

On Optimal Anchor Placement for Efficient Area-based Localization in Wireless Networks

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Abstract—Area-based localization is a simple and efficient approach, where each node estimates its position based on proximity information to some special nodes with known location, called anchors. Based on the anchors' coordinates, each node first determines its residence area and then approximates its position as the centroid of that area. Therefore, the accuracy of the estimated position depends on the size of the residence area; the smaller the residence area is, the better the accuracy is likely to be. Because the size of the residence area mainly depends on the number and the positions of anchor nodes, their deployment should be carefully considered in order to achieve a better accuracy while minimizing the cost. For this purpose, in this paper we conduct a theoretical study on anchor placement for a very popular area based localization approach. We determine the optimal anchor placement pattern for increased accuracy and how to achieve a particular accuracy goal with the least anchor count. Our analytical results are further validated through simulation.

Keywords—Area based localization, anchor placement, triangular lattice, optimization

I. INTRODUCTION

Localization is one of the most important services, on which numerous wireless network applications relay heavily. For example, applications like indoor navigation, object tracking, environmental habitat, and controlled irrigation, require the knowledge of the individual node's position to achieve their design goal. Since GPS signals are not available in buildings or too expensive for a large outdoor wireless network, a wide range of solutions were proposed, each of them is designed to meet specific application requirements or to suit the adopted technology.

Existing localization schemes can be classified into two main categories; range-based and range-free. Range-based schemes [16], [19], [20] work by measuring the point-to-point distance or angle between each pair of communicating nodes, or between a node and an anchor (a reference node with a known position). Range-based solutions, generally require the use of a specific ranging hardware, which in turn need a special configuration and tuning to get the desired ranging quality. On the other hand, range-free localization [4], [9], [14], [17], [21] does not require point-to-point measurements and generally uses connectivity to estimate an approximate distance or uses received signal strength indication (RSSI) information to infer the near-far relationships between nodes and anchors. Area-based localization is a special class of

range-free methodologies in which the localization process determines a residence area within which the node is located.

Area-based localization is deemed effective in applications in which determining the exact position is not a must, the cost and complexity of range estimation hardware is unwarranted, and/or for which the increased communication and computation overhead of multi-iteration is not bearable [10], [11], [23]. The localization methodology relies on the presence of a set of anchors such that some of them can reach the nodes to be localized. Existing area-based localization algorithms employ one of four types of primitive geometric shapes to draw the nodes' residence areas, namely, a triangle [9], a ring [14], a half-symmetric lens [13], and a circle [21]. In general, the quality of the measured residence area is highly dependant on to the number and position of the anchors that a node hears from and on the geometric shape that the algorithm pursues. For this reason, it is very important to carefully place sufficient number of anchors to ensure certain level of localization accuracy.

In this paper we seek to analytically determine the optimal anchor placement strategy that ensures the best localization accuracy using the least anchor count. For this study, we consider the circle-based localization approach [21] and we analytically derive the best anchor placement pattern. We further confirm our results by extensive simulation. To the best of our knowledge, little attention, if any, has been paid by the research community to the problem of optimal anchors placement for area-based localization. The main contributions of this paper are as follow:

- We derive a condition for achieving the best accuracy, i.e., least average residence area, for a set of m neighboring anchors.
- We derive the optimal number of anchors which ensure that all nodes in the network can be localized.
- We determine the least anchor count and best placement pattern to achieve best accuracy when each node is to be reachable by two anchors.

The rest of this paper is organized as follow. In Section II we formulate the anchor placement problem and give some preliminary analysis. Section III presents the anchor placement pattern for the best localization accuracy. We validate the analysis results by simulation in Section IV. Section V summaries related works on anchor placement problem and area-based localization. Finally, Section VI concludes the paper.

II. PROBLEM FORMULATION AND ANALYSIS

Area-based localization algorithms, first determine the smallest possible area where a node is located in and then calculate the centroid of that area which will represent the estimated node position. To determine the residence area, the position and the proximity information to a set of neighboring nodes, called anchors, are required. Basically, the anchor position and proximity are used to define a possible area within which a node can be. The intersection of areas, concluded based on the proximity to the individual anchors, defines the residence area for that node. The accuracy of the estimated position depends on the size of the residence area; the smaller the size of the residence area is the better the accuracy is likely to be. Obviously the accuracy can be boosted by increasing the population of anchors in the vicinity of a node.

In this paper, we define optimality in terms of *anchor number* and *localization accuracy (residence area)*. Therefore, we opt to find the optimal anchor placement that achieves the best accuracy (minimal residence area) using the fewest anchors. In the balance of this section we first study how the anchors position affects the average size of the residence area, and derive a condition for minimizing the average. Then we use such a condition to determine an anchor placement pattern.

A. Minimizing the Average Residence Area

Let N be the number of nodes, which are uniformly deployed in an area of size T . Let A be the set of k anchors that will be deployed in the network, $A = \{A_1, A_2, \dots, A_k\}$. According to the anchors' positions, the deployment area will be divided into a set S of m non-overlapped regions, $S = \{x_1, x_2, \dots, x_m\}$ and $x_1 + x_2 + \dots + x_m = T$. Each node in the network will be then located within one of the regions in S , which will represent its residence area. Let X denote the random variable that represents the size of the node's residence area.

Theorem 1. *Let $E[X]$ be the expected residence area of any deployed node, and let m be the number of distinct sub-regions that constitute the total deployment area of size T . Then the minimal expected area is achieved iff all the sub-regions are equals.*

Proof: reader is referred to [1]. ■

Thus, (T/m) is the optimal choices for x_i , $i = 1, 2, \dots, m$, which means that the total area T should be divided into *equal* regions to obtain the minimum expected residence area (best localization accuracy). Based on Theorem 1, we study the optimal anchor placement for circle-based localization algorithm, which is a very popular area based localization approach. We first give a brief description of circle-based localization approach, and then we show the optimal anchor placement strategy.

B. Circle-based Localization Algorithms

In this class of algorithms [12], [21], a node calculates its residence area based on the assumption that the radio coverage is modelled as a perfect circle with a known radius R . So, each node can deduce whether it is within the areas covered by the radio range of its neighbor anchors. The overlapping region of

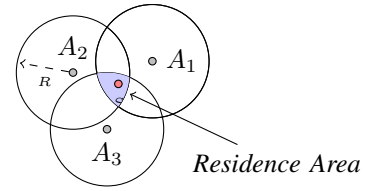


Fig. 1. The residence area in circle-based localization algorithm, where A_1, A_2 and A_3 are neighbor anchors of node S .

all circles, ranges, of anchors that node hears from, defines the node's residence area (see Fig. 1). The node then estimates its position as a centroid of its residence area.

The first requirement of circle-based localization, is that each node must have a sufficient number of neighboring anchors to determine its residence area. Ensuring that the deployment area is covered by at least one anchor node will enable each node in the network to determine its position with the range of at least one anchor. Moreover, as studied in the previous section, the best localization accuracy can be only achieved when the deployment area is equally subdivided. To ensure such subdivision, a deterministic deployment of anchor nodes is then required.

C. Deterministic Anchors Deployment

As pointed out in the previous section, to ensure that every node can estimate its position using the circle-based localization approach, it has to be within the range of at least one anchor. With the interest in minimizing the cost in terms of required anchor count, the anchor placement problem can thus be mapped to the problem of achieving full area coverage using the fewest discs of radius R , often referred to in the realm of wireless sensor networks as 1-coverage problem [3]. It is well-known that the optimal deterministic placement pattern that ensures 1-coverage is by placing nodes, anchors in our case, at the vertices of a triangular lattice [3]. The side of the triangular lattice is equal to $\sqrt{3}R$, where R is the communication range and represent also the coverage range, as we look to ensure that each node must fall within the communication range of a set of anchors. In area-based localization, it is not necessary that anchor nodes be in the neighboring of each other, then the vertices of the triangular lattice can be at distance larger than R , without affecting the algorithm performance. Therefore, the optimal number of anchors to ensure that all nodes will have at least one neighbor anchor, is by placing the anchors at the vertices of a triangular lattice of side equal to $\sqrt{3}R$. If we consider a rectangular deployment area of size $L \times W$, and if the triangular lattice is oriented parallel to the long side L (as depicted in Fig. 2), then the least number of anchors m that cover the whole deployment area can be calculated as follow:

$$m = \left\lceil \frac{2L}{\sqrt{3}R} \times \frac{W}{3R} \right\rceil = \left\lceil \frac{2LW}{3\sqrt{3}R^2} \right\rceil \quad (1)$$

Moreover, when ignoring the intersections, the triangular lattice pattern divides the deployment area into equal sub-regions of circular geometric form with radius R ; such a pattern also yields the best accuracy for circular-based localization with single anchor cover (see Fig. 2).

Similarly when more anchors can be deployed to boost the nodes' positions accuracy, the anchors can be placed at the

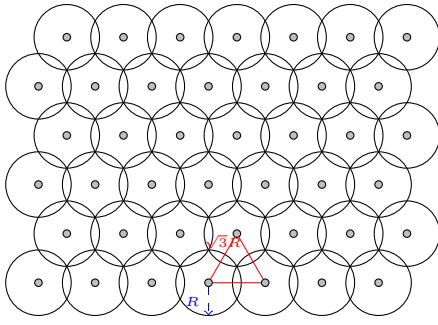


Fig. 2. Triangular lattice pattern for optimal 1-coverage

vertices of a second triangular lattice. It has been proven that by using k triangular lattice layers, k -coverage is achieved with optimal number of anchors [3], which means that each node will have k neighboring anchors. However, for localization this is not sufficient to achieve the best accuracy. The question is; *how to place the different triangular lattice layers in order to achieve the best accuracy in circle-based localization algorithm?* In the next section we study the pattern of a second triangular lattice for 2-coverage and better accuracy. For the rest of the paper we refer to the first Triangular Lattice Anchor layer by 1-TLA, and the second layer by 2-TLA.

III. ACCURACY UNDER TWO-ANCHOR COVERAGE

In this section we study the localization accuracy when each node can be reached by two anchors and determine the optimal anchor placement strategy. As pointed out in the previous section, the second layer of anchors again will form a triangular lattice (2-TLA). When adding a 2-TLA each region x , obtained from the 1-TLA, will be divided into multiple sub-regions. As shown in Fig. 3, the 2-TLA, with dashed blue circles, divides each circle of the 1-TLA into three equal sub-regions (we ignore the small intersections of the circles of the same layer). However, by shifting the 2-TLA to any direction, each circle in the first layer, will be divided into four sub-regions instead of three (see Fig. 4(b)). The interesting question is how to place the 2-TLA so that the localization accuracy is maximized. This section opts to address such a question.

Theorem 2. *If the deployment area is divided into m equal sub-regions (by the first anchors, layer), and if each of the sub-regions will be also divided into l sub-regions (by a second anchors, layer). Then the expected residence area will be further reduced.*

Proof: reader is referred to [1]. ■

Therefore, we can conclude that, if each of the sub-region of one layer is subdivided, the expected residence area will be further reduced. Moreover, as proved in Section II-A, the best positioning of the second layer, for a given number of sub-regions, is that which ensure that each circle in the first layer will be *equally* divided, i.e., each of the m regions is divided into l equal sub-regions. When the second layer divides each circle into three equal parts, this ensures that the whole deployment area is equally partitioned and thus, and according to Theorem 1, the best accuracy in this case is achieved. In the other hand, when the second layer divides each circle of the first layer into four sub-regions, the residence area can be

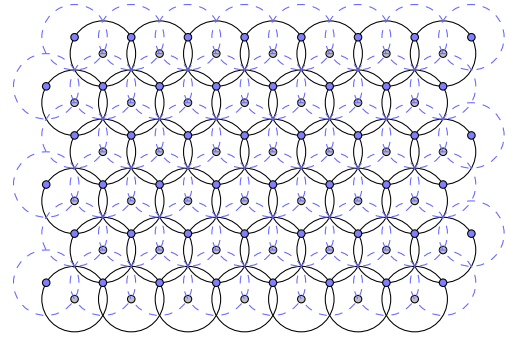


Fig. 3. Triangular lattice pattern for optimal 2-coverage, first layer with solid black circles and a second layer with dashed blue circles

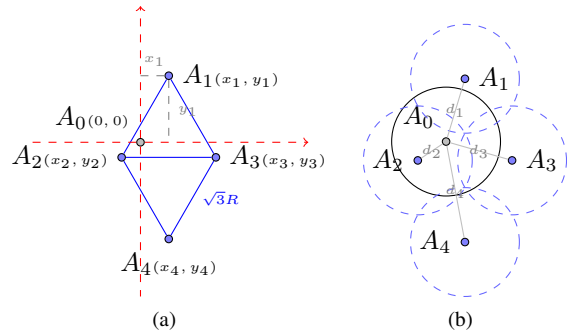


Fig. 4. (a) The coordinates of the vertices in a triangular lattice. (b) Illustrating the distance effect on the size of the intersection areas

further reduced according to Theorem 2 above. However, no positioning of the second layer will achieve an equal four subdivisions. Therefore, no optimality can be guaranteed when we divide the first layer into four non equal sub-regions. In other words, it is not clear which pattern yields better accuracy, three equal subdivisions or four non-equal subdivisions. In the balance of this section we derive, the optimal positioning of the second layer to achieve the best accuracy, and we compare the accuracy between three and four subdivisions.

A. Four-Subdivisions Analysis

Let us consider four anchors from the 2-TLA, namely A_1, A_2, A_3 and A_4 at the coordinate $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) , respectively. Assume that anchor A_0 from the first layer is at position $(0, 0)$, as shown in Fig. 4(a). According to the position of the second layer with respect to the position of A_0 , the covered region by anchor A_0 will be subdivided into different four subregions. We can notice, as shown in Fig. 4(b), that the intersection of the two regions covered by any two anchors is strongly related to the distance between the two anchors; *the more the distance is, the smaller the intersection area becomes*. Based on theorem 1, if we map the distances d_1, d_2, d_3 , and d_4 (see Fig 4(b)) to the resulting intersection areas $y_i, i = 1, 2, 3, 4$, then minimizing the expected residence area becomes equivalent to minimizing the sum of the distances square. In Section IV we validate this analysis through simulation, where we show the correlation between the minimum sum of the distance square and the corresponding minimum average residence area. The problem

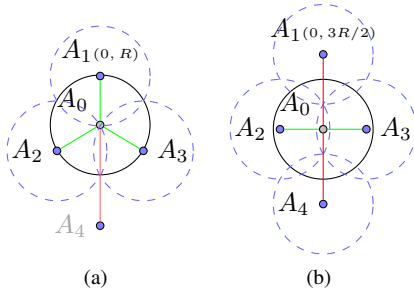


Fig. 5. The optimal anchor placement pattern for (a) 3-subdivision and (b) 4-subdivision

can be thus formulated as follow:

$$\begin{aligned} \text{Min} \quad & \sum_{i=1}^4 d_i^2 \\ \text{s.t.} \quad & d_i > 0, \quad i = 1, 2, 3, 4 \\ & d_i < 2R, \quad i = 1, 2, 3, 4 \end{aligned}$$

d_i here, represents the Euclidean distance between anchor A_0 and A_i , where $i = 1, 2, 3, 4$, and R is the radius of the coverage circle. Because the four anchors of 2-TLA are at the vertices of a triangular lattice, each of the anchor's coordinates can be represented as a function of the coordinates of another anchor. Let us represent the four coordinates with respect to the coordinate of anchor A_1 . According to Fig. 4(a),

$$\begin{aligned} x_2 &= x_1 - \frac{\sqrt{3}}{2}R, & y_2 &= y_1 - \frac{3}{2}R \\ x_3 &= x_1 + \frac{\sqrt{3}}{2}R, & y_3 &= y_1 - \frac{3}{2}R \\ x_4 &= x_1, & y_4 &= y_1 - 3R \end{aligned}$$

By substituting the previous equations into the objective function, we obtain:

$$\begin{aligned} \sum_{i=1}^4 d_i^2 &= (x - 5\sqrt{3})^2 + (y - 15)^2 + x^2 + (y - 30)^2 + \\ &\quad (x + 5\sqrt{3})^2 + (y - 15)^2 + x^2 + y^2 \end{aligned} \quad (2)$$

Using the Nelder-Mead numerical method [15], the solution to this optimization problem is $x_1 = 0$ and $y_1 = 3R/2$. According to the optimal position of anchor A_1 , the position of the second layer will be as shown in Fig. 5(b). The covered area of anchor A_0 is subdivided into two sets of two subregions of the same size. In the next section we compare the expected residence area when the first layer is subdivided by the 3-sub-regions optimal pattern (Fig. 5(a)) and by 4-sub-regions optimal pattern (Fig. 5(b)).

B. Optimal Anchor Placement

In order to obtain the expected residence area for each pattern, the size of the different sub-regions must be calculated. For the ease of calculation and presentation, let us approximate the circular coverage area of an anchor with a hexagon. Note that each hexagon is regular and has a side equals to R . In what follows, we will consider that the size of the deployment area

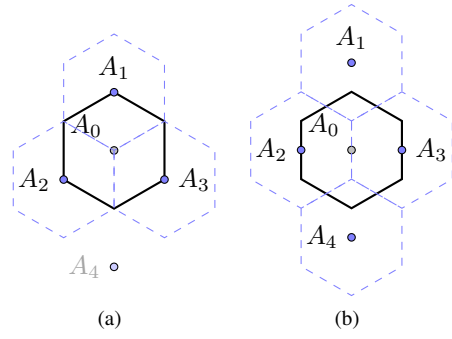


Fig. 6. (a) Hexagon centred at A_0 subdivided into 3 equal regions. (b) Hexagon centred at A_0 subdivided into 4 regions.

T approximately equals to the coverage area of the hexagons of the first layer. In other words, we assume that $T \approx n \frac{3\sqrt{3}}{2} R^2$, where n is the number of anchors in the 1-TLA.

Fig. 6(a) shows the hexagon approximation of the 3-sub-regions optimal pattern. In this case, the area of each first layer anchor A_i is subdivided into three sub-regions $x_{i,j}$. The size of each sub-region equals to one third of the area of a hexagon and is given by:

$$x_{i,j} = \frac{\sqrt{3}}{2} R^2$$

Therefore, the expected residence area for the 3-sub-regions pattern, where $T = n \frac{3\sqrt{3}}{2} R^2$, is given by:

$$\begin{aligned} E[X] &= \frac{1}{T} \sum_{i=1}^n \sum_{j=1}^3 x_{i,j}^2 = \frac{3n}{T} \left(\frac{\sqrt{3}}{2} R^2 \right)^2 \\ &= \frac{\sqrt{3}}{2} R^2 = 0.86 R^2 \end{aligned} \quad (3)$$

For the case of 4-sub-regions, each hexagon at anchor A_i will be divided into four sub-regions, namely, $x_{i,1}, x_{i,2}, x_{i,3}$ and $x_{i,4}$, where $x_{i,1} = x_{i,2}$ and $x_{i,3} = x_{i,4}$. According to Fig. 6(b), the size of each sub-region is given by:

$$x_{i,1} = x_{i,2} = \frac{5\sqrt{3}}{8} R^2 \quad \text{and} \quad x_{i,3} = x_{i,4} = \frac{\sqrt{3}}{8} R^2$$

Thus the expected residence area in this case is given by;

$$\begin{aligned} E[X] &= \frac{1}{T} \sum_{i=1}^n \sum_{j=1}^4 x_{i,j}^2 \\ &= \frac{n}{T} \left(2 \left(\frac{5\sqrt{3}}{8} R^2 \right)^2 + 2 \left(\frac{\sqrt{3}}{8} R^2 \right)^2 \right) \\ &= \frac{13}{8\sqrt{3}} R^2 = 0.93 R^2 \end{aligned} \quad (4)$$

From equations (3) and (4), we can conclude that the expected residence area, when each hexagon of the first layer is divided by three, as shown in Fig. 6(a), is smaller than that of dividing it by four, as shown in Fig. 6(b). Therefore, the pattern that divides the first layer into 3 equal sub-regions represents the optimal deployment pattern. To validate and confirm our

result, we conduct an extensive simulation in the next section.

IV. EVALUATION

In this section, the performance of the obtained theoretical optimal pattern is compared through simulation to a wide range of alternatives patterns. The different patterns are evaluated in terms of the following metrics:

- 1) **Mean residence area (μ):** defined as the average of the residence area of the individual nodes, obtained by executing the circle-based localization algorithm.
- 2) **Standard deviation (σ):** indicates how far the residence areas can be from the average value.

In the simulation environment, which is developed in Python, n nodes are deployed using a uniform random distribution within an area of size $100 \times 100m^2$. We first deploy a 1-TLA, where about 42 anchors are required to cover the entire deployment area, and we also deploy a 2-TLA. We study the effect of the position of the second layer on the size of obtained residence area, as a mean to assess the localization accuracy. In the simulation results, each calculated metric represents the average of 100 executions. The number of nodes and the node's transmission range are set to 300 and $10m$, respectively. We show 3 sample patterns for placing the 2-TLA, and the corresponding mean residence area and standard deviation, as well as the optimal pattern for comparison.

From Fig. 7, we can confirm that the best pattern for the 2-TLA is the one that subdivides the first layer into 3 equal sub-regions (by ignoring the small intersection between circles) as depicted in Fig. 7(a). The value of the mean residence area and the standard deviation are the least compared to the other patterns, which confirms our analysis in Section III-B. We can notice also that the standard deviation is proportional to the expected residence area; the smaller the standard deviation is, the better the accuracy gets. This confirms our theoretical results about the way that the deployment area should be divided equally.

To confirm our analysis in Section III-A, about the correlation between the sum of distances square and the minimum expected residence area, we conduct the following experiment. Let A_0 be an anchor from the first layer with coordinate $(0, 0)$, and let A_1, A_2, A_3 and A_4 be four anchors from the 2-TLA where their coverage ranges intersect with that of anchor A_0 (see Fig. 4(b)). Note that because of the triangular lattice pattern, each circle from the first layer can intersect with at most four anchor coverage ranges of a 2-TLA. According to the position of the 2-TLA, the four anchors can be at different distances from A_0 , and yet stay within 0 and $2R$. We note that if we take two adjacent anchors from the second layer, for example anchors A_1 and A_4 , then if the proximity of one of them to A_0 increases, the other becomes further from A_0 . In order to show the correlation between the sum of the four distances squares and the expected residence area, we plot the mean residence area as a function of the sum of the four distances to A_0 as well as the corresponding standard deviation. We plot the 95% confidence interval on the graphs. According to the analysis made in Section III-A, the minimum

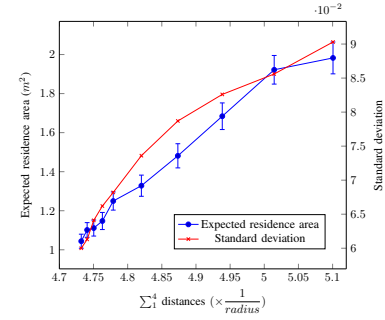


Fig. 8. The Average residence area and the standard deviation, when varying the position of the 2-TLA

sum of the distance squares is achieved when two adjacent of the four anchors are at distance $1.5R$ from A_0 and the two other are at distance $0.866R$ from A_0 . Thus the minimum sum of the four distances is $4.73R$, which is depicted in Fig. 8. From that figure, we can confirm that the minimum sum of the distances corresponds to the minimum expected residence area and also to the minimum standard deviation.

V. RELATED WORKS

Optimal node placement has been investigated in the context of different applications and under various objectives and constraints [22]. It is generally a very challenging problem that has been proven to be NP-Hard for most of the formulations of sensor deployment [5], [8]. To tackle such complexity, several heuristics have been proposed to find sub-optimal solutions [8], [18]. However, to the best of our knowledge, there are no works dealing with the problem of optimal anchors placement for area-based localization. Most of the existing studies about anchor placement assume a multilateration localization algorithms [2], [6], [7]. For example, based on an analytical proof the authors of [2] propose to place anchors uniformly around the perimeter of a rectangular area for better accuracy. In [7], anchors are positioned at the corners of the deployment area with a constraint that all nodes must be within the convex hull of anchors. On the other hand, some recent work in [6], studies the minimum number of anchors for optimal localization under power allocation constraint.

For area-based localization, the RSSI values are not used in the process; instead the near-far effect is exploited to define an area where a node resides. In general, the approaches can be categorized based on the geometric shape that is used to outline the boundary of the residence area of a node, as follows:

Circle based localization: In this class of algorithms [12], [21], a node calculates its residence area based on the assumption that the radio coverage is modelled as a disc with a known radius R . So, a node can deduce whether it is within the areas covered by the radio range of its neighboring anchors. The intersection of all circles centred at the neighboring anchors, determines the final node's residence area

Triangle based localization: He et al., [9] have proposed to redraw the node residence area as a set of triangles made up of vertices formed by all possible subsets of three neighbouring anchors. A belonging test is then applied to check whether a node is inside each of the formed triangles or not. The

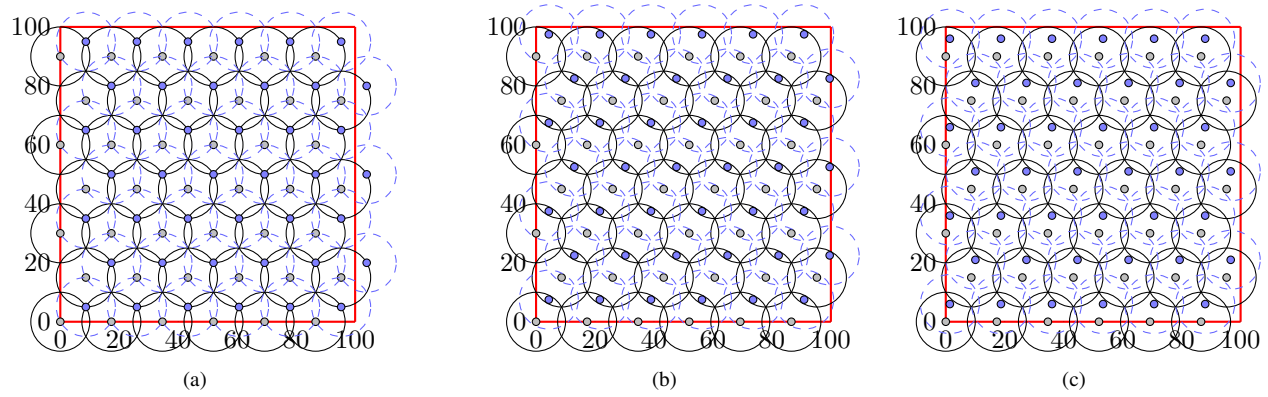


Fig. 7. Different pattern for the second layer (a) optimal pattern [$\mu = 0.8$, $\sigma = 3.0$], (b) [$\mu = 0.92$, $\sigma = 3.17$] and (c) [$\mu = 0.99$, $\sigma = 5.02$]

node residence area is greatly reduced and redefined as the intersection of the triangles which the node resides in.

Half-symmetric lens based localization: In [13], Lasla et al. have developed HSL, a localization algorithm that yields a residence area shaped by the intersection of multiple half-symmetric lens. Based on the RSSI information, each node draws a set of symmetric lens area using the position of its neighboring anchors and checks its presence within each of them. The residence area is refined by intersecting all the half-symmetric lens areas where the node is located.

In this paper we focused on the circle-based methods given its popularity. Our future plan includes extending our analysis to the other approaches.

VI. CONCLUSIONS

In this paper, we have conducted an analytical study to determine the optimal anchor placement that ensures both the best accuracy and least anchor count when area based localization is employed. The analysis is focused on circle-based schemes. We have proven that the best localization accuracy for a given number of anchors can be achieved if the intersection of the basic localization pattern, e.g., circle, yields symmetric shapes with equal sizes. We have also shown the optimal anchor placement pattern that ensure the least number of anchors. We have then combined these two analytical results to derive the optimal pattern that achieve the highest localization accuracy with least number of anchors. The simulation experiments have confirmed our analytical results. Our future plan includes extended the analysis to factor k-anchor coverage and to consider other geometric shapes.

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