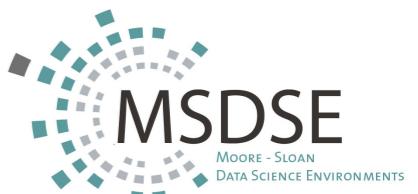


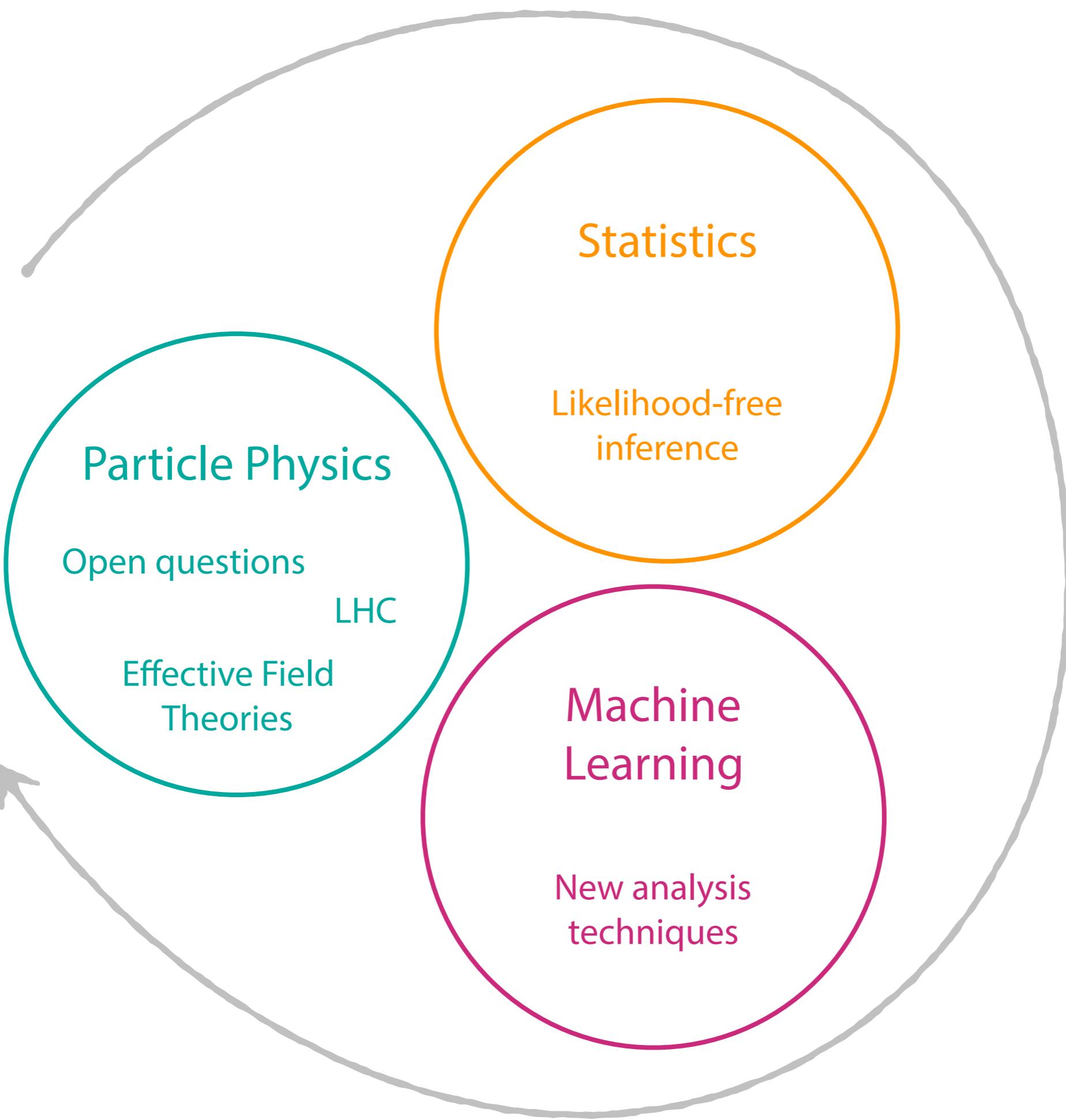
# Learning from the LHC, Effectively

Johann Brehmer

Center for Cosmology and Particle Physics & Center for Data Science, NYU

CityTech physics seminar  
May 10, 2018





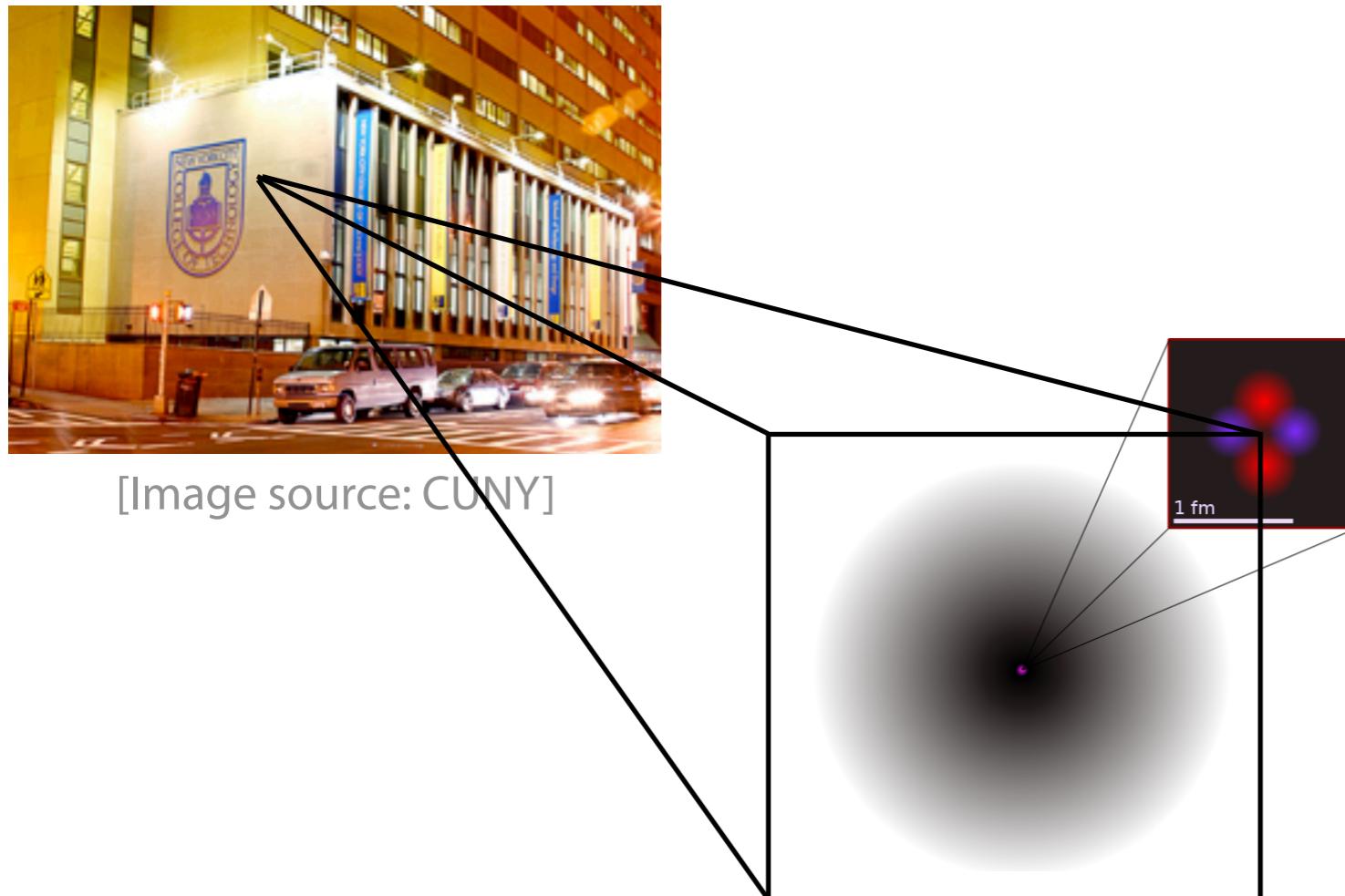
Big questions  
for a large collider

# The Standard Model of particle physics (SM)



[Image source: CUNY]

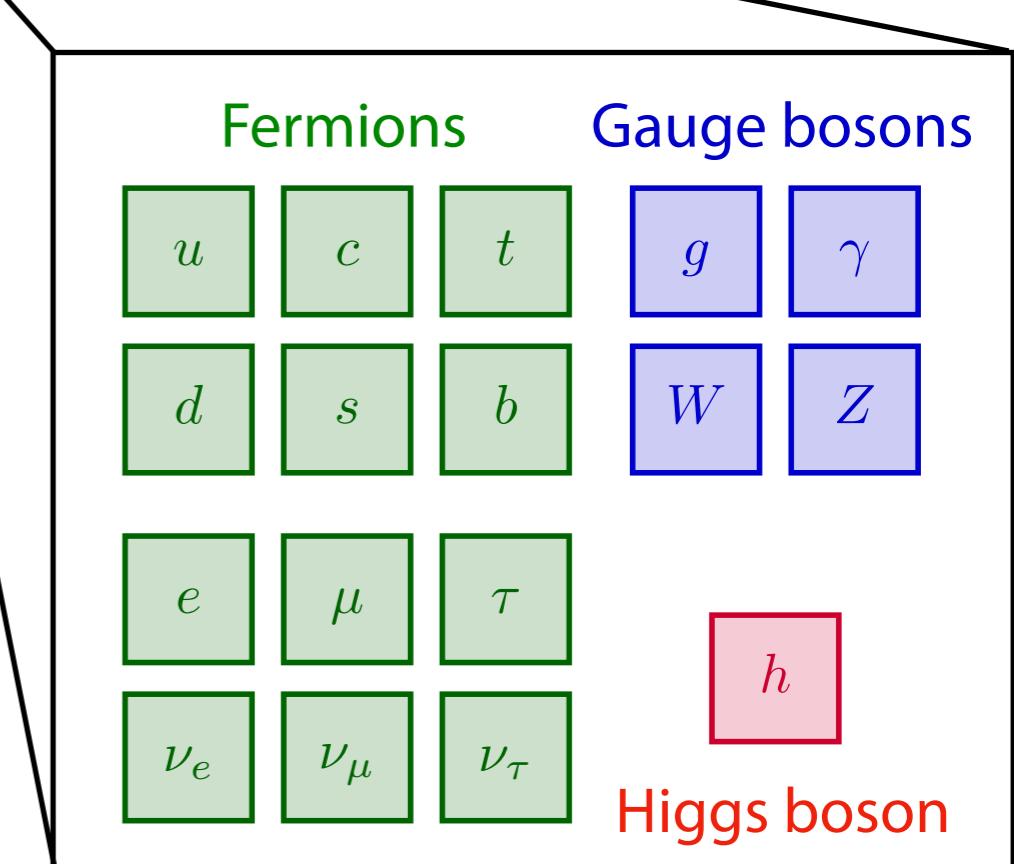
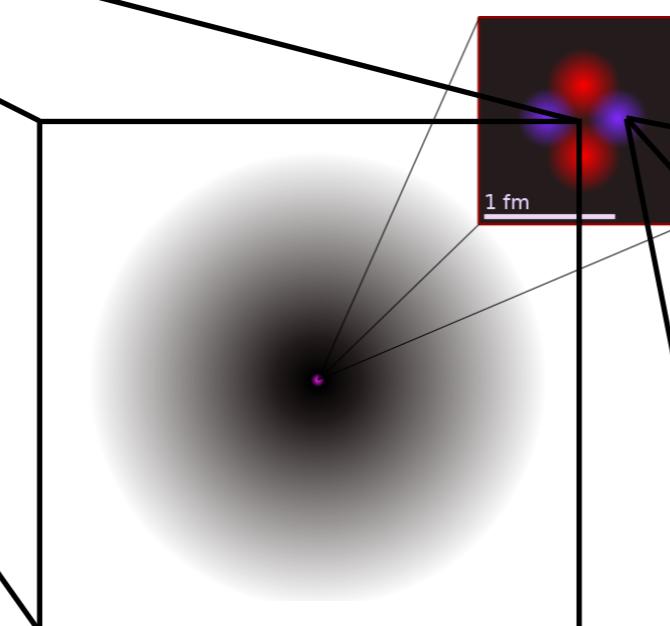
# The Standard Model of particle physics (SM)



# The Standard Model of particle physics (SM)



[Image source: CUNY]



# SM track record

- ✓ All observed elementary particles
- ✓ Electromagnetism, weak force, strong force
- ✓ In agreement with virtually every experimental test, up to  $\sim 10^{-10}$  precision

# SM track record

- ✓ All observed elementary particles
  - x Gravity
  - x Dark matter
  - x Baryon asymmetry
  - x (Neutrino masses)
- ✓ Electromagnetism, weak force, strong force
- ✓ In agreement with virtually every experimental test, up to  $\sim 10^{-10}$  precision
  - x Few experimental anomalies  
(Muon g-2, proton radius, ...)
  - x Complicated structure, many parameters
  - x Unexplained smallness of some parameters  
(Hierarchy problem, strong CP problem, cosmological constant problem, ...)

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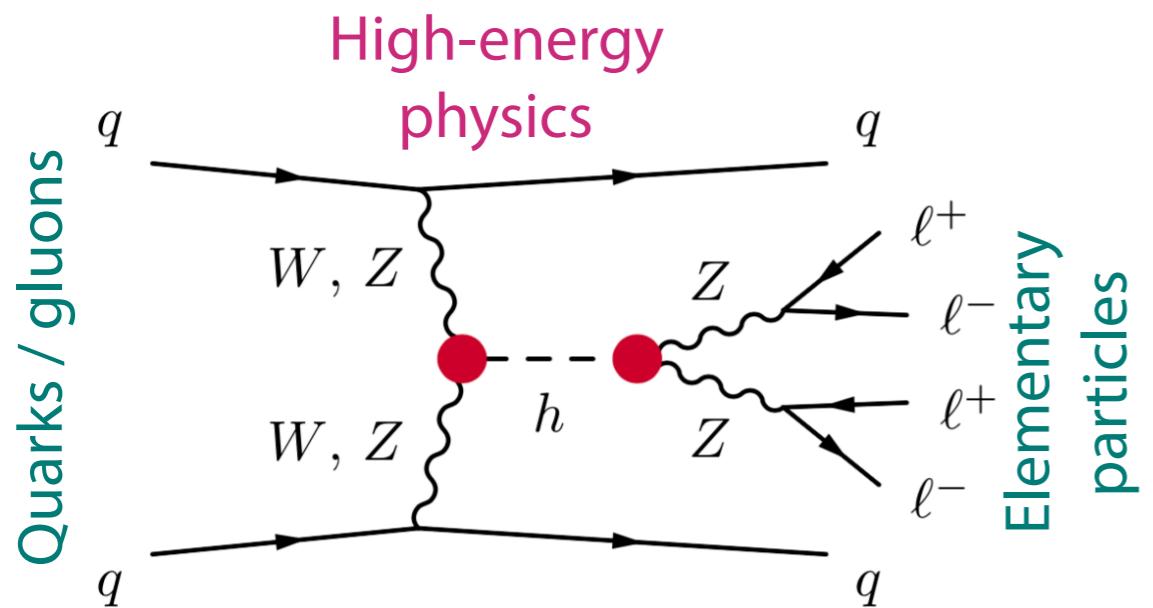
Many models of new physics to answer the open questions

⇒ Look for them at the LHC!

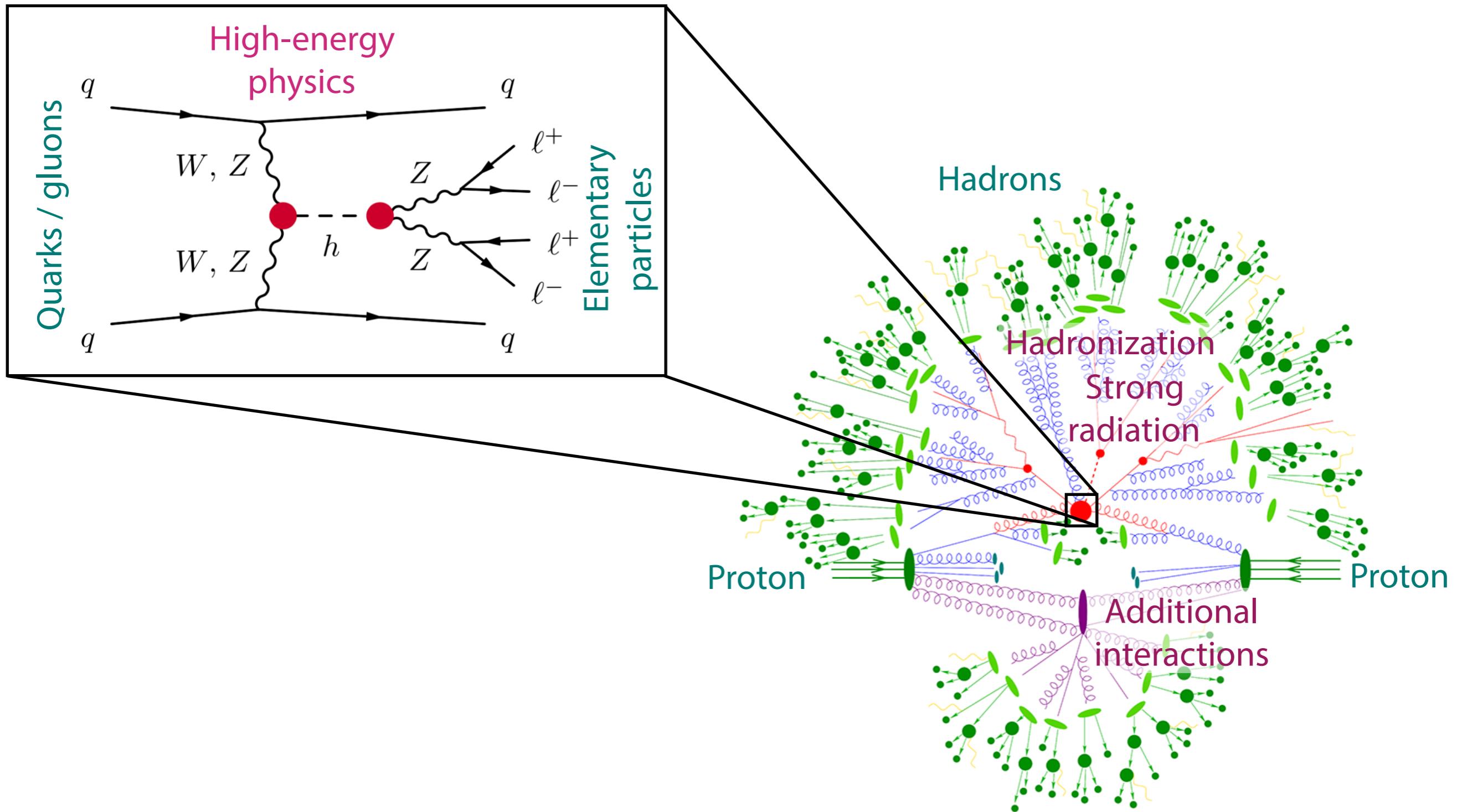




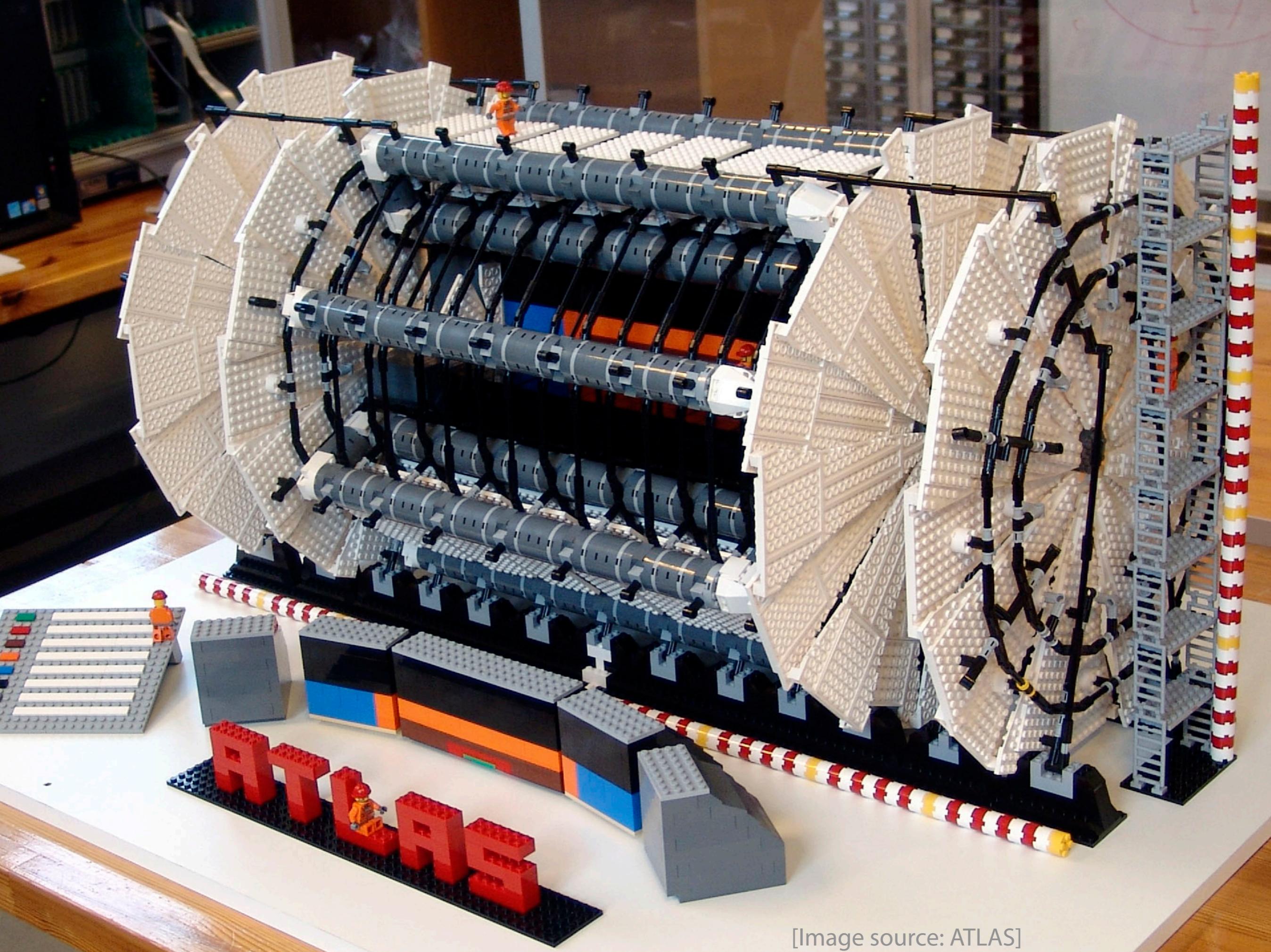
# LHC processes



# LHC processes

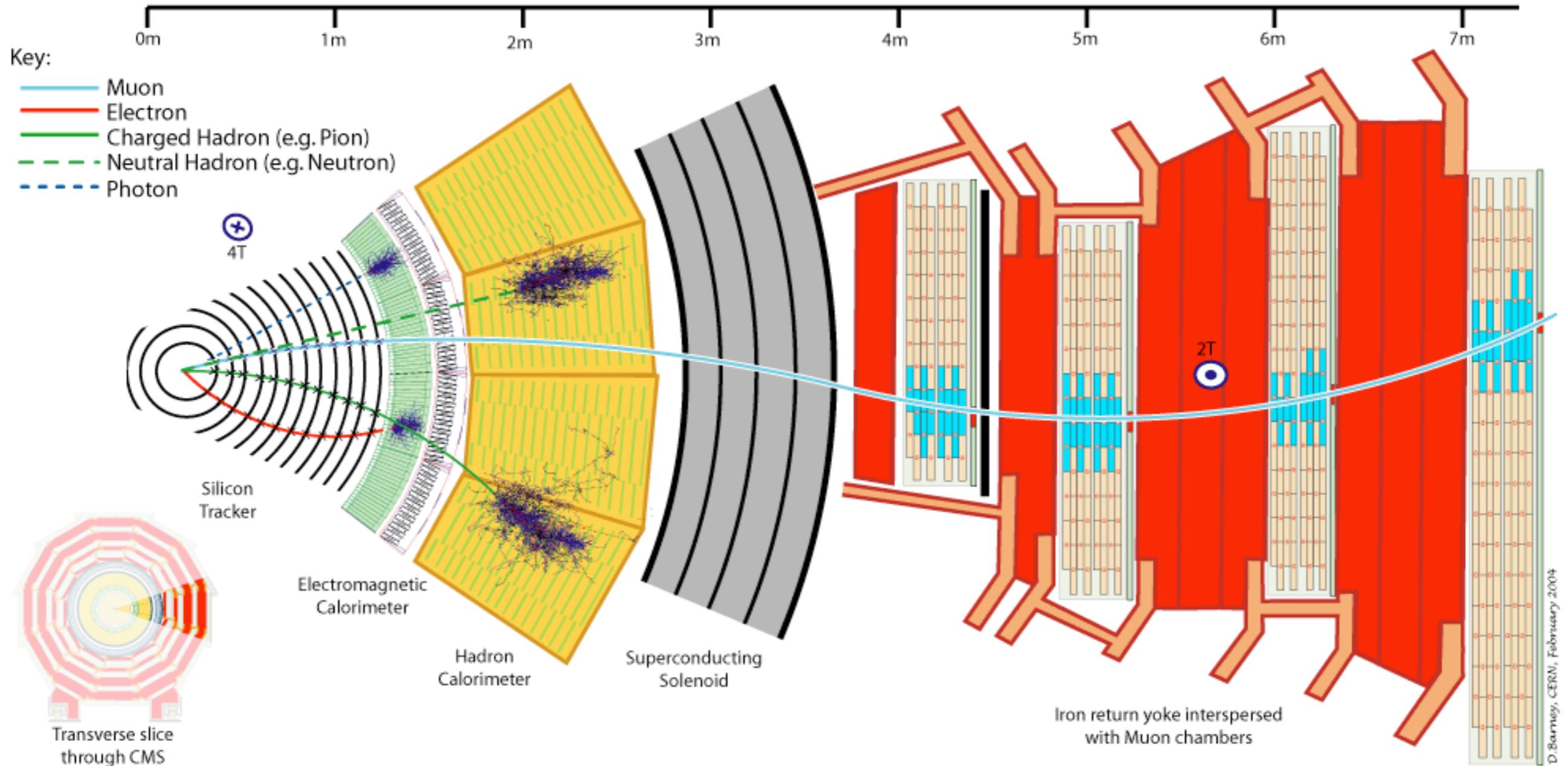


[Image source: F. Krauss]



[Image source: ATLAS]

# Particle, meet detector



[Image source: CMS]

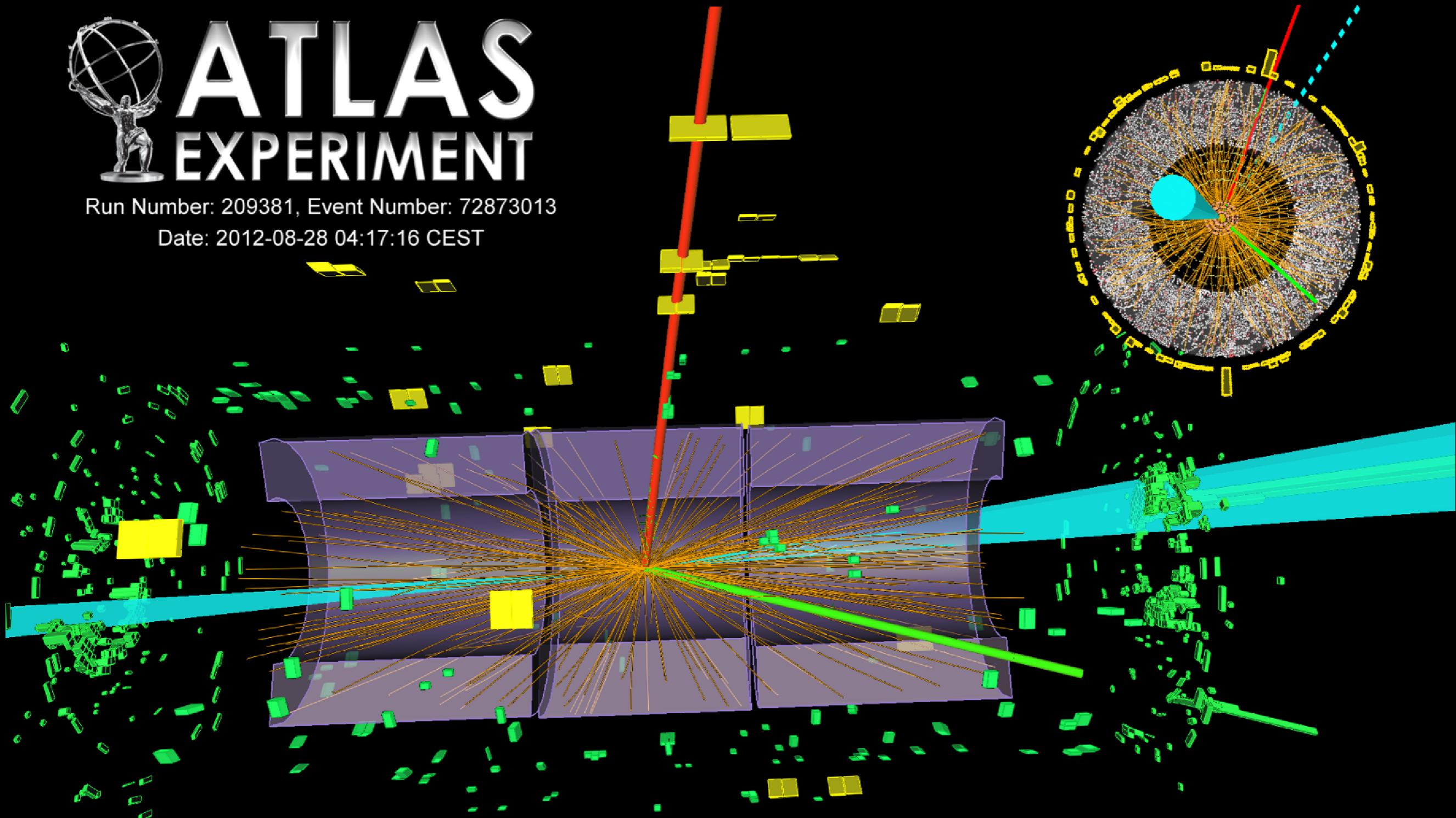
9/42



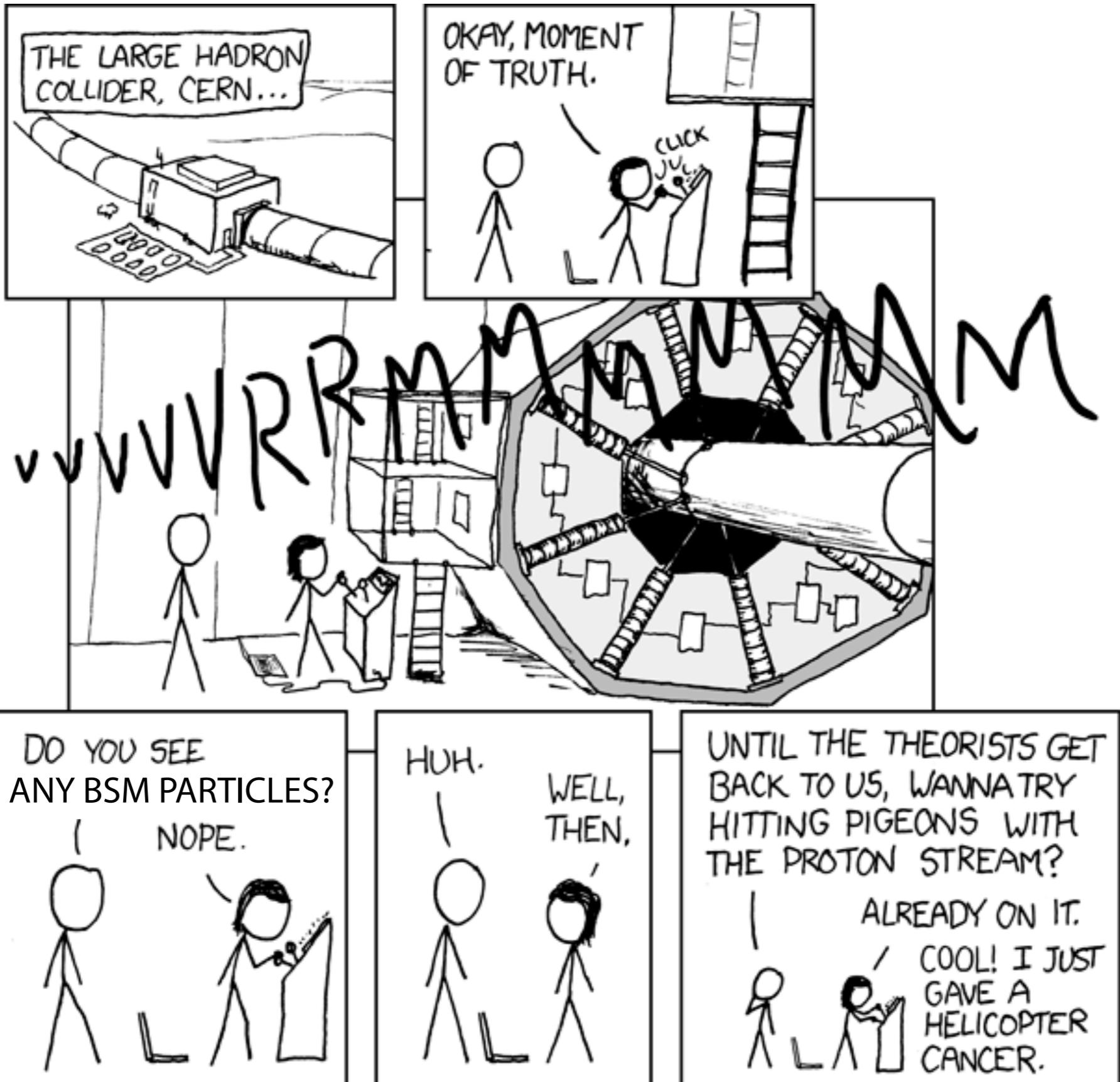
# ATLAS EXPERIMENT

Run Number: 209381, Event Number: 72873013

Date: 2012-08-28 04:17:16 CEST



[Image source: ATLAS]



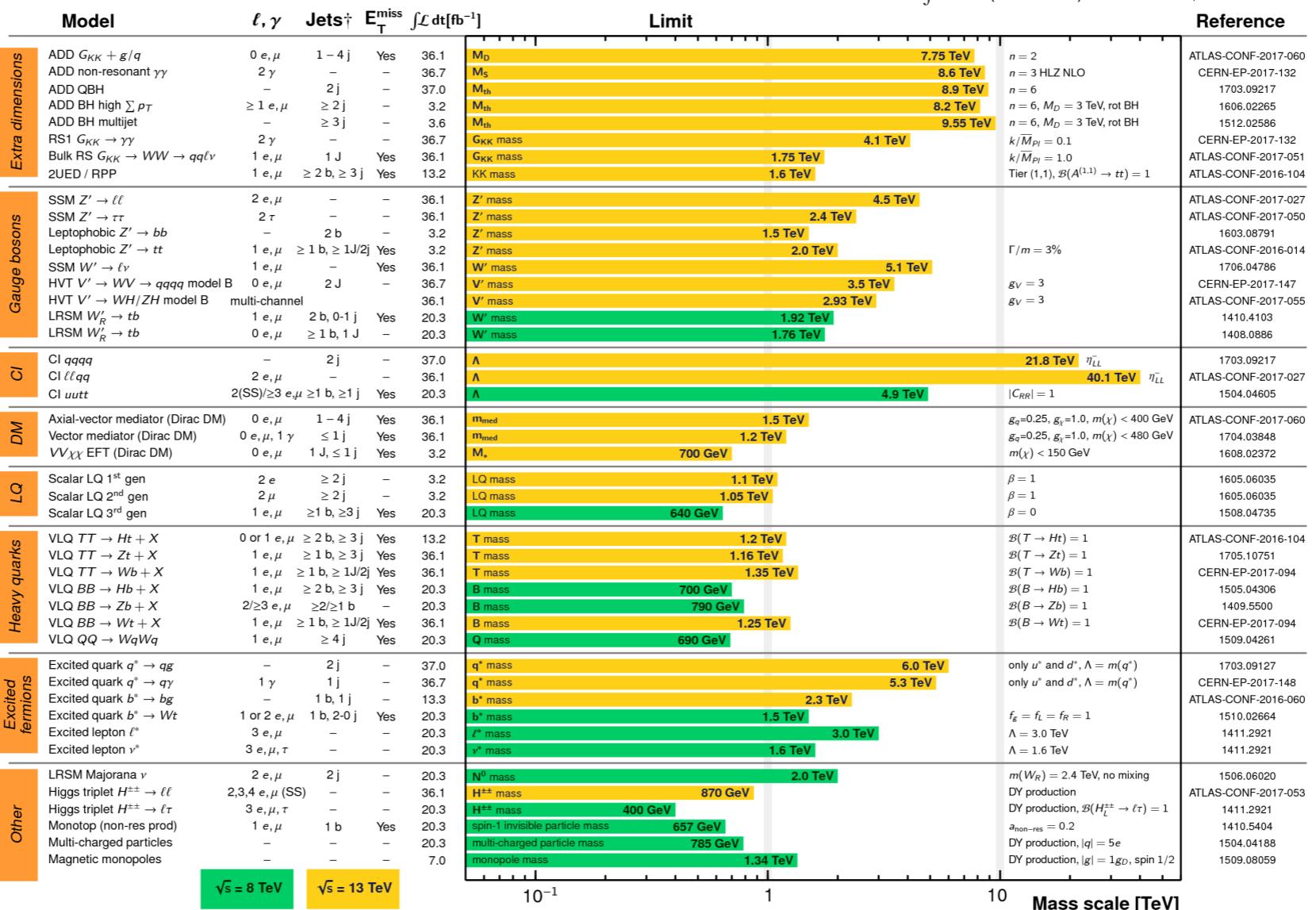
[Source: xkcd]

# Where is everyone?

Models of new physics

## ATLAS Exotics Searches\* - 95% CL Upper Exclusion Limits

Status: July 2017



\*Only a selection of the available mass limits on new states or phenomena is shown.

<sup>†</sup>Small-radius (large-radius) jets are denoted by the letter j (J).

Mass of new particles

[Source: ATLAS]

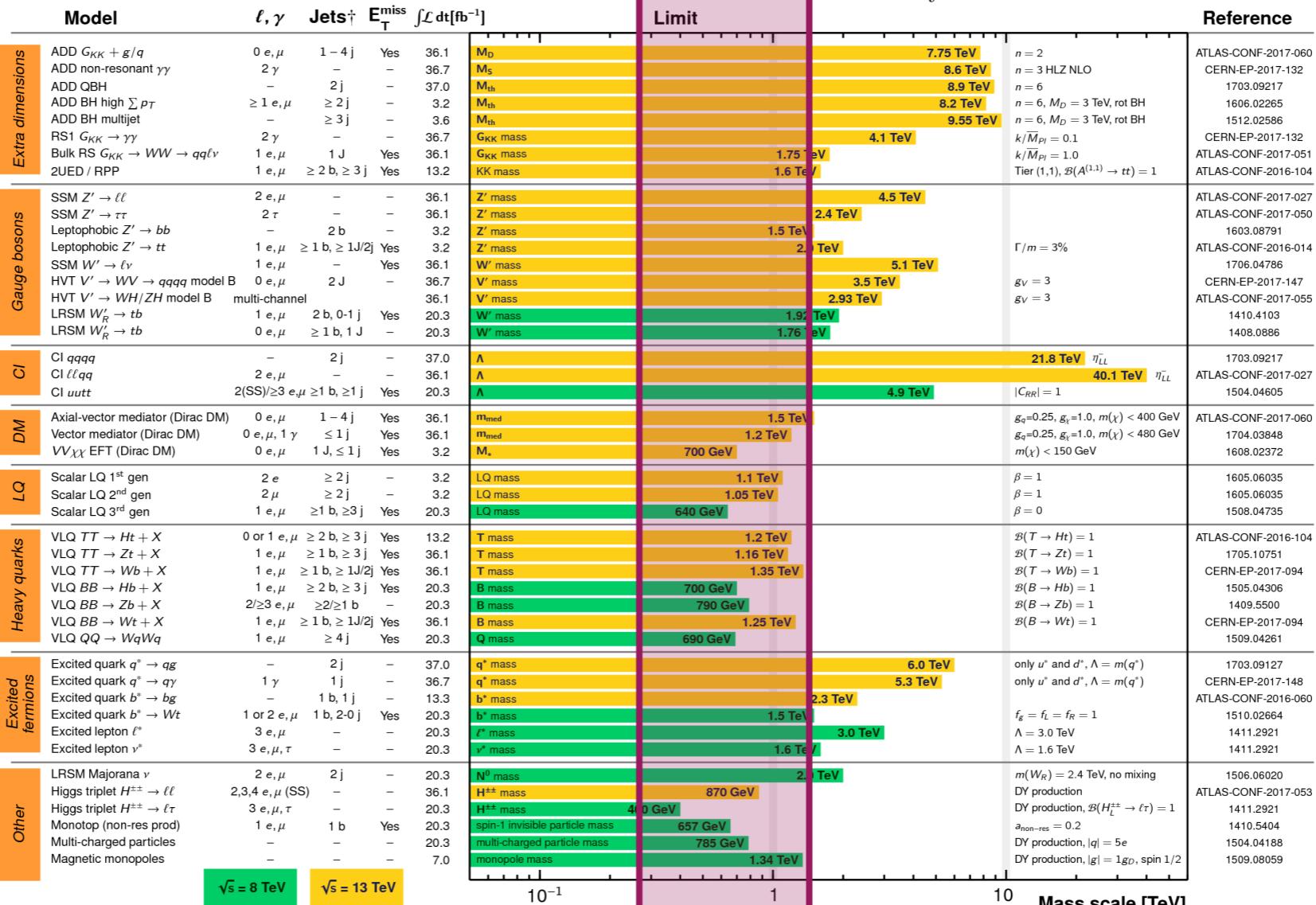
# Where is everyone?

New physics “naturally” expected

Models of new physics

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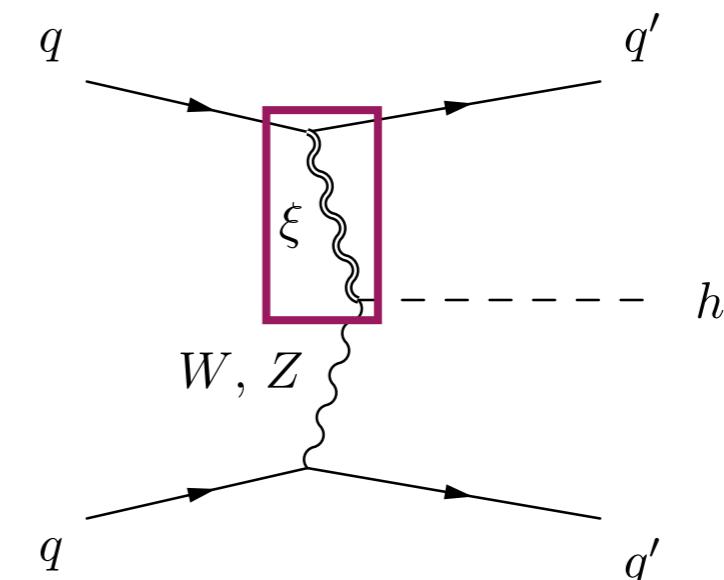
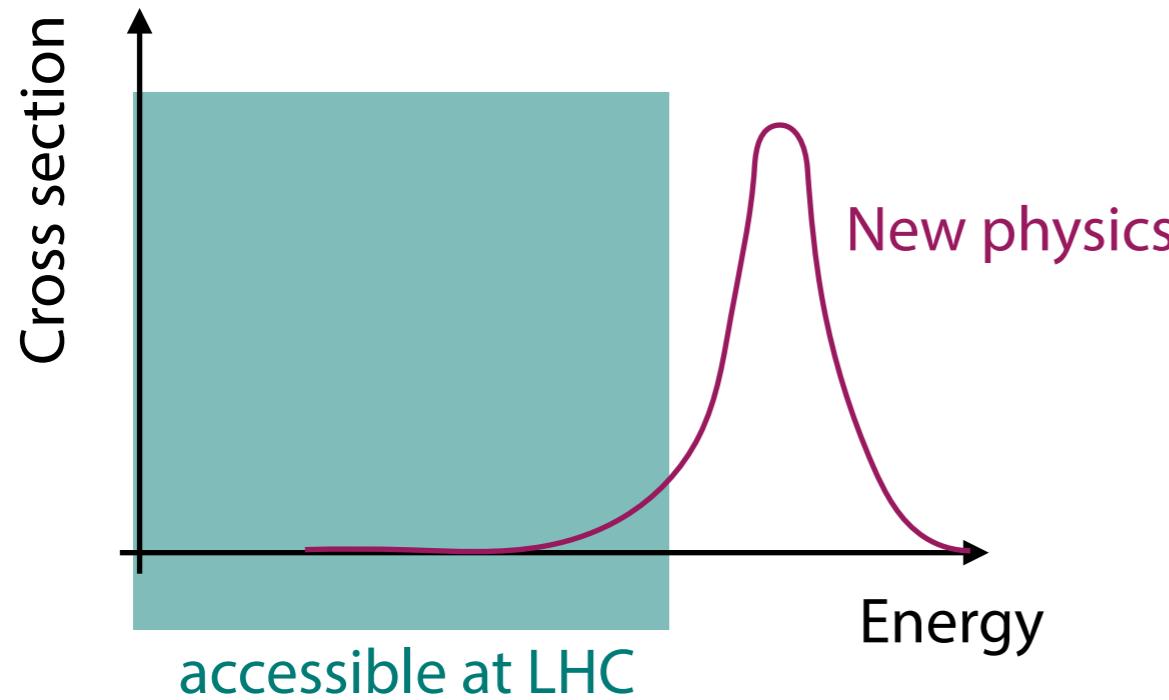
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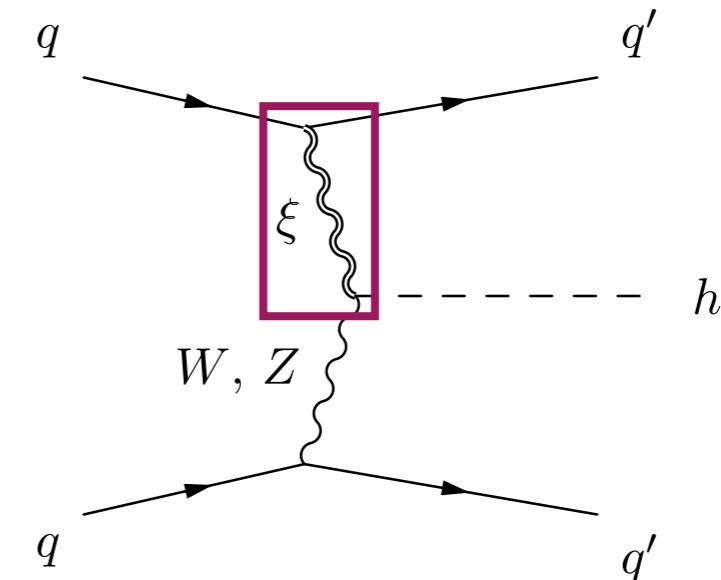
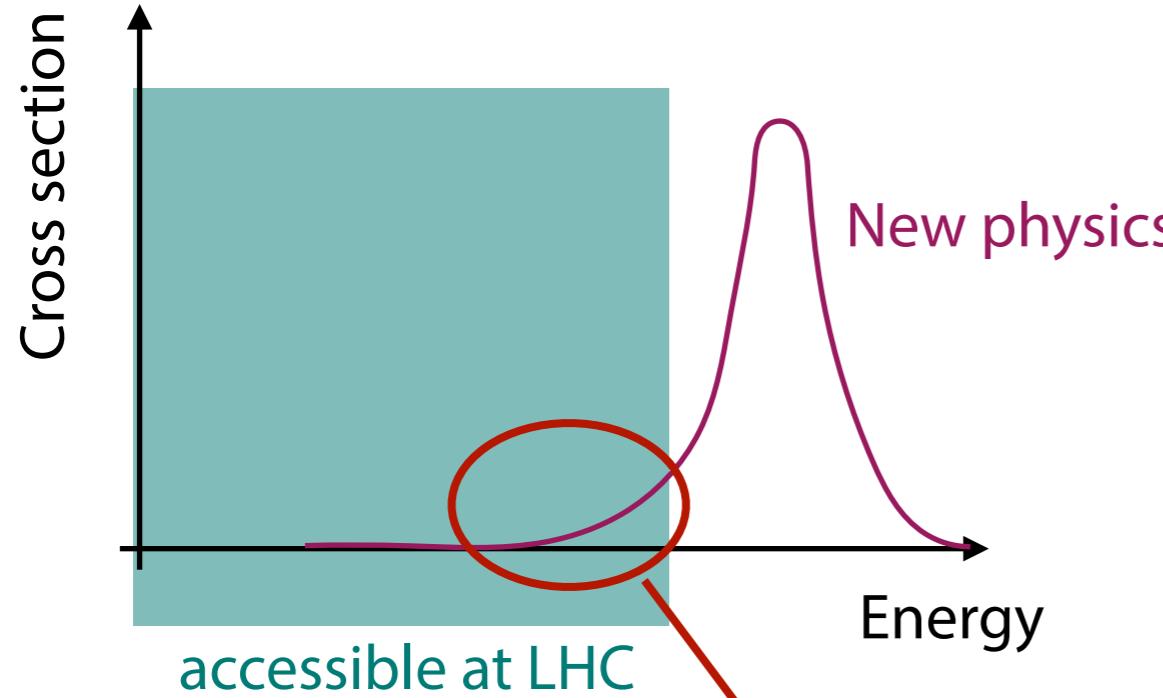
⇒ New physics is heavier than hoped for

[Source: ATLAS]

# Indirect effects of heavy new physics



# Indirect effects of heavy new physics

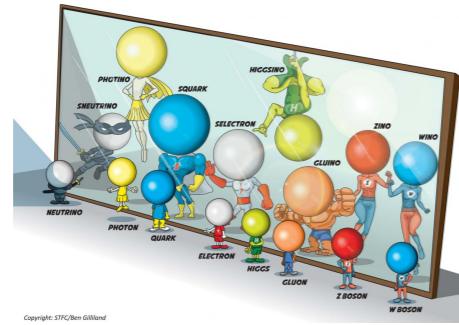


Can we find such effects in the interactions of SM particles?

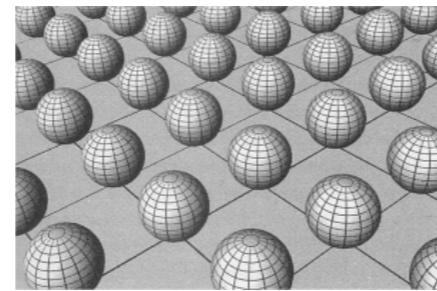
# Effective field theory

Energy

$\Lambda$



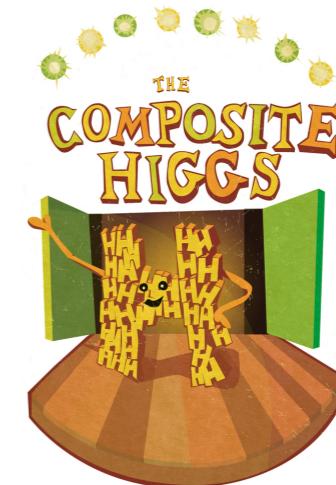
Supersymmetry?



Extra dimensions?



Dragons?



Composite Higgs?

...

$E \ll \Lambda$

LHC Higgs

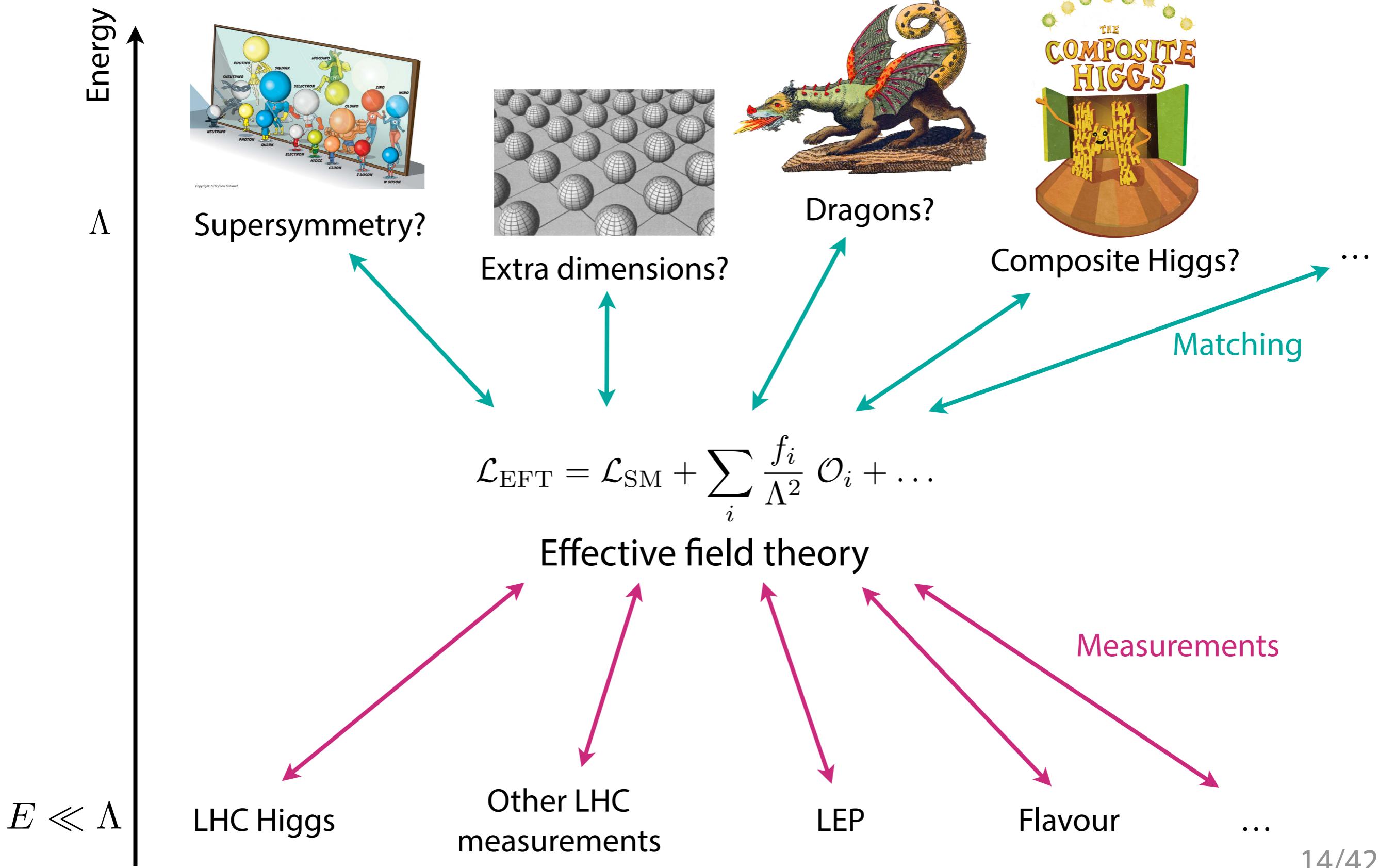
Other LHC  
measurements

LEP

Flavour

...

# Effective field theory



[W. Buchmuller, D. Wyler 85;  
B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek 1008.4884; ...]

# SMEFT (Standard Model Effective Field Theory)

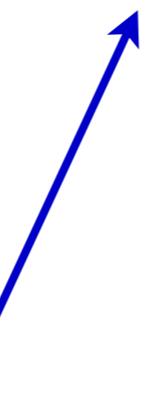
$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i + \dots$$

# SMEFT (Standard Model Effective Field Theory)

## Operators

- all possible interactions between SM particles mediated by new physics
- fixed by SM particles + SM symmetries + expansion in  $1/\Lambda$ , independent of high-energy physics
- affect rates + kinematics

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---

$\mathcal{O}_{\phi,1} = (D_\mu \phi)^\dagger \phi \phi^\dagger D^\mu \phi$	$\mathcal{O}_{GG} = (\phi^\dagger \phi) G_{\mu\nu}^a G^{\mu\nu a}$
$\mathcal{O}_{\phi,2} = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi)$	$\mathcal{O}_{BB} = -\frac{g'^2}{4} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\phi,3} = \frac{1}{3} (\phi^\dagger \phi)^3$	$\mathcal{O}_{WW} = -\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^a W^{\mu\nu a}$
$\mathcal{O}_{\phi,4} = (\phi^\dagger \phi) (D_\mu \phi)^\dagger D^\mu \phi$	$\mathcal{O}_{BW} = -\frac{gg'}{4} (\phi^\dagger \sigma^a \phi) B_{\mu\nu} W^{\mu\nu a}$
	$\mathcal{O}_B = \frac{ig'}{2} (D^\mu \phi)^\dagger D^\nu \phi B_{\mu\nu}$
	$\mathcal{O}_W = \frac{ig}{2} (D^\mu \phi)^\dagger \sigma^a D^\nu \phi W_{\mu\nu}^a$

---

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## Wilson coefficients

- precise measurement of these parameters is one of the most important goals of the LHC
- can be translated to high-energy physics parameters

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## Higher-order terms

- suppressed by additional factors of  $E^2 / \Lambda^2$

## Wilson coefficients

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**What makes the EFT  
measurements so  
difficult?**

# LHC processes

Parameters  
of interest

Theory  
parameters

$$\theta$$



Evolution

# LHC processes

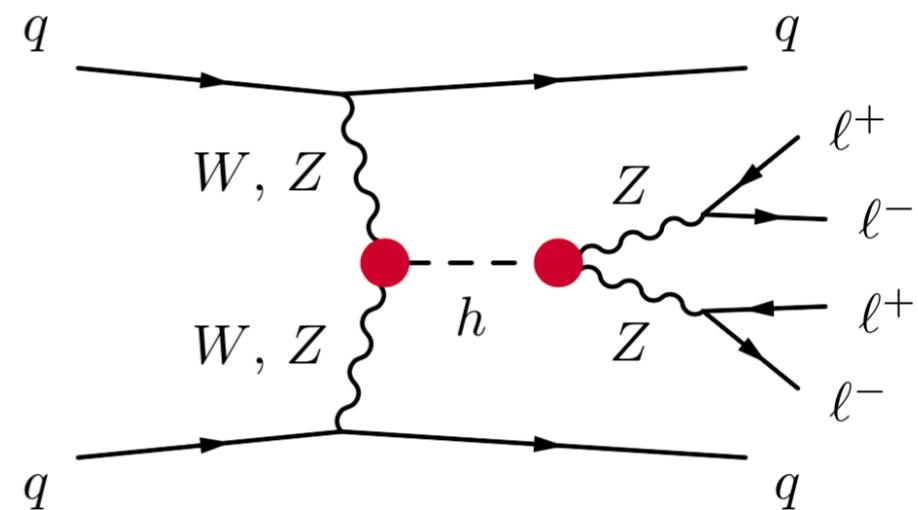
Latent variables

Parton-level  
momenta

Parameters  
of interest

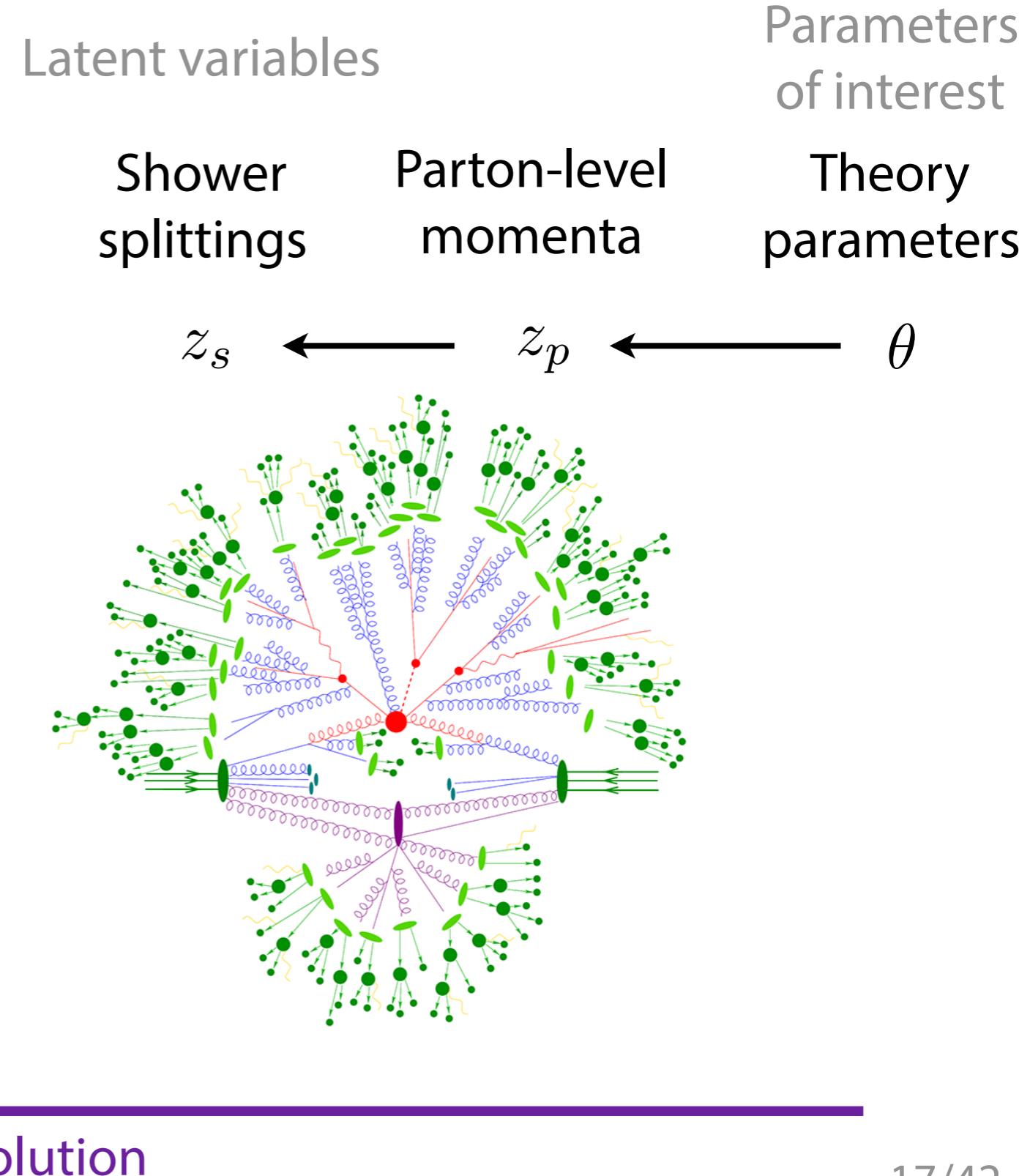
Theory  
parameters

$$z_p \leftarrow \theta$$



Evolution

# LHC processes

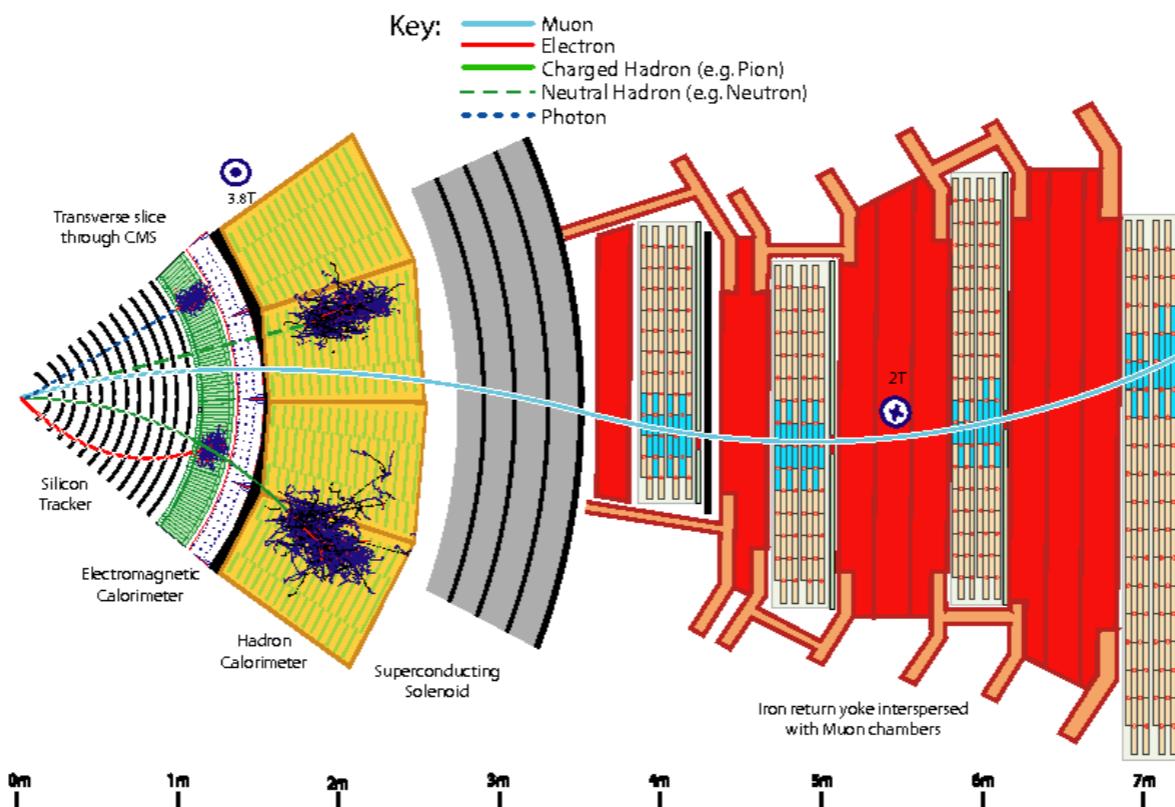


# LHC processes

Latent variables      Parameters  
of interest

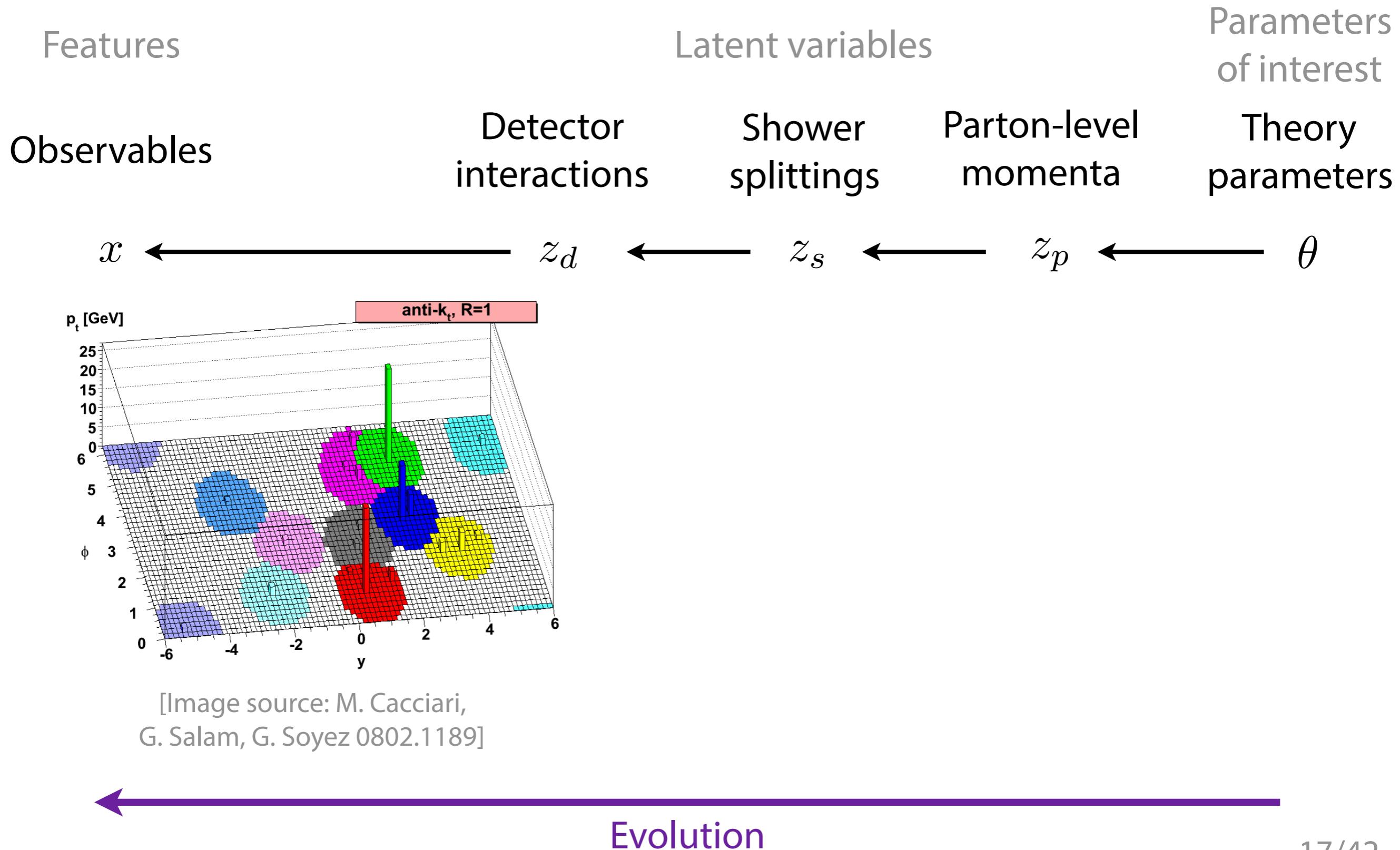
Detector interactions	Shower splittings	Parton-level momenta	Theory parameters
-----------------------	-------------------	----------------------	-------------------

$$z_d \longleftrightarrow z_s \longleftrightarrow z_p \longleftrightarrow \theta$$

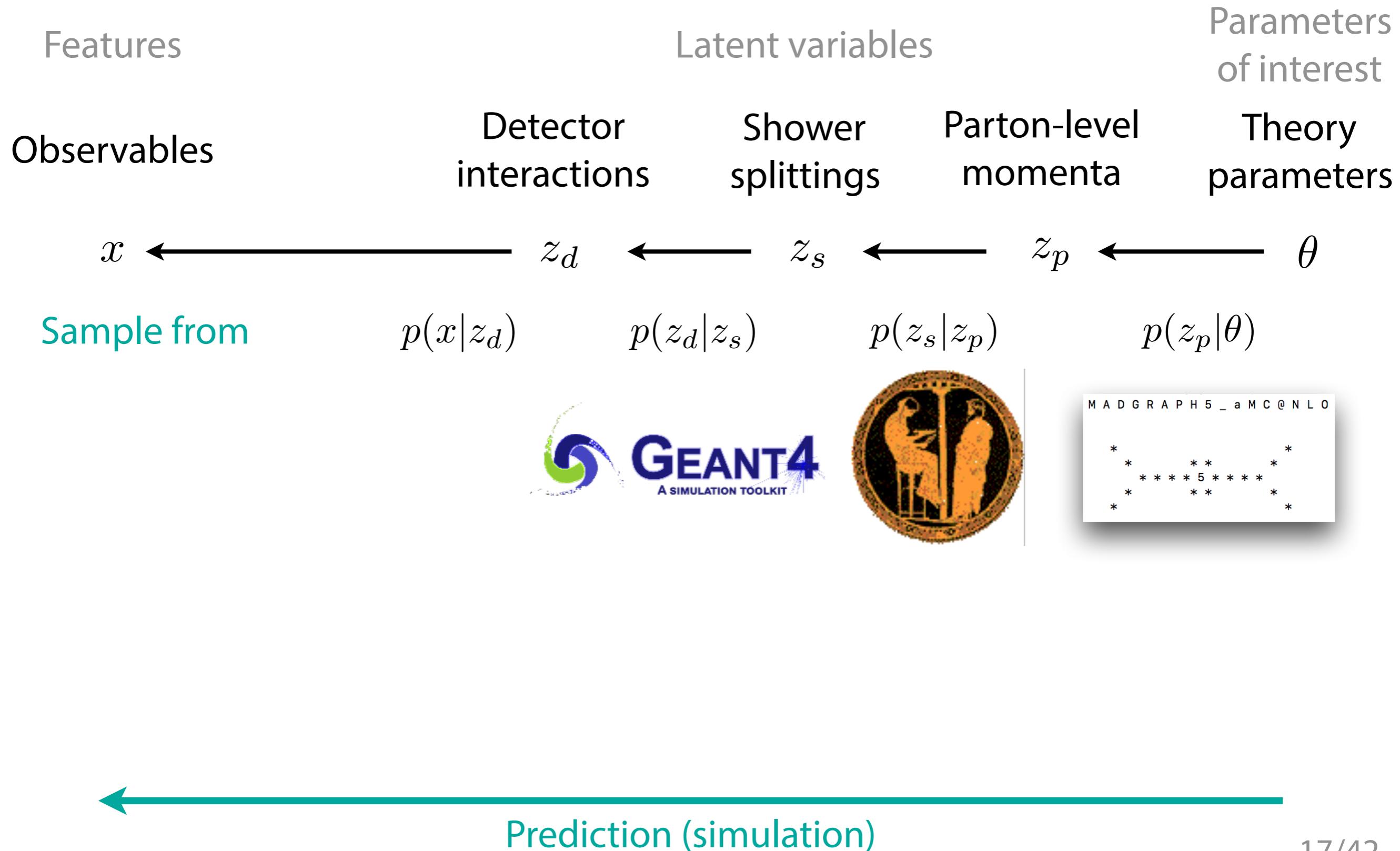


Evolution

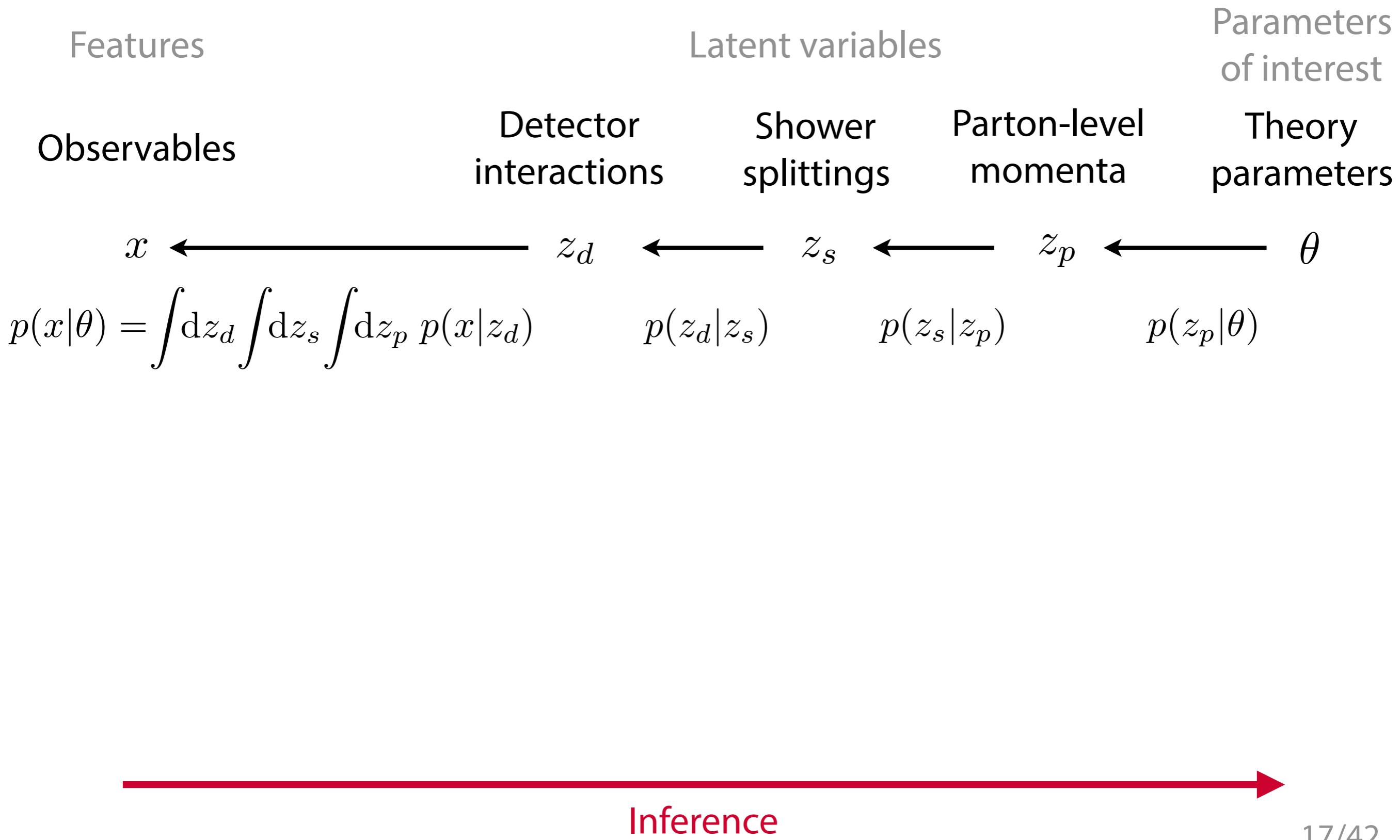
# LHC processes



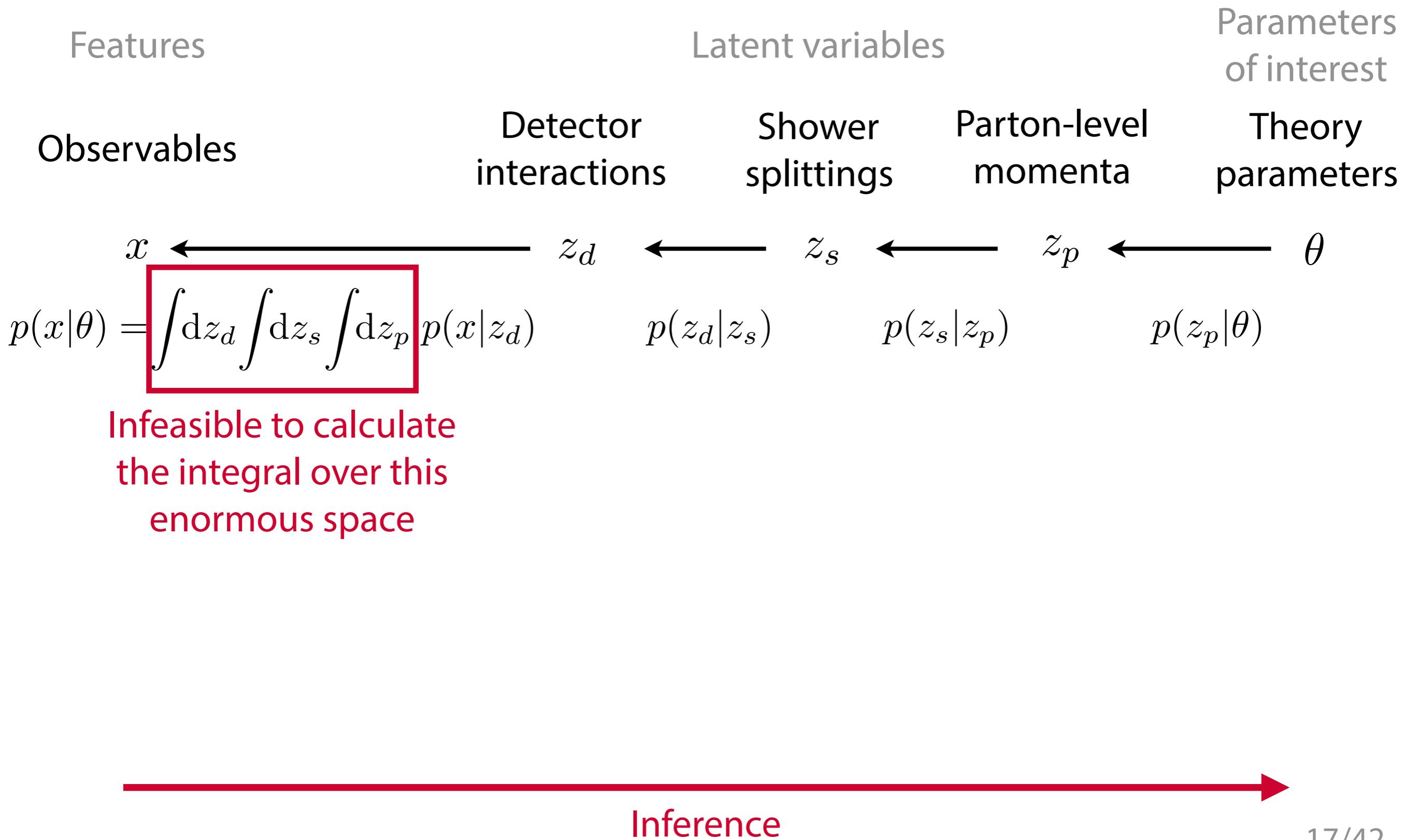
# LHC processes



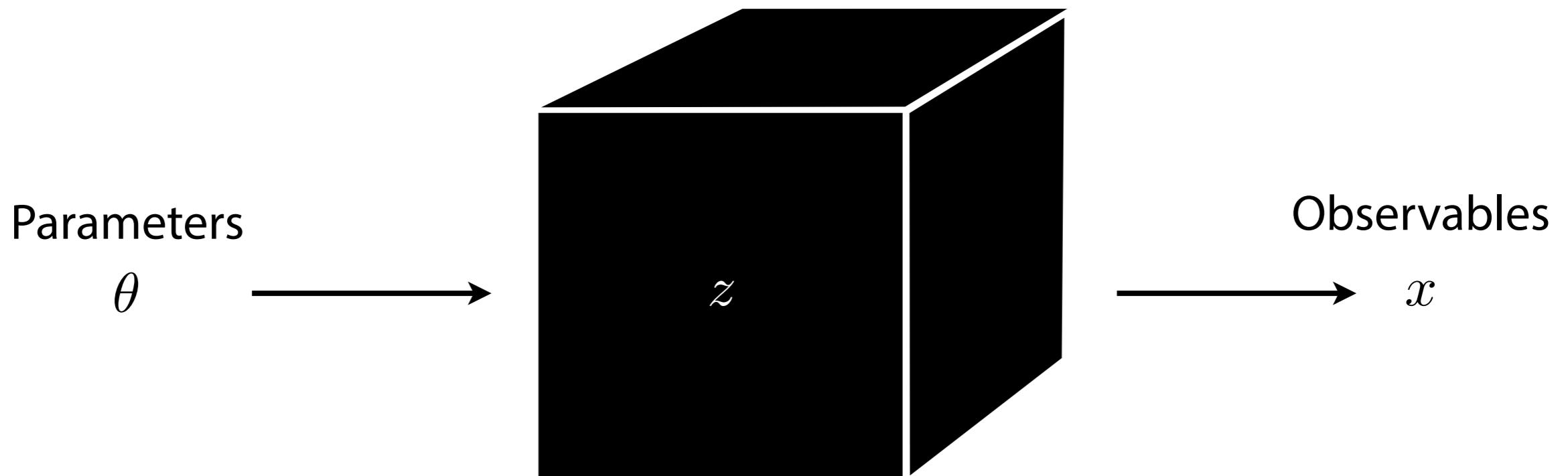
# LHC processes



# LHC processes



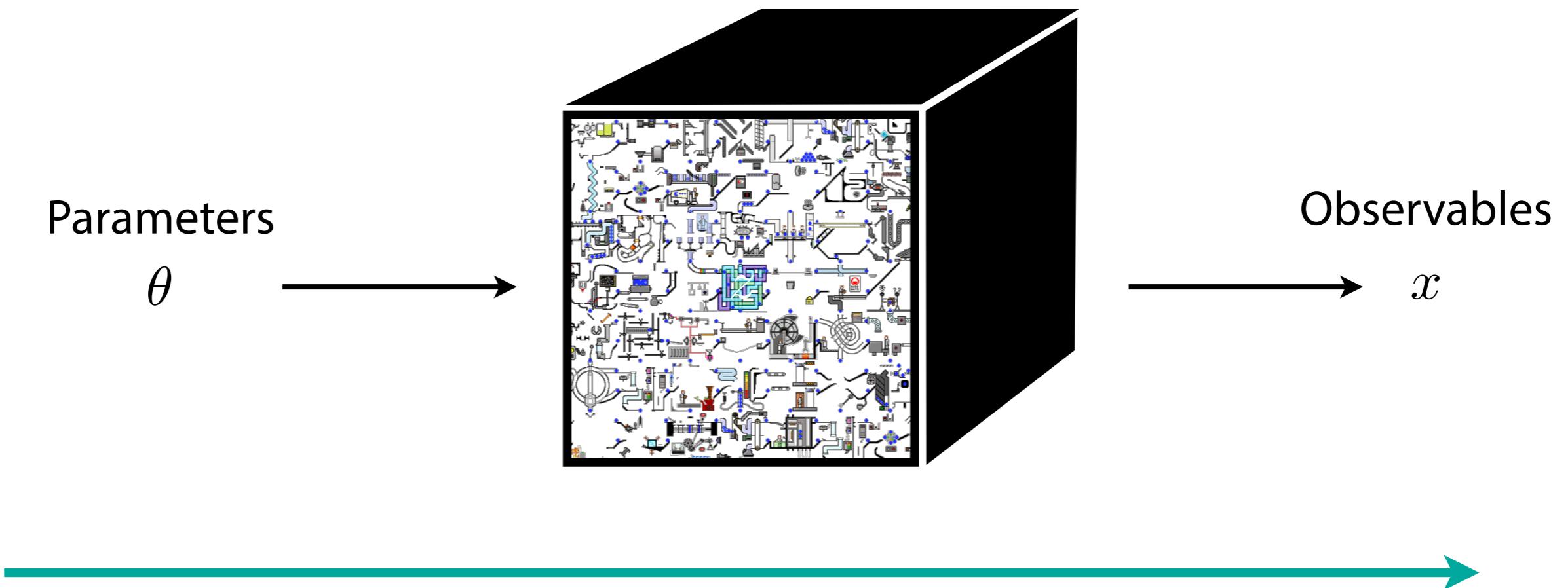
# Likelihood-free inference



Prediction (simulation):

- Well-understood mechanistic model
- Simulator can generate samples

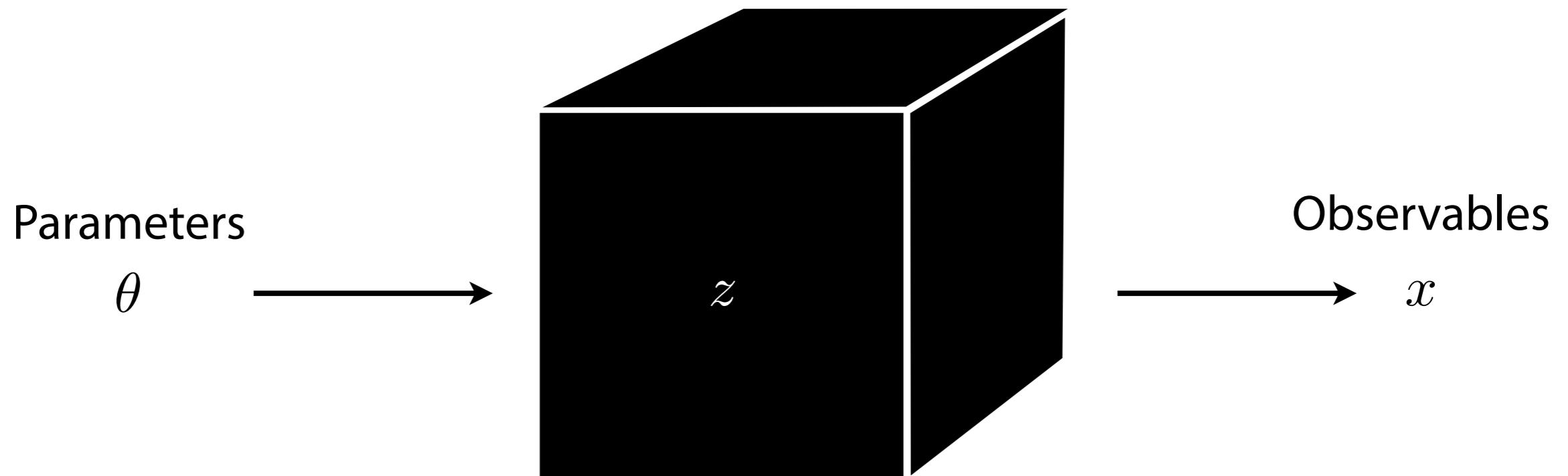
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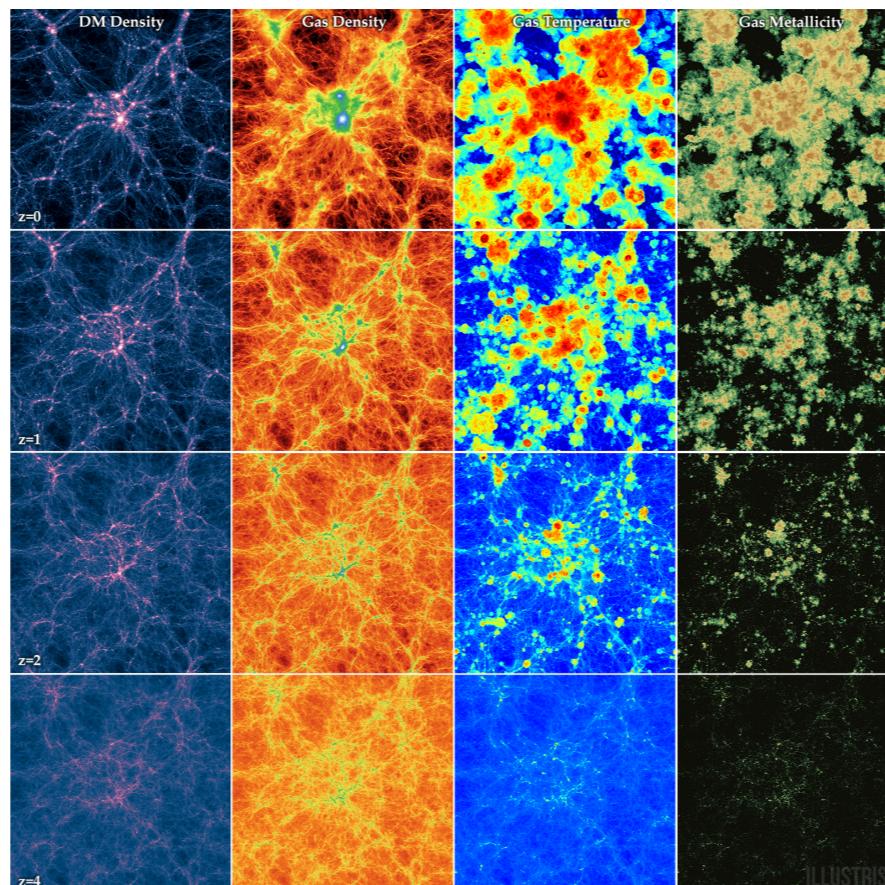
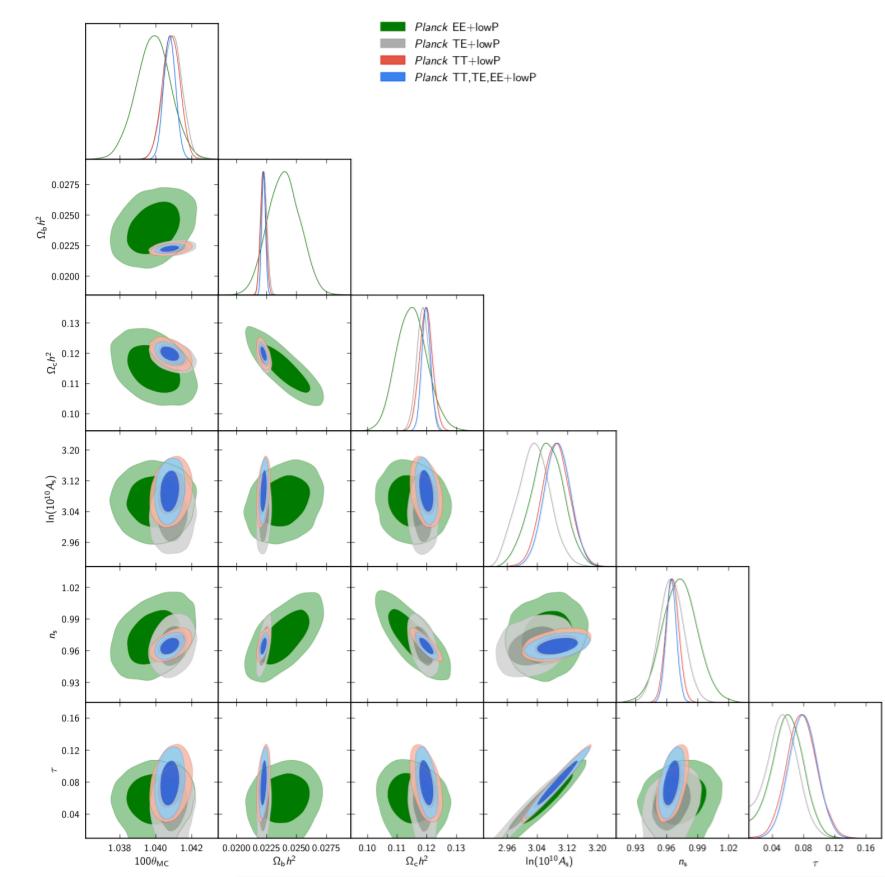
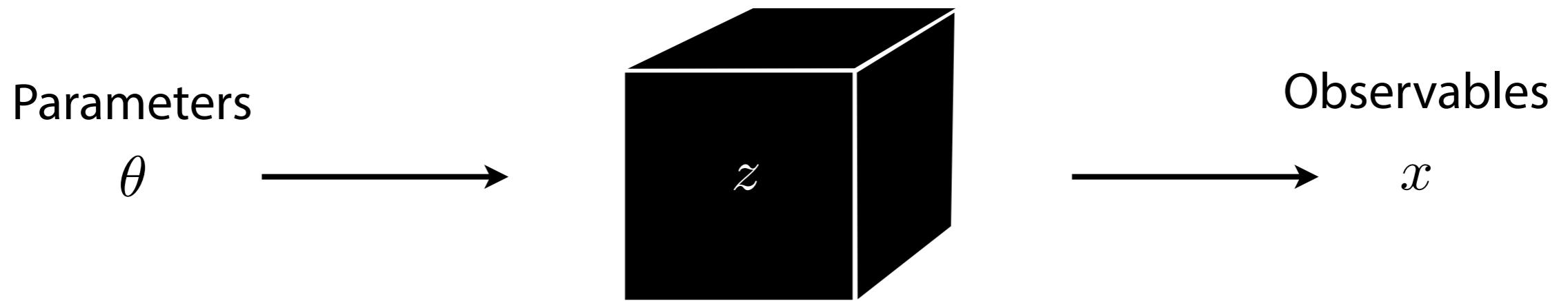
Prediction (simulation):

- Well-understood mechanistic model
- Simulator can generate samples

Inference:

- Likelihood function  $p(x|\theta)$  is intractable
- Goal: estimator  $\hat{p}(x|\theta)$

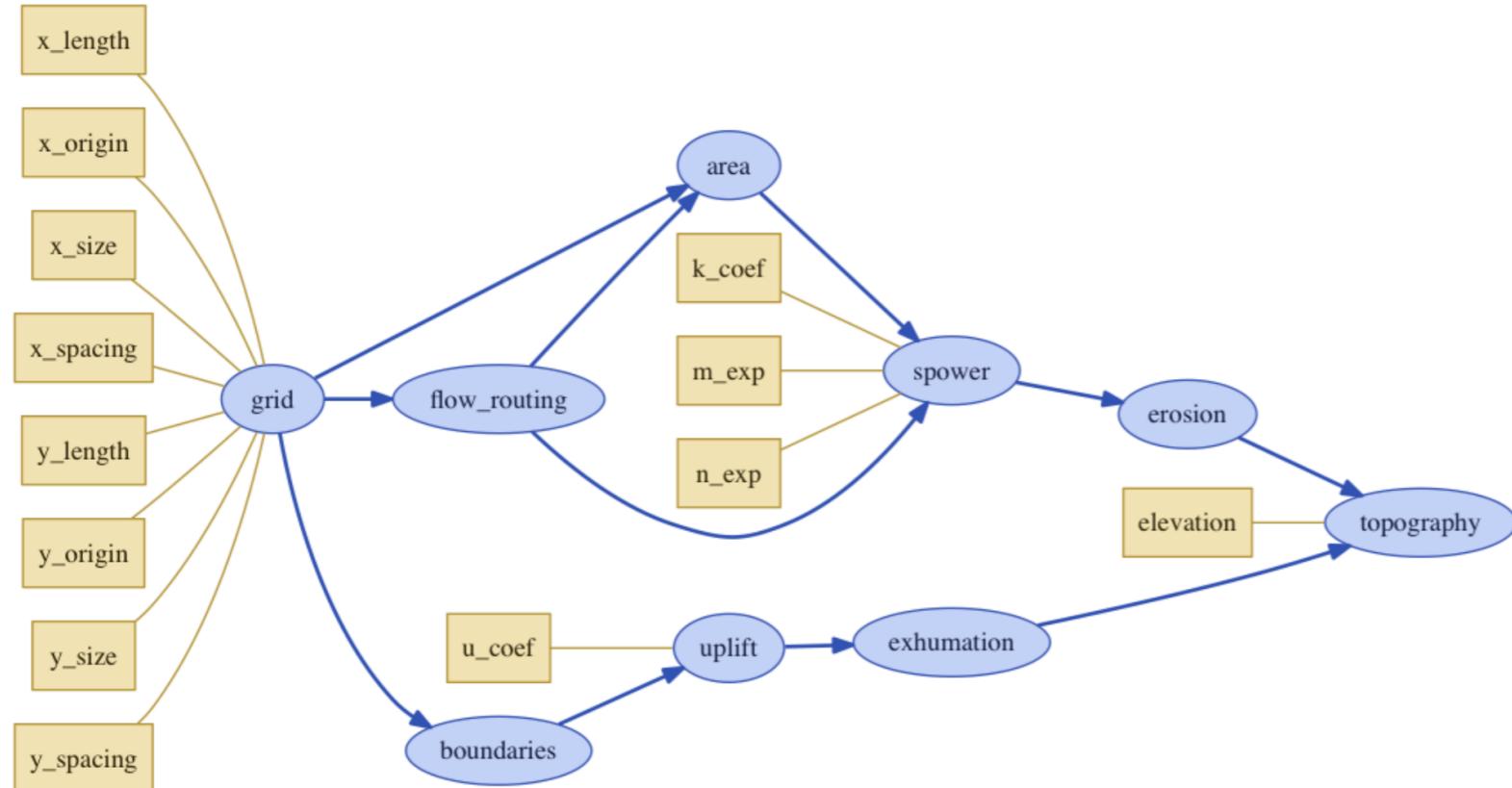
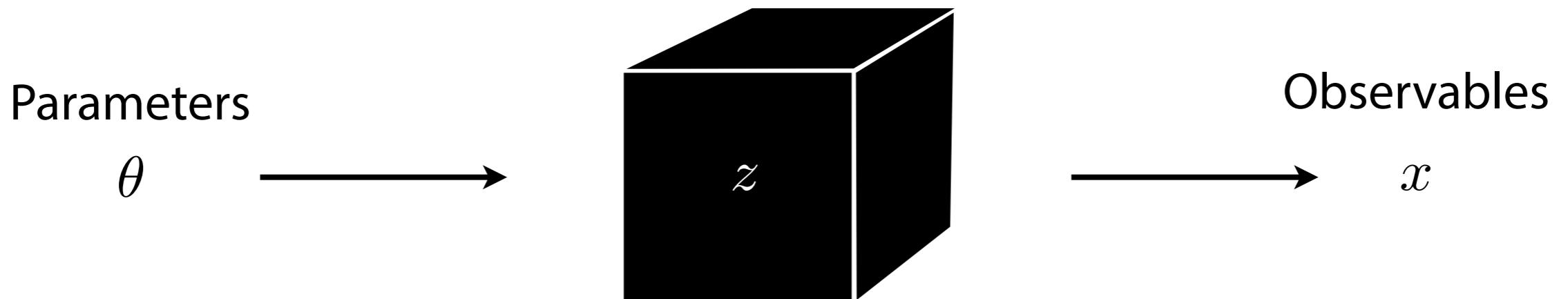
# Cosmological N-body simulations



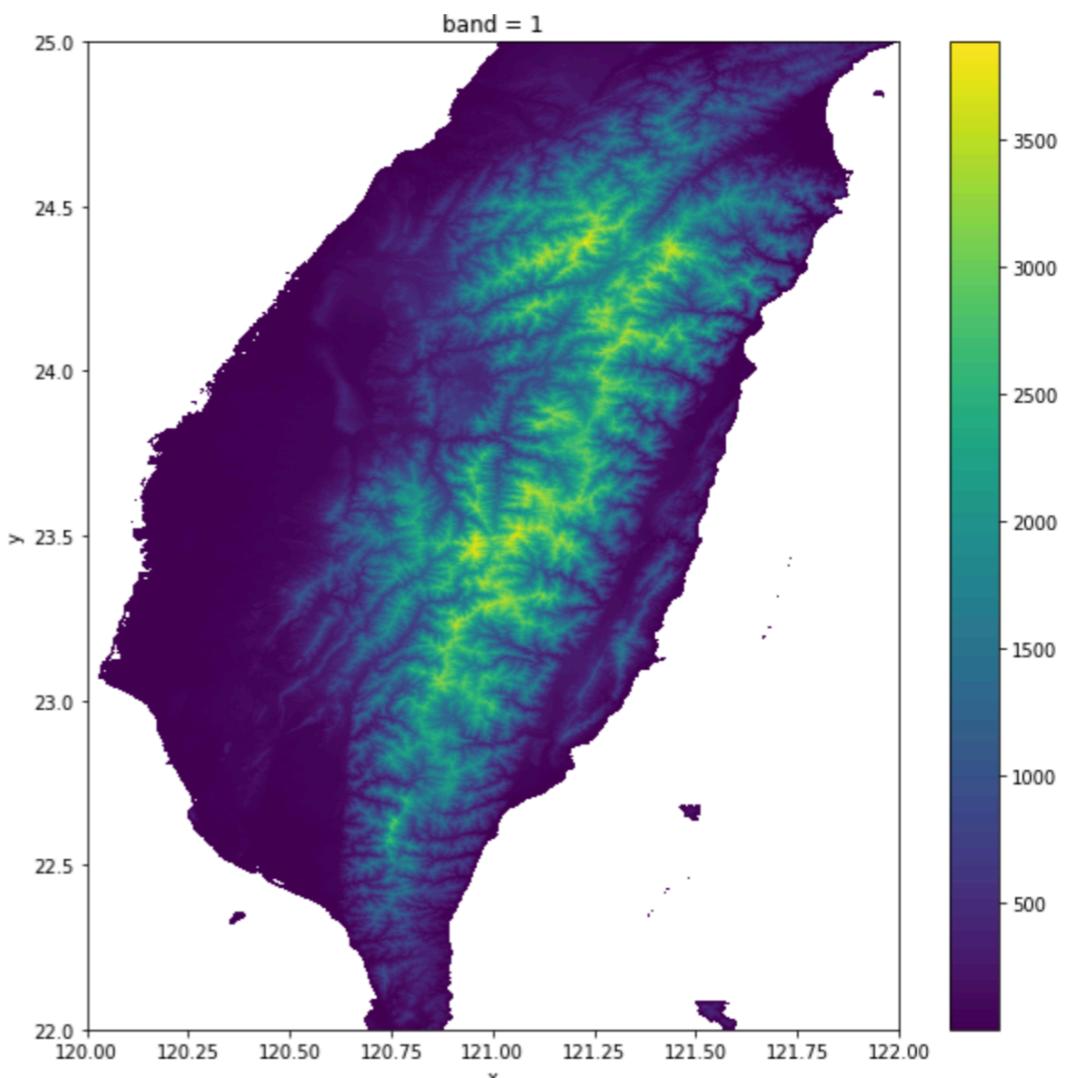
[Source: Planck 1502.01589]

[Source: Illustris 1405.2921]

# Computational topography

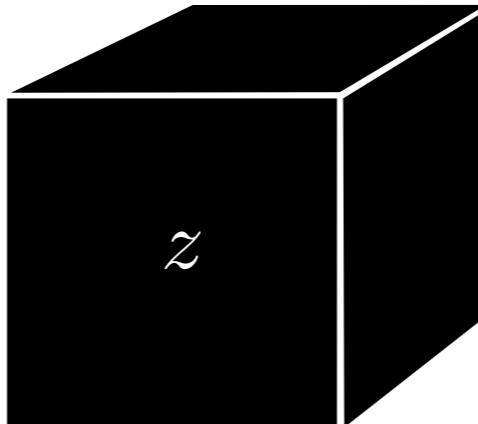


We create a simulation setup for this model, run it, and then plot the final topography (after 1 million years of simulation).



# Epidemiology

Parameters

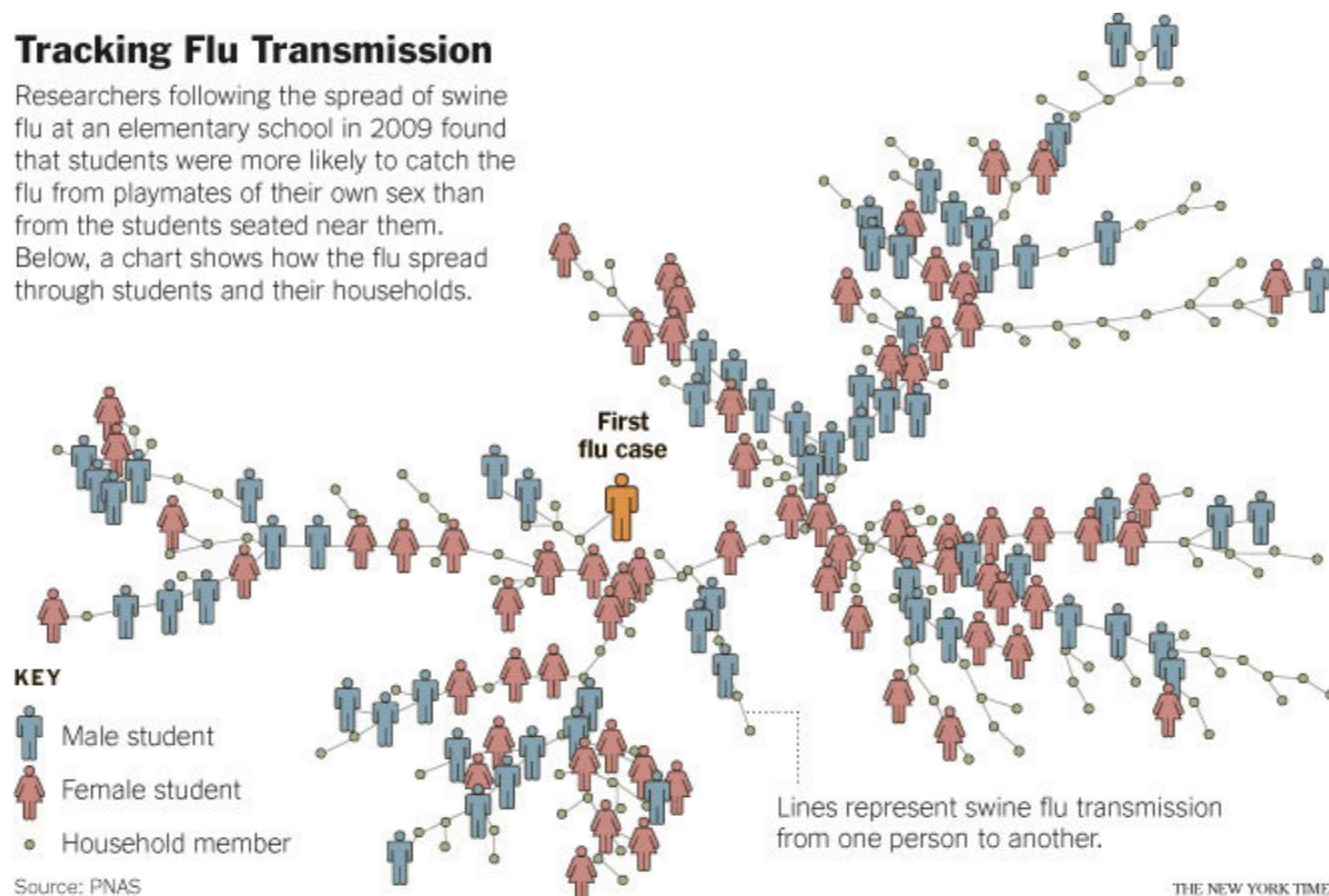
 $\theta$ 

Observables

 $x$ 

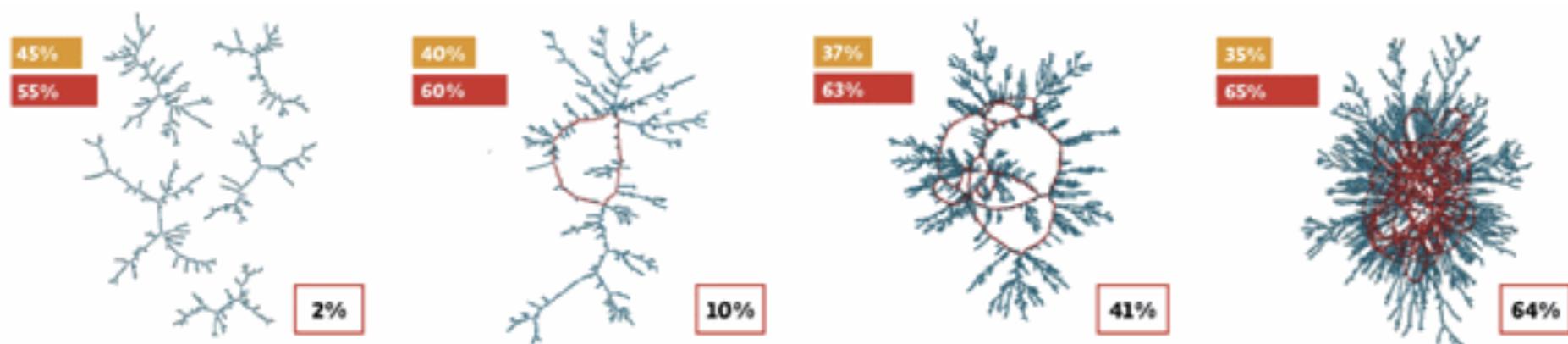
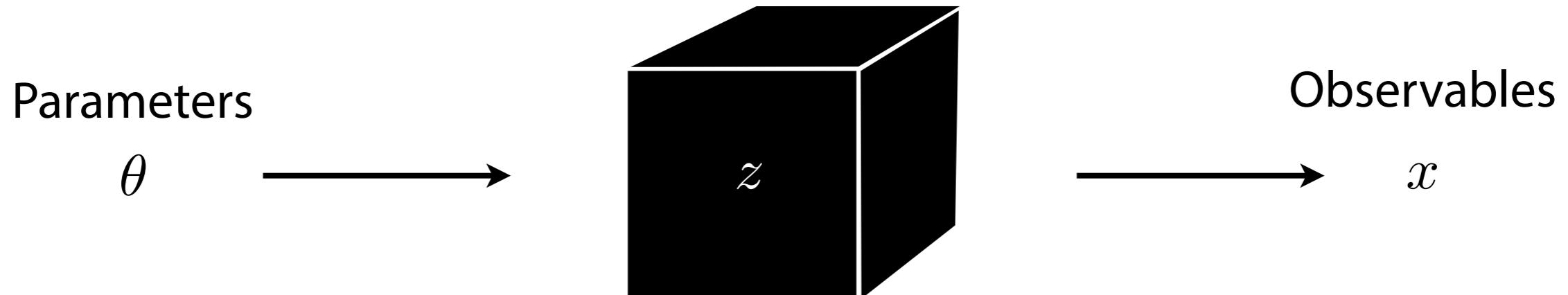
## Tracking Flu Transmission

Researchers following the spread of swine flu at an elementary school in 2009 found that students were more likely to catch the flu from playmates of their own sex than from the students seated near them. Below, a chart shows how the flu spread through students and their households.



[Source: New York Times]

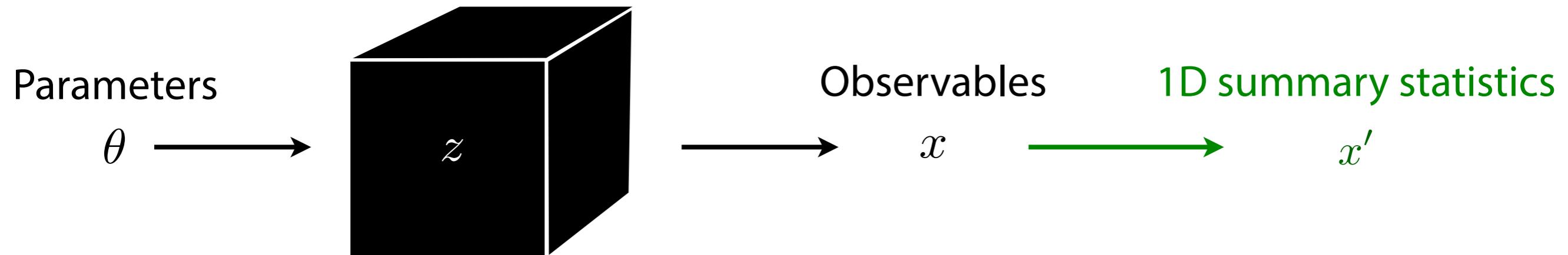
# Epidemiology



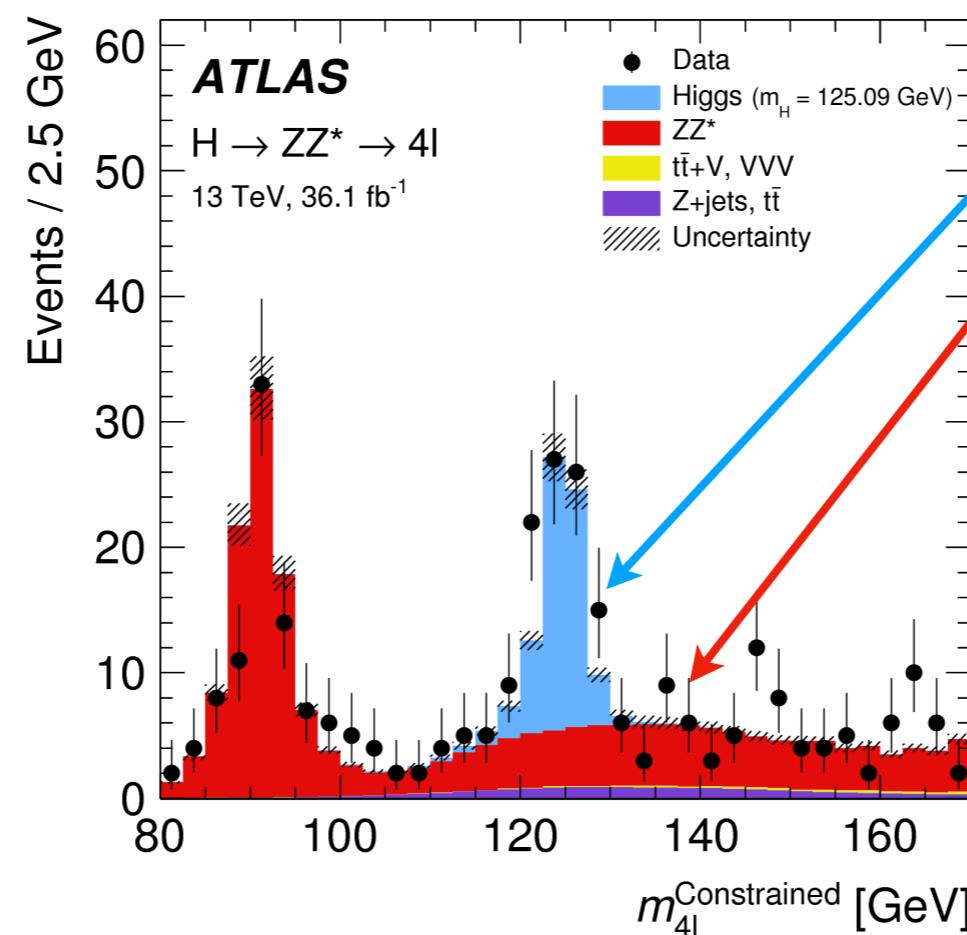
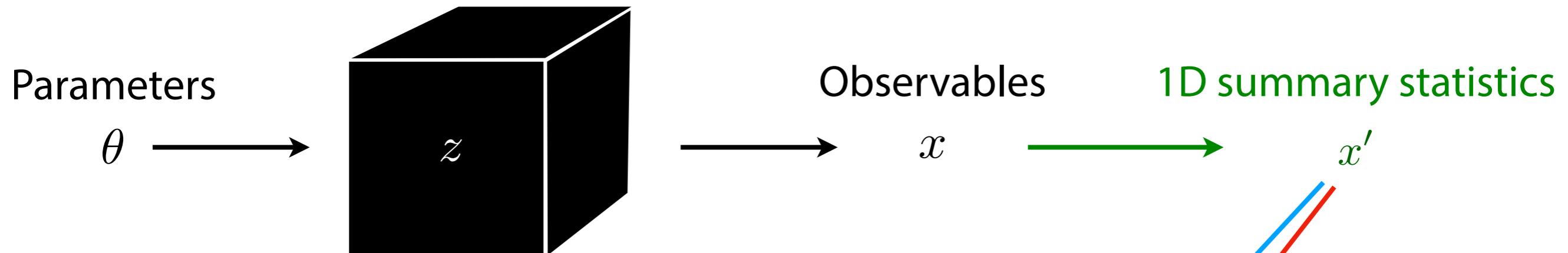
Source: Morris, et al. The Relationship Between Concurrent Partnerships and HIV Transmission, 2008. See [www.aidstar-one.com/](http://www.aidstar-one.com/).

[Source: Morris et al 2008]

# Why has that not stopped us so far?



# Why has that not stopped us so far?



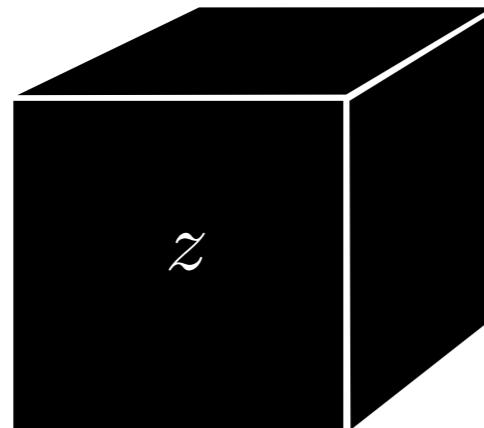
Run simulator for  
different  $\theta$ ,  
fill histograms

$x'$

# Why has that not stopped us so far?

Parameters

$$\theta \longrightarrow$$



Observables

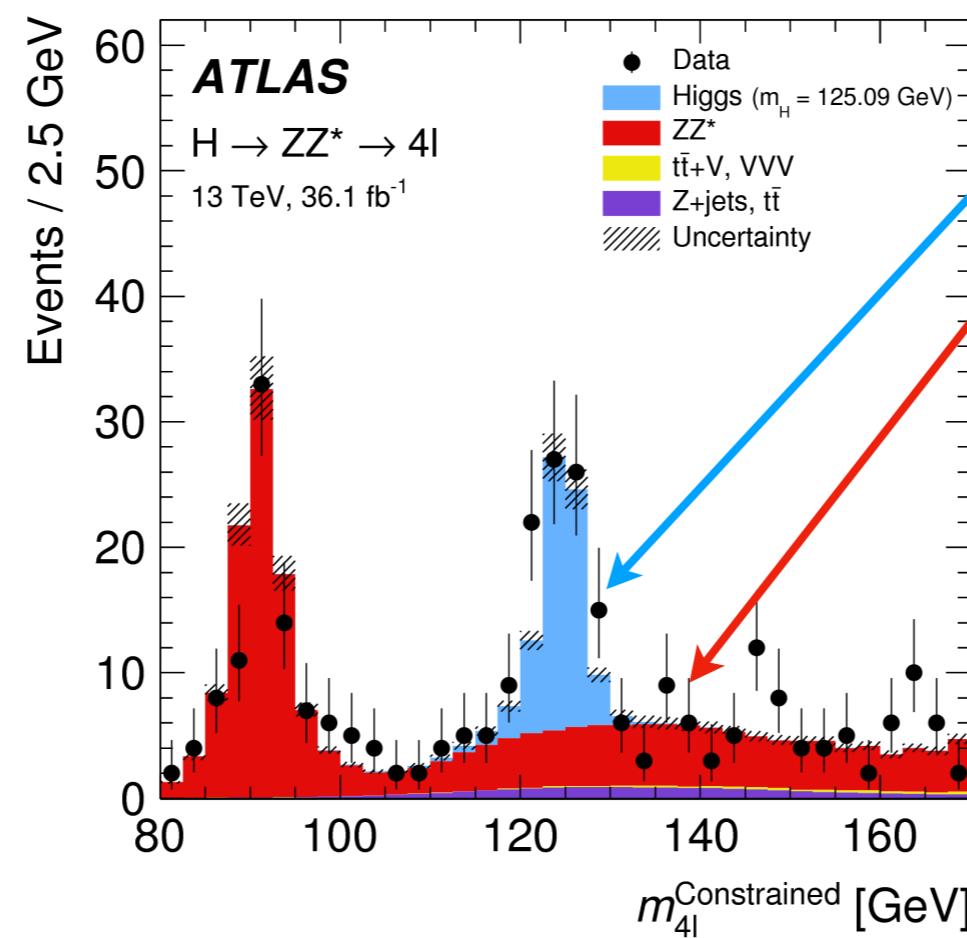
$$\longrightarrow$$

$$x$$

1D summary statistics

$$x'$$

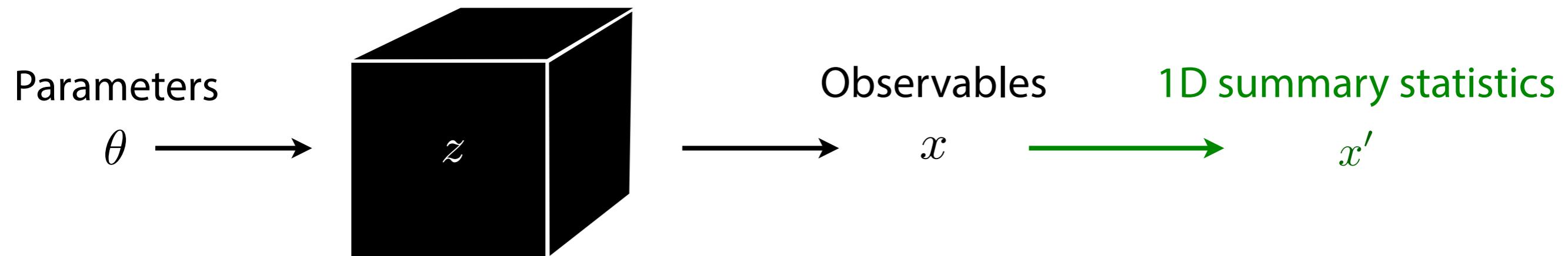
$$\hat{p}(x|\theta) = p(x'|\theta) =$$



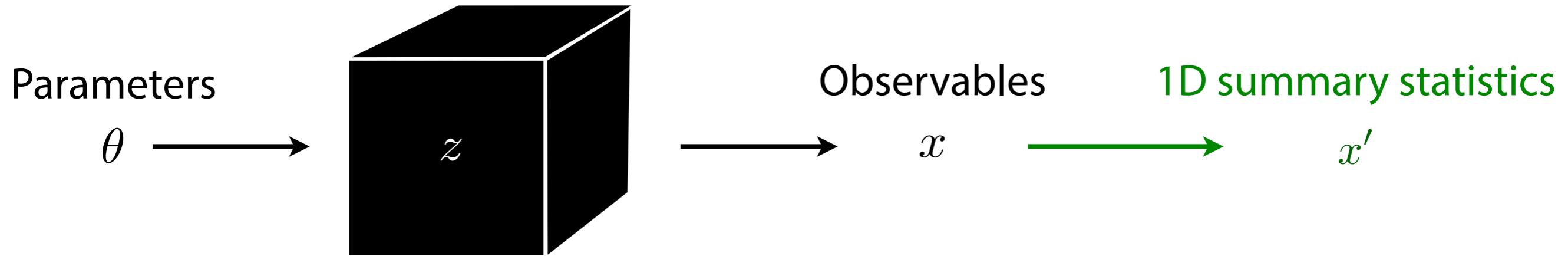
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$$x'$$

# Does the histogram method scale?



# Does the histogram method scale?

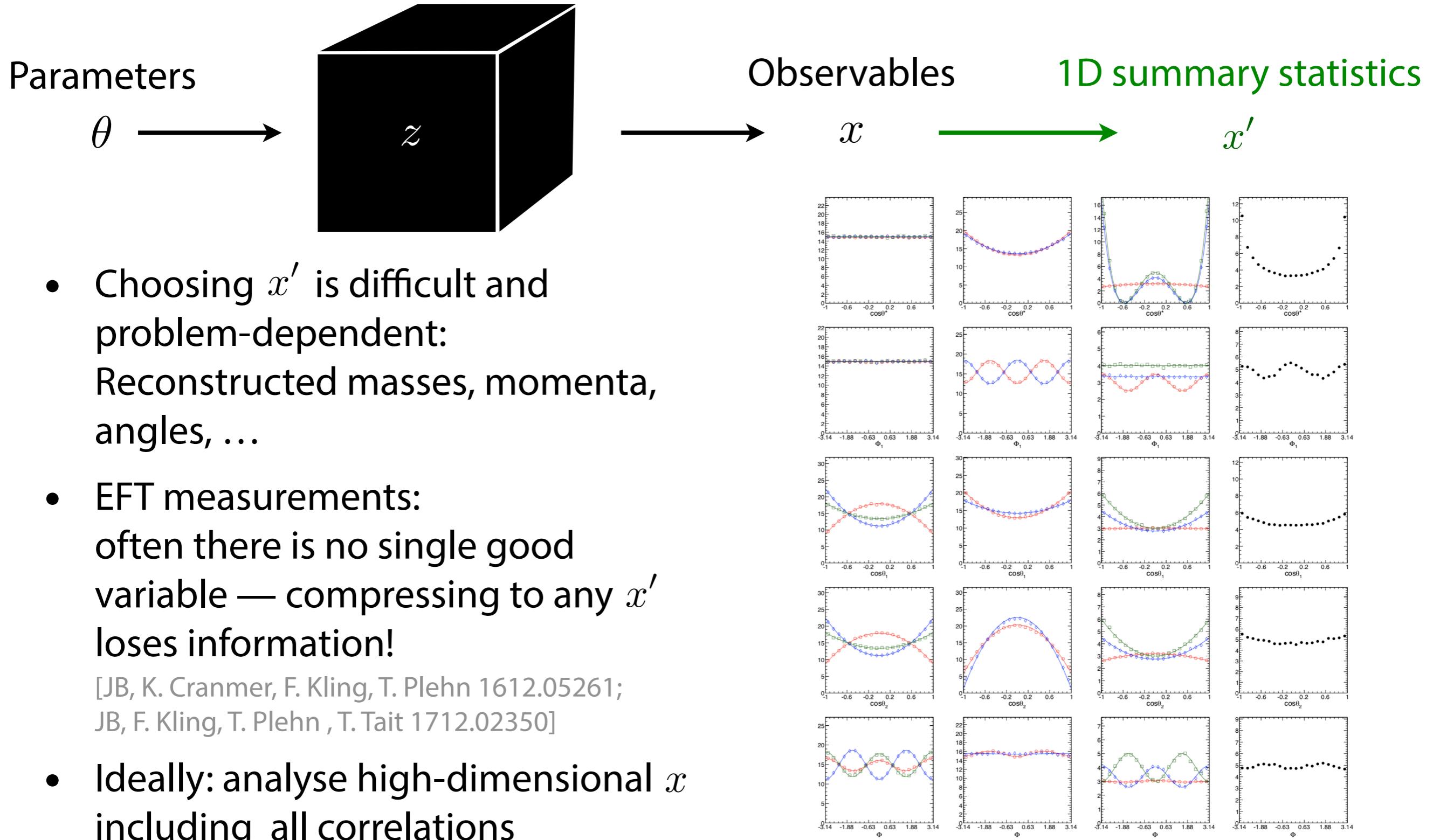


- Choosing  $x'$  is difficult and problem-dependent:  
Reconstructed masses, momenta,  
angles, ...
- EFT measurements:  
often there is no single good  
variable — compressing to any  $x'$   
loses information!

[JB, K. Cranmer, F. Kling, T. Plehn 1612.05261;  
JB, F. Kling, T. Plehn , T. Tait 1712.02350]

- Ideally: analyse high-dimensional  $x$   
including all correlations

# Does the histogram method scale?

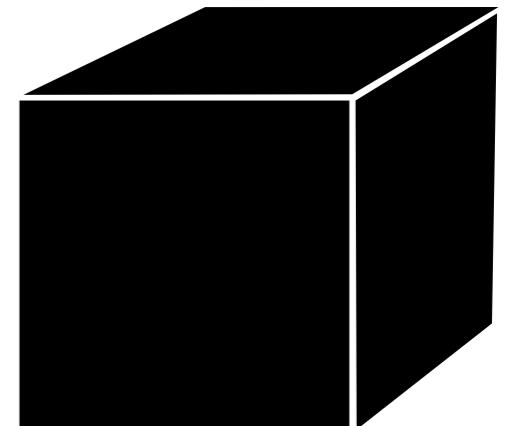


[Source: Bolognesi et al. 1208.4018

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# An incomplete list of established methods

- Histograms of individual kinematic variables  
Summary statistics
- Approximate Bayesian Computation  
Summary statistics
- Machine Learning techniques  
CARL, autoregressive models, normalizing flows, ...
- Matrix Element Method / Optimal Observables  
Neglect or approximate shower + detector,  
explicitly calculate integral



$$\hat{p}(x|\theta) = \int dz_p \tilde{p}(x|z_p) p(z_p|\theta)$$

**How can we get more  
information out of the  
data?**

# Learning Effective Theories, Effectively

- New analysis techniques
  - Based on high-dimensional  $x$   
(no summary statistics necessary!)
  - Leverages structure of particle physics processes + power of machine learning
- Appropriate for large-scale EFT measurements at LHC
  - Scales well to many observables and parameters
  - Supports full shower & detector simulation

# Learning Effective Theories, Effectively

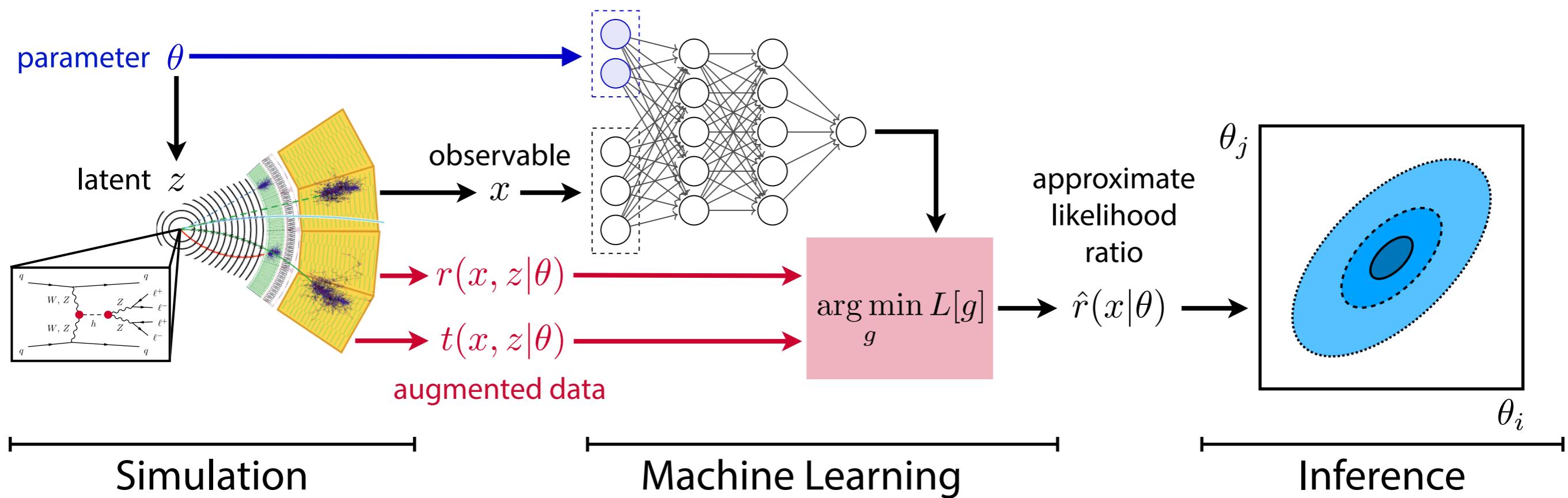
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- Appropriate for large-scale EFT measurements at LHC
  - Scales well to many observables and parameters
  - Supports full shower & detector simulation
- Goal: estimator  $\hat{r}(x|\theta_0, \theta_1)$  for likelihood ratio

$$r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

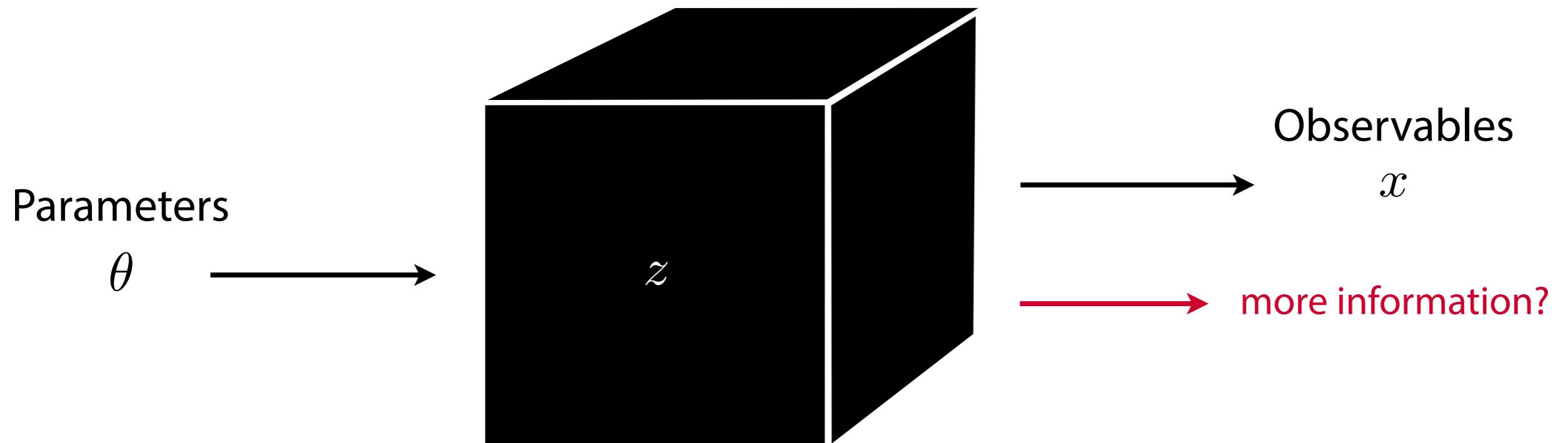
↑  
observables  
↓  
theory parameters

("How much more likely is the observation  $x$  assuming  $\theta_0$  compared to  $\theta_1$ ?")

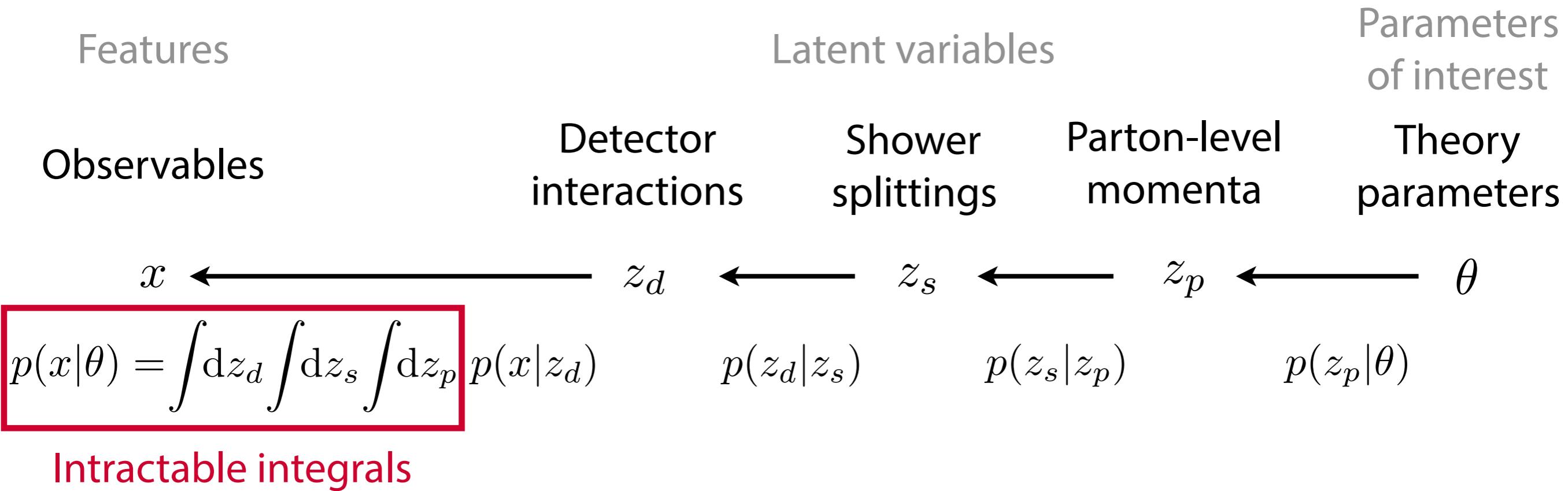
# Sneak Preview



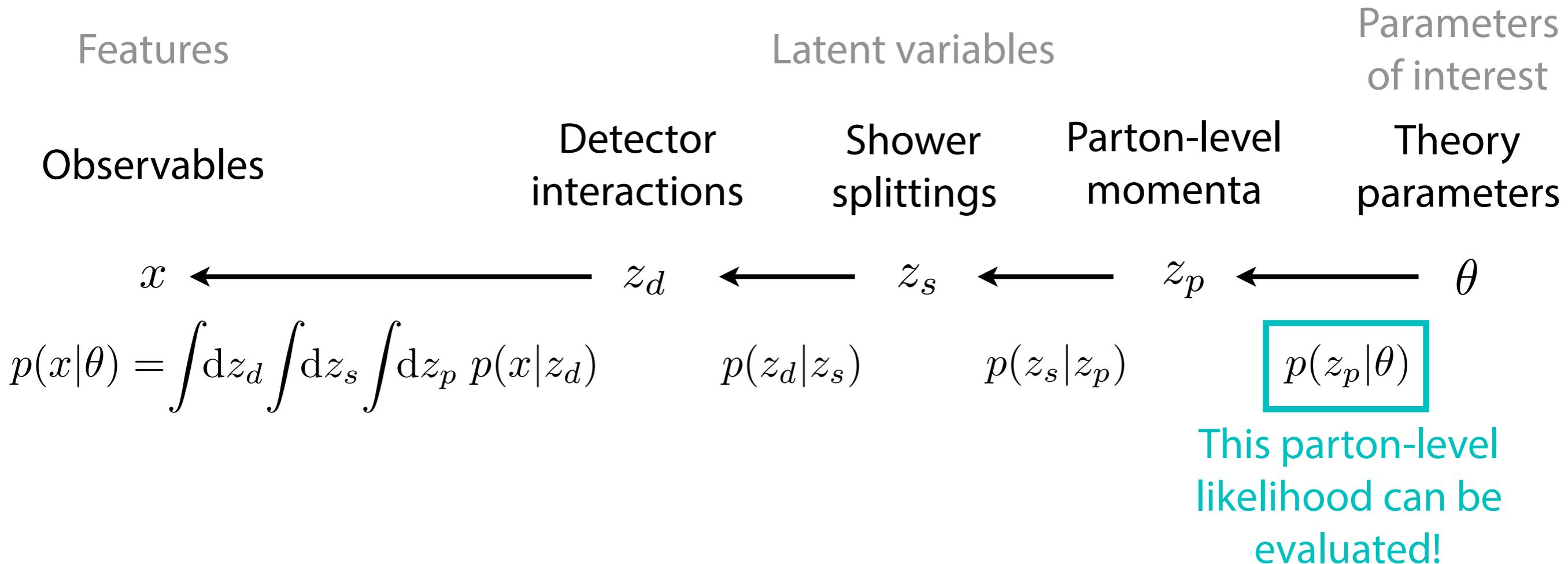
# Opening the black box



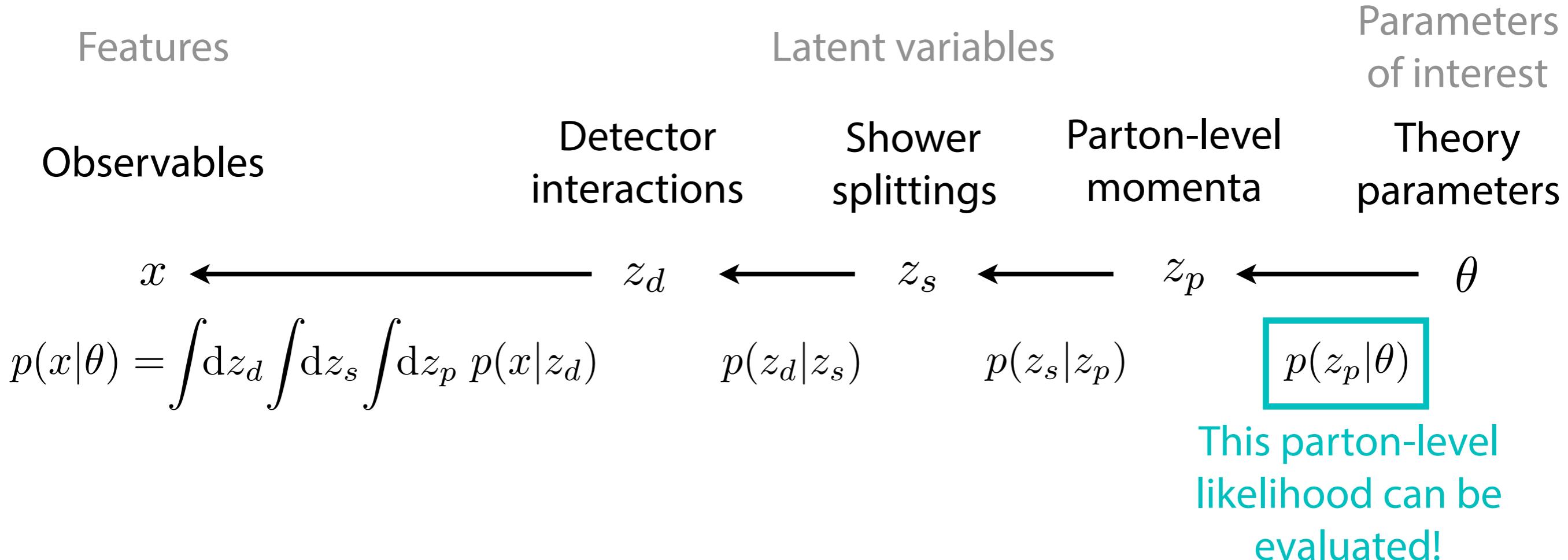
# Opening the black box



# Opening the black box



# Opening the black box



⇒ We can calculate the “joint” likelihood ratio conditional on a specific evolution:

$$r(x, z | \theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p | \theta_0)}{p(x, z_d, z_s, z_p | \theta_1)} = \frac{p(x|z_d)}{p(x|z_d)} \frac{p(z_d|z_s)}{p(z_d|z_s)} \frac{p(z_s|z_p)}{p(z_s|z_p)}$$

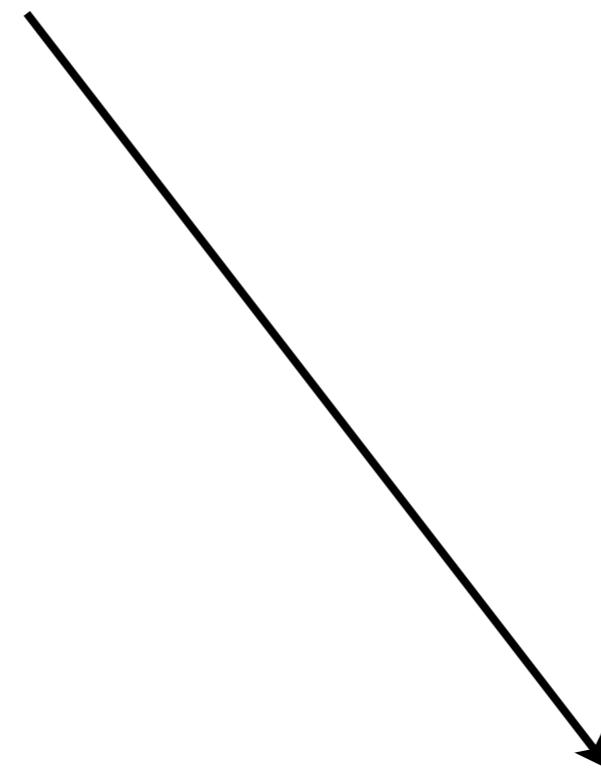
$$\frac{p(z_p|\theta_0)}{p(z_p|\theta_1)}$$

(“How much more likely is this specific evolution assuming  $\theta_0$  compared to  $\theta_1$ ? ”)

# From joint likelihood ratio to likelihood ratio

We have **joint likelihood ratio**

$$r(x, z|\theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p|\theta_0)}{p(x, z_d, z_s, z_p|\theta_1)}$$



We want **likelihood ratio**

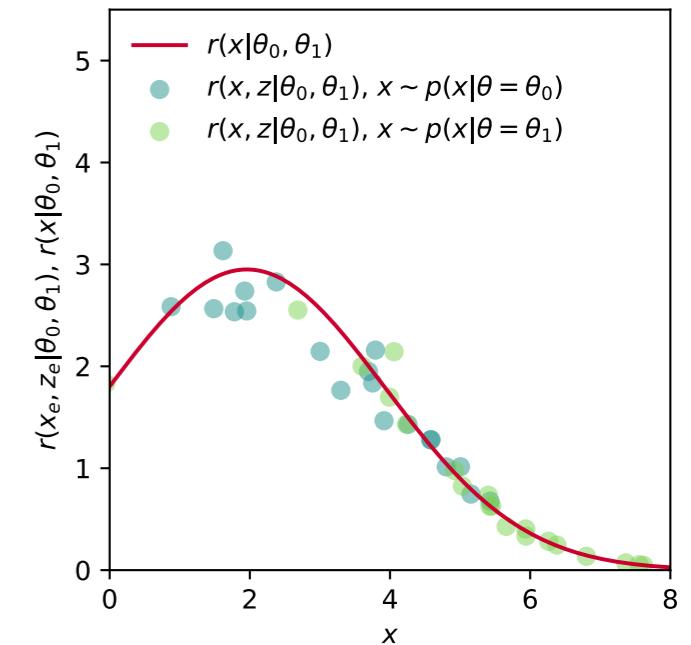
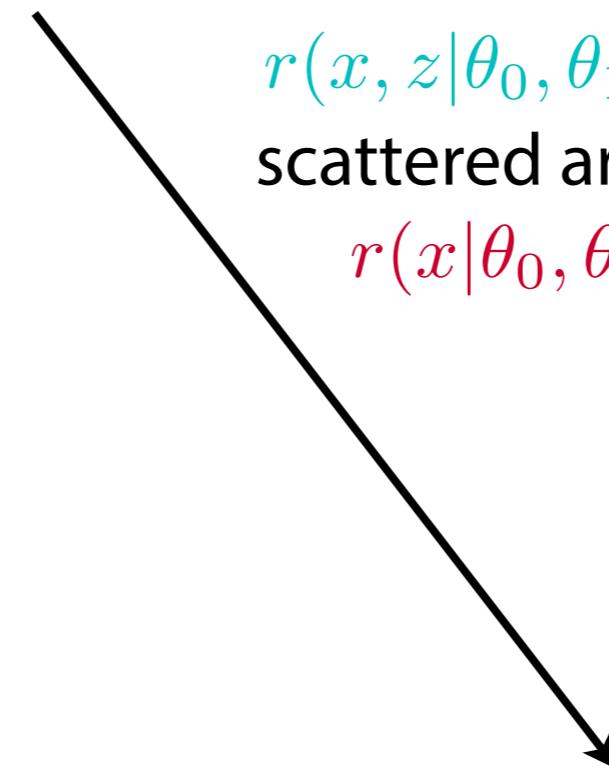
$$r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

# From joint likelihood ratio to likelihood ratio

We have **joint likelihood ratio**

$$r(x, z|\theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p|\theta_0)}{p(x, z_d, z_s, z_p|\theta_1)}$$

$r(x, z|\theta_0, \theta_1)$  are  
scattered around  
 $r(x|\theta_0, \theta_1)$



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$$r(x|\theta_0, \theta_1) \equiv \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

# From joint likelihood ratio to likelihood ratio

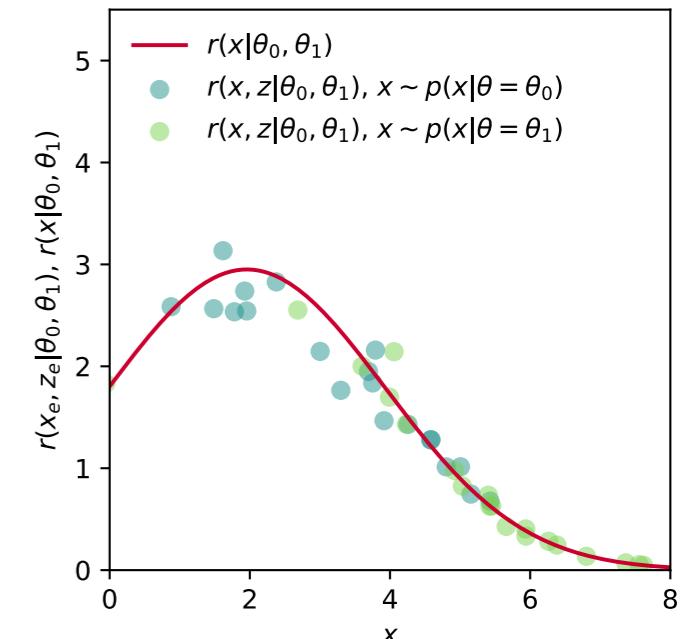
We have **joint likelihood ratio**

$$r(x, z|\theta_0, \theta_1) \equiv \frac{p(x, z_d, z_s, z_p|\theta_0)}{p(x, z_d, z_s, z_p|\theta_1)}$$

With  $r(x, z|\theta_0, \theta_1)$ ,  
we can define a functional  $L[g]$  that is  
minimized by  $r(x|\theta_0, \theta_1)$ :

$$\hat{r}(x|\theta_0, \theta_1) = \arg \min_g L[g] \rightarrow r(x|\theta_0, \theta_1)$$

$r(x, z|\theta_0, \theta_1)$  are  
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 $r(x|\theta_0, \theta_1)$

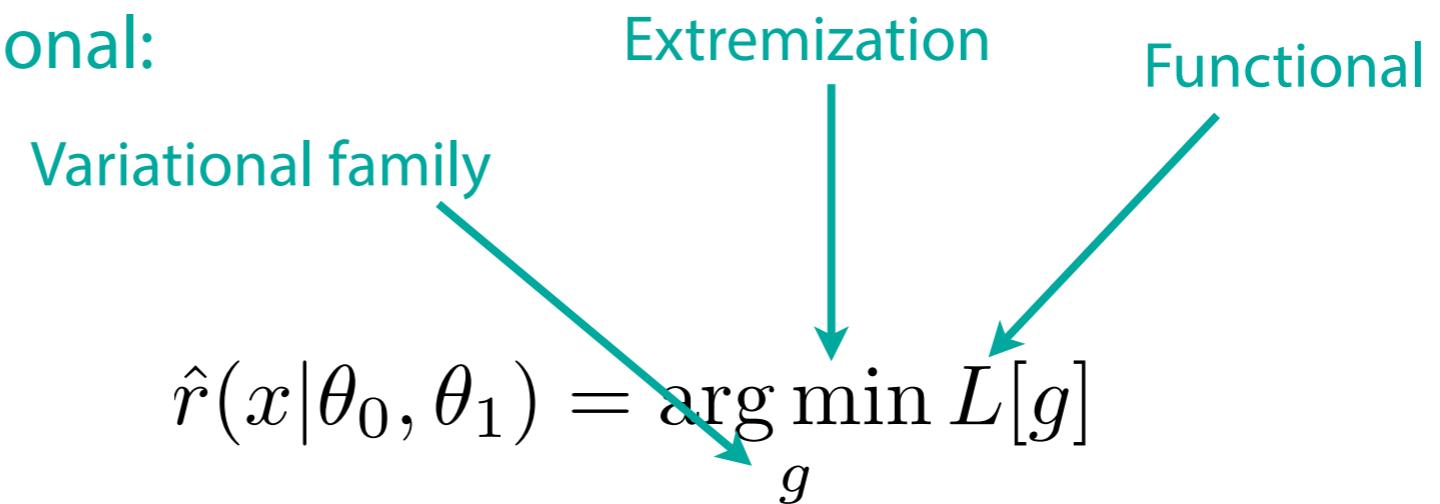


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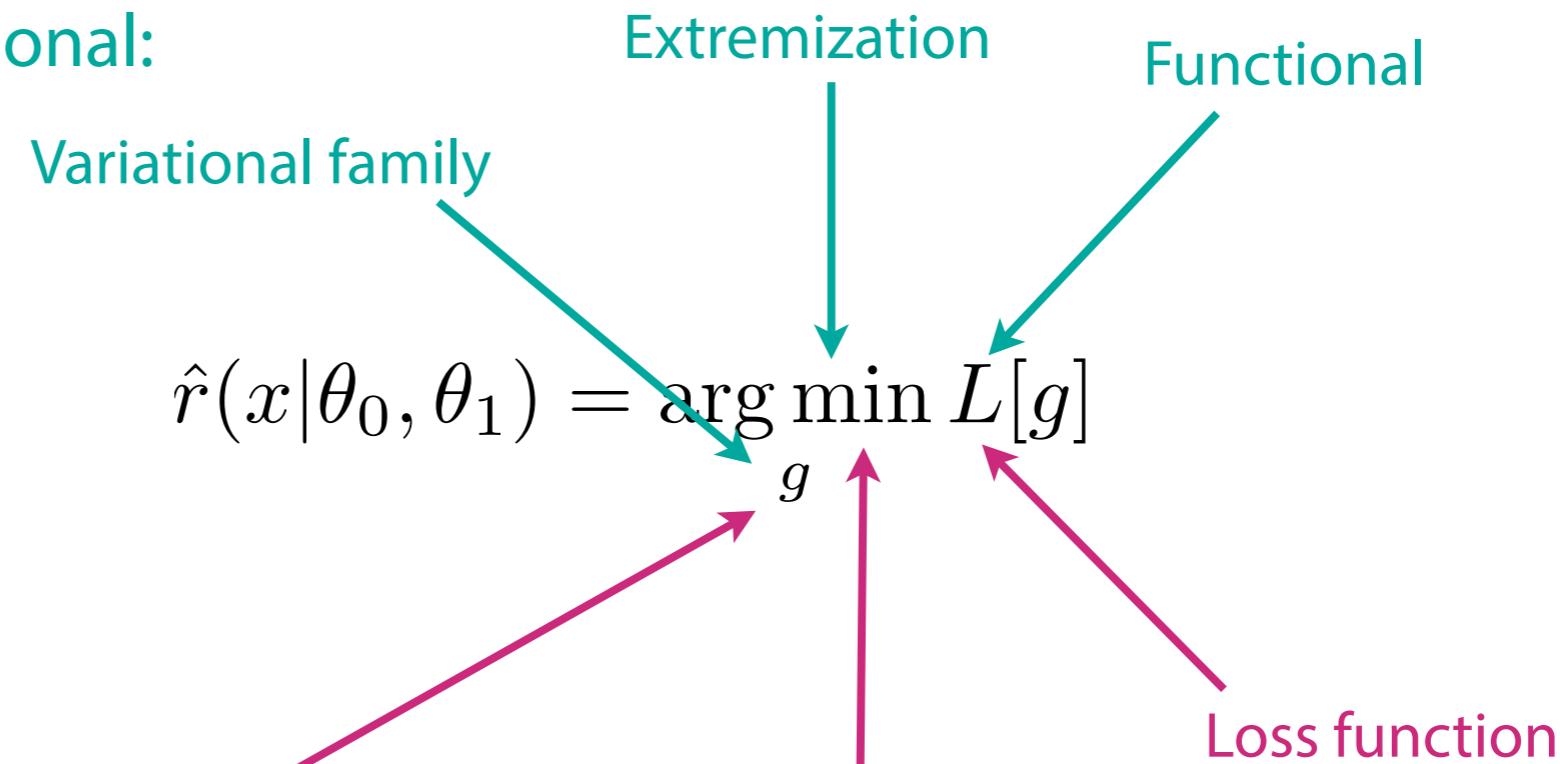
# Machine Learning = applied calculus of variations

Need to minimize a functional:

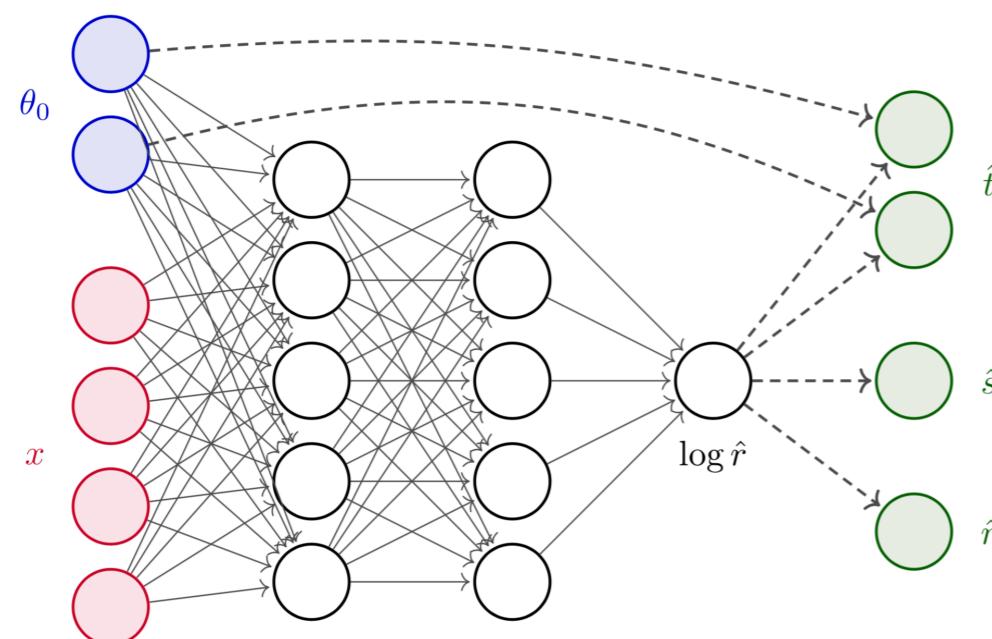


# Machine Learning = applied calculus of variations

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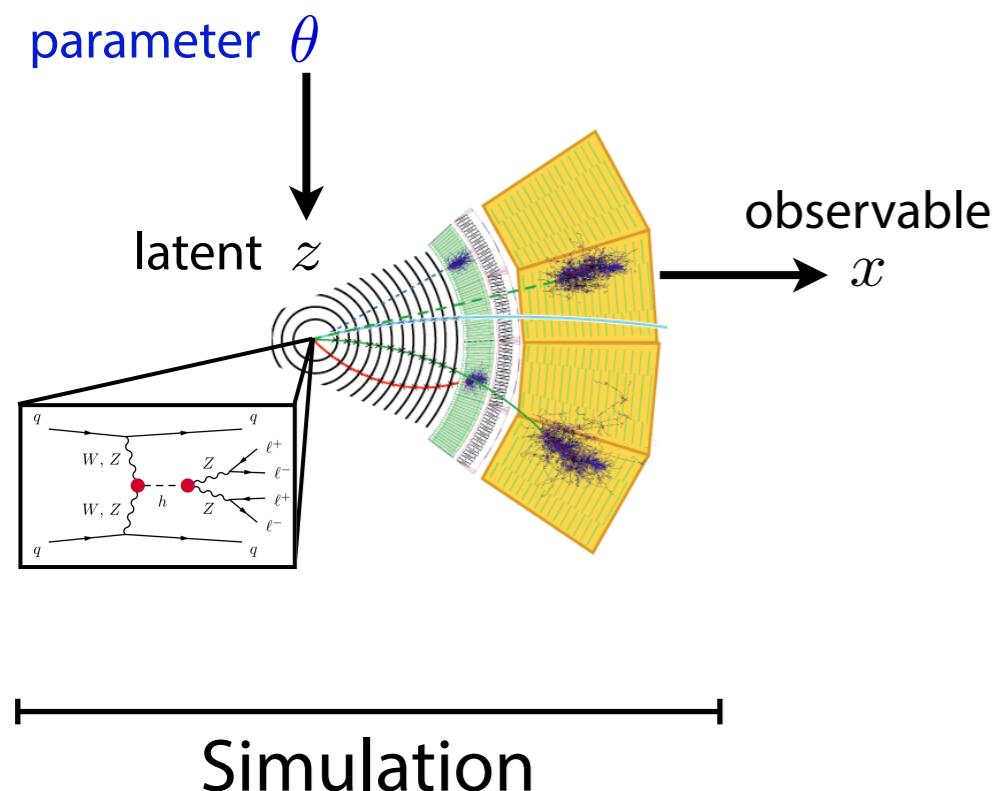


This is exactly what  
Machine Learning does!

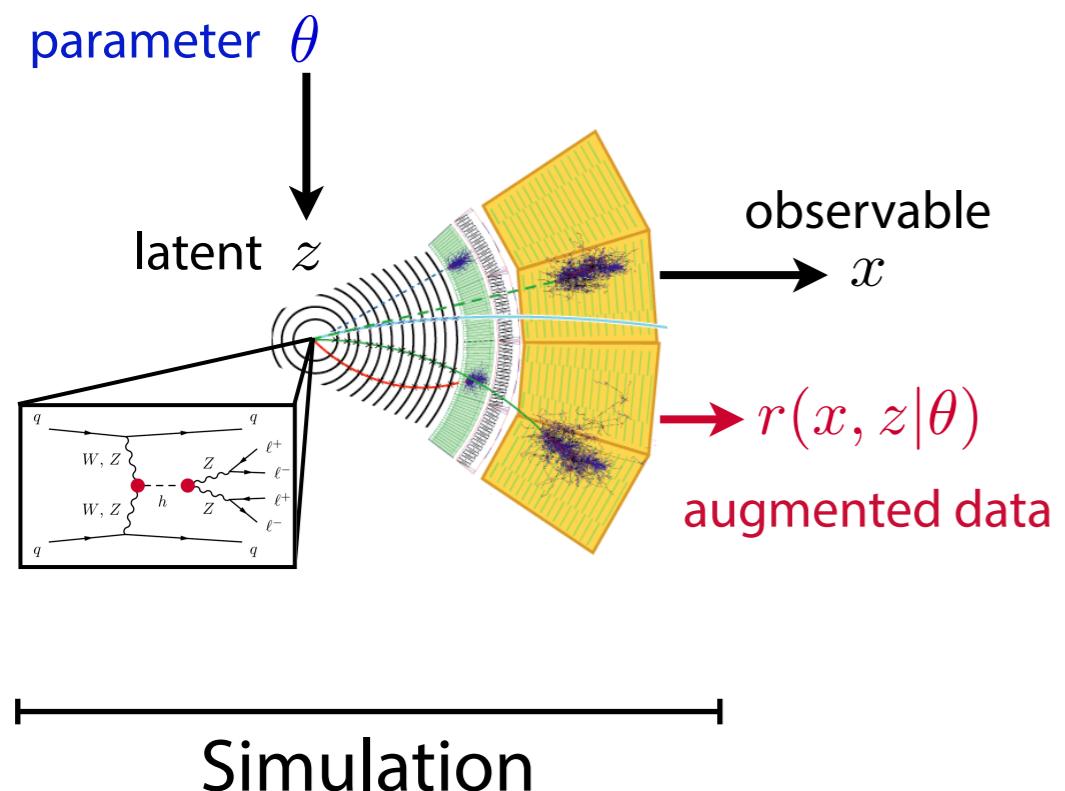


$\Rightarrow$  We implement  $\hat{r}(x|\theta_0, \theta_1)$   
as a Neural Network trained on the  
data available from the simulator

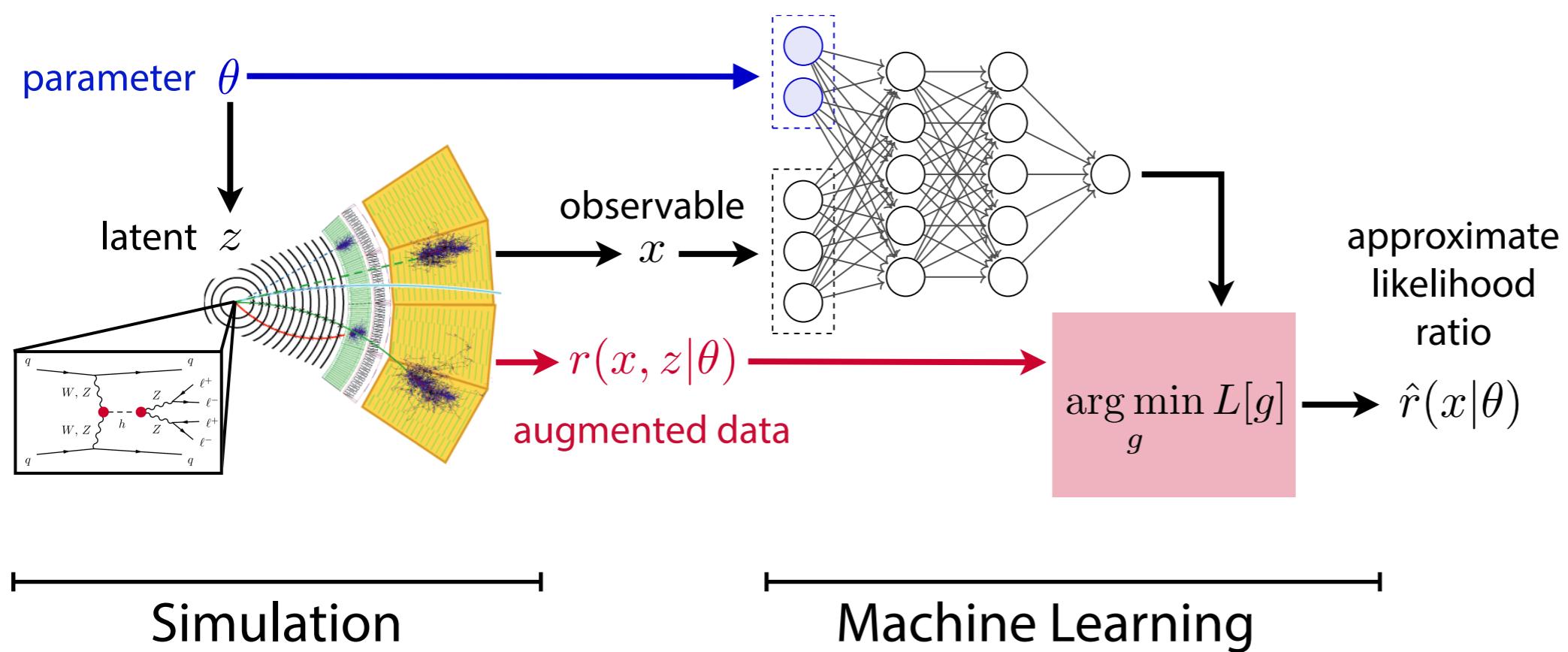
# What we have so far



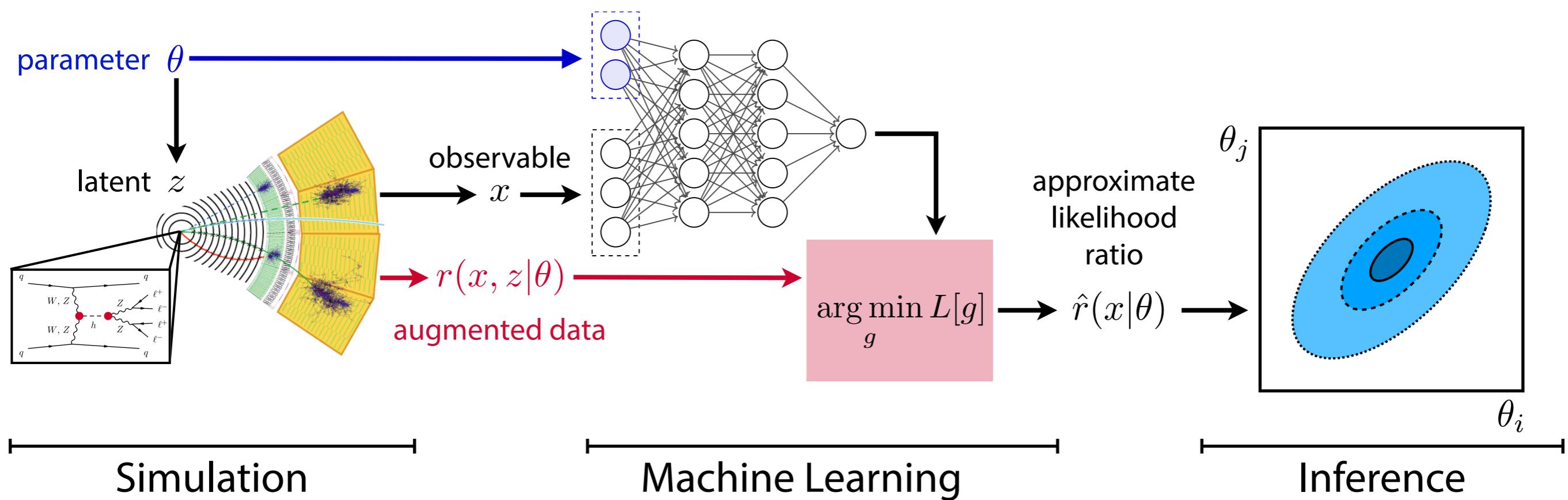
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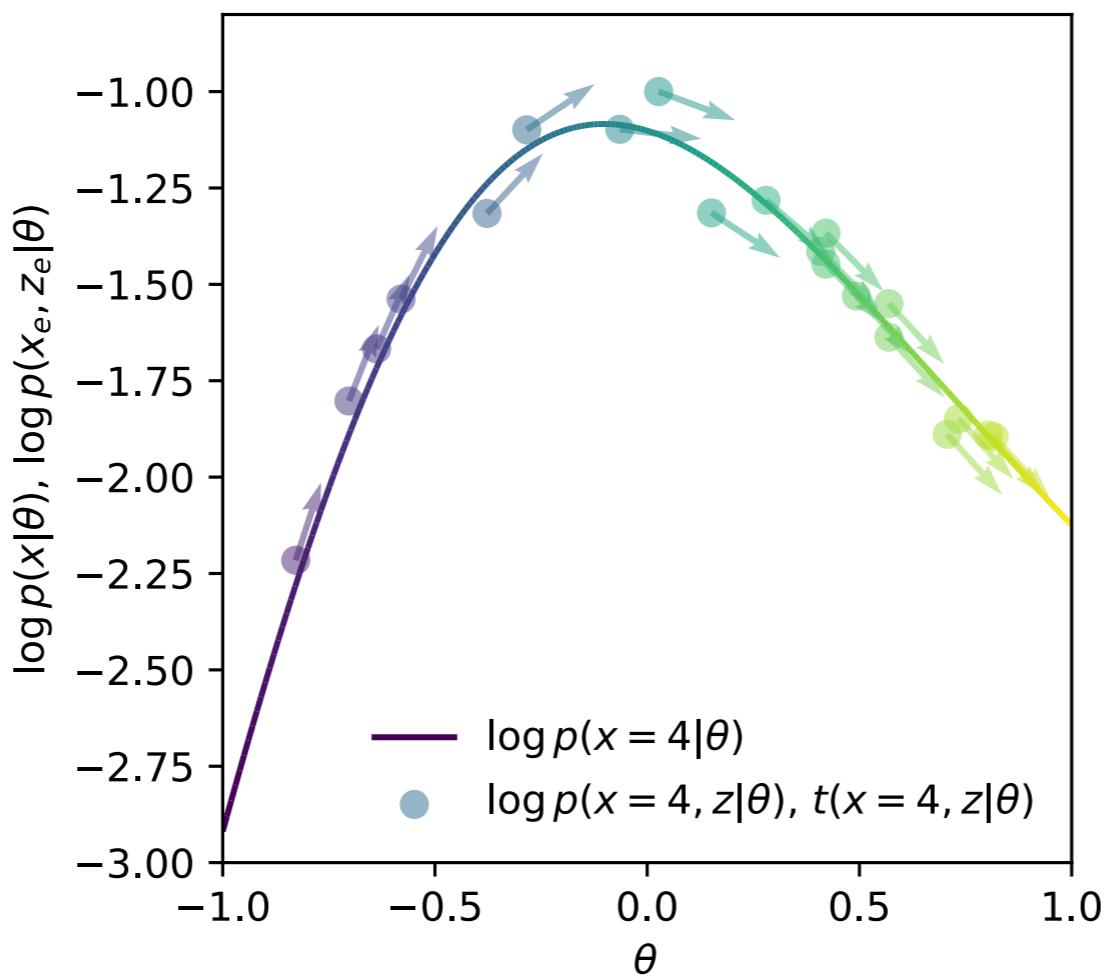
# What we have so far



# One more piece: the score

Score  $t(x|\theta_0) \equiv \nabla_{\theta} \log p(x|\theta) \Big|_{\theta_0}$  : tangent vectors of likelihood function

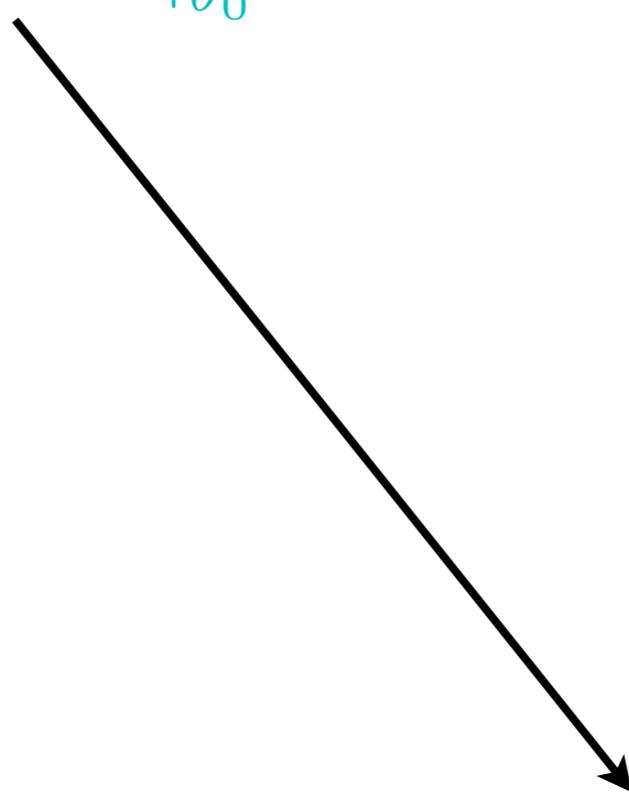
("How much more likely does the observation  $x$  get when I move in  $\theta$  space?")



# Learning the score

Similar to the joint likelihood ratio,  
we can calculate the **joint score**

$$t(x, z|\theta_0) \equiv \nabla_{\theta} \log p(x, z_d, z_s, z|\theta) \Big|_{\theta_0}$$



We want **score**

$$t(x|\theta_0) \equiv \nabla_{\theta} \log p(x|\theta) \Big|_{\theta_0}$$

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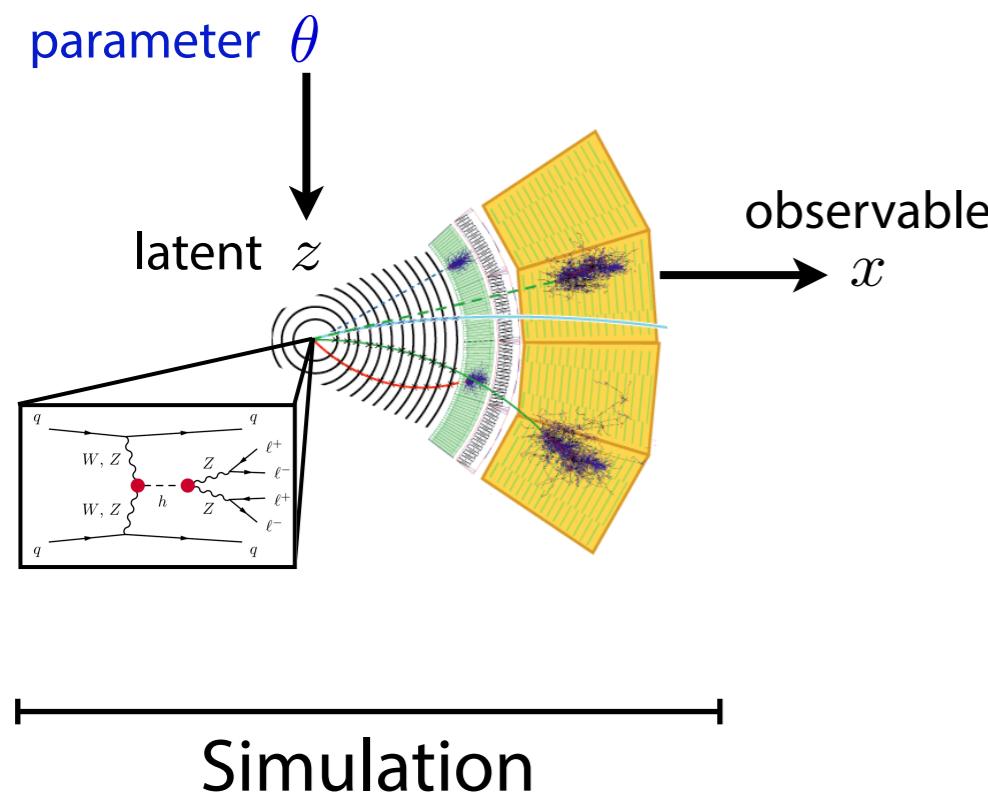
From  $t(x, z|\theta_0)$ ,  
we can learn  $\hat{t}(x|\theta_0)$ :

$$\hat{t}(x|\theta_0) = \arg \min_g L[g] \rightarrow t(x|\theta_0)$$

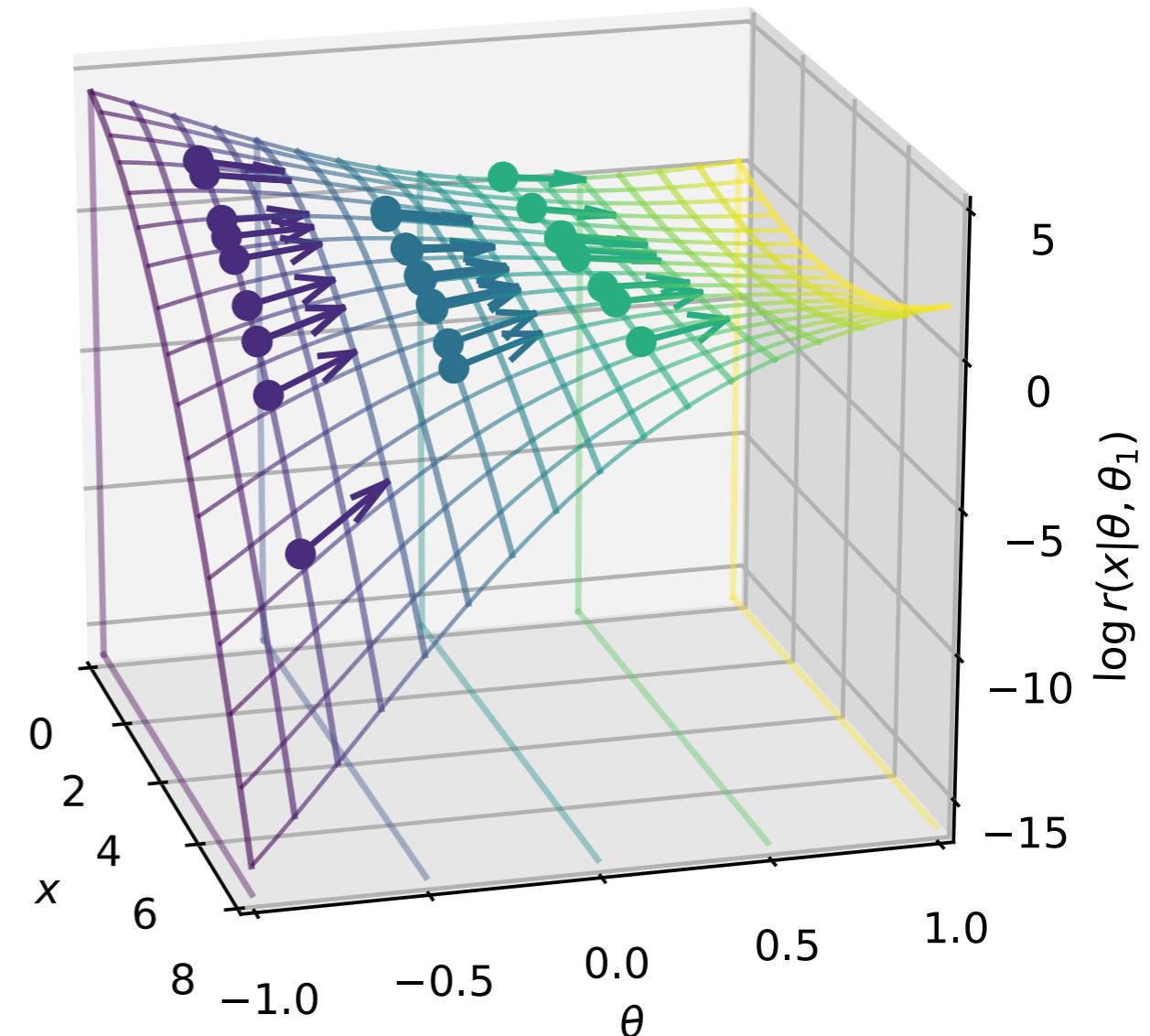
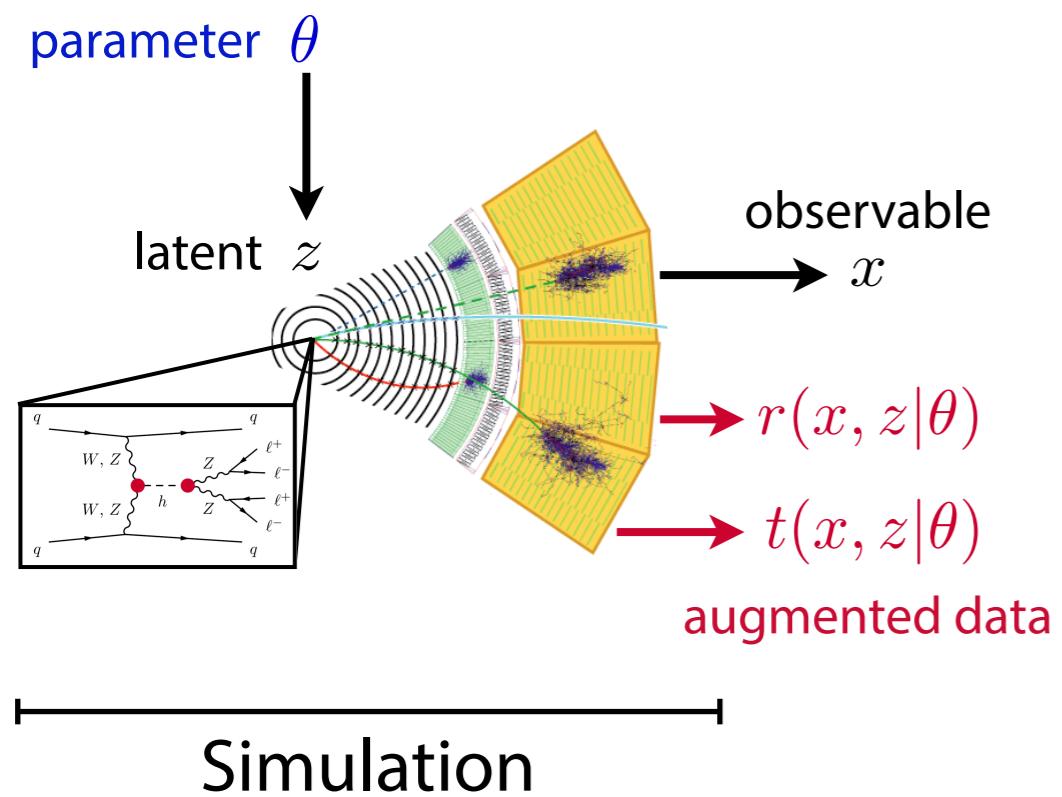
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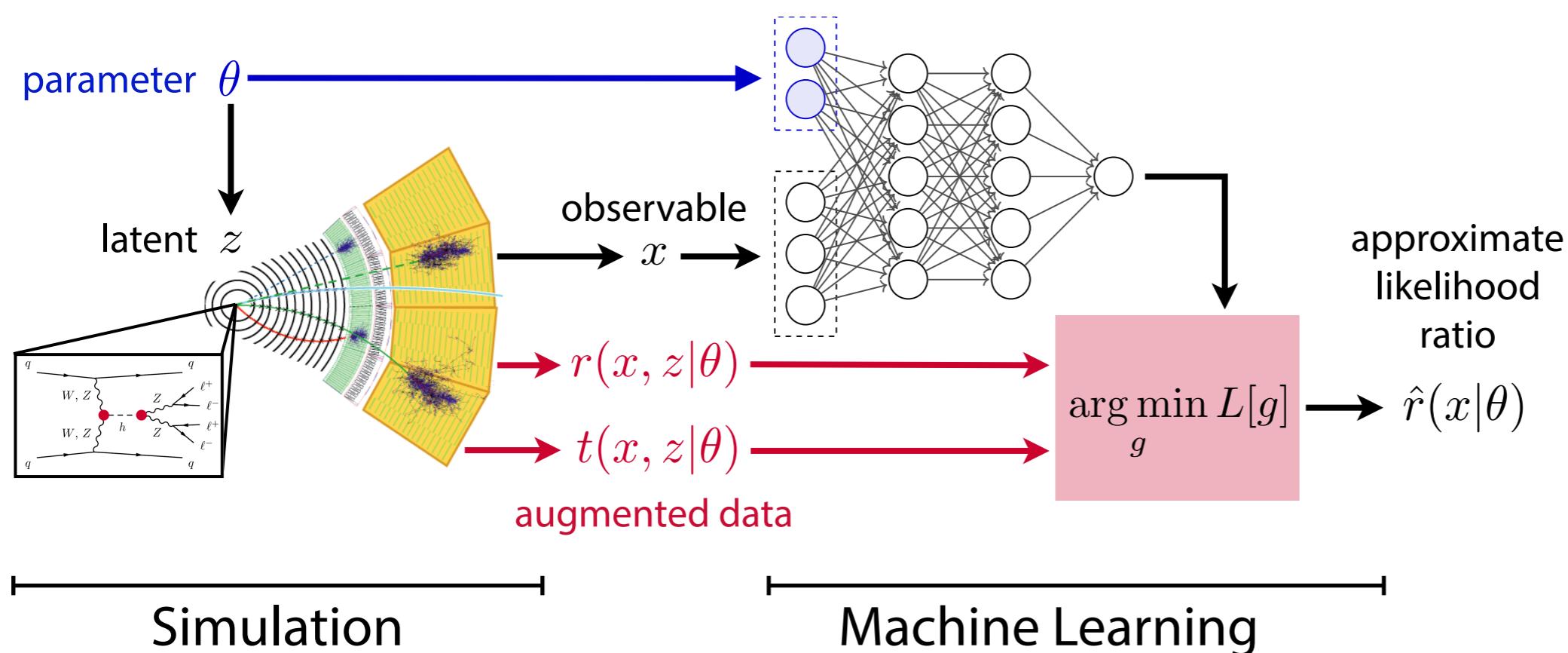
# RASCAL (Ratio And SCore Approximate Likelihood ratio)



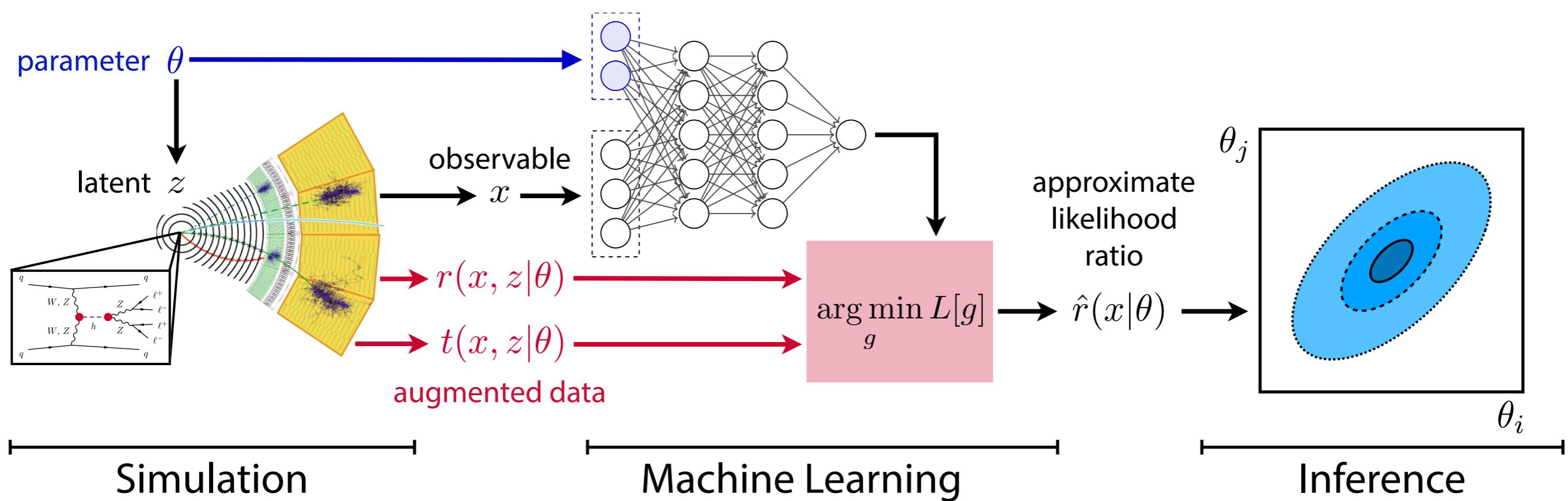
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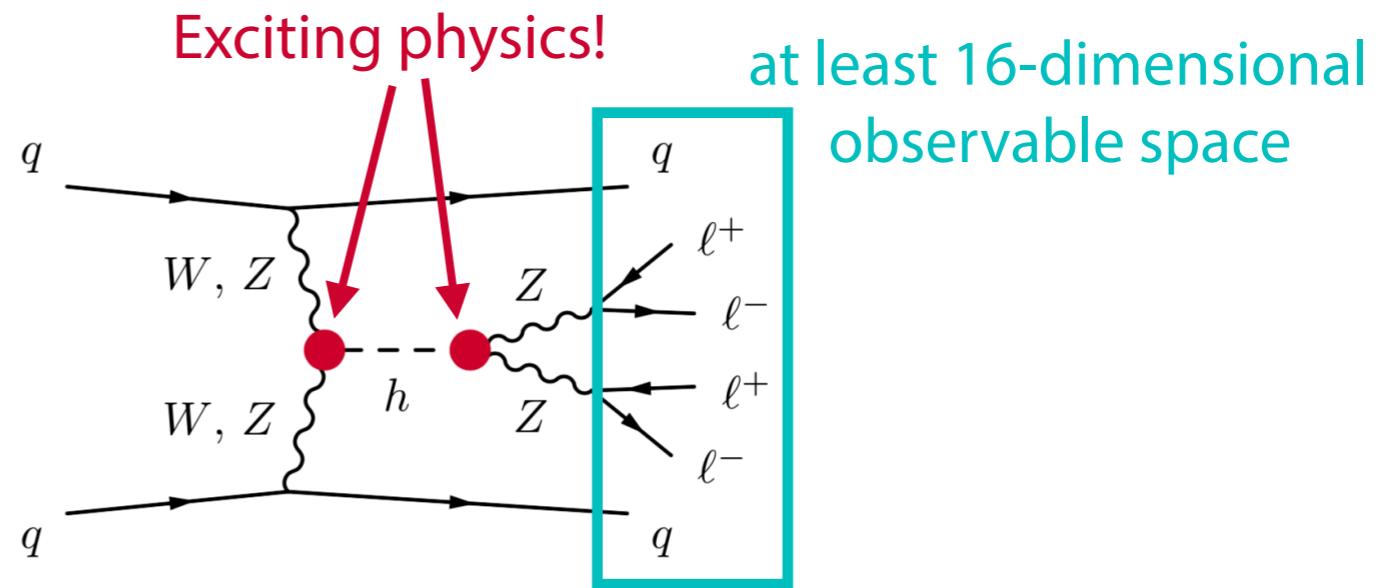
# RASCAL (Ratio And SCore Approximate Likelihood ratio)



# How well does it work?

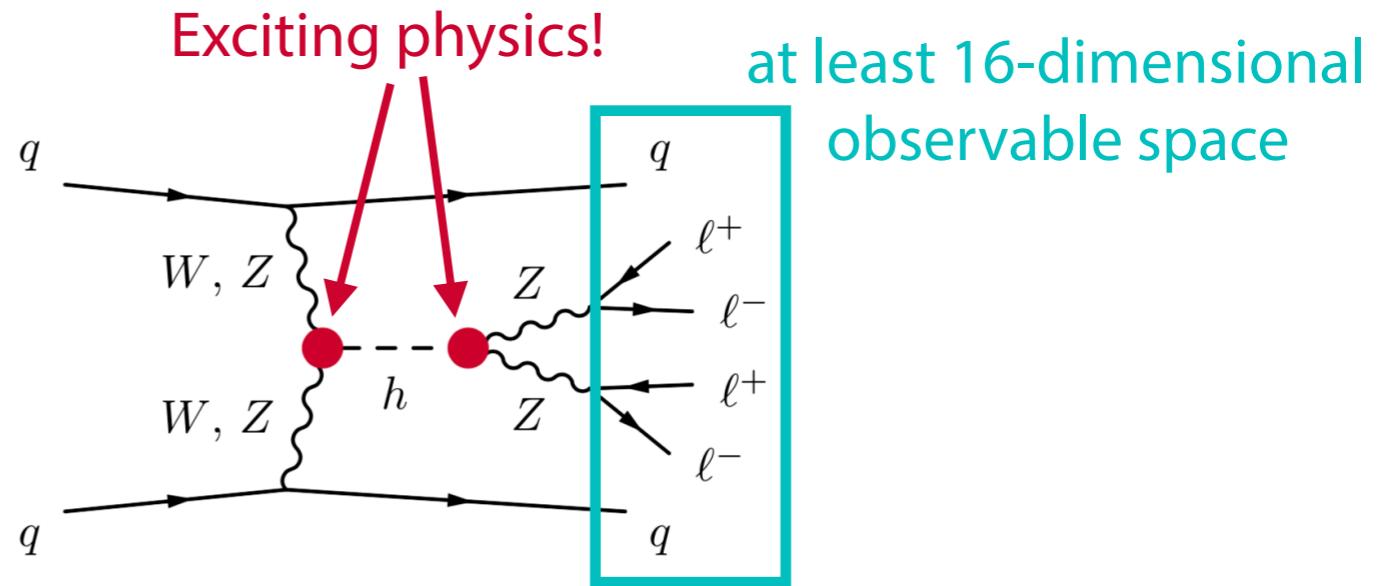
# Proof of concept

- Higgs production in weak boson fusion:



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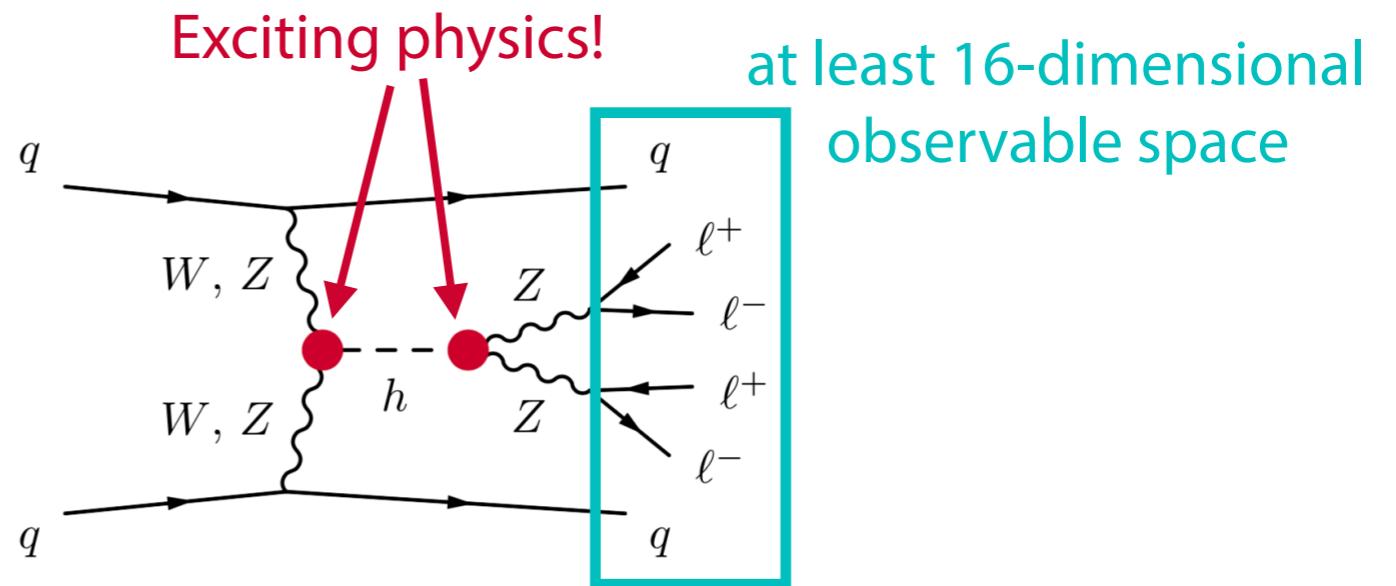


- Goal: constraints on two theory parameters

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \boxed{\frac{f_W}{\Lambda^2}} \underbrace{\frac{ig}{2} (D^\mu \phi)^\dagger \sigma^a D^\nu \phi W_{\mu\nu}^a}_{\mathcal{O}_W} - \boxed{\frac{f_{WW}}{\Lambda^2}} \underbrace{\frac{g^2}{4} (\phi^\dagger \phi) W_{\mu\nu}^a W^{\mu\nu a}}_{\mathcal{O}_{WW}}$$

# Proof of concept

- Higgs production in weak boson fusion:



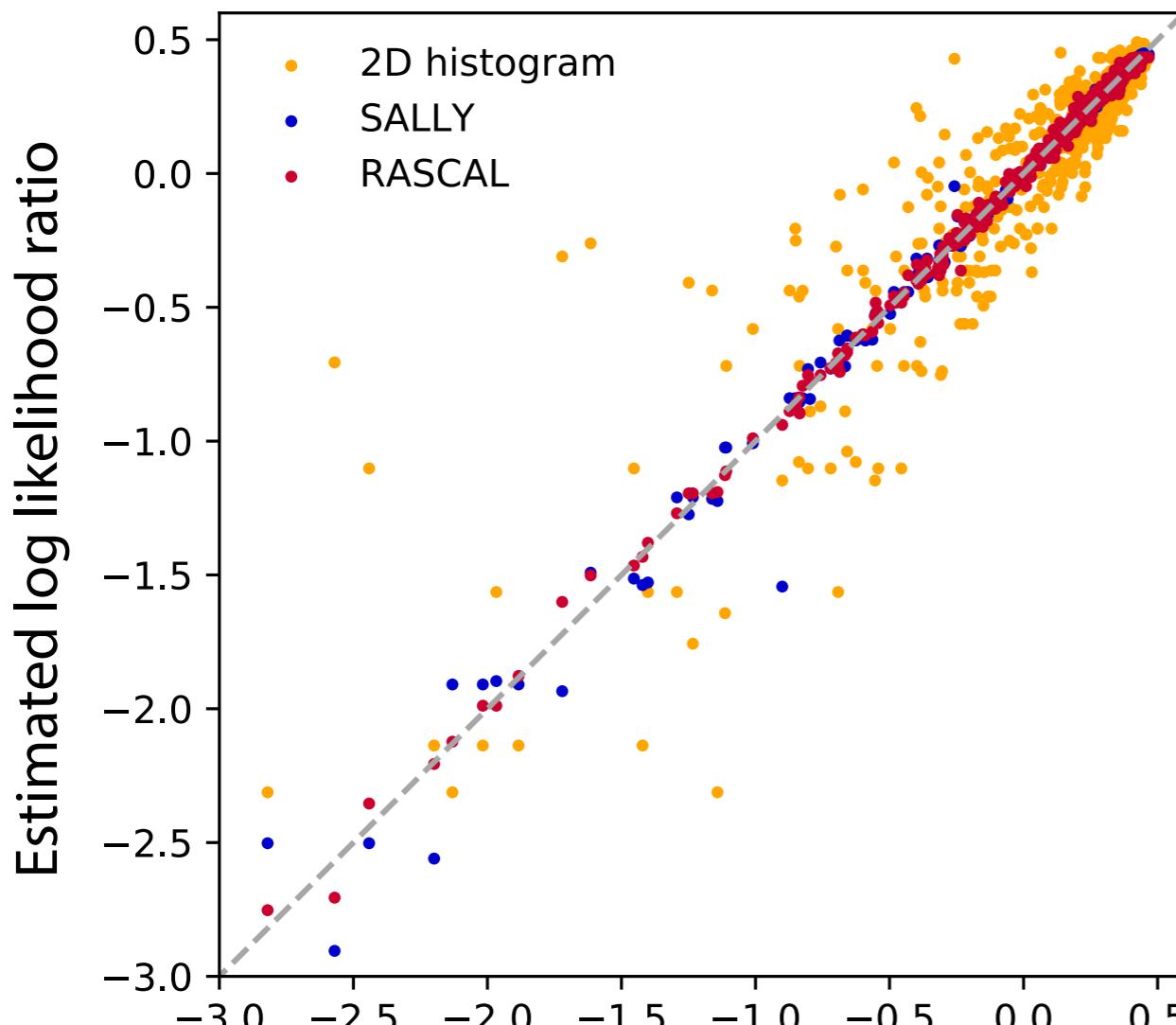
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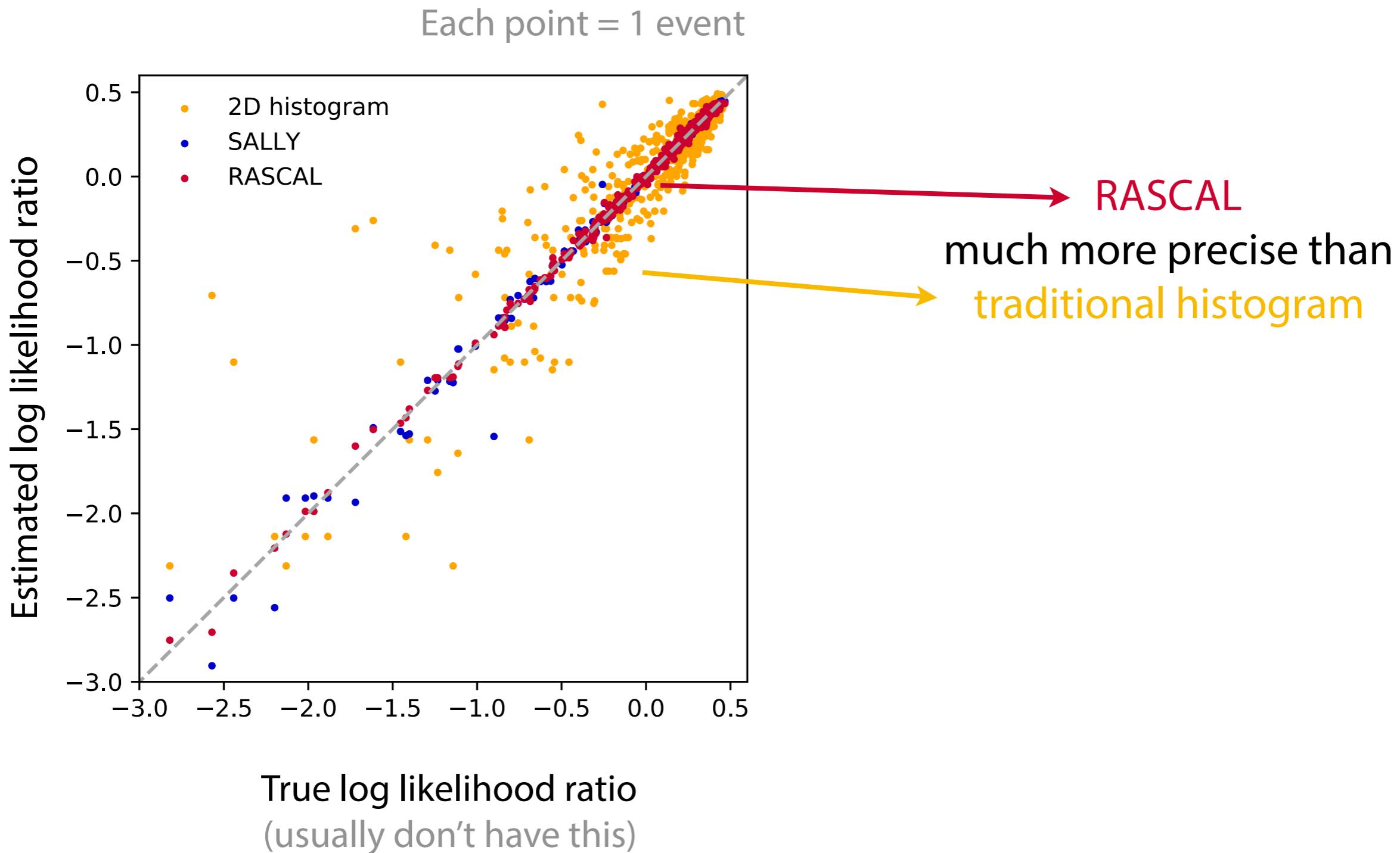
- Two setups:
  - Simplified setup in which we can compare to true likelihood
  - “Realistic” simulation with approximate detector effects
- Simulation: MadGraph [J. Alwall et al. 1405.0301] + MadMax [K. Cranmer, T. Plehn hep-ph/0605268; T. Plehn, P. Schichtel, D. Wiegand 1311.2591]

# Precise likelihood ratio estimates

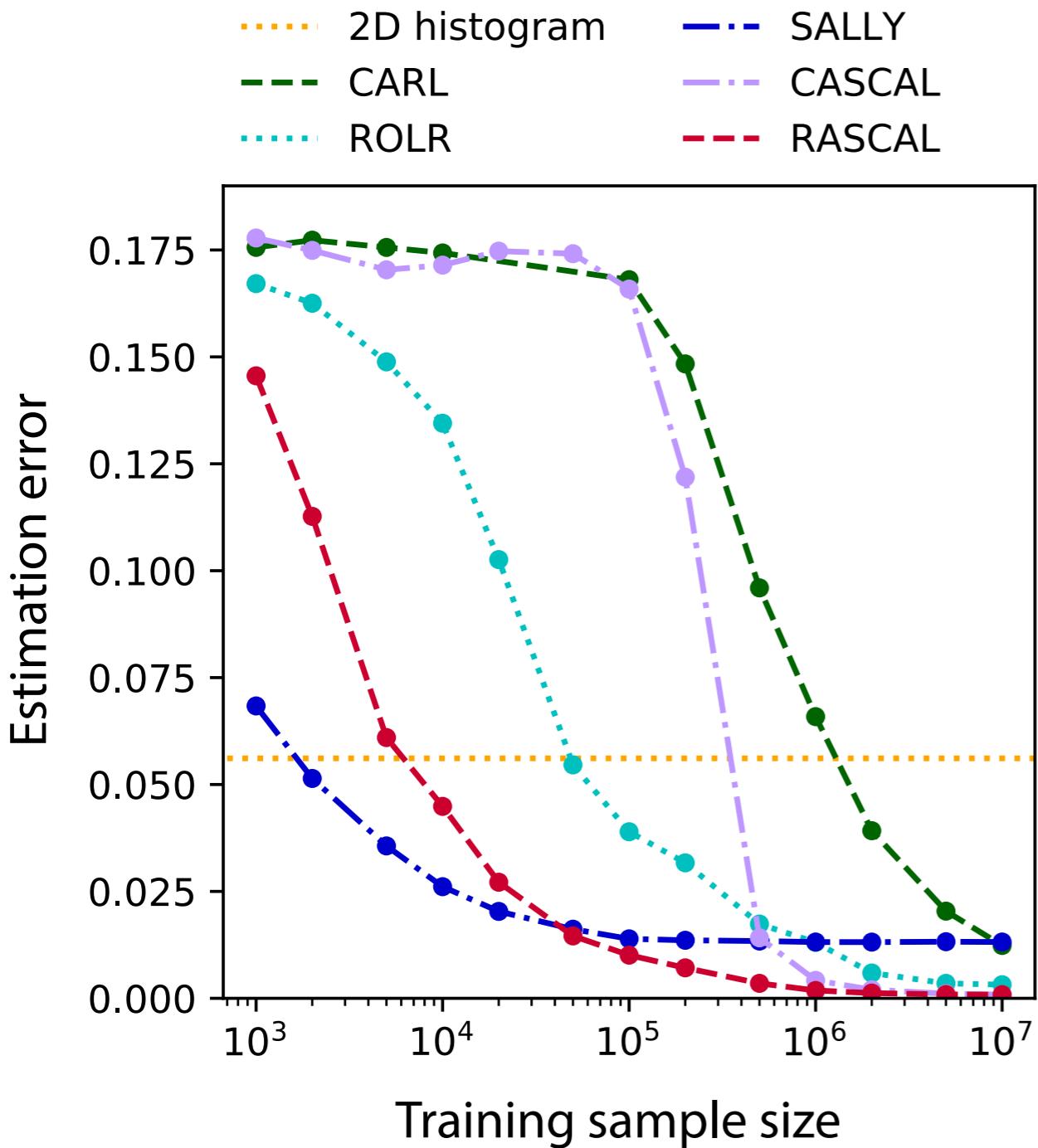
Each point = 1 event



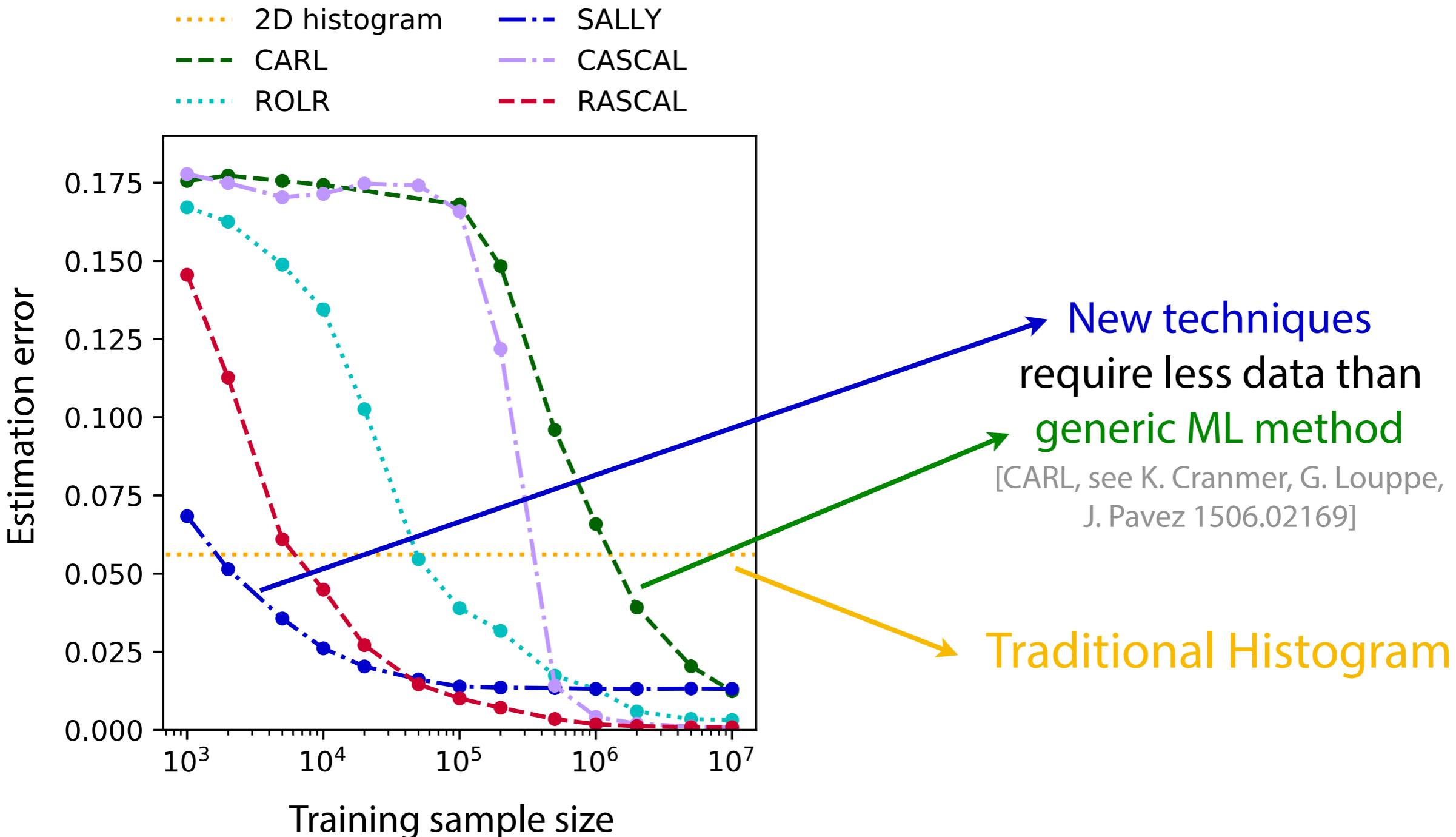
# Precise likelihood ratio estimates



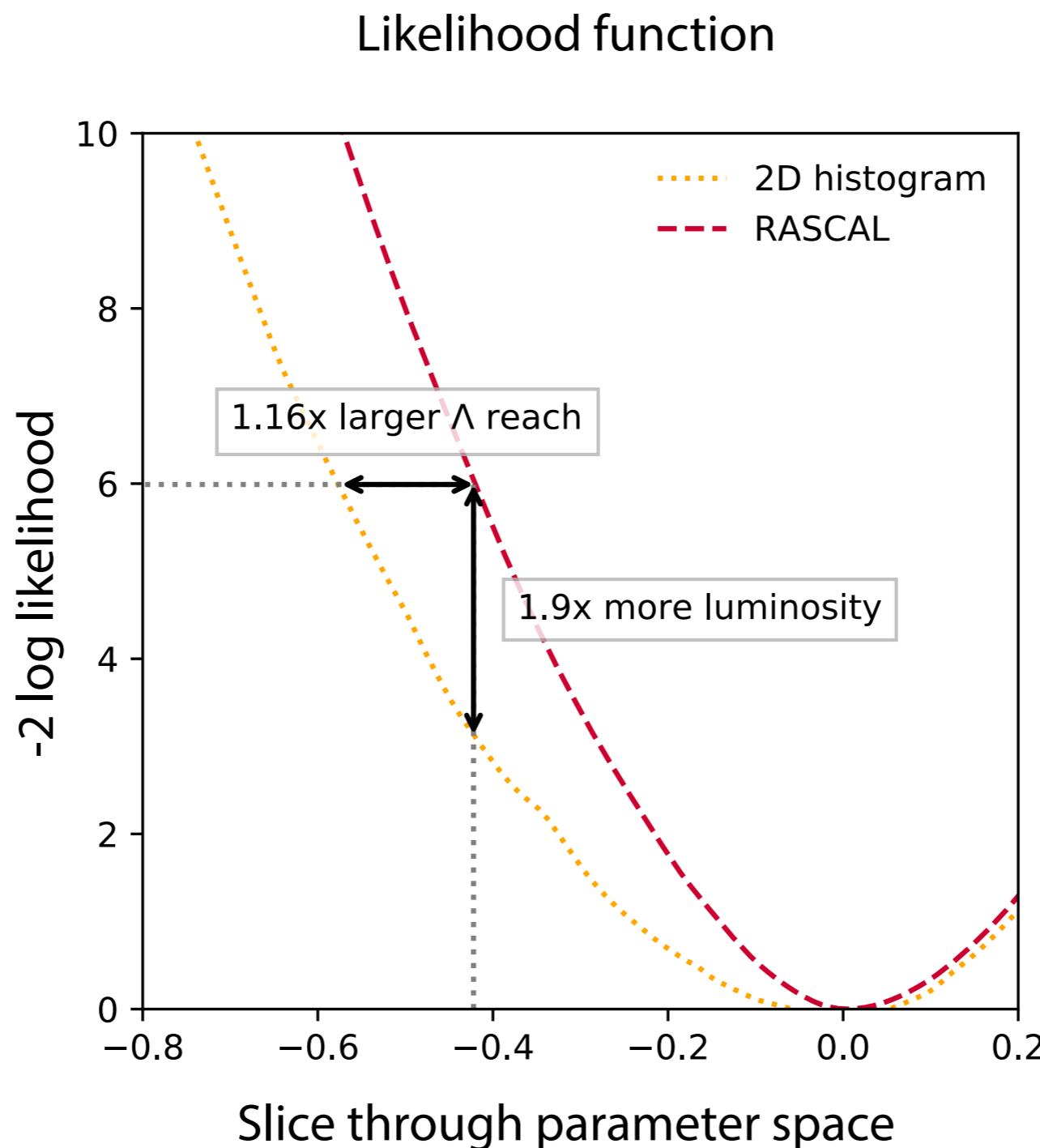
# Less training data needed



# Less training data needed



# Better sensitivity

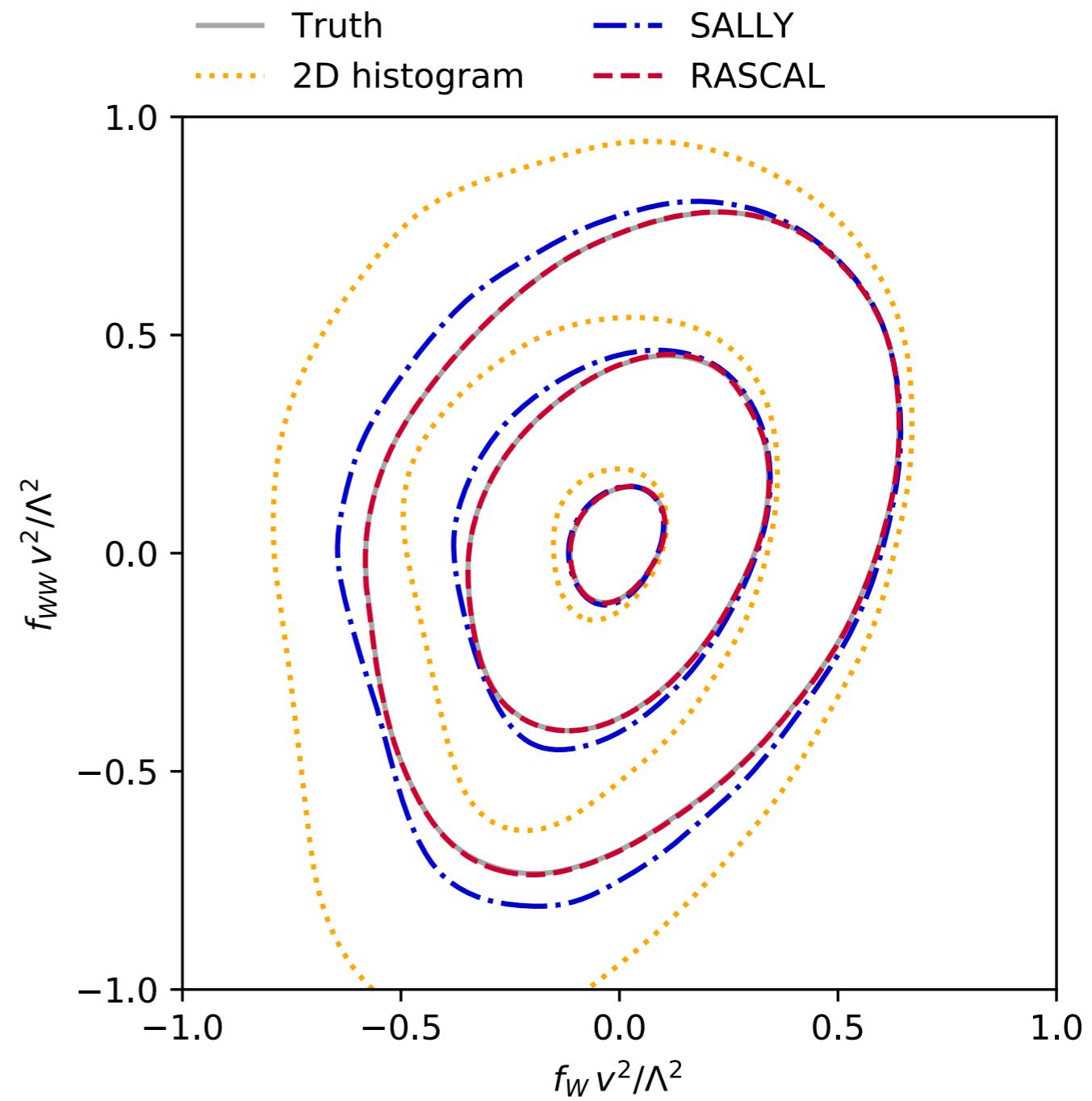


36 events, assuming SM

39/42

# Stronger bounds

Expected exclusion limits at 68%, 95%, 99.7% CL



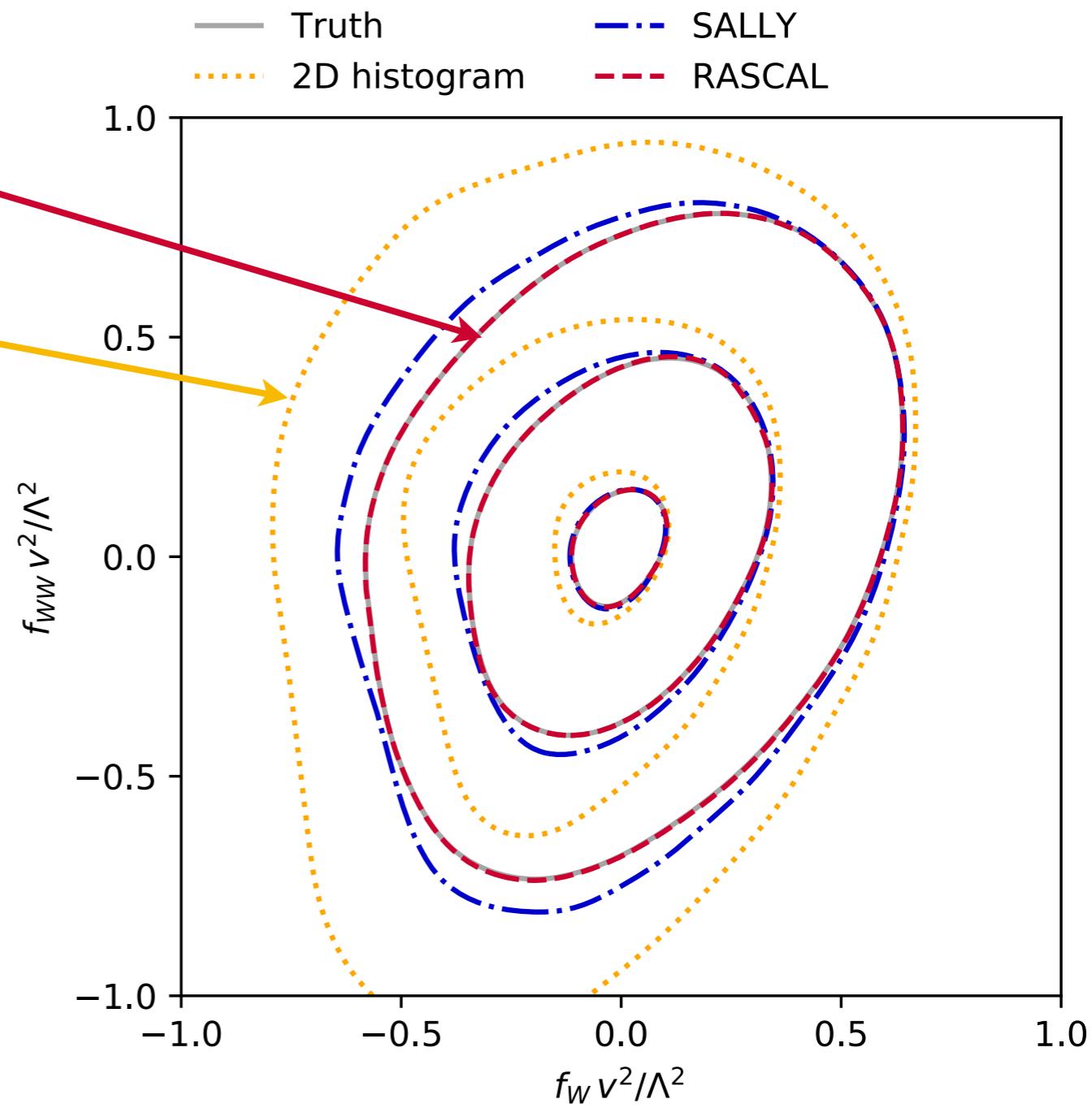
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40/42

# Stronger bounds

RASCAL  
enables stronger  
limits than  
traditional histogram

Expected exclusion limits at 68%, 95%, 99.7% CL



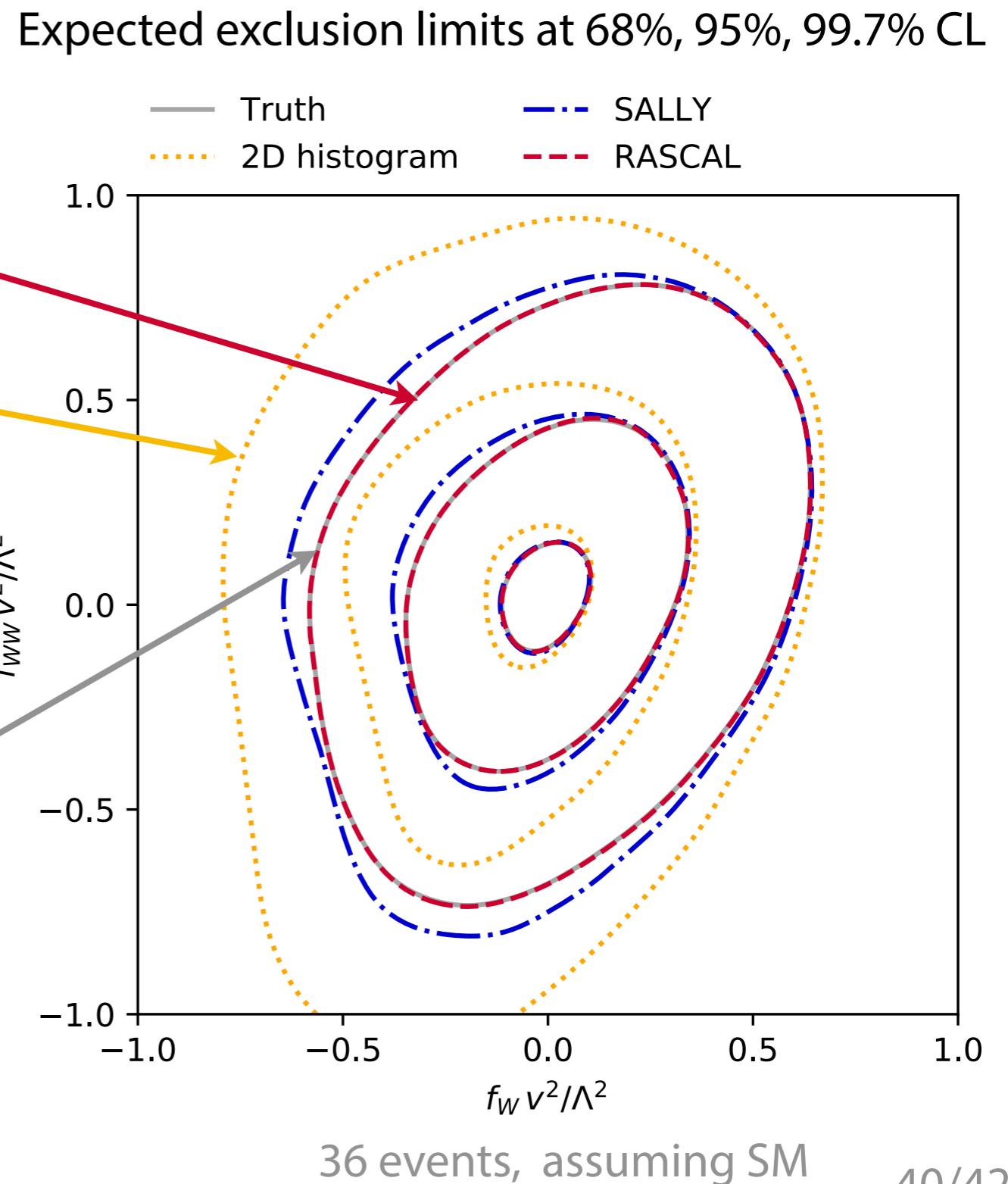
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40/42

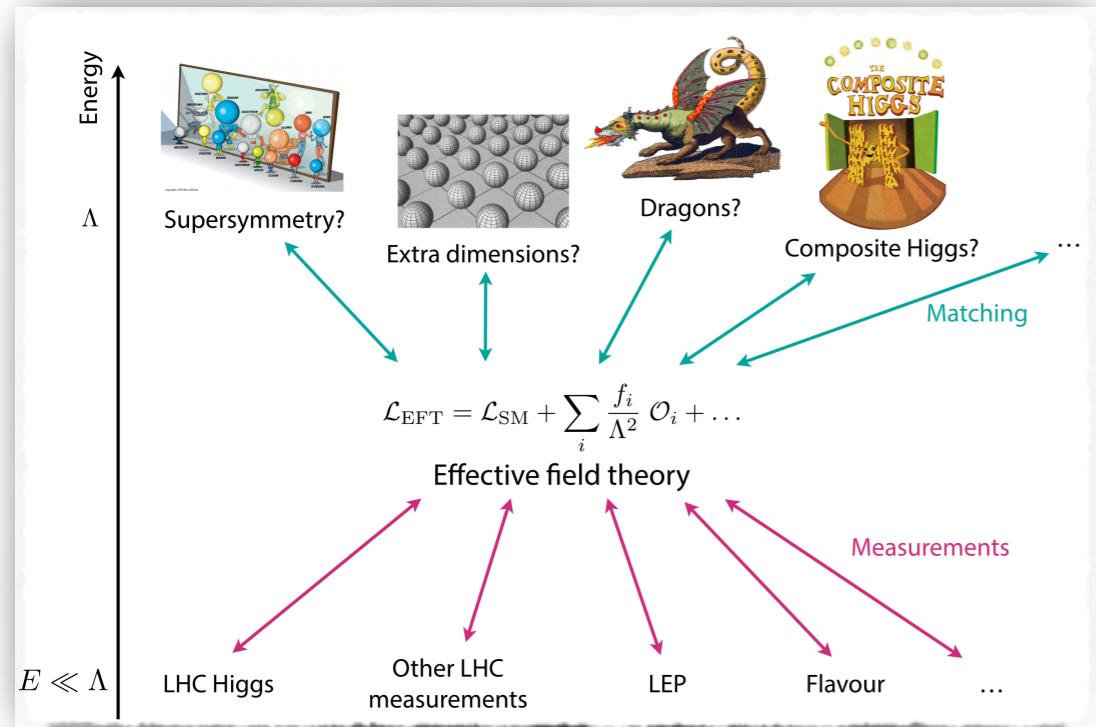
# Stronger bounds

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Limits from RASCAL  
virtually indistinguishable  
from true likelihood  
(usually we don't have that)

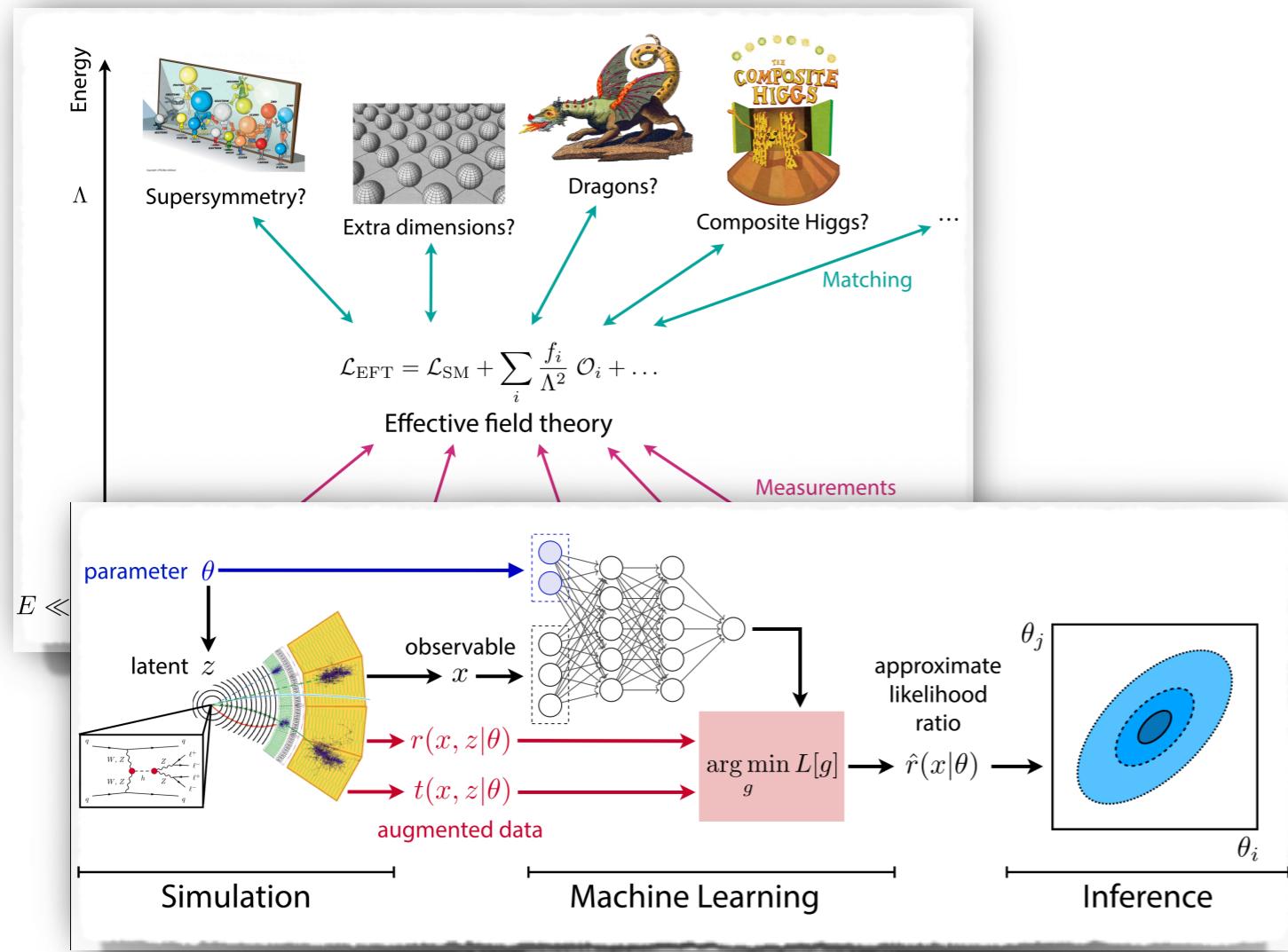


# Conclusions



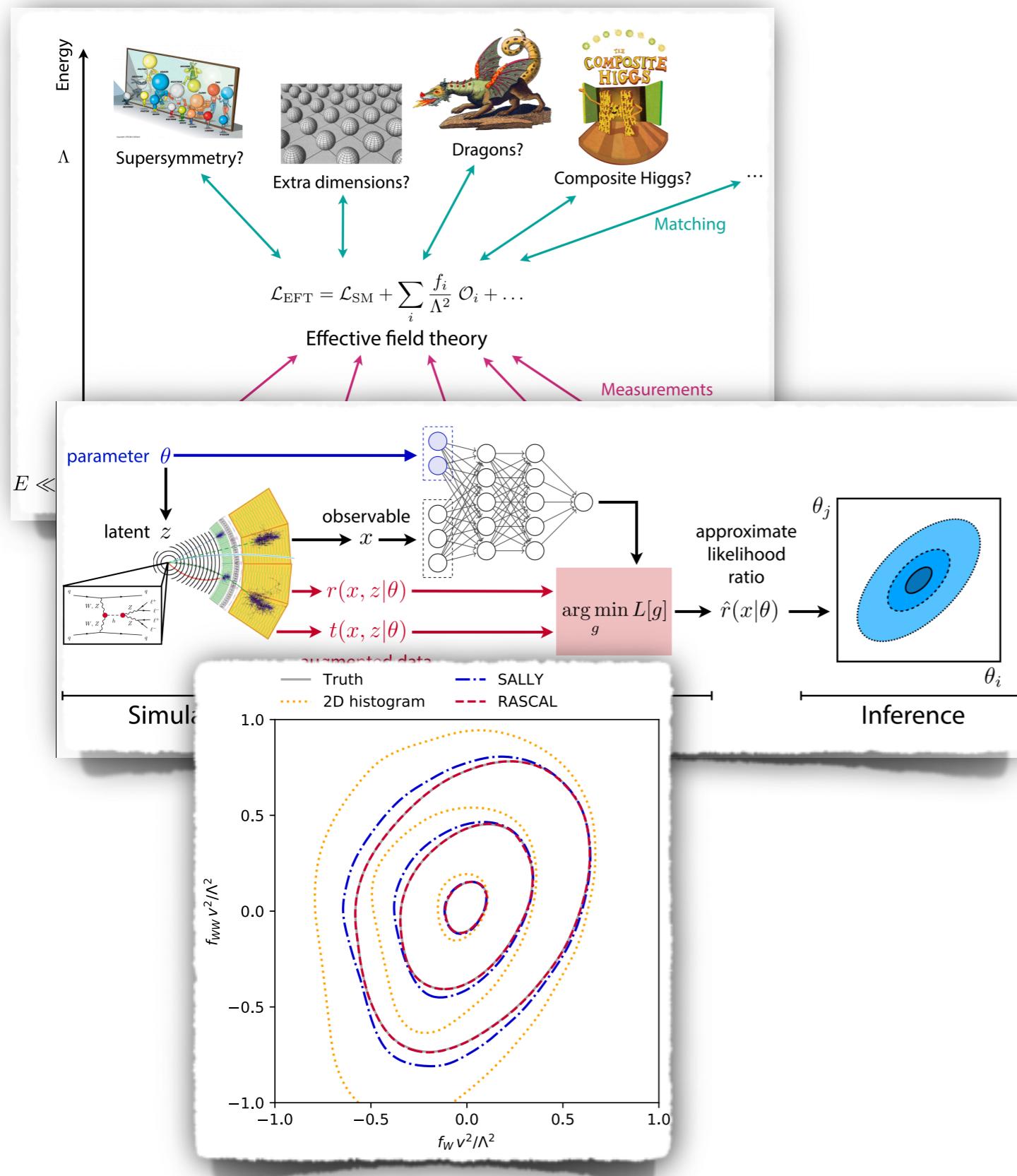
- EFT measurements at LHC might point us to exciting new physics

# Conclusions



- EFT measurements at LHC might point us to exciting new physics
- New analysis techniques for difficult measurements:  
Knowledge of physics structure + power of Machine Learning

# Conclusions



- EFT measurements at LHC might point us to exciting new physics
- New analysis techniques for difficult measurements:  
Knowledge of physics structure + power of Machine Learning
- First tests:  
stronger bounds with less data

# Collaborators



Kyle Cranmer



Gilles Louuppe



Juan Pavez



Tilman Plehn



Felix Kling



Tim Tait

1805.00020

## A Guide to Constraining Effective Field Theories with Machine Learning

Johann Brehmer,<sup>1</sup> Kyle Cranmer,<sup>1</sup> Gilles Louppe,<sup>2</sup> and Juan Pavez<sup>3</sup>

<sup>1</sup>*New York University, USA*

<sup>2</sup>*University of Liège, Belgium*

<sup>3</sup>*Federico Santa María Technical University, Chile*

(Dated: 30th April 2018)

We develop, discuss, and compare several inference techniques to constrain theory parameters in collider experiments. By harnessing the latent-space structure of particle physics processes, we extract extra information from the simulator. This augmented data can be used to train neural networks that precisely estimate the likelihood ratio. The new methods scale well to many observables and high-dimensional parameter spaces, do not require any approximations of the parton shower and detector response, and can be evaluated in micro-

1805.00013

## Constraining Effective Field Theories with Machine Learning

Johann Brehmer,<sup>1</sup> Kyle Cranmer,<sup>1</sup> Gilles Louppe,<sup>2</sup> and Juan Pavez<sup>3</sup>

<sup>1</sup>*New York University, USA*

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(Dated: 30th April 2018)

We present powerful new analysis techniques to constrain effective field theories at the LHC. By leveraging the structure of particle physics processes, we extract extra information from Monte-Carlo simulations, which can be used to train neural network models that estimate the likelihood ratio. These methods scale well to processes with many observables and theory parameters, do not require any approximations of the parton shower or detector response, and can be evaluated in microseconds. We show that they allow us to put significantly stronger bounds on dimension-six operators than existing methods, demonstrating their potential to improve the precision of the LHC legacy constraints.

## INTRODUCTION

Precision constraints on indirect signatures of physics beyond the Standard Model (SM) will be an important part of the legacy of the Large Hadron Collider (LHC) experiments. A key component of this program are limits on the dimension-six operators of the SM Effective Field Theory (SMEFT). Processes relevant to these measurements are often sensitive to a large number of EFT coefficients, which predict subtle kinematic signatures in high-dimensional phase spaces.

Traditionally, such signatures are analysed by focussing on a few hand-picked kinematic variables. This approach discards any information in the remaining directions of phase space. Well-chosen

Thanks to Kyle Cranmer for letting me steal inspiring many slides!

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# Deleted scenes

# Dictionary

Symbol	Physics meaning	Machine learning abstraction
$x$	Set of all observables	Features
$v$	One or two kinematic variables	Low-dimensional summary statistics /engineered feature
$z \equiv z_{\text{parton}}$	Parton-level four-momenta	Latent variables
$z_{\text{shower}}$	Parton shower trajectories	Latent variables
$z_{\text{detector}}$	Detector interactions	Latent variables
$z_{\text{all}} = (z_{\text{parton}}, z_{\text{shower}}, z_{\text{detector}})$	Full simulation history of event	All latent variables
$\theta$	Theory parameters (Wilson coefficients)	Parameters of interest
$\hat{\theta}$	Best fit for theory parameters	Estimator for parameters of interest
$p(x \theta)$	Distributions of observables given theory parameters	Intractable likelihood
$p(z \theta)$	Parton-level distributions from matrix element	Tractable likelihood of latent variables
$p(x z)$	Effect of shower, detector, reconstruction	Intractable density defined through stochastic generative process
$r(x \theta_0, \theta_1)$	Likelihood ratio between hypotheses $\theta_0, \theta_1$ , see Eq. (11).	
$\hat{r}(x \theta_0, \theta_1)$	Estimator for likelihood ratio	
$t(x \theta)$	Score, see Eq. (14).	
$\hat{t}(x \theta)$	Estimator for score	
$x_e, z_e$	Event	Data point
$\theta_o$	Wilson coefficient for one operator	Individual parameter of interest
$\theta_c, w_c(z), p_c(x)$	Morphing basis points, coefficients, densities, see Eq. (6).	

# For the Bayesians among us

$$\begin{aligned}\hat{r}(x|\theta) &\approx \frac{p(x|\theta)}{\int d\theta' p(x|\theta') p(\theta')} \\ p(\theta|x) &= \frac{p(x|\theta) p(\theta)}{\int d\theta' p(x|\theta') p(\theta')} \\ &\approx \hat{r}(x|\theta) p(\theta)\end{aligned}$$

# Available information

Quantity		General likelihood-free	Particle physics
Samples	$\{x_e\}$	✓	✓
Likelihood	$p(x_e \theta)$		
Likelihood ratio	$r(x_e \theta_0, \theta_1)$		*
Score	$t(x_e \theta)$		*
Latent state	$\{x_e, z_e\}$		✓
Joint likelihood	$p(x_e, z_{\text{all},e} \theta)$		
Joint likelihood ratio	$r(x_e, z_{\text{all},e} \theta_0, \theta_1)$		✓
Joint score	$t(x_e, z_{\text{all},e} \theta)$		✓

# Variational calculus

$$\begin{aligned} L[\hat{g}(x)] &= \int dx dz \, \textcolor{red}{p}(x, z | \theta) |g(x, z) - \hat{g}(x)|^2 \\ &= \int dx \underbrace{\left[ \hat{g}^2(x) \int dz \, \textcolor{red}{p}(x, z | \theta) - 2\hat{g}(x) \int dz \, \textcolor{red}{p}(x, z | \theta) g(x, z) + \int dz \, \textcolor{red}{p}(x, z | \theta) g^2(x, z) \right]}_{F(x)} \end{aligned}$$

$$0 = \frac{\delta F}{\delta \hat{g}} \Big|_{g^*} = 2\hat{g} \underbrace{\int dz \, \textcolor{red}{p}(x, z | \theta)}_{=\textcolor{red}{p}(x | \theta)} - 2 \int dz \, \textcolor{red}{p}(x, z | \theta) g(x, z)$$

$$g^*(x) = \frac{1}{\textcolor{red}{p}(x | \theta)} \int dz \, \textcolor{red}{p}(x, z | \theta) g(x, z)$$

# Loss functions

- Ratio estimator:

$$L[\hat{r}(x|\theta_0, \theta_1)] = \frac{1}{N} \sum_{(x_e, z_e) \sim \textcolor{red}{p}(x, z|\theta_1)} |r(x_e, z_{\text{all},e}|\theta_0, \theta_1) - \hat{r}(x|\theta_0, \theta_1)|^2$$

- Score estimator:

$$L[\hat{t}(x|\theta)] = \frac{1}{N} \sum_{(x_e, z_e) \sim \textcolor{red}{p}(x, z|\theta)} |t(x_e, z_{\text{all},e}|\theta) - \hat{t}(x|\theta)|^2$$

- Combined (RASCAL):

$$\begin{aligned} L[\hat{r}(x|\theta_0, \theta_1)] &= \frac{1}{N} \sum_{(x_e, z_e, y_e)} \left[ y_e |r(x_e, z_e|\theta_0, \theta_1) - \hat{r}(x|\theta_0, \theta_1)|^2 \right. \\ &\quad + (1 - y_e) \left| \frac{1}{r(x_e, z_e|\theta_0, \theta_1)} - \frac{1}{\hat{r}(x|\theta_0, \theta_1)} \right|^2 \\ &\quad \left. + \alpha y_e |t(x_e, z_e|\theta) - \hat{t}(x|\theta_0)|^2 \right] \end{aligned}$$

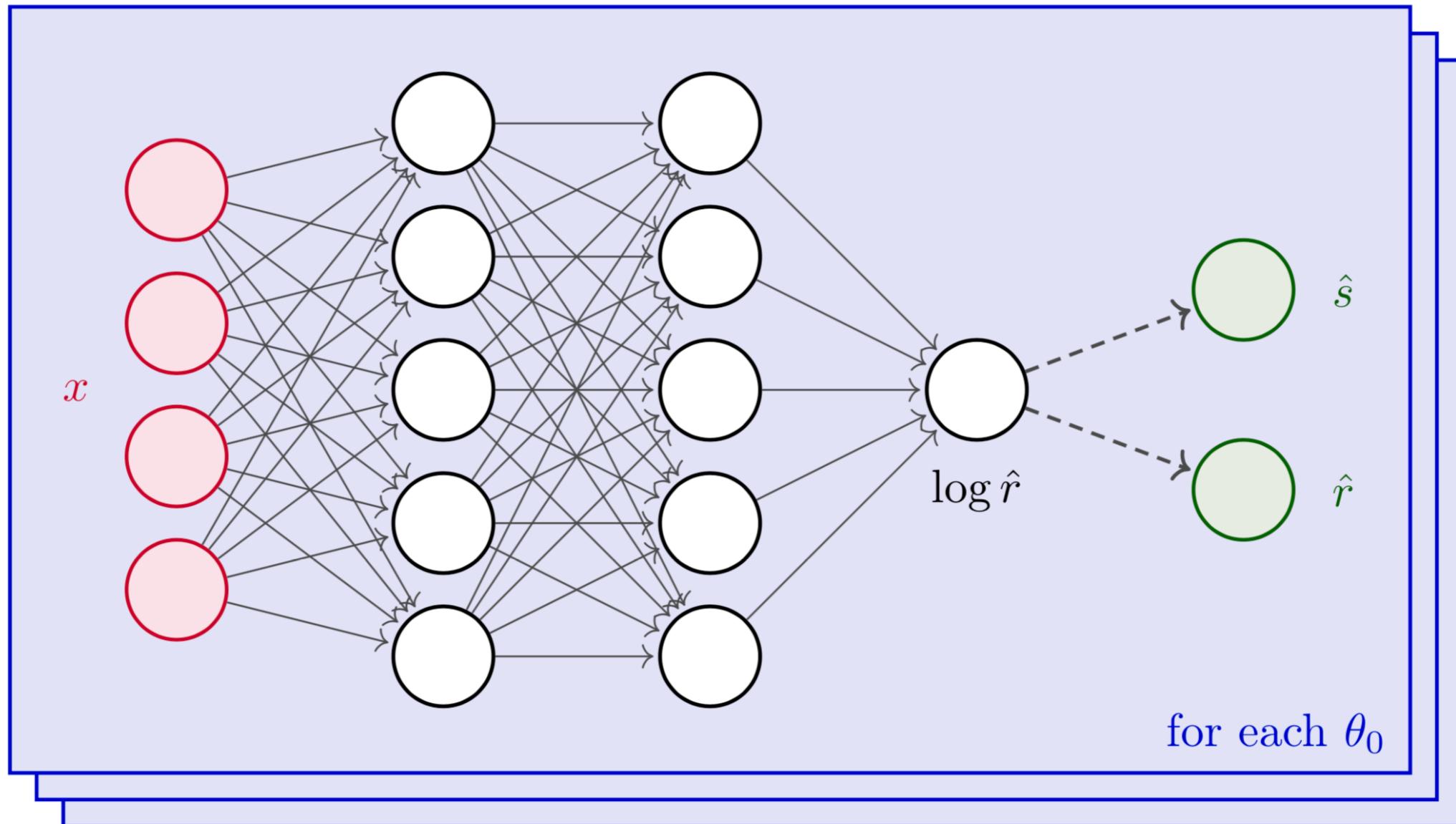
# Three classes of estimators

- Point by point
- (Agnostic) parameterized
- Morphing-aware:

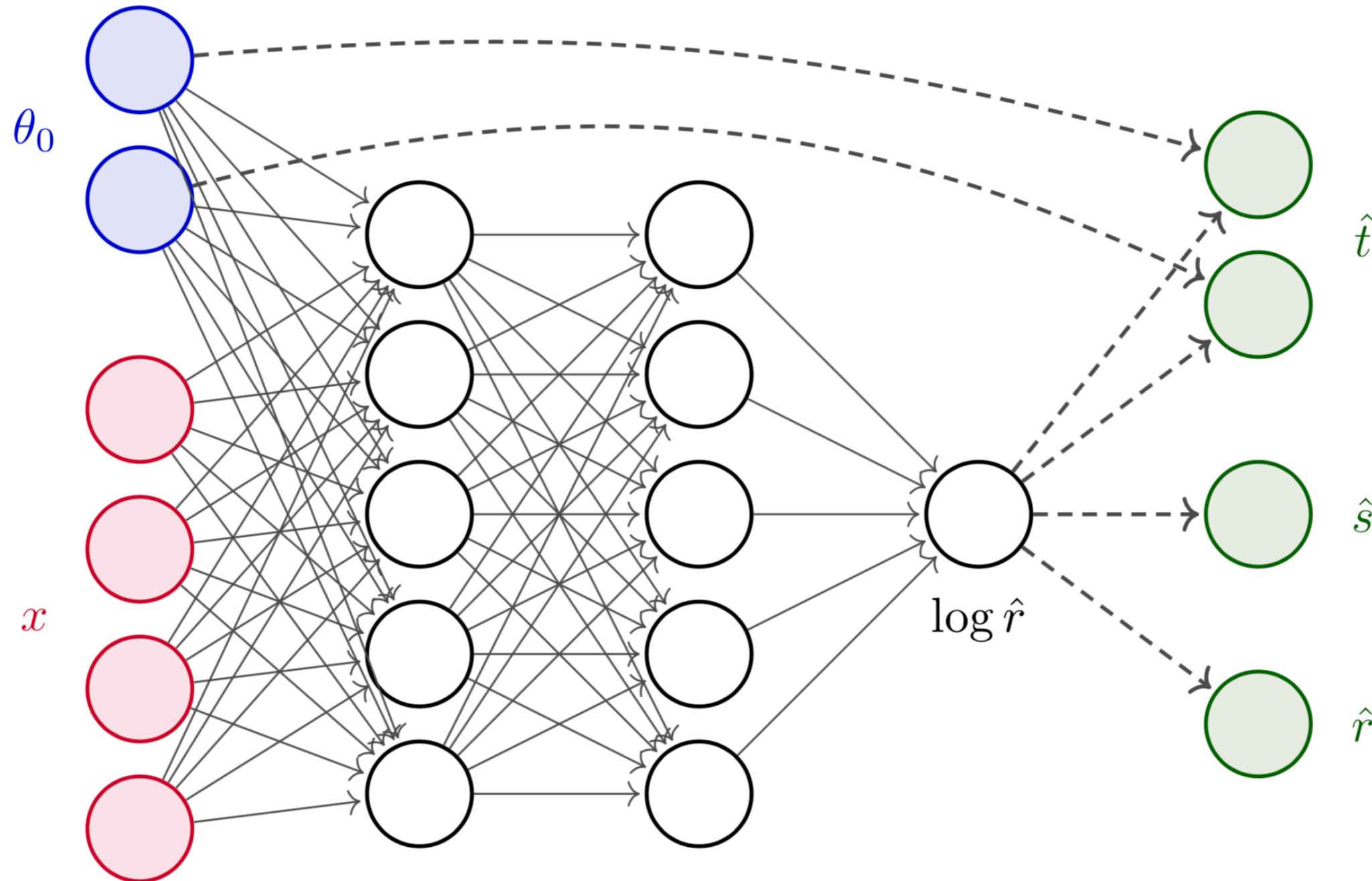
$$\textcolor{red}{p}(x|\theta) = \sum_c w_c(\theta) \textcolor{red}{p}_c(x)$$

$$\hat{r}(x|\theta_0, \theta_1) = \sum_c w_c(\theta_0) \hat{r}_c(x)$$

# Point by point

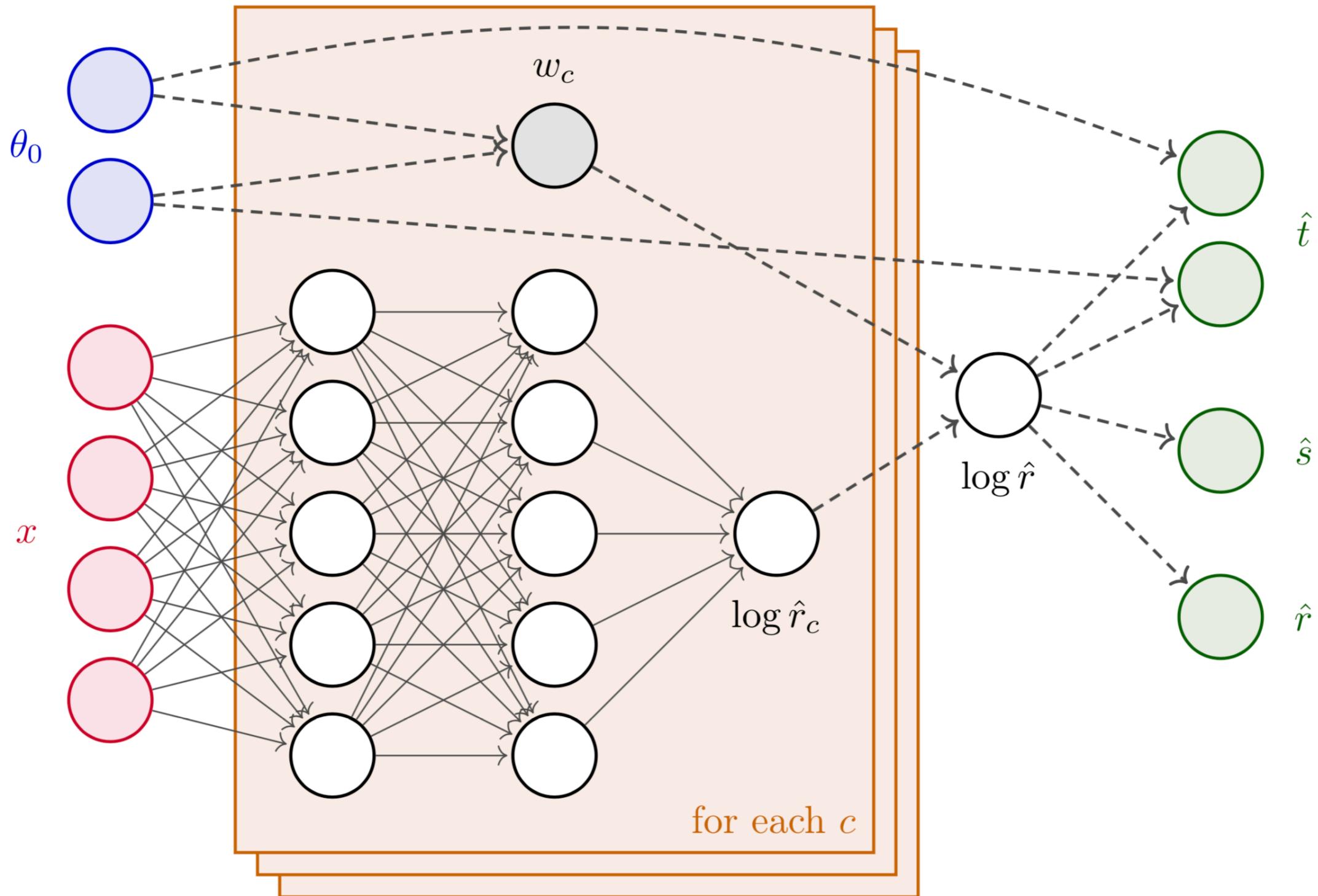


# (Agnostic) parameterized

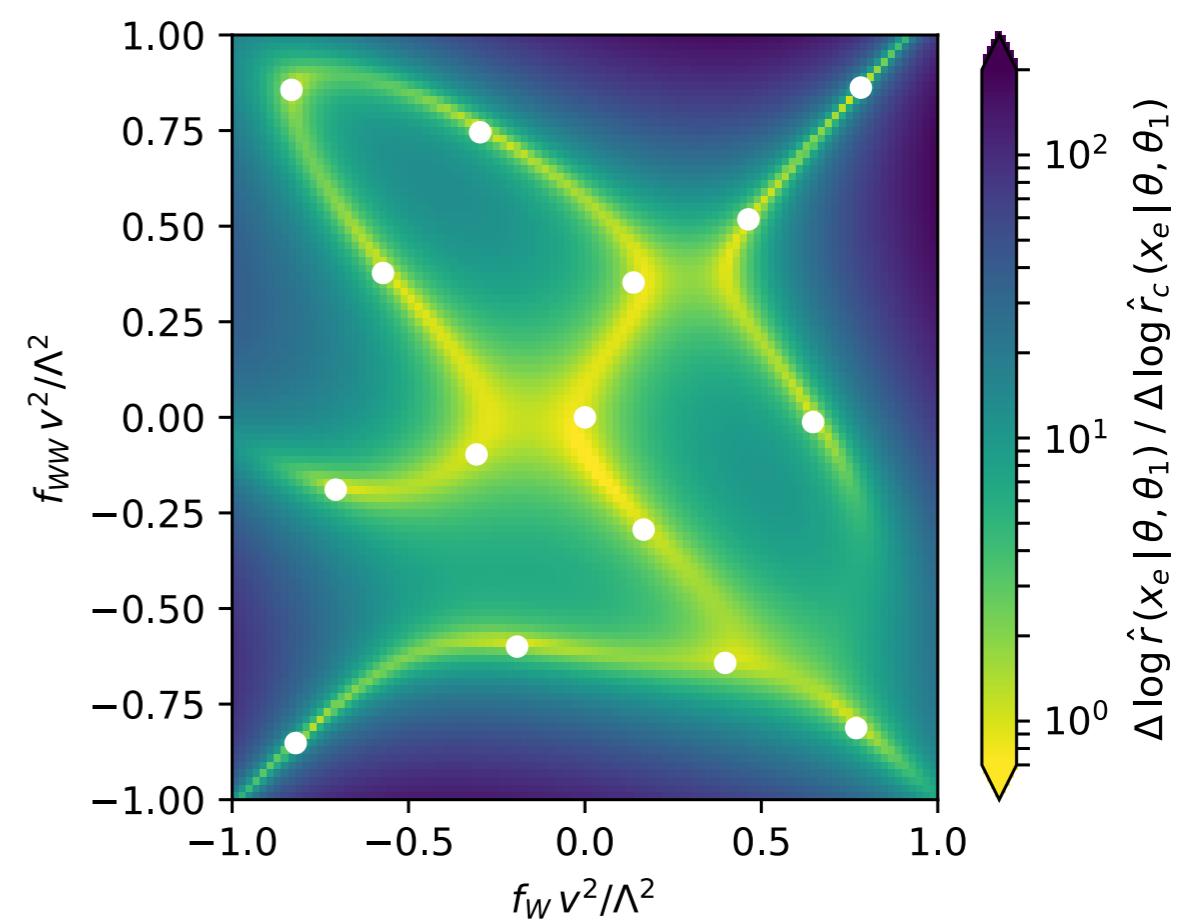
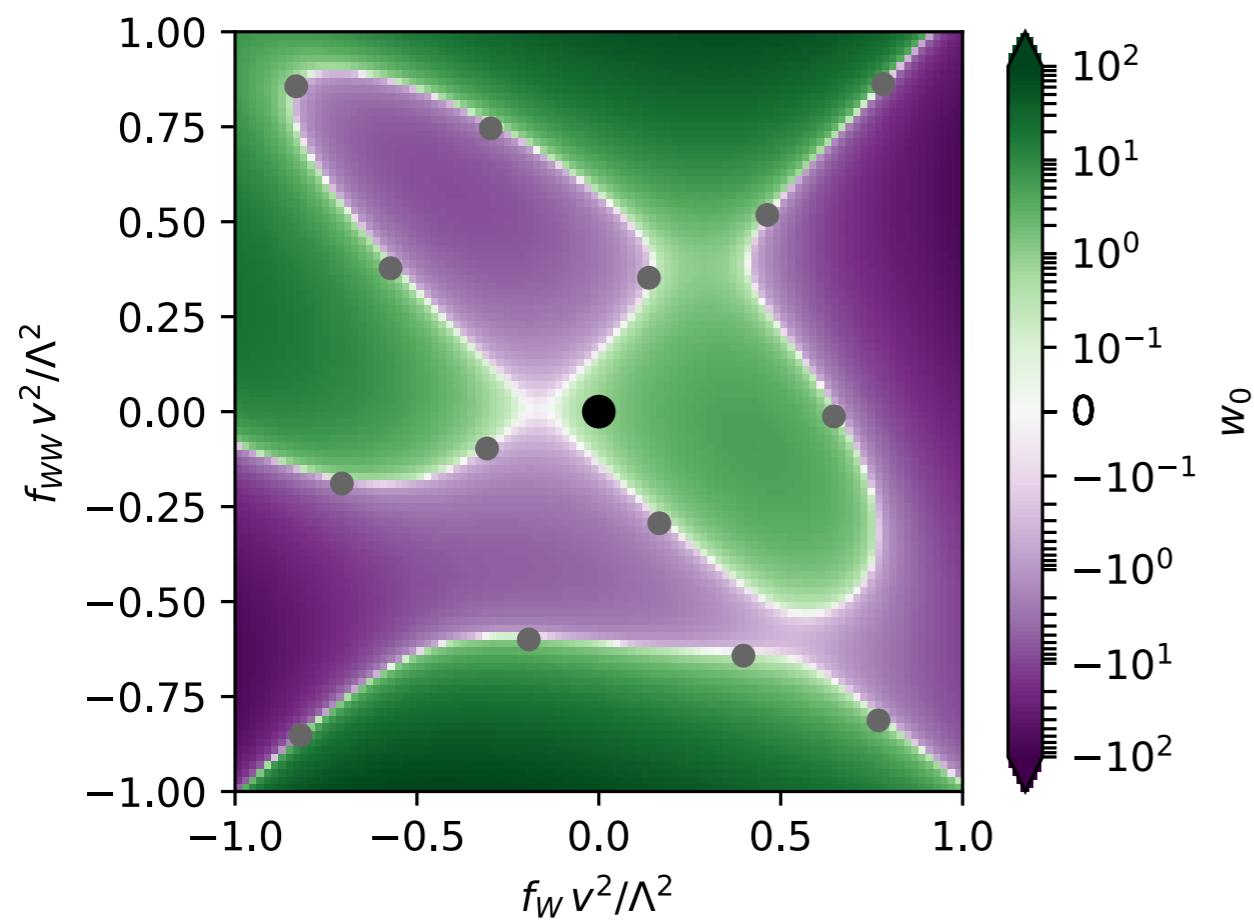


# Morphing-aware

$$\hat{r}(x|\theta_0, \theta_1) = \sum_c w_c(\theta_0) \hat{r}_c(x)$$



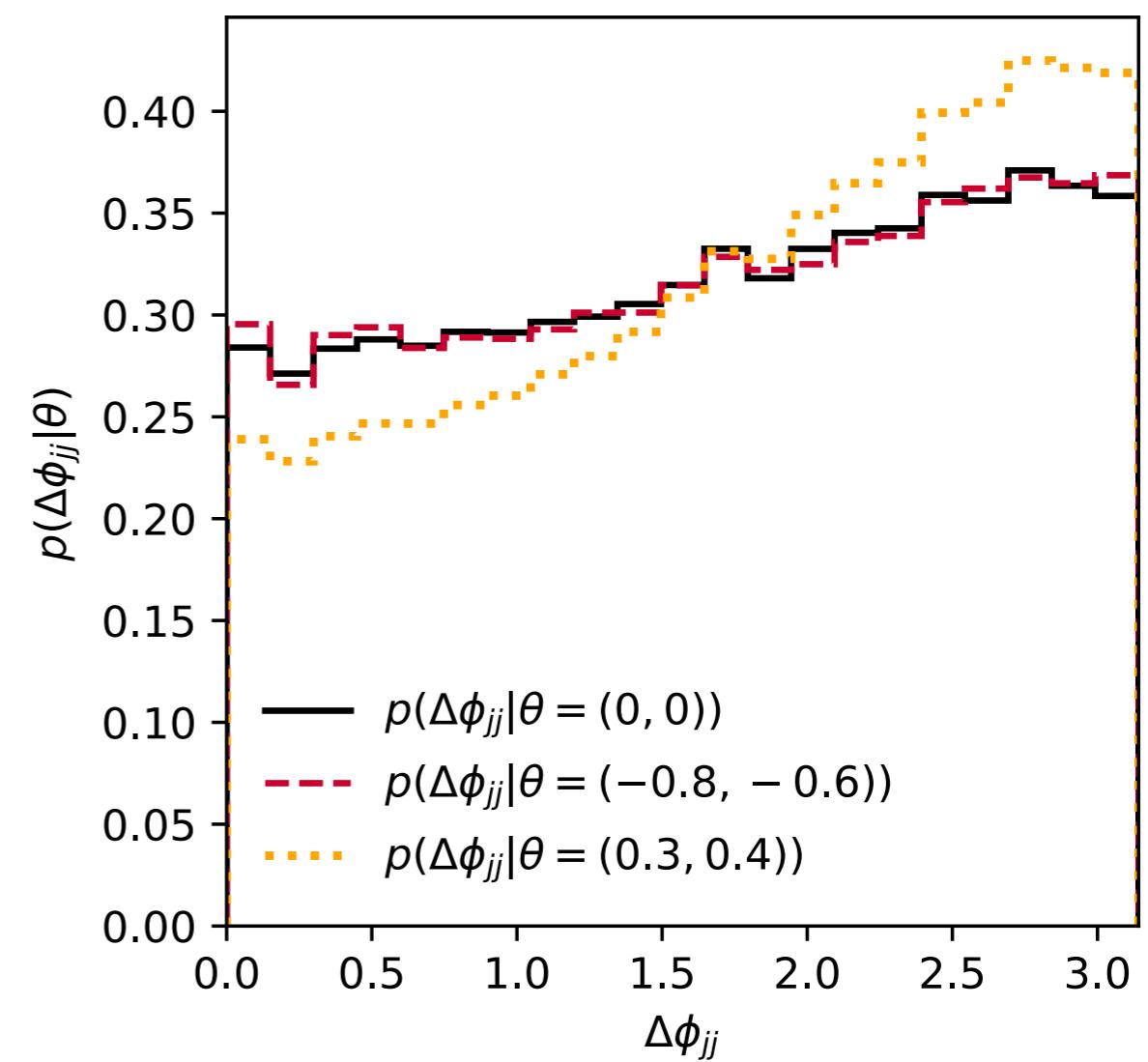
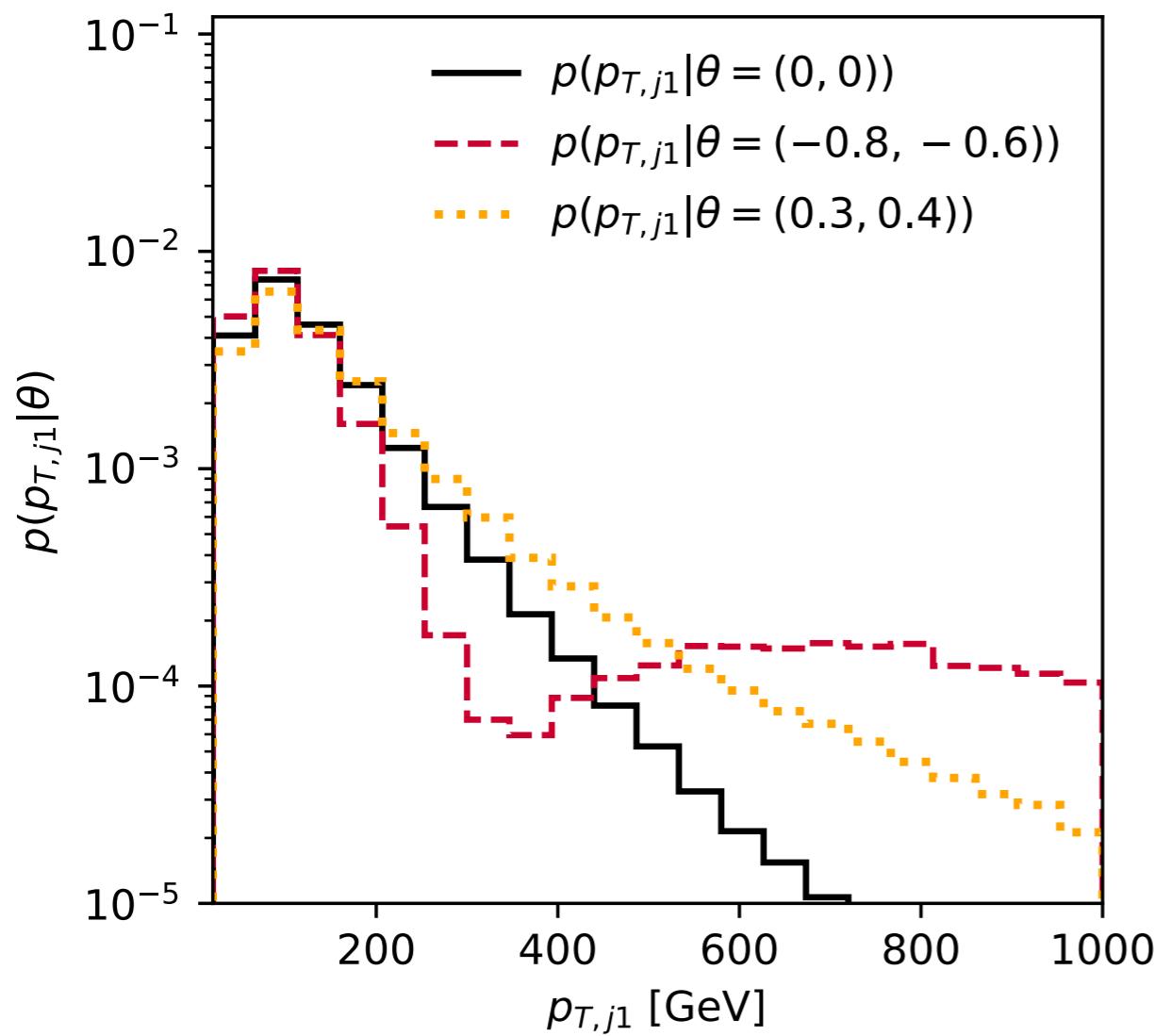
# Morphing coefficients



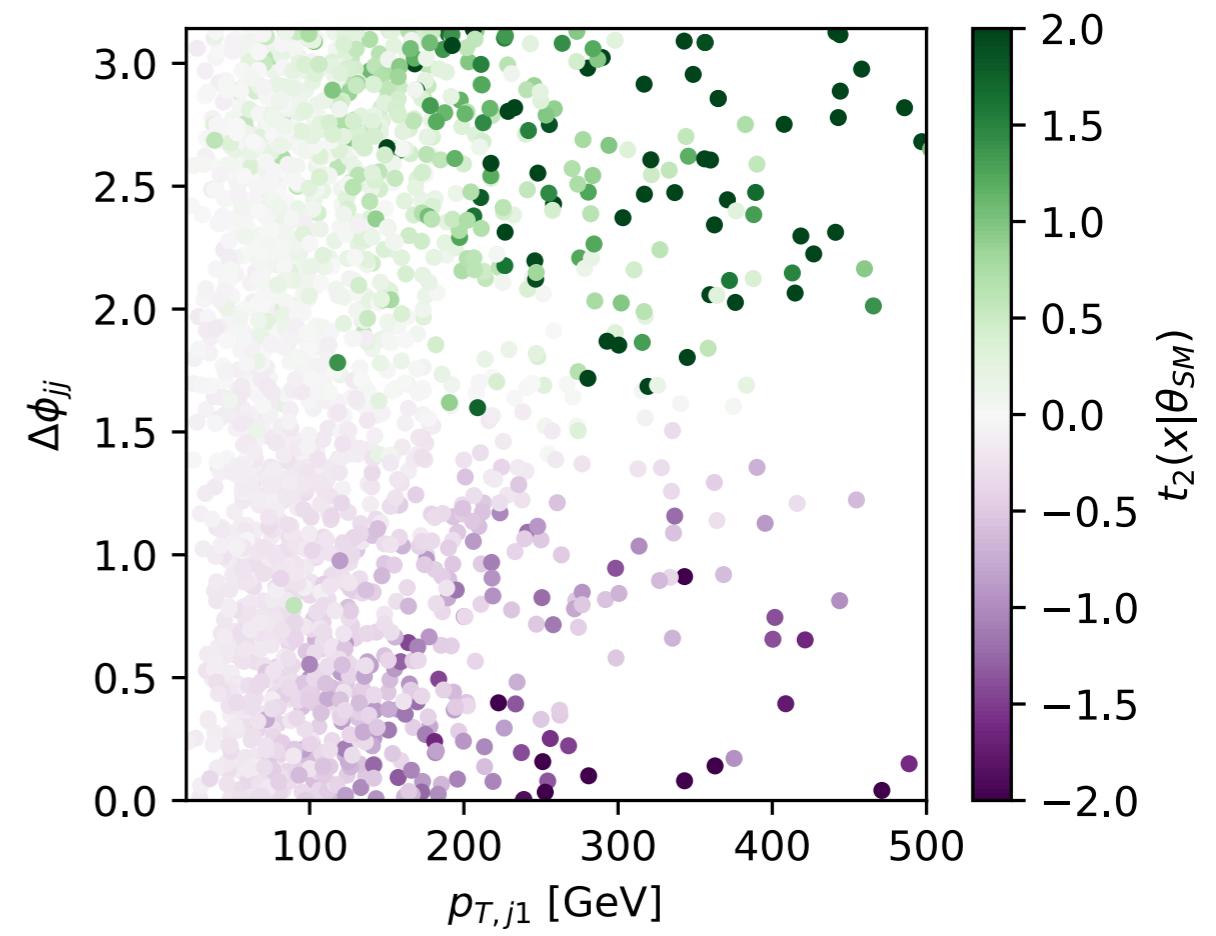
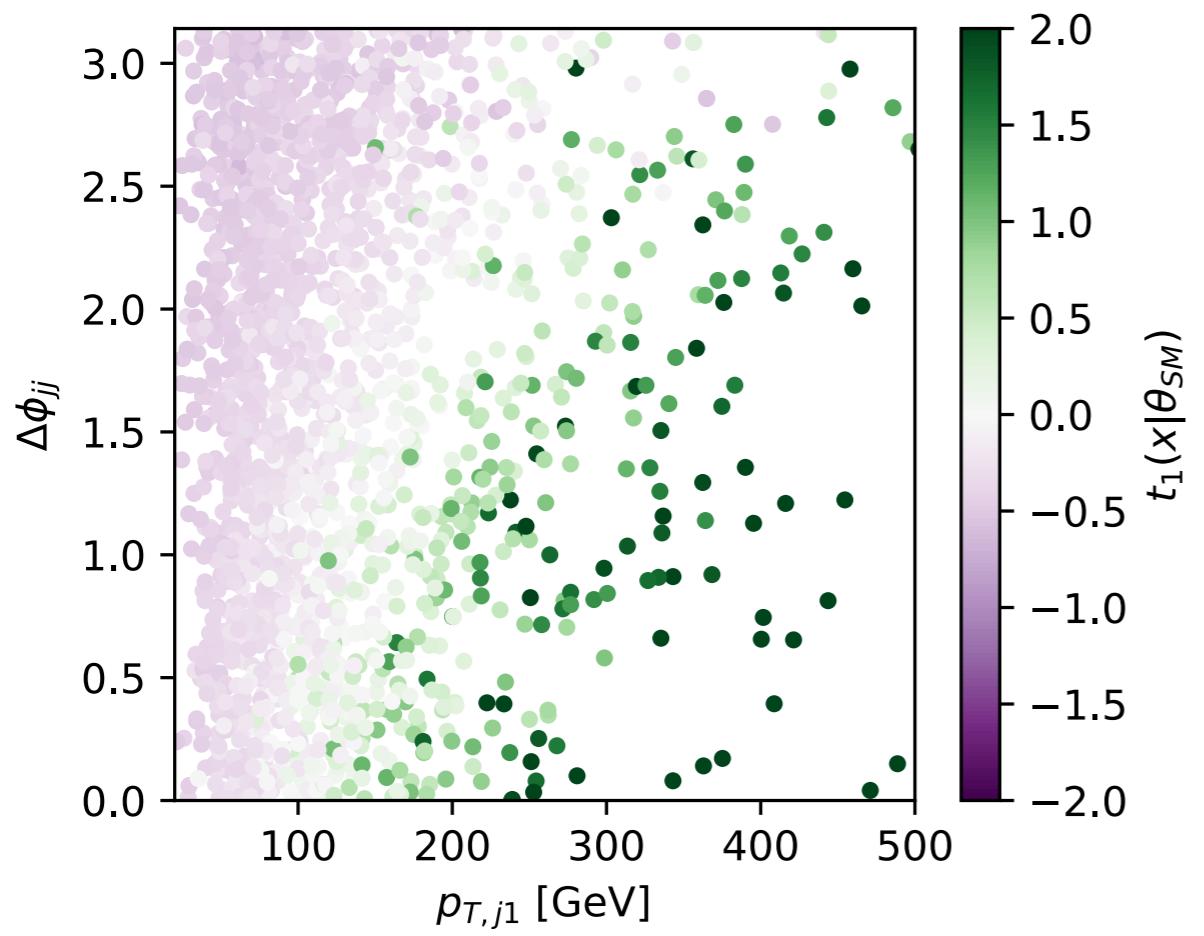
# Analysis techniques

Strategy	Estimator versions			Loss function			Asymptotically exact
	PbP	Param	Aware	CE	Ratio	Score	
Histograms	✓		(✓)				
AFC	✓		(✓)				
CARL	✓	✓	✓	✓			✓
ROLR	✓	✓	✓		✓		✓
SALLY	✓		(✓)			✓	
SALLINO	✓		(✓)			✓	
CASCAL		✓	✓	✓		✓	✓
RASCAL	✓	✓	✓	✓	✓		✓

# Distributions



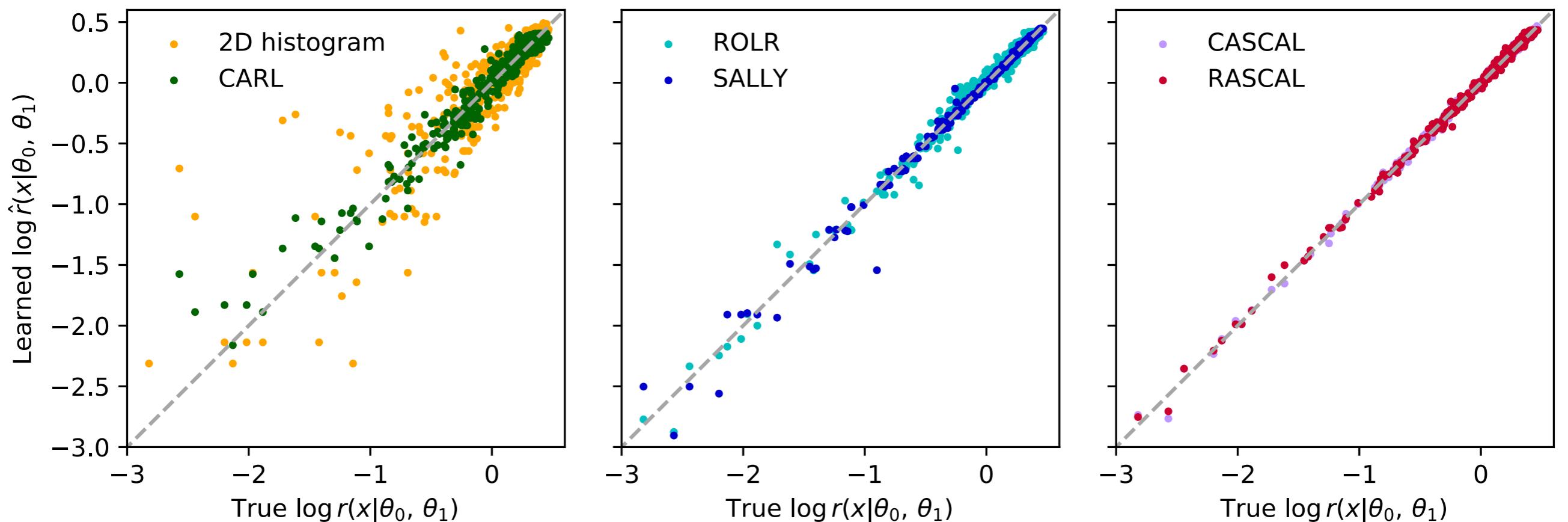
# Score vs observables



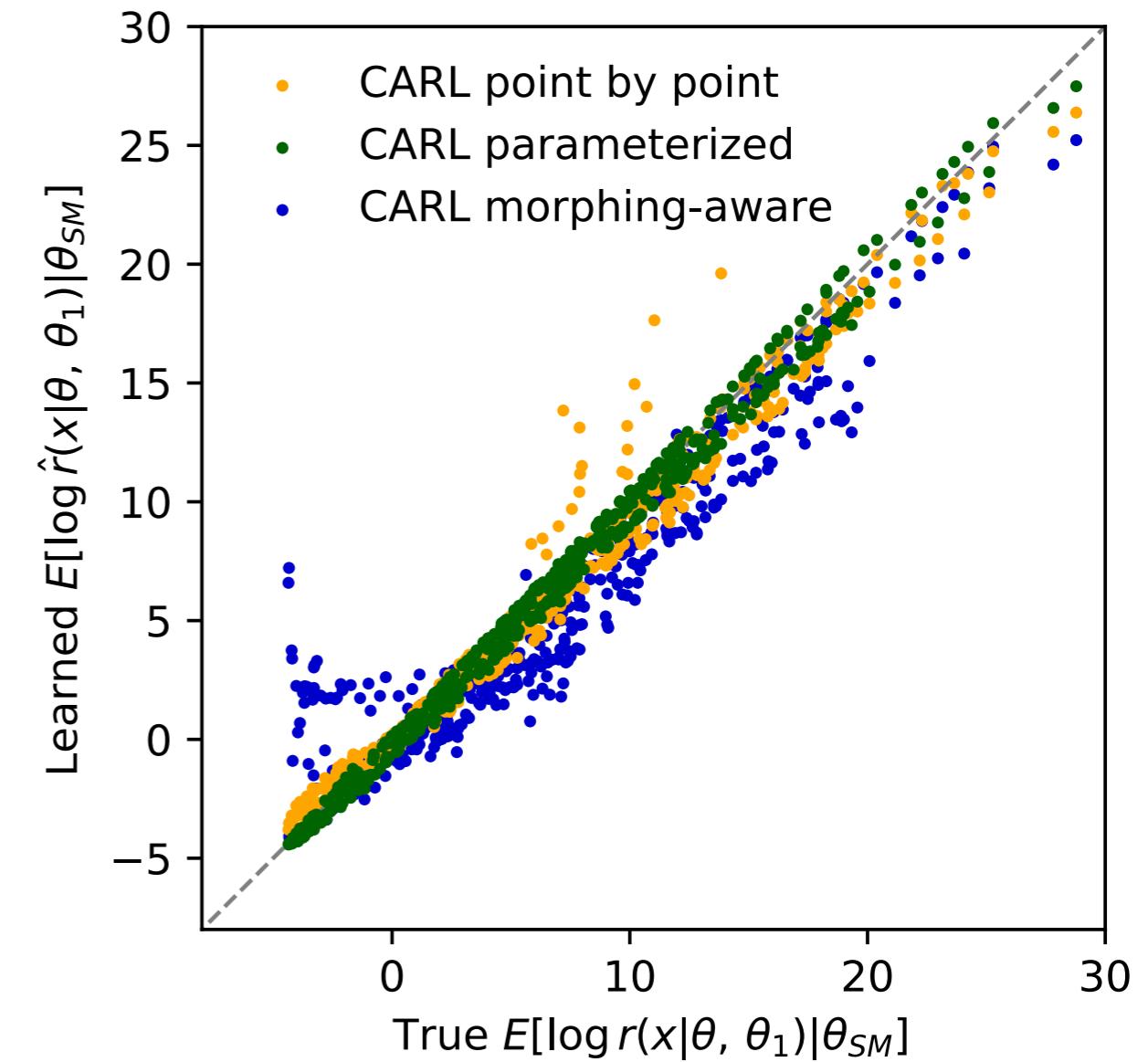
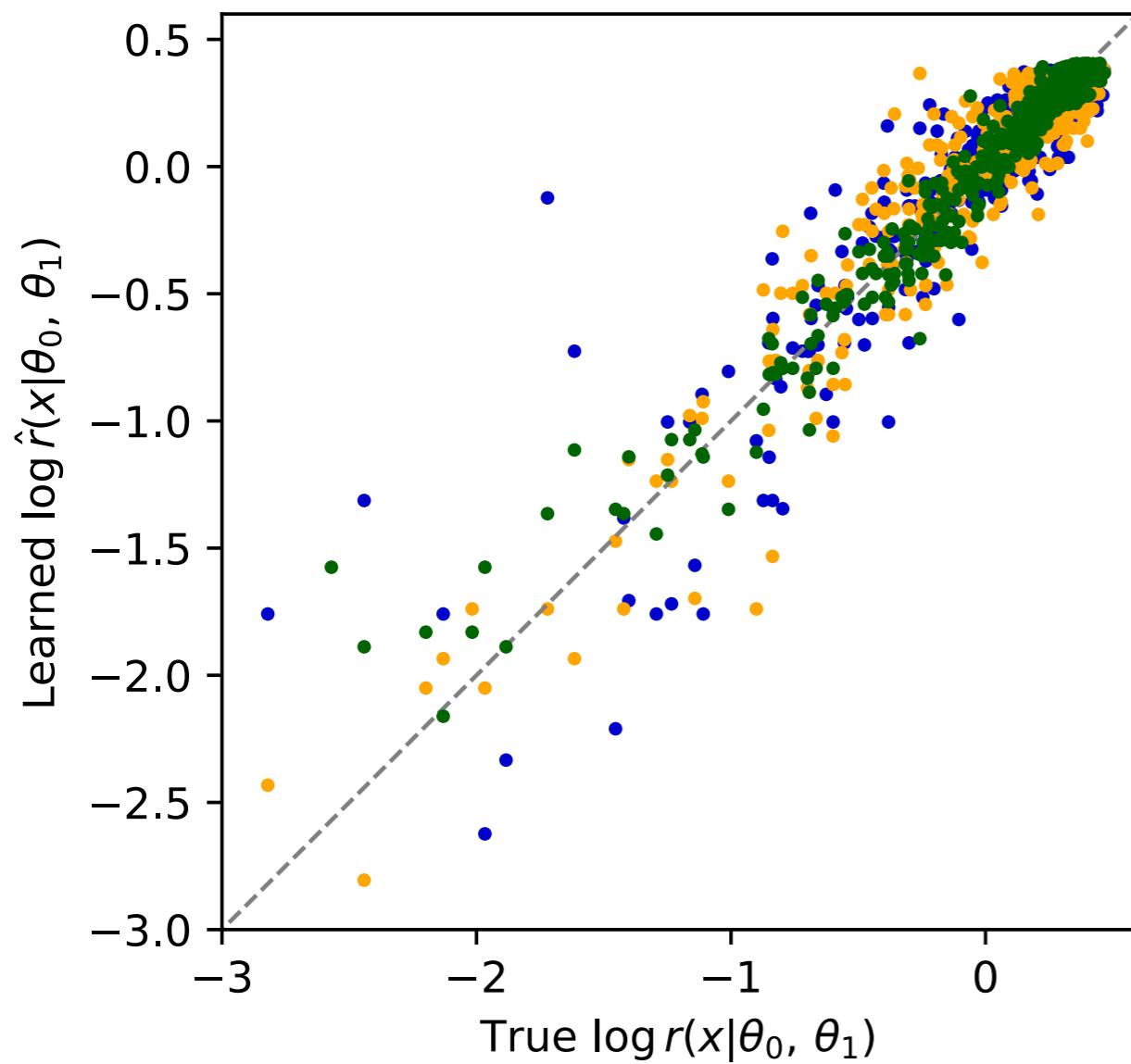
# Results

Strategy	Setup	Expected MSE		Figures
		All	Trimmed	
Histogram	$p_{T,j1}, \Delta\phi_{jj}$	<b>0.056</b>	0.0106	✓
	$p_{T,j1}$	0.088	0.0230	
	$\Delta\phi_{jj}$	0.160	0.0433	
AFC	$p_{T,j1}, \Delta\phi_{jj}$	0.059	<b>0.0091</b>	
	$p_{T,j1}, m_{Z2}, m_{jj}, \Delta\eta_{jj}, \Delta\phi_{jj}$	0.078	0.0101	
CARL (PbP)	PbP	0.030	0.0111	Fig. 12
CARL (parameterized)	Baseline	0.012	<b>0.0026</b>	✓
	Random $\theta$	<b>0.012</b>	0.0028	
	Baseline	0.076	0.0200	Fig. 12
CARL (morphing-aware)	Random $\theta$	0.086	0.0226	
	Morphing basis	0.156	0.0618	
ROLR (PbP)	PbP	0.005	0.0022	
ROLR (parameterized)	Baseline	0.003	0.0017	✓
	Random $\theta$	<b>0.003</b>	<b>0.0014</b>	
	Baseline	0.024	0.0063	
ROLR (morphing-aware)	Random $\theta$	0.022	0.0052	
	Morphing basis	0.130	0.0485	
SALLY		<b>0.013</b>	<b>0.0002</b>	✓
SALLINO		0.021	0.0006	
CASCAL (parameterized)	Baseline	<b>0.001</b>	<b>0.0002</b>	✓
	Random $\theta$	0.001	0.0002	
	Baseline	0.136	0.0427	
	Random $\theta$	0.092	0.0268	
	Morphing basis	0.040	0.0081	
RASCAL (parameterized)	Baseline	0.001	0.0004	✓
	Random $\theta$	<b>0.001</b>	<b>0.0004</b>	
	Baseline	0.125	0.0514	
RASCAL (morphing-aware)	Random $\theta$	0.132	0.0539	
	Morphing basis	0.031	0.0072	

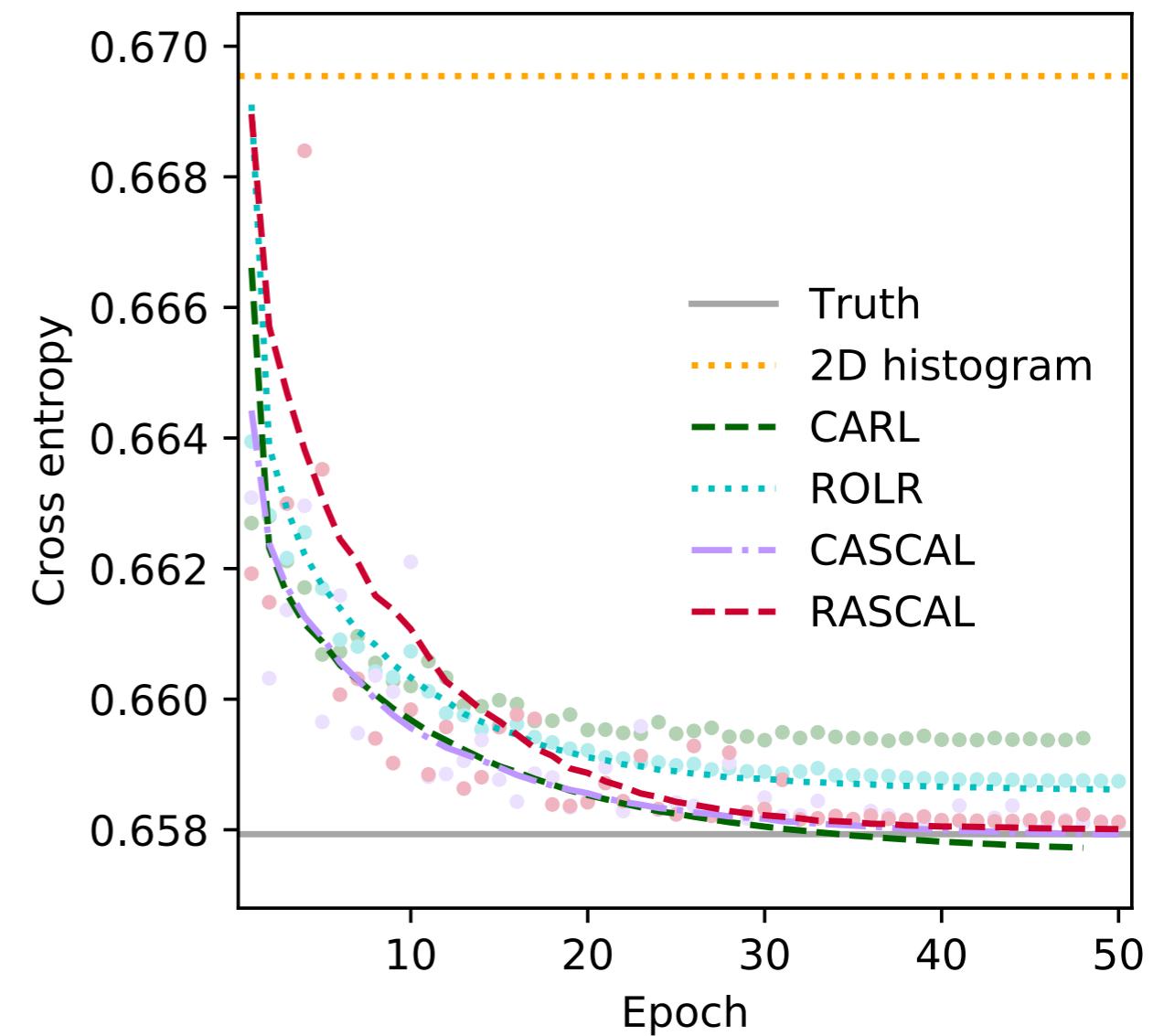
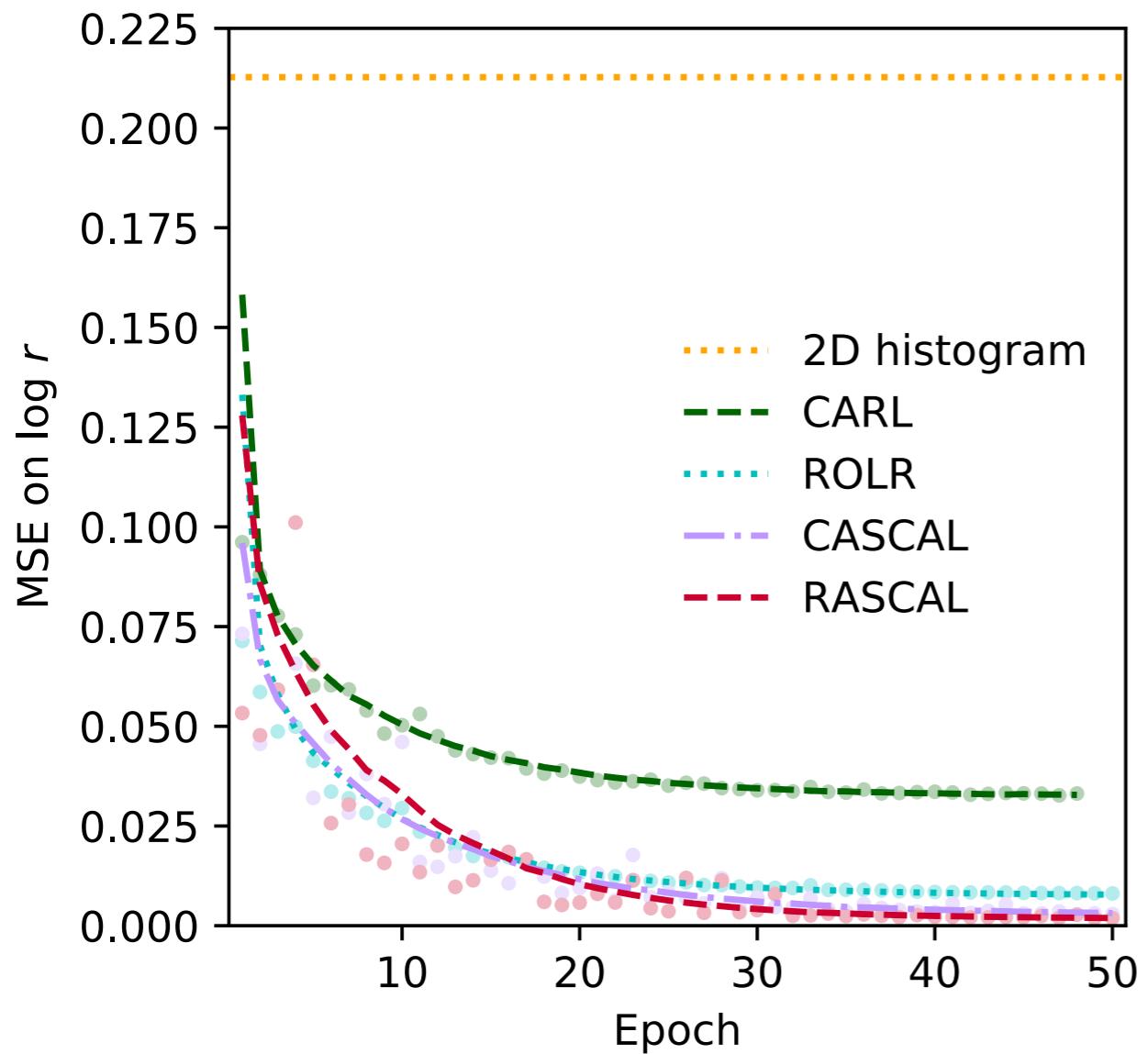
# Precision of estimates



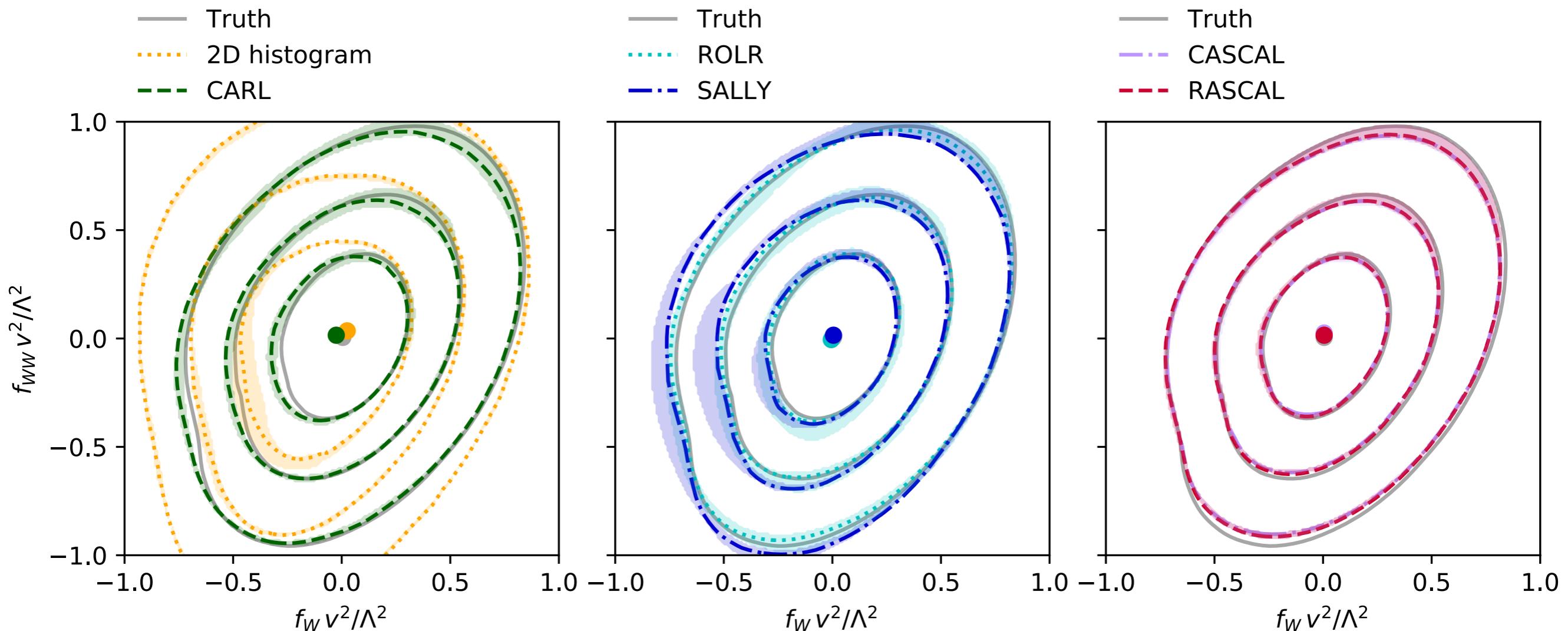
# PbP vs parameterized vs morphing-aware



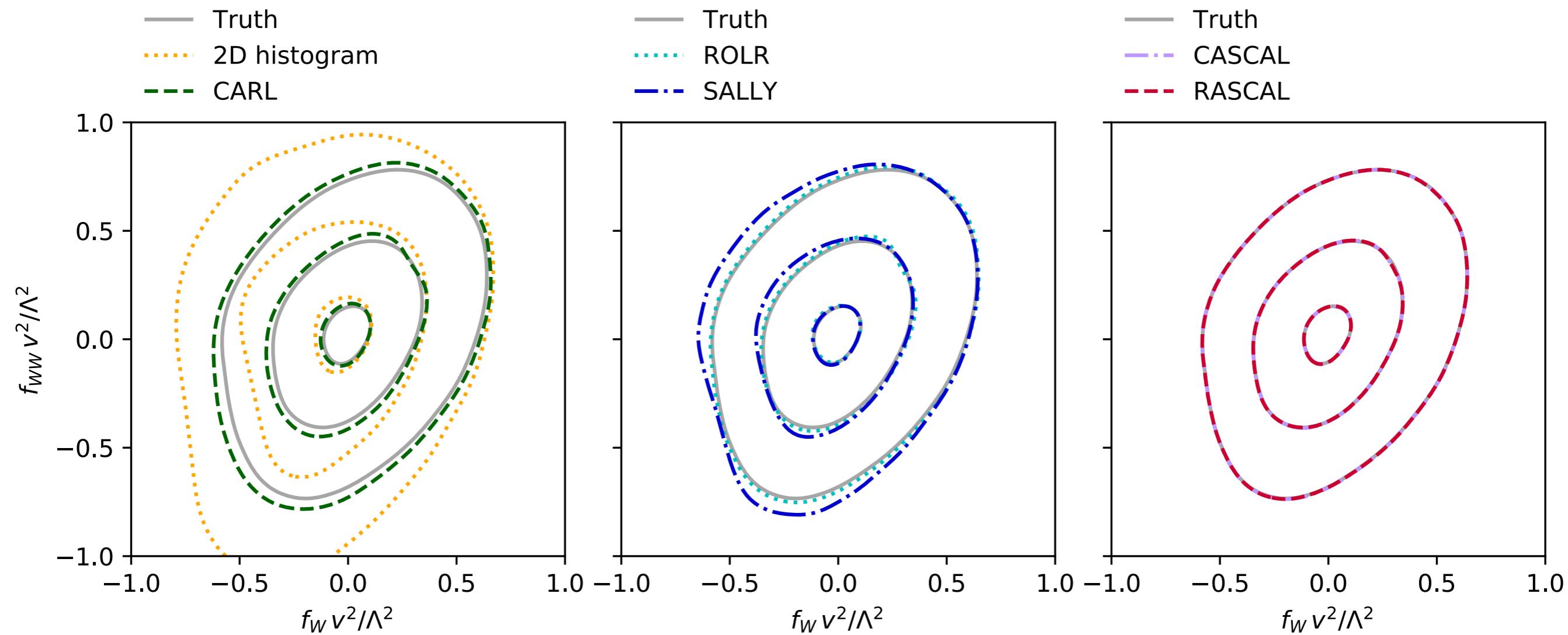
# Learning curves

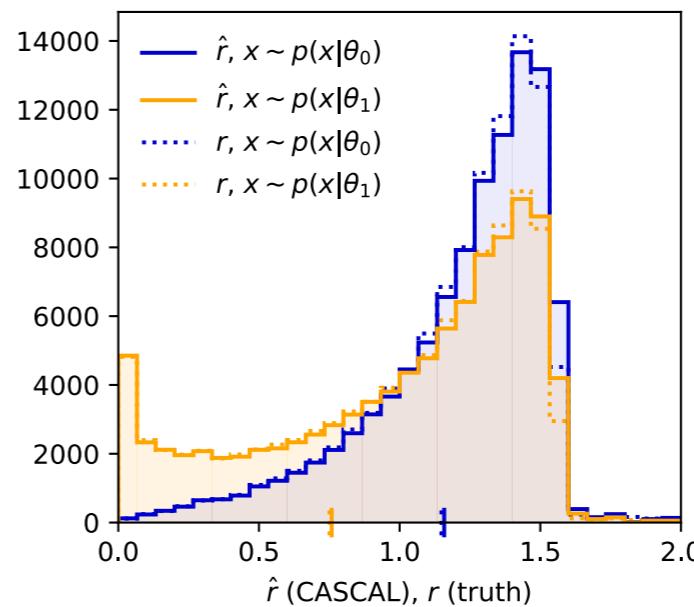
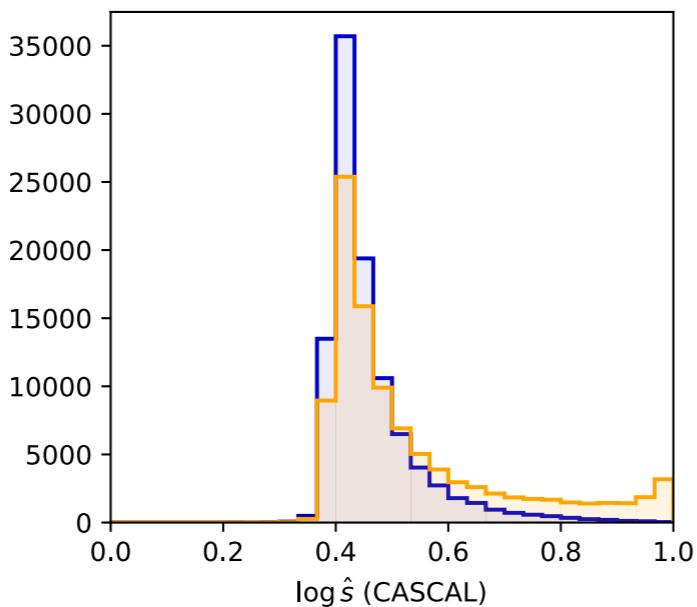
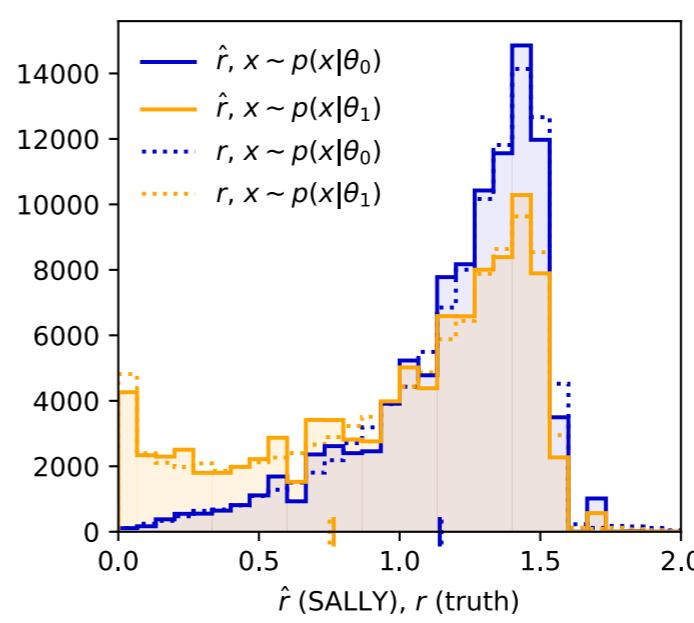
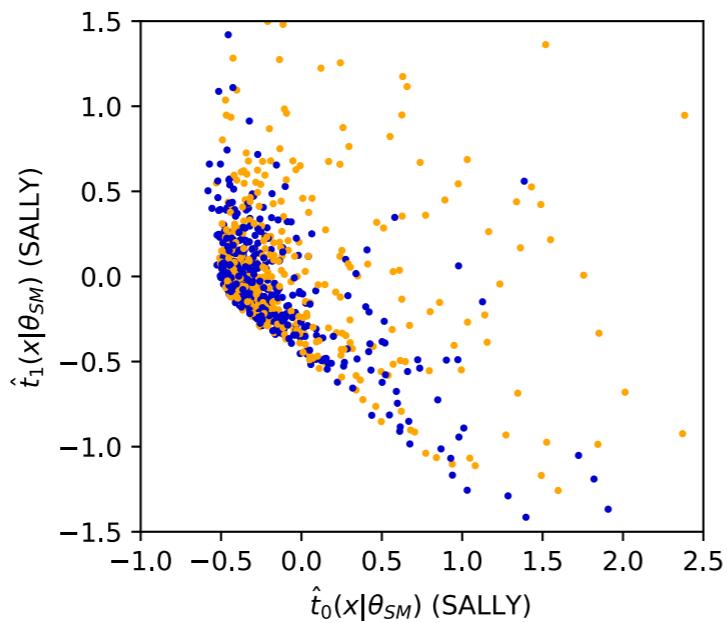
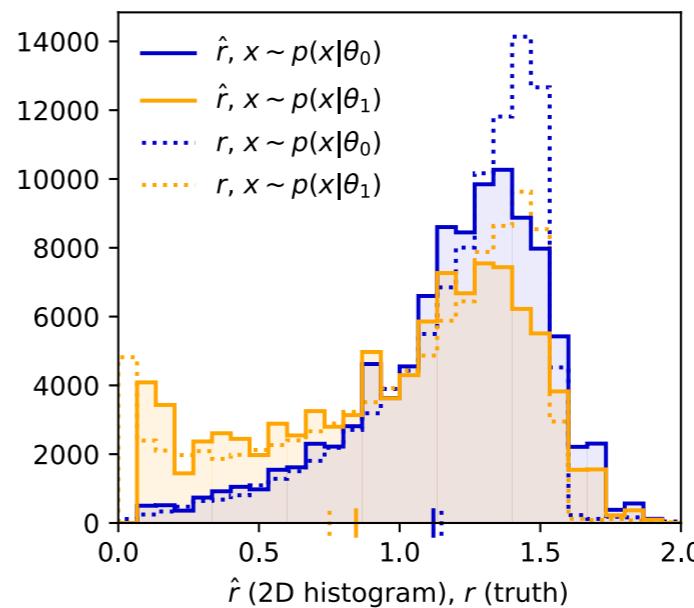
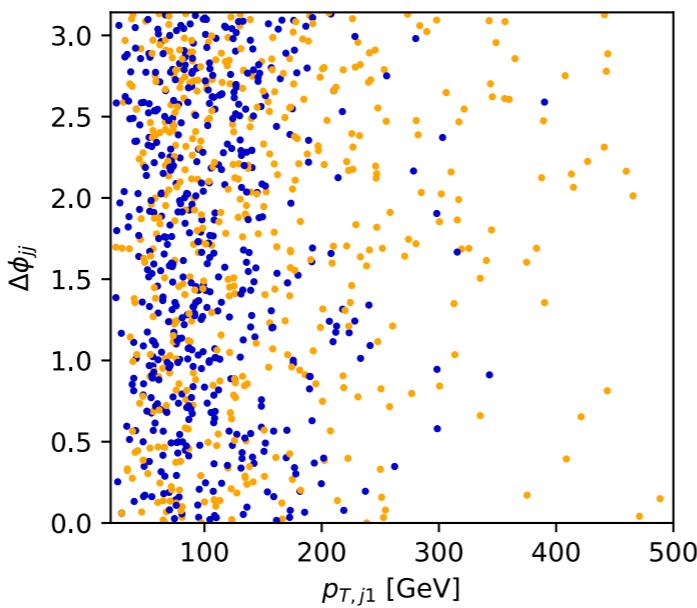


# Expected limits (asymptotics)

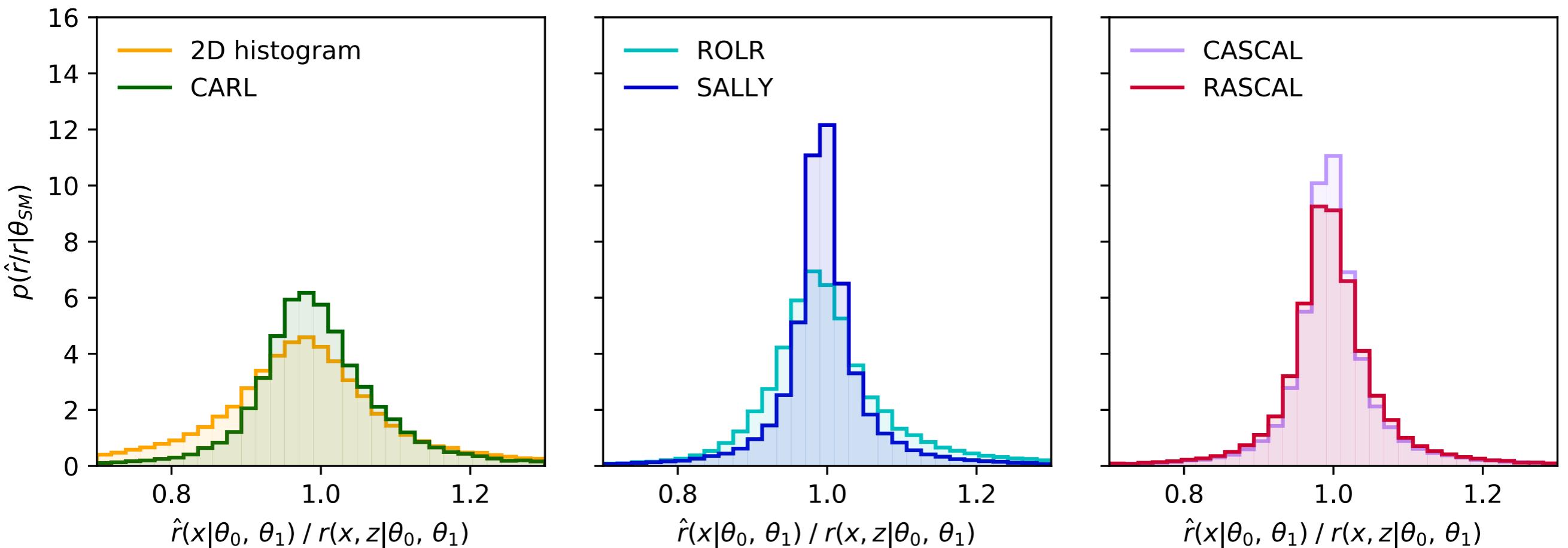


# Expected limits (Neyman constr.)

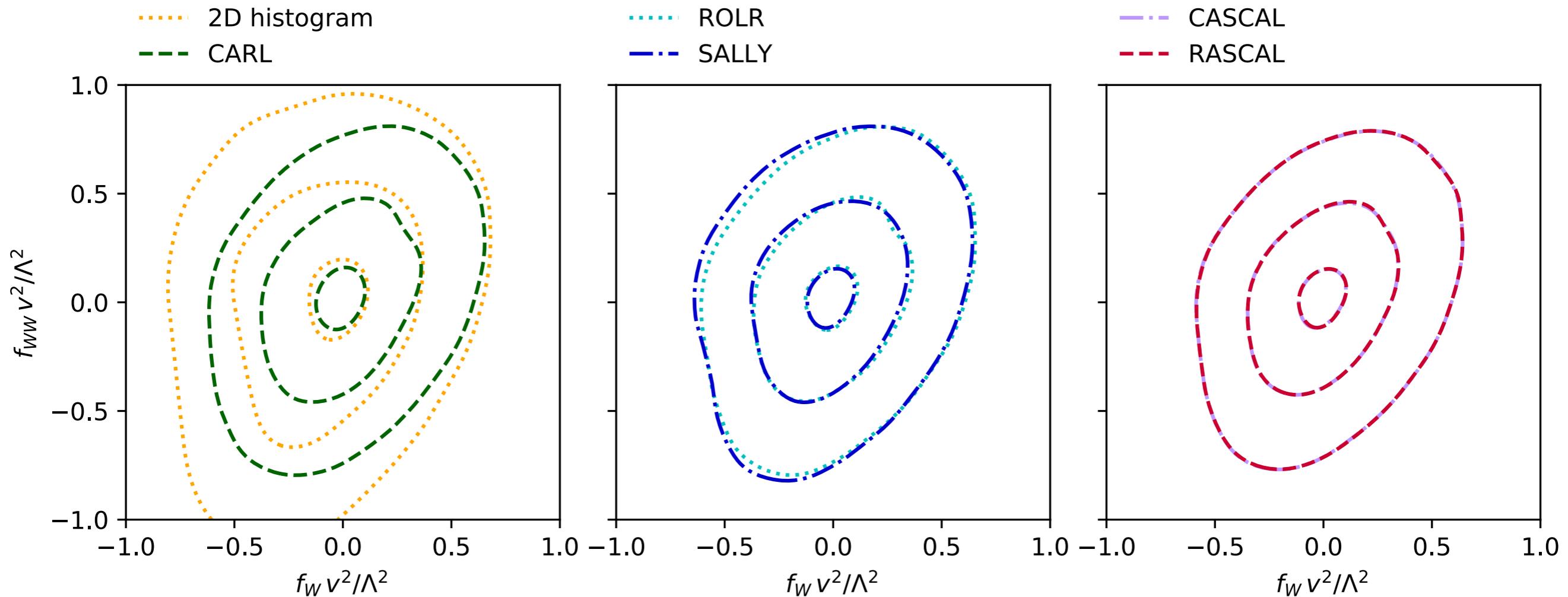




# Detector effects



# Constraints with detector effects



The End