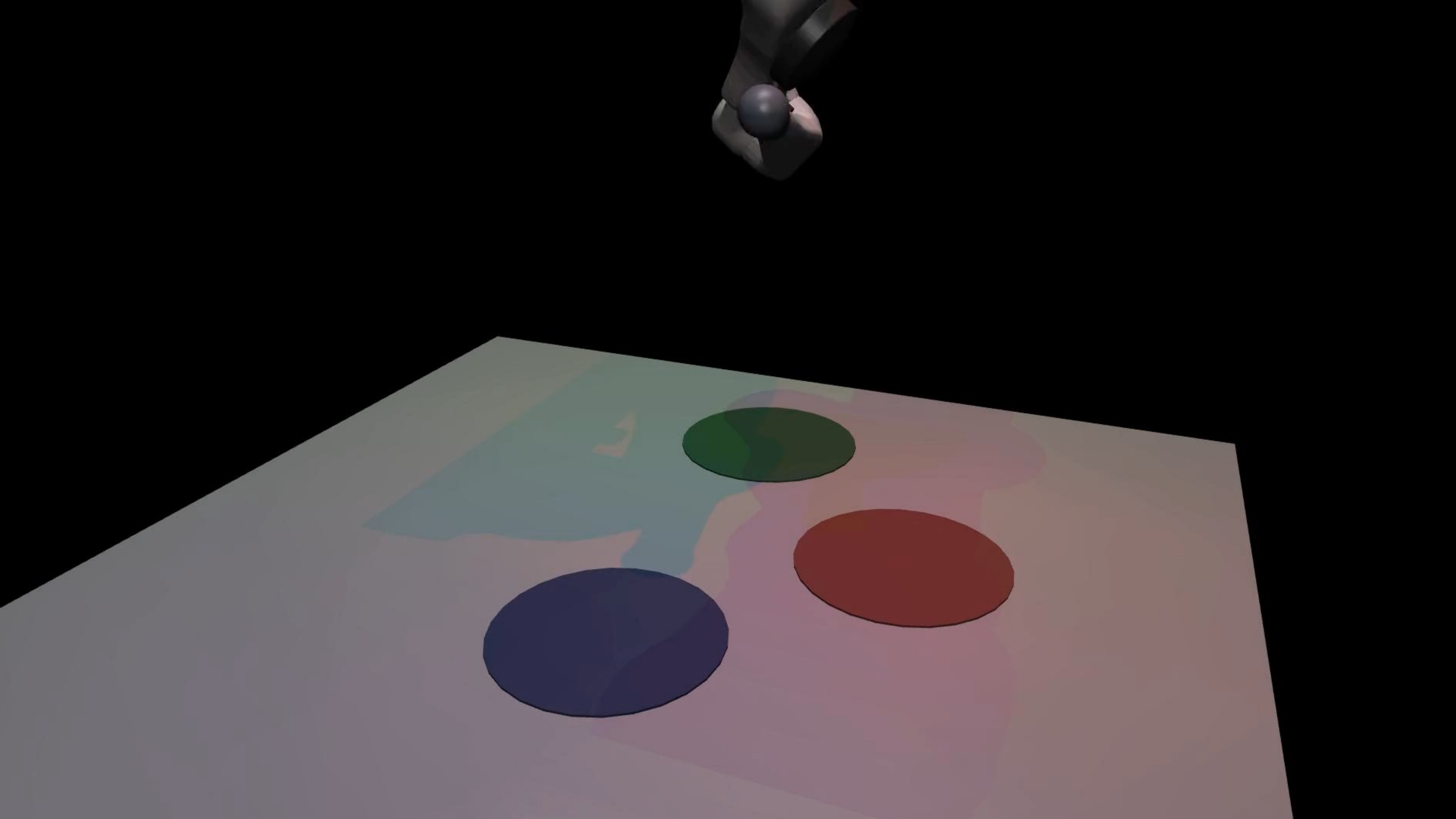
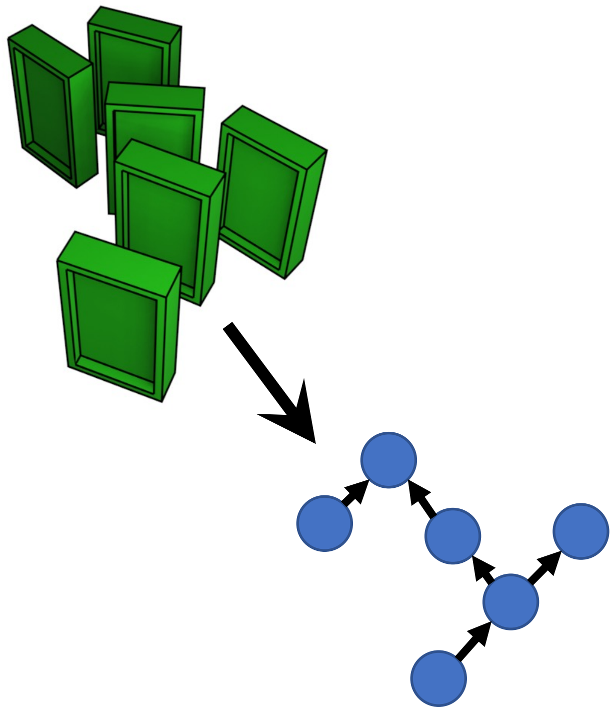
Causal representations and how to learn them

Johann Brehmer

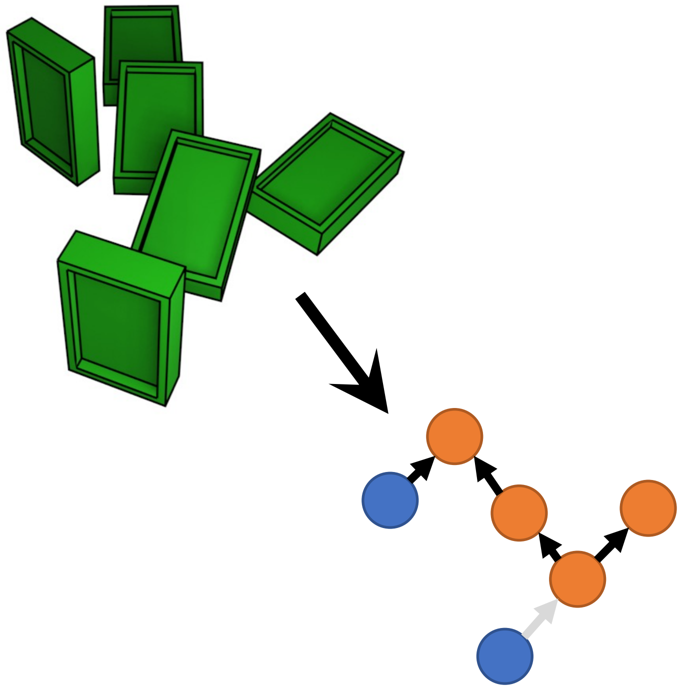
Qualcomm Technologies Netherlands B. V.

Work with Pim de Haan, Phillip Lippe, and Taco Cohen

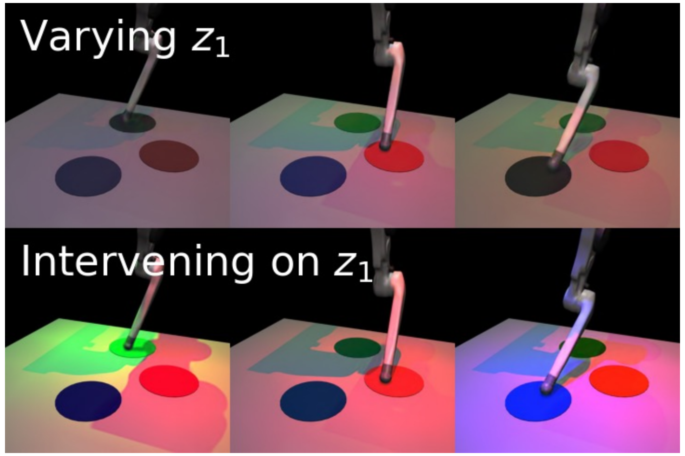




Can we **learn causal variables & causal structure from pixels**, without labels?



We prove: this is possible with **weak supervision**, when observing effects of interventions

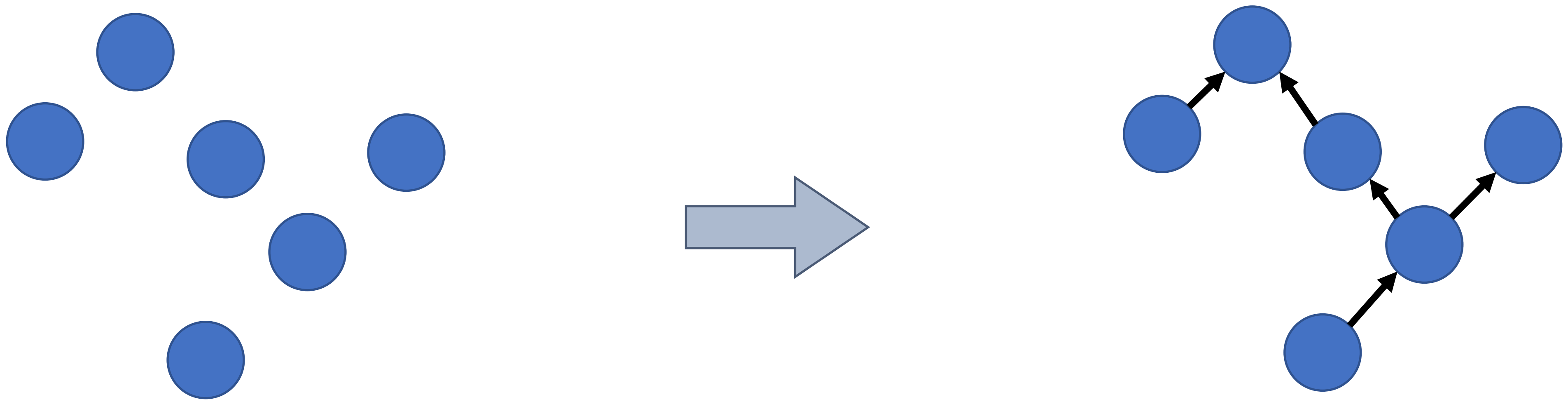


In practice, **implicit latent causal models** can identify the causal structure in image datasets

Problem

Can we learn causal representations from pixels?

Causal discovery / inference

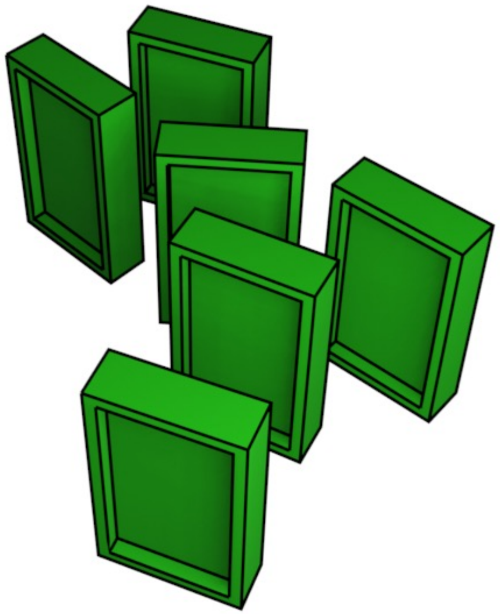


Given: dataset in terms of
high-level causal variables

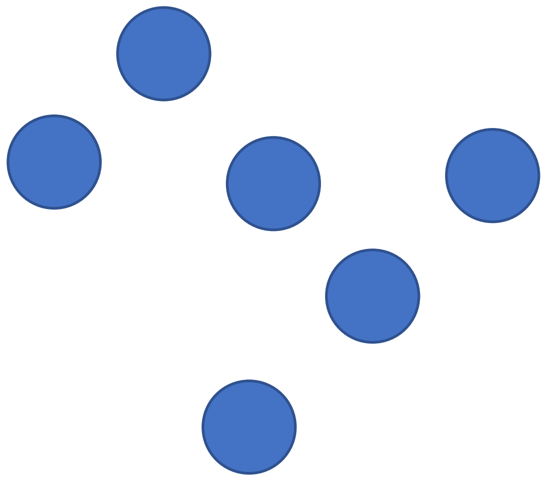
Goal: learn the
causal structure

But: what if we don't observe the causal variables?

Disentangled representation learning



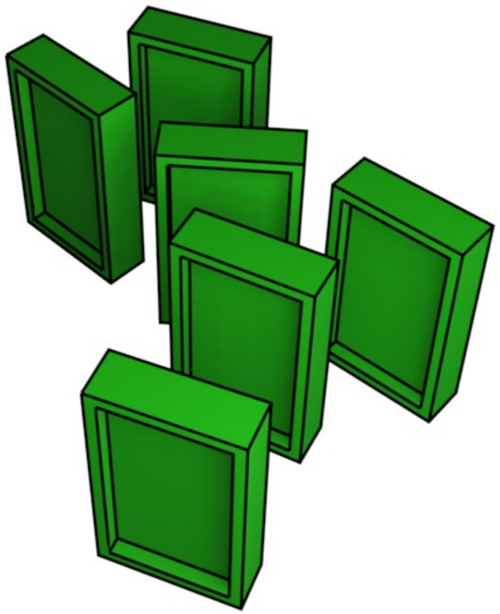
Given: **low-level, unstructured data representation**
(e.g. pixels)



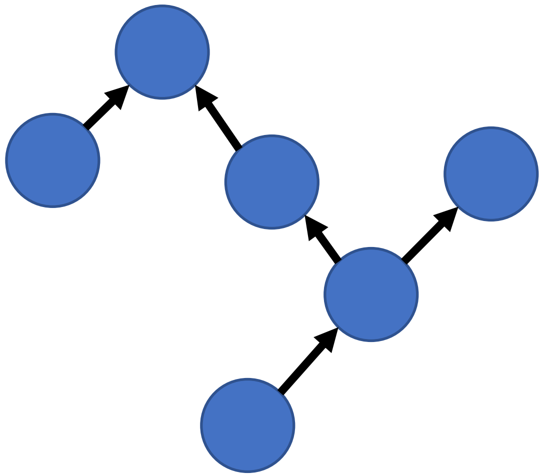
Goal: learn encoder to **high-level variables**
(e.g. object positions, states, ...),
usually **assuming independence**

But: useful high-level concepts are rarely independent

Causal representation learning

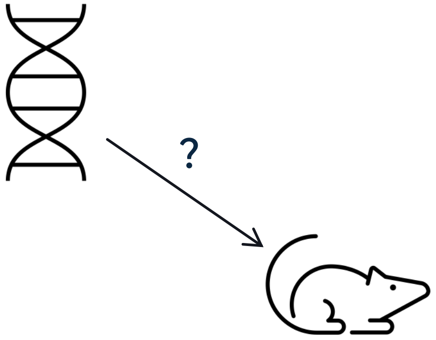


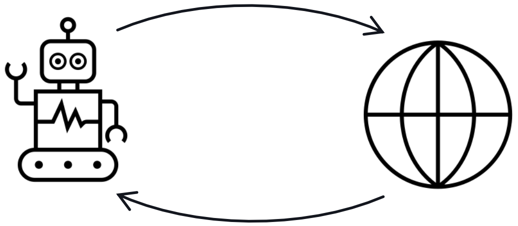
Given: **low-level, unstructured data representation** (e.g. pixels)

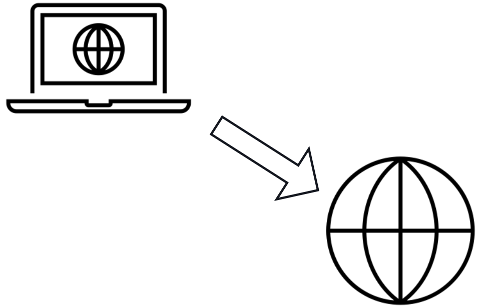


Goal: learn encoder to **high-level variables** (e.g. object positions, states, …) **and their relations / causal structure**

Why learn causal representations?







Causal structure may be of **scientific interest**

Causal representations are **abstractions** that may be **useful for planning**

Causal models may be more **robust to changes**

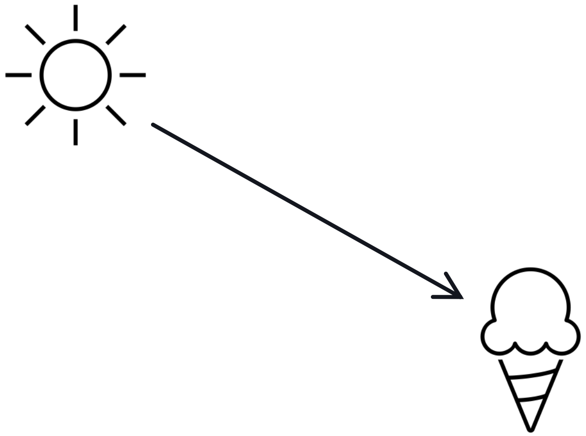
Arguably, these potential benefits have not yet been clearly demonstrated

[Recent review: B. Schölkopf et al, "Towards causal representation learning", IEEE Advances in Machine Learning and Deep Neural Networks 2021]

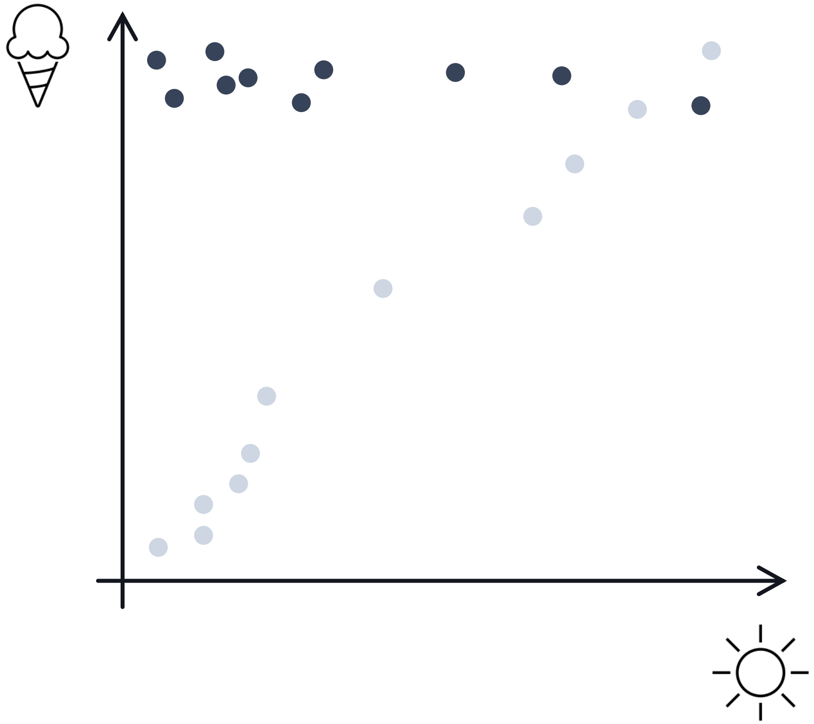
Background

Causality and identifiability

Causality



Semantically, causal models label relations between random variables as **cause-effect relations**



Functionally, causal models describe **probability distributions and how they change** under changing conditions

Structural causal models (SCMs)

- **SCM:**

Causal variables — Noise variables with base distribution $\varepsilon_i \sim p_i(\varepsilon_i)$

$$z_1 = f_1(\varepsilon_1) \qquad z_2 = f_2(\varepsilon_2; z_1)$$

Causal mechanisms — Causal parents in acyclic graph

- **Solution:**

$$z = s(\varepsilon) \;\Rightarrow\; z \sim p_z(z)$$

Solution function (= successively applying causal mechanisms)

Observational distribution

- **Interventions:**

$$f_i(\varepsilon_i; z_{\text{parents}}) \to \tilde{f}_i(\varepsilon_i) \qquad \Rightarrow\; z \sim \tilde{p}_z^i(z)$$

New mechanism (perfect intervention: no parents)

Interventional distribution

Identifiability

- An representation / SCM $\mathcal{M}$ is **identifiable** if

Any two model
(from some family)

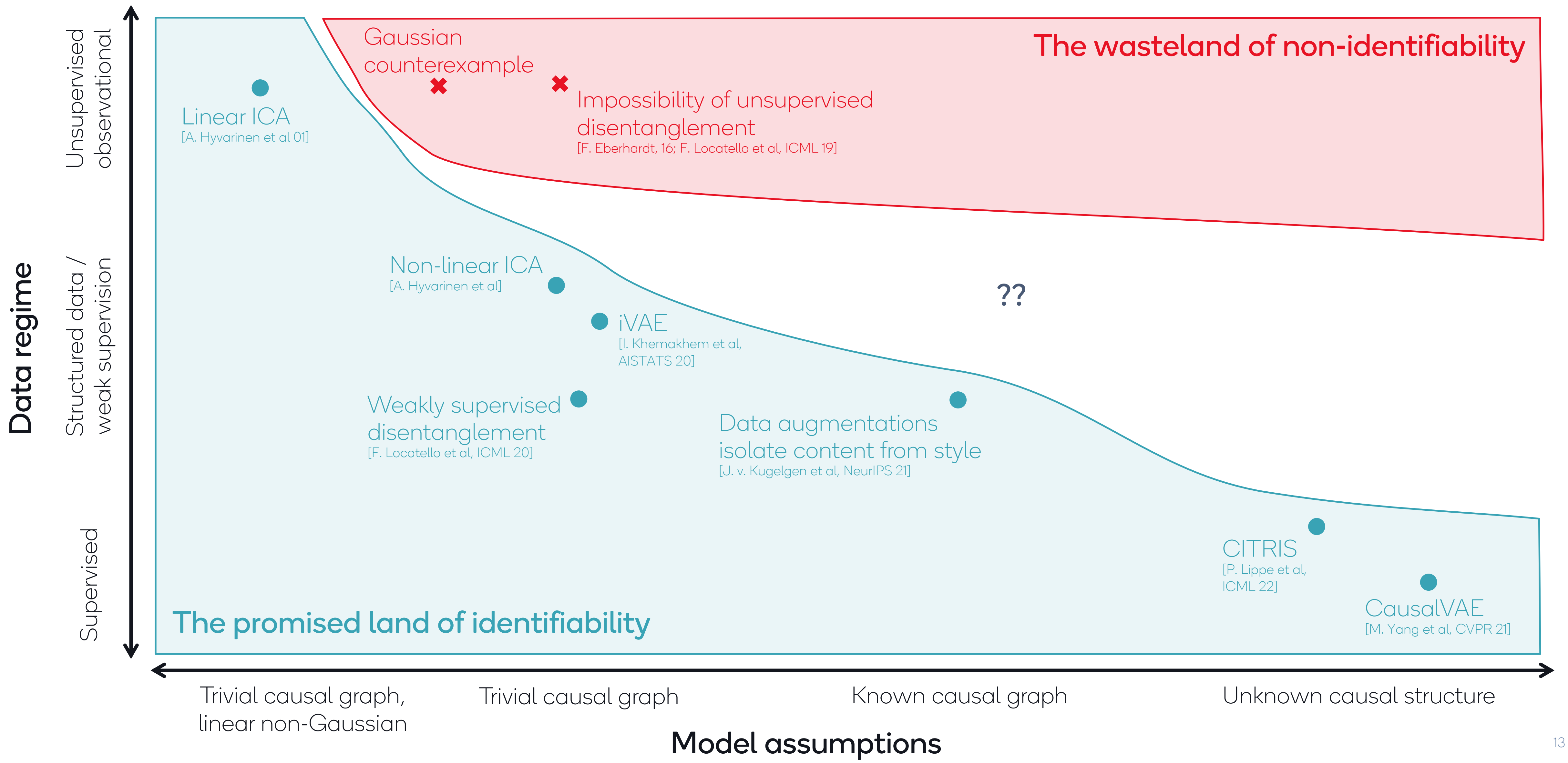
$$p_{\mathcal{M},x}(x) = p_{\mathcal{M}',x}(x) \implies \mathcal{M} \sim \mathcal{M}'$$

Data regime
(e.g. observational
distribution on pixel level)

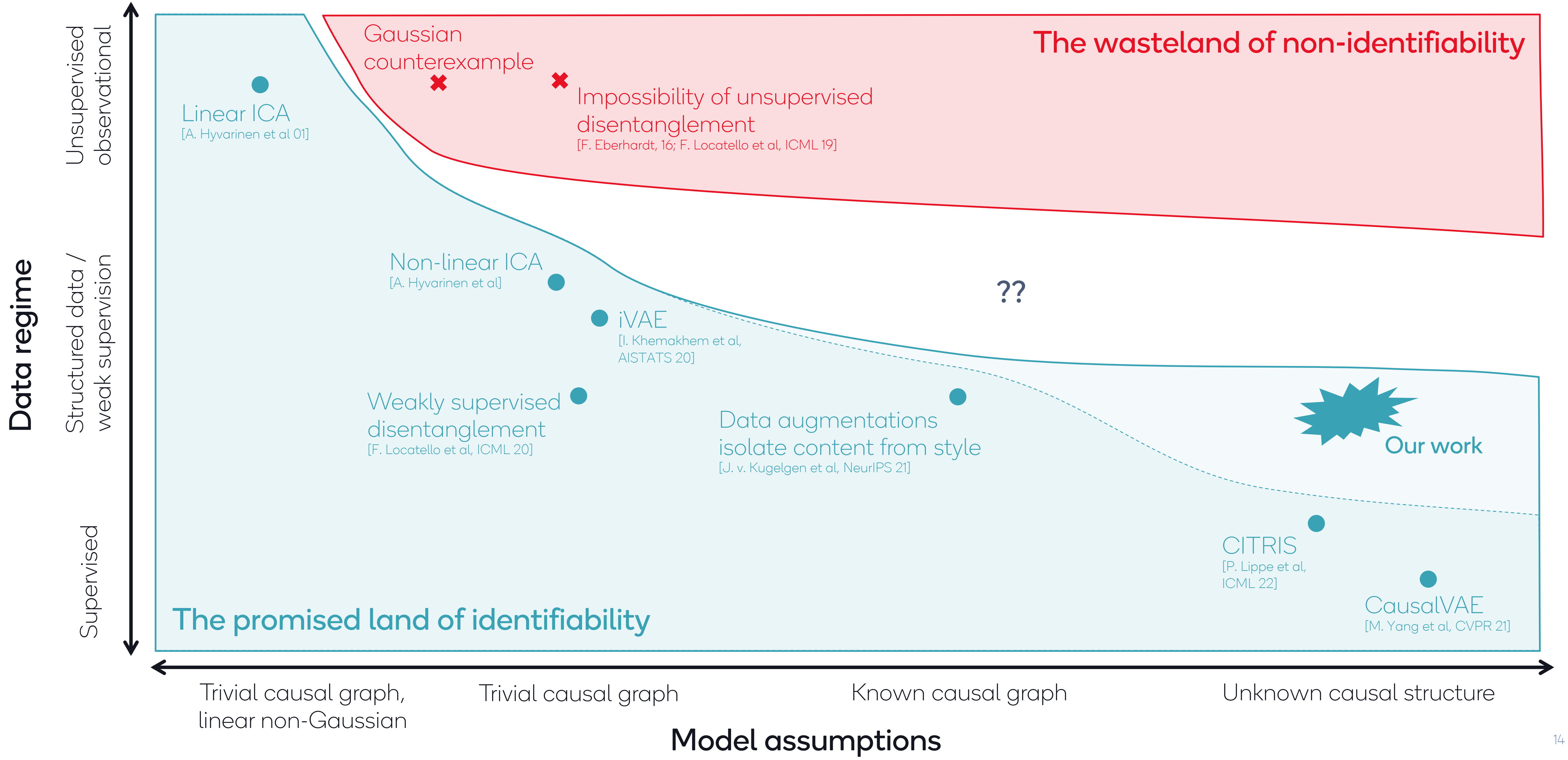
Equivalence relation
(e.g. same up to
permutations)

- Identifiability means we can **find ground-truth causal structure** through maximum-likelihood training
  - if it is within the specified model family
  - up to the equivalence relation
  - in the limit of infinite data
  - assuming perfect training

When are causal representations are identifiable?



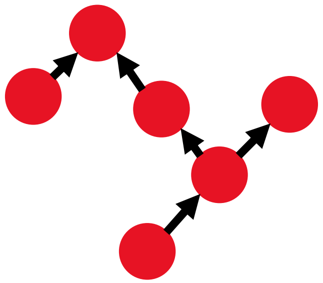
When are causal representations are identifiable?



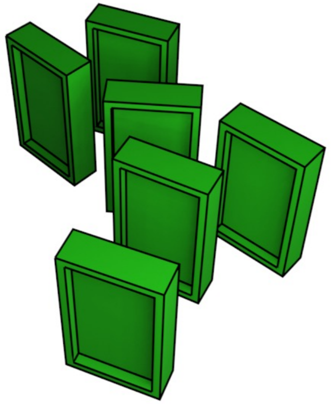
Theory

Causal representations can be identified from weak supervision

Latent causal model



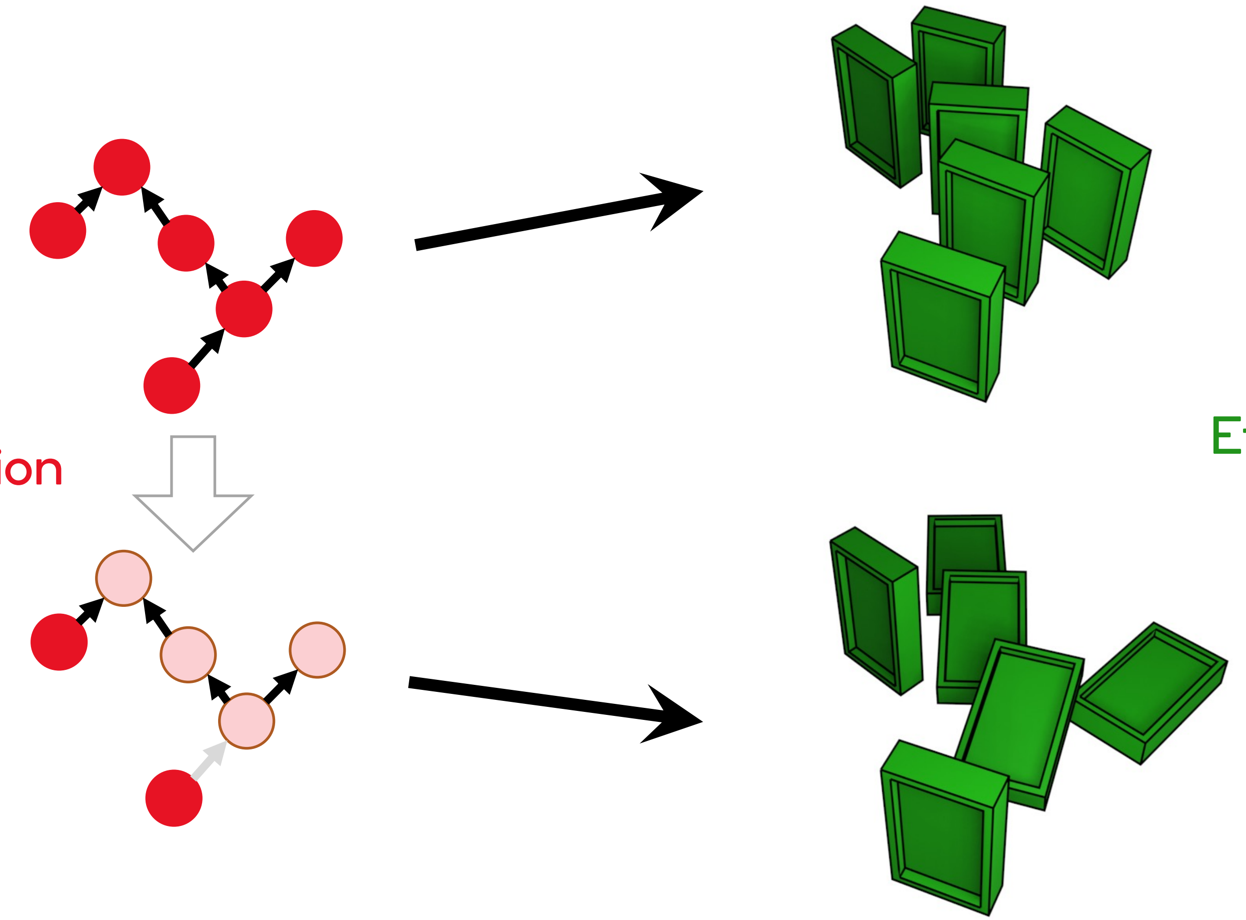
Decoder / rendering function



High-level variables with a structural causal model between them

Low-level data (pixels)

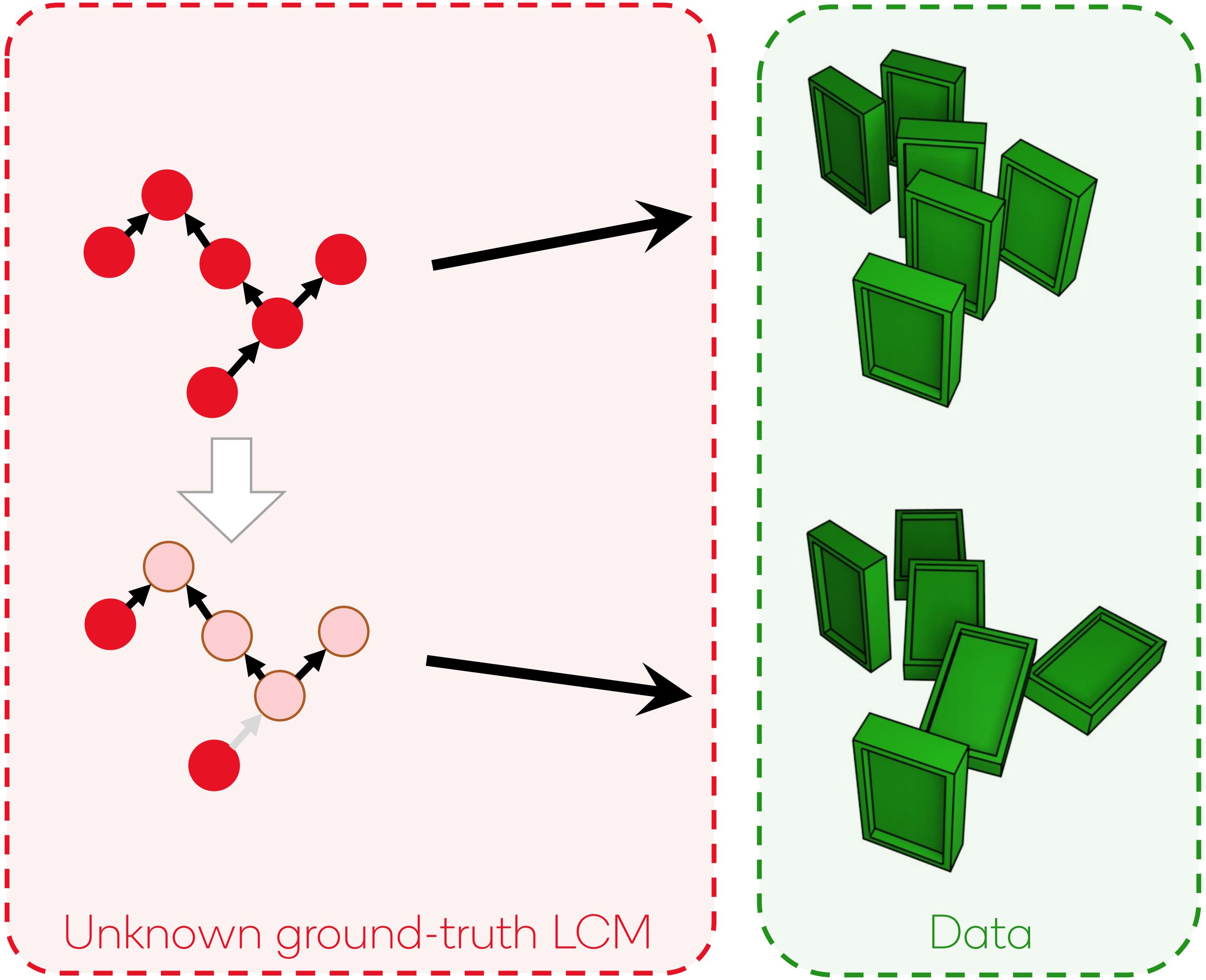
Interventions



Intervention

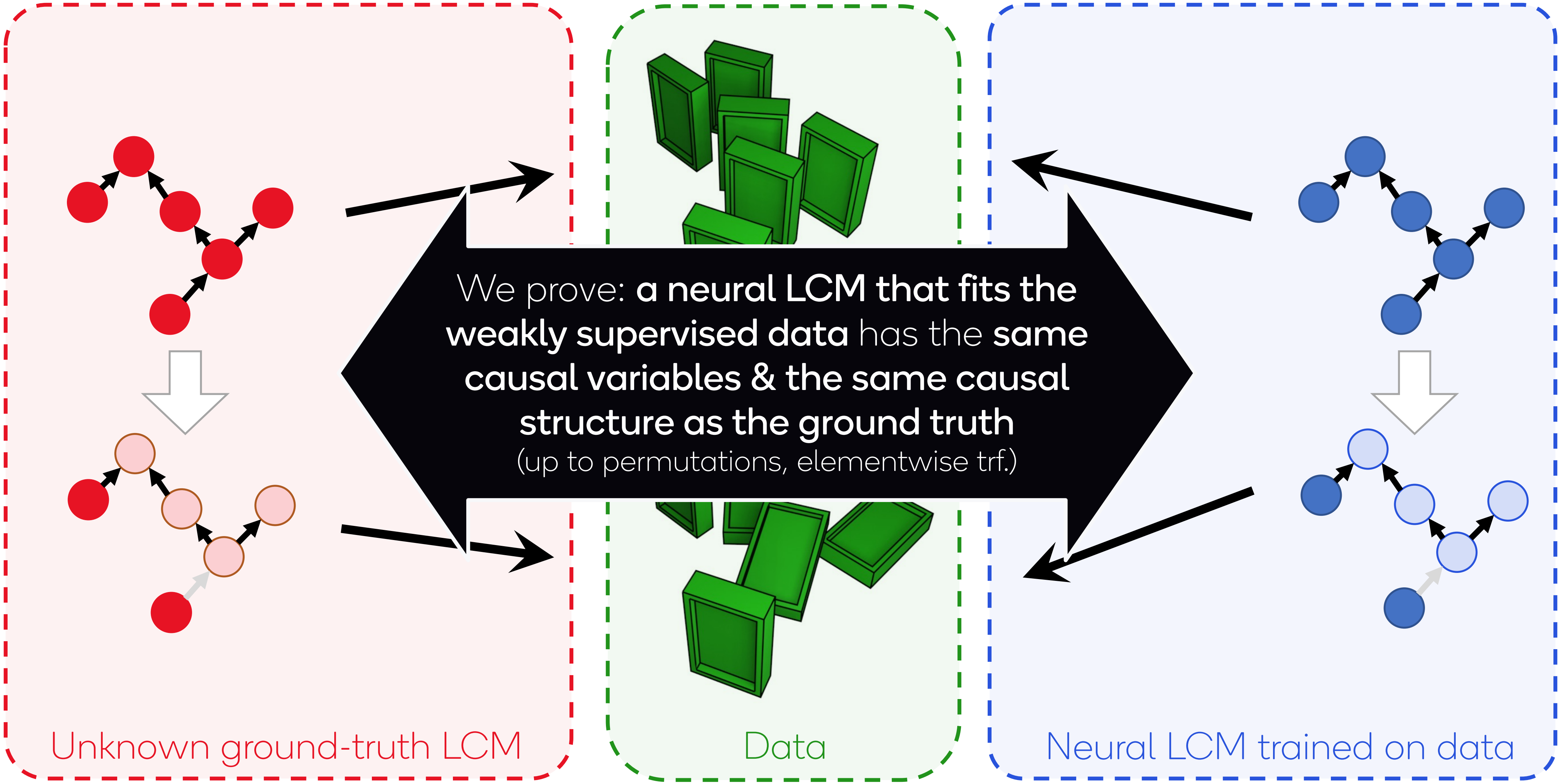
Effect of intervention
in data space

Weakly supervised data setting



- We assume access to **data pairs of the system before and after interventions**
  - Equivalent to counterfactuals
  - Causal abstraction of time-series data
- Otherwise, **no labels**
  - Only pixel-level data is observed
  - Intervention targets are unknown

Identifiability theorem



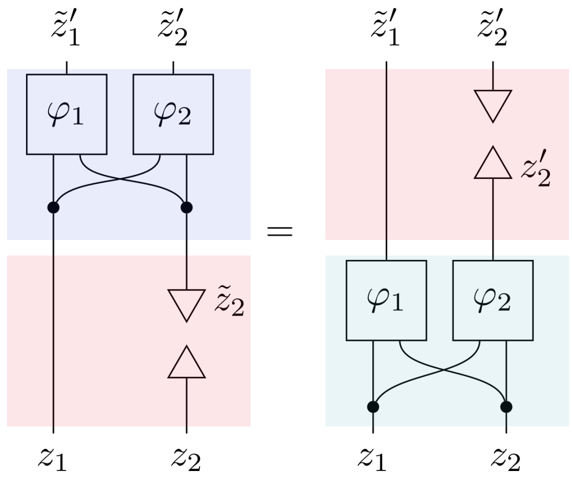
Proof sketch

1. Consider two LCMs with causal variables z and z', both matching the data.
   Define $\varphi : z \to z'$.

2. Interventions commute with φ:

$$\begin{tikzcd} \mathcal{Z} \arrow[r, "\text{intervene}"] \arrow[d, "\varphi"] & \tilde{\mathcal{Z}} \arrow[d, "\varphi"] \\ \mathcal{Z}' \arrow[r, "\text{intervene}"] & \tilde{\mathcal{Z}}' \end{tikzcd}$$

3. We assume perfect interventions. Then then $\tilde{z}_i'$ is independent of z_j . For 2 variables:



4. We assume $\mathbb{R}$-valued variables. Statistical independence then implies functional independence. Thus, $\varphi_i(z_i, z_j)$ must be constant in z_j.

5. Since this holds for any i, φ must be a permutation plus elementwise transformations.

6. Finally, we can show that the causal graphs and intervention targets in the two models are consistent with this transformation.

7. Thus the two models are isomorphic.

Assumptions

Assumption	Possible relaxation
Weakly supervised data is available	Maybe (first results)
Causal variables are $\mathbb{R}$-valued	Maybe (some ideas)
Causal mechanisms are diffeomorphic	Difficult
No hidden confounders	Difficult
Decoder is deterministic	Plausible (as in iVAE)
Interventions are perfect (Post-intervention values of intervention targets are independent of pre-intervention state)	Difficult (counterexamples)
Interventions are complete (The dataset contains interventions on any single causal variable)	Relaxation to n-target interventions plausible (incomplete interventions $\rightarrow$ partial identifiability)

Practice

Implicit is better than explicit

Explicit and implicit representations of causal structure

Explicit representation
through graph & causal mechanisms:

Causal variables

Noise variables with
base distribution
$\varepsilon_i \sim p_i(\varepsilon_i)$

$$z_i = f_i(\varepsilon_1; z_{\text{parents}})$$

Causal mechanisms

Causal parents
in acyclic graph

Implicit representation
through solution function:

Causal variables

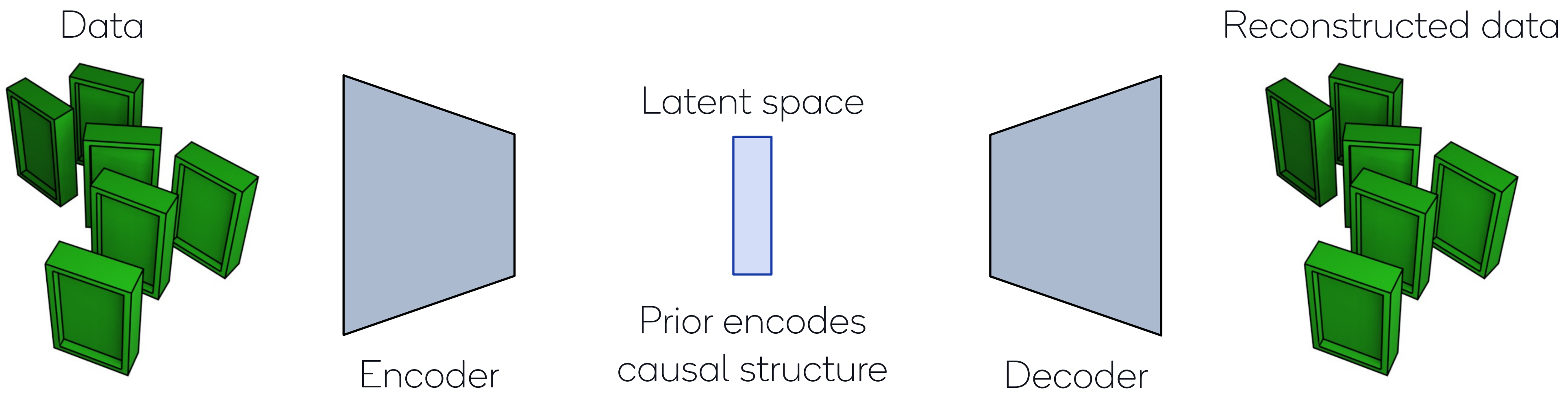
Noise variables with
base distribution
$\varepsilon_i \sim p_i(\varepsilon_i)$

$$z = s(\varepsilon)$$

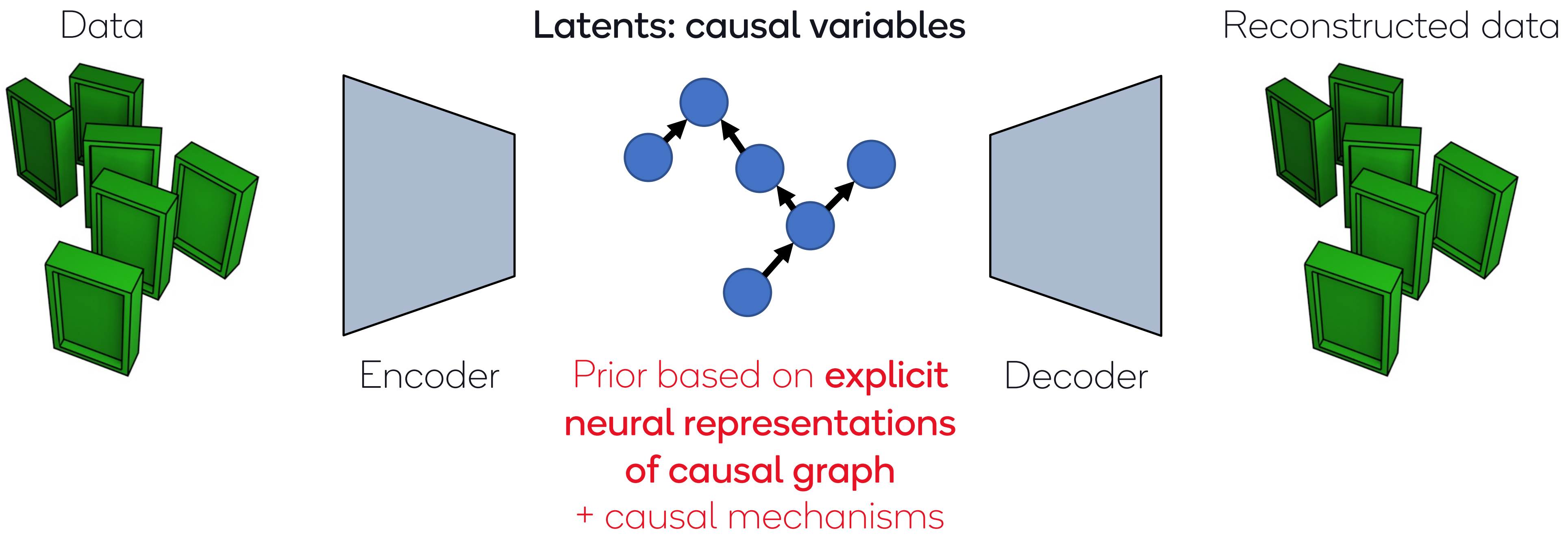
Solution function
(= successively applying
causal mechanisms)

Under our assumptions, explicit and implicit
representation **contain the same information**

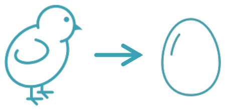
Operationalizing latent causal models



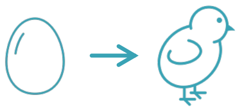
Explicit latent causal models



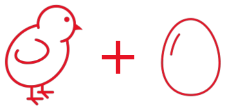
Explicit latent causal models in practice



Easy to learn graph given representations



Easy to learn representations given graph

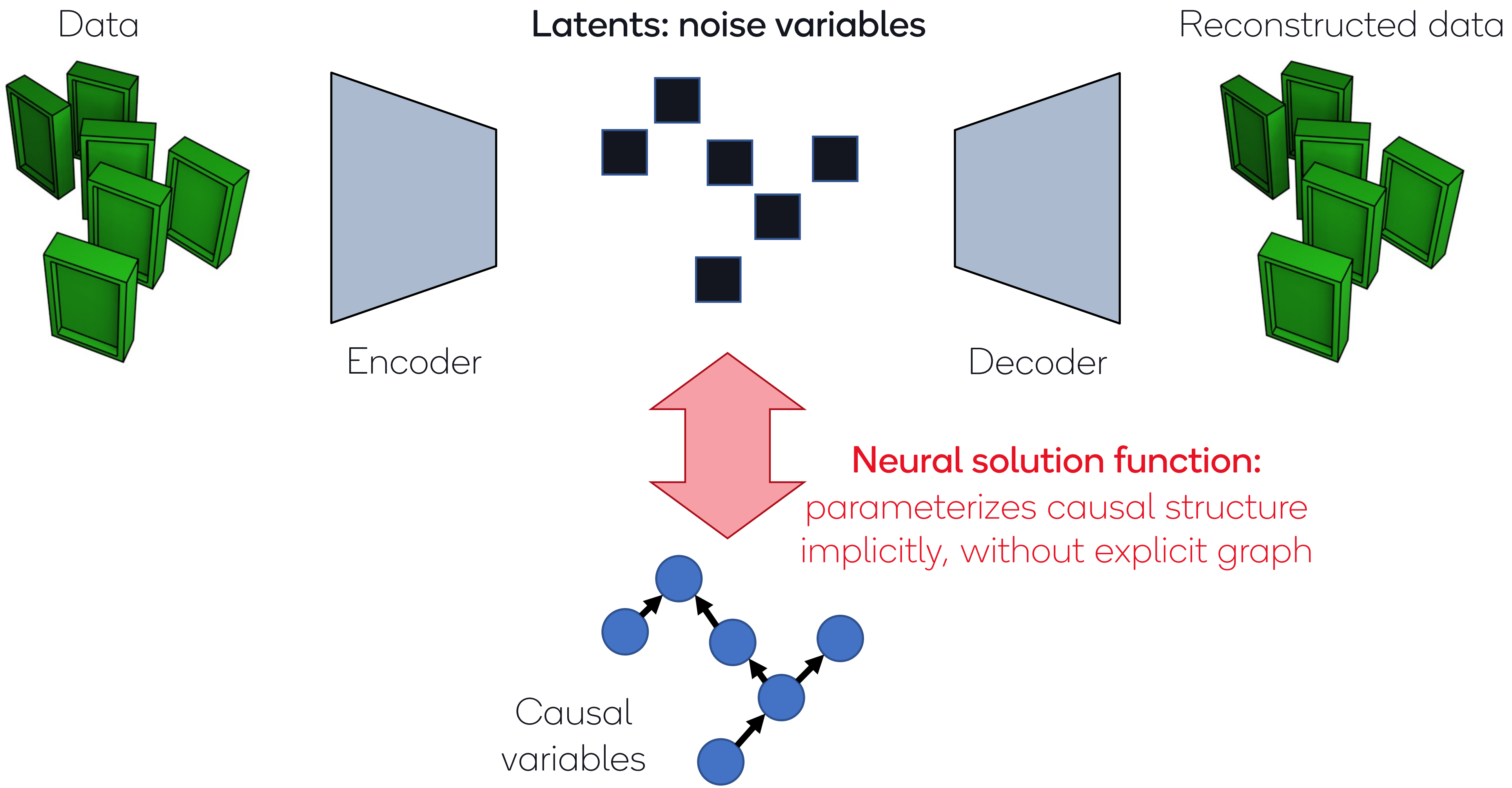


Difficult to learn graph and representation simultaneously

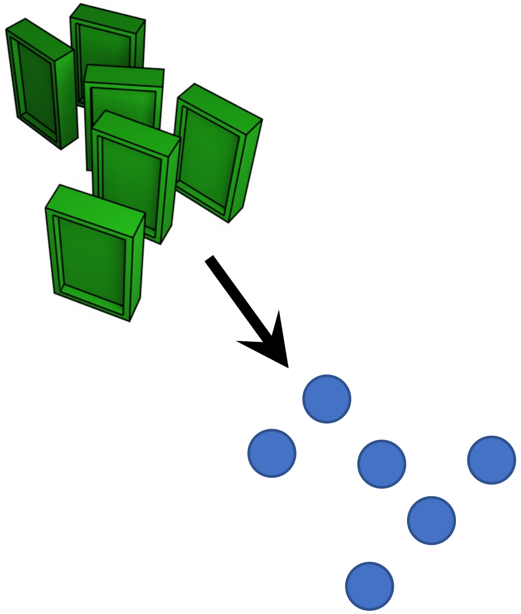
(Evidence for **local minima** in the loss landscape corresponding to wrongly oriented graph edges)

⇒ **don’t learn a graph if you don’t have to**

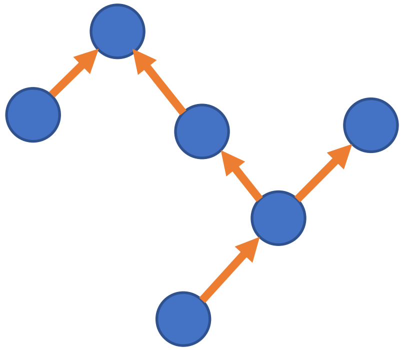
Implicit latent causal models



What can you do with ILCMs?

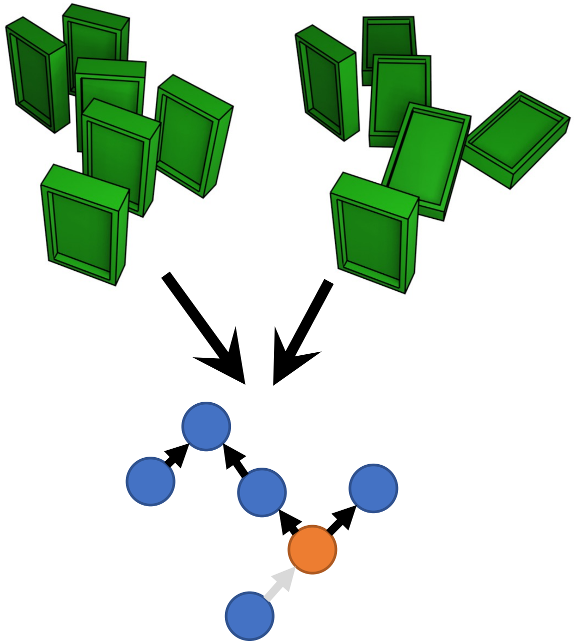


Map pixels to causal variables

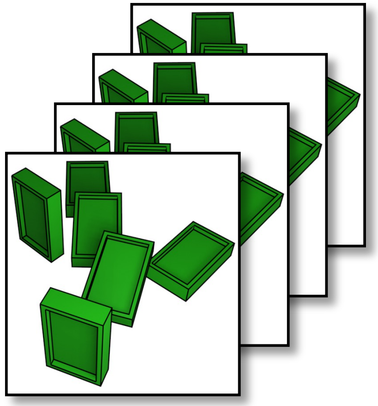


Find the causal graph

- ILCM-E: with off-the-shelf causal discovery algorithm ENCO
- ILCM-H: with our new heuristic



Infer interventions from data pairs

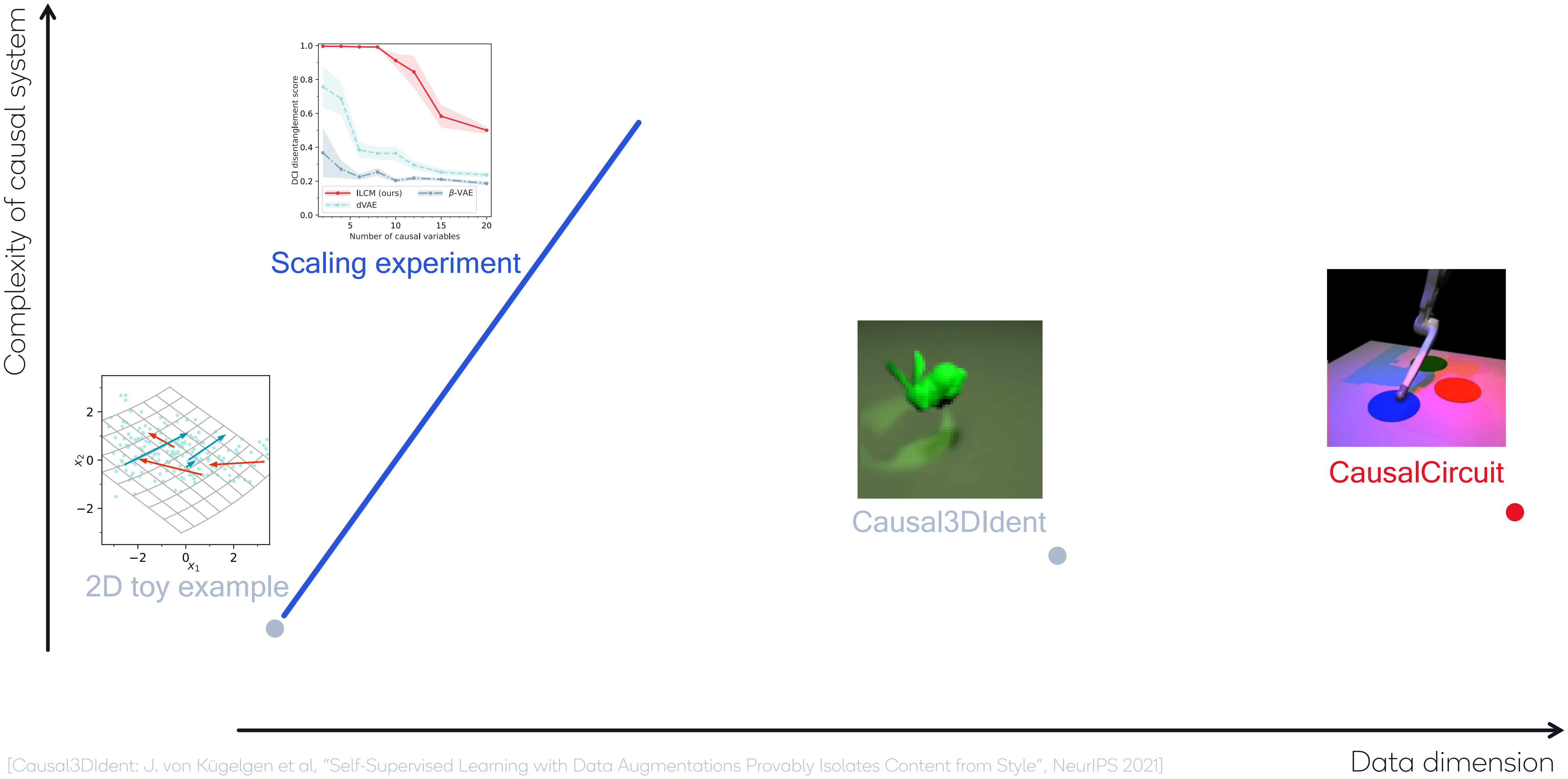


Generate observational, interventional, and counterfactual data

Experiments

Things work, mostly

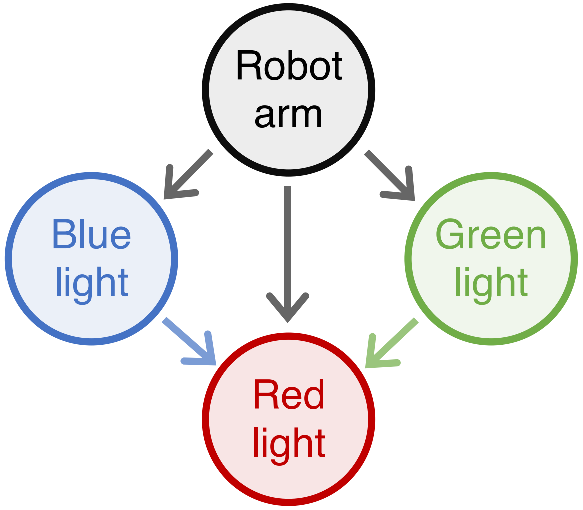
Experiments

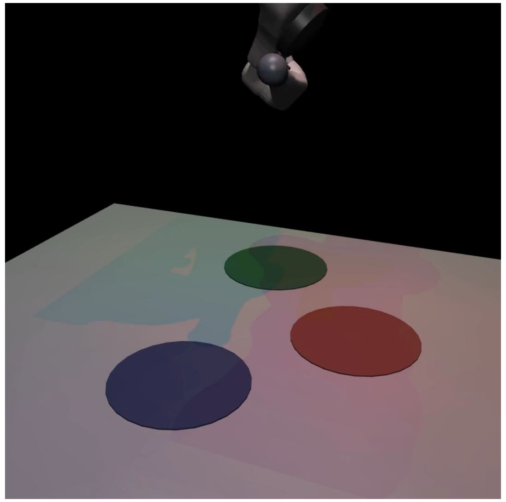


[Causal3DIdent: J. von Kügelgen et al, "Self-Supervised Learning with Data Augmentations Provably Isolates Content from Style", NeurIPS 2021]

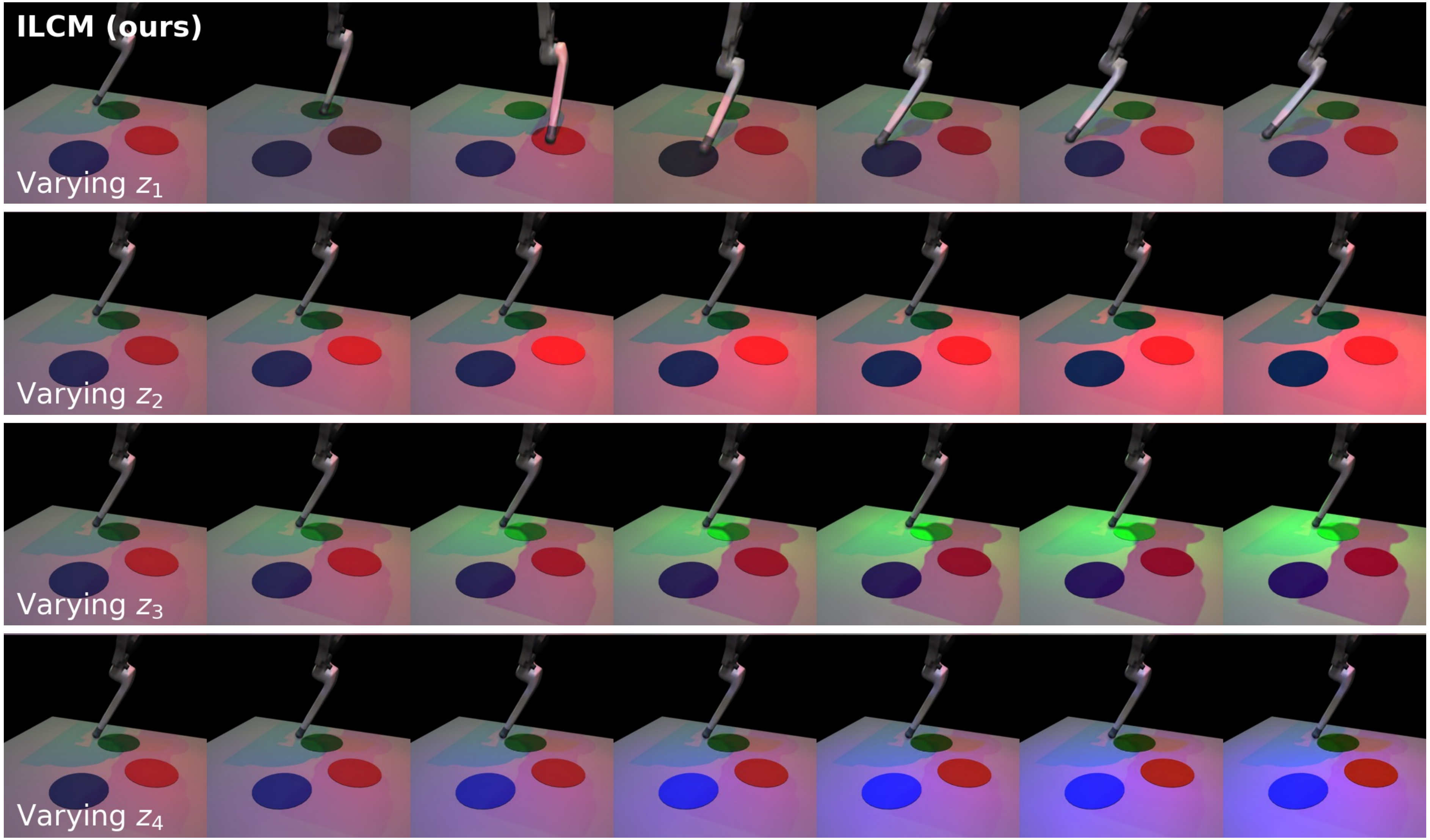
CausalCircuit

- **New dataset** with more intuitive causal structure
- **Robot arm interacts with touch-sensitive lights, which are connected with a circuit**
  - Robot arm movement based on inverse kinematic model
  - Physics + rendering with MuJoCo
  - 4 continuous causal variables: robot arm restricted to 1D arc + 3 light states
  - 512x512 images from fixed camera position
- ILCMs are trained on pre- and post-intervention data

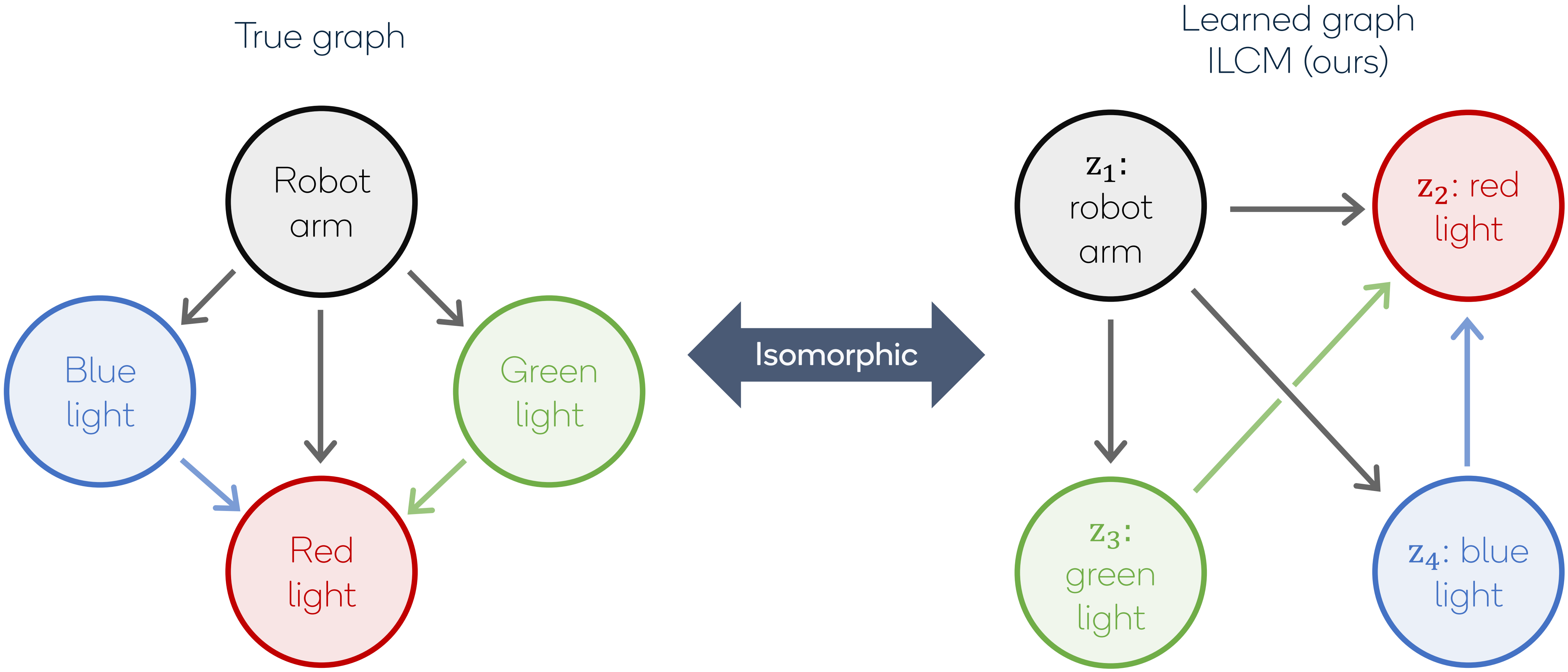




LCMs **disentangle** the causal variables

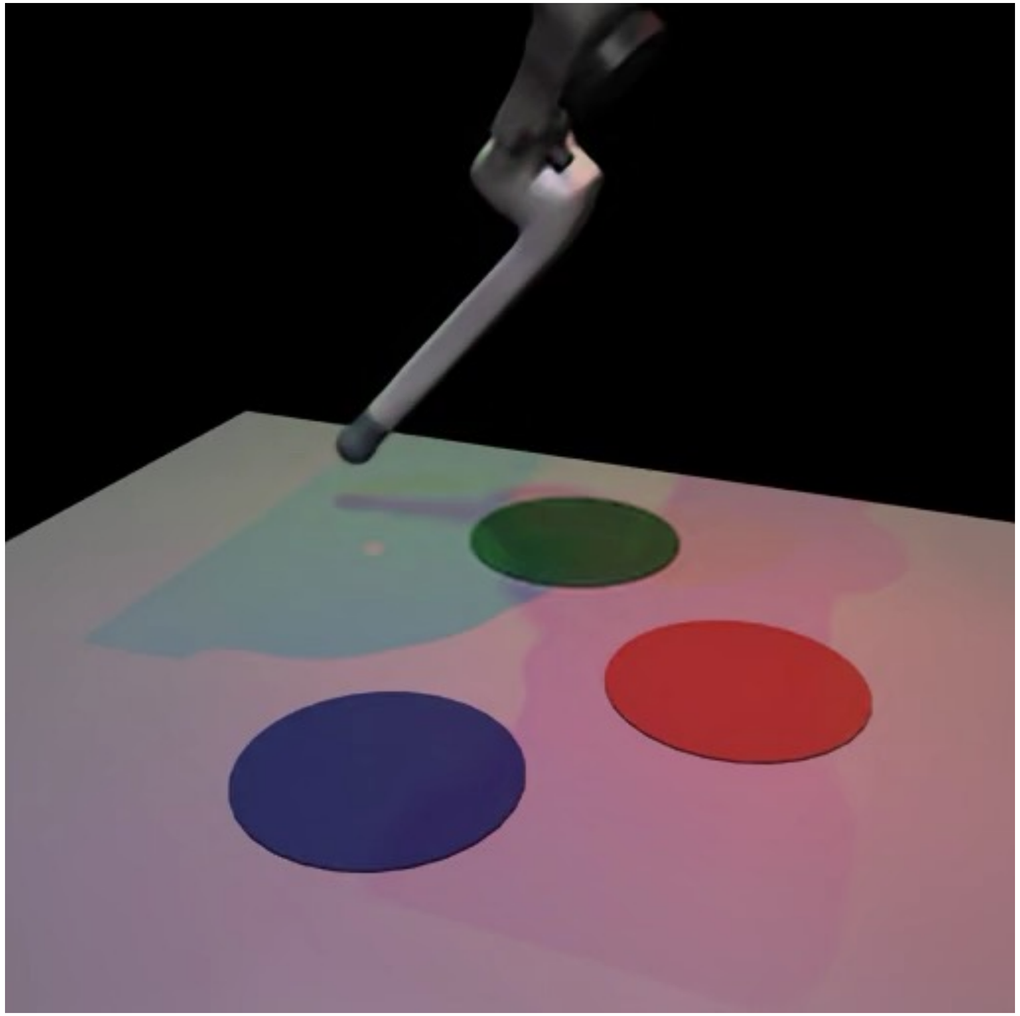


LCMs learn the **correct graph**



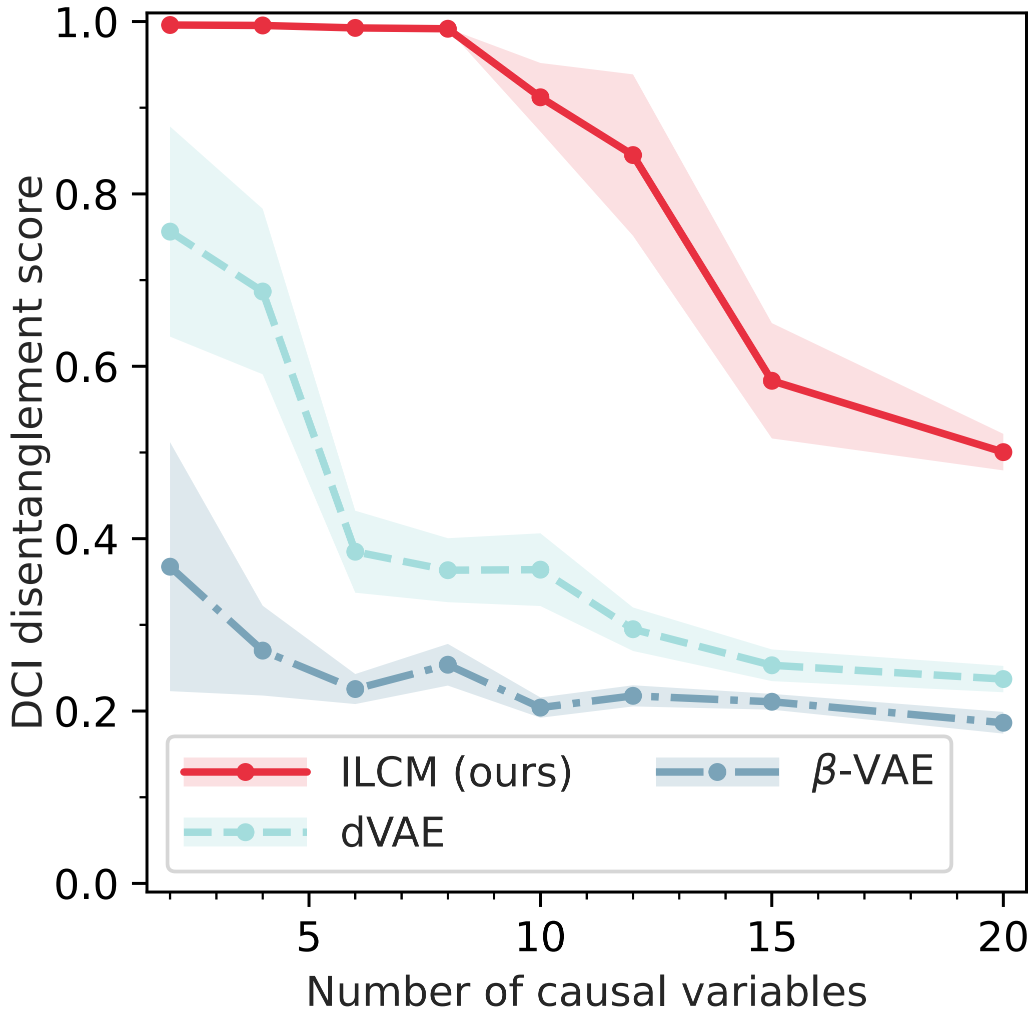
ILCMs let us **reason causally**

ILCM samples, **intervening** on a single latent (including causal effects)



Do ILCMs **scale**?

- **Toy experiment**:
  - n causal variables
  - linear causal effects
  - SO(n) decoder
- ILCM results **robust up to ~10 variables** without additional tuning

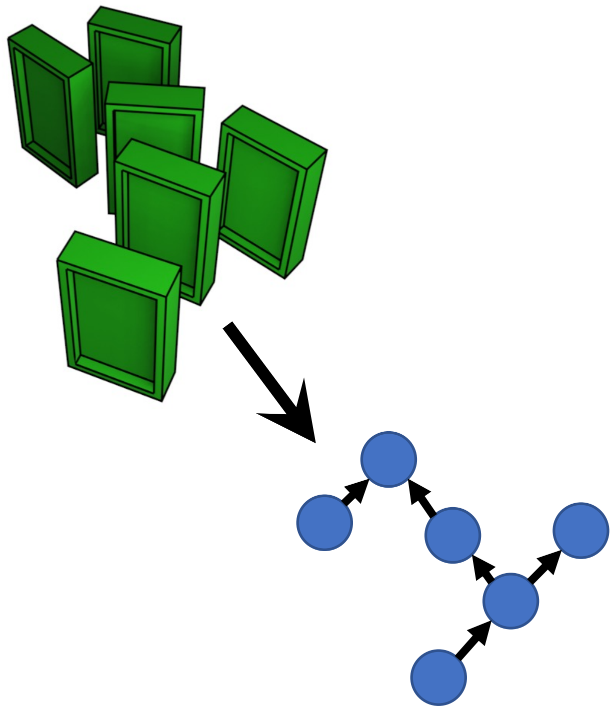


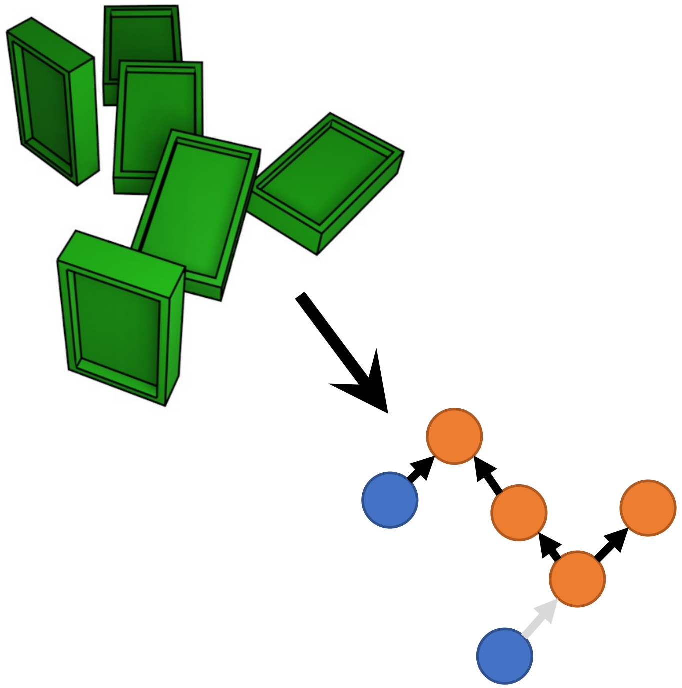
Outlook

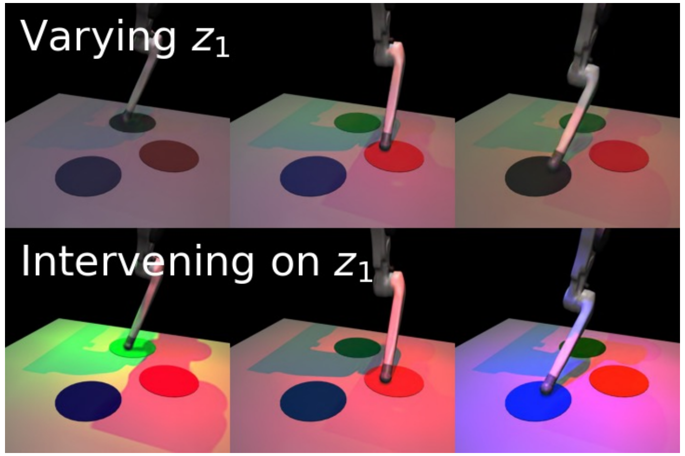
Towards useful causal representation learning

A long way to go

Where we are	Where we need to get
Identifiability theorems	**Demonstrate usefulness** on downstream tasks
Pre- & post-intervention data	**Realistic data regimes:** observational & interventional data, video data, ...
God-given interventions	**Learning intervention policies**
Fixed causal variables	**Variable scene composition**
Strict **DAG-based causality**	**Weaker relational structures**
Toy experiments (up to O(10) variables)	**Realistic experiments**







Can we **learn causal variables & causal structure from pixels**, without labels?

We prove: this is possible with **weak supervision**, when observing effects of interventions

In practice, **implicit latent causal models** can identify the causal structure in image datasets

Weakly supervised causal representation learning

JB*, Pim de Haan*, Phillip Lippe, Taco Cohen

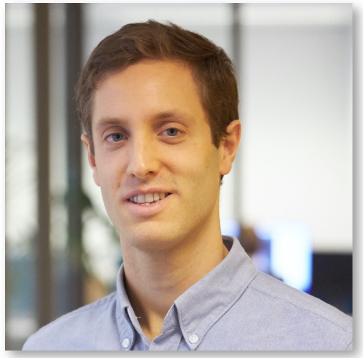
*equal contribution

NeurIPS 2022

arXiv:2203.16437

Pim de Haan

Phillip Lippe



Taco Cohen

Towards causal representation learning
Bernhard Schölkopf, Francesco Locatello, Stefan Bauer, Nan Rosemary Ke, Nal Kalchbrenner, Anirudh Goyal, Yoshua Bengio
IEEE 2021, arXiv:2102.11107

Weakly-supervised disentanglement without compromises
Francesco Locatello, Ben Poole, Gunnar Rätsch, Bernhard Schölkopf, Olivier Bachem, Michael Tschannen
ICML 2020, arXiv:2002.02886

Self-supervised learning with data augmentations provably isolates content from style
Julius von Kügelgen, Yash Sharma, Luigi Gresele, Wieland Brendel, Bernhard Schölkopf, Michel Besserve, Francesco Locatello
NeurIPS 2021, arXiv:2106.04619

CITRIS: Causal identifiability from temporal intervened sequences
Phillip Lippe, Sara Magliacane, Sindy Löwe, Yuki M. Asano, Taco Cohen, Efstratios Gavves
ICML 2022, arXiv:2202.03169

Interventional causal representation learning
Kartik Ahuja, Divyat Mahajan, Yixin Wang, Yoshua Bengio
arXiv:2209.11924

Causal triplet: an open challenge for intervention-centric causal representation learning
Yuejiang Liu, Alexandre Alahi, Chris Russell, Max Horn, Dominik Zietlow, Bernhard Schölkopf, Francesco Locatello
CLeaR 2023, arXiv:2301.05169

Thank you

Qualcomm

Follow us on:

For more information, visit us at:

qualcomm.com & qualcomm.com/blog