusion production. As a decay we include a simple fermionic two-body decay $H \rightarrow \tau\tau$ [?], see ???. For our proof of concept we stick to a parton-level analysis at leading order. The dominant irreducible backgrounds are QCD Zjj production and electroweak Zjj production, both with the decay $Z \rightarrow \tau\tau$, and Higgs production in gluon fusion with $H \rightarrow \tau\tau$.

We do not simulate tau decays, but multiply the rates with the branching ratios for the semi-leptonic di-tau mode and assume the di-tau system can be reconstructed with the collinear approximation with a realistic resolution for $m_{\tau\tau}$. Following the procedure outlined in Refs. [? ?], we smear the $m_{\tau\tau}$ distributions using a Gaussian (with width 17 GeV) for Higgs production and a double Gaussian (where the dominant component has a width of 13 GeV) for Z production, estimated from Figure 1a of Reference [?]. The double Gaussian ensures an accurate description of the high-mass tail of the Z peak around $m_{\tau\tau} = m_H$. Otherwise, no detector effects are included. We require loose cuts

$$\begin{aligned} p_{T,j} &> 20 \text{ GeV} & |\eta_j| &< 5.0 & \Delta\eta_{jj} &> 2.0 \\ p_{T,\tau} &> 10 \text{ GeV} & |\eta_\tau| &< 2.5, \end{aligned} \quad (4.31)$$

to include as much of phase space as possible.

The different QCD radiation patterns of electroweak and QCD signal and background processes are a key feature to separate the signal from the background [?]. We take it into account through approximate individual jet veto survival probabilities [?],

$$\epsilon_{\text{WBF } H}^{\text{CJV}} = 0.71 \quad \epsilon_{\text{EW } Z}^{\text{CJV}} = 0.48 \quad \epsilon_{\text{QCD } Z}^{\text{CJV}} = 0.14 \quad \epsilon_{\text{GF } H}^{\text{CJV}} = 0.14. \quad (4.32)$$

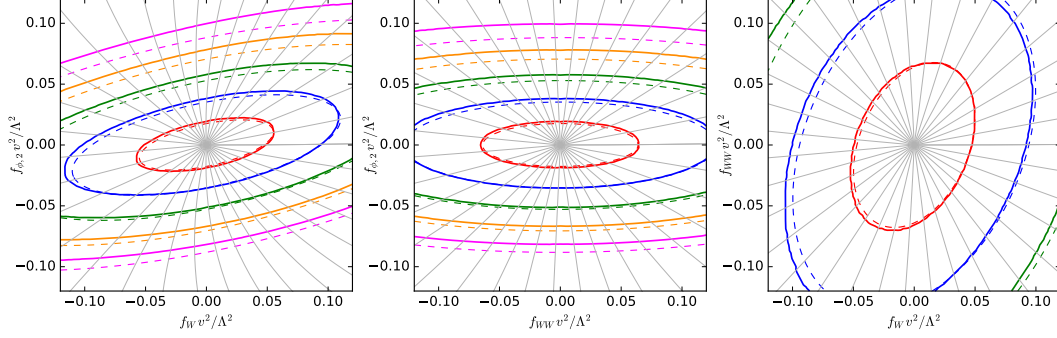


Figure 4.1: Error ellipses defined by the Fisher information in the WBF $H \rightarrow \tau\tau$ channel. We show contours of local distance $d_{\text{local}}(\boldsymbol{\theta}; \mathbf{0})$ (dashed) and global distance $d(\boldsymbol{\theta}, \mathbf{0})$ (solid). The colored contours indicate distances of $d = 1 \dots 5$. In grey we show example geodesics. The θ_i not shown are set to zero.

Since our phase space \mathbf{x} does not include any jets other than the two tagging jets, we are not sensitive to details of the central jet veto other than the relative survival probabilities and possible second-order effects that would correlate the veto with \mathbf{x} . After the event selection of Equation (4.31) and applying the CJV efficiencies, the WBF Higgs signal of 53 fb in the SM faces a dominant QCD Z background of 2.7 pb.

We consider five CP -even dimension-6 operators in the HISZ basis [? ?],

$$\begin{aligned}
 \mathcal{O}_B &= i \frac{g}{2} (D^\mu \phi^\dagger) (D^\nu \phi) B_{\mu\nu} & \mathcal{O}_W &= i \frac{g}{2} (D^\mu \phi)^\dagger \sigma^k (D^\nu \phi) W_{\mu\nu}^k \\
 \mathcal{O}_{BB} &= -\frac{g'^2}{4} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu} & \mathcal{O}_{WW} &= -\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^k W^{\mu\nu k} \\
 \mathcal{O}_{\phi,2} &= \frac{1}{2} \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi).
 \end{aligned} \tag{4.33}$$

The first four operators introduce new Lorentz structures into Higgs-gauge interactions, which translate into changed kinematic shapes. The pure Higgs operator $\mathcal{O}_{\phi,2}$ leads to a universal rescaling of all single-Higgs couplings and otherwise only affects the Higgs self-coupling. Other operators that contribute to WBF Higgs production are tightly constrained by electroweak precision data or can be removed from the basis using field redefinitions [?]. The effect of $\mathcal{O}_{\phi,2}$ on gluon-fusion Higgs production is taken into account in our analysis, while the effects from \mathcal{O}_W and \mathcal{O}_B on the subleading electroweak Zjj background are neglected.

Maximum precision on Wilson coefficients

Following ??, our model space is spanned by five dimensionless parameters

$$\boldsymbol{\theta} = \frac{v^2}{\Lambda^2} \begin{pmatrix} f_{\phi,2} \\ f_W \\ f_{WW} \\ f_B \\ f_{BB} \end{pmatrix}. \quad (4.34)$$

With these basis vectors we calculate the Fisher information for 13 TeV using a combination of MadGraph5 [?], MadMax [?], and our own MadFisher algorithm, described in Appendix ?. We absorb the all particle identification and trigger efficiencies into a single universal ε (which does not include the process-dependent CJV efficiencies). Then for our toy example we assume the integrated luminosity times universal efficiencies to be $L \cdot \varepsilon = 30 \text{ fb}^{-1}$. We find

$$I_{ij}(\mathbf{0}) = \begin{pmatrix} 3202.1 & -625.3 & -7.2 & -34.8 & 0.3 \\ -625.3 & 451.0 & -109.5 & 23.3 & -1.5 \\ -7.2 & -109.5 & 243.7 & -5.5 & 2.8 \\ -34.8 & 23.3 & -5.5 & 4.1 & -0.3 \\ 0.3 & -1.5 & 2.8 & -0.3 & 0.1 \end{pmatrix}. \quad (4.35)$$

The eigenvectors, ordered by the size of their eigenvalues, are

$$\boldsymbol{\theta}_1 = \begin{pmatrix} 0.98 \\ -0.21 \\ 0.01 \\ -0.01 \\ 0.00 \end{pmatrix} \quad \boldsymbol{\theta}_2 = \begin{pmatrix} -0.18 \\ -0.79 \\ 0.58 \\ -0.04 \\ 0.01 \end{pmatrix} \quad \boldsymbol{\theta}_3 = \begin{pmatrix} 0.12 \\ 0.57 \\ 0.81 \\ 0.03 \\ 0.01 \end{pmatrix} \quad \boldsymbol{\theta}_4 = \begin{pmatrix} 0.00 \\ -0.05 \\ 0.00 \\ 1.00 \\ -0.07 \end{pmatrix} \quad \boldsymbol{\theta}_5 = \begin{pmatrix} 0.00 \\ -0.00 \\ -0.01 \\ 0.07 \\ 1.00 \end{pmatrix}. \quad (4.36)$$

The corresponding eigenvalues are (3338, 395, 165, 2.9, 0.1), indicating that the WBF process has very different sensitivities to the five operators: $\mathcal{O}_{\phi,2}$ can be most strongly constrained and is weakly correlated with \mathcal{O}_W . It is followed by the strongly correlated \mathcal{O}_W - \mathcal{O}_{WW} plane. The sensitivity to \mathcal{O}_B and \mathcal{O}_{BB} , which only play a role in subleading Z -mediated production diagrams, is much smaller and shows very little correlation with each other and everything else.

We visualize our results as contours of the local and global distances defined in ?? for slices of parameter space in Figure 4.1. First, the contours show the maximum precision that can be attained in a measurement in this process. Without taking into account systematic uncertainties, an optimal measurement will probe the $\mathcal{O}_{\phi,2}$ direction with $\Delta g \approx 0.02$, translating into $\Lambda/\sqrt{f_{\phi,2}} \approx 1.8 \text{ TeV}$. The \mathcal{O}_W and \mathcal{O}_{WW} directions can optimally be probed at the $\Delta g \approx 0.05$ or $\Lambda/\sqrt{f_{\phi,2}} \approx 1.1 \text{ TeV}$ level.

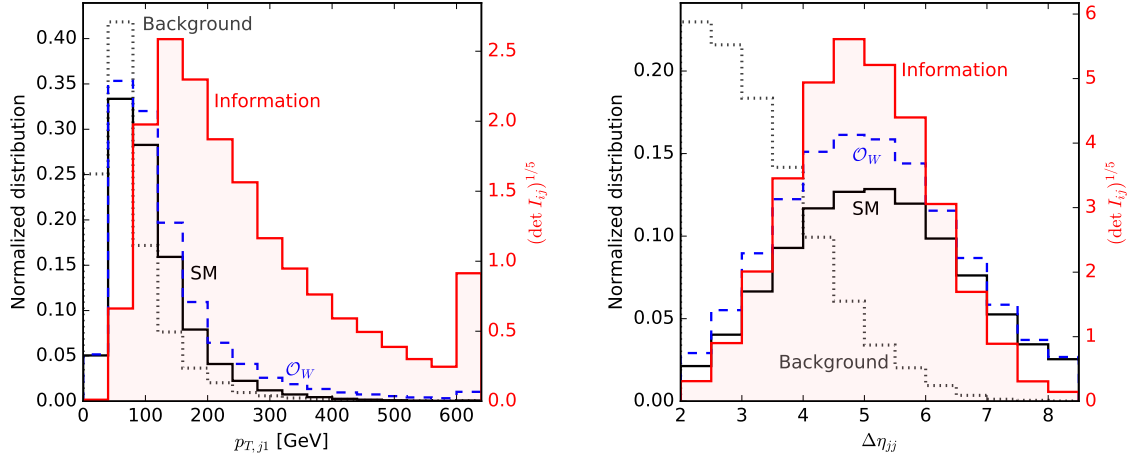


Figure 4.2: Distribution of the Fisher information in the WBF $H \rightarrow \tau\tau$ channel (shaded red). We also show the normalized SM signal (solid black) and QCD Z+jets (dotted grey) rates. The dashed blue line shows the effect of an exaggerated $f_W v^2/\Lambda^2 = 0.5$. The last bin is an overflow bin.

Comparing the local and global distances provides some insight into the role of $\mathcal{O}(1/\Lambda^4)$ effects, as discussed before. At $d = 1, 2$ the differences are small, signaling that an optimal measurement will be dominated by the linearized dimension-6 amplitudes. On the other hand, analyses based on less luminosity or requiring more stringent exclusion criteria (translating into larger distances) gain sensitivity to the squared dimension-6 terms.

Differential information

The fact that the Fisher information is additive across different phase-space regions means that we can consider the differential information with respect to phase space ($dI_{ij}/d\mathbf{x}$) or a specific kinematic variable ($dI_{ij}/d\mathbf{v}$). In Figure 4.2 we show the differential cross sections of the signal and dominant background process for typical kinematic distributions and compare it to the differential information. More distributions are shown in Appendix ??.

Obviously, the signal-to-background ratio improves for large invariant masses or the tagging jets and towards $m_{\tau\tau}$ values around the Higgs mass. The information is larger in these phase-space regions, independent of the direction in model space. On the other hand, most of our dimension-6 operators include derivatives, leading to an increasing amplitude with momentum transfer through the gauge-Higgs vertex. This momentum flow is not observable, but the transverse momenta of the tagging jets and the Higgs boson are strongly correlated with it [?]. Indeed most of the information on higher-dimensional operators comes from the high-energy tail of $p_{T,j1}$.

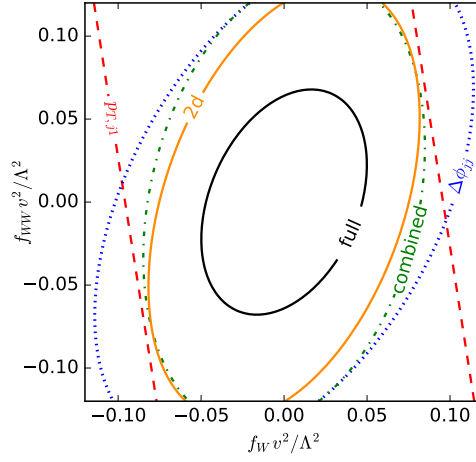


Figure 4.3: Information from histograms compared to the full information (black) in the WBF $H \rightarrow \tau\tau$ channel, shown as contours $d_{\text{local}}(\boldsymbol{\theta}; \mathbf{0}) = 1$. We include p_{T,j_1} , $\Delta\phi_{jj}$, their naive combination assuming no mutual information, and their two-dimensional histogram. The θ_i not shown are set to zero.

The rapidity difference between the tagging jets indicates a trade-off between these two effects: on the one hand, at larger rapidity distances the signal-to-background ratio clearly improves [?]. On the other hand, the largest effects from dimension-6 operators appear at smaller $\Delta\eta_{jj}$, again driven by the larger momentum transfer [?]. In the right panel of Figure 4.2 we see that the information on these operators comes from $\Delta\eta_{jj} = 3 \dots 7$. Tight cuts with the aim to remove backgrounds lose a sizable fraction of the information on dimension-6 operators.

Information in distributions

While the integrated, fully differential information defined in ?? provides us with optimal experimental results, it remains to be shown that we can access it in practice. Recent proposals using machine learning for high-dimensional likelihood fits aim to tackle exactly this problem [?]. Regardless, a relevant question is how much of this maximum information is retained in simple one-dimensional or two-dimensional distributions of standard kinematic observables \mathbf{v} .

In the presence of backgrounds, a histogram-based analysis first requires a stringent event selection. We choose the WBF cuts

$$p_{T,j_1} > 50 \text{ GeV} \quad m_{jj} > 1 \text{ TeV} \quad \Delta\eta_{jj} > 3.6. \quad (4.37)$$

This improves the signal-to-background ratio to approximately unity, but at the cost of losing discrimination power. Eventually, a histogram-based analysis will benefit from optimizing this

selection, for instance foregoing the simple cuts for a multivariate approach, going beyond the scope of this demonstration. Based on this selection, we analyze the distributions

- p_{T,τ_1} with bin size 25 GeV up to 500 GeV and an overflow bin;
- $m_{\tau\tau}$ with bin size 5 GeV in the allowed range of 105 ... 165 GeV.
- $p_{T,\tau\tau}$ with bin size 50 GeV up to 800 GeV and an overflow bin;
- p_{T,j_1} with bin size 50 GeV up to 800 GeV and an overflow bin;
- m_{jj} with bin size 250 GeV up to 4 TeV and an overflow bin;
- $\Delta\eta_{jj}$ with bin size 0.5 up to 8.0 and an overflow bin;
- $\Delta\phi_{jj} = \phi_{j_{\eta<0}} - \phi_{j_{\eta>0}}$ [°] with bin size $2\pi/20$;
- $\Delta\eta_{\tau\tau,j_1}$ with bin size 0.5 up to 8.0 and an overflow bin;
- $\Delta\phi_{\tau\tau,j_1}$ with bin size $\pi/10$.

Figure 4.3 demonstrates that virtuality measures such as the transverse momentum of the leading tagging jet mostly constrain \mathcal{O}_W , while angular correlations between the jets are more sensitive to \mathcal{O}_{WW} . Stringent constraints on the full operator space can only be achieved by combining the information in these distributions, ideally in a two-dimensional histogram.

In Figure 4.4 we extend our comparison to the information in all of the above distributions. The top panel shows the eigenvalues of the individual information matrices, and the colors indicate which operators the corresponding eigenvectors are composed of. This allows us to see which operators can be measured well in which distributions, and where blind (or flat) directions arise. In the lower panel we compare the determinants, providing a straightforward measure of the information in distributions that is invariant under basis rotations.

In general, single differential cross sections probe individual directions in phase space well, but always suffer from basically blind directions. To maximize the constraining power on all operators we need to combine virtuality measures and angular correlations. Even then there is a substantial difference to the maximum information in the process: the combined analysis of jet transverse momenta and $\Delta\phi_{jj}$ has a new physics reach in the $\mathcal{O}_{\phi,2}$ - \mathcal{O}_W - \mathcal{O}_{WW} space of 0.9 TeV, compared to 1.2 TeV for the fully differential cross section. Under our simplistic assumptions this corresponds to roughly three times more data. Half of this loss in constraining power is due to information in background-rich regions discarded by the WBF cuts, and half is due to non-trivial kinematics not captured by the double differential distributions.

4.4.2 Weak-boson-fusion Higgs to four leptons

Another question we can approach with information geometry is how much the non-trivial decay mode $H \rightarrow 4\ell$ adds to the WBF production analyzed in Section 4.4.1. For this particularly clean channel, shown in ??, the backgrounds are not the limiting factor, so we omit them for our toy

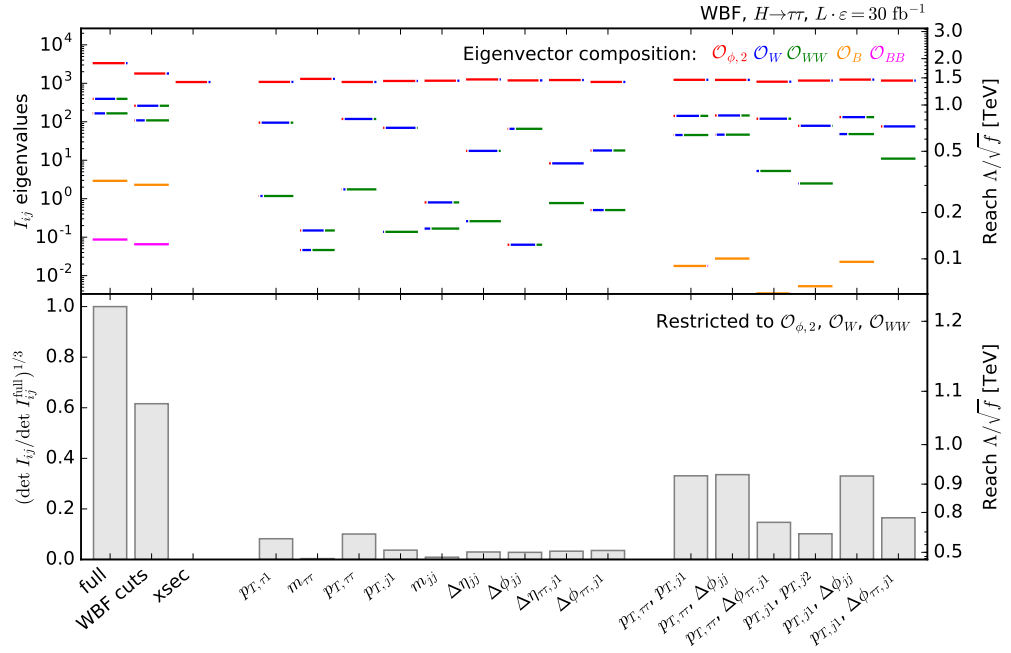


Figure 4.4: Fisher information for the WBF $H \rightarrow \tau\tau$ channel exploiting the full phase space, after the cuts in Equation (4.37), and for several kinematic distributions. The top panel shows the eigenvalues, the colors denote the composition of the corresponding eigenvectors. The right axis translates the eigenvalues into a new physics reach for the corresponding combination of Wilson coefficients. In the bottom panel we show the determinants of the Fisher information restricted to $\mathcal{O}_{\phi,2}$, \mathcal{O}_W , and \mathcal{O}_{WW} , normalized to the full information. Again, the right axis translates them into a new physics reach.

study. This also allows us to avoid smearing the $m_{4\ell}$ distribution. At parton level we apply the generator-level cuts

$$\begin{aligned} p_{T,j} &> 20 \text{ GeV} & |\eta_j| &< 5.0 \\ p_{T,\ell} &> 10 \text{ GeV} & |\eta_\ell| &< 2.5, \end{aligned} \quad (4.38)$$

with $\ell = e, \mu$. The SM cross section after these cuts is 0.36 fb.

Maximum precision on Wilson coefficients

Again we study the five-dimensional space of CP -even Wilson coefficients given in Equation (4.34). For increased luminosity, $L \cdot \varepsilon = 100 \text{ fb}^{-1}$, we find the SM information

$$I_{ij}(\mathbf{0}) = \begin{pmatrix} 144.3 & -27.3 & -11.5 & -1.6 & -0.7 \\ -27.3 & 50.9 & -9.1 & 6.7 & -0.2 \\ -11.5 & -9.1 & 36.9 & -1.2 & 1.0 \\ -1.6 & 6.7 & -1.2 & 1.9 & -0.1 \\ -0.7 & -0.2 & 1.0 & -0.1 & 0.1 \end{pmatrix} \quad (4.39)$$

with the eigenvectors

$$\begin{aligned} \theta_1 &= \begin{pmatrix} 0.96 \\ -0.25 \\ -0.08 \\ -0.02 \\ 0.00 \end{pmatrix} & \theta_2 &= \begin{pmatrix} -0.16 \\ -0.79 \\ 0.58 \\ -0.11 \\ 0.02 \end{pmatrix} & \theta_3 &= \begin{pmatrix} 0.21 \\ 0.54 \\ 0.81 \\ 0.09 \\ 0.02 \end{pmatrix} & \theta_4 &= \begin{pmatrix} 0.02 \\ 0.14 \\ 0.01 \\ -0.99 \\ 0.04 \end{pmatrix} & \theta_5 &= \begin{pmatrix} 0.00 \\ -0.00 \\ -0.03 \\ 0.04 \\ 1.00 \end{pmatrix}. \end{aligned} \quad (4.40)$$

and the eigenvalues (152.4, 52.8, 27.8, 1.0, 0.0).

When we compare the above outcome to our earlier WBF production analysis we see that the decay $H \rightarrow 4\ell$ does not increase the sensitivity to \mathcal{O}_B or \mathcal{O}_{BB} ; both of them are still basically blind directions. We visualize the information geometry in the remaining directions in Figure 4.5. The differences between local and global distances are much larger than in the $H \rightarrow \tau\tau$ channel. This is because the tiny $H \rightarrow 4\ell$ branching fraction decreases the new physics reach and with it the hierarchy of scales in our effective Lagrangian. This means that the squared dimension-6 amplitudes are numerically more relevant.

Production vs decay kinematics

Focusing on the question how the decay analysis improves our global information, we disentangle the effects on the production and decay vertices in Figure 4.6. As well known for the LHC, the

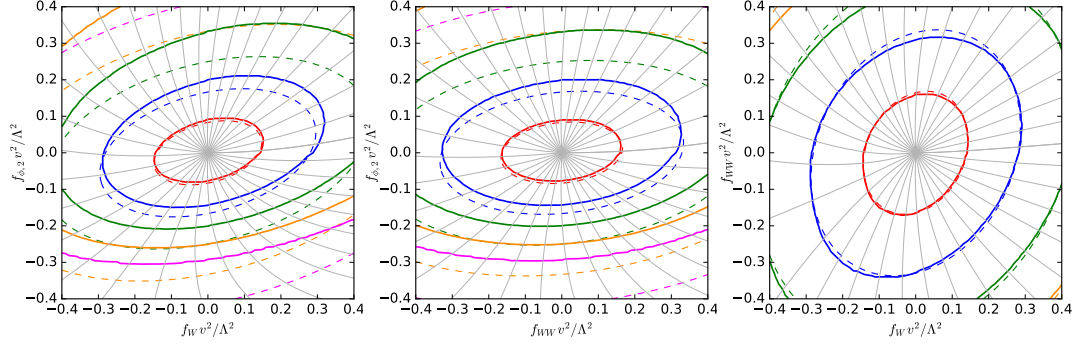


Figure 4.5: Error ellipses defined by the Fisher information in the WBF $H \rightarrow 4\ell$ channel. We show contours of local distance $d_{\text{local}}(\boldsymbol{\theta}; \mathbf{0})$ (dashed) and global distance $d(\boldsymbol{\theta}, \mathbf{0})$ (solid). The colored contours indicate distances of $d = 1 \dots 5$. In grey we show example geodesics. The θ_i not shown are set to zero.

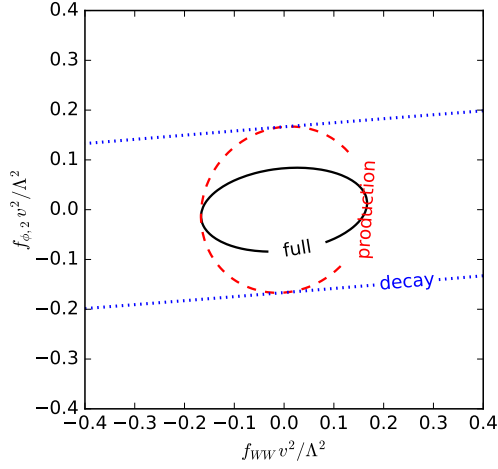


Figure 4.6: Information in the WBF $H \rightarrow 4\ell$ channel from including dimension-6 operators only in the production vertex (red), only in the decay vertex (blue), and both (black). The information is visualized as local contours $d_{\text{local}}(\boldsymbol{\theta}; \mathbf{0}) = 1$. The θ_i not shown are set to zero.

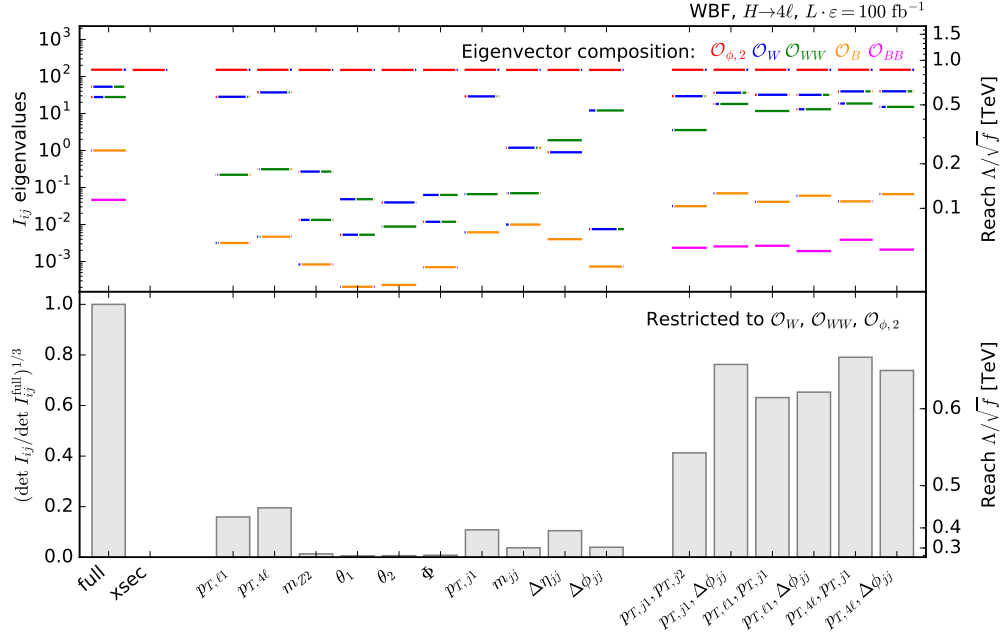


Figure 4.7: Fisher information for the WBF $H \rightarrow 4\ell$ channel exploiting the full phase space, after the cuts in Equation (4.37), and for several kinematic distributions. The top panel shows the eigenvalues, the colors denote the composition of the corresponding eigenvectors. The right axis translates the eigenvalues into a new physics reach for the corresponding combination of Wilson coefficients. In the bottom panel we show the determinants of the Fisher information restricted to $\mathcal{O}_{\phi,2}$, \mathcal{O}_W , and \mathcal{O}_{WW} , normalized to the full information. Again, the right axis translates them into a new physics reach.

production-side analysis benefits from a large momentum flow through the Higgs vertex, while the momentum flow through the decay vertices is bounded by the Higgs mass (neglecting off-shell phase space regions). For momentum-dependent operators this disadvantage is not compensated by the complex $H \rightarrow 4\ell$ decay kinematics. Consequently, the Higgs decay only improves the reach in the $\mathcal{O}_{\phi,2}$ direction, corresponding to a change in the total rate. This operator also affects many other total Higgs rates, so we conclude that the complex $H \rightarrow 4\ell$ kinematics does not play a significant role as part of a global analysis.

In complete analogy to Figure 4.4 for the WBF production, we compare the information in different distributions for CP -even operators in Figure 4.7. The standard tagging jet observables are complemented by five observables characterizing the 4ℓ decay kinematics,

- p_{T,ℓ_1} ;
- $p_{T,4\ell}$;

- m_{Z_2} for the lower-mass reconstructed Z boson;
- $\cos \theta_1 = \hat{p}_{\ell_1^-} \cdot \hat{p}_{Z_2} \big|_{Z_1}$ defined in terms of unit-3-vectors \hat{p} , and analogously $\cos \theta_2^*$;
- $\cos \Phi = (\hat{p}_{\ell_1^-} \times \hat{p}_{\ell_1^+}) \cdot (\hat{p}_{\ell_2^-} \times \hat{p}_{\ell_2^+})$, defined in the ZZ or Higgs rest frame [?].

In all cases we use at least ten bins and include underflow and overflow bins where applicable.

In our quantitative analysis we find similar patterns as in the $\tau\tau$ mode. The key observables are again transverse momenta and jet angular correlations. Without the complication of removing backgrounds efficiently, the combined analysis of these variables comes close to the maximum information: a two-dimensional histogram of jet transverse momenta and $\Delta\phi_{jj}$ probes new physics scales up to 650 GeV, while for a fully differential analysis the maximum probed new physics scale is close to 700 GeV. Under our assumptions, this difference roughly corresponds to 25% more data. The decay kinematics and its angular observables do not help significantly or change the picture qualitatively. This shows again how much the sensitivity of the decay vertices to dimension-6 operators is limited by the restriction of the momentum flow to the Higgs mass. This is not accidental: the reason behind this role of the momentum dependence is that for all operators shown in Equation (4.34) with the exception of $\mathcal{O}_{\phi,2}$, gauge invariance forces us to include the field strength tensor instead of the gauge boson field, automatically introducing a momentum dependence.

4.4.3 Higgs plus single top

Our final example is Higgs production with a single top with $H \rightarrow \gamma\gamma$ and a hadronic top decay. As shown in ??, diagrams where the Higgs is radiated off a W boson interfere destructively with diagrams with a top-Higgs coupling, making this channel a direct probe of the sign of the top Yukawa coupling [?]. We stick to a parton-level analysis at leading order in the five-flavor scheme. For our toy example we include only one of the dominant backgrounds, single top production with two photons, and in particular ignore the multi-jet background. The subleading $t\bar{t}\gamma\gamma$ background populates qualitatively different phase-space regions from the single-top signal and can be suppressed with an appropriate event selection [?]. We smear the $m_{\gamma\gamma}$ distribution of the signal process with a Gaussian of width 1.52 GeV estimated from Figure 6b of Reference [?], and do not include any other detector effects. Our basic event selection requires

$$\begin{array}{llll}
 p_{T,j} > 20 \text{ GeV} & |\eta_j| < 5.0 & \Delta R_{jj} > 0.4 & 152 \text{ GeV} < m_{bjj} < 192 \text{ GeV} \\
 p_{T,\gamma} > 10 \text{ GeV} & |\eta_\gamma| < 2.5 & \Delta R_{\gamma j}, \Delta R_{\gamma\gamma} > 0.4 & 120 \text{ GeV} < m_{\gamma\gamma} < 130 \text{ GeV},
 \end{array} \tag{4.41}$$

leading to a SM tH cross section of 0.10 fb and a background of 0.22 fb.

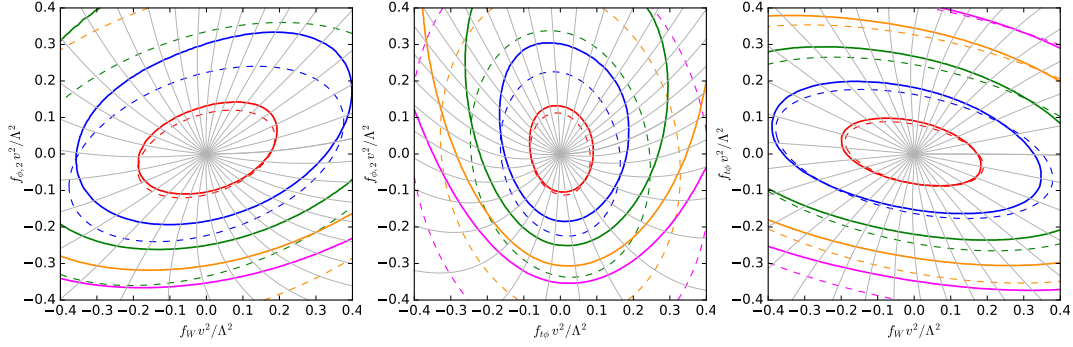


Figure 4.8: Error ellipses defined by the Fisher information in Higgs plus single top production. We show contours of local distance $d_{\text{local}}(\boldsymbol{\theta}; \mathbf{0})$ (dashed) and global distance $d(\boldsymbol{\theta}, \mathbf{0})$ (solid). The colored contours indicate distances of $d = 1 \dots 5$. In grey we show example geodesics. The θ_i not shown are set to zero.

We consider four CP -even dimension-6 operators

$$\begin{aligned} \mathcal{O}_W &= i \frac{g}{2} (D^\mu \phi)^\dagger \sigma^k (D^\nu \phi) W_{\mu\nu}^k & \mathcal{O}_{t\phi} &= (\phi^\dagger \phi) (\bar{Q}_3 \tilde{\phi} t_R) + \text{h.c.} \\ \mathcal{O}_{WW} &= -\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^k W^{\mu\nu k} & \mathcal{O}_{\phi,2} &= \frac{1}{2} \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi). \end{aligned} \quad (4.42)$$

The operators \mathcal{O}_W and \mathcal{O}_{WW} affect the production amplitudes where the Higgs couples to a W , while $\mathcal{O}_{t\phi}$ re-scales the top Yukawa coupling. Both, \mathcal{O}_{WW} and $\mathcal{O}_{\phi,2}$ also affect the $H \rightarrow \gamma\gamma$ decay.

Maximum precision on Wilson coefficients

We calculate the Fisher information in terms of the dimensionless parameters

$$\boldsymbol{\theta} = \frac{v^2}{\Lambda^2} \begin{pmatrix} f_{\phi,2} \\ f_W \\ f_{WW} \\ f_{t\phi} \end{pmatrix} \quad (4.43)$$

for 13 TeV and an integrated luminosity times efficiencies of $L \cdot \varepsilon = 300 \text{ fb}^{-1}$ and find

$$I_{ij}(\mathbf{0}) = \begin{pmatrix} 80.1 & -18.7 & -957.0 & 13.2 \\ -18.7 & 32.6 & 221.7 & 27.0 \\ -957.0 & 221.7 & 11446.1 & -146.0 \\ 13.2 & 27.0 & -146.0 & 150.3 \end{pmatrix}. \quad (4.44)$$

The eigenvectors are

$$\boldsymbol{\theta}_1 = \begin{pmatrix} 0.08 \\ -0.02 \\ -1.00 \\ 0.01 \end{pmatrix} \quad \boldsymbol{\theta}_2 = \begin{pmatrix} 0.00 \\ -0.23 \\ -0.01 \\ -0.97 \end{pmatrix} \quad \boldsymbol{\theta}_3 = \begin{pmatrix} -0.02 \\ 0.97 \\ -0.02 \\ -0.23 \end{pmatrix} \quad \boldsymbol{\theta}_4 = \begin{pmatrix} 1.00 \\ 0.02 \\ 0.08 \\ -0.01 \end{pmatrix} \quad (4.45)$$

with corresponding eigenvalues (11532, 155, 21.3, 0.1). The best constrained direction along \mathcal{O}_{WW} corresponds to the combination of Wilson coefficients that affects the $H \rightarrow \gamma\gamma$ decay in addition to production effects, which will already be tightly constrained once a tH measurement is feasible. The orthogonal direction in the $\mathcal{O}_{\phi,2}$ - \mathcal{O}_W plane is for all practical purposes blind. Even with the assumed sizeable event rate corresponding to 300 fb^{-1} , the sensitivity to \mathcal{O}_W and $\mathcal{O}_{t\phi}$ is limited, with some mixing between the two operators.

We visualize this maximum sensitivity to dimension-6 operators in Figure 4.8. With the exception of \mathcal{O}_{WW} , an optimal measurement can probe all operators at the $\Delta g \approx 0.1 \dots 0.2$ level, equivalent to $\Lambda/\sqrt{f_{\phi,2}} \approx 600 \dots 750 \text{ GeV}$. There are large differences between local and global distances already at the $d = 2$ level, implying that a measurement of this channel will always be sensitive to the squared dimension-6 terms.

Differential information

In Figure 4.9 we show the distribution of this information over phase space. More distributions are shown in Appendix ?? . As expected, the information is concentrated in the $m_{\gamma\gamma} \sim m_H$ peak and in the high-energy tails of transverse momenta. Studying angular correlations between the Higgs system and the top decay products, we find that the region $\Delta\eta_{\gamma\gamma,bjj} \lesssim 3$ contains a lot of discrimination power.

Information in distributions

In a next step, we compare this full information to the reduced information in one-dimensional and two-dimensional distributions of kinematic observables. We now require harder cuts

$$p_{T,j_1} > 50 \text{ GeV} \quad p_{T,\gamma} > 50, 30 \text{ GeV} \quad 122 \text{ GeV} < m_{\gamma\gamma} < 128 \text{ GeV}, \quad (4.46)$$

which reduces the background to the level of the signal. We then analyze the distributions [? ?]

- p_{T,γ_1} with bin size 25 GeV up to 400 GeV and an overflow bin;
- $m_{\gamma\gamma}$ with bin size 1 GeV in the allowed range of 123 ... 127 GeV;
- $p_{T,\gamma\gamma}$ with bin size 40 GeV up to 600 GeV and an overflow bin;
- $\Delta\phi_{\gamma\gamma}$ with bin size $\pi/10$;
- p_{T,j_1} with bin size 40 GeV up to 400 GeV and an overflow bin;

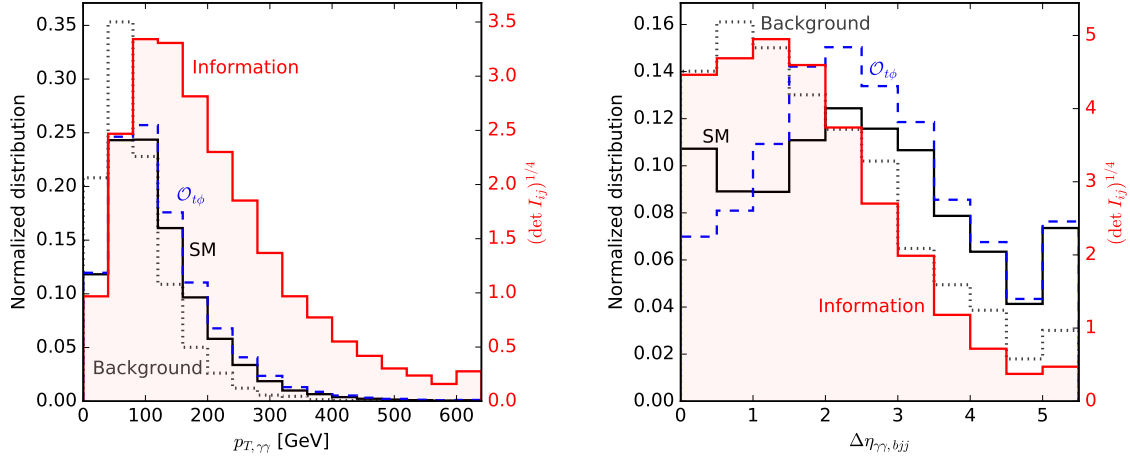


Figure 4.9: Distribution of the Fisher information in the Higgs plus single top channel (shaded red). We also show the normalized SM signal (solid black) and single-top background (dotted grey) rates. The dashed blue line shows the effect of $f_{t\phi} v^2/\Lambda^2 = 0.2$. The last bin is an overflow bin.

- $p_{T,b}$ with bin size 40 GeV up to 400 GeV and an overflow bin;
- $p_{T,bjj}$ with bin size 40 GeV up to 600 GeV and an overflow bin;
- $\Delta\phi_{\gamma\gamma,b}$ with bin size $\pi/10$;
- $\Delta\eta_{\gamma\gamma,b}$ with bin size 0.5 up to 5.0 and an overflow bin;
- $m_{\gamma\gamma bjj}$ with bin size 100 GeV up to 1500 GeV and an overflow bin;
- $p_{T,\gamma\gamma bjj}$ with bin size 40 GeV up to 400 GeV and an overflow bin;
- $\Delta\phi_{\gamma\gamma,bjj}$ with bin size $\pi/10$;
- $\Delta\eta_{\gamma\gamma,bjj}$ with bin size 0.5 up to 5.0 and an overflow bin.

As in the WBF case, different observables probe different Wilson operators. Figure 4.3 shows that the di-photon transverse momentum constrains mostly the \mathcal{O}_W direction, while the rapidity separation between the Higgs and top systems is more sensitive to $\mathcal{O}_{t\phi}$.

In Figure 4.11 we compare the eigenvalues, eigenvectors and determinants of the information matrices in all of the above distributions. We confirm that the photon observables mostly probe changes in the Higgs-gauge coupling from \mathcal{O}_W , while a rescaled top Yukawa will be visible in the properties of the top decay products. Distributions of the properties of the b jet consistently contain significantly less information than the corresponding distributions for the reconstructed top system. The rapidity difference between the $\gamma\gamma$ system and the reconstructed top provides a particularly good probe of this operator [?]. Combining this variable with the transverse

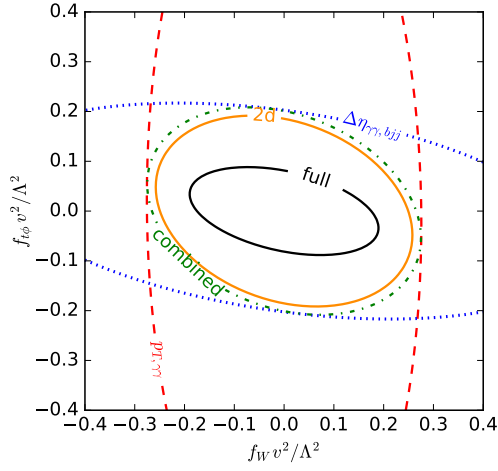


Figure 4.10: Information from histograms compared to the full information (black), shown as contours $d_{\text{local}}(\boldsymbol{\theta}; \mathbf{0}) = 1$. We include $p_{T,\gamma\gamma}$, $\Delta\eta_{\gamma\gamma,bjj}$, their naive combination assuming no mutual information, and their two-dimensional histogram. The θ_i not shown are set to zero.

momentum of the $\gamma\gamma$ system we can probe new physics scales in the \mathcal{O}_W - $\mathcal{O}_{t\phi}$ plane around 550 GeV in the \mathcal{O}_W - \mathcal{O}_{WW} - $\mathcal{O}_{\phi,2}$ space, compared to 700 GeV for the fully differential cross section. This corresponds to almost three times more data under our simplifying assumptions.

4.5 CP violation in the Higgs sector

4.6 Technical questions

4.6.1 Systematic uncertainties

We demonstrate this for WBF Higgs production in the $\tau\tau$ mode in ???. We assign a 5% or 10% Gaussian uncertainty on the overall signal rate, representing for instance missing higher orders, pdf or efficiency uncertainties. This significantly reduces the information in the total rate, and thus mostly the expected precision in the $\mathcal{O}_{\phi,2}$ direction. In ??? we show how the information in various distributions is affected by such an uncertainty. The new physics reach in the $\mathcal{O}_{\phi,2}$ direction is reduced by 800 GeV.

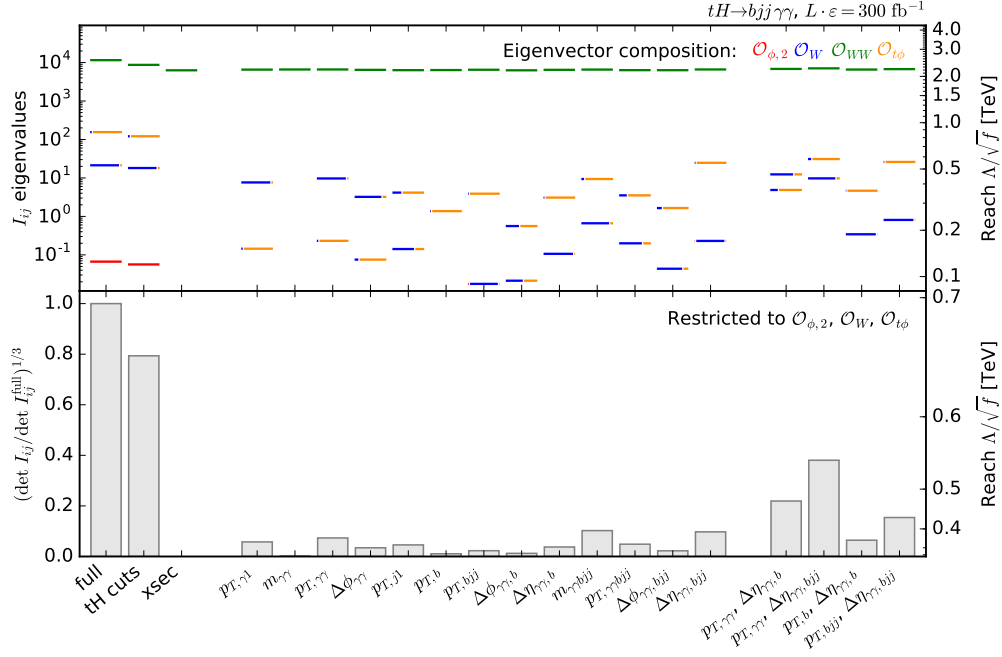


Figure 4.11: Fisher information for the Higgs plus single top channel exploiting the full phase space, after the cuts in Equation (4.46), and for several kinematic distributions. The top panel shows the eigenvalues, the colors denote the composition of the corresponding eigenvectors. The right axis translates the eigenvalues into a new physics reach for the corresponding combination of Wilson coefficients. In the bottom panel we show the determinants of the Fisher information restricted to $\mathcal{O}_{\phi,2}$, \mathcal{O}_W , and $\mathcal{O}_{t\phi}$, normalized to the full information. Again, the right axis translates them into a new physics reach.

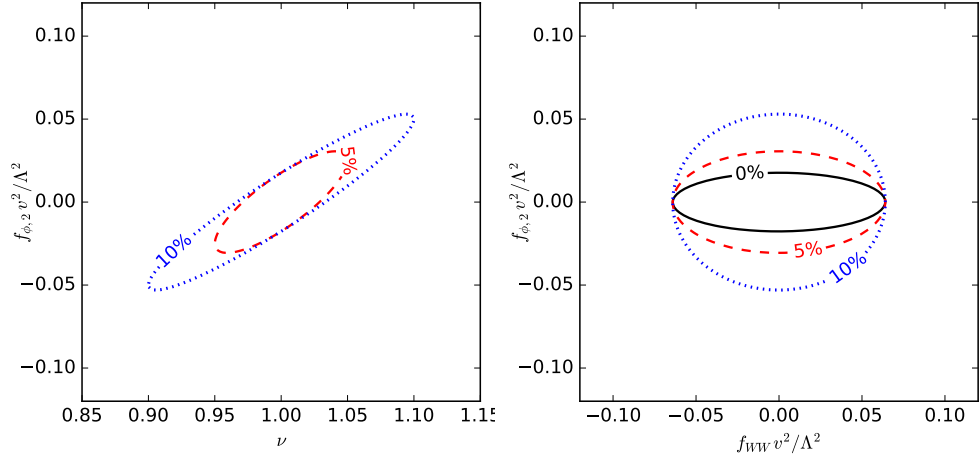


Figure 4.12: Effects of Gaussian uncertainties of 5% and 10% on the total signal rate. In the left panel we show the expected error ellipse $d_{\text{local}}((g, \nu); \mathbf{0}) = 1$ in the plane spanned by a physical parameter and the nuisance parameter ν rescaling the signal rate. In the right panel we show the error ellipses in the \mathcal{O}_W - $\mathcal{O}_{\phi,2}$ plane after profiling over this systematic uncertainty.

4.6.2 Comparison with other tools

Likelihood ratio

There is some subtlety in the relationship between standard likelihood ratio tests and the Fisher distance. We anticipate that the confidence intervals in θ will continue to be based on likelihood ratio tests. While both are invariant to reparametrization of θ , non-linear terms in θ that lead to curvature in the information geometry can break the one-to-one relationship between the expected value of the log-likelihood ratio and the Fisher information distance.

In ?? we compare the two tools. As an example, we study WBF Higgs production in the $\tau\tau$ mode and sample parameter points θ in the \mathcal{O}_W - \mathcal{O}_{WW} plane. For each of these points we calculate the local and global distance from the SM defined by the Fisher information, as well as the expected log-likelihood ratio

$$q(\theta_b, \theta_a) \equiv -2 E \left[\log \frac{f(\mathbf{x}|\theta_b)}{f(\mathbf{x}|\theta_a)} \middle| \theta_a \right]. \quad (4.47)$$

For small significance deviations, the local and especially the global distances are almost exactly equal to the expected likelihood ratio, with differences only becoming visible around the 3σ level. This demonstrates that different statistical tools probe the same physics and can be chosen based on convenience.

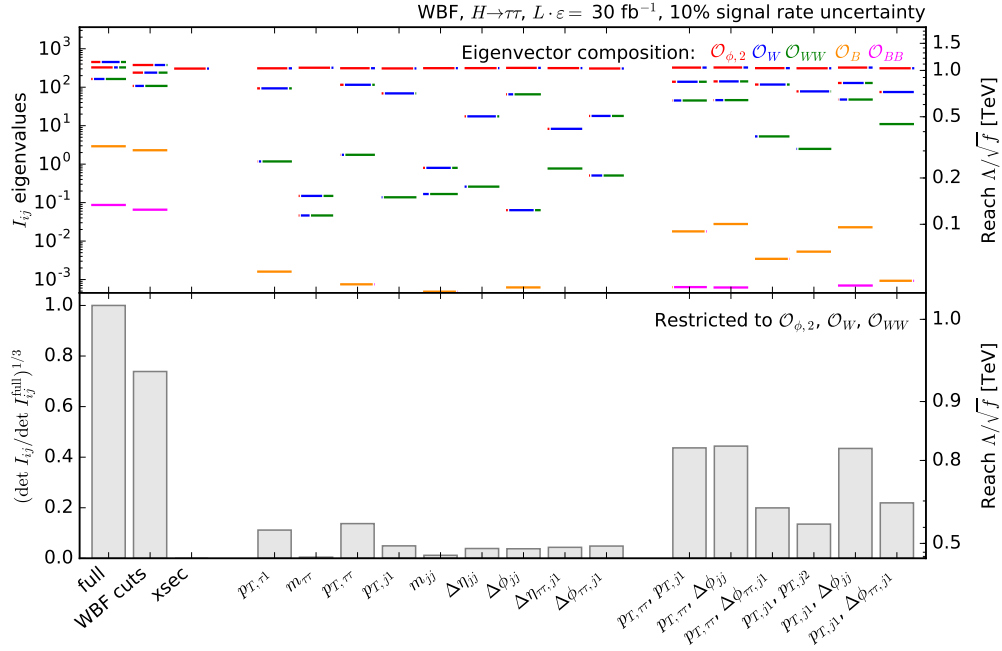


Figure 4.13: Fisher information for the WBF $H \rightarrow \tau\tau$ channel profiled over a 10% signal rate uncertainty. We compare the information for the full phase space, after the cuts in Equation (4.37), and for several kinematic distributions. The top panel shows the eigenvalues, the colors denote the composition of the corresponding eigenvectors. The right axis translates the eigenvalues into a new physics reach for the corresponding combination of Wilson coefficients. In the bottom panel we show the determinants of the Fisher information restricted to $\mathcal{O}_{\phi,2}$, \mathcal{O}_W , and \mathcal{O}_{WW} , normalized to the full information. Again, the right axis translates them into a new physics reach.

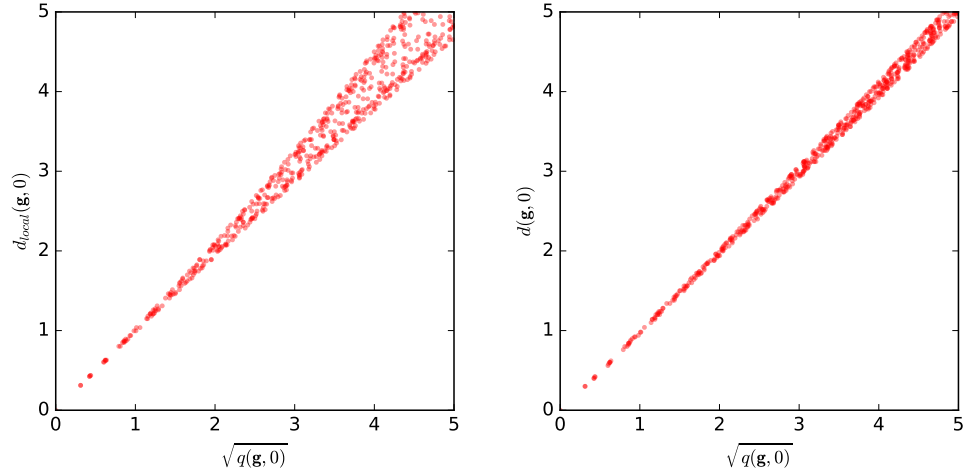


Figure 4.14: Comparison of the local (left) and global (right) distances defined by the Fisher information with the expected local likelihood ratio. We use WBF Higgs production in the $\tau\tau$ mode and sample parameter points in the \mathcal{O}_W - \mathcal{O}_{WW} plane.

Ambient geometry

4.7 Conclusions

We have used information geometry to calculate the maximum sensitivity of Higgs measurements to dimension-6 operators, to understand the structure of the observables, and to discuss how to improve these measurements. Our approach is based on the Fisher information matrix, which according to the Cramér-Rao bound defines the maximum precision that can be achieved in a measurement. Unlike traditional multivariate analysis techniques, it is designed for continuous, high-dimensional parameter spaces like effective field theories. We have demonstrated how the Fisher information can be reliably calculated using Monte-Carlo techniques.

Going beyond global statements, the Fisher information can be studied differentially to understand how the discriminating power is distributed over phase space, which helps guide event selection strategies. Moreover, we can also calculate the information contained in a subsets of kinematic distributions. This helps us determine which observables are the most powerful, and allows us to compare the constraining power in conventional analyses with one or two variables to that in more complex multivariate analyses.

Our first testing ground was Higgs production in weak boson fusion with decays into a tau pair or into four leptons. Crucial information comes from the high-energy tails as well as from angular correlations between jets. Decay kinematics hardly adds any information, since the momentum flow is limited by the Higgs mass and gauge invariance forces us to include operators

with a momentum dependence. Tight cuts on the rapidity separation of the tagging jets throw away a large amount of discrimination power. Under idealized conditions, conventional analyses based on a simple event selection and standard kinematic distributions can probe new physics scales around 900 GeV in the early phase of Run II. Multivariate analyses have the potential to significantly enhance the sensitivity and probe new physics scales of up to 1.2 TeV.

In Higgs production with a single top we find that kinematic properties of the Higgs decay products and observables in the top system provide orthogonal information. The transverse momenta of the di-photon system as well as the rapidity separation of the $\gamma\gamma$ and bjj systems are powerful observables. But even with HL-LHC data these distributions are only sensitive to new physics scales around 550 GeV, while a multivariate analysis might be able to probe scales up to 700 GeV.

To summarize, information geometry provides a powerful and intuitive tool that can help understand the phenomenology of models with a continuous, high-dimensional parameter space, and in turn can be used to optimize measurement strategies. We have demonstrated this approach in different Higgs channels for dimension-6 operators, but it can easily be translated to other processes and models.

Chapter 5

Conclusions

*Can we actually know the universe? My God, it's
hard enough finding your way around in
Chinatown.*

— W. Allen [203]

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Without doubt, the three years of my PhD were an amazing time.

For that, I first and foremost want to thank my adviser Tilman Plehn: for taking me on as a student, for working with me on intriguing questions, and for disagreeing with me on countless questions over coffee. I want to express my sincere gratitude to him for always having my back, including coming into the office on a Sunday morning to get me a backup laptop when mine broke down days before a deadline. And last but certainly not least, I am grateful to him for clearing the way to an academic career for me, even though I finally decided against this path.

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...

Appendices

*Did you know that in my PhD defence, I didn't
answer a single question correctly?*

— T. Plehn [204]

Here we collect technical details and intermediate results that are not too interesting on their own, but should be included for completeness and reproducibility. We begin with our conventions for the Standard Model and effective operators. Appendix A.2 contains some additional details on the models considered in Chapter 3. Finally, in Appendix A.3 we derive some of the key results of Chapter 4.

Large parts of this appendix has been published previously, as part of the same articles that contained much of the main body of this thesis. In particular, Appendices A.1 and A.2 was published in References [2, 3], while Appendix A.3 contains material published in Reference [5].

A.1 Effective field theory conventions

A.1.1 Standard Model setup

We begin with our conventions for the Standard Model, which also apply to all models of new physics. The SM Lagrangian, invariant under a $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, is given

Fermions	Representation					
				$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
LH quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	3	2	$\frac{1}{6}$
RH up-type quarks	u_R	c_R	t_R	3	1	$\frac{2}{3}$
RH down-type quarks	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
LH leptons	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
RH leptons	e_R^-	μ_R^-	τ_R^-	1	1	-1

Table A.1: SM fermions and their representation under the SM gauge group.

by

$$\begin{aligned}
 \mathcal{L}_{\text{SM}} = & -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_f \bar{f} i \not{D} f + (D^\mu \phi)^\dagger (D_\mu \phi) \\
 & - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \sum_{\text{generations}} \left(\gamma_u \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_L \tilde{\phi} u_R + \gamma_d \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}_L \phi d_R + \gamma_\ell \begin{pmatrix} \bar{\nu} \\ \bar{\ell} \end{pmatrix}_L \phi \ell_R + \text{h. c.} \right).
 \end{aligned} \tag{A.1}$$

The fields in this Lagrangian are a scalar ϕ transforming as the $(\mathbf{1}, \mathbf{2}, 1/2)$ representation of the SM gauge group; the gauge bosons G_μ^a , W_μ^a , and B_μ ; and the fermions given in Table A.1. We usually leave out generation (flavour) indices and simply denote up-type quarks, down-type quarks, charged leptons, and neutrinos with u , d , ℓ , ν ; and the left-handed doublets with Q and L . The field strength tensors are defined as

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \tag{A.2}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \varepsilon^{abc} W_\mu^b W_\nu^c, \tag{A.3}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{A.4}$$

The covariant derivatives are defined according to the gauge charges and with a conventional minus sign in front of the gauge term, for instance

$$\begin{aligned}
 D_\mu \phi &= \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a - ig' \frac{1}{2} B_\mu \right) \phi, \\
 D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L &= \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^a}{2} W_\mu^a - ig' \frac{1}{6} B_\mu \right) \begin{pmatrix} u \\ d \end{pmatrix}_L.
 \end{aligned} \tag{A.5}$$

These expressions include the coupling constants corresponding to the SM gauge group, g_s , g , and g' ; the structure constants f^{abc} and ε^{abc} of $SU(3)$ and $SU(2)$, as well as the Pauli matrices σ^a and the Gell-Mann matrices λ^a . Finally, the scalar Higgs potential contains two real parameters μ^2 and λ , and the Yukawa couplings y_f are complex-valued matrices in flavour space.

For $\mu^2 < 0$, the potential for ϕ has a minimum at the non-zero vacuum expectation value (VEV)

$$v^2 \equiv 2|\langle\phi\rangle|^2 = -\frac{\mu^2}{\lambda}. \quad (\text{A.6})$$

Using some of the gauge freedom, we can choose that the vacuum expectation value of ϕ points into the lower component of the doublet. Expanding the four physical degrees of freedom w_a and h around this minimum, we have

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -w_2 - iw_1 \\ v + h + iw_3 \end{pmatrix}. \quad (\text{A.7})$$

Plugging this into Equation (A.1), and diagonalising the mass matrices, one sees that the gauge bosons W^a and B combine with the Goldstone bosons w_a to the mass eigenstates

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left((W_\mu^1 - \frac{1}{g v} \partial_\mu w^1) \pm i(W_\mu^2 - \frac{1}{g v} \partial_\mu w^2) \right), \quad (\text{A.8})$$

$$Z_\mu = c_W (W_\mu^3 - \frac{1}{g v} \partial_\mu w^3) - s_W B_\mu, \quad (\text{A.9})$$

$$A_\mu = s_W W_\mu^3 + c_W B_\mu \quad (\text{A.10})$$

with weak mixing angle (or Weinberg angle)

$$c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad s_W \equiv \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad (\text{A.11})$$

and masses

$$m_W = \frac{g v}{2}, \quad m_Z = \frac{g v}{2 c_W}. \quad (\text{A.12})$$

The remaining degree of freedom from ϕ is the physical Higgs boson h with a mass

$$m_h^2 = -2\mu^2 = 2\lambda v^2. \quad (\text{A.13})$$

Inserting Equation (2.5) into the Yukawa couplings also yields fermion masses

$$m_f = \frac{y_f v}{\sqrt{2}}. \quad (\text{A.14})$$

Since the Yukawa couplings are not flavour-diagonal, one has to diagonalise these mass matrices in flavour space, ultimately leading to the CKM matrix. But this does not affect the Higgs couplings, and in this thesis we can safely assume flavour-diagonal Yukawa couplings.

With respect to the remaining gauge group $U(1)_Q$ with the gauge boson A_μ and the gauge coupling

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} = g s_W = g' c_W, \quad (\text{A.15})$$

the fermions carry the electromagnetic charge

$$q = y + \frac{\sigma_3}{2}, \quad (\text{A.16})$$

where the second term is shorthand for its eigenvalue in the case of $SU(2)_L$ doublets and zero for the singlets. More important for this thesis, the terms that lead to mass terms for the weak gauge bosons and fermions also yield couplings between the Higgs boson and these particles. Expressed in terms of the masses, these couplings read

$$g_{hff} = -\frac{m_f}{v} \quad (\text{A.17})$$

for the fermions and

$$g_{hW^+W^-} = \frac{2m_W^2}{v}, \quad g_{hZZ} = \frac{m_Z^2}{v} \quad (\text{A.18})$$

for the vector bosons.

A.1.2 HISZ basis

For the dimension-six operators of linear Higgs effective field theory, we nearly entirely follow the conventions of References [21, 80, 81], which is based on the Hagiwara-Ishihara-Szalapski-Zeppenfeld basis (HISZ) [78]. Our framework is defined in Section 2.3.2. The Lagrangian is given in Equation (2.51), and the operators are listed in Tables 2.1 to 2.4. We discuss the phenomenology of these operators in Section 2.3.3.

A.1.3 SILH basis

Another common basis for dimension-six operators was developed in References [76, 77] and is usually names “Strongly Interacting Light Higgs” or “SILH” after the first of these publications. It

$\mathcal{O}_H^{\text{SILH}} = \partial^\mu (\phi^\dagger \phi) \partial_\mu (\phi^\dagger \phi)$	$\mathcal{O}_{HB}^{\text{SILH}} = (D^\mu \phi^\dagger) (D^\nu \phi) B_{\mu\nu}$
$\mathcal{O}_T^{\text{SILH}} = (\phi^\dagger \overleftrightarrow{D}^\mu \phi) (\phi^\dagger \overleftrightarrow{D}_\mu \phi)$	$\mathcal{O}_{HW}^{\text{SILH}} = (D^\mu \phi^\dagger) \sigma^k (D^\nu \phi) W_{\mu\nu}^k$
$\mathcal{O}_6^{\text{SILH}} = (\phi^\dagger \phi)^3$	$\mathcal{O}_B^{\text{SILH}} = (\phi^\dagger \overleftrightarrow{D}^\mu \phi) (\partial^\nu B_{\mu\nu})$
$\mathcal{O}_u^{\text{SILH}} = (\phi^\dagger \phi) (\phi^\dagger \cdot \bar{Q}_L) u_R + \text{h. c.}$	$\mathcal{O}_W^{\text{SILH}} = (\phi^\dagger \sigma^k \overleftrightarrow{D}^\mu \phi) (D^\nu W_{\mu\nu}^k)$
$\mathcal{O}_d^{\text{SILH}} = (\phi^\dagger \phi) (\phi \bar{Q}_L) d_R + \text{h. c.}$	$\mathcal{O}_g^{\text{SILH}} = (\phi^\dagger \phi) G_{\mu\nu}^a G^{\mu\nu a}$
$\mathcal{O}_\ell^{\text{SILH}} = (\phi^\dagger \phi) (\phi \bar{L}_L) l_R + \text{h. c.}$	$\mathcal{O}_\gamma^{\text{SILH}} = (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}$

Table A.2: Dimension-six operators in the SILH basis relevant for Higgs physics.

is based on the Lagrangian

$$\begin{aligned}
 \mathcal{L}_{\text{SILH}} = & \mathcal{L}_{\text{SM}} + \frac{c_H}{2v^2} \mathcal{O}_H^{\text{SILH}} + \frac{c_T}{2v^2} \mathcal{O}_T^{\text{SILH}} - \frac{c_6 \lambda}{v^2} \mathcal{O}_6^{\text{SILH}} \\
 & + \frac{igc_W}{2m_W^2} \mathcal{O}_W^{\text{SILH}} + \frac{ig'c_B}{2m_W^2} \mathcal{O}_B^{\text{SILH}} + \frac{igc_{HW}}{m_W^2} \mathcal{O}_{HW}^{\text{SILH}} + \frac{ig'c_{HB}}{m_W^2} \mathcal{O}_{HB}^{\text{SILH}} \\
 & + \frac{g'^2 c_\gamma}{m_W^2} \mathcal{O}_\gamma^{\text{SILH}} + \frac{g_s^2 c_g}{m_W^2} \mathcal{O}_g^{\text{SILH}} - \sum_f \frac{c_f}{v^2} \gamma_f \mathcal{O}_f^{\text{SILH}}
 \end{aligned} \tag{A.19}$$

with the Wilson coefficients c_i and the operators $\mathcal{O}_i^{\text{SILH}}$ defined in Table A.2. Note that this Lagrangian is not of the form given in Equation (2.12): the dimension-six operators are suppressed by powers of m_W or v instead of the cutoff scale Λ .

These operators can be translated to our conventions (the HISZ basis) with the following dictionary:

$$\begin{aligned}
 \mathcal{O}_H^{\text{SILH}} &= 2\mathcal{O}_{\phi 2}, & \mathcal{O}_{HB}^{\text{SILH}} &= -\frac{2i}{g'} \mathcal{O}_B, \\
 \mathcal{O}_T^{\text{SILH}} &= 2\mathcal{O}_{\phi 2} - 4\mathcal{O}_{\phi 1}, & \mathcal{O}_{HW}^{\text{SILH}} &= -\frac{2i}{g} \mathcal{O}_W, \\
 \mathcal{O}_6^{\text{SILH}} &= 3\mathcal{O}_{\phi 3}, & \mathcal{O}_B^{\text{SILH}} &= \frac{2i}{g'} (\mathcal{O}_{BB} + \mathcal{O}_{BW} - 2\mathcal{O}_B), \\
 \mathcal{O}_u^{\text{SILH}} &= \mathcal{O}_u, & \mathcal{O}_g^{\text{SILH}} &= \mathcal{O}_{GG}, \\
 \mathcal{O}_d^{\text{SILH}} &= \mathcal{O}_d, & \mathcal{O}_W^{\text{SILH}} &= \frac{2i}{g} (\mathcal{O}_{WW} + \mathcal{O}_{BW} - 2\mathcal{O}_W), \\
 \mathcal{O}_\ell^{\text{SILH}} &= \mathcal{O}_\ell, & \mathcal{O}_\gamma^{\text{SILH}} &= -\frac{4}{g'^2} \mathcal{O}_{BB}.
 \end{aligned} \tag{A.20}$$

For the Wilson coefficients, we find

$$\begin{aligned}
 c_H &= \frac{v^2}{\Lambda^2} \left(\frac{1}{2} f_{\phi 1} + f_{\phi 2} \right), & c_{HB} &= \frac{v^2}{\Lambda^2} \frac{g^2}{8} (f_B + 2f_{BW} - 2f_{WW}), \\
 c_T &= -\frac{v^2}{\Lambda^2} \frac{1}{2} f_{\phi 1}, & c_{HW} &= \frac{v^2}{\Lambda^2} \frac{g^2}{8} (f_W + 2f_{WW}), \\
 c_6 &= -\frac{v^2}{\Lambda^2} \frac{1}{3\lambda} f_{\phi 3}, & c_B &= \frac{v^2}{\Lambda^2} \frac{g^2}{4} (f_{WW} - f_{BW}), \\
 c_u &= -\frac{v^2}{\Lambda^2} \frac{1}{y_u} f_u, & c_W &= -\frac{v^2}{\Lambda^2} \frac{g^2}{4} f_{WW}, \\
 c_d &= -\frac{v^2}{\Lambda^2} \frac{1}{y_d} f_d, & c_g &= \frac{v^2}{\Lambda^2} \frac{g^2}{4g_s^2} f_{GG}, \\
 c_\ell &= -\frac{v^2}{\Lambda^2} \frac{1}{y_\ell} f_\ell, & c_\gamma &= \frac{v^2}{\Lambda^2} \frac{g^2}{16} (f_{BW} - f_{BB} - f_{WW}). \tag{A.21}
 \end{aligned}$$

A.2 Model fine print

A.2.1 Singlet extension

The singlet model is defined in Equations (3.23) and (3.24) in Section 3.3.2. Ignoring the Goldstones, the scalar doublet and singlets fields can be expanded into components as

$$\begin{aligned}
 \phi &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ v + \phi^0 \end{pmatrix}, \\
 S &= \frac{1}{\sqrt{2}} (v_s + s^0), \tag{A.22}
 \end{aligned}$$

where $v \equiv \sqrt{2}\langle\phi\rangle = 246$ GeV and $v_s \equiv \sqrt{2}\langle S\rangle$ denote their respective VEVs. The minimisation condition for this potential can be used to eliminate the parameters $\mu_{1,2}$ in favour of v and v_s . ϕ^0 and s^0 mix to form a light (h) and a heavy (H) mass eigenstate,

$$\begin{aligned}
 h &= \phi^0 \cos \alpha - s^0 \sin \alpha, \\
 H &= \phi^0 \sin \alpha + s^0 \cos \alpha, \tag{A.23}
 \end{aligned}$$

where

$$\tan(2\alpha) = \frac{\lambda_3 v v_s}{\lambda_2 v_s^2 - \lambda_1 v^2}. \tag{A.24}$$

Their masses are

$$m_{h,H}^2 = \lambda_1 v^2 + \lambda_2 v_s^2 \mp |\lambda_1 v^2 - \lambda_2 v_s^2| \sqrt{1 + \tan^2(2\alpha)} \tag{A.25}$$

with $m_H^2 \approx 2\lambda_2 v_s^2 \gg m_h^2$ in the limit $v^2 \ll v_s^2$.

A.2.2 Two-Higgs-doublet model

Model setup

We analyse the most general gauge invariant, CP -even theory for two scalar fields with an additional \mathbb{Z}_2 symmetry, as defined in Equation (3.39) in Section 3.3.3. The Higgs mass-eigenstates follow from the set of rotations

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = R(\alpha) \begin{pmatrix} h_1^0 \\ h_2^0 \end{pmatrix}, \quad \begin{pmatrix} w^0 \\ A^0 \end{pmatrix} = R(\beta) \begin{pmatrix} a_1^0 \\ a_2^0 \end{pmatrix}, \quad \begin{pmatrix} w^\pm \\ H^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} h_1^\pm \\ h_2^\pm \end{pmatrix}, \quad (\text{A.26})$$

with

$$\phi_k = \begin{pmatrix} h_k^+ \\ \frac{1}{\sqrt{2}}(v_k + h_k^0 + i a_k) \end{pmatrix} \quad (\text{A.27})$$

and

$$R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (\text{A.28})$$

Since the two doublets contribute to giving masses to the weak gauge bosons, custodial symmetry will impose tight constraints on the viable mass spectrum of the model [205–213]. Analytic relations linking the different Higgs masses and mixing angles with the Lagrangian parameters in Equation (3.39) can be found e. g. in Appendix A of [106]. The conventions

$$0 < \beta < \pi/2 \quad \text{and} \quad 0 \leq \beta - \alpha < \pi \quad (\text{A.29})$$

guarantee that the Higgs coupling to vector bosons has the same sign in the 2HDM and in the SM. As we will next show, the decoupling limit implies that the light Higgs interactions approach the alignment limit, where $\cos \beta \sim |\sin \alpha|$ and the couplings become SM-like [169].

Couplings

The tree-level coupling shifts of the light Higgs follow from these rotations and are given in Eqs. (3.40) to (3.44). The light Higgs coupling to a charged Higgs pair reads

$$\frac{g_{h^0 H^+ H^-}}{g_{hhh}^{\text{SM}}} = \frac{1}{3m_{h^0}^2} \left[\sin(\beta - \alpha) (2m_{H^\pm}^2 - m_{h^0}^2) + \frac{\cos(\alpha + \beta)}{\sin(2\beta)} \left(2m_{h^0}^2 - \frac{2m_{12}^2}{\sin \beta \cos \beta} \right) \right], \quad (\text{A.30})$$

with $g_{hhh}^{\text{SM}} = -3m_h^2/v$.

The loop-induced couplings are more involved, giving

$$1 + \Delta_g = \frac{1}{A_{gg}^{\text{SM}}} \left[\sum_{f=t,b} (1 + \Delta_f) A_f(\tau_f) \right], \quad (\text{A.31})$$

$$1 + \Delta_\gamma = \frac{1}{A_{\gamma\gamma}^{\text{SM}}} \left[\sum_{f=t,b} N_C Q_f^2 (1 + \Delta_f) A_f(\tau_f) + Q_\tau^2 (1 + \Delta_\tau) A_f(\tau_\tau) + (1 + \Delta_W) A_v(\tau_W) - g_{h^0 H^+ H^-} \frac{m_W s_W}{e m_{H^\pm}^2} A_s(\tau_{H^\pm}) \right], \quad (\text{A.32})$$

where A_{xx}^{SM} are the corresponding contributions in the SM. The conventional loop form factors read

$$\begin{aligned} A_s(\tau) &= -\frac{\tau}{2} [1 - \tau f(\tau)] &= 1/6 + \mathcal{O}(\tau^{-1}), \\ A_f(\tau) &= \tau [1 + (1 - \tau) f(\tau)] &= 2/3 + \mathcal{O}(\tau^{-1}), \\ A_V(\tau) &= -\frac{1}{2} [2 + 3\tau + 3(2\tau - \tau^2) f(\tau)] &= -7/2 + \mathcal{O}(\tau^{-1}), \end{aligned} \quad (\text{A.33})$$

where

$$f(\tau) = \begin{cases} -\frac{1}{4} \left[\log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2 & \text{for } \tau < 1 \\ \left[\arcsin \frac{1}{\sqrt{\tau}} \right]^2 & \text{for } \tau \geq 1, \end{cases} \quad (\text{A.34})$$

and $\tau_x = 4m_x^2/m_{h^0}^2$.

Matching

The effect of the second doublet on the phenomenology of the light Higgs consists purely of shifted couplings Δ_x . This allows us to match the dimension-six model by setting equal the coupling shifts from the full model, given by Eqs. (3.40) to (3.44) and (A.32), to the corresponding couplings in the dimension-six Lagrangian, Equation (2.73).

In a second step we then expand in $1/\Lambda$ and keep terms up to $\mathcal{O}(1/\Lambda^2)$, where Λ is defined in the unbroken phase for the default matching or as the physical mass m_{A^0} in the ν -improved matching, as described in Section 3.3.3.

This defines the effective model in a straightforward way. For more details see Appendix A.3 of Reference [2].

A.2.3 Scalar top partners

Model setup

The potential for the simplified scalar top-partner model consists of three parts,

$$\begin{aligned} \mathcal{L}_{\text{top partners}} \supset & (D_\mu \tilde{Q})^\dagger D^\mu \tilde{Q} + (D_\mu \tilde{t}_R)^* D^\mu \tilde{t}_R - \underbrace{M^2 \tilde{Q}^\dagger \tilde{Q} - M^2 \tilde{t}_R^* \tilde{t}_R}_{\mathcal{L}_{\text{mass}}} \\ & - \underbrace{\kappa_{LL} (\phi \cdot \tilde{Q})^\dagger (\phi \cdot \tilde{Q}) - \kappa_{RR} (\tilde{t}_R^* \tilde{t}_R) (\phi^\dagger \phi)}_{\mathcal{L}_{\text{Higgs}}} - \underbrace{[\kappa_{LR} M \tilde{t}_R^* (\phi \cdot \tilde{Q}) + \text{h. c.}]}_{\mathcal{L}_{\text{mixing}}}. \end{aligned} \quad (\text{A.35})$$

We use the customary notation for the $SU(2)_L$ invariant product $\phi \cdot \tilde{Q} \equiv \epsilon_{ab} \phi^a \tilde{Q}^b$, with the help of the antisymmetric pseudo-tensor $\epsilon^{ab} \equiv (i\sigma^2)^{ab}$ such that $\epsilon^{12} = -\epsilon^{21} = 1$.

The term $\mathcal{L}_{\text{Higgs}}$ gives rise to scalar partner masses proportional to the Higgs vev, mirroring the supersymmetric F -term contribution to the squark masses. By a similar token, the explicit mass terms $\mathcal{L}_{\text{mass}}$ are analogous to the squark soft-SUSY breaking mass terms. $\mathcal{L}_{\text{mixing}}$ is responsible for the mixing between the gauge eigenstates, reminiscent of the MSSM A -terms. In the absence of an underlying supersymmetry, the Lagrangian features no equivalent of the D -term contributions.

Collecting all bilinear terms from Equation (A.35), we get

$$\mathcal{L}_{\text{top partners}} \supset (\tilde{t}_L^* \tilde{t}_R^*) \mathcal{M}_{\tilde{t}} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \quad (\text{A.36})$$

with the mass matrix $\mathcal{M}_{\tilde{t}}$ given in Equation (3.52). Assuming all parameters in Equation (A.35) to be real, m_{ij} can be diagonalised through the usual orthogonal transformation $R(\theta_{\tilde{t}})$ given in Equation (A.28). This rotates the gauge eigenstates $(\tilde{t}_L, \tilde{t}_R)$ into the mass basis $(\tilde{t}_1, \tilde{t}_2)$,

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = R(\theta_{\tilde{t}}) \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}. \quad (\text{A.37})$$

The physical scalar partner masses and the mixing angle are then given by

$$\begin{aligned} m_{\tilde{t}_1}^2 &= M_{LL}^2 \cos^2 \theta_{\tilde{t}} + M_{RR}^2 \sin^2 \theta_{\tilde{t}} + 2M_{LR}^2 \sin \theta_{\tilde{t}} \cos \theta_{\tilde{t}}, \\ m_{\tilde{t}_2}^2 &= M_{LL}^2 \sin^2 \theta_{\tilde{t}} + M_{RR}^2 \cos^2 \theta_{\tilde{t}} - 2M_{LR}^2 \sin \theta_{\tilde{t}} \cos \theta_{\tilde{t}}, \end{aligned} \quad (\text{A.38})$$

$$\tan(2\theta_{\tilde{t}}) = \frac{2M_{LR}^2}{M_{LL}^2 - M_{RR}^2}. \quad (\text{A.39})$$

As we assume the right-handed bottom partner \tilde{b}_R to be heavy and thus decoupled, the sbottom-like scalar eigenstate \tilde{b}_L undergoes no mixing and can be readily identified with the physical eigenstate.

Matching

We compute the effective action at one loop with the help of the covariant derivative expansion [65, 67], which is fully consistent with our mass degeneracy setup. Since the Lagrangian Equation (A.35) lacks any linear terms in the heavy scalar fields $\Psi \equiv (\tilde{Q}, \tilde{t}_R^*)$, the tree-level exchange of such heavy partners cannot generate any effective interaction at dimension six. Our results are in agreement with References [65, 69, 124] and given in Equation (3.55).

A.2.4 Vector triplet

Our final example is the heavy vector triplet defined in ???. This Lagrangian includes conventional factors of coupling constants, introduced for a convenient power counting in certain UV embeddings [195]: insertions of V and ϕ are weighted by one factor of g_V , while SM gauge bosons come with a factor of g_w . In addition, the coupling to fermions is weighted with an additional factor of g_W^2/g_V^2 . For simplicity, it is assumed that the fermion current in ??? is universal.

We can add an explicit kinetic V - W mixing term

$$\mathcal{L} \supset c_{WV} \frac{g_W}{2g_V} D_{[\mu} V_{\nu]}^a W^{\mu\nu a} \quad (\text{A.40})$$

to the Lagrangian in Equation (3.57). As it turns out, this term is redundant and can be removed with field redefinitions, shifting its effects into the other model parameters [195, 214]. For more details, see Appendix A.5 of Reference [2].

In this model, seven gauge eigenstates W^a , B , V^a mix into mass eigenstates A , W^\pm , Z , ξ^\pm , and ξ^0 . The photon is as usual defined as the component that remains massless during EWSB, $A_\mu = c_w B_\mu + s_w W_\mu^3$, where the Weinberg angle is linked to the gauge couplings $e = g_W s_w = g' c_w$. For the remaining fields we have to diagonalise the neutral and charged mass matrices, giving the mass eigenstates

$$\begin{aligned} Z_\mu &= \cos \theta_N (-s_w B_\mu + c_w W_\mu^3) + \sin \theta_N V_\mu^3, \\ \xi_\mu^0 &= -\sin \theta_N (-s_w B_\mu + c_w W_\mu^3) + \cos \theta_N V_\mu^3, \\ W_\mu^\pm &= \cos \theta_C \frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}} + \sin \theta_C \frac{V_\mu^1 \mp V_\mu^2}{\sqrt{2}}, \\ \xi_\mu^\pm &= -\sin \theta_C \frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}} + \cos \theta_C \frac{V_\mu^1 \mp V_\mu^2}{\sqrt{2}}. \end{aligned} \quad (\text{A.41})$$

The corresponding mass eigenvalues read

$$m_{Z/\xi^0}^2 = \frac{1}{2} \left[\hat{m}_V^2 + \hat{m}_Z^2 \mp \sqrt{(\hat{m}_Z^2 - \hat{m}_V^2)^2 + c_H^2 g_V^2 \hat{m}_Z^2 \hat{v}^2} \right] \\ = \begin{cases} \hat{m}_Z^2 \left(1 - \frac{c_H^2 g_V^2}{4} \frac{\hat{v}^2}{\hat{m}_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4) \right) \\ \hat{m}_V^2 \left(1 + \frac{c_H^2 g_V^2}{4} \frac{\hat{v}^2}{\hat{m}_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4) \right) \end{cases}, \quad (\text{A.42})$$

and

$$m_{W^\pm/\xi^\pm}^2 = \frac{1}{2} \left[\hat{m}_V^2 + \hat{m}_W^2 \mp \sqrt{(\hat{m}_W^2 - \hat{m}_V^2)^2 + c_H^2 g_V^2 \hat{m}_W^2 \hat{v}^2} \right] \\ = \begin{cases} \hat{m}_W^2 \left(1 - \frac{c_H^2 g_V^2}{4} \frac{\hat{v}^2}{\hat{m}_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4) \right) \\ \hat{m}_V^2 \left(1 + \frac{c_H^2 g_V^2}{4} \frac{\hat{v}^2}{\hat{m}_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4) \right) \end{cases}. \quad (\text{A.43})$$

For the mixing angles, we find

$$\tan(2\theta_N) = \frac{c_H g_V \hat{v} \hat{m}_Z}{\hat{m}_V^2 - \hat{m}_Z^2} = \frac{c_H g g_V}{2 c_w} \frac{\hat{v}^2}{\hat{m}_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4), \\ \tan(2\theta_C) = \frac{c_H g_V \hat{v} \hat{m}_W}{\hat{m}_V^2 - \hat{m}_W^2} = \frac{c_H g g_V}{2} \frac{\hat{v}^2}{\hat{m}_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4), \quad (\text{A.44})$$

or

$$\sin \theta_C = \frac{c_H g g_V}{4} \frac{v^2}{M_V^2} + \mathcal{O}(\hat{v}^4/\hat{m}_V^4). \quad (\text{A.45})$$

Here we use the definitions

$$\hat{m}_Z = \frac{g_W \hat{v}}{2 c_w}, \quad \hat{m}_W = \frac{g_W \hat{v}}{2}, \quad \hat{m}_V^2 = M_V^2 + g_V^2 c_{VVHH} \hat{v}^2, \quad (\text{A.46})$$

where \hat{v} is the actual vev of ϕ , which does not necessarily have the SM value of $v = 2m_W/g \approx 246$ GeV.

This mixing affects the weak current interactions: instead of being simply governed by the $SU(2)_L$ coupling constant g_W , the physical $Wf f'$ couplings are now determined by

$$g = \cos \theta_C g_W - \sin \theta_C c_F \frac{g_W^2}{g_V} = g_W \left(1 - \frac{c_F c_H g_W^2}{4} \frac{v^2}{M_V^2} \right) + \mathcal{O}(v^4/M_V^4). \quad (\text{A.47})$$

The relation between \hat{v} and v can be read off from Equation (A.43):

$$\frac{\hat{v}}{v} = 1 + \frac{c_H^2 g_V^2}{8} \frac{v^2}{M_V^2} - \frac{c_F c_H g_W^2}{4} \frac{v^2}{M_V^2} + \mathcal{O}(v^4/M_V^4). \quad (\text{A.48})$$

Finally, note that to ensure compatibility with electroweak precision measurements, the neutral and charged states ξ^0 and ξ^\pm have to be nearly mass-degenerate,

$$m_{\xi^0} \approx m_{\xi^\pm} \equiv m_\xi. \quad (\text{A.49})$$

In practice, we set up our model in the m_W - g scheme, i. e. based on the input parameters g , m_W , α , m_{h^0} , α_s , the vector triplet couplings g_V and c_i , as well as the physical mass m_{ξ^\pm} . The remaining Lagrangian parameters, mixing angles, and couplings are calculated by solving Equations (A.42) and (A.43) iteratively.

Couplings

The mismatch between g and g_w as well as between v and \hat{v} leads to a shift of the Higgs couplings to fermions,

$$\begin{aligned} \Delta_f &= \frac{g_W}{g} \frac{v}{\hat{v}} - 1 = \frac{1}{\cos \theta_C - c_F \frac{g_W}{g_V} \sin \theta_C} \frac{v}{\hat{v}} - 1 \\ &= c_H^2 \frac{g_V^2 v^2}{8M_V^2} + c_F c_H \frac{g^2 v^2}{4M_V^2} + \mathcal{O}(M_V^{-4}), \end{aligned} \quad (\text{A.50})$$

and to (on-shell) W bosons,

$$\begin{aligned} \Delta_W &= \frac{1}{gm_W} \left(\frac{\cos^2 \theta_C g^2 \hat{v}}{2(\cos \theta_C - c_F \frac{g_W}{g_V} \sin \theta_C)^2} - c_H \frac{\sin \theta_C \cos \theta_C g g_V \hat{v}}{\cos \theta_C - c_F \frac{g_W}{g_V} \sin \theta_C} \right. \\ &\quad \left. + 2 c_{VVHH} \sin^2 \theta_C g_V^2 \hat{v} \right) - 1 \\ &= c_H^2 \frac{3g_V^2 v^2}{8M_V^2} + c_F c_H \frac{g^2 v^2}{4M_V^2} + \mathcal{O}(M_V^{-4}). \end{aligned} \quad (\text{A.51})$$

Matching

Since we are only interested in tree-level effects, we can construct the effective theory in our default matching procedure with the classical equation of motion for V_μ^a , corresponding to the first term in Equation (2.28). To simplify the notation, we define currents

$$J_F^{\mu,a} \equiv \bar{f}_L \gamma^\mu \sigma^a f_L, \quad J_H^{\mu,a} \equiv \phi^\dagger \sigma^a \overleftrightarrow{D}^\mu \phi. \quad (\text{A.52})$$

This Euler-Lagrange equation then reads

$$[\partial^\mu \partial^\nu - g^{\mu\nu} \partial^2 - M_V^2] V_\nu^a = g_V c_H J_H^{\mu,a} + \frac{g_W^2}{2g_V} c_F \sum_F J_F^{\mu,a} + \mathcal{O}(V^2) \quad (\text{A.53})$$

or

$$V^{\mu,a} = -\frac{1}{M_V^2} \left[g_V c_H J_H^{\mu,a} + \frac{g_W^2}{2g_V} c_F \sum_F J_F^{\mu,a} \right] + \mathcal{O}(p_V^2/\tilde{M}_V^4) + \mathcal{O}(V^2), \quad (\text{A.54})$$

where we neglect higher powers of V that are irrelevant for our analysis.

Inserting this into the Lagrangian in Equation (3.57), we find the effective theory

$$\mathcal{L}_{\text{EFT}} \supset -\frac{g_W^4 c_F^2}{8g_V^2 M_V^2} J_F^{\mu,a} J_\mu^{F,a} - \frac{g_V^2 c_H^2}{2M_V^2} J_H^{\mu,a} J_\mu^{H,a} - \frac{g_W^2 c_F c_H}{2M_V^2} J_H^{\mu,a} J_\mu^{F,a}. \quad (\text{A.55})$$

$J_F^{\mu,a} J_\mu^{F,a}$ only contains four-fermion operators irrelevant for our analysis. The remaining current products can be expressed in terms of dimension-six operators as

$$\begin{aligned} J_H^{\mu,a} J_\mu^{H,a} &= -\frac{1}{2} (\mathcal{O}_{\phi,2} - 2\phi^\dagger \phi (D_\mu \phi)^2) \\ &= -\frac{1}{2} \left[3\mathcal{O}_{\phi,2} - 12\lambda \mathcal{O}_{\phi,3} - \sum_f y_f \mathcal{O}_f \right], \end{aligned} \quad (\text{A.56})$$

$$\begin{aligned} J_F^{\mu,a} J_\mu^{H,a} &= \frac{1}{2} \mathcal{O}_{\phi F}^{(3)} \\ &= \frac{-2}{g_W^2} [\mathcal{O}_{WW} + \mathcal{O}_{BW} - 2\mathcal{O}_W] + 3\mathcal{O}_{\phi,2} - 12\lambda \mathcal{O}_{\phi,3} - \sum_f y_f \mathcal{O}_f. \end{aligned} \quad (\text{A.57})$$

Here we have used the equations of motions in Equations (2.54) to (2.56) to bring the operators to our HISZ basis.

Combining the pieces, we finally arrive at

$$\mathcal{L}_{\text{EFT}} \supset \left(\frac{g_V^2 c_H^2}{4M_V^2} - \frac{g_W^2 c_F c_H}{2M_V^2} \right) \left[3\mathcal{O}_{\phi,2} - 12\lambda \mathcal{O}_{\phi,3} - \sum_f y_f \mathcal{O}_f \right] + \frac{c_F c_H}{M_V^2} [\mathcal{O}_{WW} + \mathcal{O}_{BW} - 2\mathcal{O}_W], \quad (\text{A.58})$$

corresponding to the Wilson coefficients given in Equation (3.61).

A.3 Fisher information derivations

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